UBS

Just Enough Interest Rates And Foreign Exchange

Joe Troccolo, Managing Director
UBS Financial Markets Education

Preface

Everyone involved in the financial markets needs to know something about interest rates and foreign exchange. Even people who deal entirely with equities are affected through the influence of interest rates on equity prices and foreign exchange fluctuations on corporate balance sheets and income statements. Unfortunately most books on these subjects are written for specialists in the rates and foreign exchange market and tend to be too complete. The effect of this is that someone who wants to know "just enough" has to sift through a lot of information to find the truly important parts. Its probably true that with most forms of expression – books, movies, music, paintings and even everyday conversation, knowing what to leave out is at least as important as knowing what to include. This "short" book is our answer to the question of what everyone needs to know about interest rates and foreign exchange. If, having read through the book, you feel we included something unnecessary, feel free to forget it! If you find there is something you really needed to know but we left it out, first ask yourself if everyone needs to know it. If so, let us know and we'll include it next time.

I would like to thank my colleagues in UBS Financial Markets Education for their help and encouragement in the course of writing this book. Particularly I would like to thank Spencer Morris who proof read the text exhaustively. Naturally any errors are the responsibility of the author. Should any errors be found please send us an email or write to us at the following address and if your discovery is verified, we will send you a unit of your country's currency whose value most closely approximates one pound sterling.

Joe Troccolo
Managing Director
UBS Financial Markets Education
1 Finsbury Avenue
London EC2M 2PP
United Kingdom

Email: sh-fme@ubs.com

Contents

Section 1 – Interest Rates on Deposits and Discount Securities	1
The World of Interest Rates	1
Answers to Exercises	11
Section 2 – Bonds	12
Yield To Maturity	16
The Price-Yield Relationship	19
Bond Risk Measures	22
Duration	25
Worksheet – Bond Pricing, Duration, Convexity	33
Section 3 – Forward Rates and Eurodollar Futures	35
FRAs	35
Futures	36
Forward Rates	39
Answers to Exercises	41
Bond Futures	42
Section 4 – Interest Rate Swaps	45
Answers to Exercises	52
Section 5 – Building a Yield Curve	53
The US Treasury Market	53
Bond Yields	59
Formulas and Bootstrapping the Yield Curve	61
The Relationship Between Forward, Zero and Par Rates	64
Practical Construction of the Treasury Yield Curve	67
LIBOR Curve	68
Worksheet – Yield Curve Relationships	69
Worksheet -Stripping and Recombining	72
Exercise – Swaps and Forward Start Swaps	73
Answers to Exercises	74
Section 6 – Just Enough Foreign Exchange	79
Bids and Offers	81
Spot and Forward Transactions	83
Forward Points	85
Interest Rate Parity	89
Currency Futures	93
Currency Bond Swaps	94
Appendix 1 – Interest Rate and Bond Taxonomy	102
Appendix 2 – Day Basis	106
Global Disclaimer	112

Section 1
Interest Rates on Deposits and Discount Securities

The world of interest rates

Man (humankind, if you like) may be the measure of all things, but money is the measure of all investments. Money, though, is the one good that you enjoy most when you get rid of it, presumably by spending it on something you want. Misers may get enjoyment from just looking at a stack of bills, or running their hands over a pile of Kruggerands, or maybe just looking at a large bank balance, but, given our preferences, most of us would just as soon buy things. One reason we don't spend all our money on immediate consumption (or maybe you do), is that we are willing to wait until a future time when we might have a greater need to buy, but a lesser ability to do so i.e. we "save for a rainy day". Another reason for not spending money now is that there may not be so much you need, or want, to buy. If you don't have much money you are likely to spend all of it; but if you have a lot of it you might get to the point where there isn't really anything you want! You've bought the Porsche and the Rolex watch, you have your clothes tailor-made, you're renting the $50,000 a month penthouse and you get take-out from Le Cirque 2000 most nights. There just isn't anything left! In the meantime there are other people who need to spend money now but don't have it. Maybe someone wants to make a major purchase like a car or a house, or a college education. Or maybe the government wants to build a new highway or a business wants to expand. These potential borrowers are willing to pay you more money in the future if you will let them use your money from now until then. Maybe some of them will approach you directly, probably relatives and old college friends. But you don't often find the CFO of a major corporation in your lobby waiting for you to come out in the morning. Well, not most mornings anyway. So to make the picture complete we need someone to channel the money from you, the investor, to the borrower. This is usually done by a bank or some other type of financial institution. In this sense the interest rate world is really a pretty simple place. So why do you need a book on interest rates?

Like other markets, the world of interest rates is populated with individuals and institutions with widely varying motives for being there. But there are three primary themes among the participants – they need to borrow money, or they have money to invest or they want to intermediate between the first two. The most important of these are the investors. The astonishing variety of types of investments available in the interest rate markets is due to the differing appetites of the investors. The borrowers – corporations, governments, banks and other financial institutions, along with their advisers – the investment banks, will tailor their borrowing to suit the investor who is lending the money. It is the wide range of instruments, pricing conventions and uses for the instruments that has lead to this book. We have tried to keep it as short as possible while still covering the most interesting and important aspects of rates. For no extra charge we have included a section on foreign exchange, as the interest rate and foreign exchange markets are so closely related.

Risk

The defining characteristics of interest rate investments are maturity and risk. Investors choose the form of their investment primarily based on these two dimensions. Maturity simply refers to how long the investor is lending the money for – a day, a week, a year or some longer period of time. Risk refers to both the uncertainty of the return the investor will get on the investment, which can range from no uncertainty to nearly total and whether the borrower will fail to live up to one or more of the obligations represented by the investment. The first type of risk we call *market risk* and the second is *credit risk*.

Actually an investor might not even think that he/she is lending money. For example, if you buy a bond in the secondary market, you are not lending any money to the issuer of the bond. Rather it is the original buyer of the bond who did that. What you are doing is taking that investment over from someone who no longer wants it. Still, once you have bought the bond you have assumed all the risk the original investor did. So from a practical point of view you are the lender now.

Basic principles

Let's begin with some obvious principles. The amount of interest received must become larger as the time lapse between when it is lent and when it is returned grows larger. That is, if you borrow for a year, you will have to pay more than if you borrow for just 6 months. A second principle, so obvious it seems unnecessary to state, is that the more you borrow, the more you pay. That is, the amount of interest you pay on £100,000 is more than the amount you would pay on £50,000.

Cash flows

Cash flows involve amounts of cash and timing. If you receive £500 today and repay £510.35 in 3 months, then the first cash flow is £500, which you take in and the second is the £510.35 which you pay out. The £500 is referred to as a Present Value (PV) because it is valued as of today. The £510.35 is called a Future Value (FV) because it is deferred until a future date.

Calculating interest

Suppose you went to a bank and asked to borrow £500 for 3 months and the banker told you that you would need to repay £510.35 at the termination of the loan. Wouldn't you wonder where the number £510.35 came from? Or suppose you wanted to make an investment rather than a loan. You wish to invest £750 for 9 months. The banker says you can receive £771.25 in 9 months. But he gives you an alternative. Instead he says, you can receive £750 in 9 months if you will give him £731 today? Which is the better deal? How would you figure it out? The answer is by using an interest rate.

Figure 1

Add-on interest rates

When we know the present value, we usually arrive at the future value by adding the interest to the initial amount. The computation of the amount of interest is done using an interest rate. The word ”rate” implies that the result will be proportional to the amount borrowed or invested, just like the distance you travel in a car is proportional to the ”rate” of speed at which you travel. Continuing that analogy, if you were driving at a rate of 100kph, we could not say how far you have travelled until we knew how long i.e. for how much time, you had travelled at that rate. It is the same with interest. We need to know both the ”speed” i.e. the rate and the time as well as the present value, in order to know how much interest there will be.

Example

We lend £500 for 3 months at a rate of 8.28% per annum. The ”8.28% per annum” is the interest rate. It says that for every £1 invested, you would receive £1.0828 in a year’s time. But we are not investing £1 for 1 year; we are investing £500 for 3 months. Thus the 8.28% must be adjusted for the amount - £500 - and the time - 3 months - of the loan. One of our principles said that the amount of interest should be proportional to the amount of the loan. This means we need to multiply the amount - £500 - by the rate - 8.28% - (expressed as a decimal: 0.0828). But the other principle said that the interest must also be proportional to the amount of time of the loan - 3 months -. So we must also multiply by the time (expressed as fraction of a year: $^3/_{12}$).
Finally then, the amount of interest is:

$$500 \times 0.0828 \times {}^3/_{12} = 10.35$$

So the future value, which we receive in 3 months time is:

$$500 + 10.35 = 510.35.$$

Exercise 1

Present Value (Investment)	Interest Rate	Time	Future Value
£1,000	8.50%	1 year	
$2,600	6.25%	6 months	
€30,000	5.10%	1 month	

Interest rate problems

Several types of problems commonly occur when working with interest rates:

1. **Solving for Future Value, given Present Value, Interest Rate and Time.**
   How much will you have to repay if you borrow £150,000 for 8 months at an interest rate of 9% per annum?

2. **Solving for Present Value, given Future Value, Interest Rate and Time.**
   You need to have $10,000 in 1 year's time in order to make a down payment on a new car. If you can earn interest at 6.50% per annum, how much would you have to deposit today in order to have $10,000 in one year?

3. **Solving for the Interest Rate, given Present Value, Future Value and Time.**
   A UK corporate is selling commercial paper (a security) that will pay the owner £1,000 6 months from now. If the paper costs £980 to purchase today, what interest rate is the investor earning?

4. **Solving for Time, given Present Value, Future Value and the Interest Rate.**
   How long will the money have to be on deposit if we want to invest £6,000 and want to earn £337.50 in interest, if the bank is offering an interest rate of 7.50% per annum?

1. Solve for Future Value

The first of these is the type of problem we started with. We solved it by first computing the interest then adding it on to the Present Value, i.e.

$$\text{Future Value} = \text{Present Value} + \text{Interest}$$

If you didn't faint, you have just survived an encounter with an equation! The equation represents a relationship between the three quantities – FV, PV and I. You have now survived an encounter with mathematical shorthand. It's quicker to write PV than Present Value. Continuing to push you, recall that we calculated the Interest by multiplying the present value by the rate and the time, i.e.

$$\text{Interest} = \text{Present Value} \times \text{Interest Rate} \times \text{Time}$$

Combining the two equations (will this never end!), we get

$$FV = PV + PV \times r \times t$$

For our next trick, we can "simplify" the right hand side of the equation by writing:

$$FV = PV(1 + rt)$$

which is very succinct*. Since you saw how we built up this equation, we hope it is not mysterious to you. Let's see how we would use it on one of our previous problems.

If we deposit £500 for 3 months at a rate of 8.28%, then the amount we receive back (FV) is:

$$FV = 500(1 + 0.0828 \times 3/12)$$

*Notice how we dropped the 'x' from r x t; when two quantities (r,t) are written together, they are assumed to be multiplied.

Parentheses () indicate that the quantity inside must be calculated first before the final result can be obtained:

$$FV = 500 \times 1.0207 = 510.35$$

where the 1.0207 = 1 + 0.0828 x $^3/_{12}$.

The reason for writing the relationship between FV, PV, r and t in symbolic form is that the rest of the problems can be solved using (gasp) algebra.

2. Solve for PV

$$FV = PV(1 + rt)$$

$$PV = \frac{FV}{1 + rt}$$

You need to have £10,000 in 1 years time in order to make a down payment on a new car. If you can earn interest at 6.50% per annum, how much would you have to deposit today in order to have £10,000 in one year?

$$PV = \frac{10,000}{1 + 0.065} = 9,389.67$$

3. Solve for r

$$r = \frac{1}{t}\left(\frac{FV}{PV} - 1\right)$$

A UK corporate is selling commercial paper (a security) that will pay the owner £1,000 six months from now. If the paper costs £980 to purchase today, what interest rate is the investor earning?

$$r = \frac{1}{1/2}\left(\frac{1,000}{980} - 1\right) = 0.0408 = 4.08\%$$

4. Solve for t

$$t = \frac{1}{r}\left(\frac{FV}{PV} - 1\right)$$

How long will the money have to be on deposit if we want to invest £6,000 and want to earn £337.50 in interest, if the bank is offering an interest rate of 7.50% per annum?

$$t = \frac{1}{0.0750}\left(\frac{6,337.50}{6,000} - 1\right) = 0.75 = 9 \text{ months}$$

Summary of Formulas:

$$FV = PV(1 + rt)$$

$$PV = \frac{FV}{1 + rt}$$

$$r = \frac{1}{t}\left(\frac{FV}{PV} - 1\right)$$

$$t = \frac{1}{r}\left(\frac{FV}{PV} - 1\right)$$

Exercise 2 Fill in the missing items

Present Value	Future Value	Interest Rate	Interest	Time
	5,000	6.45%		6 months
450			5	3 months
550		575		8 months
2,400	2,505.60	5.50%		

Day basis

We have presented a simplified view of interest rate calculations. Most calculations are more complicated because of market conventions which regulate exactly how the calculations are to be done. This involves how the number of days that earn interest are calculated and whether the year is on a 360 or 365 day count. These topics are taken up in the appendix. If you are interested, you can look there for more details. In this section we will "keep it simple" and just use fractions of a year.

Discount rates

Some securities are sold at a discount to their face value. This means that the security pays a specified amount at the maturity of the instrument and the buyer pays something less than that so-called face amount today. In our previous terminology, the security has a specific Future Value and the Present Value is today's price. This would all be very simple except that the convention in the market is to quote the price today in terms of an annual discount rate. Here is what this means. If the security pays £100 in 1 year and the discount rate is given as 8.00%, it means that today's price (the PV) is found by simply subtracting the discount from the face value i.e. the price = 100 – 100 x 0.08 = 92. This is not the same thing as using the 8.00% as if it were an add-on interest rate. If it had been an add-on rate we would have found the present value using our previous formula: PV = 100/1.08 = 92.59. This would be wrong! In the world of discount securities interest rates are given as discounts not add-ons. So we must use them as they are intended. Let's do another example.

Example

A 3 month Treasury bill is being sold at a 6.00% annual discount. How much would you pay for $25,000 face value of this bill?

Before we solve this problem, let's be sure we know what is meant. The security in question is a Treasury bill. The US Treasury borrows money by issuing i.e. selling these bills. The bill pays a specific amount ($25,000 in this case) at a specific time (3 months). The investor buys the bill at less than its face amount ($25,000 is the face amount). To find the price we subtract the amount of the discount from the face value of the bill. As usual we need a rate to work with, 6.00% in this case and a time period, 3 months. Then:

$$\text{Price} = \text{PV} = \text{Face} - \text{Discount}$$

$$\text{Price} = 25{,}000 - 25{,}000 \times 0.0600 \times 3/12 = 24{,}625$$

Notice that the amount of the discount was determined by both the rate (6.00%) and the time (1/4 of a year). The greater the period of time, the greater the discount would be. The time period of the bill would have had to have been an entire year for the discount to be a full 6.00% (which would have been $1,500 instead of just $375).

Since you are so used to formulas by now, we can determine the correct expression for discount securities:

$$\text{Price} = \text{PV} = \text{Face} - \text{Discount}$$

$$\text{Price} = \text{Face} - \text{Face} \times r \times t$$

$$\text{Price} = \text{Face} \times (1 - rt)$$

We have a common "backwards" problem too. Note that Face is the FV here.

Solve for r, given PV, FV and t

$$r = \frac{1}{t}\left(1 - \frac{PV}{FV}\right)$$

Example

A piece of commercial paper will pay off $5,000 in 4 months. If its market price is $4,850, what is the discount rate?

$$r = \frac{1}{4/12}\left(1 - \frac{4{,}850}{5{,}000}\right) = 0.0900 = 9.00\%$$

Exercise 3: Fill in the missing items

Face Value	Discount Rate	Time	Price
£5,000	8.50%	8 months	
€1,000,000	6.20%	3 months	
¥2,000,000,000		6 months	1,910,000,000

Comparing rates

Remember this problem we posed early on?

You wish to invest £750 for 9 months. The banker says you can receive £771.25 in 9 months. But he gives you an alternative. Instead he says, you can receive £750 in 9 months if you will give him £731 today? Which is the better deal?

Since you are investing £750 for 9 months and receiving £771.25, we know that we can solve for the interest rate:

$$r = \frac{1}{9/12}\left(\frac{771.25}{750} - 1\right) = 0.0378 = 3.78\%$$

The second alternative is to invest 731 and get £750 in 9 months. We can solve for the implied interest rate:

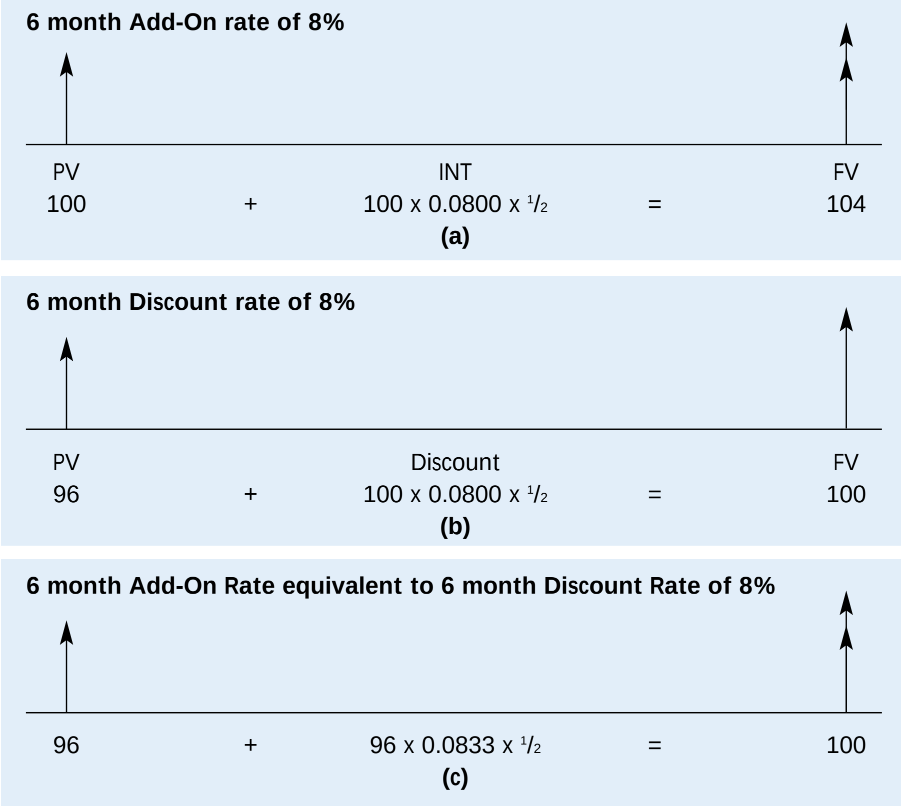
$$r = \frac{1}{9/12}\left(\frac{750}{731} - 1\right) = 0.0347 = 3.47\%$$

So the first alternative is better. But let's go one step further. The second alternative is really a discount security. You are told that its face value is £750 and its price is £731, since we know the time to when the security pays off is 9 months, we can solve for the discount rate:

$$r = \frac{1}{9/12}\left(1 - \frac{731}{750}\right) = 0.0338 = 3.38\%$$

That's probably one too many rates for you! What is the relationship between the answers to the last two problems? In the last problem we treated the investment as a discount security and found that the discount rate was 3.38%. That is, if we started with the face value of the security (£750) and discounted it by that rate (3.38%) we would get its price (£731). Alternatively, if we thought of the investment as being like a deposit, then if we started with the deposit amount (£731) and used an add-on rate of 3.47%, we would arrive at its future value (£750). The two rates, 3.47% and 3.38%, are obviously two ways to get the same result. However one is used as an add-on and the other is used as a discount. We can see, though, that they are equivalent.

Figure 2 Add-On, Discount and Equivalent Rates



The usefulness of this equivalence is in comparing investments. If one investment is on an add-on basis and the other is priced at a discount, we must convert one rate to the other type in order to compare them and see which is better. That is, we must find the add-on rate that gives the same answer as the discount rate or the other way around. This is all we have to say on this topic for now, but if you were to look in a financial newspaper like the Wall Street Journal at the prices of US Treasury bills, you will see that the discount rate is given and along with it, the equivalent add-on interest rate. This is for the convenience of the newspaper's readers since they have the opportunity of either investing their money at their local savings institution, which would use an add-on rate, or buying US Treasury bills, which are priced at a discount. By looking in the Wall Street Journal and comparing to the local savings rate, they can see which gives a better return.

Example

A bank is advertising a rate of 7.50% for 3 month deposits. A highly rated US Corporation is selling its 3 month commercial paper at a discount of 7.40%. Which investment gives a better return?

Suppose you were going to buy $1,000 face value of the commercial paper. You would have to pay

$$1{,}000 - 1{,}000 \times 0.0740 \times {}^{3}/_{12} = \$981.50$$

today.

The implied add-on rate is:

$$r = \frac{1}{3/12}\left(\frac{1{,}000}{981.50} - 1\right) = 0.0754 = 7.54\%$$

so the return on the commercial paper is better than the return on the bank deposit. Assuming the corporation has a credit rating at least as good as the bank, we would be better off investing in the CP (Commercial Paper).

Exercise 4:

Time	Discount Rate	Price (per £1,000)	Add-On Rate
4 months	8.00%		
1 year	5.25%		
6 months	4.80%		
9 months	6.00%		

In each case assume you have a discount security with a face value of £1000. Find its price, then determine which add-on rate would give you the same return on your initial investment. The first problem is solved for you:

$$\text{Price} = 1{,}000 - 1{,}000 \times 0.0800 \times 4/12 = 973.33$$

$$r = \frac{1}{4/12}\left(\frac{1{,}000}{973.33} - 1\right) = 0.0822 = 8.22\%$$

Market interest rates

LIBOR

The market for deposits is an inter-bank market. Each day banks borrow from and lend to each other and their customers. The benchmark rates for deposits and loans are referred to as LIBOR, which stands for the London Inter-Bank Offered Rate, the rate at which large banks will lend (offer funds) to one another. The rate at which the bank will borrow is called LIBID and the average of the two is called LIMEAN. The significance of these rates, which are quoted for terms of overnight to one year, is that they are visible to all market participants and so form a reference point for all other cash rates. This rate exists in all the major currencies such as USD, GBP, JPY and EUR.

Table 1

Euro Deposit Rates on 19 August 1999

	Euro	Yen	Swiss	Sterling	USD	HKD	MXN
1 week	2.10	0.0275	0.88	4.84	5.26	5.97	19.00
1 month	2.57	0.033	0.911	4.98	5.30	6.12	20.25
3 months	2.65	0.0575	1.063	5.08	5.43	6.37	20.70
6 months	2.86	0.27	1.427	5.39	5.81	7.03	22.75
1 year	3.16	0.2862	1.651	5.73	5.92	7.50	24.40

Answers to Exercises

Exercise 1:

Present Value (Investment)	Interest Rate	Time	Future Value
£1000	8.50%	1 year	**£1,085**
$2600	6.25%	6 months	**$2,681.25**
€30000	5.10%	1 month	**€30,127.50**

Exercise 2:

Present Value	Future Value	Interest Rate	Interest	Time
4843.79	5,000	6.45%	**156.21**	6 months
450	**455**	**4.44%**	5	3 months
550	575	**6.82%**	**25**	8 months
2400	2,505.60	5.50%	**105.60**	**0.8 years (9.6 months)**

Exercise 3

Face Value	Discount Rate	Time	Price
£5000	8.50%	8 months	**£4,716.67**
€1000000	6.20%	3 months	**€984,500**
¥2,000,000,000	**9.00%**	6 months	¥1,910,000,000

Exercise 4

Time	Discount Rate	Price (per £1000)	Add-On Rate
4 months	8.00%	**973.33**	**8.22%**
1 year	5.25%	**947.50**	**5.54%**
6 months	4.80%	**976**	**4.92%**
9 months	6.00%	**955**	**6.28%**

Section 2
Bonds

Bonds are issued by corporations, banks, financial institutions - such as insurance companies and brokerages, governments, government agencies and so-called ”supranationals” such as the World Bank. A bond is just a market-based method for these entities to borrow money. A bond really represents a loan, but whereas loans are ”taken out” from a bank, bonds are securities and so are ”sold” in a market. In this section we are going to, as usual, ignore the fine details of how bonds are originated and sold. For now we want to go over some of the basic terminology of the bond market.

Principal Amount

When a bond is sold the principal amount is the total amount of money being borrowed. Generally the bond market only deals in significant sizes of many millions of dollars. A typical size for a bond offering by a corporation would be 100 to 200 million dollars, but there have been many bonds of over 1 billion dollars. Bonds are used by corporations to fund their capital investments, to expand their operations and frequently to make acquisitions of other corporations. The World Bank raises money to fund loans to underdeveloped countries, while the IMF, which is part of the World Bank, uses funds to help stabilise the economic systems in developing countries. The most significant feature of the principal amount is that it is used as the basis for computing the interest payments to be made.

Interest

The most common type of bond pays a fixed interest rate. The interest rate is also called the coupon rate, which refers back to the days when bonds were physical pieces of paper. In those days, the interest payments were symbolised by coupons attached to the physical security. When it was time for one of the payments to be made, the bond holder would ”clip” the coupon off and bring or send it to the issuer’s bank for payment.

When the interest rate is of the fixed type, interest is calculated by multiplying the coupon rate times the principal amount and the time period covered by the payment. The usual day basis* used is either 30/360 or actual/365, but in the US Treasury market, which is the largest single bond market in the world, the convention is actual/actual. As usual these are described later in more detail.

Floating Rate

Some bonds have an interest rate that is reset every period (just like the interest rate swaps discussed in Section 4). These bonds are usually called Floating Rate Notes (FRNs). The most common reference rate for resetting the interest rate is LIBOR, the London Interbank

---

* See the Appendix – Day Basis

Offered Rate, or one of its relatives in other currencies such as EURIBOR (LIBOR for payments in Euro).

Maturity Date

On the maturity date, the issuer of the bond will repay the principal amount of the bond as well as any coupon that may be due on that date.

Example

➤ Issuer	XYZ Corporation
➤ Principal Amount	USD250,000,000
➤ Payment Date	15 July 1999
➤ Coupon Rate	6.50%
➤ First Coupon	15 January 2000
➤ Frequency	semi-annual
➤ Day Basis	30/360
➤ Maturity Date	15 July 2004

This is an example of a five year, fixed coupon bond issued by a corporation. The principal amount would be received by the company on 15 July 1999 and the full amount would be repaid on 15 July 2004. The coupon payments are made every 6 months on 15 July and 15 January. For the detail minded, when the day on which a payment is meant to be made falls on a non-business day, which generally means the banks are not open, the payment rolls forward to the next business day. In the event the next business day would be in the next month, however, the payment is usually rolled back to the previous business day. This method of determining when a payment happens is an example of a Business Day Convention. Because this bond uses the 30/360 method of counting days, each coupon period will contain 180 days (ignoring the non-business day problem) so one-half of the coupon will be paid out at each coupon date.

The principal amount of the bond is also referred to as the ”par” amount. When the price of the bond, which fluctuates, happens to equal the par value of the bond we say the bond is ”trading at par”. The coupon on the bond is then called the par coupon. Prior to a bond being issued, we might ask ”what would the par coupon be?” What this question asks is what coupon would the bond need to have in order for buyers to be willing to pay the full amount for the bond? The higher the coupon, the more attractive the bond will be to investors but the worse for the borrower. Conversely, a low coupon would be good for the borrower but unattractive to an investor. Because the borrower usually needs the money and the investor usually has lots of choices for investing, the requirements of the investors usually predominate in determining what the interest payment will be. The rate the investor will demand in turn depends on the general level of rates and the specific credit quality of the borrower. The general level of rates refers to the rate being paid by the highest quality issuers such as governments and the credit quality refers to the perceived ability of the borrower to make all the bond payments as scheduled.

