Interest Rates

Warning

Please sign your name in the space below.

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INTRODUCTION: INTEREST RATES

COURSE OVERVIEW

The purpose of this Interest Rates workbook is to introduce basic interest-rate products and ideas. This workbook builds on concepts related to money-market deposits and loans, which have been introduced in the Basic Treasury workbook. In the first four units, we focus on concepts related to interest-rate instruments. The three basic areas of the time value of money concept are introduced, as well as the different conventions for calculating prices and yields of interest-rate instruments and for converting one type of yield into another for comparison purposes.

The forward market is a multibillion dollar market for transactions which fix the values of commodities or financial instruments on future dates. In this market, companies and individuals transform risk inherent in their businesses. Commodity forward contracts have existed around the world for centuries and, in the last decade, financial forward contracts have become popular.

The development and structure of interest-rate forward contracts and how they are used by banks and their customers is the focus of the last four units of this workbook. In addition, we identify the risks that forward agreements create for both parties. (A discussion of foreign exchange forward contracts may be found in the Foreign Exchange workbook.)

COURSE OBJECTIVES

When you complete this workbook, you will be able to:

- Recognize the basic interest-rate products
- Determine the future value, present value, and internal rate of return of cash flows
- Apply present value and internal rate of return concepts to the pricing and yield quotes on fixed-rate securities

- Compare interest-rate quotes by converting between standard quoting conventions
- Understand the conditions which have created the need for an interest-rate forward market
- Recognize the typical structure of an FRA and why this structure is used
- Identify settlement procedures for FRAs
- Recognize the use of FRAs to meet customer needs
- Understand how FRAs are used internally by banks
- Identify the major forms of interest-rate forward products
- Recognize the risks involved in interest-rate forward transactions from the perspective of the customer and the bank

THE WORKBOOK

This self-instruction workbook has been designed to give you complete control over your own learning. The material is divided into workable sections, each containing everything you need to master the content. You can move through the workbook at your own pace, and go back to review ideas that you didn't completely understand the first time. Each unit contains:

Unit Objectives –	which point out important elements in the unit that you are expected to learn.
Text –	which is the "heart" of the workbook. Here, the content is explained in detail.
Key Terms –	which are relevant to the topic and important for you to know. They appear in **bold face** the first time they appear in the text and also appear in the Glossary.
Instructional Mapping –	terms or phrases in the left margin which highlight significant points in the lesson.

Progress Checks – which do exactly that — check your progress. Appropriate questions are presented at the end of each unit, or within the unit, in some cases. You will not be graded on these by anyone else; they are to help you evaluate your progress. Each set of questions is followed by an Answer Key. If you have an incorrect answer, we encourage you to review the corresponding text and find out why you made an error.

In addition to these unit elements, the workbook includes the:

Glossary – which contains all of the key terms used in the workbook.

Index – which helps you locate the glossary item in the workbook.

Each unit covers a certain aspect of interest-rate forwards. The units are:

- UNIT 1 – Basic Interest-Rate Products
- UNIT 2 – Time Value of Money
- UNIT 3 – Determining the Price and Yield of Fixed-Rate Securities
- UNIT 4 – Converting Between Quoting Conventions
- UNIT 5 – Fundamentals of Interest-Rate Forwards
- UNIT 6 – Forward Rate Agreement (FRA) Transactions
- UNIT 7 – A Bond Forward Agreement — The REPO
- UNIT 8 – Risks of Interest-Rate Forward Transactions

Since this is a self-instructional course, your progress will not be supervised. We expect you to complete the course to the best of your ability and at your own speed.

Now that you're familiar with the workbook elements and know what to expect, please turn to Unit One and begin. Good Luck!

Unit 1

UNIT 1: BASIC INTEREST-RATE PRODUCTS

INTRODUCTION

In this unit, we identify the basic interest-rate products. They may be viewed as standard bank deposits and bank loans or corporate / government securities that have similar characteristics to deposits and loans. The primary distinctions among the interest-rate products are the nature of the interest-rate payments (fixed or variable), the maturity of the agreements, and the ease with which the agreements can be transferred to another party.

