Hedging a trade for PCA component neutrality

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Suppose I am given a set of financial instruments, e.g. {1Y, 2Y, ..., 30Y} interest rate swaps or {Barclays, Lloyds, .. } FTSE100 companies. It doesn't matter which so let's go with IRS.

I have historical datapoints so I can calculate a PCA decomposition and return eigenvectors ordered by largest eigenvalue, representing the Principal Components:

$$\mathbf{E} = [\mathbf{e_1} : \mathbf{e_2} : .. : \mathbf{e_n}]$$

Say that I now have a portfolio of risk positions, or a specific trade strategy, e.g. I am going to pay a 2Y5Y10Y butterfly:

$$\mathbf{t} = \begin{array}{c} 1Y \\ 2Y \\ 3Y \\ 4Y \\ 5Y \\ 6Y \\ .. \\ 10Y \end{array} \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 2 \\ 0 \\ .. \\ -1 \end{bmatrix}$$

My risk exposure to each Principal Component is calculated by:

$$\mathbf{E^T t}$$

If I want to hedge against the first principal component, or the first and the second what should I do?

That is I seek an adjustment trade $\mathbf{x}$ to the portfolio $\mathbf{t}$ such that, for example;

$$\mathbf{E^T(t+x)} = \begin{bmatrix} 0 \\ 0 \\ \alpha \\ \beta \\ .. \end{bmatrix}$$

fixed-income interest-rates portfolio-management hedging pca

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asked Nov 10, 2019 at 14:52

Attack68 ♦
11.9k 2 20 51

Let's say I have a view that that long term yields are going to increase faster than short term yields for whatever reason so I enter a steepener. We know the most likely movement in the curve tends to be a parallel shift (i.e. first PC, which generally explains most of the movement of the UST curve). Even if I structured the trade to be DV01 neutral, I may not be fully neutralized against a parallel shift so using PC1 rotation could help better manage risk – Joel Alcedo Nov 10, 2019 at 19:44

1 Answer

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Simple Directionality Spread Trade Hedge

If the sum of the risks of the trade t **are zero** (as in the case of the 2Y5Y10Y spread trade) that immediately gives a starting point from which to make a simple calculation for an adjustment.

For example if one assumes that the first principal component is the outright market driver and that the factor loadings represent a relative volatility of an instrument with the outright market, then

> dividing the trade positions by the factor loadings will remove this component.

$$\frac{\mathbf{t}}{\mathbf{e_1}} \cdot \mathbf{e_1} = \sum_i t_i = 0$$

where the division is done element wise. In this case the adjustment $\mathbf{x}$ is the difference:

$$\mathbf{x} = \frac{\mathbf{t}}{\mathbf{e_1}} - \mathbf{t} \quad \implies (\mathbf{t} + \mathbf{x}) = \frac{\mathbf{t}}{\mathbf{e_1}}$$

Note that this is not the only way to make this calculation. It is not a unique solution; this method has the advantage of being relatively easy with a transparent assumption about its design.

More General Case

In the case that the risks of $\mathbf{t}$ do not sum to 0, or one wishes to hedge against more than one principal component we can consider other options.

A reasonable concept is to suggest that one seeks $\mathbf{x}$ such that it is as small as possible, and the change to $\mathbf{t}$ is therefore minimal in some sense.

One might also need to make a decision whether other instruments are allowed in $\mathbf{x}$, or if we must stick to 2Y 5Y or 10Y. Suppose we stick with only those 3 instruments for now and we formulate the optimisation problem with respect to the l_2 norm:

$$\min_{\mathbf{x}} f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathbf{T}}\mathbf{I}\mathbf{x}$$
$$\text{subject to} \quad \mathbf{e_1^T}(\mathbf{t}+\mathbf{x}) = 0 \quad (\text{1st PC hedged})$$

This is a quadratic program with an equality constraint solvable via the KKT conditions:

$$\nabla L(\mathbf{x}, \lambda) = \begin{bmatrix} \mathbf{I} & \mathbf{e_1} \\ \mathbf{e_1^T} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{e_1^T t} \end{bmatrix} = 0$$

With small rearranging, the formula for block matrix inversion and cancellation we are left with the result:

$$\begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{I} - \mathbf{e_1 e_1^T} & \mathbf{e_1} \\ \mathbf{e_1^T} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ -\mathbf{e_1^T t} \end{bmatrix}$$

or

$$\mathbf{x} = -\mathbf{e_1 e_1^T t} \quad \Longrightarrow \quad (\mathbf{t}+\mathbf{x}) = (\mathbf{I} - \mathbf{e_1 e_1^T})\mathbf{t}$$

Degrees of Freedom

The above two methods will give different results but both are valid under their inherent assumptions. The general method can be used to neutralise more than one component, for example if you want to neutralise PC1 and PC2 (the optimisation constraints are extended). But doing so reduces the degrees of freedom in the solution. For example in a 3 instrument configuration if PC1 and PC2 are made neutral the only valid solution is for the trade to be a multiple of PC3.

Share Improve this answer Follow edited Nov 10, 2019 at 20:29 answered Nov 10, 2019 at 14:52

Attack68 ♦
11.9k 2 20 51

How do you know that the only solution under PC1 & PC2 constraints is a multiple of PC3 ? It seems easier at all and a better hedge, to be neutral at PC1 & PC2 with a easier solution, no ? – Benjamin Freoa Feb 21 at 19:20

In a 3 axis basis (i.e 3 PCs), if 2 of those axes are set to zero, the only mathematically valid position is to then be along the 3rd axis, i.e. have risk only to PC3. – Attack68 ♦ Feb 21 at 22:15

Thank you @Attack68! The risk on PC3 is a 'fatalility" with this method so ... Great to know. Do you have the mathematical solution to this problem or a paper on it ? Do you know if some Utility Functions are used to hedge it ? – Benjamin Freoa Feb 26 at 13:58