

# **Advanced Statistics**

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# Textbooks

- ❑ **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer
- ❑ **Elementary Statistics: Picturing the World**, 6<sup>th</sup> Edition, Ron Larson and Betsy Farber
- ❑ **Elementary Statistics**, 13<sup>th</sup> Edition, Mario F. Triola

# Reference books

- ❑ **Probability and Statistical Inference, Ninth Edition,** Robert V. Hogg, Elliot A. Tanis, Dale L. Zimmerman
- ❑ **Probability Demystified,** Allan G. Bluman
- ❑ **Practical Statistics for Data Scientists: 50 Essential Concepts,** Peter Bruce and Andrew Bruce
- ❑ **Schaum's Outline of Probability,** Second Edition, Seymour Lipschutz, Marc Lipson
- ❑ **Python for Probability, Statistics, and Machine Learning,** José Unpingco

# References

Readings for these lecture notes:

- ❑ Probability & Statistics for Engineers & Scientists, Ninth edition, Ronald E. Walpole, Raymond H. Myer
- ❑ Elementary Statistics, Tenth Edition, Mario F. Triola

These notes contain material from the above resources.

# Distribution of marks

**Midterm Examination:** 35 points

**Final Examination:** 40 points

**Sessional Marks:** 25 points, distributed as follows:

- **Assignments:** = 4 assignments × 0.5 points each = 2 points
- **Quizzes:** 8 quizzes × 1.5 points each = 12 points
- **Project Presentation:** 2.5 points
- **Paper Submission (IEEE format):** 7.5 points

**Note:** Failure to submit the paper will result in a **score of zero for the project presentation.**

**Group size:** 1 to 4 students

# Group Rules

**Group Size and Structure:** Groups must consist of 1 to 4 students, with a preference for smaller groups.

**Contribution and Submission:** Each group must ensure equitable contributions from all members. Only one submission per group is required for both the project and the paper.

**Presentation Requirements:** During the project presentation, each group member must **clearly articulate their individual contributions**.

**Conflict Resolution:** Any disputes or concerns regarding uneven contributions should be reported to the instructor before the project submission for appropriate mediation

## Target Journals

Some of the journals that are relevant to health care and the medical field, based on computer science.

1. **Medical Decision Making**, JCR Impact Factor (2017-18) = 2.793
2. **Health Informatics Journal**, JCR Impact Factor (2017-18) = 2.297
3. **Informatics for Health and Social Care**, JCR Impact Factor (2017-18) = 1.137
4. **Health Care Analysis**, , JCR Impact Factor (2017-18) = 1.043
5. **International Journal of Health Care Quality Assurance**, JCR Impact Factor (2017-18) = 1.218

## Target Journals

Some of the journals that are relevant to education, based on computer science.

1. **Computers & Education**, JCR Impact Factor (2017-18) = 5.627
2. **Computer Applications in Engineering Education**, JCR Impact Factor (2017-18) = 1.435
3. **Journal of Computing in Higher Education**, JCR Impact Factor (2017-18) = 1.870
4. **Acm Transactions on Computing Education**, , JCR Impact Factor (2017-18) = 1.356
5. **Assessment & Evaluation In Higher Education**, JCR Impact Factor (2017-18) = 2.473
6. **Educational Assessment Evaluation and Accountability**, JCR Impact Factor (2017-18) = 1.772
7. **Computer Applications in Engineering**