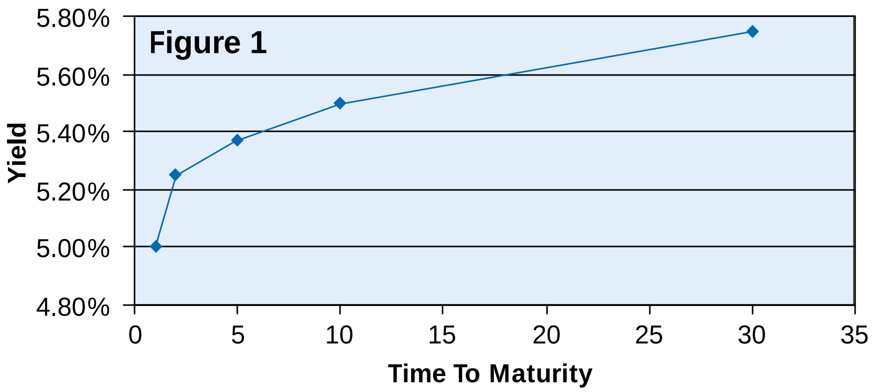
Yield Curves

The coupon rate on a bond is the payment the investor is demanding in order to be willing to buy the bond. If we restrict ourselves temporarily to bonds issued by highly trusted borrowers such as the US Treasury, then we can concentrate on the term structure of interest rates. This refers to the fact that bonds with different times to maturity will often have different interest rates. For example, if the US Treasury were to consider issuing bonds of maturities 1, 2, 5, 10 and 30 years, the coupons might be:

Maturity	Coupon
1 year	5.00%
2 years	5.25%
5 years	5.375%
10 years	5.50%
30 years	5.75%

Technically, a bond issued by the US Treasury with an original maturity of 10 years or less is called a Note, but we will ignore this distinction for now. The different interest rates that apply to different times to maturity are an example of what we referred to as "term structure" and if we were to draw a graph like Figure 1 with time to maturity on the X axis and Coupon Rate on the Y axis, we would have a representation of the "yield curve". The coupon rate on a bond trading at par is also it's yield, a term we will explain later.

Yield Curve



There are a number of theories as to why the yield curve takes on a particular "shape". In this case the shape is thought of as "normal" in that it is upwardly sloping. This means that investors require a higher rate for a longer maturity bond. One theory says that investors require a higher rate for a longer term investment because they are "tying up" their money for a longer time. But since the curve sometimes takes on a flat or even inverted shape (short term rates higher than long term rates) the theory is far from perfect. We will see later that long term bonds are inherently riskier than short term bonds and this too would indicate that investors would need to get a promise of higher interest as compensation. However the short term rate is often controlled by actions and policies of the central bank which can lead to either very high or very low short term rates. Over the years we have seen short term rates in the US as high as 20% and in Japan as low as nearly 0%. We will take the yield curve as, in the terms of economists, "exogenous" or more simply, outside the scope of our work and so just treat it as something which is given to us to work with.

Bond Price

As noted above, a bond has a principal amount, which is the amount of money being borrowed. In the market we do not usually have much reason to refer to the principal amount. We are more concerned with the market price of the bonds. If the principal amount is USD250 million, it is unlikely that any one person or institution will buy all of the bonds. Each buyer of a bond is a lender to the issuer. The bonds themselves will be bought in amounts that are smaller than the entire size of the issue. Usually there will be some smallest amount of the bond that will be offered for sale to investors. Because the bond market is mostly a professional and institutional market, the smallest size is usually 1 million dollars. It may be possible for individual investors to buy smaller amounts of the bond through their broker but we will ignore this ”retail” market. Also most bonds are no longer physical pieces of paper, rather they are kept in book-entry form and bought and sold through banks or brokers. The price of the bond in the market is quoted as a percentage. A price of 105, for example, would mean that an investor would need to pay 105% of the principal amount of the bond in order to buy it. If the investor wanted to buy 5 million dollars of principal, or face value of the bond, the price would be 5.25 million dollars. If, instead, the investor wanted to buy 10 million dollars of face value, the price would be 10.5 million dollars. This way of quoting the bond price is convenient because it can be used to calculate the actual price to be paid for any face value of bonds. This is analogous to quoting stock on a per share basis. If the price of IBM is USD200 then the investor pays 200 multiplied by the number of shares bought. Just think of percent of par as being the fundamental unit of price.

Accrued Interest

By its nature a bond is a lot like a savings account. When you put money in the bank you expect to be paid interest. Even if the bank credits your account just once a year, if you were to withdraw your money part way through the year, you would expect to receive the interest already earned. This is the concept of accrued interest. If a bond has an annual 6% coupon, the issuer is promising to pay 6% interest every year, on the coupon payment date. If the entire bond issue was owned by just one person or institution and if the bonds were never traded in the secondary market, there would be no problem with this. But as a matter of fact, after the bonds are issued there will be secondary trading i.e. trading by people who did not buy the bond originally from the issuer. Each time the bond is traded, the buyer of the bond is required to pay the amount of interest that has accrued from the last coupon payment to the trade date. This is because when the issuer pays the coupon, the full amount of the coupon will go to the person who owns the bond on the coupon payment date, no matter how long they have actually owned the bond, even if it is as short as one day. This idea of accruing a benefit like interest is different from the case of dividends on stock. If you own a stock for years and then sell it the day before it goes ”ex dividend” you will not receive any part of that dividend. Basically the reason for this is that investors realise that corporations are not required to pay dividends and even if they do, the amount of the dividend is uncertain. So there would be no basis for determining how much had ”accrued”. Rather than try to make something up, it is customary for whoever owns the share on the ”record date” to receive the entire dividend. But as noted this is not the case in the bond market.

Calculation of Accrued Interest

This is done using whatever convention was stated when the bond was issued. But it always amounts to multiplying the coupon rate times the face amount of bonds owned times the number of interest days. In the bond market, this calculation is done by all of the systems used to price and trade bonds such as Bloomberg® or Reuters®. However, the bond price as quoted in the market does not include the accrued interest, rather the interest is added to the stated price. So if the bond price is 105 and there is 0.75 of accrued interest, the full price of the bond is 105.75. In the bond market,

105 is the ”clean price”
105.75 is the ”dirty price” or the ”invoice price”

Par, Premium and Discount

When a bond is issued the coupon reflects the level of interest rates at that time. If the bond has a fixed coupon then, by definition, it will not change over the life of the bond. However the interest rate environment can change due to economic or other forces. When that happens, the bond coupon may not be in line with the new conditions. In this case the bond price will change to compensate for those new conditions. For example if the economy starts to experience a period of inflation, investors will demand higher rates of interest. If newly issued bonds have coupons that are higher than those on previously issued bonds, the older bonds will suffer a decrease in price, as they are no longer as desirable. Conversely if there is little inflation or less demand for borrowing, then bonds issued when rates were high may start to look attractive and their prices will rise. The bond market uses this terminology to describe the relationship of the bond's price to its interest rate:

A bond trading at a premium has a price greater than 100.
A bond trading at par has a price of 100.
A bond trading at a discount has a price less than 100.

Yield To Maturity

We really can't go much farther without discussing this important concept. The standard way the market measures a bond is by its yield to maturity. Keep in mind that this number, although it is universally accepted, only represents a theory of bond value. The yield to maturity is not a guarantee of performance and is subject to many fluctuations. In particular, the yield to maturity, which is forward-looking, should not be confused with the idea of rate of return, which is backward-looking. Many people make the mistake of thinking that if a bond has a yield to maturity of 8%, then if they buy it they will earn an 8% rate of return. Later we will discuss what the rate of return means and why the two values – yield and return – do not need to coincide.

We will begin by simply describing what the yield to maturity is and how it is used. Later we will have much to say about how it should or should not be interpreted.

A bond can be thought of as a series of cash flows. We generally would regard a cash flow we will receive at a future time to be not as valuable as a cash flow we would receive today.

This is simply the difference between future value and present value. When we are looking at a future value and we want to calculate a present value we need to have an interest rate to use for the purpose of calculation. The type of interest rate that is used to discount the cash flows from a bond, depends on the type of bond. If the bond pays an annual coupon then we would use an annual-compounded rate to do the discounting. This means that if, for example, the bond pays a cash flow of $100 in two years and we want to discount (present value) this amount using an 8% interest rate, our calculation would be:

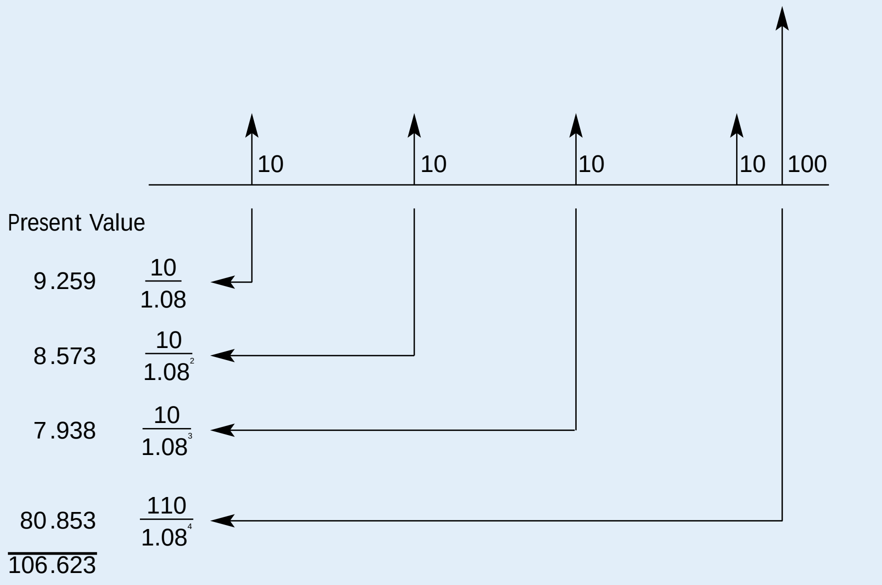
$$\frac{100}{(1 + 0.08)^2} = 85.73$$

This is what is meant by an annual-compounded rate.

Example

Suppose we have a bond that pays a 10% coupon annually and matures in 4 years. Let's present value the cash flows assuming a "yield" of 8%.

Figure 2



Definition

The *yield to maturity* of a bond is the single interest rate such that the total present value of the bond's cash flows calculated using that interest rate equals the bond's price.

Which comes first, the price or the yield to maturity? Bonds are traded on price. That is, if you buy a bond, the dealer will tell you how much money you have to pay for it. But the market "thinks" in terms of yield. In fact a typical statement from a salesperson would be "if you buy this bond you'll get a yield of 8%".

Example

Suppose the yield of the bond in our previous example was 9%. How would the price change? Since we would be using a higher interest rate with which to discount the cash flows, the price would certainly be less. The calculation below verifies this.

Time	present	1 year	2 years	3 years	4 years
Cash Flow		10	10	10	110
Present Value	103.24	9.17	8.42	7.72	77.93

Exercise 1

Fill in the table below if the yield to maturity of the bond is 7% rather than 8% or 9%.

Time	present	1 year	2 years	3 years	4 years
Cash Flow		10	10	10	110
Present Value					

Coupon Bond Pricing Formula

For bonds which pay an annual coupon:

$$\text{Price} = \sum_{k=1}^{k=n} \frac{cF}{(1+y)^k} + \frac{F}{(1+y)^n}$$

F = Face Value
y = yield to maturity
c = coupon rate
n = years to maturity

If the bond pays a coupon more than once in a year, then the formula is:

$$\text{Price} = \sum_{k=1}^{k=m \times n} \frac{\frac{cF}{m}}{(1+\frac{y}{m})^k} + \frac{F}{(1+\frac{y}{m})^{m \times n}}$$

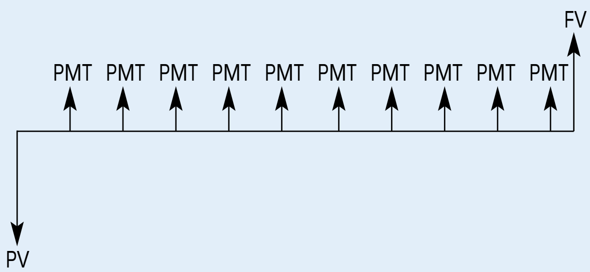
where interest is paid m times per year.

Example

A 5 year bond with a 6%, semi-annual coupon has a yield to maturity of 4%. What is its price? This exercise involves doing 10 calculations. Clearly it would be better done by a calculator. We can use the HP12C to get the answer.

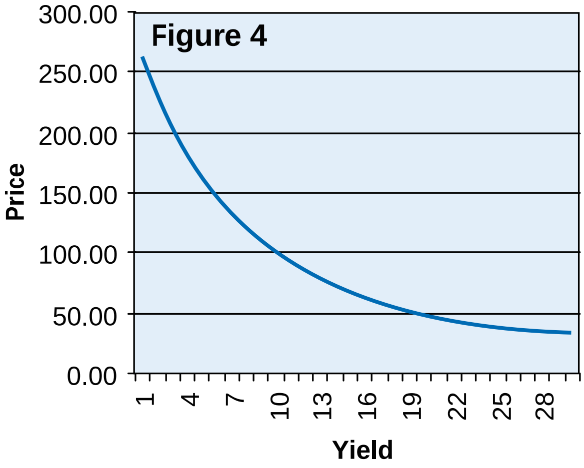
Figure 3

Keystroke	Display	Notes
f FIN	0	clear the financial registers
10 n	10	10 payments
3 PMT	3	each one is 3
100 FV	100	principal repayment
2 i	2	yield/periods per year
PV	-108.98	price is shown as negative indicating you would pay this to receive the cash flows



The HP12C does financial calculations using a cash-flow approach. The diagram illustrates how the calculator views the problem and the terms it uses for its calculations. The calculator distinguishes between the last coupon payment, which is part of the PMT sequence and the repayment of the bond principal which is FV, even though it assumes the coupon and principal payments occur at the same time.

Price of 10% 20 year semi-annual bond



The Price-Yield Relationship

As the graph in Figure 4 shows, the price of a fixed coupon bond decreases as the yield increases. This makes sense because we are present valuing each cash flow with a higher interest rate, which means the denominator of each term in the expression for the bond price is larger, which makes the value of the fraction smaller. Notice though that the relationship between yield and price is not linear.

Table 1

Yield (%)	1	3	5	7	9	11	13	15
Price	262.78	204.71	162.76	132.03	109.20	91.98	78.78	68.51

When the yield is increased from 1% to 3%, the bond price drops by about 58 points or 22%, while if we increase the yield from 13% to 15%, the price decreases by about 10 points or 13%. Notice also that when the yield is 10%, the bond price is 100 (this can be seen from the graph). Recall that we talked earlier about par, premium and discount bonds. Another way to think about this is that a bond will be priced at par if its yield is equal to its coupon. Premium bonds have coupons greater than their yield, while the yield on a discount bond is greater than the coupon.

Yield From Price

If the bond in the previous example, which was a 20 year bond with a semi-annual coupon of 10%, is trading for 88 in the market, what is its yield? From the graph and the chart below it, we can guess that the yield must be more than 11% but less than 13%. But how could we find its exact value?

Determining the yield of a bond from its price is not an easy mathematical problem to solve. We could use various approximations which give good results or we could use a process of trial and error, trying different yields until we got one that resulted in a price close to 88. For example, we could try 12%. Using the HP12C:

Figure 5

40 n	40 n
100 FV	100 FV
5 PMT	5 PMT
6 i	5.75 i
PV -84.95	PV -88.35

Since 12% was too large, we then tried 11.5%. Note that since the coupon is semi-annual we had to use 40 periods (2 x 20) , half the coupon (5) and half the proposed yield (6 or 5.75). We could go on like this, by next trying 11.75, but fortunately the HP12C can solve this problem for us. All we have to do is ask! This time put in the price (as a negative number) for the PV.

Figure 6

40 n
100 FV
5 PMT
-88 PV

i –> 5.775
2 x –> 11.55

This shows the yield to maturity is 11.55%. Once again, we had to multiply the answer (5.775) by 2 since the coupon was semi-annual and the value that the HP12C gave us was the one-period interest rate.

Understanding Yield To Maturity

Yield to maturity is useful because it is so well accepted in the market. It is worth analysing more carefully though to see what insight it can give us about bond value. Let's use our first example, which was a 4 year 10% annual coupon bond that was priced at 106.623, thus yielding 8%:

$$106.623^* = \frac{10}{1.08} + \frac{10}{1.08^2} + \frac{10}{1.08^3} + \frac{110}{1.08^4}$$

A criticism of yield to maturity is that it seems to imply that the yield curve is "flat" i.e. interest rates are the same for investments of any maturity. This does seem to be implied by the coupon bond pricing formula. However this criticism is not truly correct as we will see later when we discuss the yield curve in more detail. Still, it is true that the formula uses

*The exact value is 106.6243 if you use a bond calculator the same rate for each payment. We can gain some insight into the equation by looking at it in a different way:

$$106.623(1.08^4) = 10(1.08^3) + 10(1.08^2) + 10\ (1.08) + 110$$

Let's look at the two sides of the equation separately. The left hand side is the amount we would have in four years if we put 106.623 into a bank deposit that paid 8% interest on an annual-compounded basis. The right hand side has four terms. The first one, $10(1.08^3)$, is the amount we would have four years from now if we were able to invest the first coupon, which we will receive in one year, for three additional years in an account paying that same 8% interest. Similarly the next two terms represent the amount in an account four years from now if we are able to reinvest the coupons we receive at 8% from the time we receive them to the end of the four year period. The last term, 110, is simply the final payment from the bond with its last coupon. As these are received in four years time, no reinvestment of those cash flows is necessary.

So a way to understand yield to maturity is that buying a bond is equivalent to investing its price in a deposit that pays interest equal to the bond's yield to maturity, compounded with the same frequency as the bond's coupon is paid. In other words, we would be indifferent between buying the bond and investing its price in such an account. Note that this analysis indicates that the bond's yield to maturity is an implied reinvestment rate for its coupons. This makes clear that reinvestment risk is a significant feature of coupon bonds.

But we're not done yet! Everyone does talk about reinvestment of the coupons as part of the risk of the bond and we are not going to disagree with that here. But let's have one more look at that bank account – the one that pays 8% interest on an annual compounded basis. Suppose we do put the bond's price into that account and then each year withdraw 10 from it and reinvest what remains. Here are the cash flows*:

Table 2

Time	0	1	2	3	4
Amount	106.6243	115.1542	113.5666	111.8519	110
Withdraw	0	10	10	10	110
Remains		105.1542	103.5666	101.8579	0

What this says is that the bond's price placed in the account at 8% interest can exactly duplicate the bond's cash flows. Replicating one instrument with another is the foundation of arbitrage, a theme we will return to later.

Summary

We have seen three ways to understand yield to maturity:

1. The single interest rate that equates the bond's price to the sum of the present values of its cash flows.

---

*We used values correct to four decimal places to avoid rounding errors.

2. The assumed reinvestment rate of the bond's coupons.
3. The equivalent rate of interest on a bank deposit of the bond's price.

All of these different ways to understand the yield to maturity concept help to clarify the strengths of this measure as well as pointing to some of its weaknesses.

Bond Risk Measures

Return versus Yield

Sometimes these two terms are used interchangeably, but they are quite different. Yield, as we mentioned above, is a forward looking concept. You can see this from our discussion of yield to maturity. The yield to maturity of a bond can be viewed as the implied reinvestment rate of the bond's coupons. Return is retrospective i.e. a backward-looking concept. It applies to any type of investment not just bonds. The idea is simple. When you enter into an investment what really matters are the cash flows you get from it and what you do with them. For example if you buy a stock, you might want to look at it on some time horizon such as 5 years. Over that 5 year period as you hold the stock, you may receive cash flows in the form of dividends. You can either put them in a deposit or you could use them to purchase more shares. The decision you make will affect the performance of your investment. At the end of the 5 years you could look at the cash value of your position at that time and ask, what annual-compounded rate of interest would I need to have been paid on my original investment to have this final amount of cash? That interest rate would be your realised annualised rate of return.

Example

You buy a share for 100, which pays a dividend at the end of each year. When you receive a dividend you reinvest it in the shares. Your investment experience over the 5 year period is shown below:

Table 3

Time	0	1	2	3	4	5
Share price	100	110	105	95	125	135
Dividend	–	4	4	5	5	6
Number of Shares	1	1.03636	1.07446	1.12709	1.16709	1.21153
Cash Value	100	114.00000	112.81818	107.07359	145.88631	163.55721
Return*	–	14%	6.2%	2.3%	9.9%	10.3%

As the table shows, after 5 years the cash value of your position is 163.56, which is the cash value of the approximately 1.2 shares you own. You have earned an annual compounded 10.3% rate of return. Notice that the annual rate of return over the 5 year period was quite volatile. You earned 14% in your first year, but lost money in the second and third. Fortunately the stock price came back in years 4 and 5.

*Annual-compounded return

Exercise 2

Fill in the table again but assume that instead of reinvesting the dividend into the share, you deposit the cash amount of the dividend in an account that pays 5% annual interest.

Share price	100	110	105	95	125	135
Dividend	–	4	4	5	5	6
Number of Shares	1	1	1	1	1	1
Cash Value	100					
Return	–					

Returning to bonds, suppose that the current yield for 20 year bonds is 9% so that a 20 year bond with a 9% annual coupon would be priced at par.

Calculator Note

Before we get into our main discussion let's explain the use of the HP12C in doing return calculations. We already know that 9% is the implied reinvestment rate for the coupons on our bond, but if we are able to reinvest those coupons at 9%, how much cash will we have in 20 years? The HP12C can easily handle this calculation shown in Figure (7a):

Figure (7a)
20 n
0 PV
9 PMT
9 i
FV -> -460.44

Figure (7b)
20 n
-100 PV
0 PMT
560.44 FV
i -> 9%

Now if we add in the repayment of the bond principal, we will have 560.44 in total at the end of 20 years. So what rate of return did we earn? Figure (7b) shows the calculation. Remember we are asking, if we put the bond's price into a deposit that pays an annual compounded rate, what rate would we need to be paid to have the same amount in cash after 20 years? The unsurprising answer is 9%.

What if yields were suddenly to go up to 10% immediately after we bought the bond? Then the bond's price would drop to 91.49 (check this on your calculator). Conversely, if the 20 year rate were to go down to 8%, the bond's price would go up to 109.82. This illustrates the price risk of owning the bond, due to the change in yield. We can see this price risk in the pricing formula for the bond:

Figure 8

$$\text{Price} = \sum_{k=1}^{k=n} \frac{cF}{(1+y)^k} + \frac{F}{(1+y)^n}$$

A larger yield (y) will give a lower price, while a smaller yield will give a higher price. Recall once again that the yield is the implied reinvestment rate for the coupons. We verified above that if we are able to reinvest the coupons at 9% for our sample bond, then the realised return does equal the yield.

Suppose now that rates go to 8% and that we are only able to reinvest the bond's coupons at this reduced rate. How much cash will we have in 20 years and what will our realised rate of return be? Clearly the return should be less than 9% and the calculation shown below confirms this.

Figure 9

20 n	20 n
0 PV	-100 PV
9 PMT	0 PMT
8 i	FV 511.86
FV –> -411.86	i –> 8.51%

Similarly, if rates go to 10% and we are able to reinvest the coupons at that rate, we would expect that in 20 years, our realised return would be more than 9%. You should verify as an exercise that the return would, in fact, be 9.51%.

These calculations illustrate the reinvestment risk of a bond. Note that the two risks, price risk and reinvestment risk work in opposite directions. When yields go up, it is bad for the bond price but good for the reinvestment of the coupons. Conversely when rates go down, the bond price will increase but the total coupon reinvestment will be less.

Holding Period Return

In the previous section we considered what would happen to a bond and our rate of return under two interest rate and time scenarios. If rates change and we immediately sell the bond we profit if rates have gone down and lose if rates have gone up; if instead we hold the bond to maturity, the opposite is true. Think of these as two different holding periods for our investment, either 1 day or to maturity. What if we hold the bond for some intermediate amount of time? We will explore this idea in this section.

The amount of time from when we buy the bond to when we terminate the investment, either by selling it or because it matures, is called the holding period. We assume that while we hold the bond we will reinvest any coupons we get at whatever the current rate of interest happens to be. In order to conduct our investigation we have to make some simplifying assumptions. Let's not worry right now about how realistic these are. We will discuss that later. If we can understand how things work in a simple environment we can use it to extend our understanding to more realistic situations. At the very least we should understand the limitations of our analysis.

We will assume that the yield curve is flat. This means that all bonds have the same yield, irrespective of their maturity. We also assume that when the yield curve changes, it does so instantaneously and in a way that affects all maturities in the same way i.e., a parallel shift. Finally we assume that we buy the 20 year 9% bond for par and that instantly the yield curve moves to a new level and stays there for the entire holding period.

Table 4 below shows the realised rates of return for different holding periods for three interest rate levels.

Table 4

(a)

Rate	8%										
Holding period	0	2	4	6	8	10	12	14	16	18	20
Bond price	109.82	109.37	108.85	108.24	107.54	106.71	105.75	104.62	103.31	101.78	100.00
Reinvested coupons	0.00	18.72	40.56	66.02	95.73	130.38	170.79	217.93	272.92	337.05	411.86
Total value	109.82	128.09	149.41	174.27	203.27	237.09	276.54	322.56	376.23	438.84	511.86
Holding period return	–	13.18%	10.56%	9.70%	9.27%	9.02%	8.85%	8.72%	8.63%	8.56%	8.51%

(b)

Rate	9%										
Holding period	0	2	4	6	8	10	12	14	16	18	20
Bond price	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Reinvested coupons	0.00	18.81	41.16	67.71	99.26	136.74	181.27	234.17	297.03	371.71	460.44
Total value	100.00	118.81	141.16	167.71	199.26	236.74	281.27	334.17	397.03	471.71	560.44
Holding period return	–	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%

(c)

Rate	10%										
Holding period	0	2	4	6	8	10	12	14	16	18	20
Bond price	91.49	91.80	92.18	92.63	93.19	93.86	94.67	95.64	96.83	98.26	100.00
Reinvested coupons	0.00	18.90	41.77	69.44	102.92	143.44	192.46	251.77	323.55	410.39	515.47
Total value	91.49	110.70	133.95	162.07	196.11	237.29	287.12	347.42	420.38	508.66	615.47
Holding period return	–	5.21%	7.58%	8.38%	8.78%	9.03%	9.19%	9.30%	9.39%	9.46%	9.51%

As we can see in (a), when rates move down to 8%, we have a high rate of return if we have a short holding period. For example, if we hold the bond for 6 years we earn 9.70%. Conversely if rates move to 10% as in (c) we have a low rate of return if the holding period is short. For that same 6 year holding period the return would be just 8.38%. The most interesting thing in the table though is that if we have a ten year holding period, the realised return is just about the same irrespective of whether rates go down to 8%, up to 10% or stay the same at 9%. This observation leads us to the main topic of this section – duration.

Duration

Duration is a concept that is important for both bonds and swaps. We will see applications of duration in many parts of the world of fixed income and interest rates. We will look at duration in these ways:

- Price Risk versus Reinvestment Risk
- Geometric View
- As a weighted average of time

In the preceding example we saw that a 10 year holding period ”immunises” the bond owner to a sudden, sustained parallel shift in the yield curve. This is our first view of duration.

Price Risk Versus Reinvestment Risk

In the early years of a bond’s life price risk is dominant, while as the time the bond is held approaches maturity, reinvestment risk is dominant. Duration is the point in time where price risk is equal to and opposite in sign to reinvestment risk. That is, duration is the holding period for which the bond’s return does not depend on the level of interest rates. Naturally one has to be careful in using this idea. Duration is an instantaneous risk measure. As time goes on and interest rates do change so will duration, a feature we discuss below.

