Notes for Modelling the Prompt Emission

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1 Inputs

- M_{BH}: mass of the Black Hole
- M_{NS}: mass of the Neutron Star
- $\chi_{BH} = c|\mathbf{S}|/(GM_{BH}^2)$: dimensionless spin of the Black Hole. NS spin is assumed to be low, since binary formation takes enough time to allow the NS to spin down.
- ι_{tilt}: angle made by the spin of the BH with respect to the binary's angular momentum vector. Usually taken to be 0 for simplicity.
- $C_{NS} = GM_{NS}/(R_{NS}c^2)$: this is usually prescribed by using the EoS, which is essentially a M-R relation or a density (ρ) - tidal deformability (Λ) relation.

2 Important Formulae

2.1 Baryonic Mass

For the neutron star, we can also define the baryonic mass M_b, which is the (ADM) mass *plus* the binding energy it takes to bring together N baryons. It is hence defined as (from Lattimer & Prakash, 2001):

$$M_b = M_{NS} + B.E. = M_{NS}\left(1 + \frac{0.6C_{NS}}{1 - 0.5C_{NS}}\right) \quad (1)$$

2.2 Effective Tidal Deformability

We can also define the system's *effective* tidal deformability $\tilde{\Lambda}$, which is simply "a mass weighted combination of the dimensionless quadrupolar tidal deformabilities of the binary components" (see Raithel et al., 2018). For a NSBH binary, we have:

$$\tilde{\Lambda} = \frac{16}{13}\frac{(M_{NS} + 12M_{BH})M_{NS}^4\Lambda_{NS}}{(M_{NS} + M_{BH})^5} \quad (2)$$

Furthermore, the term Λ_{NS} can be expressed as (Flanagan & Hinderer, 2008):

$$\Lambda_{NS} = (2/3)k_2 C_{NS}^{-5} \quad (3)$$

where $k_2 = (3/2)G\lambda R_{NS}^{-5}$ is the dimensionless tidal Love number[1].

2.3 C-Love relations

In Yagi & Yunes, 2017, the authors find an approximately universal (i.e. EoS-independent)) relation between the NS compactness and the tidal deformability expressed as:

$$C_{NS} = \sum_{k=0}^{2} a_k (\ln \Lambda_{NS})^k \quad (4)$$

Basically this is a quadratic equation in $\ln \Lambda_{NS}$ with the coefficients $a_0 = 0.360$, $a_1 = -0.0355$, $a_2 = 0.000705$.

3 The Main Formulae

3.1 M_{out}

This is the mass left outside the apparent horizon of the Black Hole post-merger. Also referred to as the remnant mass M^{rem} and prescribed as a multiple of the baryonic mass, in which case it is defined as $\hat{M}^{rem} = M^{rem}/M_{NS}^b$. Mainly this is what the model given in Foucart et al., 2018 talks about. We also need the normalized ISCO radius:

$$\begin{aligned}\hat{R}_{ISCO} &\equiv R_{ISCO}/M_{BH} \\ &= 3 + Z_2 - \mathrm{sgn}(\chi_{BH})\sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}\end{aligned} \quad (5)$$

where $Z_1 = 1 + (1 - \chi_{BH}^2)^{1/3}[(1 + \chi_{BH})^{1/3} + (1 - \chi_{BH})^{1/3}]$ and $Z_2 = \sqrt{3\chi_{BH}^2 + Z_1^2}$. Now, we have:

$$M_{out} = M_{NS}^b\left[\max\left(\alpha\frac{1 - 2\rho}{\eta^{-1/3}} - \beta\hat{R}_{ISCO}\frac{\rho}{\eta} + \gamma, 0\right)\right]^{\delta} \quad (6)$$

Here, $\alpha = 0.308, \beta = 0.124, \gamma = 0.283$ and $\delta = 1.536$, which are the parameters derived from Least-squares fitting this model to a suite of NR simulations. Also,

[1]Here, λ is the quadrupolar polarisability, representing the ratio of the induced quadrupole moment Q_{ij} to the applied tidal field E_{ij} i.e. $Q_{ij} = -\lambda E_{ij}$.

$\rho = (15\Lambda)^{-1/5} \sim C_{NS}$. Note that "...this prediction performs as well as the compaction-based model (Eq. 4 of Foucart et al., 2018) for $M_{rem} \leq 0.2 M_\odot$, but *has larger errors for simulations using single-polytrope equations of state and producing more massive remnants*".