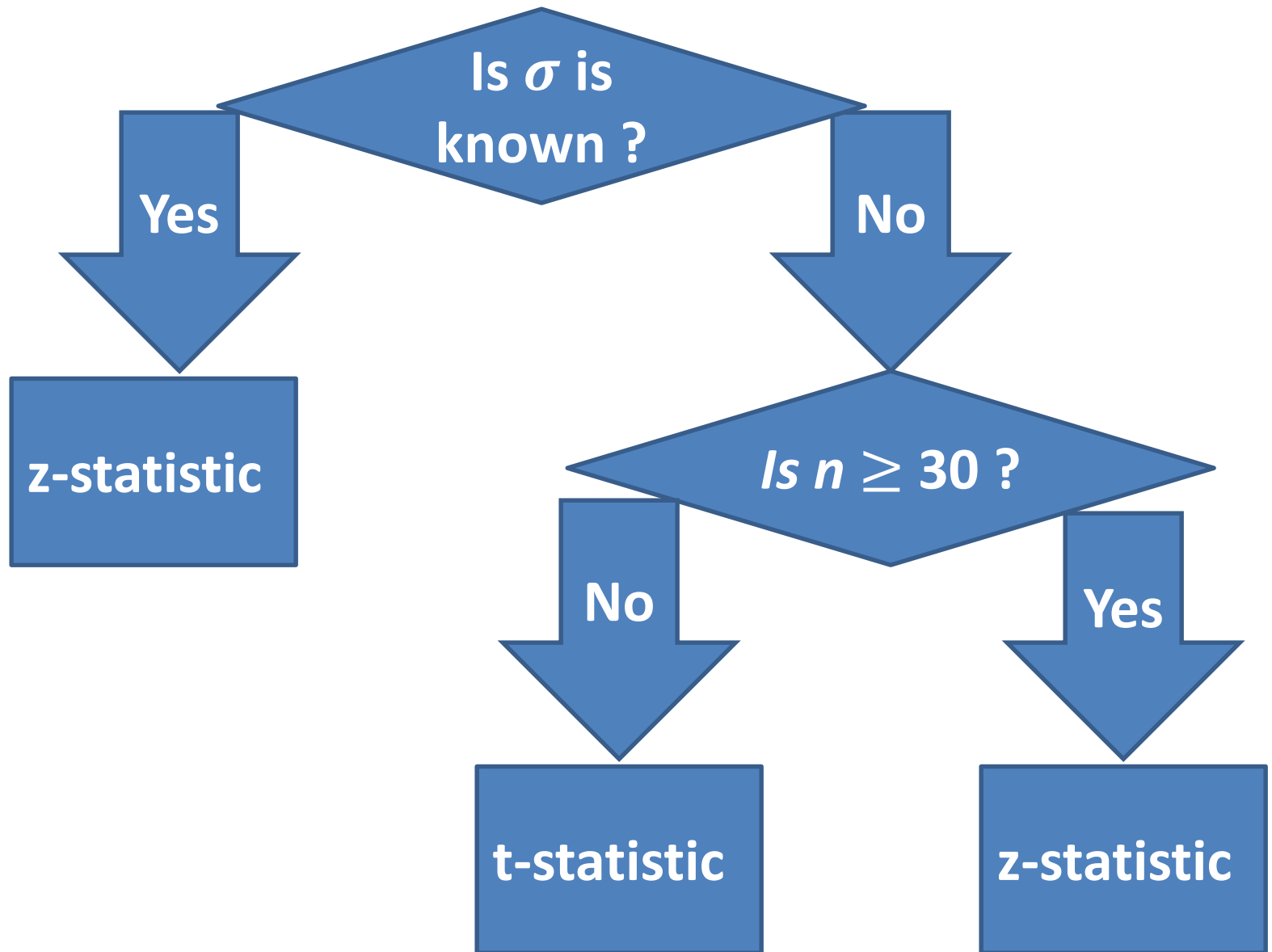
Is  $\sigma$  is known ?

Yes

If either the population is normally distributed or  $n \geq 30$ , then use the standard normal distribution or Z-test

No

If either the population is normally distributed or  $n \geq 30$ , then use the  $t$ -distribution or t-test



When both  $n < 30$  and the population is *not normally distributed*, we *cannot* use the standard normal distribution or the *t*-distribution.

The two main activities of inferential statistics are using sample data to

(1) *estimate* a **population parameter**, and

(2) test a **hypothesis or claim** about a **population parameter**

# Hypothesis

- ❑ In statistics, a **hypothesis** is a **claim** or **statement** about a **property of a population**.
- ❑ A **hypothesis test** (or **test of significance**) is a **standard procedure** for **testing a claim** about a property of a population.

# Examples of Hypotheses

- ❑ **Business** A newspaper headline makes the claim that most workers get their jobs through **networking**.
- ❑ **Medicine** Medical researchers claim that the **mean body temperature** of healthy adults is not equal to 98.6°F.
- ❑ **Aircraft Safety** The Federal Aviation Administration claims that the **mean weight** of an airline passenger (with carry on baggage) is greater than the **185 lb** that it was 20 years ago.

**Type I Error** : Rejection of the null hypothesis when it is true is called a **type 1 error**.

**Type II error** : Nonrejection of the null hypothesis when it is false is called a **type II error**.

	$H_0$ is true	$H_0$ is false
Do not reject $H_0$	Correct decision	<b>Type II error</b>
Reject $H_0$	<b>Type I error</b>	Correct decision

Verdict	Truth About Defendant	
	Innocent	Guilty
Not Guilty	Justice	Type II error
Guilty	Type I error	Justice



# Type I Error (False Positive)

## Definition:

A Type I error occurs when we **reject a null hypothesis that is actually true**.

In other words, we detect an effect or difference that doesn't actually exist.

## Example:

In a medical trial, concluding a drug works when it actually does not. This could lead to approving an ineffective drug.