Geometric View

A simple physical analogy is often used to illustrate another way to understand duration. Imagine that we put weights along a plank (like a see-saw). Each point along the plank represents a point in time where we receive a cash flow from the bond, either coupon or principal. The weight is not the cash flow itself, but the present value of that cash flow. For simplicity we will assume that we discount all the cash flows using the bond’s yield to maturity. So at time t (expressed in years) we place a weight equal to:

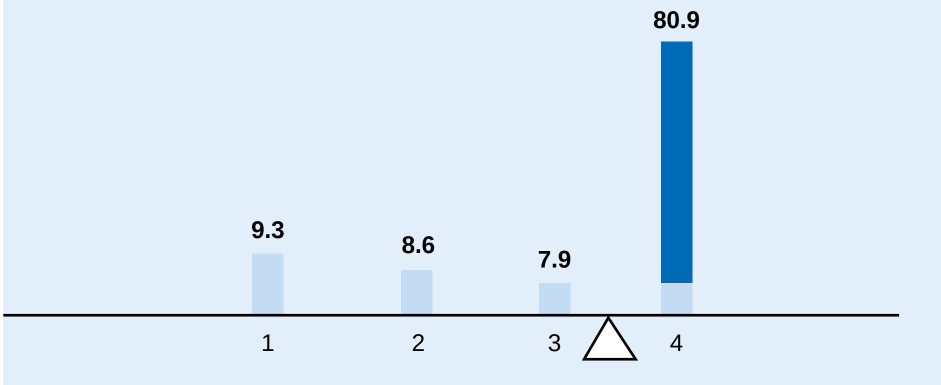
$$\frac{\text{Cash flow}_t}{(1 + y)^t}$$

where, as usual, y denotes the yield on the bond*.

Example

Consider a 4 year 10% annual coupon bond which is currently yielding 8%. The distribution of weights is shown below.

Figure 10



The point along the plank where we could put a fulcrum and have the whole system in balance is the bond’s duration. This turns out to be about 3.5 years. We will see in the next section how to calculate this number directly. For now we can verify that the ”moments of force” around that point are equal, which they must be to have the system balance.

* This form is correct for annual coupon bonds. For other bonds we would make the appropriate change to the denominator.

Weights to the left tend to make the plank rotate counter-clockwise, while weights to the right tend to make it rotate clockwise. The moment of force associated with a weight is the weight multiplied by its distance from the fulcrum. The calculation below shows that 3.5* is the correct point for the fulcrum.

$$9.3 \times 2.5 + 8.6 \times 1.5 + 7.9 \times 0.5 = 80.9 \times 0.5$$

An intuitive interpretation of this number is that the bond reacts to changes in yields as if it were a zero-coupon bond whose maturity is equal to the duration. This is similar to the physical reasoning that although the earth is really a large sphere, we can often treat it as if all its mass were concentrated at a single point. Since our 4 year 10% bond has a duration of 3.5 years, it should "act like" a zero coupon bond with that maturity. Let's illustrate what "act like" means.

Table 5

	3.5 year zero coupon bond	4 year 10% bond
Priced at 8%	76.38	106.62
Priced at 9%	73.96	103.24

Notice that when rates went from 8% to 9%, the zero coupon bond changed by:

$$\frac{76.38 - 73.96}{76.38} = 0.0317 = 3.17\%$$

The 4 year 10% bond changed by:

$$\frac{106.62 - 103.24}{106.62} = 0.0317 = 3.17\%$$

This illustrates that bonds with the same duration have the same percentage price sensitivity to rate changes.

The fulcrum analogy makes it easy to understand some important relationships between:

- Duration and Coupon
- Duration and Time to Maturity
- Duration and Yield Level

We will examine these one at a time.

Duration and Coupon

First consider a zero-coupon bond. Since there is only one cash flow, the only place to put a fulcrum is at the time to maturity of the bond. Thus the duration of a zero-coupon bond equals its time to maturity.

*3.503 is more accurate

Which would have a longer duration, the 4 year 10% bond we considered earlier or a 4 year 6% bond? Lowering the coupon makes the initial weights (present value of coupons) smaller. It also makes the final weight (present value of coupon plus principal) slightly smaller but the effect is dwarfed by the return of principal. In order to keep the system in balance we would need to move the fulcrum to the right (this decreases the moment of force from the final cash flow). So lowering the coupon lengthens the duration. Conversely if we increased the coupon to 12%, the three initial coupons would be larger and the fulcrum (duration) would move to the left (become less). Numerically the durations are shown below.

Table 5

Coupon	6%	10%	12%
Duration	3.66	3.51	3.44

Duration and Time to Maturity

What would happen if we changed our 4 year 10% bond to a 5 year 10% bond? Now there would be one more coupon (weight). Also the return of face value would move 1 year to the right. Since the final payment is so large, moving one year to the right will greatly increase its force (the force was 0.5 x 80.9, now it is 1.5 x 74.9). This will cause the fulcrum to move to the right i.e. as the time to maturity increases, so does the duration. The size of this effect on duration diminishes as the time to maturity continues to increase because the size of the weight gets smaller as we move it out in time. For example, if the bond has a 20 year maturity, the final payment of 110 would have a present value (at 8%) of only 23.6, which is less than the total present value of the first three coupons (25.77). This suggests that although the duration increases, the rate of increase lessens as we extend the maturity. Figure 11 illustrates this.

The black horizontal line is both a ceiling and a limit to the duration of a par or premium bond (coupon ≥ yield). The level at which the line is drawn is (1+y)/y, where y is the current yield level. This value is the duration of a so-called perpetual bond – one that pays a coupon indefinitely but never repays its principal. At 8% this would be 13.5 years. One useful observation suggested by the picture is that the difference in duration between two bonds with time to maturity 20 and 25 years is less than the difference in duration between two bonds with maturities of 10 and 15 years, assuming all the bonds have the same coupon and are yielding the same.

Bonds with coupons less than the current yield are called discount bonds because they will be priced at less than par. Discount bonds with short maturities tend to act like zero coupon bonds, but the resemblance diminishes as the maturity increases. Eventually even discount bonds will have durations that approach the theoretical limit. The full picture is shown overleaf.

Duration and Yield Level

What happens as the general level of yields changes? Recall that the 'ceiling' in our picture was at the level (1+y)/y, where y is the current level of yields.

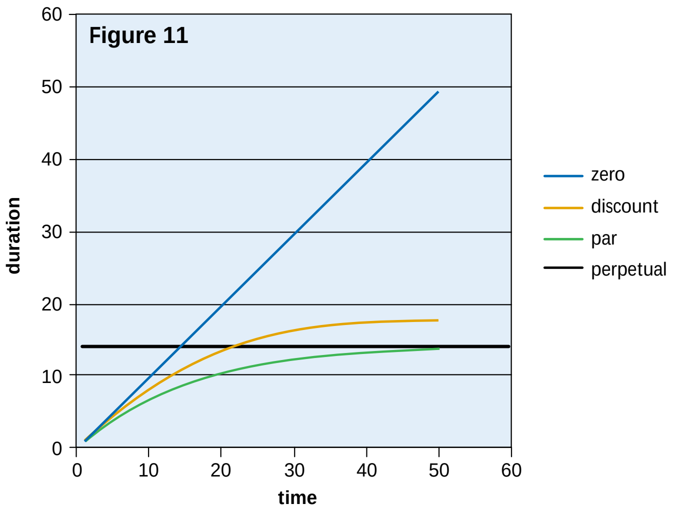
Yield level	6%	8%	10%
Duration limit	17.67 years	13.50 years	11.00 years

So as the level of rates goes down, the limit of duration of par and premium bonds goes up; conversely, as the level of yields goes up, the limit of the duration goes down. This makes sense because, at a lower level of yields, the cash flows are being discounted at a lower rate. This has a greater effect on payments that are farther out in time, giving them greater weight. This causes us to have to move the balancing point to the right i.e. to a larger number of years.

The table below summarises the relationships we have discussed:

Increase in	Coupon	Time to maturity	Level of yields
Effect on duration	decreases	increases*	decreases

Figure 11



We've postponed the mathematics of duration until a good understanding of duration was achieved. In order to do quantitative work though, we have to work with the math.

Weighted Average of Time

Duration is a weighted average of time. This is what we learned from the geometric picture of duration. The mathematics that goes along with the picture is:

Table 6

$$\text{Duration} = \frac{1}{P}\,\frac{cF}{1+y} \times 1 + \frac{1}{P}\,\frac{cF}{(1+y)^2} \times 2 + \cdots + \frac{1}{P}\,\frac{cF+F}{(1+y)^n} \times n$$

where F = face amount
c = coupon rate
n = time to maturity
y = yield
P = price

*There are some minor exceptions - please don't send us any e-mail about it.

The weight at time t is the proportion of the bond's price that is attributable to the present value of the payment at that time. By multiplying each time by the appropriate weight, we get a weighted average of time. This particular form of duration was proposed by Macauley and so is called Macauley's duration.

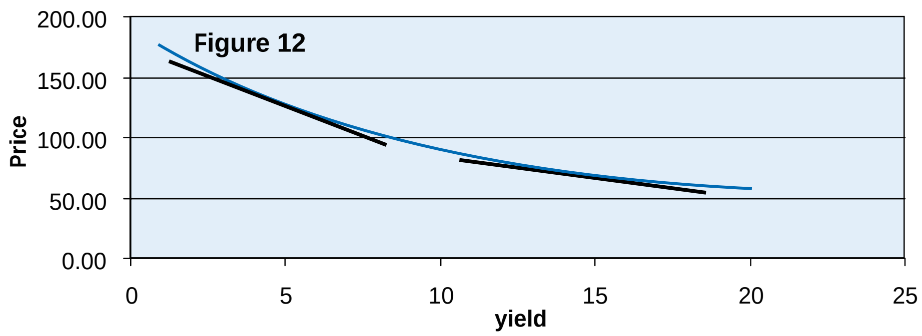
Price Sensitivity

Duration is often used to determine the price sensitivity of a bond that results from a change in yield. If we let ΔP stand for a change in price and Δy stand for a change in yield, we want a way to calculate ΔP/Δy. This expression is a rate of change. In calculus we learn that rates of change are calculated from derivatives i.e. ΔP/Δy is approximately equal to the derivative of price as a function of yield. Figure 12 illustrates geometrically the following mathematical relationship:

$$\frac{dP}{dy} = -\frac{1}{1+y} PD$$

where the left hand side is the rate of change of the bond's price as a function of the level of yield. The reason for the negative sign is that as yield increases, price decreases i.e., there is an inverse relationship between price and yield.

10 year 9% semi annual bond



Because of the presence of the 1/(1+y) factor, we are lead to modify the idea of duration and replace it by:

$$D_{mod} = \frac{1}{1+y} D$$

and call it modified duration. This gives a cleaner equation:

$$\frac{dP}{dy} = -PD_{mod}$$

How can we use this? As we mentioned, ΔP/Δy is approximately equal to dP/dy , so

$$\frac{\Delta P}{\Delta y} \approx -PD_{mod} \quad \text{or} \quad \Delta P \approx -PD_{mod}\,\Delta y$$

The equation above tells us what happens to price on the vertical axis when we change the yield on the horizontal axis. In Figure 12 the slope of the line tangent to the graph is $- PD_{mod}$.

Example

A 20 year annual coupon bond is yielding 8% and so is priced at par. How much will the bond's price decrease if the yield increases to 8.10%?

We used a bond calculator to find the modified duration of the bond. The result was 9.9 years. The change in the yield is 0.1% or 0.001 decimal form. So the approximate price change is:

$$\Delta P = - 100 \times 9.9 \times 0.001 = - 0.99$$

and the new price should be about 99.01. You can check this on the HP12C. The actual new price is 99.025, so the approximation is quite good.

PVBP

The most common measure used for price sensitivity is the price value of a basis point (PVBP). This measures how much a bond's price will change if its yield changes by 1 basis point. Since a basis point is 0.0001, it is simple to compute the PVBP:

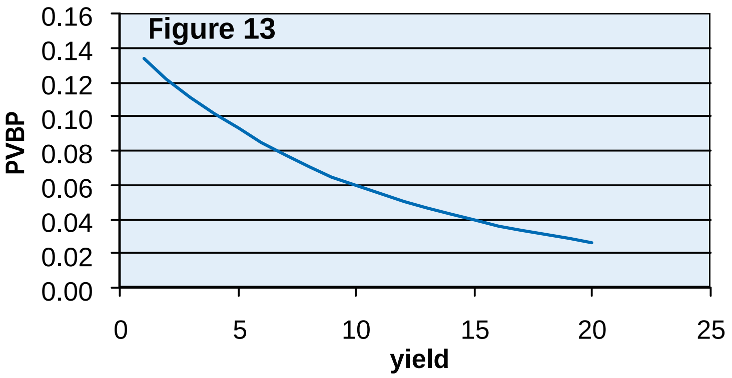
$$PVBP = PD_{mod} \times 0.0001$$

In the example above the PVBP is:

$$100 \times 9.9 \times 0.0001 = 0.099$$

Traders often refer to this as the bond's price delta. It is the rate at which the bond's price changes as yields change.

10 year 9% semi annual bond



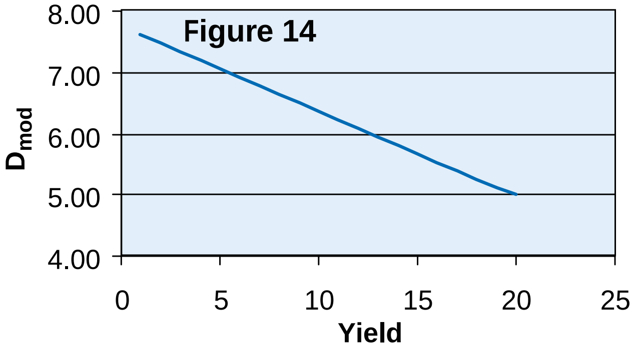
Convexity

A bond is more sensitive to yield changes when yields are low than when yields are high. This is illustrated in the next example. This feature is referred to as convexity.

Example

A 10 year semi-annual bond with a 9% coupon is priced at 122.32 to yield 6%. If yields change to 5.90%, the bond's price would become 123.17, so that it increases by 0.85. As a percentage of the bond's original price this increase is 0.69%. At high yields the bond is less sensitive to yield changes. If the same bond were yielding 10% so that its price was 93.77 and yields moved to 9.90% then the new price would be 94.37. This is a increase of 0.60 in absolute terms and 0.64% as a percentage of the price. In this case both the bond's percentage price sensitivity (duration) and its absolute price sensitivity (pvbp) are different at 10% than they are at 6%. These changes are called convexity and gamma, respectively:

10 year 9% semi annual bond



Convexity is the change in a bond's duration due to a change in yield.

$$\text{Convexity} = \frac{\partial D}{\partial y}$$

Gamma is the change in a bond's price delta (pvbp) due to a change in the bond's yield.

$$\Gamma = \frac{\partial \Delta}{\partial y}$$

When yields increase a bond's price decreases and its duration also decreases. This decrease in duration is a good thing for bond holders since they are losing value as yields rise. The fact that duration decreases as yields increase means that further increases in yield will cause less loss than changes at lower levels. Similarly, as yields increase the bond's price delta decreases (in absolute sense – it is a negative number). Because the delta is the product of the price and the modified duration, both of which decrease as yields go up, the bond's price delta is more affected than the duration. This is even better for the bond holder as the delta measures the absolute loss the investor is incurring with yield increases. The table below shows the prices and sensitivities of the bond in our example at different levels of yield.

Table 5

Yield	2	4	6	8	10	12	14	16	18	20
Macauley Duration	7.63	7.49	7.34	7.18	7.00	6.81	6.62	6.42	6.21	6.00
Modified Duration	7.48	7.20	6.92	6.64	6.36	6.08	5.81	5.53	5.26	5.00
pvbp	0.1220	0.1015	0.0847	0.0710	0.0597	0.0504	0.0427	0.0363	0.0310	0.0266

Worksheet – Bond Pricing, Duration, Convexity

The information below is from the Bloomberg® Screen (CT Govt) on 29 July 1993.

	Security				Market		
	coupon	maturity	bid	offer	ytm	durr	risk
2 year	$4^1/_4$	7/95	100-7	100-8	4.12	1.93	1.90

The current 2 year note, the $4^1/_4$s of 7/95 is offered at **$100^8/_{32}$.** Asked yield is **4.12%**

a. Present value each of the bond's cash flows at the stated yield. Add the present values to verify the bond price

Time (years)	0.5	1.0	1.5	2
Cash Flow				
Present Value				

b. The duration of the note is 1.93 years. Verify this.

Time (years)	0.5	1.0	1.5	2
Cash Flow pv				
Weight				
CF* x Weight				

The weight for any cash flow is the proportion of the bond's price that is represented by that cash flow:

$$\frac{1}{P} \quad \frac{CF^*}{(1+ r/2)^t}$$

Duration (Macauley) is the weighted average of time:

Macauley Duration=

Modified Duration is:

$$D_{mod} = \frac{1}{1 + r/2} D_{Macauley}$$

Modified Duration =

c. The **risk** of the bond is 1.90

Risk = amount the price will change if the bond's yield changes by 1 bp

Risk = pvbp

The pvbp can be calculated from D_{mod} and the bond's price:

Risk = 0.0001 x P x D_{mod}

Calculate: Risk =

d. On 29 July the current 2 year note was offered at 4.12% ($100^8/_{32}$);

* CF means cash flow

On 27 July the current 2 year note was offered at 4.32% ($99^{55}/_{64}$).

Actual price change:

Calculated price change(using pvbp):

Summary:

- **Duration** measures the **percentage** price **sensitivity** of a bond to changes in yield.
- **Convexity** measures the **change** in **duration** as yields change.
- **Delta** measures the bond's **price sensitivity** to changes in yield.
- **Gamma** measures the **change** in **delta** as yields change.

Section 3
Forward Rates and Eurodollar Futures

There are two important interest rate contracts that relate to deposits or loans that will take place at a future date. These are forward rate agreements (FRAs) and their exchange-traded equivalents, Euro interest rate futures, which we will refer to for convenience as Eurodollar futures even if they happen to be on some interest rate other than the US dollar. We will describe the idea of these instruments and then talk about how they work in practice.

Facts about Futures

- Futures are contracts traded on Exchanges such as:
  - LIFFE (London International Financial Futures Exchange)
  - CME (Chicago Mercantile Exchange)
  - IMM (International Monetary Market – part of CME)
  - SIMEX (Singapore Monetary Exchange)
- The underlying is a commodity, currency, interest rate, bond or index.
- The contract is for a standardised amount and quality.
- Only a small number of delivery (settlement) dates are available.
- Trades are done by Exchange members but the exchange itself acts as the counterparty to both sides.
- A combination of a long position and a short position in the same contract and held in the same account offset one another, resulting in no position.
- Traders are required to post (deposit) margin to guarantee their performance on the contract.
- Futures are marked to market daily with profits and losses realised immediately through the margin account.
- If the contract is settled into an actual underlying for which the "standard" grade is either non-existent or difficult to obtain or deliver (like live hogs or Treasury bonds), the party who is short the contract will have some choice as to what to deliver and the exact timing of delivery.

FRAs

When a bank's customer wants to arrange today to agree an interest rate that they can use (for borrowing or lending) at some future date, the customer can enter into a forward rate agreement (FRA) with the bank. The agreement would allow the customer to know today that they could, for example, borrow USD 10,000,000 at a rate of 5.50% for a 6 month period beginning 3 months from today. This would be referred to in the market as a 3x9 FRA, since the period of the loan begins in 3 months and ends in 9 months. The customer is said to have "bought" the FRA for reasons that will become clear later. The bank in this case has not agreed to actually lend any money to the customer. Rather the bank has agreed to compensate the customer if the borrowing rate in 3 months time should turn out to be more than the agreed upon 5.50%. In return the customer has taken on the opposite obligation. That is, if the rate in 3 months is less than 5.50%, the customer will be liable to make a payment to the bank. In this way the rate the customer wants has been "locked in".

Here are some typical FRA quotes:

Table 1

Term	Rate
1x4	5.00%
2x5	5.25%
3x6	5.45%
1x7	5.05%
2x8	5.30%
3x9	5.50%

The term of the FRA tells you when the interest rate period begins and when it ends. So the 2x5 FRA is a 3 month rate starting in 2 months; a 1x7 FRA is a 6 month rate beginning in 1 month.

How would a customer use an FRA? Suppose the customer needs, as we said above, to borrow USD10,000,000 for 6 months starting in 3 months. The customer approaches an investment bank and gets a quote of 5.50% for the 3x9 FRA. The customer buys the FRA on a "notional" amount of 10,000,000. The amount is called notional because, as we mentioned, the bank has not agreed to lend the customer any money. Suppose now that we have arrived at the maturity date of the FRA in 3 month's time. If the customer still needs to borrow money, he will go to his commercial bank or some other lender and arrange for a 6 month loan of USD10,000,000 at the prevailing market rate. For simplicity we assume the rate the customer receives from his lender is the same as the rate that is used to "settle" the FRA. Suppose that rate turns out to be 6.00% i.e. the 6 month rate has gone up. Then the customer will have to pay his lender 6.00% on his loan. To compensate him for this, the investment bank that sold him the FRA will make a payment to him of 0.50% i.e. 50 basis points on his $10,000,000 for the 6 month period of the loan. This payment will make up for the additional interest above 5.50% the customer will have to pay his lender.

Conversely if the market interest rate had moved in the customer's favour to 5.30%, say, then the customer will be able to obtain his loan at this more favourable rate. However the investment bank will expect a payment from the customer of 0.20% i.e. 20 basis points on the 6 month $10,000,000 "notional". This additional payment will result in the customer effectively paying 5.50% on his loan. So once he has traded the FRA, the rate of 5.50% has been determined. The customer has no downside or upside exposure.

Futures

Eurodollar futures are the exchange-traded equivalent of FRAs. However, they have more restrictions because of the need for standardisation of contracts on the futures exchange. The Euro interest rate futures are available only on 3 month interest rates (some 1 month rates are available in dollars only), while the FRAs can be on 3 month, 6 month or 1 year rates and even on periods of tailor-made length. Also the ED futures contracts have only 4 dates during the year on which they expire. The dates are in the months of March, June, September and December. These dates are meant to correspond to the beginning of the loan period that is being hedged. So if a customer has a loan that begins on a date that is not the expiry date of a futures contract, the customer either has to use an FRA or accept the "basis risk" of using a contract that does not exactly match his exposure. Futures do enjoy the advantage of greater liquidity, especially in the nearer maturity contracts and there is virtually no credit risk associated with these futures contracts.

Marking To Market

A fundamental feature of futures contracts, not just ED futures, is a process known as marking-to-market. This is a prudent measure on the part of the exchange to reduce, nearly to 0, the credit risk that the exchange takes on from its customers. It works in the following way. When two people trade a contract on the futures exchange, the exchange agrees to guarantee performance. Consequently, each day the exchange forces anyone who has sustained a loss on the futures contract to realise the loss by making a payment to the exchange. The exchange then passes this payment on to those people who have the corresponding gain on the contract. Because of this, there is no need at the end of the contract for the kind of settling up that occurs with FRAs. On the last day the exchange just passes along the final day's mark to market. At that point everyone with a position has realised through the daily accumulation of the marks to the market, the full value of their gain or loss on the contract.

In order to illustrate this we have to take note of a peculiarity of the ED futures contract. The price of the contract is quoted as (100 – interest rate). So when the future is priced at 94.50, the underlying three month interest rate is 5.50%. Suppose then that a Corporate Treasurer sells 10 June Eurodollar futures contracts at a price of 94.50. The corresponding rate is 5.50% and the notional underlying of each of the 10 contracts is $1,000,000. The Treasurer has done this because she needs to borrow money for 3 months at the time that the futures contract expires in June. She hopes to lock in a rate of 5.50% on that loan. Suppose that on the day the Treasurer sells the contract, the US Federal Reserve raises short term interest rates and the futures market responds by selling the June ED future down to 94.00. The corresponding rate is 6.00% and reflects the markets expectation, based on the Fed action, of what the 3 month rate will be in June. The Treasurer has a gain of 50 basis points on the contract because she had a short position. The price decrease is a profit to her. The next day the exchange will credit her "margin" account with the profit she made on the 50 basis point mark to market.

Margin

The margin account is a deposit with the exchange of a sum of money, or marketable securities, that assures the exchange that the customer will make good on any losses from the mark to market. You can contrast this to what happens if you buy shares of equity. If you pay £100 for the shares and the price decreases so that your shares are only worth

£90, your loss of £10 is real but not realised unless you sell the shares. This is an important distinction not just psychologically but also possibly from a tax point of view, since it applies equally well to profits. In the futures markets all gains and losses are realised when they occur. The amount of margin the exchange requires depends on the size of the contract and its volatility. The size of the contract determines the monetary value of a price move and the volatility determines the likely amount the price will move. The exchange reserves the right to increase or decrease the amount it requires for margin.

FRA and Future Settlement

Settlement of futures is actually easier to describe. Because the underlying notional is $1,000,000 and the interest rate is a 3 month rate, the exchange has specified that each basis point change in the ED futures contract is worth $25. This can be rationalised as the value of 1 basis point on $1,000,000 for one quarter of a year. Note that one basis point is 0.01%, which is represented as a decimal by 0.0001.

$$\text{Basis Point} = 0.0001 \times 1000000 \times 1/4 = 25$$

Example

The Treasurer sold 10 futures at a price of 94.50 and the price subsequently closed for the day at 94.00. So the day's mark-to-market is:

10 contracts x 50 basis points x $25 per basis point = $12,500

The Treasurer would receive this into her margin account the next day. The payment does not necessarily come from the party that bought the contract from the Treasurer. That person may well have closed out the position by selling the contract later in the day. Instead the exchange collects the losses from everyone who is long the contract at the end of the trading day and passes out the required amounts to the people who were short. If for any reason someone who was long the contract, and therefore had a loss, did not have enough money in their margin account to pay their loss, the exchange would issue an immediate call for the loser to pay up and bring their margin account back to its full initial level. Failure to do so would result in the exchange closing out the contract at the current price and initiating legal action to recover the loss from the defaulting parties clearing firm. The clearing firm is an organisation that guarantees the trades of its members. This structure protects the exchange from losses.

Exercise

Indicate the daily mark to market for the following trade

Note: EDM1 is the exchange symbol for the **E**uro**D**ollar futures contract which expires in June (**M**) of 200(**1**)

Customer buys 20 EDM1 contracts at a trade price of 94.00 on day 0

Day	Settlement Price	Mark to Market
0	94.10	
1	94.15	
2	94.05	
3	93.90	
4	94.00	

Suppose the customer closes out the position by selling the 20 contracts on day 4 at a price of 94.00. What is the customer's accumulated mark-to-market?

FRA Settlement

FRAs are settled differently from futures because they do not have a mark to market. Therefore gains or losses accumulate over the length of the contract and have to be paid out when the contract is settled. So now suppose that our Treasurer, instead of selling ED futures, had decided to use FRAs as a hedge. Because the Treasurer is exposed to rates going up, she needs to have a position that will have a profit when this happens. The correct action then would have been to buy the 3x6 month FRA that expired at the time that the ED future expired. The underlying notional would be $10,000,000. On the trade date, suppose the Treasurer is able to buy the FRA at a rate of 5.50%. Suppose further that on the day the FRA expires, the 3 month rate is 6.50%, 100 basis points higher than when the Treasurer bought it. Then the Treasurer has a profit of 100 basis points on $10,000,000 for a 3 month period. The value of a basis point is not so simple as for futures. First of all, FRAs use an actual day count i.e. the actual number of days in the 3 month period covered by the FRA. Let us suppose for simplicity that this is 90 days exactly. Even then there is still a complication. Although each basis point is worth $25 just like with the future, FRA settlement is discounted by the current interest rate. So the actual amount the Treasurer would receive is:

$$\text{FRA payment} = \frac{100 \text{ basis points x } 10 \text{ contracts x } \$25 \text{ per basis point}}{1 + 0.065/4} = \$24,600.25$$

Once again we can rationalise the result. The Treasurer will have to pay the higher rate of 6.50% to borrow the $10,000,000. But the additional interest will not have to be paid out until the loan matures in 3 months time. Since she receives the FRA payment now rather than later, she can either reduce the amount she borrows or she can place the payment in an account that bears interest at the 6.50% rate. Either way she has been correctly compensated for the increase in the interest rate. Futures contracts do not use this method because of the daily mark to the market and because the need for simplicity, i.e. $25 per basis point, is paramount on the exchange.