3.2 M_{dyn}

Also referred to as the ejecta mass, M_{ej}. Expressed as a multiple of the total baryonic mass of the NS. From Kawaguchi et al., 2016 this is expressed as:

$$
\begin{aligned}
M_{ej} = M_{NS}^b \cdot \max\{a_1 Q^{n_1}(1 - 2C_{NS})C_{NS}^{-1} \\
- a_2 Q^{n_2} \tilde{R}_{ISCO}(\chi_{eff}) \\
+ a_3 \left(1 - \frac{M_{NS}}{M_{NS}^b}\right) + a_4, 0\}
\end{aligned}
\tag{7}
$$

with the parameters $a_1 = 4.464 \times 10^{-2}, a_2 = 2.269 \times 10^{-3}, a_3 = 2.431, a_4 = -0.4159, n_1 = 0.2497, n_2 = 1.352$ as derived from least-square fitting to a suite of numerical relativity driven NSBH merger simulations. Note that this suite only considered those mergers with $\chi_{eff} \leq 0.9$, where $\chi_{eff} = \chi_{BH} \cos \iota_{tilt}$ although Lovelace et al., 2013 showed that very massive ejecta ($M_{ej} \gtrsim 0.2\ M_\odot$) can be produced in cases where the Black Hole spin is very high ($\chi_{eff} \sim 0.97$).

3.3 M_{disc}

This is just taken to be the following:

$$
M_{disc} = \max\{M_{out} - M_{dyn}, 0\} \tag{8}
$$

3.4 Relativistic Jet Launching

The luminosity extracted via the Blandford-Znajek mechanism from the accretion of the post-merger matter is given by :

$$
L_{BZ} \propto \frac{G^2}{c^3} M_{BH}^2 B^2 \Omega_H^2 f(\Omega_H) \tag{9}
$$

where B is the magnetic field at the BH event horizon, $0 \leq \Omega_H \leq 1/2$ is the dimensionless angular frequency at the horizon[2], given by:

$$
\Omega_H = \frac{\chi_{BH}}{2(1 + \sqrt{(1 - \chi_{BH}^2)}} \tag{10}
$$

and $f(\Omega_H) = 1 + 1.38\Omega_H^2 - 9.2\Omega_H^4$ is a high-spin correction factor. Now, assuming that the Kelvin-Helmholtz and Magneto-rotational instabilities amplify the magnetic field, which reaches an equipartition with the disc energy density, we have approximately that :

$$
B^2 \propto \frac{c^5}{G^2} \dot{M} M_{BH}^{-2} \tag{11}
$$

where $\dot{M}$ is the mass accretion rate, and hence the previous equation becomes :

$$
L_{BZ} \propto \dot{M} c^2 \Omega_H^2 f(\Omega_H) \tag{12}
$$

The kinetic energy of the jet released afterwards is given by $E_{K,jet} = L_{BZ} \times t_{acc}$, where t_{acc} is the disc accretion time, given by $t_{acc} = (1 - \xi_w - \xi_s) M_{disc}/\dot{M}$. We thus have :

$$
E_{K,jet} = \epsilon (1 - \xi_w - \xi_s) M_{disc} c^2 \Omega_H^2 f(\Omega_H) \tag{13}
$$

where ϵ is a fudge factor[3] depending on the large-scale geometry of the magnetic field, disc aspect ratio and the ratio of the magnetic field energy density to disc pressure at saturation.
Furthermore, assuming that the intrinsic structure of the jet is as follows:

$$
\begin{aligned}
\frac{dE(\theta)}{d\Omega} &= E_c e^{-(\theta/\theta_{c,E})^2}; \\
\Gamma(\theta) &= (\Gamma_c - 1) e^{-(\theta/\theta_{c,\Gamma})^2} + 1;
\end{aligned}
\tag{14}
$$

where $\Gamma_c = 100$, $\theta_{c,E} = 0.1$ rad, $\theta_{c,\Gamma} = 0.2$ rad and $E_c = E_{K,jet}/\pi\theta_{c,E}^2$, we can compute the apparent structure of the jet, as seen by an observer at a viewing angle of θ_v as:

$$
E_{iso}(\theta_v) = \eta \int \frac{\delta^3}{\Gamma} \frac{dE}{d\Omega} d\Omega \tag{15}
$$

Here, as is customary, the authors of Barbieri et al, 2019 assume that $\eta = 10\%$ of the kinetic energy of the jet is converted into gamma-rays and radiated.

[2]In this formula, the term χ_{BH} is the spin parameter of the final BH, which is computed using the results of Pannarale, 2013.
[3]See end of page 7 of Barbieri et al., 2019 for an explanation of the value that they choose to set it to.