## Real-life analogy:

Declaring someone guilty when they are actually innocent.

# Type II Error (False Negative)

## Definition:

A Type II error occurs when we fail to reject a null hypothesis that is false. In other words, we miss detecting an effect or difference that is present.

## Example:

In a drug trial, concluding the drug has no effect when it actually works. This could prevent a useful drug from being approved.

## Real-life analogy:

Declaring someone innocent when they are actually guilty.

# Summary of Errors

	True State ( $H_0$ is True)	True State ( $H_0$ is False)
Reject $H_0$	Type I Error ( $\alpha$ )	Correct Decision
Fail to Reject $H_0$	Correct Decision	Type II Error ( $\beta$ )

# Medical Testing and Security Systems

## Type I Error (False Positive)

- **Medical Testing:**
- **Scenario:** A diagnostic test for a disease.
- **Example:** A patient is diagnosed with a disease, but they do not have it. The test falsely detects the condition.
- **Security Systems:**
- **Scenario:** Airport security screening for prohibited items.
- **Example:** A security system flags a harmless item as a threat, leading to unnecessary checks and delays.

# Spam Email Filters and Criminal Justice

## Type I Error (False Positive)

- **Spam Email Filters:**
- **Scenario:** Email filters detecting spam.
- **Example:** A legitimate email is marked as spam and sent to the junk folder, potentially missing important messages.
  
- **Criminal Justice:**
- **Scenario:** A legal trial.
- **Example:** A court convicts an innocent person of a crime they did not commit, a false positive.

# COVID-19 Testing and Machine Learning Models

## Type I Error (False Positive)

- **COVID-19 Testing:**
- **Scenario:** Testing for COVID-19 infection.
- **Example:** A test result comes back positive, but the person does not have COVID-19, causing unnecessary quarantine.
  
- **Machine Learning Models:**
- **Scenario:** Classifying images as cats or dogs.
- **Example:** A machine learning model classifies a picture of a dog as a cat, a false positive.

# Fire Alarm and Pregnancy Test

## Type I Error (False Positive)

- **Fire Alarm System:**
- **Scenario:** A fire alarm detects smoke.
- **Example:** The alarm goes off due to burnt toast, even though there is no real fire, a false signal.
  
- **Pregnancy Test:**
- **Scenario:** A home pregnancy test.
- **Example:** A test indicates pregnancy, but the woman is not pregnant, leading to confusion and emotional distress.

# Scientific Research and Face Recognition Technology

- **Scientific Research:**
- **Scenario:** A study testing the effect of a drug.
- **Example:** Researchers conclude that a drug is effective, but in reality, the drug has no real effect.
  
- **Face Recognition Technology:**
- **Scenario:** A facial recognition system for security.
- **Example:** The system identifies an innocent person as a suspect, leading to wrongful detainment.



# What is a Type II Error?

A Type II error, or false negative, occurs when a test fails to detect an effect or relationship that actually exists. This means concluding there is no effect when there is one.

# Medical, Spam, and Product Quality Examples

**Type II error, or false negative**

## **1. Medical Testing:**

- A test fails to detect cancer in a patient who has it.
- Consequences: Delayed treatment.

## **2. Spam Detection:**

- A spam email is classified as legitimate.
- Consequences: Harmful messages.

## **3. Product Quality Testing:**

- A faulty product passes quality control.
- Consequences: Defective products reach customers.

# COVID-19, Jury Trial, and Statistical Testing

Type II error, or false negative

## 4. COVID-19 Testing:

- An infected person tests negative.
- Consequences: Continued spread of virus.

## 5. Jury Trial:

- A guilty defendant is acquitted.
- Consequences: Justice is not served.

## 6. Statistical Testing:

- A test fails to detect a true relationship.
- Consequences: Incorrect research findings.

# Summary

- Type II errors occur in various contexts when a test or system fails to detect an existing issue. Their consequences can range from minor inconveniences to significant societal impacts.