Exercise

In each case indicate the amount of payment due on the FRA and who pays. Assume that 3 month FRAs cover exactly 90 days and 6 month FRAs cover exactly 180 days. The first example is done for you. Note that a basis point on a 3 month rate is worth $25 per million, but a basis point on a 6 month rate is worth $50.

Type of FRA	Price Traded	Customer notional (millions)	Rate at Settlement	Payment	From
2x5	6.00%	Sells 10	6.25%	6153.85	Cust
3x6	6.10%	Sells 20	6.15%		
2x8	8.00%	Buys 30	7.75%		
3x9	7.50%	Buys 25	7.80%		

Forward Rates

Because of the actions of the players in the interest rate markets – which includes hedgers such as commercial banks and corporate treasurers as well as money market fund managers and speculators such as hedge funds and investment banks – the FRAs and futures represent the consensus of the market as to what the future interest rates will be. So if the March 2002 ED futures contract trades at a price of 93.25, it is because the market view is that the 3 month interest rate in March 2002 will be 6.75%. These contracts are liquid and easy to trade. Their prices quickly adapt to any changes in market variables and market sentiment. So people concerned with the future of interest rates are constantly watching the ”forward curve” which consists of the rates implied by the various ED futures contracts and the FRAs.

Eurodollar Futures

Contract	Eurodollar Future
Traded:	IMM, LIFFE, SIMEX
Underlying:	$1,000,000 90 day time deposit
Delivery Months:	March(H), June(M), September(U), December(Z)
Settlement:	Cash, determined by 3-month LIBOR on expiry
Quotation:	100-Interest Rate
Minimum Price Change:	1 basis point, value = $25, referred to as a ”tick”

Euro interest rate futures contracts trade on the interest rates of many different countries. They are among the most successful financial futures contracts ever devised. Listed below are some of these contracts and the underlying notional amount for each. All are contracts on 3 month interest rates, except as noted, and are settled against the 3 month LIBOR rate prevailing at the time the contract expires.

Currency	Notional	Contracts
USD	USD1m	10 years of 3 month futures
	USD3m	2 months of 1 month futures
EUR	EUR1m	5 years i.e. 20 contracts
GBP	GBP500k	18 months
JPY	JPY100m	18 months
MXN*	MXN2m	12 months

* Underlying is a 91 day Treasury Bill

There are futures on a number of other currencies also.

Answers to Exercises

Exercise

In each case below specify which party will make a payment on the FRA and how much that payment will be. The first one is done for you.

Type of FRA	Price Traded	Customer	Rate at Settlement	Payment	From
2x5	6.00%	Sells	6.25%	**0.25%**	**Cust**
3x6	6.10%	Sells	6.15%	**0.15%**	**Cust**
2x8	8.00%	Buys	7.75%	**0.25%**	**Cust**
3x9	7.50%	Buys	7.80%	**0.30%**	**Cust**

Exercise

Indicate the daily mark to market for the following trade

Customer buys 20 EDM7 contracts at a trade price of 94.00 on day 0

Day	Settlement Price	Mark to Market
0	94.10	**+$5000**
1	94.15	**+$2500**
2	94.05	**-$5000**
3	93.90	**-$7500**
4	94.00	**+$5000**

Suppose the customer closes out the position by selling the 20 contracts on day 4 at a price of 94.00.
What is the customer's accumulated mark-to-market?

The customer's accumulated mark to the market is 0. This makes sense since the contract was closed out at the same price as it was initiated.

Exercise

In each case indicate the amount of payment due on the FRA and who pays. Assume that 3 month FRAs cover exactly 90 days and 6 month FRAs cover exactly 180 days. The first example is done for you. Note that a basis point on a 3 month rate is worth $25 per million, but a basis point on a 6 month rate is worth $50.

Type of FRA	Price Traded	Customer notional (millions)	Rate at Settlement	Payment	From
2x5	6.00%	Sells 10	6.25%	**6153.85**	**Cust**
3x6	6.10%	Sells 20	6.15%	**2462.14**	**Cust**
2x8	8.00%	Buys 30	7.75%	**36101.08**	**Cust**
3x9	7.50%	Buys 25	7.80%	**36092.40**	**Bank**

Bond Futures

The futures contract that trades on the Chicago Board of Trade (CBOT) is another successful financial futures contract. Having said that it is also a somewhat complicated contract. The complication arises because the contract is written on a "nominal" bond. The reason for this is that there are a great many US Treasury bonds but they differ with respect to their coupon rate and their time to maturity. The CBOT could have made a contract that was either cash-settled or was marked to a bond index but it did not do so. Rather, a person who is short the contract during the delivery month is given the choice of a number of different bonds to deliver to fulfil the contract. The person who is short the contract can also decide on which day of the delivery month to make delivery. As with all futures contracts, anyone who is long or short the contract can trade out of it by reversing their position. However with the bond futures contract there is a day in delivery month where the contract ceases to trade but there remain several days after the last day of trading to month end. During that time someone who is short will have to make delivery since they can not reverse the contract any more.

Sound complicated? We've only just begun! Because there is just one futures price but there are many bonds that could be delivered there must be a way to adjust the amount that the person delivering the bond will be paid. We will now describe as briefly as possible how that works.

The nominal bond that the contract is based on is (since March 2000) a 6% semi-annual coupon US Treasury bond. There are four futures delivery months, as with many of the exchange traded futures – March, June, September and December. The "near" month contract tends to be the most liquid until delivery month approaches, then the next expiring contract becomes more liquid. For example in January, the March contract will be the most liquid future but toward the end of February, the June contract will take its place.

The bond futures contract is used by the usual suspects – bond portfolio managers wanting to hedge their positions, banks wanting to position themselves on interest rate changes, investors who sometimes find the futures market more liquid, more responsive or more convenient than the cash market and others trying to arbitrage cash bonds versus futures. We will talk shortly about the opportunity that portfolio managers or pension funds who are naturally long bonds sometimes have to pick up yield by selling their cash bonds and replacing them with futures.

Back to the Future

What would the price of an 8% 20 year semi-annual coupon bond be if its yield were 6%? If you use your bond calculator you will find the answer is 123.11 or 1.2311 for each $1 of face value. You now know, roughly, how to calculate the "conversion factor" for delivering a cash bond in fulfilment of the futures contract. More specifically, to calculate the conversion factor:

Determine the time to maturity of the bond measured from the first day of the delivery month, then

- Round the time to maturity down to the nearest quarter of a year, then
- Calculate the price based on a 6% yield.

Alternatively:

- Look up the conversion factor on Bloomberg®.

Example

Calculate the conversion factor for the 8s of November 2021 if delivered against the March 2000 bond future.

> Time to maturity from 1 March 2000 is 21 years 8 months and 14 days
> Rounded down to 21 years 6 months
> On the HP 12C:
> n = 43 (number of coupons to be paid)
> PMT = 4
> FV = 100
> i = 3
> PV = -123.98

So the conversion factor is 1.2398.

The closing price for the March 2000 Bond future on 14 February 2000 was 94-04 (which means 94 $^4/_{32}$ = 94.125). Since it is not yet delivery month, it is not possible to deliver the 8s of November 2021 against this future. However we can calculate the ”invoice price”, which is the price that the person delivering the bond would receive if it could be delivered today*:

Invoice price = Future x Conversion Factor = 94.125 x 1.23982 = 116.70

The closing cash price of the bond on that day was 117 $^8/_{32}$. There are two main reasons for the difference between the cash price and the invoice price. The first is that there are two weeks to go before delivery can occur and about 6 weeks to the last day on which delivery can occur. During this time if you have a long position in the cash bond, financed in the repo market, you may enjoy ”positive carry”, meaning that the coupon interest that you earn by owning the bond outweighs the interest cost it takes to finance it. The second reason is that a short futures position has embedded options that allow for the person who is short the future to make a profit under some circumstances. Because of these options, the future tends to trade at a lower price than it otherwise might.

As mentioned above, the fact that the bond future trades below what might be considered its ”fair” value if there were no optionality to it, presents a possible opportunity for funds

---

*The person delivering the bond would also be paid any accrued interest on the bond.

that have long bond positions. Suppose for example that a portfolio manager or pension fund owned the 8s of November 2021. The manager could sell the bond in the market for its cash price of 117.25 and replace it by buying futures. At expiry, the fund manager could either sell the future or take delivery of a bond. If the options in the future have expired worthless, then the manager will pick up several basis points in yield. But this is not in any way riskless! If it does work out, then the manager has just been compensated for having sold the options. It could just as well turn out that the options work against the manager's position and a loss results. Still a manager with a view on interest rates may decide it is worth the risk. We will not go into this topic any further but will leave it for a more advanced treatment.

Section 4
Interest Rate Swaps

Interest rate swaps are probably the most useful of all financial instruments. Although seemingly simple on the surface, they have so much flexibility that they turn up in an astonishing number and variety of applications. We will begin with a simple description.

In an interest rate swap, two parties agree to exchange the interest payments that are due on an underlying loan of the same size. The size of the loan is called the ”notional” amount because, as with an FRA, there is no money actually being lent by either party to the other. Now there would be no point to the parties exchanging payments if there were no potential for those payments to be different. In fact, one of the parties always pays at the same rate throughout the period of the swap. This party is called, naturally, the fixed rate payer. The other party’s obligation under the swap is to make an interest payment that is determined by some observable market interest rate, which over the period of the swap, may change from one reset to the next. The term reset refers to the process by which the rate is observed and a new value established. It is common for interest rate swaps to reset to the LIBOR rate for the currency of the swap. Thus in a swap, we observe the so-called floating interest rate at regular intervals such as three months or six months, called the tenor of the swap. If the floating rate is greater than the fixed rate, then the floating rate payer, who is usually referred to instead as the fixed rate receiver, will need to make a payment to the fixed rate payer. Of course the opposite happens when the floating rate is less than the fixed rate. This process continues for the life, or maturity, of the swap. So here are the specifications of a typical swap:

Fixed Rate Payer	UBS
Fixed Rate Receiver	GoodCorp
Notional Amount	£100 million
Tenor	6 months
Maturity	5 years
Effective Date	15/04/1997
Maturity Date	15/04/2002
Floating Rate	6 month GBP LIBOR
Fixed Rate	6.50%

This swap involves 10 payments with one payment occurring every six months. The last of these payments would take place on the maturity date, 15 April 2002. The first payment would be determined at the first observation date, (11 April 1997, which is two business days prior to 15 April 1997), but would not be paid until six months later, 15 October 1997. Note that these dates would be modified if they do not happen to be a business day in the UK. We will not concern ourselves with these details here.

Example

On 11 April 1997 the six month GBP LIBOR rate is observed to be 5.25%. The amounts due on the swap are:

From UBS	100,000,000 x 0.065/2	= £3,250,000
From GoodCorp	100,000,000 x 0.0525/2	= £2,625,000

(For the detail conscious – the floating rate would probably be on an actual/365 day basis, but we are NOT going to be concerned with this.)
It would be pointless for GoodCorp to pay UBS and for UBS to simultaneously pay GoodCorp, so the standard documentation that governs swap agreements would specify that UBS should, in this case, make a payment of the net amount, £625,000 to GoodCorp. This payment would take place on 15 October 1997, the end of the period i.e. swaps are settled in arrears.

A final important note on this example is that on the maturity date 15 April 2002 the only payment due is the last net interest payment. The notional amount of £100,000,000 is not involved in the swap other than as the basis for the interest calculations.

Exercise

Here are the 6 month forward rates for the next five reset dates, based on the Eurosterling interest rate futures at the time the swap was priced*:

	4/97	**10/97**	**4/98**	**10/98**	**4/99**
rate	5.25%	5.45%	6.00%	6.30%	6.50%

If the 6 month rate turns out to be exactly what is being ''predicted" by today's forward rates, what will be the swap payments associated with these resets?

What inspires swap agreements? In this instance it may be that GoodCorp has issued a 5 year bond in the Euro market but in reality prefers to borrow funds at a floating rate. This may be because they have a matching floating rate asset or simply because they want to keep the duration of their liabilities low (you learned about the duration concept in the section on bonds). So if they preferred floating rate funds, why didn't they just borrow the money from a bank or issue a bond with a coupon that resets periodically – a floating rate note? This is a complicated question, or rather a simple question with a complicated answer. Borrowers want to pay as little as possible for the funds they borrow. The job of an investment bank is to help the borrower find the most efficient i.e. lowest cost, way of borrowing money. In order to do this the bank will look in every interest rate market and at every possible way of raising the funds. Based on its research and experience it will recommend a particular solution to the borrower. Here UBS may have found an "arbitrage" opportunity for GoodCorp whereby they can raise floating rate funds more cheaply indirectly through the swap market than they could have by borrowing at a floating rate directly.

---

* See the section on the yield curve for details of how forward rates are determined in the market.

You will have noticed in the exercise you did above that UBS was always paying out to the counterparty. This does not seem like a good deal for the bank. Why are we willing to do this? It must be that we expect at some point to ”catch up”. In fact you probably noticed that the payments we expected to make became less and less as time went forward. If we were able to see the rest of the forward curve i.e. the next 5 forward rates, it would indicate that we expected to begin receiving payments from GoodCorp. If we were then to calculate the present value of all the payments, the ones we paid out and the ones we received, we would find that they exactly balance out. This is the basic principle of swap valuation:

If an interest rate swap is properly priced at market rates, the net present value of the expected cash flows is 0.

As mentioned above, interest rate swaps are an important tool in financial markets. They have become completely commoditised in the less than 20 years of their existence. All major investment banks will quote swaps to their customers and to other investment banks, credit limits permitting, in good size and at a tight bid-offer spread in most major currencies. A typical swap quote would look like:

6.50 | 6.55

for a 5 year swap versus 6 month GBP LIBOR. The meaning of the quote is that the market-maker is ready to enter into a 5 year swap as the payer of the fixed rate of 6.50% versus receiving 6 month GBP LIBOR. If the customer wants to pay the fixed rate, then the bank requires a spread of 5 basis points i.e. needs to receive fixed at 6.55%. As in any market, the bid-offer spread is a measure of the risk the market-maker takes on by being willing to enter into a trade as either the buyer or seller.

We can illustrate now why GoodCorp might be able to use the swap market to borrow money more cheaply. Suppose that GoodCorp is able to issue a 5 year bond in the Eurobond market with a 6.40% coupon. Simultaneous with the bond issue, GoodCorp enters into the swap with UBS in which it receives fixed at 6.50% (that is the bid in the swap market). Each time a coupon payment is due, GoodCorp will receive the 6.50% payment from UBS and pass on just 6.40% to its bondholders. In return GoodCorp will have to pay GBP LIBOR to UBS. The net result of this is that GoodCorp pays a floating rate of 6 month GBP LIBOR minus 10 basis points. This sort of sub-LIBOR funding is the holy grail of the bond market. The job of investment banks like UBS is to find opportunities like this one for their clients.

Exercise

Here are some swap quotes:

Maturity	Tenor	Currency	Quote
5 yrs	6 month	EUR	5.25|5.30
5 yrs	6 month	USD	6.38|6.40
10 yrs	3 month	GBP	7.00|7.15

Suppose that SingleBCorp can borrow 5 year USD from a bank if it pays 6 month USD LIBOR plus 50 basis points or it can issue a 5 year bond with a coupon of 7.20%. SingleBCorp would really rather borrow $50,000,000 at a fixed rate but it thinks that the 7.20% coupon is too high. How could the swap market help them?

Swap Documentation

The International Swap Dealers Association (ISDA) has written standard documentation for interest rate (and currency) swaps that is in wide use. The documentation covers netting, swap terminology, etc. This cuts down on potential misunderstandings and duplicate paperwork.

Swap Quotes

Dollar swaps are often quoted as a spread to the on-the-run Treasury bond with the same maturity as the swap. For example, a 7 year semi-annual pay swap versus 6 month LIBOR might be quoted by a market-maker as:

$$T + 55 \mid T + 58$$

55 and 58 are the bid and offer swap spreads in terms of basis points of yield. Suppose that the 7 year Treasury is currently yielding 6.50%. This means that the marketmaker whose quote this is will enter into a swap as the payer of fixed and will pay:

$$6.50 + 0.55 = 7.05\%$$

Or will enter into a swap as the payer of the floating rate and receive:

$$6.50 + 0.58 = 7.08\%$$

A 3 basis point spread on a 7 year swap is fairly tight but realistic. This is a measure of how efficient, liquid, and commoditised the swap market has become.

In other currencies where the underlying government bond market is not so liquid or well-developed, the swap may be quoted as a pure yield. For example, a 5 year Swiss franc swap might simply be quoted as 5.35 | 5.39. One advantage to the T + spread quote is that it focuses on the swap spread which reflects credit quality versus the U.S. Treasury (among other things) and therefore isolates an important market variable.

Swap Calculations

The fixed side of the swap may be paid in a number of ways. The most common way is 30/360 i.e., each month is assumed to have 30 days and each year to have 360 days. Sometimes the fixed side is paid on an actual/365 or actual/actual basis. We are not concerned with the details here so we'll just assume 30/360. For a semi-annual pay swap, this will mean that the fixed side will pay one half of its coupon on each pay date. The floating-rate side will usually be paid on an actual/365 or actual/360 basis, like an ordinary simple interest rate.

Example:

UBS has entered into the following swap as the payer of fixed:

Notional Principal	$50,000,000
Maturity	5 years
Floating Rate	6 month LIBOR
Fixed Rate	8%

Suppose that today is April 15, 1999, the day on which the swap becomes effective (settlement date). No payment is due today but we observed the current value of 6 month LIBOR two business days ago. Suppose this happened to be 6.50%. The first swap payment is due in 6 months, on 15 October 1999. The number of days between the two dates (15/04/99 and 15/10/99) is 183. The fixed rate payment is just:

$$50{,}000{,}000 \times 0.04 = 2{,}000{,}000$$

while the floating payment is:

$$50{,}000{,}000 \times 0.0650 \times \frac{183}{360} = 1{,}652{,}083.33$$

In this case, the fixed rate payer (UBS) would pay the difference of $347,916.67. This is typical at the beginning of a swap since short term rates tend to be lower than long term rates in a "normal" interest rate environment.

Swap Offsets

We have already noted that the periodic payments on the swap are customarily netted to decrease the risk at payment time. In addition, a bank will frequently sign a master agreement with its major swap counterparties that governs the entirety of the swap positions between them. The agreement will include the netting provision for individual payments as well as a provision as to what will happen in the case of a default. Commonly, if a counterparty defaults, then the bank would want to declare all of its swap positions with that counterparty to be in default. The swaps would then be individually valued and the net value of the swap book would have to be paid by one party to the other. We'll talk about valuing swaps later. For now, just suppose that UBS has entered into 3 swaps with Bob's Bank of Brooklyn. Bob's Bank goes into default by failing to make a payment due on a swap with UBS (other events can trigger a default; there does not have to be failure on a payment). At the time of the default, these are the swaps between the parties and their remaining time to maturity:

	UBS	Bob's Bank
Swap 1	Pays 8% $10,000,000 notional 5 years maturity	Pays 6 month LIBOR
Swap 2	Pays 6 month LIBOR $50,000,000 notional 2 years to maturity	Pays 7%
Swap 3	Pays 3 month CP – 150bp $20,000,000 notional 4 years to maturity	Pays 6.1%

These swaps were entered into at various times in the past, so the fixed payments reflect the interest rate environment at the time the swap was established. Today these swaps are valued relative to current market rates. This is done similarly to valuing a bond. Suppose the value of the swaps today, from UBS's point of view, are:

Swap 1	-250,565
Swap 2	189,422
Swap 3	75,189

This means that Swap 1 is unfavourable to UBS. Apparently the current 5 year swap coupon is less than 8%. Conversely, Swaps 2 and 3 are in UBS‘s favour. The total value of the swap portfolio is $14,046. UBS can declare, according to its master agreement, that all three swaps are immediately terminated. UBS could then go into the swap market and replace Bob‘s Bank with other counterparties at today's market rates. Neglecting "friction" such as bid-ask spreads and transaction costs, this would have a net cost to the bank of about $14,000. UBS now becomes a creditor of Bob‘s Bank for that amount. Where UBS stands in relation to other creditors depends on the master agreement and local laws. UBS may have to write off the $14,000, but it does not have any further exposure to Bob. Banks without netting agreements have to worry about "cherry-picking". What this means is that if Bob should go into bankruptcy, the receiver could look at the three swaps and decide that Swap 1 is an asset and demand payment of its value from UBS, but that Swaps 2 and 3 are debts that Bob cannot afford to pay. In that case, UBS would become a creditor with an exposure of $264,611 and would also have to pay $250,565!

Swap Variations

There are a number of common variations to the "plain-vanilla" fixed-versus-floating swap. Here is a representative sample:

Amortising Swap — the notional principal decreases (amortises) at each reset date. This could be used to match a real estate mortgage, for example. If the notional principal increases rather than decreases, then the swap is accreting.

Roller-coaster Swap — the notional principal varies both up and down over the life of the swap. This could be used to match funding needs that have a seasonal component.

LIBOR set in arrears — as noted above, the floating payment due at the end of a period is usually determined by observing the floating rate at the beginning of the period. In this type of swap, the LIBOR payment is determined at the end of the period by observing LIBOR one or two business days before the payment date.

Basis Swap both sides of the swap may be indexed to a floating rate e.g. 6 month LIBOR versus 3 month LIBOR or versus 6 month CP. In the case of 6 month LIBOR versus 3 month LIBOR, one side makes payments every three months.

Some of these variations have good practical uses related to matching the hedging needs of customers. Others are risk management tools used by banks that run a large swap book. For example, if UBS pays fixed versus 6 month LIBOR with one customer and receives fixed versus 6 month CP with another, then it is effectively paying CP and receiving LIBOR. A basis swap would get the bank out of the risk that the spread between these two rates might change.

There are endless variations on the swap theme, but you now have a good idea of swap basics.

Answers to Exercises

Exercise

Here are the forward rates for the next five reset dates, based on the Eurosterling interest rate futures at the time the swap was priced:

	4/97	**10/97**	**4/98**	**10/98**	**4/99**
rate	5.25%	5.45%	6.00%	6.30%	6.50%

If the 6 month rate turns out to be exactly what is being "predicted" by today's forward rates, what will be the swap payments associated with these resets?

	4/97	**10/97**	**4/98**	**10/98**	**4/99**
UBS	pays 625000	pays 525000	pays 250000	pays 100000	pays 0

Exercise

Here are some swap quotes:

Maturity	Tenor	Currency	Quote
5 yrs	6 month	EUR	5.25|5.30
5 yrs	6 month	USD	6.38|6.40
10 yrs	3 month	GBP	7.00|7.15

Suppose that SingleBCorp can borrow 5 year USD from a bank if it pays 6 month USD LIBOR plus 50 basis points or it can issue a 5 year bond with a coupon of 7.20%. SingleBCorp would really rather borrow $50,000,000 at a fixed rate but it thinks that the 7.20% coupon is too high. How could the swap market help them?

SingleBCorp should borrow 5 year USD from its bank and agree to pay USD LIBOR + 0.50%. Then it should enter into an interest rate swap, with UBS perhaps, as follows:

Notional	**$50000000**
Maturity	**5 years**
Tenor	**6 months**
Floating Rate	**USD LIBOR**
SingleBCorp	**pays 6.40%**
UBS	**pays USD LIBOR**

At each coupon date, SingleBCorp will:

pay	**LIBOR + 0.50% to its bank**
pay	**6.40% to UBS**
receive	**USD LIBOR from UBS**
net pays	**6.90%**

In this way SingleBCorp has saved 30 basis points on its borrowing, compared to issuing a five year bond.

Section 5
Building a Yield Curve

In the last two sections we discussed a lot of different types of rates and bonds, but how can these be observed in the market? In this section we will discuss the rates from the cash market, while in a later section we will take up the rates that we observe in the derivatives market.

The US Treasury Market

The US Treasury borrows money by selling securities, which are considered free of default risk by investors. The US Treasury only borrows in USD. Because of the strong economic conditions in the US that prevailed starting somewhere around 1991, the US Treasury has made some changes in the auction schedule for its debt, but in general it auctions off (sells) Treasury Bills, Treasury Notes and Treasury Bonds. We have mentioned these previously, but it is useful to review them in the context of yield-curve building.

Treasury Bills

These are discount securities i.e., are sold at less than their face value and then pay off their entire face value at maturity. The terms of T-Bills are 13 weeks, 26 weeks and 1 year. They are priced on an a/360, discount basis. There is a T-Bill auction every week but not every maturity is offered every week. Potential investors can either submit a competitive bid, in which the bidder, usually a primary dealer acting for itself or on behalf of customers, bids for a face amount of the bill at a discount given to 2 decimal places, or a non-competitive bid, by which they agree to buy the bill at a price determined by the Treasury. For example UBS might bid for USD100,000,000 of a 26 week bill at a 5.75% discount. If the bid is accepted, UBS would pay:

$$100{,}000{,}000 \times \left(1 - 0.0575 \times \frac{182}{360}\right) = 97{,}093{,}056$$

for the bills, which it would then sell on to its customers, who might be money market mutual funds, state or local governments or financial institutions. Any investor can submit a non-competitive bid for a face amount of T-Bills of at least USD10,000. The Treasury does not charge a commission and the bid can be submitted to any Federal Reserve Bank.

Treasury Notes and Bonds

These are coupon bearing securities of maturities 2, 3, 5, 10 and 30 years. The securities with original maturity of 10 years or less are called notes, while the 30 year securities are called bonds. There are no other differences between them. They all pay their interest on a semi-annual basis. The interest accrues on an actual/actual basis (see the Appendix for details). The notes and bonds are auctioned off in much the same way as the T-Bills. Potential buyers submit bids to two decimal places for a specific face value of the securities.

Because the notes are coupon securities, the Treasury has to set the coupon rate, which it does in the form of a whole number plus a fraction such as 6 3/8 or 7 1/4. The successful buyer gets the note for a price implied by the yield. For example if the bidder buys USD50,000,000 of a 10 year note for a yield of 6.40% when the note coupon is set at 6 3/8 (6.375%) then the price paid will be a little less than par. Using a bond calculator, the buyer would pay 99.82 per 100 face value.

Treasury Strips

Many investors would like to buy long-dated discount securities such as a 20 year zero coupon bond. This would be a Treasury security that would be purchased at a steep discount to its face value and would not pay any cash flows until it matures in 20 years. At that point it would pay out its face value. For example if it were priced at a 6.00% semi-annual yield, the price of the bond would be 30.66 for each 100 face value. Such a long-dated zero is called a Treasury Strip, because of the way it is created – by the process of "stripping" US Treasury bonds.

Bond Stripping

It is an axiom of the financial markets that if investors want something and are willing to pay for it, bankers will find a way to give it to them. The US Treasury does not issue discount securities with an original maturity of more than one year. This may be because it would be too tempting for legislators to fund their current projects with money that would not have to be repaid until long after they had left office. But whatever the reason, this is a problem for investors. Discount securities are attractive because of their low cost and lack of reinvestment risk – no payout until maturity so there is nothing to reinvest. The US Treasury now allows buyers of its bonds to sell the individual coupons as separate securities. Suppose for example that the Treasury issues a 30 year bond with a 6% coupon. Then if a bank were to buy $100,000,000 face value of this bond, it will own coupons that will pay $3,000,000 in interest every 6 months for the next 30 years. Each of these coupons is a marketable security. The bank can sell any amount of face value of these coupons to investors. If the bond is worth par, so that 6% is the correct yield then $100 face value of the last coupon, which will not be paid for another 30 years would sell for approximately*

$$\frac{100}{1.03^{60}} = 16.97$$

(Note that there are 60 semi-annual periods to maturity and the convention in the US Treasury market is to use semi-annual compounding.)

Recombining a Bond

Just as bonds can be stripped, they can be recombined. If a bank buys all of the coupons and the principal payment due at maturity, it can present them to the US Treasury and get back the original bond. This is not an easy thing to do but it can occasionally be profitable.