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the basic interest-rate products
- Identify the characteristics of bank and nonbank interest-rate products

BASIC INTEREST-RATE PRODUCTS

Bank and nonbank products

The basic interest-rate products can be differentiated into bank and nonbank product lines:

- Bank product lines include deposits and loans (e.g. credit lines)
- Nonbank products include debt securities (e.g. government or corporate bonds)

Both bank products and nonbank products are characterized by maturity, structure, and type of rate. Product categories include:

- Short-term fixed rate
- Long-term fixed rate
- Long-term variable (or floating) rate

BANK PRODUCTS

Deposits

Short-term, fixed-rate deposits

Most bank deposits have a short contractual maturity, usually six months or less. A *call* deposit has no stated maturity and can be withdrawn by the customer at any time. Common maturities for other short-term deposits are: one week, one month, three months, and six months.

Example

Let's look at an example of a typical short-term deposit. A customer places 100 on deposit for six months at an interest rate of 8% per annum (p.a.). The beginning of the contractual period is referred to as the *spot date*. The "100" amount is called the *principal amount* on deposit. At the end of the six-month deposit period, the bank returns the principal of 100 and an amount of interest calculated from the 100 principal at the 8% p.a. interest rate. For example, the interest amount may be 100 x (.08 / 2) = 4.

From the depositor's perspective, the cash flows resulting from this transaction include an initial outflow of 100 and an inflow of 104 after six months. The customer's perspective of these two cash flows is illustrated on a time line in Figure 1.1.

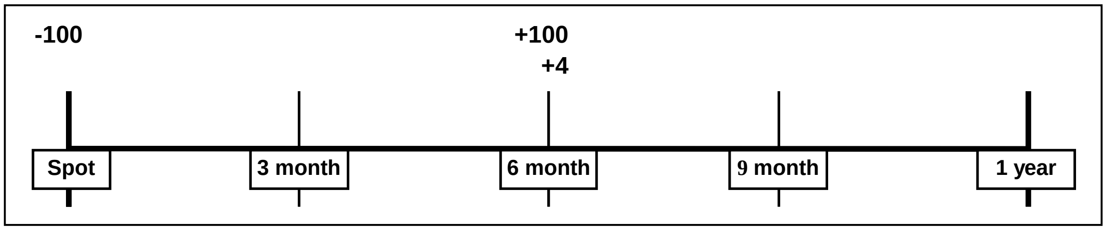

Figure 1.1: Six-Month Deposit from the Customer's Perspective

Longer-term, fixed-rate deposits

Deposits may be structured for longer periods of time (rarely more than five years). A deposit with a maturity of more than one year is referred to as a *term* deposit. Interest on the deposits may be paid periodically or accrued. An accrual is the acknowledgment of a growing interest liability when a portion of the interest-earning period has elapsed but the interest payment is not contractually due.

The maturities that are common for short-term deposits also are common interest-payment or interest-accrual periods for longer-term deposits. Monthly and quarterly periods are the most common. Generally, the larger the amount of the principal and the longer the tenor of the deposit, the longer the interest-payment period. The time line from the customer's perspective for a four-year deposit at 8% p.a. with interest accruals or payments every six months is illustrated in Figure 1.2.

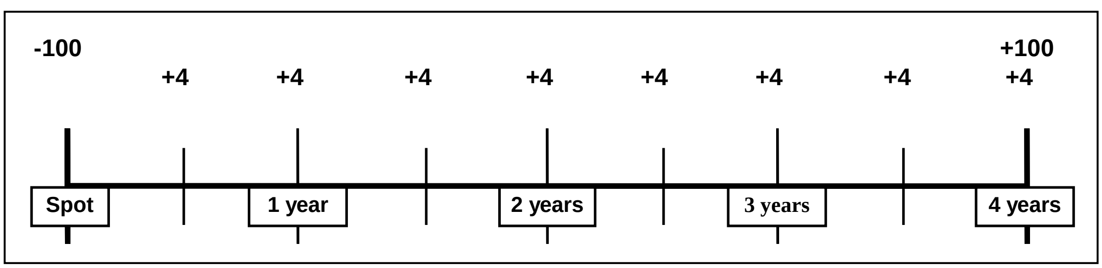

Figure 1.2: Four-Year, Fixed-Rate Deposit from the Customer's Perspective

Rolled-over, variable-rate deposits

Many short-term deposits are *rolled over* when they mature and the money remains in the bank for years. One difference between a four-year deposit and rolling over a deposit every six months for four years is that the deposit rates may change each time the deposit is rolled over. When the earnings rate changes from period to period, we call it a *variable* rate or a *floating* rate. Since the deposit rates may change for each roll-over period, the short-term, fixed-rate deposit becomes a four-year, variable-rate deposit.