# Approach to Hypothesis Testing with Fixed Probability of Type I Error

1. State the **null** and **alternative** hypotheses.
2. Choose a fixed **significance level  $\alpha$** .
3. **Test statistic** to be used it
4. **Calculations**
5. **Critical region**
6. **Conclusion**

# Area under the Normal Curve [1]



Table A.3 Areas under the Normal Curve

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

# Area under the Normal Curve [2]

Table A.3 (continued) Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

$H_0$	Value of Test Statistic	$H_1$	Critical Region
$\mu = \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}; \sigma \text{ known}$	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$z < -z_\alpha$ $z > z_\alpha$ $z < -z_{\alpha/2} \text{ or } z > z_{\alpha/2}$
$\mu = \mu_0$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}; v = n - 1,$ $\sigma \text{ unknown}$	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$t < -t_\alpha$ $t > t_\alpha$ $t < -t_{\alpha/2} \text{ or } t > t_{\alpha/2}$
$\mu_1 - \mu_2 = d_0$	$z = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}};$ $\sigma_1 \text{ and } \sigma_2 \text{ known}$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$z < -z_\alpha$ $z > z_\alpha$ $z < -z_{\alpha/2} \text{ or } z > z_{\alpha/2}$
$\mu_1 - \mu_2 = d_0$	$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p \sqrt{1/n_1 + 1/n_2}};$ $v = n_1 + n_2 - 2,$ $\sigma_1 = \sigma_2 \text{ but unknown,}$ $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t < -t_\alpha$ $t > t_\alpha$ $t < -t_{\alpha/2} \text{ or } t > t_{\alpha/2}$
$\mu_1 - \mu_2 = d_0$	$t' = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{s_1^2/n_1 + s_2^2/n_2}};$ $v = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}},$ $\sigma_1 \neq \sigma_2 \text{ and unknown}$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t' < -t_\alpha$ $t' > t_\alpha$ $t' < -t_{\alpha/2} \text{ or } t' > t_{\alpha/2}$
$\mu_D = d_0$ paired observations	$t = \frac{\bar{d} - d_0}{s_d/\sqrt{n}};$ $v = n - 1$	$\mu_D < d_0$ $\mu_D > d_0$ $\mu_D \neq d_0$	$t < -t_\alpha$ $t > t_\alpha$ $t < -t_{\alpha/2} \text{ or } t > t_{\alpha/2}$



# A null hypothesis $H_0$ vs the alternative hypothesis $H_1$

A **null hypothesis  $H_0$**  is a statistical hypothesis that contains a statement of equality, such as  $\leq$ ,  $\geq$  **or**  $=$

The **alternative hypothesis  $H_1$**  is the complement of the null hypothesis. It is a statement that must be true if  $H_0$  is false and it contains a statement of **strict inequality**, such as  $>$ ,  $<$ , **or**  $\neq$

# A null hypothesis $H_0$ vs the alternative hypothesis $H_1$

To write the null and alternative hypotheses, **translate the claim** made about the **population parameter** from a **verbal statement** to a **mathematical statement**.

Then, write its complement. For instance, if the claim value is  $k$  and the population parameter is  $\mu$ , then some possible pairs of null and alternative hypotheses are

$$\begin{cases} H_0: \mu \leq k \\ H_1: \mu > k \end{cases}$$

$$\begin{cases} H_0: \mu \geq k \\ H_1: \mu < k \end{cases}$$

$$\begin{cases} H_0: \mu = k \\ H_1: \mu \neq k \end{cases}$$

# A null hypothesis $H_0$ vs the alternative hypothesis $H_1$

Regardless of which of the **three pairs of hypotheses** you use, you always assume  $\mu = k$  and examine the **sampling distribution** on the basis of this assumption.

Within this sampling distribution, you will determine whether or not a **sample statistic** is unusual.

# Null Hypothesis

The **null hypothesis** (denoted by  $H_0$ ) is a statement that the value of a **population parameter** (such as proportion, mean, or standard deviation) is equal to some **claimed value**. Here are some typical null hypotheses of the type:

$$H_0: p = 0.5 \quad H_0: \mu = 98.6 \quad H_0: \sigma = 15$$

□ We test the null hypothesis directly in the sense that we **assume it is true** and reach a conclusion to either reject  $H_0$  or fail to reject  $H_0$ .

# Alternative Hypothesis

□ The **alternative hypothesis** (denoted by  $H_1$  or  $H_a$  or  $H_A$ ) is the statement that the parameter has a value that somehow differs from the null hypothesis. The symbolic form of the alternative hypothesis must use one of these symbols:

**$<$  or  $>$  or  $\neq$  .**

# Alternative Hypothesis

Here are nine different examples of alternative hypotheses involving proportions, means, and standard deviations:

❑ **Means:**  $H_1: \mu > 98.6$   $H_1: \mu < 98.6$   $H_1: \mu \neq 98.6$

❑ **Proportions:**  $H_1: p > 0.5$   $H_1: p < 0.5$   $H_1: p \neq 0.5$

❑ **Standard Deviations:**  $H_1: \sigma > 15$   $H_1: \sigma < 15$   $H_1: \sigma \neq 15$

# Note About Always Using the Equal Symbol in $H_0$ :

- ❑ A few textbooks use the symbols  $\geq$  and  $\leq$  in the null hypothesis  $H_0$ , but most professional journals use only the equal symbol for equality.
- ❑ We conduct the hypothesis test by assuming that the proportion, mean, or standard deviation is *equal to* some specified value so that we can work with a single distribution having a specific value.

