CURVE OF GROWTH

The equivalent width of a spectral line is a combination of the column density N of the ion and the Doppler b parameter. The Doppler b parameter is a measure of the broadening of lines. If the line broadening is purely thermal, then b-parameter is given by

$$b = \sqrt{\frac{2kT}{m}} \tag{1}$$

If the line broadening is due to both thermal motion as well as non-thermal motion of the ions that are responsible for producing the absorption line (or emission line), then

$$b = \sqrt{b_t^2 \ + \ b_{nt}^2} \tag{2}$$

where b_t is the thermal Doppler broadening component and b_{nt} is the non-thermal broadening.

Curve of Growth : The behavior of the equivalent width of a line with respect to changes to N and b is called the curve of growth (CoG, for short). As the optical depth of a line increases (meaning, as the column density increases), the equivalent width also increases. The precise functional dependence is sensitive to the optical depth at the line core τ_0. From here onward, our discussion will be for absorption lines.

With increase in column density, τ_0 increases, the absorption line becomes deeper, the equivalent width of the line increases. This continues until all the photons from the background source are fully absorbed are thus removed from the incoming light beam. At this point, the absorption line is considered *saturated.* As τ_0 (i.e., column density) increases further, very little additional light is removed from the incoming beam. The change in equivalent width with increase in column density will be small. This continues until the damping wings begin to show in the absorption line corresponding to very large values of τ_0. This entire behavior of a spectral line's change in equivalent width with respect to optical depth (i.e., column density) is captured in the curve of growth.

The three regimes of the curve of growth are *linear* regime, *logarithmic* also called *flat* regime, and *square root* regime. The functional forms for each regime is as follows:

1. Linear Regime:

$$W \propto N, \quad \text{where } \tau << 1 \tag{3}$$

$$W = \frac{\pi e^2}{mc^2}\,\lambda^2 N f \tag{4}$$

   where W is the equivalent width, N is the column density, λ and f are the wavelength and oscillator strength of a chosen line, and m and e the mass and charge of electron.

2. Logarithmic Regime:

   This corresponds to when the optical depth values are roughly in the range of $10 \leq \tau_0 \leq 10^3$.

$$W = b\frac{\lambda}{c}\,\left(\ln\Big[\frac{\sqrt{\pi}\ e^2}{m\ c}\ \frac{Nf\lambda}{b}\Big]\right)^{1/2} \tag{5}$$

3. Square Root Regime:

   This regime corresponds to very high levels of optical depth of $\tau_0 > 10^4$, where the absorption in the wings begins to dominate. The line profile deviates from a Gaussian because of the presence of the Lorentzian wings. In this regime, the relationship between equivalent width of a given line and the column density of the ionic species responsible for the absorption line is given by

$$W \sim 2\sqrt{\pi\ \tau_0} \tag{6}$$

   where the optical depth at the line center τ_0 is

$$\tau_0 = \left(\frac{\sqrt{\pi}e^2}{mc}\ \frac{Nf\lambda}{b}\right) \tag{7}$$

The functional dependence of $W \propto \sqrt{N}$ gives this regime the name *square root* or *damping* part of the curve of growth.

Lab Assignment :

Produce the CoG plot for the Lyα line for column densities in the range of $N(\text{H}\,\textsc{i}) = 10^{12} - 10^{21}$ cm^{-2} and b parameters in the range of $15 - 90$ km s^{-1} in steps of 15 km s^{-1}. The CoG plot should have logarithm to the base 10 along the X-axis and equivalent width in units of Angstrom along the Y-axis. The Y-axis can be a log axis to accommodate the wide range of values that you will get. Given that the rest-frame wavelength of Lyα line is $\lambda = 1215.67$ Å and oscillator strength is $f = 0.416$.