* This is not exactly correct since we need to use the 30 year zero rate not the coupon rate, but it gives you the idea of how big the discount is.

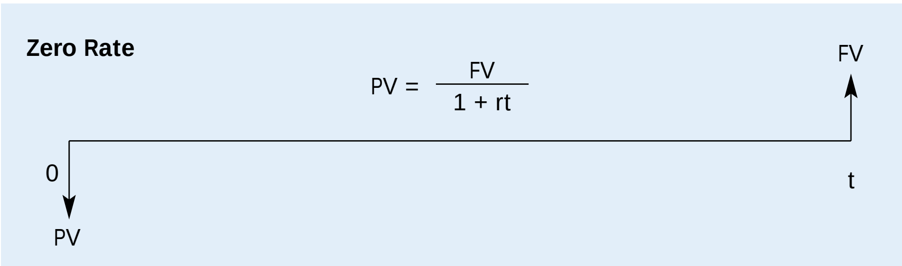
If the price of recombining the bond is less than the price for which the whole bond is trading in the market, then the bank could buy the strips and simultaneously sell the whole bond. When the strips are reconstituted, they can be used to make delivery on the sold bond. In order to do this ”arbitrage” effectively a bank would be likely to need a trading desk that was dedicated to trading strips. The importance of this activity is that it tends to keep the strip market and the bond market in close agreement. This has important implications for constructing the US Treasury yield curve.

Yield Curve Construction

We are familiar now with all the concepts and instruments we need to discuss this important topic. Our discussion will be at a level of detail appropriate to an introductory treatment like this one. Anyone who is involved in writing programs or having to take risk based on yield curves must have a good understanding of how they are built. The methods that are used in banks to build yield curves are refinements of the ones we will discuss here, but which must take into account, as we will not, all the pricing conventions and market peculiarities that occur in the ”real” world.

Zero Rates

In earlier sections we discussed three different types of interest rates – zero rates, forward rates and coupon rates. Let’s just review them briefly using some cash flow diagrams. Zero rates correspond to a pair of cash flows, the first of which occurs on the spot date, which we will just think of as today or day 0 and the second of which occurs at some point in the future, at a time we will designate * as t. Graphically:



The rate r in the diagram is the zero rate that applies from now to time t. If we were writing an academic book we would write this rate r as r(0,t) to be really explicit as to when it begins: 0 and when it ends: t. That is really just too fussy for this book.

Example

In the picture, you can see that we have assumed that we would express the rate as a simple one period rate – you see this from the way present value (PV) is calculated from future value (FV). We could of course use any type of rate such as semi-annual compounded or continuous. When we do actual examples from different interest rate markets we will use the type of rate that is common in those markets. For example, in the

---

* If the presence of a variable causes you to become faint, you might like to read our introduction to mathematics, available separately.

Eurodollar bond market, it is usual to use annual compounding, while in the US Treasury market, semi-annual compounding is the convention.

Example

The zero rate for various time periods is shown below. We use the rates to calculate the present value of $1 (a generic figure, it doesn't necessarily mean USD.)

Time	¼ year	½ year	¾ year	1 year
Rate	8.00%	8.25%	8.40%	8.50%
PV of 1	0.9804	0.9604	0.9407	0.9217

The present value of 1 to be received at time t is also called the time t discount factor and that is how we will refer to it. Once we know the discount factors the actual rates themselves become irrelevant. The rates are really only a way to get the discount factors. If the rates had been expressed in some other form such as a/360 or a/365 or continuous, we would have used a different type of calculation but the end result is still the discount factor. Why are we so interested in discount factors? Because once we have discount factors for every point in time we can value any stream of cash flows, no matter how complicated.

Example

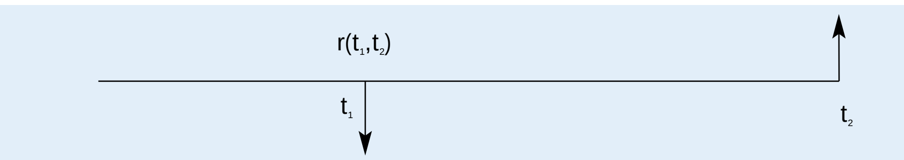
A retail store expects quarterly receipts of 50,000 for the first quarter, 75,000 for the second, 100,000 for the third and 50,000 for the fourth. What is the total present value of these receipts?

$$50{,}000 \times 0.9804 + 75{,}000 \times 0.9604 + 100{,}000 \times 0.9407 + 50{,}000 \times 0.9217 = 261{,}205$$

There seems to be a "catch" in that last statement – we need discount factors for every point in time. But where will we get them? We will defer the answer to that question until we have completed our discussion of different types of rates. Just keep in mind that this is a crucial issue.

Forward Rates

These rates are "like" zero rates in that only one pair of cash flows are involved. The difference is that the first cash flow does not occur now, at time 0, but at some point in the future, which we regrettably will have to refer to as time t_1, using the subscript 1 to refer to the time of that first cash flow, while the second one will be referred to as (did you guess?) time t_2. The diagram below shows the cash flows.



Example

A pension fund owns a bond which will mature (pay back its face value) on 15 April, which is 1 month from now. It would like to invest the proceeds in a 3 month investment (to mature on 15 July). UBS quotes a rate of 7.00% for a 1 x 4 forward deposit. What does this mean? The bank has agreed to take in the maturing bond as a deposit in 1 month and pay a 7% interest rate for the ensuing 3 month period*. At the end of that time, 4 months from now, the bank would repay the deposit plus interest. So, suppose the amount of face value plus coupon that the fund receives is $6,000,000. Then after 4 months the fund would receive:

$$6{,}000{,}000 \times \left(1 + 0.0700 \times \frac{91}{360}\right) = 6{,}106{,}166.66$$

The cash flows are shown below.

Note that the money is placed on deposit one month *from* today and the deposit matures four months *from* today but the rate is agreed on *today*. This forward-looking time frame is the essence of a forward rate.

Terminology: The bank is buying a deposit and is said to be long the forward rate or to have "bought" the forward rate; conversely, the fund has sold, i.e. placed, the deposit and is short the forward rate or to have "sold" the forward rate.

Suppose that the following rates are being quoted today:

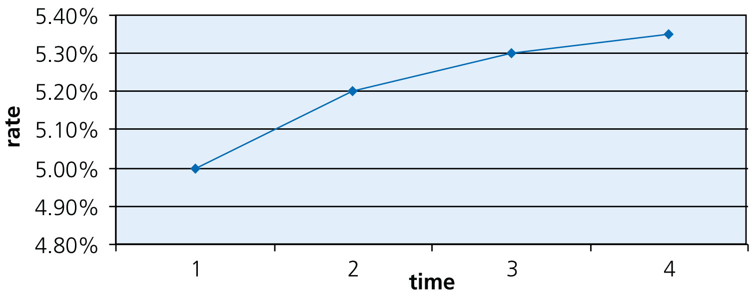
Time period (t_1,t_2)	**(0,0.25)**	**(0.25,0.50)**	**(0.50,0.75)**	**(0.75,1.00)**
Rate	5.00%	5.20%	5.30%	5.35%

We show these rates graphically in the figure overleaf.

The first rate, 5.00%, is a rate that applies from a period starting today and ending in a quarter of a year, so it is just an ordinary zero rate. The next rate 5.20%, takes up where the previous rate left off, that is, it starts in 3 months and ends in 6 months. Then the rate of 5.30% is valid from 6 months to 9 months while the last rate of 5.35% applies to the final 3 month period from 9 months to the end of the year. How can we use these rates?

Suppose that an investor has $100,000 on hand and expects to receive payments of $10,000 every three months for the next 9 months (i.e. there are 3 payments of $10,000).

*This is *not* an FRA. The bank is really accepting the funds as a deposit. But since the bank has agreed the rate, it is exposed to a change in rates. So it might decide to hedge its exposure with an FRA.



If the investor "locks-in" the rates above, what amount of money will the investor have at the end of the year? We will assume that the investor can "roll over" the accumulated deposit plus interest and the new payment at each interval.

Now	3 months	6 months	9 months	1 year
100000	111,250	122,696	134,322	136,119

$100{,}000 \times (1 + 0.0500 \times {}^{3}/_{12}) + 10{,}000 = 111{,}250$

$111{,}250 \times (1 + 0.0520 \times {}^{3}/_{12}) + 10{,}000 = 122{,}696$

$122{,}696 \times (1 + 0.0530 \times {}^{3}/_{12}) + 10{,}000 = 134{,}322$

$134{,}322 \times (1 + 0.0535 \times {}^{3}/_{12}) = 136{,}119$

Implied Spot Rates

The forward rates can also be used to infer zero rates. For example, suppose we wanted to quote a single rate for a deposit received today and repaid in one year. If we were to place $1 on deposit today at 5.00% for 3 months and roll it over at 5.20% for the next 3 months, then at 5.30% then at 5.35%, then at the end of the year, the final amount would be:

$$1 \times (1 + 0.0500 \times {}^{3}/_{12}) \times (1 + 0.0520 \times {}^{3}/_{12}) \times (1 + 0.0530 \times {}^{3}/_{12}) \times (1 + 0.0535 \times {}^{3}/_{12}) = 1.0532$$

which is the same result as if we had deposited the $1 for 1 year at 5.32%. So 5.32% is the one year zero rate that is implied by the forward rates. Implied in this context means that the rate is consistent with the forward rates.

Let's do one more – infer the spot rate for the first 6 months:

$$1 \times (1 + 0.0500 \times {}^{3}/_{12}) \times (1 + 0.0520 \times {}^{3}/_{12}) = 1.0256625$$

This means that after a ${}^{1}/_{2}$ year we would have 1.0256625. What zero rate would have given the same result?

$$1 + r_{0.5} \times {}^{6}/_{12} = 1.0256625$$

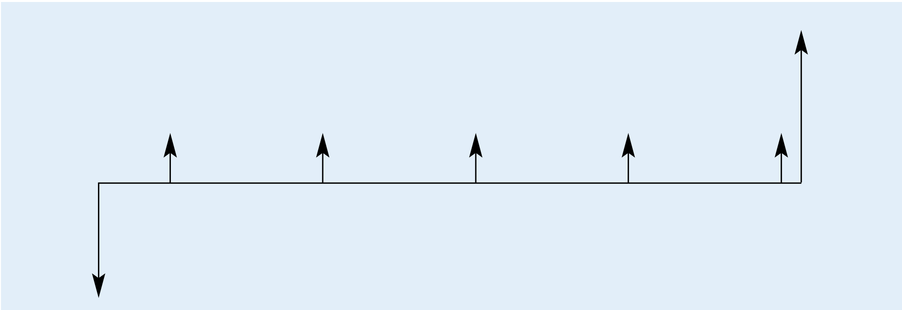
$$r_{0.5} = 0.0256625 \times 2 = 0.051325 = 5.13\%$$

The zero rates for 3, 6, 9 and 12 months are shown in the table below:

Time (quarters)	1	2	3	4
FV of 1	1.0125	1.0256625	1.039252528	1.053152531
Implied zero	5.00%	5.13%	5.23%	5.32%

Bond Yields

For our last example of types of interest rates, we look at bonds. Bond prices are quoted as a percentage of the bond's face value. For example the price might be 98, which means that 1,000,000 face value of the bond would cost 980,000, while a price of 102 would mean that 1,000,000 face value would cost 1,020,000. When the bond's price is 100, so that the face value and the price are the same, the bond is said to be trading at par. The coupon on the bond is then called the par coupon. A bond of course is a series of cash flows and we can represent them graphically:



The small arrows represent the coupons and are paid at regular intervals. The bond's face value is paid out at maturity and is represented by the large arrow on the right. This is not to scale for most bonds as the coupon is usually a small percentage of the face. The downward pointing arrow at the left is the bond‘s price. The interest rate in this case is the bond‘s yield, which is the rate that can be used to present value all of the bond‘s cash flows and arrive at the bond's price. In the case where the bond is trading at par, the yield will equal the bond's coupon rate.

Zero, Forward and Par Rates

We have already seen that if we have forward rates, then we can use them to infer zero rates. Could we also infer par rates? To see that we can, let's assume that the following rates are given for rates for periods of one year:

Period	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)
Rate	7.00%	7.50%	7.90%	8.20%	8.40%

Our first step is to get the discount factors. We can get them by compounding the forward rates:

one year discount factor

$$\frac{1}{1.0700} = 0.9346$$

two year discount factor

$$\frac{1}{1.0700} \times \frac{1}{1.0750} = \frac{0.9346}{1.0750} = 0.8694$$

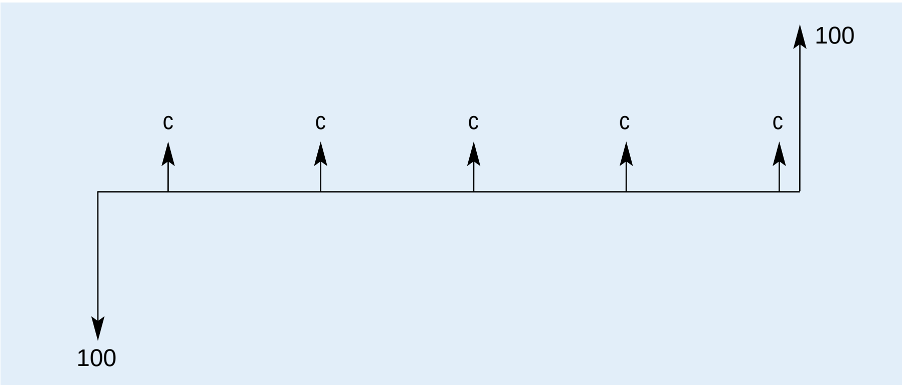
three year discount factor

$$\frac{0.8694}{1.0790} = 0.8057$$

we can get the four year and five year discount factors in the same way. The results are shown below.

Period	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)
Rate	0.0700	0.0750	0.0790	0.0820	0.0840
Discount factor	0.9346	0.8694	0.8057	0.7447	0.6870

Now that we have the discount factors, we can present value any set of cash flows. Let's use them to determine what the coupon should be on a five year bond, if we want it to trade for par. Since it is to trade for par, it's price is 100 and this must be the total present value of the bond's cash flows.



Although we don't know the coupon (yet), we can write an equation that relates the price to the cash flows, using the discount factors:

$$100 = 0.9346 \times c + 0.8694 \times c + 0.8057 \times c + 0.7447 \times c + 0.6870 \times c + 0.6870 \times 100$$

$$100 = (0.9346 + 0.8694 + 0.8057 + 0.7447 + 0.6870) \times c + 68.70$$

$$100 = 4.0413^{*} \times c + 68.70$$

$$c = \frac{100 - 68.70}{4.0413} = 7.75$$

(There was a little "algebra" involved in the second step, sorry.)

*difference due to rounding

The number 4.0413 was the sum of the 1,2,3,4 and 5 year discount factors and is called, appropriately, the cumulative discount factor. The same method could be used to determine the par coupon for bonds of maturities 1, 2, 3, and 4 years. The results are shown below:

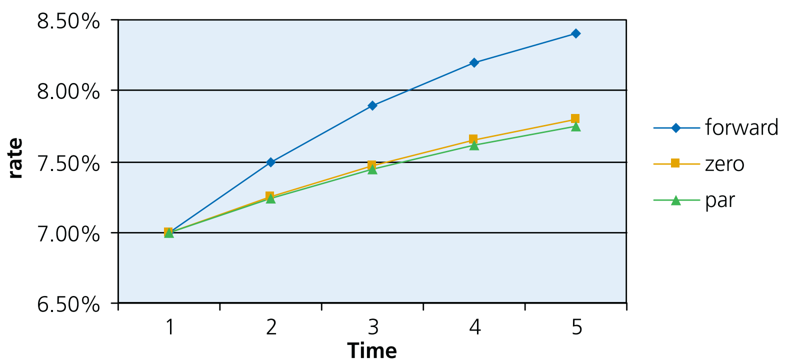
Period	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)
Rate	7.00%	7.50%	7.90%	8.20%	8.40%
Discount factor	0.9346	0.8694	0.8057	0.7447	0.6870
Cumulative df	0.9346	1.8040	2.6097	3.3543	4.0413
Par coupon	7.00%	7.24%	7.44%	7.61%	7.75%

(any numerical differences between this table and previous calculations are due to rounding)

Lastly we could put the forward, implied zero and par rates all on the same graph to compare them.

Years	1	2	3	4	5
Forward	7.00%	7.50%	7.90%	8.20%	8.40%
Zero	7.00%	7.25%	7.47%	7.65%	7.80%
Par	7.00%	7.24%	7.44%	7.61%	7.75%

Forward, Zero and Par Rates



In the figure above we see the three types of rates graphed. The graphs are referred to as curves called, respectively, the forward curve, the zero curve and the (par) yield curve. The particular configuration shown is considered "normal" by the professionals in the rates markets. Normal means that the curves are all upward-sloping, meaning that rates that are either farther into the future, such as 7.90%, the one year rate in two year's time, or which cover a longer period of time such as 7.47%, the three year zero rate, are higher than the rates that are closer in time, like 7.50%, the one year rate in one year's time or cover a shorter period of time such as 7.25%, the two year zero rate. But normality is not a law – there are plenty of instances in which the curves are either flat (rates are the same for all maturities) or inverted (short term rates higher than long term rates).

Formulas and Bootstrapping the Yield Curve

In our discussion above, we kept the formulas to a minimum (we really did!). But for the sake of completeness and in case you want to go more deeply into the math we will now present the relevant relationships.

First the notation:

r_t	the zero rate to time t, where t is expressed in years
$r_{i \times j}$	the forward rate from time i to time j
c_t	the coupon on a t year bond valued at par
df_t	the discount factor to time t
cdf_t	the cumulative discount factor to time t

(These formulas assume annual compounding.)

Zero Rates from Forward Rates

$$r_n = [(1 + r_1) \times (1 + r_{1 \times 2}) \times (1 + r_{2 \times 3}) \times \cdots \times (1 + r_{(n-1) \times n})]^{1/n} - 1$$

Discount Factors from Forward Rates

$$df_n = \frac{1}{1 + r_1} \times \frac{1}{1 + r_{1 \times 2}} \times \frac{1}{1 + r_{2 \times 3}} \times \cdots \times \frac{1}{1 + r_{(n-1) \times n}}$$

Cumulative discount factors

$$cdf_n = df_1 + df_2 + \ldots + df_n = \sum_{i=1}^{i=n} df_i$$

Par Yields From Discount Factors

$$c_n = \frac{100 - 100 \times df_n}{cdf_n}$$

Forward Rates From Discount Factors

$$r_{i \times (i+1)} = \frac{df_i}{df_{i+1}} - 1$$

Forward Rates From Zero Rates

$$r_{i \times (i+1)} = \frac{(1+r_{i+1})^{i+1}}{(1+r_i)^{i}} - 1$$

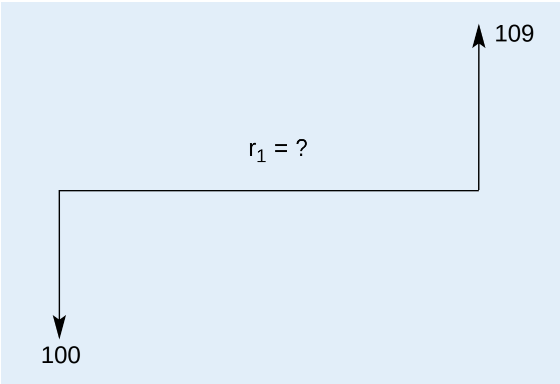
Bootstrapping the Yield Curve

Suppose we know the yields on par bonds and want to determine spot and forward rates that are consistent with those yields. This problem is a little like the problem of determining a bond's yield from its price. Remember that getting the price from the yield was easy but getting the yield from the price was more difficult. We saw above that once we have discount factors we can get the bond's yield (coupon for a par bond) very simply. There should be a way to "go backwards". And there is! We will now describe the method known as "bootstrapping".

Suppose these are the yields (coupons) on par bonds of the indicated maturities:

Years	1	2	3	4	5
Yield	9.00%	8.50%	8.20%	8.00%	7.90%

For the sake of variety, we are using an "inverted" curve as our example i.e. the longer term yields are lower than the shorter term ones.
Because the one year bond coupon is 9%, that must also be the one year zero rate. We can see this because the price of the bond is par and must equal its present value using the one year zero rate:



$$100 = \frac{109}{1 + r_1}$$

$$1 + r_1 = \frac{109}{100} = 1.09$$

$$r_1 = 0.09 = 9.00\%$$

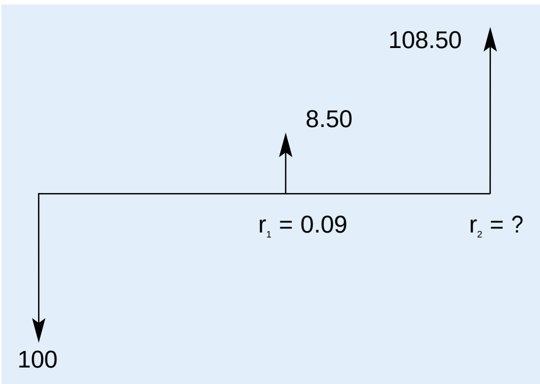
Now how will we find the two year zero rate? We will use the one year zero rate and the two year par yield of 8.50%. Since a two year bond with an 8.50% coupon is worth 100:

$$100 = \frac{8.50}{1.09} + \frac{108.50}{(1 + r_2)^2}$$

$$\frac{92.20}{108.50} = \frac{1}{(1 + r_2)^2}$$

$$(1 + r_2)^2 = \frac{108.50}{92.20} = 1.17677$$

$$r_2 = \sqrt{1.17677} - 1 = 0.08479$$
$$= 8.479\%$$



O.K. we admit it, the math was a little annoying. If it bothers you, then you should probably not be bootstrapping curves. Just be aware that this is what any computer program you might use goes through to get the zero and forward rates.

We can continue in the same way, using all the zero rates derived up to a time, together with the next par yield to get the next zero rate. We'll just do one more (promise!). We use the one and two year zero rates together with the three year par yield to get the three year zero rate.

$$100 = \frac{8.20}{1.09} + \frac{8.20}{(1.08479)^2} + \frac{108.20}{(1+r_3)^3}$$

$$(1 + r_3)^3 = \frac{108.20}{100 - \frac{8.20}{1.09} - \frac{8.20}{(1.08479)^2}} = \frac{108.20}{85.5088} = 1.26537$$

$$r_3 = 1.26537^{1/3} - 1$$

$$r_3 = 0.0816 = 8.16\%$$

(Maybe you should think about taking a nap now?)

The result of this process is shown in the table below.

Years	1	2	3	4	5
Yield	9.00%	8.50%	8.20%	8.00%	7.90
df	0.9174	0.8498	0.7903	0.7365	0.6856
Zero	9.00%	8.48%	8.16%	7.95%	7.84%
Cdf	0.9174	1.7672	2.5575	3.2940	3.9796

Whether or not you followed the math, you should realise an important implication of these formulas and procedures:

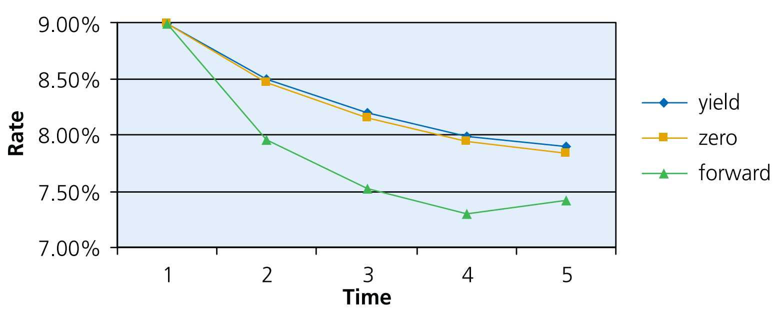
Principle: Any one of the three curves – zero, forward or par, completely determines the other two.

The Relationship Between Forward, Zero and Par Rates

The figure below is the graph of these rates from our last example.

Years	1	2	3	4	5
Yield	9.00%	8.50%	8.20%	8.00%	7.90%
Zero	9.00%	8.48%	8.16%	7.95%	7.84%
Forward	9.00%	7.96%	7.53%	7.31%	7.42%

Inverted Curve



When the curves were "normal" i.e., upward-sloping, the forward curve was above the zero curve and the zero curve was above the par curve. Also the vertical distance between the forward curve and the zero curve was much greater than that between the zero curve and the par curve. The latter statement is also true in the instance of an inverted curve, but the relationship between forward, zero and par curves is itself inverted. The forward curve is below the zero curve, which is itself below the par curve. The reason for these relationships can be seen through the way in which the rates were calculated, starting from the forward rates. To get the two year zero rate we average the first two forward rates. The average is not a straightforward arithmetic average, but a little more complicated:

$$(1 + r_2) = \sqrt{(1 + r_{0 \times 1}) \times (1 + r_{1 \times 2})}$$

This is a kind of "geometric" average. Even though it is 1 plus the rate that is averaged rather than the rate itself, it is easy to see* from the equation that the two year spot rate must be less than or equal to the larger of the two forward rates and greater than or equal to the smaller of the two forward rates. When it is plotted on the graph, the two year rate is plotted at time 2 so it is to the right of the one year forward and on the same vertical line as the two year forward. So if the curve is normal and the one year rate is less than the two year rate, then the two year zero is above the one year rate (the smaller of the two) and below the two year forward (the larger of the two).

When the curve is inverted, then the two year forward rate is less than the one year rate, so the two year zero is below the one year rate (the larger of the two) and above the two year forward (the smaller of the two). This relationship is entirely the same as we move farther out the curve. However the distance between the zero curve and the forward curve tends to get larger because the averaging at each step is between the new forward rate and the average of all the previous rates. In the case of a normal curve the average of all the previous rates would be less than the rate immediately preceding the one we are calculating. For example if we were calculating the 5 year zero, the two terms being averaged are not the 4 year forward and the 5 year forward, but the average of the first 4 years of forwards and the 5 year forward. That average is less than the 4 year forward so the distance tends to get larger. This is not a hard and fast rule since the average of the first 4 years is weighted more heavily than the last one year. You can see this in the inverted curve example where the distance between the 5 year zero and the 5 year forward is less than at the 4 year point.

Note that the zero curve and the par curve are much closer to each other than the forward curve and the zero curve. In fact it is a common mistake to think that these two curves are the same. Zero rates are used to discount single cash flows. The par rate is not really a discounting rate, although it is often used that way in yield-to-maturity based calculations.

In a normal yield curve environment, if we were to discount all of a bond's cash flows using the zero rate that applies to the bond's maturity i.e. use the five year zero to discount all the cash flows of a five year bond, it is clear that this rate is too high and would therefore

* Don't you hate it when someone says something is easy to see? It usually means it's not.

give a bond price that is too low. This is because only the very last cash flow of the bond should be discounted at that rate. The earlier cash flows should be discounted at lower rates since the earlier zeros are lower. In order to discount the entire set of cash flows correctly, if we are determined to use a single rate, it would have to be somewhat lower than the final zero rate. It doesn't have to be a lot lower though because the most important cash flow, the return of face value, comes at the end and is correctly discounted using the last zero rate. We need to lower the rate just enough to make up for "over-discounting" the earlier coupons. But since this lower rate will also be applied to the return of par, lowering it just a little is enough. Of course as we go farther out in time, that last cash flow becomes less important and the amount we have to lower the final zero rate increases. Thus the distance between the zero curve and the par curve tends to get larger as we move farther out the maturity spectrum.

Forward Yields

Although this is an advanced topic, it is so important in the investment banking business, particularly in trading, that we need to discuss it even in this elementary text. Let's go back to the idea of forward rates. If we say that we are willing to lend money today at 5%, but will agree to lend money at 5.25% if the loan starts a year from now, what is the reason? In agreeing today to do something a year from now we are taking on risk. If rates a year from now turn out to be 6%, then we have agreed to too low a rate. So where did the rate come from? Banks, investors and speculators in the rates markets are constantly trading their expectations of the future level of interest rates. Those expectations are reflected in the prices of futures contracts, which we discussed earlier. So it is not too great a stretch of the imagination to say that the current level of forward rates reflects the expectations of market participants as to the future level of rates.