# Note About Forming Your Own Claims (Hypotheses):

- ❑ If you are conducting a study and want to use a hypothesis test to *support* your claim, the claim must be worded so that it becomes the alternative hypothesis (and can be expressed using only the symbols  $<$  or  $>$  or  $\neq$  ).
- ❑ You can never support a claim that some parameter is *equal to* some specified value



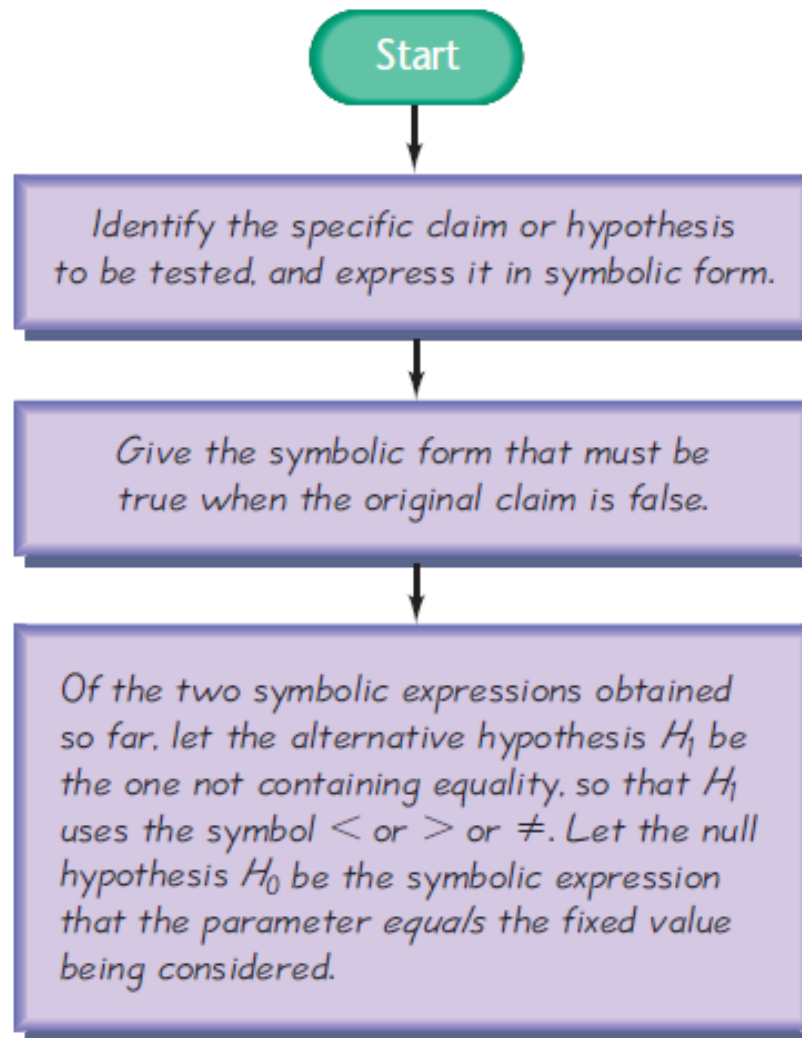
- ❑ For example, if you have developed a gender-selection method that increases the likelihood of a girl, state your claim as  $p > 0.5$  so that your claim can be supported.
- ❑ (In this context of trying to support the goal of the research, the **alternative hypothesis** is sometimes referred to as the **research hypothesis**.)
- ❑ You will assume for the purpose of the test that  $p = 0.5$ , but you hope that  $p = 0.5$  gets rejected so that  $p > 0.5$  is supported.

# The Null and Alternative Hypotheses

The structure of hypothesis testing will be formulated with the use of the term **null hypothesis**, which refers to any hypothesis we wish to test and is denoted by  $H_0$ . The rejection of  $H_0$  leads to the acceptance of an **alternative hypothesis**, denoted by  $H_1$ .

The alternative hypothesis  **$H_1$  usually represents the question to be answered or the theory to be tested**, and thus its specification is crucial. The null hypothesis  $H_0$  *nullifies or opposes*  $H_1$  and is often the logical complement to  $H_1$ .

# Identifying $H_0$ and $H_1$



**Figure 1**

# Identifying $H_0$ and $H_1$

- Note that the **original statement** could become the **null hypothesis**, it could become the **alternative hypothesis**, or it might not correspond exactly to either the **null hypothesis** or the **alternative hypothesis**.