Variable-rate term deposits

This variable return also can be achieved with the four-year (term) deposit product by resetting the interest rate every six months. Variable-rate deposits have the same length, or tenors, and interest-rate periods as described for fixed-rate deposits.

The more illiquid the currency, the more likely it is that term deposits will have variable rates. This removes interest-rate risk from the bank and from the depositor. A variable-rate term deposit is illustrated in Figure 1.3. Each rate (R) is set at the beginning of the appropriate payment period and paid or accrued at the end of the payment period. (R_1 is set at the beginning of the first six-month period and paid at the end of that period; R_2 is set at the beginning of the second six-month period, etc.)

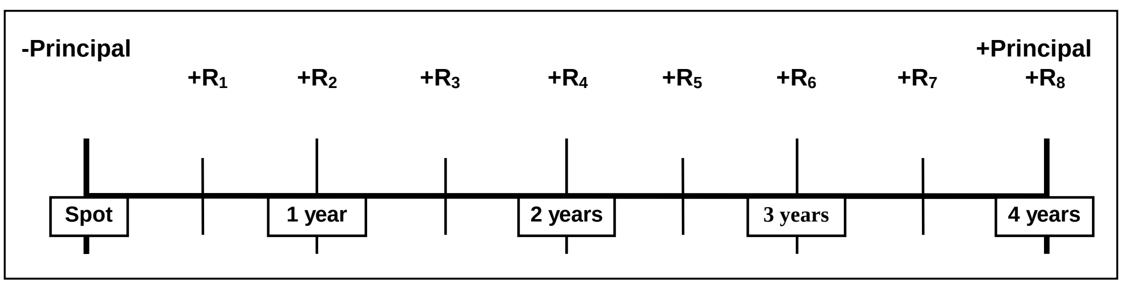

Figure 1.3: Four-Year, Variable-Rate Deposit from the Customer's Perspective

From the perspective of the bank, these deposits are a source of funding, as you can see in Figure 1.4.

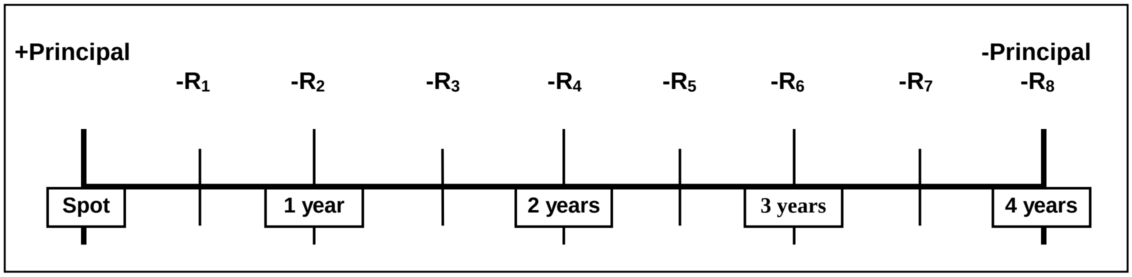

Figure 1.4: Variable-Rate Deposit from the Bank's Perspective

By comparing Figures 1.3 and 1.4, you can see that one party's investment (deposit) is the other party's funding (loan). Therefore, we can look at deposits and loans as two sides of the same product, and everything that we have said about the tenors and interest payments of deposits also applies to loans.

Loans

Deposit vs. loan

You can think of a loan from a bank to a customer as the bank *depositing* the principal with the customer. Of course, *deposit* generally applies when the recipient of the funds is some form of government-regulated financial institution. *Loan* is used in almost every other case.

The three types of deposits introduced in the previous section (short-term fixed, long-term fixed, long-term variable) also exist as lending products. Most bank loans are short-term, fixed-rate loans as shown in Figure 1.5.

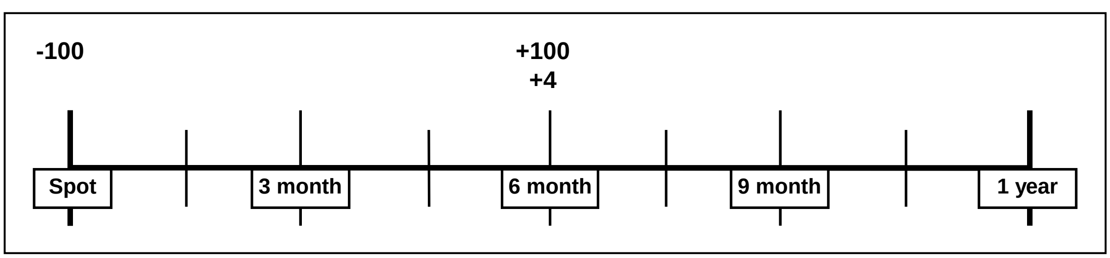

Figure 1.5: Six-Month Loan from the Bank's Perspective

Longer-term bank loans are more common than longer-term deposits, and they generally require variable-rate payments (Figure 1.6).