Now we saw how it was pretty straightforward to use the forward rates to determine the zero and par rates. Suppose that the current forward rates are as shown below:

Period	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)	(5,6)
Rate	7.00%	7.50%	7.90%	8.20%	8.40%	8.50%

We have just added one more year to a previous example. Now suppose that one year from now, the new forward curve is the one that was "predicted" by today's curve. This means that today's one year rate of 7.00% has "dropped off" and been replaced by 7.50% which was predicted last year as being the new one year zero rate. Similarly the new (1,2) forward rate, we will suppose is 7.90%. That is, we are supposing that the forward curve next year is just today's forward curve moved one year forward. This curve is shown below.

Forward Curve in one year

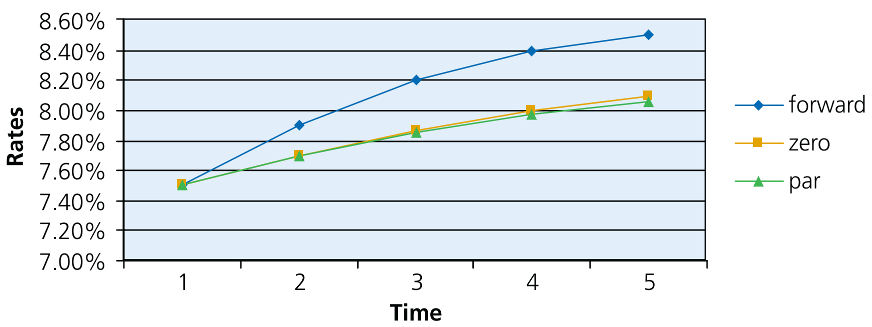
Period	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)
Rate	7.50%	7.90%	8.20%	8.40%	8.50%

We can use the same method to calculate the zero and par curves as we did before; we just use the new, assumed, forward rates. The table and graph below show the rates that we "expect" for next year.

Forward, Zero and Par Curves in one year

Years	1	2	3	4	5
Forward	7.50%	7.90%	8.20%	8.40%	8.50%
Zero	7.50%	7.70%	7.87%	8.00%	8.10%
Par	7.50%	7.69%	7.85%	7.97%	8.06%

1 year From Now



The par yields that are predicted to happen if the forward curve "comes to pass" are called the forward yields and the curve is referred to as the (one year) forward yield curve. Many investors are concerned about forward yields because they indicate the levels at which they could agree today to fund themselves in the future. In the options markets, the forward yields are the "at-the-money" strikes for interest rate options.

Practical Construction of the Treasury Yield Curve

US Treasury bonds are coupon securities so they "belong" on the par, or yield, curve. So if we were to observe the yields of these bonds in the market, each one would correspond to a point on the curve. Unfortunately, it is very difficult to draw a smooth curve through the many yields that exist. At any given time there may be dozens of US Treasury bonds and notes trading in the market. As an alternative one could use just the so called "on-the-run" bonds which are the most recently auctioned notes of their maturities. Since the Treasury only auctions off 2,3,5,10 and 30-year bonds though we would have exactly the opposite problem as before, namely we have only 5 points on the curve and we could draw many smooth curves through them. There are other practical problems as well having to do with liquidity and repo (funding) rates. We won't pursue this topic any further and just stop by remarking that it is possible to construct the US Treasury yield curve from the yields on existing Treasury bonds. Since we would be starting with the coupon bonds, we would use a bootstrapping procedure of the type we discussed earlier to carry out the calculations.

The Treasury strips correspond to zero rates. They have the advantage that they come in convenient evenly-spaced intervals of 3 months all the way out 30 years. This means that they could be used to construct a fairly smooth curve which we could then use to price any other securities. Unfortunately even this procedure has some difficulties associated with it, but that's just the way it is in the real world. If only the world were the way academics suppose it to be*, we wouldn't have to worry about the messy one we actually live in. Simply note that the zeros define the zero coupon curve. As we mentioned above, the process of arbitrage will keep the strips in line with the whole bonds. So whatever the choice of instruments used to build the curve – strip prices or bond prices, the resulting yield curve should be approximately the same.

Treasury Futures

Although there are futures that trade on T-Bills and bonds (bond futures were discussed in the previous section), they are not very useful for building the forward curve – there are not enough of them and we noted that there are peculiarities in their pricing. So the forward curve is determined either by the coupon curve, using the bootstrapping technique, or by the zero curve.

LIBOR Curve

Using Swaps

The LIBOR curve would describe the required yield for borrowers who are not as risk-free as the US Treasury. The benchmark curve for LIBOR is the swap curve. Every day investment banks make markets to their clients and to each other on interest rate swaps of different maturities from 1 year to 30 years. This process of quoting in the market is one of price discovery whereby the banks express their views on the level of rates. In a number of markets, including the major currencies such as EUR, USD, and other G7 currencies, the market for interest rate swaps is liquid enough to use to build a curve. Since these rates are equivalent to bond yields, we would use a bootstrapping procedure to carry out the calculations.

Using Futures

The short-term interest rate futures, also discussed in the previous section, can be used to observe the forward curve directly. Particularly in USD there are enough futures to build a curve out to 10 years. In other currencies there are not enough contracts to do this, so that option is not open to us and we have to rely on the swap market.

Using Zeros

There are very few zero coupon bonds available apart from US Treasury strips and some strips in the UK Gilt market, so it is not really possible to directly observe zero coupon rates outside of the US Treasury market.

Multiple Choice

In general traders will build the LIBOR curve from a combination of deposit rates, Euro interest rate futures and swap rates. This requires a sophisticated algorithm (detailed step-by-step procedure) to carry it out. This provides employment for a number of mathematicians and physicists, thus promoting the public welfare and potentially reducing the size of the prison population.

---

* Like the old joke about the economist who was trapped in a deep well. How did he get out? He just assumed he had a ladder.

Worksheet – Yield curve relationships

Time (years)	1	2	3	4	5
Forward Rate	0.0600	0.0650	0.0675	0.0690	0.0700
Discount Factor					
Cumulative Discount Factor					
Coupon					

Use the rates above for this problem.

Starting with the forward rates:

1. Use the current zero rate and the one year forward to get the **two year zero rate.**

$$r_2 = \sqrt{(1 + r_1)(1 + r_{1x2})} - 1$$

2. Use the zero rate, one year forward and two year forward to get the **three year zero rate.**

$$r_3 = \sqrt[3]{(1 + r_1)(1 + r_{1x2})(1 + r_{2x3})} - 1$$

3. Use the zero, one year forward, two year forward and three year forward to get the **four year zero rate.**

4. Now get the **5 year zero rate.**

5. Use the zero rates to get the **discount factors** – the discount factors embody all of the yield curve information.
6. Use the discount factors to get the bond coupons:

   Two year: $\mathbf{100 = c \times df_1 + c \times df_2 + 100df_2}$

   similarly for the 3, 4 and 5 year coupons.

Bootstrapping

Time (years)	1	2	3	4	5
Coupon	0.0600	0.0624	0.0640	0.0651	0.0660
Discount Factor					
Cumulative Discount Factor					
Forward Rate					

Starting with the coupons: Calculate the one year discount factor from the one year zero rate.

1. Use the 1 year df and the two year coupon to get the **two year df:**

$$100 = c \times df_1 + (c+100) \times df_2$$

2. Use the two year df to get the **two year zero rate:**

$$r_2 = [1/df_2]^{1/2} - 1$$

3. Use the 1 year df, the 2 year df and the three year coupon to get the **3 year df:**

$$100 = c \times df_1 + c \times df_2 + (c+100) \times df_3$$

4. Use the three year df to get the **three year zero rate:**

$$r_3 = [1/df_3]^{1/3} - 1$$

5. Now get the 4 and 5 year zero rates in the same way.

6. Determine each of the forward rates:

$$r_{1x2} = df_1/df_2 - 1$$

$$r_{2x3} = df_2/df_3 - 1$$

Starting with the strips:

NOTE: These are annual not semiannual.

Time (years)	1	2	3	4	5
Price (Decimal)	94.34	88.58	82.98	77.62	72.55

The prices/100 are the discount factors:

Time (years)	1	2	3	4	5
Strip (DF)					
Zero Rate					
Cumulative Discount factor					
Coupon					

1. Use the 1 year df to get the one year zero rate:

$$\mathbf{r_1 = [1/df_1] - 1}$$

2. Use the two year df to get the two year zero rate:

$$\mathbf{r_2 = [1/df_2]^{1/2} - 1}$$

3. Now determine the 3, 4 and 5 year zero rates.

4. Now get the coupons:

$$\text{2 year: } \mathbf{100 = c \times df_1 + c \times df_2 + 100df_2}$$

5. Derive the forward rates, just as in the bootstrapping worksheet.

Worksheet – Stripping and Recombining

When we determine a yield curve should we use Zero Rates or Par Coupons?

Our principal objective is to determine a zero rate curve, because once we have it we can determine all other rates. We can build the US Treasury zero curve from market instruments in two different ways:

1. Directly from the T-bills and Strips.
2. Indirectly from the Treasury bonds and notes.

Does it matter?
Do you get the same curve?
If you don't is there an arbitrage?

Stripping a bond:

1. Buy a bond
2. Sell the individual coupons and the bond principal as separate securities.

Reconstituting a bond (Recombining):

1. Buy all of the coupon cash flows and the bond principal separately.
2. Sell the package as the whole bond.

Notes:

1. Any coupon that pays its value on a given date is fungible with any other coupon that pays on that date, regardless of the bond to which it was originally attached.
2. Bond principals are not fungible with coupons.
3. Accrued interest must be included in any trade.

Given these prices and yields*:

Security	bid	offer	yield
$4^5/_8$s Aug 95	101-01	101-03	4.07
Aug 93 ci	99-25	99-25	3.24
Feb 94 ci	98-04	98-05	3.34
Aug 94 ci	96-08	96-09	3.58
Feb 95 ci	94-03	94-04	3.90
Aug 95 ci	92-04	92-05	3.98
Aug 95 np	92-02	92-03	4.01

coupon	0.04625
today's date	20-Jul-93
next coupon	15-Aug-93
days to next coupon	26
last coupon	15-Feb-93
days between coupons	181

At what prices could you strip or recombine the bond? Assume you have to sell on the bid and buy on the offer.

* From the closing prices in the WSJ Wednesday July 21, 1993

Exercise – Swaps and Forward Start Swaps

Years	1	2	3	4	5
Forward Rate	0.0400	0.0475	0.0525	0.0550	0.0570
Zero Rate	0.0400	0.0437	0.0467	0.0487	0.0504
Discount factor	0.9615	0.9179	0.8721	0.8267	0.7821
Cumulative Discount Factor	0.9615	01.8795	2.7516	3.5783	4.3604
Coupon	0.0400	0.0437	0.0465	0.0484	0.0500

a) What is the current rate for a 3 year, annual coupon, interest rate swap?

b) What is the 1 year forward 3 year swap rate?

c) What is the 2 year forward 3 year swap rate?

Answers to Exercises

Time (years)	1	2	3	4	5
Forward Rate	0.0600	0.0650	0.0675	0.0690	0.0700
Discount Factor					
Cumulative Discount Factor					
Coupon					

Starting with the forward rates:

1. Use the current zero rate and the one year forward to get the **two year zero rate.**

$$r_2 = \sqrt{(1 + r_1)(1 + r_{1x2})} - 1 \quad r_2 = \sqrt{1.06 \times 1.065} - 1 = 0.062497$$

2. Use the zero rate, one year forward and two year forward to get the **three year zero rate.**

$$r_3 = \sqrt[3]{(1 + r_1)(1 + r_{1x2})(1 + r_{2x3})} - 1 \quad r_3 = \sqrt[3]{1.06 \times 1.065 \times 1.0675} - 1 = 0.064162$$

3. Use the zero, one year forward, two year forward and three year forward to get the **four year zero rate.**

$$r_4 = \sqrt[4]{1.06 \times 1.065 \times 1.0675 \times 1.0690} - 1 = 0.065370$$

4. Now get the **5 year zero rate.**

$$r_5 = \sqrt[5]{1.06 \times 1.065 \times 1.0675 \times 1.0690 \times 1.07} - 1 = 0.066294$$

5. Use the zero rates to get the **discount factors** – the discount factors embody all of the yield curve information.

   You can either calculate the discount factors from the zero rates themselves:

$$df_i = \frac{1}{(1+r_i)^i} \quad \text{so } df_5 = \frac{1}{(1.066294)^5} = 0.725463$$

or they can be obtained by using the forward rates:

$$df_5 = \frac{1}{1.06 \times 1.065 \times 1.0675 \times 1.0690 \times 1.0700} = 0.725463$$

time	1	2	3	4	5
discount factor	0.9434	0.8858	0.8298	0.7762	0.7255

6. Use the discount factors to get the bond coupons:

Two year: $\mathbf{100 = c \times df_1 + c \times df_2 + 100df_2}$

$$c_4 = \frac{100 - 100 \times 0.7762}{0.9434 + 0.8858 + 0.8298 + 0.7762} = 6.51$$

$$c_5 = \frac{100 - 100 \times 0.7255}{0.9434 + 0.8858 + 0.8298 + 0.7762 + 0.7255} = 6.60$$

Bootstrapping

Any slight differences in the answers in this problem compared to rates given in the previous one are solely due to rounding. If you want to verify this, do not round off any values that you calculate, or write a spreadsheet to implement the solution. The solutions shown here were rounded to 4 decimal places.

Time (years)	1	2	3	4	5
Forward Rate	0.0600	0.0624	0.0640	0.0651	0.0660
Discount Factor					
Cumulative Discount Factor					
Coupon					

Starting with the coupons: Calculate the one year df from the one year zero rate.

$$df_1 = \frac{1}{1.06} = 0.9434$$

1. Use the 1 year df and the two year coupon to get the **two year df:**

$$\mathbf{100 = c \times df_1 + (c{+}100) \times df_2}$$

$$df_2 = \frac{100 - 6.24 \times 0.9434}{100 + 6.24} + 0.8859$$

2. Use the two year df to get the **two year zero rate:**

$$\mathbf{r_2 = [1/df_2]^{1/2} - 1}$$

$$r_2 = \left(\frac{1}{0.8859}\right)^{1/2} - 1 = 0.0624$$

3. Use the 1 year df, the 2 year df and the three year coupon to get the **3 year df:**

$$\mathbf{100 = c \times df_1 + c \times df_2 + (c{+}100) \times df_3}$$

$$100 = 6.40 \times 0.9434 + 6.40 \times 0.8859 + 106.40 \times df_3$$

$$df_3 = 0.8298$$

4. Use the three year df to get the **three year zero rate:**

$$\mathbf{r_3 = [1/df_3]^{1/3} - 1}$$

$$r_3 = \left(\frac{1}{0.8298}\right)^{1/3} - 1 = 0.0642$$

5. Now get the 4 and 5 year zero rates in the same way.

$$r_4 = \left(\frac{1}{0.7762}\right)^{1/4} - 1 = 0.0654$$

$$r_5 = \left(\frac{1}{0.7255}\right)^{1/5} - 1 = 0.0663$$

6. Determine each of the forward rates:

$$\mathbf{r_{1x2} = df_1/df_2 - 1}$$

$$r_{1x2} = \frac{0.9434}{0.8859} - 1 = 0.0649$$

$$\mathbf{r_{2x3} = df_2/df_3 - 1}$$

Starting with the strips:

NOTE: These are annual not semiannual.

Time (years)	1	2	3	4	5
Price (Decimal)	94.34	88.58	82.98	77.62	72.55

The prices/100 **are** the discount factors:

Time (years)	1	2	3	4	5
Strip (DF)					
Zero Rate					
Cumulative Discount Factor					
Coupon					

Although you will have done some of the same calculations in the previous problems, you should see that the method here demonstrates the fact that any one of these curves (zero, forward, coupon) determines the others.

1. Use the 1 year df to get the one year zero rate:

$$\mathbf{r_1 = [1/df_1] - 1}$$

$$r_1 = \frac{1}{0.9434} - 1 = 0.0600$$

2. Use the two year df to get the two year zero rate:

$$\mathbf{r_2 = [1/df_2]^{1/2} - 1}$$

$$r_2 = \left(\frac{1}{0.8858}\right)^{1/2} - 1 = 0.0625$$

3. Now determine the 3, 4 and 5 year zero rates.

4. Now get the coupons:

2 year: $\mathbf{100 = c \times df_1 + c \times df_2 + 100df_2}$

$$c_2 = \frac{100 - 100 \times 0.8858}{0.9434 + 0.8858} = 6.24$$

same as for the forward rates because we only used the forwards to get the discount factors

5. Derive the forward rates, just as in the bootstrapping worksheet.

Worksheet – Stripping and Recombining

Given these prices and yields*:

Security	bid	offer	yield
4 5/8s Aug 95	101-01	101-03	4.07
Aug 93 ci	99-25	99-25	3.24
Feb 94 ci	98-04	98-05	3.34
Aug 94 ci	96-08	96-09	3.58
Feb 95 ci	94-03	94-04	3.90
Aug 95 ci	92-04	92-05	3.98
Aug 95 np	92-02	92-0	4.01

coupon	0.04625
today's date	20-Jul-93
next coupon	15-Aug-93
days to next coupon	26
last coupon	15-Feb-93
days between coupons	181

At what prices could you strip or recombine the bond? Assume you have to sell on the bid and buy on the offer.

Let's assume we want to buy 100 million face value of the bond and strip it i.e. sell the individual cash flows.

We will pay 100,000,000 x (1.0109375) + (0.04625/2) x 100,000,000 x 155/181 = 103,074,067.70.

We now own 100,000,000 face value of the bond, which means we have these face amounts of the various coupons and principal:

Maturity	Face Amount	Market Bid
Aug 93 ci	2,312,500 (100,000,000 x 0.04625/2)	2,307,441.41 (2,312,500 x 0.9978125)
Feb 94 ci	2,312,500	2,269,140.63
Aug 94 ci	2,312,500	2,225,781.25
Feb 95 ci	2,312,500	2,175,917.97
Aug 95 ci	2,312,500	2,130,390.63
Aug 95 np	100,000,000	92,062,500.00
		103,171,171.90

On this trade we could make 103,171,171.90 - 103,074,067.70 = 97,104.20! Go for it!

*From the closing prices in the WSJ Wednesday July 21, 1993

Exercise – Swaps and Forward Start Swaps

Years	1	2	3	4	5
Forward Rate	0.0400	0.0475	0.0525	0.0550	0.0570
Zero Rate	0.0400	0.0437	0.0467	0.0487	0.0504
Discount factor	0.9615	0.9179	0.8721	0.8267	0.7821
Cumulative Discount Factor	0.9615	01.8795	2.7516	3.5783	4.3604
Coupon	0.0400	0.0437	0.0465	0.0484	0.0500

a) What is the current rate for a 3 year, annual coupon, interest rate swap?
b) What is the 1 year forward 3 year swap rate?
c) What is the 2 year forward 3 year swap rate?

a) From the table above, the 3 year coupon is **4.65%**

1 year forward

Time	1	2	3	4
Forward rate	0.0475	0.0525	0.0550	0.0570
Zero Rate	0.0475	0.0500	0.0517	0.0530
Discount Factor	0.9547	0.9070	0.8597	0.8134
Cumulative Discount Factor	0.9547	1.8617	2.7214	3.5348
Coupon	0.0475	0.0499	0.0515	0.0528

b) This table starts from the 1 year forward curve and derives the "expected" rates in one year. The expected 3 year coupon one year from now is **5.15%**

2 years forward

Time	1	2	3
Forward rate	0.05250	0.05500	0.05700
Zero Rate	0.05250	0.05375	0.05483
Discount Factor	0.95012	0.90059	0.85202
Cumulative Discount Factor	0.95012	1.85071	2.70273
Coupon	0.05250	0.05372	0.05475

c) This table starts with the 2 year forwards and derives the expected rates in two years. The expected three year rate two years from now is **5.475%**

Section 6
Just Enough Foreign Exchange

A modern economy could not exist without a common medium of exchange. Without money we would have to work directly for all our daily needs – an hour of teaching in return for dinner! Some things would be virtually impossible – how would we ”save” enough to ”buy” a house for example? What would we be saving and who would want to accept it in return for a building? This is so obvious as to appear comical but our current form of money, dependent so heavily on a system of banking, is a modern invention. In ancient times money existed only in physical form such as coins made of precious metal. Now most money is never seen but exists only as numbers in accounts in financial institutions. Our employer pays us not by giving us stacks of bills but by transferring an amount from its bank account to our bank account. A check may be used to assist in the transfer but it is not money itself, only an instruction to a bank.

We use money of course to buy things – food, clothing, travel, education, entertainment and all the other things we need or want and can afford. Each thing we buy has a price given in terms of the basic unit of money. That basic unit has different names in different countries and languages – dollar, pound, kroner, ringitt, rouble, zloty are just a few of the names. Prior to the introduction of the euro in 1999, there were even more names, some of which still are used to refer to the legacy currencies such as Deutschmark, lira, French Franc and so on.

Money is more than just a convenience. In each country its money has legal status. If you owe a debt of USD100, the creditor must accept US dollars in payment. He cannot insist that you pay in gold or silver – the US dollar is ”legal tender for all debts public and private”. However if you happen to be a British retailer who owes USD10 million to a US manufacturer of compact discs, you can not pay your debt in British pounds even though pounds are legal tender in the UK. You will first have to exchange – sell your pounds to buy US dollars - then use the dollars to pay the debt.

This is the basis of the foreign exchange market. In order to facilitate the movement of goods, services and capital across international borders, banks provide a market for foreign exchange – the exchange of one country’s money for that of another.

Let’s first discuss some of the practical issues involved in foreign exchange. The price of many currencies are given in terms of the US dollar. For example, it might cost CHF1.5000 or JPY120.00 to buy one USD (we will always use a 3 letter code to denote a particular currency – see the following table). Each currency will be quoted to a precision that is customary in the inter-bank market. For many currencies this will be 4 decimal places, but for JPY it is just 2.

Currencies can also be quoted without reference to the US dollar. For example the price of one EUR in terms of CHF might be 1.5750 and the price of one GBP in terms of JPY might be 184.00.

The symbol for an exchange rate has the form A|B which means the amount of currency B that is equivalent to one unit of currency A. For example USD|CHF is the price of one USD in terms of CHF and GBP|EUR is the number of euro it takes to buy one pound sterling.

Every profession has its own vocabulary and verbal shorthand. This is an aid to efficient communication between professionals but a barrier to understanding for others. This is not an issue of intelligence, just familiarity. If we want to talk about foreign exchange we need to be familiar with the terms that are used in the market by professionals.

The USD and the EUR have comparatively great importance in the FX market, just as formerly the most important currency pair was USD|DEM. Most of the attention in the foreign exchange market is concentrated on EUR|USD and USD|JPY. So the euro might be quoted as 1.0500, i.e. USD1.0500 = EUR1 and a trader would reply "one-oh-five, the figure". In this phrase 1.05 is the "handle" and "the figure" means that the last two digits are 0. The smallest price change in the quote is called a "pip". So the difference between 1.0500 and 1.0505 is 5 pips. A "big" figure is 100 pips. If the price change on the day so far is for the euro to go from 1.0300 to 1.0500 (one oh three the figure to one oh five the figure) the euro has gone up two big figures.

Since 1.0500 is the price of one EUR in terms of USD, buying at this price means to buy euros and therefore to simultaneously sell dollars. In this there is a similarity to and a difference from the ordinary way in which we understand prices. If a German buys a car for EUR60,000, she buys the car and "sells" euros, but we really don't think of it like that, although we could. If instead a German retailer exchanges USD10,500,000 for EUR10,000,000 we think of it as an exchange but the FX market calls it "buying euro-dollar", i.e. buying euro-dollar means to buy euros and sell dollars. Since an FX trade always results in receiving one currency and giving another, the terms buy and sell are conventional. The convention is that when you are receiving the quoted currency (in A|B, A is the quoted currency), you are buying and when you give the quoted currency you are selling. So someone who buys euro-sterling is selling GBP and buying EUR, since the euro is quoted against the pound e.g. EUR|GBP = 0.6689.

There is a hierarchy of which currencies are quoted. Sterling used to be quoted in all currencies including USD. This was based on the historical importance of Sterling and on the way Sterling was once denominated in shillings and pence with 240 pence to the pound. It would be difficult to express a currency price in terms of a non-decimal unit – one dollar = 10 shillings and 6 pence? But now although GBP is quoted against many currencies it is not against the euro.

The EUR as noted is generally quoted. Prior to the introduction of the euro all of the European currencies were given in terms of the dollar, the most important currency pair being dollar-mark. However the euro is now given in terms of the dollar. Because the value of the USD has been very close to 1 euro, there is potential for some confusion. A quote of 1.0500 for the euro means that it takes USD1.0500 to buy one euro, not the other way around! Some typical quotes are shown below.

GBP|USD	1.6000	called ”cable”
EUR|GBP	0.6690	euro-sterling
GBP|JPY	192.00	sterling-yen
EUR|USD	1.0500	the euro
USD|CHF	1.5000	dollar-swiss, or Swissy
USD|JPY	120.00	dollar-yen
EUR|CHF	1.5750	euro-swiss
EUR|JPY	123.00	euro-yen

Morning Chat at UBS July 1999

07:25	flechtg	mng all. EURSFR another range day ahead? looks that way but overall still prefer to buy around 1.6050-55 lvl for a break of 1.6080 and a move towards 1.6120. sell below 1.6020. have fun
07:29	kreutlc	eur usd i expect range trading 1.0485 – 1.0550, overall i think 1.0530-50 is a great lvl to go short , short term target 1.0425 in the next 72 hours, good luck
07:41	rungayj	euro gbp looks rangey would think offers 0.6655 area and good buying interest mid 20s would like to buy cable 1.5790-00 area

Trading rooms used to be very noisy, with spot dealers constantly yelling at each other and with strategists and economists making announcements over loudspeakers. The advent of computer ”chat” lines has changed that considerably. Most salespeople and traders have several public and private chat channels open at all times and are constantly ”listening” to what is happening in all areas of the market. The box above contains an excerpt from an FX chat line at UBS late in July 1999 at a time when EUR|USD was exhibiting a lot of movement. In the previous weeks the exchange rate had come close to touching ”parity” i.e., EUR|USD = 1, but after Japanese central bank intervention on USD|JPY, the euro rallied against the dollar. Combined with better economic news from Germany, the effect was to drive the euro up to around 1.0500.

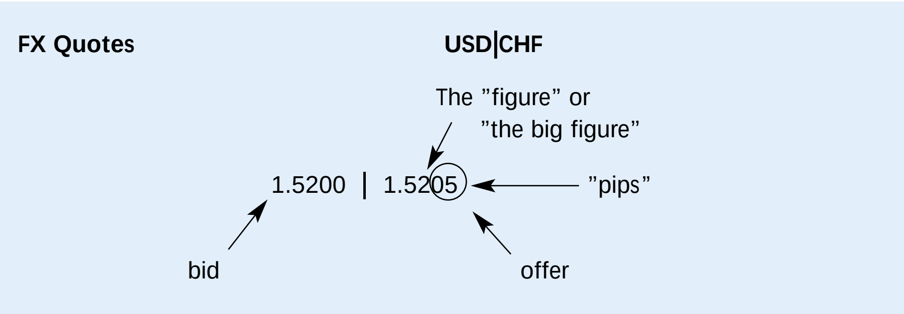
Bids and Offers

FX traders provide liquidity to customers and thus expose themselves to risk. The size of that risk can be measured by the size of the trader’s position and the size of the likely movement in the underlying currency rate while the trader has the position. A trader will quote two prices for any currency rate – the bid, which is the rate the trader is prepared to pay and the offer, which is the rate at which the trader is willing to sell. A dollar-swiss trader might quote:

1.5200 | 1.5205

Since the trader is buying and selling dollars, the lower price is the number of Swiss francs the trader will pay to receive one USD and the higher price is the number of Swiss francs he must be paid to give one USD. The 5 pip difference between the bid and the offer is called the bid-offer spread. In addition to giving a quote, the trader will also indicate the amount of the underlying currency he is willing to trade at that price. In some markets the minimum size is understood e.g. 5 million USD, but the trader will often say, or be asked, explicitly what size the quote is good for.