- ❑ For example, we sometimes test the validity of someone else's claim, such as the claim of the **Coca-Cola Bottling Company** that “**the mean amount of Coke in cans is at least 12 oz.**” That claim can be expressed in symbols as  $\mu \geq 12$ .
- ❑ In Figure 1 we see that if that **original claim is false**, then  $\mu < 12$ . The **alternative hypothesis** becomes  $\mu < 12$ , but the **null hypothesis** is  $\mu = 12$ .
- ❑ We will be able to address the **original claim** after determining whether there is sufficient evidence to reject the null hypothesis of  $\mu = 12$ .

**EXAMPLE Identifying the Null and Alternative Hypotheses** Refer to Figure 8-2 and use the given claims to express the corresponding null and alternative hypotheses in symbolic form.

- a. The **proportion** of workers who get jobs through networking is **greater than 0.5**.
- b. The **mean weight** of airline passengers with carry-on baggage is at **most 195 lb** (the current figure used by the Federal Aviation Administration).
- c. The standard deviation of IQ scores of actors is equal to **15**.

**SOLUTION** See Figure 1, which shows the three-step procedure.

- In Step 1 of Figure 1, we express the given **claim as  $p > 0.5$** . In Step 2 we see that if  **$p > 0.5$  is false**, then  **$p \leq 0.5$**  must be true. In Step 3, we see that the expression  **$p > 0.5$**  does not contain equality, so we let the **alternative hypothesis  $H_1$**  be  **$p > 0.5$** , and we let  $H_0$  be  $p = 0.5$ .
  
- In Step 1 of Figure 1, we express **“a mean of at most 195 lb”** in symbols as  **$\mu \leq 195$** . In Step 2 we see that if  **$\mu \leq 195$  is false**, then  **$\mu > 195$**  must be true. In Step 3, we see that the expression  **$\mu > 195$**  does not contain equality, so we let the alternative hypothesis  $H_1$ :  **$\mu > 195$**  be and we let  $H_0$  be  **$\mu = 195$**

c. In Step 1 of Figure 1, we express the given claim as  **$\sigma = 15$** . In Step 2 we see that if  $\sigma = 15$  is false, then  $\sigma \neq 15$  must be true. In Step 3, we let the alternative hypothesis  $H_1$  be  $\sigma \neq 15$ , and we let  **$H_0$  be  $\sigma = 15$** .



# Single Sample: Tests Concerning a Single Mean

**Example:** A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.

## **Solution:**

$n = 100$  (sample size)

$\bar{x} = 71.8$  (sample mean)

$\sigma = 8.9$  (population standard deviation)

$\alpha = 0.05$  (level of significance)

1. **We state our hypothesis as:**

$$H_0: \mu = 70 \text{ years}$$

$$H_1: \mu > 70 \text{ years (one sided test)}$$

2. **The level of significance is set**  $\alpha = 0.05$ .

3. **Test statistic to be used is**

$$Z_{\text{cal}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

4. **Calculations:**

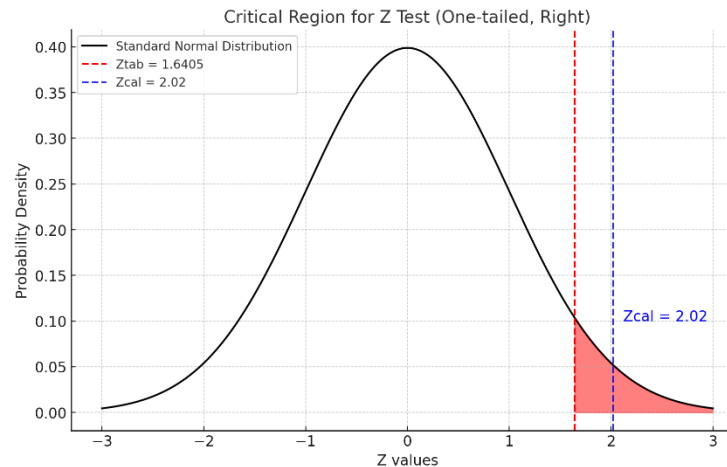
$$Z_{\text{cal}} = 2.02$$

## 5. Critical region:

$$Z_{\text{cal}} = 2.02$$

$$Z_{\text{cal}} > Z_{\text{tab}}$$

$$\text{Where } Z_{\text{tab}} = Z_{\alpha} = Z_{0.05} = 1.6405 \quad \because 1 - \alpha = 1 - 0.05 = 0.95$$



6. **Conclusion:** Since calculated value of Z is greater than the tabulate value of Z, so we are unable to accept  $H_0$

**Example:** A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of **8 kilograms** with a **standard deviation of 0.5** kilogram. Test the hypothesis that  $\mu = 8$  kilograms against the alternative that  $\mu \neq 8$  kilograms if a random sample of **50 lines** is tested and found to have a mean breaking strength of **7.8 kilograms**. Use a 0.01 level of significance.