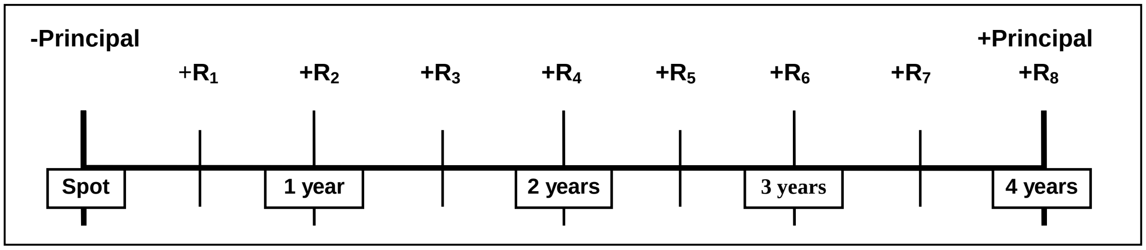

Figure 1.6: Variable-Rate Loan from the Bank's Perspective

In more developed financial markets, term loans (maturities of more than one year) may have fixed interest rates (Figure 1.7).

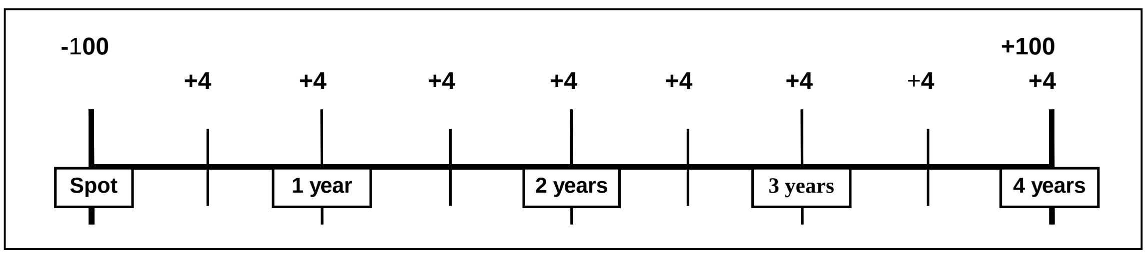

Figure 1.7: Fixed-Rate Loan from the Bank's Perspective

The tenor of a term loan varies according to currency, type of borrower, and the bank making the loan. Generally, when a bank grants a term loan, it requires a security interest in a capital asset of the borrower (e.g. real estate).

Summary

In this section, we described the basic bank interest-rate products — deposits and loans. The time line examples illustrate that one party's deposit is the other party's loan and, therefore, both products share the same basic characteristics. We continue with a discussion of nonbank interest-rate products and show the characteristics they have in common with bank interest-rate products.

NONBANK PRODUCTS

Debt security contracts

Several debt securities have similar characteristics to deposits and loans. These debt securities are contracts purchased by investors who may be banks, but most often are other financial institutions or individuals. It is common for an investor to pay for the security at the beginning of the contract — the amount paid is often called the **price**. Generally the contract lists:

- Amount of money to be repaid (often called the face value)
- Date when the face value is to be paid (often called the maturity)
- Periodic payments to be made (similar to interest, often called coupons)
- Dates when the coupons will be paid
- Payment instructions, conditions of default, and specification of other legal issues

Similar to deposits and loans

The similarity between debt securities and deposits and loans is illustrated by a four-year bond with semi-annual coupon (c) payments as shown in Figure 1.8. (Please note that term deposits, term loans, and debt securities do not all commonly have four-year maturities with semi-annual payments. Similar examples have been used here to emphasize the similarity of the interest-rate products.)

