Assignment. Due on : 6th Nov 2019

Write your own code for cubic spline by solving the tridiagonal matrix we discussed in class. (see DeVries, First Course in Computational Astrophysics, for arriving at the matrix yourself). Compare your results with that of scipy's spline interpolation function. Compare the speed (rather time taken by the code) and the accuracy of the results. Use the function we worked with in Quiz-1 to do the comparison. Make sure that you use 10 points from the original function to do the interpolation and compare the result with the a different set of points (this is a mistake a lot of people made in the quiz).

$$f(x) = \frac{1 + \tanh 2\alpha x}{2}.$$

The command "time" can give you the time taken to run the code. Result accuracy can be compared by looking at the residuals (difference from the interpolated vs the true value).

(5 marks)

2. You might have learnt already as part of another course that if an object moves relativistically, the arrival time of photons measured by an observer will be affected by doppler effect.
See the unnumbered equation just above equation 4.11 in Rybicki & Lightman. Assuming that the zero of observer's time is measured corresponding to R=0 where the source was at rest, the same equation can be re-written as

$$t_{obs} = \frac{R}{\beta(R)\,c}[1\text{-}\beta(R)\cos(\theta)]$$

where θ is the angle between the velocity vector and the observer's line of sight, and $\beta(R)$ is the velocity profile of the expanding source. The expansion velocity is going down, hence $\beta(R)$. Assume a spherical blob of plasma expanding with $\beta(R) = 0.9\,(R/R_0)^{-0.5}$, plot the locus of the points on its surface from where photons arrive at a given time t_{obs}. Fix a value for R_0 on your own. Vary for different t_{obs}. The best approach is normalizing the equation and making it dimensionless - leave it to you to figure out.

You will require either interpolation or root finding to do this, because ultimately what you will plot is R $\cos(\theta)$ vs R $\sin(\theta)$ for a given t_{obs}. As you vary θ, you will see that photons will correspond to different R values. For example, if you consider a uniformly moving source (i.e., a constant β), you will produce an ellipsoid as the locus. Of course, if you consider a non-relativistically moving source, you will get a sphere. See the figure below as well.

(5 marks)