## **Solution:**

$\mu = 8$  (Population mean)

$n = 50$  (Sample size)

$\sigma = 0.5$  (Population standard deviation)

$\bar{x} = 7.8$  (Sample mean)

$\alpha = 0.01$  (Level of significance)

1. **We state our hypothesis as:**

$$H_0: \mu = 8$$

$$H_1: \mu \neq 8 \quad (\text{Two sided test})$$

2. **The level of significance is set**  $\alpha = 0.01$ .

3. **Test statistic to be used is**

$$Z_{\text{cal}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

4. **Calculations:**

$$Z_{\text{cal}} = \frac{7.8 - 8}{0.5 / \sqrt{50}} = -2.83.$$

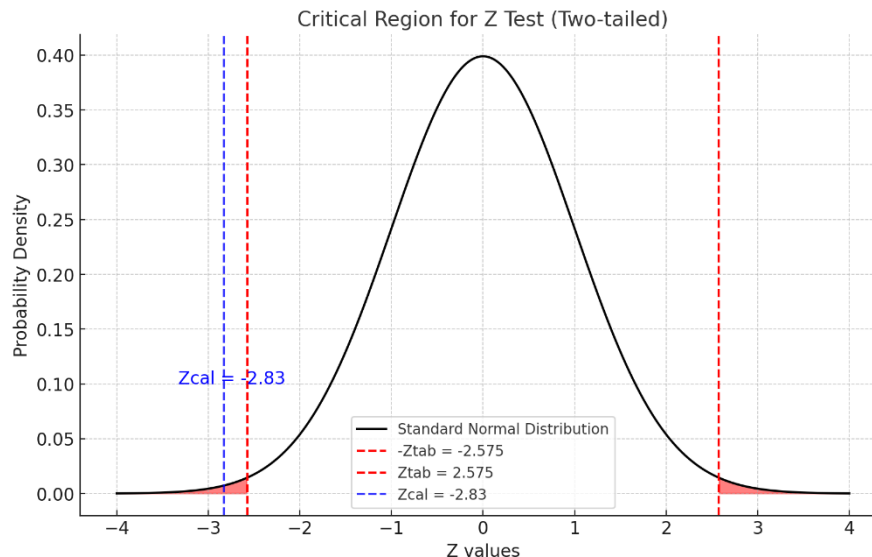
## 5. Critical region:

$$Z_{\text{cal}} = -2.83$$

$$|Z_{\text{cal}}| > Z_{\text{tab}}$$

$$\text{Where } Z_{\text{tab}} = Z_{\alpha/2} = Z_{0.05} = 2.575$$

$$\therefore 1 - \alpha = 1 - 0.01/2 = 0.995$$



6. **Conclusion:** Since calculated value of Z is greater than the tabulate value of Z, so we are unable to accept  $H_0$



**Example 10.5:** The Edison Electric Institute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of **46** kilowatt hours per year. If a random sample of **12** homes included in a planned study indicates that vacuum cleaners use an average of **42** kilowatt hours per year with a standard deviation of **11.9** kilowatt hours, does this suggest at the **0.05** level of significance that vacuum cleaners use, on average, less than **46** kilowatt hours annually? Assume the population of kilowatt hours to be normal.

# Solution

$$\mu = 46$$

(Population mean)

$$n = 12$$

(Sample size)

$$s = 11.9$$

(Sample standard deviation)

$$\bar{x} = 42$$

(Sample mean)

$$\alpha = 0.05$$

(Level of significance)

1. **We state our hypothesis as:**

$$H_0: \mu = 46$$

$$H_1: \mu < 46 \quad (\text{One tailed test})$$

2. **The level of significance is set**  $\alpha = 0.01$ .