In addition, a trader will trade with the Bank's best customers at both tighter spreads and larger size than he would do with another professional market-maker or an infrequent customer. The size of the bid-offer spread together with the dealing size of the quote are the best indication of the amount of risk involved in the trade. If the market has great liquidity, so that it is easy for the trader to offset his trade quickly at a profit, or at least at no loss, the spread will be small and the dealing size large. In times of stress, when the market is reacting to an event, such as a political crisis or an economic announcement, liquidity will evaporate, the spread will widen and only the most aggressive dealers will be willing to quote in large size.



If the trader who gave the above quote is able to buy USD1 million on his bid and subsequently sell them on his offer, he will make a profit of CHF500. However, if the market moves against him before he can "unwind" his trade, this profit may turn into a loss. Suppose for example he does buy USD1 million for CHF1.5200 per USD (he would say he was "given" USD or that his bid was "hit"). If he can't immediately sell the dollars to someone else the market might move to:

1.5190 | 1.5195

before he has an opportunity to sell. If he sells the dollars now at CHF1.5195 (he was "taken", or his offer was "lifted"), he will actually lose CHF500 on the "round trip". If his original market had been 1.5195 | 1.5205, he would have had more protection but the width of his market would make it unlikely that he would trade at all. If the trader is unprepared to take as much risk as the other traders, he can not expect to make any profits.

As a rule, an FX trader will trade only one currency pair. That is, a bank will have two different traders trading dollar-yen and swiss-yen. If there were no free flow of information in the dealing room it might be possible for a clever trader to "pick-off" the market-makers by executing a sequence of trades involving several currencies. Consider the following quotes, where we will assume no bid-offer spread just for the sake of illustrating the point.

USD | CHF 1.5000
USD | JPY 120.00
CHF | JPY 82.00

A trader seeing these quotes might do these trades as quickly as possible:

1. Sell USD5 million | Buy CHF7.5 million (1.5000)
2. Sell CHF7.5 million | Buy JPY615million (82.00)
3. Sell JPY615million | Buy USD5.125 million (120.00)

The trader is ”flat” (has no position in) CHF and JPY and has a free profit of USD125,000. This ”triangular arbitrage” will not happen because traders can always see updated quotes in real-time on electronic data services, as well as sharing price information with other traders in the dealing room.

Spot and Forward Transactions

Every trade has two distinct dates associated with it, the trade date and the settlement date. The trade date just refers to the actual date on which the trade was agreed, while the settlement date refers to the date on which the cash or security flows occur. In the FX market, a spot trade will settle two business days after the trade date, with a few exceptions. So if we agree on Monday to buy USD|CHF for spot settlement, the actual date on which we receive dollars and pay Swiss francs would be Wednesday. Since trades in foreign currency involve two different currencies, the settlement date is required to be a business day in both countries where the cash is paid. In this case the settlement day would have to be a business day in both New York and Zurich. When the settlement day for an FX transaction otherwise would be a non-business day, there are various conventions used to determine a valid date. The simplest one is ”following business day”, meaning that we just go forward to the next day that is a valid date. For example if the settlement day would fall on a Saturday or Sunday, the actual settlement day would be Monday, unless it is also a non-business day.

Example

Today’s date 9/02/1999 (Tuesday)

trade	Buy CHF100 | Sell JPY8200	
trade date	9/02/1999 (Tuesday)	
	CHF cash flow	JPY cash flow
	none	none
settlement date	11/02/1999 (Thursday)	
	CHF Cash flow	JPY Cash flow
	+100	-8200

Notice that a trader is exposed to price risk as soon as a trade is agreed, not just on the settlement day. If we buy dollar-swiss for 1.5000 and the exchange rate later in the day is 1.4990, we have lost 10 pips on our trade, even though we will not explicitly feel the loss until we reverse our trade at the lower price. However, when we mark the position to market at the end of the trading day, we will show a loss in our book, if dollar-swiss closes below 1.5000.

Spot trading takes place in the FX market for many reasons, including taking positions or simply wanting to buy foreign currencies. There is another type of trade involving foreign exchange referred to as a forward transaction. The simplest kind of forward transaction is a forward outright. This is a trade whose settlement date is beyond the spot date. If a UK importer needed to have euros in 3 month's time to pay for a shipment into the UK from Germany, he could enter into a forward transaction today that would fix the rate at which he would exchange his sterling for euros 3 month's from now. In foreign exchange, forward dates are always offset from the spot date. This means that a 3 month forward transaction would have a settlement date that was three months from the spot date, not 3 months from the trade date. In general the rule is that the forward transaction will settle on the same day of the month as the spot date, unless that day is not a valid business day, in which case the usual conventions will apply for determining the actual settlement date.

Example

Today's date 12/03/1999 (Friday)

trade date	12/03/1999	
trade	sell GBP 100|buy EUR154 **three months forward**	
spot date	16/03/1999	
	GBP Cash flow	EUR Cash flow
	none	none
settlement date	16/06/1999 (Wednesday)	
	GBP Cash flow	EUR Cash flow
	-100	+154

The second type of forward transaction is an FX swap. An FX swap involves two trades, each on a different date. For example, suppose you were the trader who took the other side of the forward trade in our last example. You have an exposure to the EUR|GBP exchange rate in three months time. To hedge this risk, you could enter into the following FX swap:

trade date	12/03/1999
trade, first leg	-GBP|EUR spot
trade, 2nd leg	+GBP|EUR 3 months forward

This trade has two settlement dates, namely the spot date 16/03/1999 and the forward date 16/06/1999.

This trade would eliminate your previous forward risk, although you now have two offsetting forward trades since the two trades do not nullify one another. Additionally you have a spot position, so you have changed your forward risk into spot risk. Since the spot market is so liquid, you could easily trade out of that position as well. If you traded at favourable prices, you should now have a profit.

Forward transactions do not take place at the same price as the corresponding spot transaction. The reason for this relates to the different interest rates in the two countries. We will defer discussion of this until after we have worked through some examples. Below you will see a table of spot prices and forward points.

	EUR|USD		GBP|USD		EUR|JPY	
	Bid	**Offer**	**Bid**	**Offer**	**Bid**	**Offer**
Spot	1.0537	1.0543	1.6011	1.6015	117.61	117.65
o/n	1	1				
t/n	2	2			-1	-1
s/n	1	1				
1 week	5	6			-5	-6
1 month	24	25	5	6	-28	-26
3 months	74	76	15	17	-82	-76
6 months	149	150	34	39	-172	-168
1 year	290	293	46	53	-351	-340

Forward Points

In the professional market, forward prices for currencies are quoted as an offset to the spot price and forward dates are always offset from the spot date. Suppose we want to trade EUR|USD 3 months forward. The quotes given above were valid on 19 August 1999, which was a Thursday. The spot date would be Monday 23 August 1999. The 3 month forward date means 3 months from 23 August, which would be Tuesday the 23rd of November. The two quotes together:

Spot	1.0537	1.0543
3 months	74	76

determine the forward price in the following way. These are the market-maker's quotes. Let's look at it from the market-maker's point of view. The trader is willing to pay USD1.0537 for EUR1.0000 or receive USD1.0543 for EUR1.0000 for the spot date. For the forward date, we add the forward points:

Bid for EUR in 3 months:	1.0537
	0.0074
	1.0611

Offer for EUR in 3 months:	1.0543
	0.0076
	1.0619

So the market for EUR|USD in the 3 month forward is:

1.0611 | 1.0619

The values 74 and 76 are called forward points, while the prices 1.0611 and 1.0619 are called the forward outrights. We'll discuss the terminology later. The point is that to arrive at the forward price we begin with the spot price and adjust it with the forward points. The convention in the way the forward points are quoted is that the rightmost digit of the forward points is aligned with the rightmost digit of the spot price. That is how we knew that 74 really meant 0.0074.

Now suppose we also wanted to trade EUR|JPY, this time 6 months forward. The spot quotes are 117.61 bid and 117.65 offered. The forward points are –172 and –168, so the forward outrights are:

Spot	117.61	117.65
6 months	-1.72	-1.68
	115.89	115.97

Once again we lined up the rightmost digit of the forward points with the rightmost digit of the spot price, so – 172 became – 1.72 and – 168 became – 1.68. Notice that because the forward points are negative, the forward outright is less than the spot price. You may sometimes see the forward points quoted without the minus signs as:

6 months	172	168

In this case the forward points look like they are reversed because, apparently, the larger number is first. When you see this, it simply means that the points are negative so you should subtract 172 or 168 from the spot price rather than add. Pricing screens these days appear to be more user-friendly than they used to be.

Exercise
What are the 3 month and 6 month forward outrights for GBP|USD?

There are some other parts of the table that need some explanation. The second line is labelled O/N, the third is T/N and the fourth is S/N. These mean, respectively,

O/N overnight/next
T/N tomorrow/next
S/N spot/next

These are quotes for 1 day (or one night, if you prefer) "swaps" in which the trader will receive a currency and repay it the next day versus another currency. For example, in an overnight swap of S/N the trader might have these cash flows:

Spot/Next Swap

Date	Spot	Next (business day after the spot date)
GBP	–	+
USD	+	–

This means that on the spot date the trader will pay out GBP and receive USD, then on the next business day will receive GBP and pay USD. In effect the trader is simultaneously lending British Pounds and borrowing US dollars for a one day period, starting with the spot date. A spot/next swap is useful for cash management purposes as are the other one day swaps, O/N and T/N. As an example, suppose the GBP|USD trader comes to the end of a trading day and has a net position of:

Spot +GPB10 -USD16

Because, after a full day's trading sometimes buying GBP against USD and sometimes selling, all the intra-day positions have offset one another except for this "residual" amount. Even if the trader does not mind carrying this position overnight, along with the risk it entails, the trader will still have to manage his/her cash position. On the spot date the trader will be taking delivery of GBP10 million and will have to pay out USD16 million. In order to "roll" this position one day, the trader would pay out the GBP and receive USD on the spot date, thereby reversing the position for the trading book. Then on the next day the trader would receive GBP and pay USD. This means that the trader's position has moved forward one day. What good does this do? A time line is helpful in understanding this (but be prepared for confusion!). We'll label today as day 0, so that the spot date is day 2 and the day after the spot date is day 3.

0	1	2	3
Today	Tomorrow	Spot	
	Today	Tomorrow	Spot

Remember that any trades that are still open at the end of day 0 will not settle until day 2, which on day 0 is the spot date. From the point of view of day 0, day 1 is tomorrow, day 2 is the spot date and day 3 is the next day after the spot date. On day 0 we assume our trader ends up with a position that is +GBP10 million and – USD16 million which will settle on the spot date (day 2). Assume also that our trader has no cash balances in either currency. The trader knows that when day 2 comes, he will receive GBP10 million into his nostro* account. Simultaneously he will have to pay out USD16 million to the counterparty. What will the trader do with the pounds and where will he get the dollars? He could go to the bank's funding desk, borrow dollars from them and lend them sterling. Instead the trader could call up the forward desk and do a Spot/Next swap. To make the trade more explicit, let's assume the forward desk is quoting a +1 pip "choice" price for internal trades. The trade the spot trader wants to do is:

Spot Date -GBP10,000,000 +USD16,000,000
Next Date +GBP10,000,000 -USD16,001,000

Where did the extra USD1,000 come from? Since the S/N quote was +1 pip, the price the trader pays in USD was 1 pip higher than the spot price. The spot price was USD1.6000 for each GBP, so 1 pip higher is 1.6001. It looks like the trader is being charged for this trade, and in fact he is. Why? We'll answer that question in detail later but the brief answer is that he is lending a currency (GBP) with a lower interest rate than the currency he is borrowing (USD).

*Nostro is the term for the account the trader keeps at a bank where he can receive or pay a particular currency like USD or GBP.

GBP			+10,000,000 **-10,000,000**	**+10,000,000**
USD			-16,000,000 **+16,000,000**	**-16,001,000**
	0 Today	1 Tomorrow Today	2 Spot Tomorrow	3 Spot

The net effect of all this activity is that on day 1, the trader starts the day with an open position for day 3, which is the new spot date. If the trader does not want to have the spot position, he can now trade out of it on day 1 using the spot market rather than the forward market. Note that the trader has accepted overnight risk, because he had a long GBP, short USD position from day 0 to day 1. If GBP weakens relative to USD overnight so that the opening price on day 1 is lower e.g., GBP|USD = 1.5990, then if the trader marks his position to market, or if he enters into an offsetting trade, he will have a loss of USD11,000 for the spot date:

GBP				-10,000,000 **+10,000,000**
USD				+15,990,000 **-16,001,000**
	0 Today	1 Tomorrow Today	2 Spot Tomorrow	3 Spot

Spot Trading versus Forward Trading

A spot foreign exchange trader tries to make a profit from trading spot. How does the trader do this? Part of the trader's profits come from the bid-offer spread. In the spot market, spreads tend to be very narrow for major currency pairs. It is not easy to make profits from bid-offer trading – which is also called providing liquidity – because the spot price doesn't stand still while trading is going on. The trader may very well buy on the bid and sell on the offer, but if the market moves against her position in between trades, she could sustain a loss greater than the spread. Still, when the market is moving and a lot of trading is going on, there is an opportunity for a nimble trader to make some money.

A second source of profits is through taking intra-day positions. In this case, rather than trying to immediately reverse open positions, the trader intentionally keeps a position that is long one currency and short another. The trader is making a bet, or to put it more politely, is predicting that the currency she is long will strengthen relative to the currency she is short. The source of this prediction may be from technical analysis i.e., by looking at charts and price trends, or from fundamental analysis i.e., analysing the basic macroeconomic forces driving the exchange rate. Other sources of trade ideas can come from observing trading activity during the day – a trader in a large dealing operation sees a lot of the activity in the market. Finally there may be a significant event about to happen in the market such as an interest rate announcement or an important economic number (trade balance, non-farm employment, inflation) and the trader may try to position herself to take advantage of the market's reaction to the event. Spot dealers generally limit their risk to one trading day. At the end of the day they want to go home "flat" i.e. with no open position. In major investment banks, the trader may well "pass the book" to another trader in a different time zone, since the foreign exchange market hardly ever stops. It then becomes the new trader's responsibility to take over the risk in the currency pair.

Forward traders, in contrast, are not expected and generally are not allowed to take spot risk. In fact a forward trader is not a foreign exchange trader at all, but rather an interest rate trader. In UBS as in many banks, forward trading is part of the short term interest rates business. A forward trader makes money through the bid-offer spread in the forward points. The bid-offer spread in the spot market represents the cost to the forward trader of hedging his spot foreign exchange risk. So when you look at the quote for a forward outright, such as the EUR|USD we determined earlier,

1.0611 | 1.0619

it may look unusually wide to you. After all the spot quote was only 6 pips wide and the forward points were 2 pips apart, but the forward outright quote is 8 pips wide i.e. the sum of the two bid-offer spreads. So it seems that a client is "giving up" both spreads. Essentially this is true, but the reason is that the client is forcing the forward trader to take a spot risk that he doesn't want. We can see this better after the much-promised discussion of interest rate parity.

Interest Rate Parity

Interest rates link the spot and forward currency markets. If a bank agrees to sell EUR forward against USD, how can it decide on the price? Unless the bank simply wants to take a position on the future value of EUR|USD, it would not want to commit itself to such a transaction without hedging i.e. protecting itself from loss.

The diagram below shows a bank's position if it has agreed to sell EUR|USD in the three month forward:

	EUR	USD
Today	0	0
3 months	–	+

The negative sign indicates that the bank has a short EUR position and the positive sign indicates that it has a long USD position. The bank is currently exposed to a strengthening of the euro relative to the dollar.

One possible way for the bank to hedge this exposure would be to buy EUR today in order to have them to sell in the future. In order to buy the EUR it could borrow dollars and sell them for EUR. In that way the bank would have an offsetting position three months from now. In three months it has the EUR it bought and can deliver them to the customer. Also, the bank owes dollars but can repay its loan with the dollars it receives from the customer.

	EUR	USD
Today	+	–
3 months	–	+

Let's work this out with the following information. To make the calculations a little easier to handle, we will not take into account the bid-offer spreads on either the exchange rate or the interest rates (but we'll let you do it in the exercises!)

Spot: USD: 1.0540
3 month rates: USD: 5.43%
EUR: 2.65%

If the transaction were done on 19 August 1999, then measured from the spot date of 23 August 1999, the three month period is 92 days in length.

Suppose the bank has agreed to receive USD1,061,900 in exchange for EUR1,000,000 from the customer in three months. Today the bank could buy the present value of EUR1,000,000, which is:

$$\frac{1{,}000{,}000}{1 + 0.0265 \times {}^{92}/_{360}} = \text{EUR}993{,}273.33$$

it will buy these EUR by borrowing enough USD to pay for them at the spot rate:

$$993{,}273.33 \times 1.0540 = \text{USD}1{,}046{,}910.09$$

The bank will have to repay it's USD loan in 3 months:

$$1{,}046{,}910.09 \times (1 + 0.0543 \times {}^{92}/_{360}) = \text{USD}1{,}061{,}437.72$$

Thus a fair quote for the forward would be to receive USD1.0615 for each EUR. Thus the bank has a small profit on its forward trade. In terms of forward points this is:

$$1.0615 - 1.0540 = 0.0075 \text{ i.e. } 75 \text{ points}$$

Notice that this is in excellent agreement with the actual forward points of 71-79. This example was an illustration of the interest rate parity relationship:

Interest Rate Parity

$$F = S \; \frac{1 + r_{foreign}t}{1 + r_{domestic}t}$$

where: F is the forward price

S is the spot price

$r_{foreign}$ is the foreign interest rate (rate for the numerator currency)

$r_{domestic}$ is the interest rate for the quoted currency

t is the time to the forward date

If we used the formula with the data above, we would get:

$$F = 1.0540 \times \frac{1 + 0.0543 \times {}^{92}/_{360}}{1 + 0.0265 \times {}^{92}/_{360}} = 1.0614$$

Interest rate parity is often represented diagrammatically by a box:

Interest Rate Parity

$$\begin{tikzcd} 1 \arrow[r] \arrow[d] & S \arrow[d] \\ (1+r_d t) \arrow[r] & S(1+r_f t) \end{tikzcd}$$

In the diagram 1 represents one unit of the domestic* currency and S is its price in the foreign currency. We could borrow one unit of the domestic currency today at the domestic interest rate and exchange it for S units of the foreign currency and invest them at the foreign interest rate. If we equate the amount of domestic currency we owe at the end of the period to the amount of foreign currency we receive, we get the interest rate parity formula.

Today: Borrow 1 unit of the domestic currency
Buy S price amount of the foreign currency
Lend the foreign currency

Forward: Owe $1 \times (1 + r_d t)$ of the domestic currency
Have $S \times (1 + r_f t)$ of the foreign currency

If F is the forward price (units of the foreign currency) for one unit of the domestic currency, then to make the values equal:

$$F(1 + r_d t) = S(1 + r_f t)$$

$$F = S \times \frac{(1 + r_f t)}{(1 + r_d t)}$$

* The term "domestic" is conventional. What we really mean is the currency that is one unit and the other currency is used as the price. These are sometimes called the quoted currency and the countercurrency, but that hardly adds anything to the discussion.

Interest rate parity has another simple interpretation. If interest rates are higher in a given currency, then investors would want to sell their own currency and invest in the higher yielding currency. For example if rates are 5.43% in the U.S. and 2.65% in Europe, European investors would rather earn the higher USD rate. To do this they have to sell euros and buy dollars. At the end of their investment period they will have earned the higher rate but are now faced with the problem of turning the dollars back into euros. If the exchange rate has not changed, the investor has truly earned the higher dollar rate. In fact, this can happen and a European investor could earn a higher rate in this way. However if the dollar has weakened relative to the euro, the investor may lose some of the additional interest earned on the dollars when they are sold for euros. It could even happen that the investor is worse off than if he had just invested in euros to start with. Looked at this way, foreign exchange can be considered an asset class, just like equity. You can invest in it and earn a return, but that return will be accompanied by risk. The investor has to decide whether the expected return is sufficient for the risk taken on.

Interest rate parity tells us the forward exchange rate that would leave the investor indifferent to investing in dollars or euros for that same time period. It is neither a prediction nor a guarantee. However, it is how the market prices forwards. Forward rates will trade in line with interest rate parity.

Forward Points Revisited

Now that we understand interest rate parity we can explain the idea behind the forward points we discussed earlier. If dollar rates are higher than euro rates, then interest rate parity shows us that it will take more dollars to buy a euro in the forward than it does in spot. Therefore the forward points will be added to the spot price. Another way to express this is to say that the market price is the price of the euro. So a spot price of 1.0540 means it costs USD1.0540 to buy EUR1. A further cost to buying the euro is giving up the higher dollar rate and earning only the lower euro rate. Thus the forward price can be thought of as:

Forward = Spot + Costs of Holding Spot – Benefits of Holding Spot

The cost minus the benefits term is the basis. It is positive because dollar rates are higher. The market also expresses this by saying that the euro is at a premium in the forward i.e. costs more dollars, or that the dollar is at a discount (worth fewer euros).

For the same reason, the EUR|JPY forward points are negative. The EUR is the quoted currency. If we buy EUR with JPY, we give up the yen rate (0.0575%) but gain the euro rate (2.65%). Thus the costs of spot (0.0575%) are less than the benefits (2.65%) of spot, so the basis is negative. The EUR is at a discount in the forward (JPY is at a premium) so we have to subtract the forward points. This is reflected in the quote for the forward points – 82 | -76. Notice that by subtracting the forward points from the spot, the bid-ask spread in the forward is 9 pips wide, while the spot spread is 4 pips wide:

117.61	117.65
–.81	–.76
116.80	116.89

If we had added the points (which would have been incorrect) the bid-ask spread in the forward would have been 1 pip wide i.e., narrower than spot. It is always true that the bid-ask spread in the forward is at least as wide as the spread in spot. Verifying this helps to prevent careless errors.

Currency Futures

Forward contracts are off-exchange agreements between two parties to conduct a transaction at some point in the future, at a price that is agreed upon today. These agreements usually cannot be offset i.e. nullified, and so generally result in an actual transaction. They can be for any amount of the underlying commodity and for any delivery date. Usually no upfront payment is required, but credit lines and sometimes collateral need to be in place. By contrast, futures contracts, as we discussed earlier, are traded on organised exchanges. They are for fixed amounts of the underlying commodity with rigid specifications as to quality if actual delivery should occur. Only a small number of delivery dates are available e.g. just four per year for most contracts on the U.S. exchanges. Since the futures markets are mostly used for price hedging purposes, actual delivery is not common. Futures contracts exist for many currency pairs with non-dollar futures being the most common i.e. futures to buy or sell currencies against the U.S. dollar. In contrast to the inter-bank market, the futures exchanges almost always express the foreign currency in terms of USD. So a typical price for the Swiss franc future on the CME would be quoted as 63.25, which would mean USD63.25 = CHF100. In the inter-bank market, the equivalent quote would be USD|CHF = 1.5810. The principal reason for this method of quoting is that the traders in Chicago prefer to have their mark-to-market settlements in USD. The contract sizes are mostly arranged so that a one ”tick” move is worth USD25 or USD12.50, depending on the currency.

Example

On the IMM (International Monetary Market) of the Chicago Mercantile Exchange, the December 1999 futures contract on EUR was due to expire on 13 December 1999. The EUR futures contract calls for delivery of EUR125,000. Naturally, there is only one ”quality” of euro that can be delivered. The settlement price on 19 August 1999 was USD1.0725. Suppose a trader bought the contract at that price. The settlement price on the next day was USD1.0761. The trader has a profit on the position of:

$$(1.0761 - 1.0725) \times 125000 = \$450$$

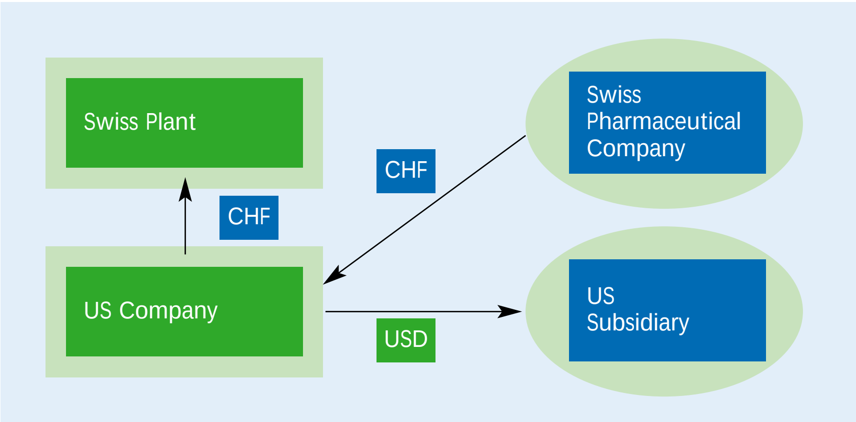
This is the daily mark-to-market of the position. The exchange will debit the margin account of traders who were short the contract and credit the account of traders who are long. So our trader would receive an inflow of USD450 to his margin account. The process takes place every day. Each trader must keep an amount in margin that is sufficient to pay the daily settlement mark-to-market. If the amount in the trader's account drops below the maintenance level set by the exchange, the trader gets a margin call from the exchange. If the margin is not replenished, the exchange will close out the trader's position. For the ECZ9* contract the required amount of margin was USD2106 for "speculators" and USD1560 for "hedgers".

Futures prices trade in line with spot prices and interest rates. This means that in markets like foreign exchange where futures are priced through arbitrage, the price of the future today has no information in it about what the price of spot in the future will be! The futures price is just the forward price of spot using today's spot price and interest rates. This has implications for using the futures market to hedge. If a European corporate with USD receivables in December 1999 sells USD forward or buys the ECZ9 contract, then the price the corporate sells its dollars for in December has been determined so they have no price uncertainty. However, if the corporate does this all the time as a strategy i.e. they sell their USD receivables forward every quarter (or whatever their hedge period is), then the volatility of their revenues in USD will be just as great as if they didn't hedge at all. This is because the futures price, or the forward price, has the same volatility as the spot price does**. The futures price behaves very much like the spot price with a time lag.

Currency Bond Swaps

Lastly, we want to discuss the FX analogs of interest rate swaps. There are two basic types of currency swaps involving the periodic exchange of currency payments. They are distinguished by whether or not an exchange of principal occurs.

Historically, bond swaps arose through the need for back-to-back or parallel loans. Suppose a Swiss pharmaceutical manufacturer has a U.S. subsidiary. Suppose the U.S. subsidiary needed dollar funding at the same time that a U.S. based company decided to build a plant in Switzerland. The U.S. company could lend dollars to the Swiss company's subsidiary and the Swiss manufacturer could lend CHF to the U.S. company. This would occur in the form of a parallel loan.



---

* ECZ9 is the Euro Currency contract ending on 13 December (Z) 1999.

** If interest rates are random and correlated to the exchange rate then the returns could be systematically more or less volatile than the spot, but the point is the same

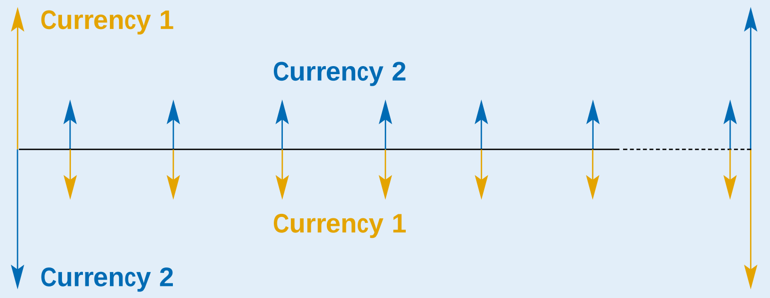
Determining the proper interest rates is straightforward. Each company is simultaneously buying a bond from the other. The coupons on those bonds should be set at the company's required cost of funds for the swap's maturity i.e. the rate of interest the company would have to pay in its domestic market to raise the funds.

