

Q No 1

Porter

Step # 01.

$$\text{Minimize : } \frac{(w_1^2 + w_2^2)}{2}$$

Constraints

$$y_i (w_1^T x + b) \geq 1$$

$$+1 (w_1(1) + w_2(1) + b) \geq 1$$

$$-1 (w_1(2) + w_2(2) + b) \geq 1$$

Step # 02.

Langrangian

$$\mathcal{L}(w, b, \alpha) = \frac{1}{2} (w_1^2 + w_2^2) - \alpha_1 (w_1 + w_2 + b) - \alpha_2 (-1)(2w_1 + 2w_2 + b)$$

Step # 03.

$$\frac{\partial \mathcal{L}}{\partial w_1} = w_1 - \alpha_1 + 2\alpha_2 = 0 \quad \text{--- (1)}$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = w_2 - \alpha_1 + 2\alpha_2 = 0 \quad \text{--- (2)}$$

$$\frac{\partial \mathcal{L}}{\partial b} = -\alpha_1 + \alpha_2 = 0 \quad \text{--- (3)}$$

From (3)

$$\alpha_1 = \alpha_2$$

Put in (1) & (2)

$$w_1 - \alpha_1 + 2\alpha_1 = 0 \Rightarrow w_1 = -\alpha_1$$

$$w_2 - \alpha_1 + 2\alpha_1 = 0 \Rightarrow w_2 = -\alpha_1$$

Step #04.

$$L\text{-dual} = \alpha_1^2 + \alpha_2^2 - \alpha_1(-\alpha_1 - \alpha_1 - 1) + \alpha_1(-2\alpha_1 - 2\alpha_1 + 1)$$

$$= \alpha_1^2 - \alpha_1(-2\alpha_1 - 1) + \alpha_1(-4\alpha_1 + 1)$$

$$= \alpha_1^2 + 2\alpha_1^2 + \alpha_1 - 4\alpha_1^2 + \alpha_1$$

$$= -\alpha_1^2 + 2\alpha_1$$

Step #05.

$$\frac{2(L\text{-dual})}{2\alpha_1} = -2\alpha_1 + 2 \geq 0$$

$$\alpha_1 \leq 1$$

$$\alpha_1 \geq \alpha_2 \geq 1$$

$$\boxed{\alpha_2 \geq 1}$$

Step # 06

$$w_1 = -\alpha_1 = -1$$

$$w_2 = -\alpha_1 = -1$$

b using 1st data point

$$1(-1) + (1) + (-1)(1) + b = 1$$

$$-2 + b = 1$$

$$b = 3$$

Step # 07.

$$w_1 x_1 + w_2 x_2 + b = 0$$

$$-1(x_1) + (-1)x_2 + 3 = 0$$

$$-x_1 - x_2 + 3 = 0$$

Step # 08.

Data # 1

$$1(-1 - 1 + 3) \geq 1$$

$$1 \geq 1$$

Satisfied

#12

$$-1(-2 - 2 + 3) \geq 1$$

$$1 \geq 1$$

Satisfied

Q NO 1

b)

Sigmoid.

$$X = \{-2, -1, 0, 1, 2\}$$

$$\sigma(x) = \frac{1}{1+e^x}$$

$$\sigma(-2) = \frac{1}{1+e^2} = 0.119$$

$$\sigma(-1) = \frac{1}{1+e^1} = 0.269$$

$$\sigma(0) = \frac{1}{1+e^0} = 0.500$$

$$\sigma(1) = \frac{1}{1+e^{-1}} = 0.731$$

$$\sigma(2) = \frac{1}{1+e^{-2}} = 0.881$$

Soft max,

$$S(x_i) = \frac{e^{x_i}}{\sum e^x}$$

$$e^1 = 2.718, e^2 = 7.389, e^3 = 20.085$$

$$\text{Sum} = 2.718 + 7.389 + 20.085$$

$$= 30.192$$

$$S(1) = \frac{2.718}{30.192} = 0.090$$

$$S(2) = \frac{7.389}{30.192} = 0.245$$

$$S(3) = \frac{20.085}{30.192} = 0.665$$

Relu.

$$\text{ReLU}(x) = \max(0, x)$$

$$X = \{-3, -1, 0, 2, 4\}$$

$$\max(0, -3) = 0$$

$$\max(0, -1) = 0$$

$$\max(0, 0) = 0$$

$$\max(0, 2) = 2$$

$$\max(0, 4) = 4$$

$$= \{0, 0, 0, 2, 4\}$$