3. **Test statistic to be used is**

$$t_{\text{cal}} = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

4. **Calculations:**

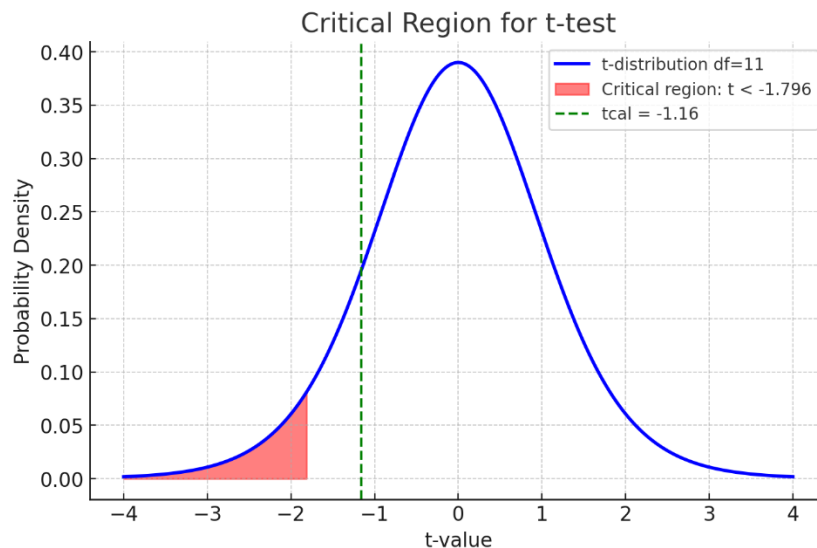
$$t_{\text{cal}} = \frac{42 - 46}{11.9 / \sqrt{12}} = -1.16$$

## 5. Critical region:

$$t_{\text{cal}} = -1.16$$

$$t_{\text{cal}} < t_{\text{tab}}$$

$$\text{Where } -t_{\text{tab}} = -t_{(\alpha, n-1)} = -t_{(0.05, 11)} = -1.796$$



6. **Conclusion:** Since calculated value of  $t_{\text{cal}}$  is greater than the tabulate value of  $t$ , so we accept  $H_0$

# Table A.4 Critical Values of the t-Distribution

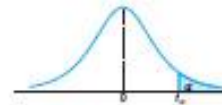


Table A.4 Critical Values of the t-Distribution

v	$\alpha$						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.378	1.963	3.078	6.314	12.708
2	0.289	0.817	1.061	1.386	1.886	2.920	4.303
3	0.277	0.884	0.978	1.250	1.638	2.353	3.182
4	0.271	0.869	0.941	1.190	1.533	2.132	2.776
5	0.267	0.859	0.920	1.156	1.478	2.015	2.571
6	0.265	0.853	0.906	1.134	1.440	1.943	2.447
7	0.263	0.849	0.896	1.119	1.415	1.895	2.365
8	0.262	0.846	0.889	1.108	1.397	1.860	2.306
9	0.261	0.843	0.883	1.100	1.383	1.833	2.262
10	0.260	0.842	0.879	1.093	1.372	1.812	2.228
11	0.260	0.840	0.876	1.088	1.363	1.796	2.201
12	0.259	0.839	0.873	1.083	1.356	1.782	2.179
13	0.259	0.838	0.870	1.079	1.350	1.771	2.160
14	0.258	0.837	0.868	1.076	1.345	1.761	2.145
15	0.258	0.836	0.866	1.074	1.341	1.753	2.131
16	0.258	0.835	0.865	1.071	1.337	1.746	2.120
17	0.257	0.834	0.863	1.069	1.333	1.740	2.110
18	0.257	0.834	0.862	1.067	1.330	1.734	2.101
19	0.257	0.833	0.861	1.066	1.328	1.729	2.093
20	0.257	0.833	0.860	1.064	1.325	1.725	2.086
21	0.257	0.832	0.859	1.063	1.323	1.721	2.080
22	0.256	0.832	0.858	1.061	1.321	1.717	2.074
23	0.256	0.832	0.858	1.060	1.319	1.714	2.069
24	0.256	0.831	0.857	1.059	1.318	1.711	2.064
25	0.256	0.831	0.856	1.058	1.316	1.708	2.060
26	0.256	0.831	0.856	1.058	1.315	1.706	2.056
27	0.256	0.831	0.855	1.057	1.314	1.703	2.052
28	0.256	0.830	0.855	1.056	1.313	1.701	2.048
29	0.256	0.830	0.854	1.055	1.311	1.699	2.045
30	0.256	0.830	0.854	1.055	1.310	1.697	2.042
40	0.255	0.829	0.851	1.050	1.303	1.684	2.021
60	0.254	0.827	0.848	1.045	1.296	1.671	2.000
120	0.254	0.826	0.845	1.041	1.289	1.658	1.980
$\infty$	0.253	0.824	0.842	1.036	1.282	1.645	1.960

# Table A.4 (continued) Critical Values of the t-Distribution

v	$\alpha$						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.385	3.634	4.032	4.773	6.889
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.189	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.328	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.078	2.196	2.358	2.468	2.617	2.860	3.373
$\infty$	2.054	2.170	2.326	2.432	2.576	2.807	3.290