On the value date of the swap, the counterparties exchange the principal amount of the swap. On each coupon date they exchange currency payments. These may be netted so that only one party makes a payment. At maturity the parties once again exchange the full principal amount of the swap. This final exchange plays a critical role in the swap. We will discuss this shortly in the context of an example. First, let us look at this type of swap in general terms.

Currency Swap – Fixed for Fixed

The mechanics and pricing of this type of swap are straightforward, but there are some interesting features. In this swap, the parties exchange principal and interest payments from fixed rate debt in two currencies. Graphically:

A Currency Bond Swap – Fixed for Fixed



The twist is that, initially, the amount of currency 1 is equivalent to the amount of currency 2 at the current spot exchange rate. But as we know, unless interest rates in the two countries are identical, one currency must weaken with respect to the other in the forward. So the amounts to be exchanged at maturity, which are the same as the amounts exchanged today, are not equivalent at today's forward price.

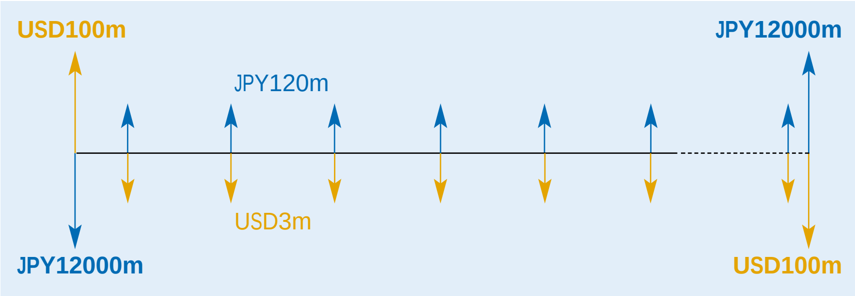
The coupons on the swap must be the par coupons for that maturity in the currency of the coupon. The reason for this is that effectively, one party is simultaneously buying a bond from and selling a bond to the other. The two bonds must be worth exactly the same at the time that the swap is made if no upfront payment is required. Since the notional amount and the actual amount of principal are the same, each bond must have a value equal to its face amount i.e. the coupon must be the par coupon.

Example:

Ten-year U.S. Treasuries are yielding 6%, while ten-year JGBs (Japanese Government Bonds) are yielding 2%. The current spot rate is JPY120 = USD1. A ten-year, semi-annual $100 million swap would have these cash flows:

A Currency Bond Swap – Fixed for Fixed

10 Year Semi-annual
USD|JPY = 120
USD Rate = 6%
JPY Rate = 2%



Today the principal amounts have the same value and although the total of all coupons together with payment of principal at maturity have the same present value, none of the individual payments are necessarily equal. The actual present values of each coupon exchange could be determined by using the spot rate curves in the two currencies; this would be the preferred method. But we can get a rough idea of the values by discounting each payment using the bond's yield (i.e. 6% on the dollar side and 2% on the yen side. For example, the first JPY coupon payment in six months time, is worth:

$$120/1.01 = \text{JPY}118.81$$

and for the dollar side:

$$3/1.03 = \text{USD}2.91$$

At today's spot rate of JPY120, the dollar coupon is worth JPY349.20. So, at the beginning of the swap, the JPY payer is getting a coupon in dollars worth more than the yen being paid out. Now consider the coupons due in four years. Their present values in JPY at today's spot rate are:

$$\frac{120}{(1.01)^8} = \text{JPY}110.82$$

and again on the dollar side:

$$\frac{3}{(1.03)^8} = \text{USD}2.37 = \text{JPY}284$$

The dollar payment is still worth more than the JPY payment, but not by as much. However, the value of the last payments are:

$$\frac{12120}{(1.01)^{20}} = \text{JPY}9932.88$$

and

$$\frac{103}{(1.03)^{20}} \quad \text{USD}57.02 = \text{JPY}6843.43$$

In this swap, the dollar payer is making net payments to the JPY payer until the very end, when the JPY payer pays more. Among other things, this points out that the dollar payer has considerable risk in the swap since the net benefit comes at the end. You should also realise that the calculations we did above are only in present value terms. Even though U.S. rates today are greater than JPY rates today, we cannot actually predict the future value of the dollar. It could happen that at some point in the future, the dollar coupon of USD3 will be worth less than the JPY120 coupon i.e. the dollar could weaken to less than JPY40, in which case the JPY payer would actually be called upon to make payments to the dollar payer. Swaps are not riskless! In this case, the dollar payer has sold a dollar bond and bought a yen bond. If the yen strengthens relative to the dollar, the swap becomes valuable to the dollar payer.

Currency Swap – Fixed for Floating

A second type of currency swap would be an exchange of a fixed rate in one currency for a floating-rate in another. However, this is actually nothing new once one understands the following simple principle:

LIBOR = LIBOR

Meaning? If today's spot rate is GBP | USD = 1.7500 then a swap of six month dollar LIBOR on USD1,750,000 versus six month Sterling LIBOR on GBP1,000,000 should trade ”flat” i.e. no spread needs to be taken on either side. A market-maker might quote this as -1|1 i.e., would pay U.S. dollar LIBOR – 1 basis point and receive Sterling LIBOR or would pay Sterling LIBOR and receive dollar LIBOR + 1 basis point. However, this ideal is rarely achieved. Because of conditions that frequently occur in the spot foreign exchange market, this type of basis swap, rarely trades flat. However, we will ignore this for now.

Given this principle of LIBOR = LIBOR, a swap of a fixed rate in GBP versus floating in dollars is the sum of two swaps:

Swap 1 pay fixed GBP and receive Sterling LIBOR

Swap 2 pay Sterling LIBOR receive dollar LIBOR

The end result is to swap the current Sterling fixed coupon versus US dollar LIBOR flat, with the notional amounts adjusted by the current spot rate. Remember that in a currency swap, principal amounts are assumed to be exchanged at the time the swap is established and at maturity.

Example

The 10 year GBP swap rate is 7% and USD1.7500 = GBP1

A 10 year swap of fixed rate Sterling versus six month dollar LIBOR on GBP20,000,000 would have the following cash flows:

Today:	Pay USD35,000,000 Receive GBP20,000,000
Every 6 months:	Pay GBP700,000 Receive six month LIBOR on USD35,000,000
In 10 Years:	Pay GBP20,000,000 Receive USD35,000,000

Just as before, we could calculate the present value of each exchange. If Sterling rates are higher in this example than US dollar rates, we would expect that Sterling would weaken relative to the dollar in the forward. Therefore, the final exchange is highly favourable to the Sterling payer, in present value terms at least. The intermediate payments are all done at implied spot rates that are determined by the dollar LIBOR curve. In an upwardly sloping yield curve environment, the dollar payer will be receiving more Sterling per dollar at the beginning of the swap than at the end since the dollars paid are determined by increasing interest rates. Thus, on this swap, the initial payments are favourable to the dollar payer but become less so through time. Finally, at the end, the last exchange is highly favourable to the Sterling payer. This final exchange is essential to making the swap work as it was described.

Currency Coupon – Only Swap

What if we wanted to exchange a series of currency payments but did not want to have an exchange of principal? This type of swap arises in the structuring of bond issues and in the professional market. Once again we will discuss this in terms of an example.

Example:

Suppose we wanted to price a ten-year U.S. dollar – JPY swap with a notional principal on the dollar side of USD100 million but with no exchange of principal. If, as above, U.S. Treasuries are yielding 6% and JGBs are yielding 2%, we can price the swap as follows.

A 2% coupon, ten-year JPY bond is worth par. Since the repayment of face value at maturity is worth (assuming semi-annual payments):

$$\frac{12000}{(1.01)^{20}} = 9834.53$$

The coupon stream must be worth 12000 – 9834.53 = JPY2165.47

If we did the same to a dollar bond we would find that the repayment at maturity on a 6% dollar bond, with a face amount of USD100 million is worth:

$$\frac{100}{(1.03)^{20}} = 55.37$$

So the coupon stream = 100 – 55.37 = USD44.63.

Since we would not exchange USD44.63 for JPY2165.46, we need to increase the notional amount of the JPY side. The JPY value of the dollar coupons is (at the current spot rate of JPY120):

$$44.63 \times 120 = \text{JPY}5355.89$$

Thus, the notional on the JPY side should be:

$$\frac{5355.89}{2165.47} \times 12000 = \text{JPY}29680\text{m}$$

The reason the JPY side must be larger than JPY12000 million, the spot equivalent of USD100 million, is that there is no final exchange of principal. Recall that in our example of a bond swap, the final exchange "made up" to the dollar payer for the coupon exchanges that had been made at an off-market forward rate. Since there is no final exchange, the dollar payer must get more JPY at each coupon exchange.

Another View

We can look at a coupon – coupon swap as being the exchange of annuities in two currencies. If we want the dollar annuity to consist of 20 semi-annual payments of 3 million dollars, then the JPY payments will have to have the same present value, calculated at today's spot rate. We can do this on the HP12C:

Using the HP12C

Keystrokes	Meaning
20 n	Number of annuity payments
3 PMT	Each one is USD3
0 FV	No "principal" payment
3 i	USD interest rate (6 months)
PV –> – 44.63	Present value of USD annuity
120 x –> – 5355.89	Multiply by spot rate to get value in JPY
PV	Set this equal to PV
1 i	JPY 6 month interest rate
PMT –> 296.80	Annuity payment in JPY

Example – A Reverse Dual Currency Samurai Bond

What? These bonds have been popular in the Japanese market. They are sometimes called "holiday" bonds.

Bond Terms:

Price	JPY10 billion
Maturity	10 years
Redemption Value	JPY10billion
Annual Coupon	AUD3.1million

The term dual currency refers to the fact that the bond has payments in two different currencies. It is a Samurai bond if it is issued in Japan by a non-Japanese issuer and it is reverse if the coupon is in the foreign currency. The reason for calling this a holiday bond is that, when you receive the coupon payment, you could use it to go on holiday in Australia.

Pricing of the bond

Suppose that 10 year rates in Japan are 2% and 10 year rates in Australia are 6%. Then a 10 year JPY bond with a 2% coupon would be worth par (100%). Suppose that the bond had a principal amount of JPY10billion. The final payment of JPY10billion has a present value of:

$$\frac{10,000,000,000}{1.02^{10}} = \text{JPY}8,203,482,999$$

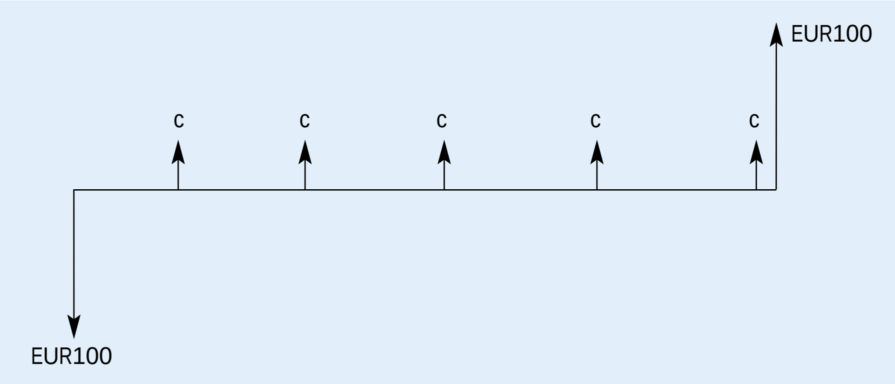
So the coupon stream needs to be worth JPY1,796,517,001. Suppose the current exchange rate is JPY78 = AUD1. Then the coupon stream is equivalent to AUD23,032,269. Finally, a 10 year annuity in AUD that has a present value of AUD23,032,269 has an annual payment of AUD3,129,347 (using the HP12). Thus the coupon of AUD3.1million is about right.

Exercise

Suppose that EUR 5 year rates are 3%, USD 5 year rates are 5% and USD1.0500 = EUR1. What should be the coupon on a 5 year bond with a face amount of EUR100 million that redeems at par in EUR but which has a coupon in USD?

Exercise – Solution

The bond looks like this:



The present value of the final cash flow is

$$\frac{100}{1.03^{5}} = \text{EUR}86.26$$

So the value of the coupon stream is EUR100-EUR86.26 = EUR13.74.

This value in USD is 13.74 x 1.0500 = USD14.43. We need to solve for the USD coupon. Since we are assuming that the USD interest rate is 5% for every maturity, we can use the HP12C:

PV	-14.43
i	5.00
n	5
FV	0
Solve for PMT	3.33

So the USD coupon should be 3.33%.

Appendix 1
Interest Rate and Bond Taxonomy

US Treasury Bond

Bonds are auctioned periodically by the US Treasury. They are backed by the "full faith and credit" of the US Government. The market considers them to be free of default risk. Original maturities of the bonds are 30 years. The bonds have a semi-annual coupon and a coupon rate which is set after the auction (see below). The bonds are sold in denominations of USD10,000 and more. There are no physical certificates for these bonds. Instead ownership is maintained through registration.

US Treasury Note

There is no difference between a bond and a note other than the original maturity. A note has an original maturity of 10 years or less. The US Treasury auctions off 2, 3, 5 and 10 year notes. They all have semi-annual coupons.

T-Bill

A treasury bill or T-Bill is a discount security. The owner pays less than the face value for the bill and receives the full face value when the bill matures. The difference between the price paid and the face value is the implied interest received. T-Bills have original maturities of 13, 26 or 52 weeks. T-Bills that mature on the same date are fungible, even if they were originally issued on different days.

Auction

The US Treasury sells its debt through an auction process. The participants in the auction are the 40 or so "primary dealers" in government bonds. UBS is one such dealer. Each dealer will bid a yield and an amount for each new issue. For example if the Treasury is auctioning new 5 year notes, a dealer might bid 5.75% for USD100 million face value. This means that the dealer wants to buy 100 million face value of the new issue and wants to receive a yield of 5.75%. Since yield and price move in opposite directions, the higher the price the lower the yield. Also the price of a bond whose yield is the same as its coupon rate is par, or 100. So if the new 5 year bond has a coupon of 5.75% and the dealer's bid is accepted, the dealer will pay USD100 million for the bonds. If another dealer bid 5.70% for 50 million and that dealer's bid was also accepted, the second dealer would be required to pay a higher price, since the bid was lower. The price of USD50 million face value of the bond would be USD50,107,448. A third dealer who bid 5.80% for USD20 million would pay less than par, if the bid was accepted. But that bid may not be accepted if the Treasury can sell all of its debt at a lower yield. The coupon rate on the note is not set until the auction is completed. The Treasury then sets the coupon so that the price of the bond will be par for most of the accepted bids.

CUSIP number

Most securities traded in the US can be identified by a CUSIP number, which is unique to the security. That is how one knows which security one owns. There is a special circumstance with US Treasury bonds though. The bond itself has a CUSIP number of course. However, each coupon on a bond has its own CUSIP number and so does the final payment of the face value of the bond. On the day the bond matures it will pay both its final coupon and its face value. However these two securities have different CUSIP numbers. A second point involves the coupons from different US Treasury bonds. If two coupons are paid on the same date, then they have the same CUSIP number. That means it is impossible to distinguish between them. For example two bonds such as the 8's of Nov 05 and the 12s of Nov 02 would both pay a coupon on 15/05/02. Both coupons have the same CUSIP number. However consider the 7's of Nov 05 and the 8's of November 05 which both mature on 15 November 2005. On that date both bonds pay a coupon. The coupons are indistinguishable. Both bonds repay their face value. The principal repayments are distinguishable, because they have different CUSIP numbers.

Treasury Strip

The US Treasury allows the owner of a bond to sell the individual coupons from the bond as separate securities. Each individual coupon is called a strip. Imagine that you buy USD100 million face value of the 8's of Aug 2008. Then you will receive USD4 million on 15/02/98, 15/08/98, 15/02/99, the 15th of February and 15th of August every year, until 15/08/05. Also on 15/08/05 you will receive USD100 million from the repayment of the bond's face value. You can sell any amount of these securities, up to the amount you are due to receive (in multiples of USD10000). For example you could sell USD3 million of the coupon due on 15/08/02 and 2 million of the coupon due 15/02/03, USD25 million of the repayment of face on 15/08/05.

T-Bill Futures

Are traded on the IMM (International Monetary Market) of the CME (Chicago Mercantile Exchange). The face amount of the future is USD1000000. The contract calls for the delivery of USD1,000,000 face value of a 13 week T-Bill. The future is quoted as 100 – discount. If a person is long the future at a price of 95, they will pay (1 — 0.05 x 91/360) x 1,000,000 = USD987,361.11 for the bill when the contract matures.

Eurodollar Interest Rate Futures

Are also traded on the CME. The face value is USD1000000. The future is quoted as 100 – rate. The contract is settled in cash on expiry. If a person is long the future at 94, and the future expires at 95, the person will receive 100 x 25 = USD2,500 on expiry. Each basis point (0.01) on the futures contract is worth USD25. This can be rationalised as 0.0001 x 1,000,000 x 90/360 = 25. However the real reason is that the exchange wants it that way.

LIBOR

London Interbank Offered Rate – this is the rate at which a money centre bank will lend dollars to another bank. The rate at which the bank would borrow dollars is called LIBID and the average of the two is called LIMEAN.

FRA

Forward Rate Agreement – an agreement between a bank and a customer based on a notional underlying amount. The smallest amount is generally USD1000000. The FRA is quoted as a rate e.g. 6.00% and for a specific period of time e.g. 3x6 meaning a 3 month rate starting in 3 month's time. If the buyer pays 6.00% for the FRA and the rate at the maturity of the FRA is 6.20%, the buyer will receive a payment from the bank of 20 basis points on the notional amount. The details of the calculation are a little complicated.

Reset

Each day there is a poll of major London banks to determine their LIBOR rate. The rates are averaged in a certain way and the resulting value is that day's LIBOR reset. This number is used to settle FRAs and set the rates for interest rate swaps.

Interest Rate Swap

An agreement to exchange net interest payments on an underlying notional amount based on a variable rate and a fixed rate. The fixed rate is set at the beginning of the swap. The variable rate is usually LIBOR. The two payments generally are due on the same date and have the same "tenor" – the amount of time in each interest rate period. The most common tenor is 6 months. Interest rate swaps are quoted in terms of the fixed rate. That is, if the swap is quoted at 6.00% versus 6 month LIBOR, then the fixed rate payer would pay 6.00% semi-annually and the floating rate payer would pay 6 month LIBOR. That LIBOR rate could be different each time it is reset.

TED Spread

The difference in interest rates between the US Treasury and the Interbank market. Usually quoted as a number of basis points. For example if the UST 2 year note is yielding 6.00% and the 2 year swap rate is 6.25%, the 2 year TED spread is 25 basis points. (TED stands for Treasuries versus EuroDollars). There is a TED spread for every maturity. The difference between the T Bill future and the EuroDollar future is one such spread. It is actually a forward spread since it represents the difference between the 3 month LIBOR rate at the expiry of the contract and the 3 month T-Bill discount rate for the same period (more or less). So if the T-Bill future is 94.00 and the ED future is 93.70, the 3 month TED spread is 30 basis points.

The TED spread is one of the most actively watched rates in the market. You should think of it as being like an underlying security itself. The difference is that in order to "buy" or "sell" it you have to trade a variety of instruments. When traders say they have bought the spread, they mean they are paying the swap rate and receiving the Treasury rate. For example a way to buy the 2 year TED spread would be to enter into a 2 year swap as the payer of the fixed rate and simultaneously buy a 2 year Treasury note. You are then paying the swap rate and receiving the Treasury rate. In order to do this you would have to finance the bond. See the entry for repo. If you buy the TED spread, then you make a profit if the spread increases i.e. the LIBOR rate goes up, the Treasury rate goes down or some other variation such as the LIBOR rate goes up more than the Treasury rate goes up. As this rate is sensitive to credit issues, the spread tends to increase when banks or other swap counterparties are viewed as having increased risk.

Repo

Bond positions are financed in the repo market, which is a market for borrowing and lending cash with bonds as collateral. The largest and most liquid of these repo markets is the one for US Treasury bonds. If you want to lend cash, you can receive a Treasury bond as collateral. If you want to buy a bond, you can lend it out in the repo market and receive the cash to fund it. Buying a bond and funding it by lending it out is called a repo (repurchase agreement). Lending cash and receiving a bond as collateral is called a reverse-repo. The rate of interest paid to finance a bond is the repo rate for that bond. It can be different depending on the particular bond. General Collateral refers to a group of bonds all of which have the same repo rate. A bond is "special" if its repo rate is lower than the rate for general collateral. This usually happens when traders want to borrow the bond and the supply is small relative to the demand for borrowing. Like other interest rates, there are different repo rates depending on the term (maturity) of the repo. The repo could be for overnight or one week or longer. Anything more than overnight is called term repo.

Appendix 2
Day Basis

A bank quotes a rate of 8% simple interest. If we leave money in this account for 90 days, how much money will the bank pay us? The amount depends on the day basis, a seemingly minor point but crucial to the correct calculation of interest. There are several methods of calculating simple interest in common use. Here are some of them.

1. Actual/360

This is often referred to as "money-market" basis. In this method, interest is calculated as the actual number of days the money is on deposit divided by 360:

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \frac{\text{Actual Days}}{360}$$

Example: If we deposit USD1,000 for 90 days in an account paying 8% using this rate basis, our interest is:

$$1000 \times 0.08 \times {}^{92}/_{360} = 20$$

If we instead leave the amount on deposit for 52 weeks i.e. 364 days, the interest is:

$$1000 \times 0.08 \times {}^{364}/_{360} = 80.89$$

This latter calculation should dispel any idea you might have had that the "360" indicated the length of the year! It simply serves to determine how much interest the account earns *per day*, in this case 1/360th of the interest rate.

2. Actual/365

In some markets, such as the UK, interest is calculated on a 365 day basis. So if we deposited £1000 in an account with an interest rate of 8%, the interest after 90 days would be:

$$1000 \times 0.08 \times {}^{90}/_{365} = 19.73$$

And for the 52 week, 364 day period:

$$1000 \times 0.08 \times {}^{364}/_{365} = 79.78$$

3. 30/360

Many banks compute interest on deposits using this convention. Describing it is a little complicated. We count the number of days the money is on deposit "assuming" that each month has exactly 30 days. Thus if the deposit is held through the end of a 31 day month, the 31st day is not counted.

Example:

Money is placed on deposit on 1 March and withdrawn 1 May. We will assume the first day earns interest and the last day does not i.e. you get interest for 1 March to 2 March but not for 1 May. How many days of interest accrue if the rate basis is 30/360?

In this case 31 March would not be counted, so the number of interest days is 60 rather than 61, even though 61 is the actual number of days. Thus, the customer is credited with 1 less day of interest. Note that if the money was placed on deposit on 1 February and withdrawn on 1 March, the customer would be credited with 30 days of interest rather than the actual number of 28 or 29.

Example:

USD1000 is placed in a deposit paying 10% simple interest for a one year period (365 days). How much interest is due at the end of the year if the day basis is actual/360, actual/365 and 30/360?

Actual/360: $1000 \times 0.10 \times {}^{365}/_{360} = 101.39$

Actual/365: $1000 \times 0.10 \times {}^{365}/_{365} = 100$

30/360: $1000 \times 0.10 \times {}^{360}/_{360} = 100$

Exercise:

Answer the same question if the deposit starts on 1 March and ends on 1 September.

Special Rule for 31st day and for last day of February

Suppose we have deposits with these dates:

First Day of Period	Last Day of Period	a/360 Days	30/360 Days
a) 3 March	31 May	89	88
b) 4 January	28 February	55	54
c) 31 January	28 February	28	28
d) 30 January	1 March	30/31	31
e) 31 March	31 May	61	60
f) 30 March	31 May	62	60
g) 29 March	31 May	63	62

a) For 30/360 we did not count 31 March, but we also did not shorten May to be 30 days, i.e. we did not treat the period as if it were 3 March to 30 May.

b) For 30/360 we did not count 31 January but we did not lengthen February to 30 days. This is because the last day of the period was 28 February.

c) 31 January was counted because it was the first day of the period, but February was again not lengthened to 30 days.

d) This time 31 January was not counted but February was lengthened to 30 days.

e) May was shortened to 30 days but 31 March was counted.

f) For 30/360, 31 March was not counted and May was shortened to 30 days.

g) 31 March not counted but May was not shortened to 30 days.

The ”rule” is

- If the last day of the period is the 31st of its month and the first day of the period is NOT either the 30th or 31st of its month, do not shorten the last month.

- If the last day of the period is the last day of February do NOT lengthen February to 30 days.

There is another day basis called: 30E/360 This is the same as 30/360 without the first part of the preceding rule i.e., the part referring to the 31st day of the month, but including the rule about the last day of February.

Global Disclaimer

This material has been prepared by UBS AG or an affiliate thereof ("UBS"). In certain countries UBS AG is referred to as UBS SA.

This material is for distribution only under such circumstances as may be permitted by applicable law. It has no regard to the specific investment objectives, financial situation or particular needs of any specific recipient. It is published solely for informational purposes and is not to be construed as a solicitation or an offer to buy or sell any securities or related financial instruments. No representation or warranty, either express or implied, is provided in relation to the accuracy, completeness or reliability of the information contained herein, nor is it intended to be a complete statement or summary of the securities markets or developments referred to in this material. It should not be regarded by recipients as a substitute for the exercise of their own judgment. Any opinions expressed in this material are subject to change without notice and may differ or be contrary to opinions expressed by other business areas or groups of UBS as a result of using different assumptions and criteria. UBS is under no obligation to update or keep current the information contained herein. UBS, its directors, officers and employees' or clients may have or have had interest or long or short positions in the securities or other financial instruments referred to herein and may at any time make purchases and/or sales in them as principal or agent. UBS may act or have acted as market-maker in the securities or other financial instruments discussed in this material. Furthermore, UBS may have or have had a relationship with or may provide or has provided investment banking, capital markets and/or other financial services to the relevant companies. Neither UBS nor any of its affiliates, nor any of UBS' or any of its affiliates, directors, employees or agents accepts any liability for any loss or damage arising out of the use of all or any part of this material. Additional information may be made available upon request.

United Kingdom and rest of Europe: Except as otherwise specified herein, this material is communicated by UBS Limited, a subsidiary of UBS AG, to persons who are market counterparties or intermediate customers (as detailed in the FSA Rules) and is only available to such persons. The information contained herein does not apply to, and should not be relied upon by, private customers. **Switzerland:** This material is distributed in Switzerland by UBS AG to institutional investors only. **United States:** In the U.S., securities underwriting, trading and brokerage activities and M&A advisory activities are conducted by UBS Securities LLC, a wholly owned subsidiary of UBS AG that is a registered broker-dealer and a member of the New York Stock Exchange and other principal exchanges and SIPC. **Canada:** This material is distributed by UBS Securities Canada Inc., a subsidiary of UBS AG and a member of the principal Canadian stock exchanges & CIPF. **Japan:** This material is distributed in Japan by UBS Securities Japan Ltd, a registered securities company, or by UBS AG, Tokyo Branch, a licensed bank. For further details of our local services, please call your regular contact at UBS in Japan. **Hong Kong:** This material is distributed in Hong Kong by UBS Securities Asia Limited or by UBS AG, Hong Kong Branch. **Singapore:** This material is distributed in Singapore by UBS Securities Singapore Pte. Ltd or UBS AG, Singapore Branch. **Australia:** Distributed by UBS AG (Holder of Australian Financial Services Licence No. 231087) and UBS Securities Australia Ltd (Holder of Australian Financial Services Licence No. 231098). **New Zealand:** This material is distributed in New Zealand by UBS New Zealand Ltd.

Produced by:
UBS

+44-20-7567 8000

Head office:
1 Finsbury Avenue
London
EC2M2PP
United Kingdom

JUN04/01446