Probability for Engineers

2021 IEOR 3658 **Midterm 1**

You may use a calculator, as well as class notes and the textbook. No collaboration, and no online resources or allowed. You may use without justification any result stated in class. Otherwise, as usual, show your work! An answer without justification will not receive full credit.

Q. 1 (18 points)**.** In the U.S., about half of the population will get married in their lifetime. About half of first marriages end in divorce. About 80% of divorced people will remarry.

(1) What fraction of people get married at least twice?

(2) What fraction of people get married at most once?

(3) Among those people who get married exactly once in their lifetime, what fraction end up divorced?

Q. 2 (18 points)**.** About 1 in every 10 shishito peppers is spicy. Robert does not like spicy food. He bought three peppers from the store, not knowing which (if any) are spicy. Before he starts cooking, Robert takes a tiny bite of each pepper to see if it is spicy. He discards all of the spicy ones, unless that would leave him empty-handed; if they are all spicy, he reluctantly keeps one (as his recipe would be too bland without any peppers). Let X be the number of peppers remaining.

(1) Find the probability mass function (PMF) of the random variable X.

(2) Find the expected value of X.

(3) Find the variance of X.

Q. 3 (20 points)**.** Let A, B and C be events, with $\mathbb{P}(C) > 0$. The concept of *conditional independence* is defined as follows. We say that "A and B are conditionally independent given C" if $\mathbb{P}(A \cap B \mid C) = \mathbb{P}(A \mid C)\,\mathbb{P}(B \mid C)$.

(1) Suppose A and B are independent. Are A^c and B necessarily independent?

(2) Suppose that A, B, and C are **pairwise** independent, and that A and B are conditionally independent given C. Are (A, B, C) necessarily jointly independent?

(3) Suppose that A, B, and C are **jointly** independent. Are A and B necessarily conditionally independent given C?

(4) Suppose A and C are disjoint. Are A and B necessarily conditionally independent given C?

For each part, either explain why the statement must be true, or give a counterexample to illustrate that it is not necessarily true.

Q. 4 (18 points)**.** Suppose you roll five (standard, six-sided) dice.

(1) Find the probability that all five numbers are distinct.

(2) Find the probability that there are more even numbers than odd numbers.

(3) Find the probability that the sum of the five numbers equals 7.

Q. 5 (16 points)**.** Suppose you post a video on your YouTube channel. Each person who views your video (independently) **likes** it with probability $1/2$, or **dislikes** it with probability $1/10$. Everyone else, who neither likes nor dislikes the video, is said to **ignore** it. On average, about three people **like** your video per hour.

(1) For a single person who views your video, what is the conditional probability that they **like** the video given that they do not **ignore** it?

(2) What is a good choice of distribution to model the numbers of **likes** your video gets within the first hour?

(3) What is a good choice of distribution to model the numbers of **likes** within the first 100 people who view your video?

(4) What is a good choice of distribution to model the numbers of **likes** your video gets before the first **dislike**?

Each answer should specify the name and parameter(s) of a distribution.

Q. 6 (10 points)**.** Arturo gets a COVID test. It says negative. As an extra precaution, he gets a second COVID test. It says positive. **Find the probability that Arturo actually has COVID.**

You may assume the following:

- About $1/10$ of people have COVID.
- The results of Arturo's first and second tests are conditionally independent given that he has COVID. That is, the event that the first test is x is conditionally independent of the event that the second is y given that Arturo has COVID, for any $x, y \in \{\text{positive, negative}\}$. See Q3 for the definition of conditional independence.
- The results of Arturo's first and second tests are also conditionally independent given that he **does not** have COVID.
- The false positive rate is $1/1000$, for both of his tests.
- The false negative rate is $2/1000$, for both of his tests.

For partial credit, identify the three relevant events and the given probabilities.

Note: These numbers are fictional, and your answer should be based on these fictional numbers, not on actual COVID statistics that you might find elsewhere.