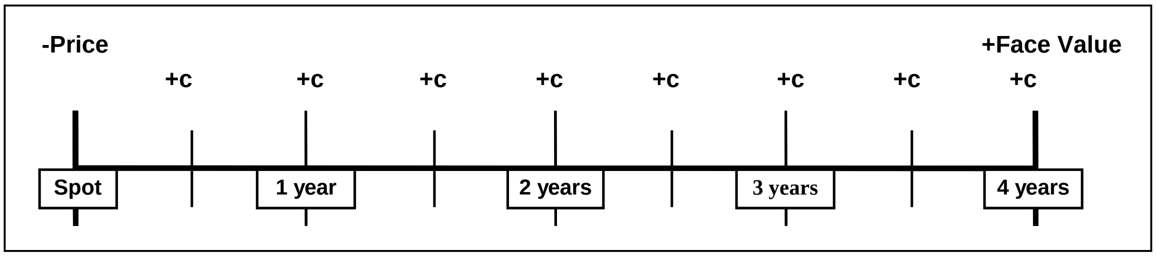

Figure 1.8: Debt Security (Bond) from the Investor's Perspective

In less developed financial markets, governments use bank loans for financing; in more financially-developed markets, governments often borrow money through the sale of securities. Debt securities for the US government carry fixed rates to maturity and generally have periodic payments at least annually. Some forms of long-term corporate and government debt in other counties have variable payments.

Government Securities

Conventions in US markets

Debt securities are called many different names in different countries. However, since the US government is the largest and most active borrower in the world, it is becoming more common for these securities to be named bills, notes, and bonds according to the conventions in the US market.

Named according to original maturity

US government debt securities are named according to their original maturity at the time they are first issued. Standard maturities include 13-week, 26-week, 52-week, 3-year, 5-year, 10-year, and 30-year instruments. Since so many securities have been issued in the past that have not yet matured, US Treasury securities exist for almost every quarterly maturity out to 30 years. In the table below, we can see the maturity range and payment period for each type of US government debt security.

Name	Original Maturity	Payment Period
Treasury Bill	Less than one year	At maturity
Treasury Note	From one year to ten years	Semi-annually
Treasury Bond	More than ten years	Semi-annually

Generally, the terms *note* and *bond* are used interchangeably.

The most common form of bond has fixed semi-annual or annual payments. In Figure 1.9, you can see the investor's cash flows for a four-year bond with annual 10% coupons. The investor pays a price of 96 for the bond, receives annual coupon payments of 10, and receives the face value of 100 at the end of four years.

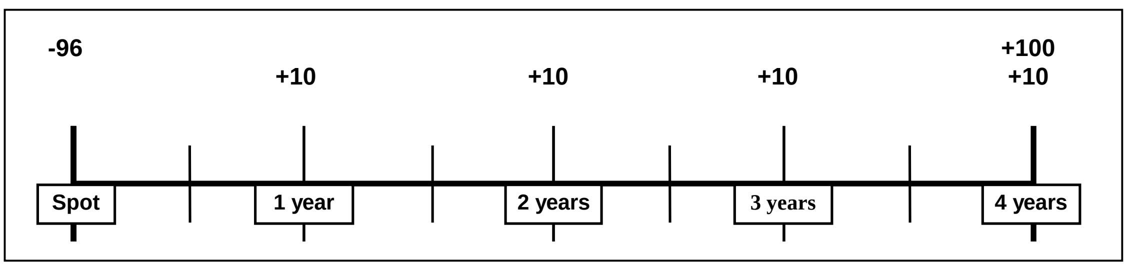

Figure 1.9: Four-Year, 10% Bond from the Investor's Perspective

Nongovernment Securities

Long-term, nongovernment (e.g. corporate) debt securities use the same terminology. In addition, corporate bonds are often classified according to their credit provisions. They are called debentures, notes, and bonds; they are either *senior* or *subordinate* to other debt instruments.

Terminology for corporate debt securities

Short-term nongovernment debt securities often have names which differ from government securities. When banks offer securities that are similar to their deposits, they are called (negotiable) certificates of deposit (NCD or CD). Many corporations issue similar securities which are generically called commercial paper (CP). Trade financing often involves a short-term security called a trade bill or, when guaranteed by a bank, a banker's acceptance (BA).

Comparison of CDs and other debt securities

CDs have a stated interest rate payable at maturity with the principal amount. Other forms of short-term securities often state the final payment of face value without a stated interest amount. As a result, prices for these securities are correspondingly lower than the amount required to purchase a CD. Since these securities can be easily traded in a secondary market, the value of the securities changes continually according to market conditions. Let's look at a comparison of bank Cds and other types of short-term securities.

A six-month CD purchased on the spot date that pays interest at maturity is illustrated in Figure 1.10.

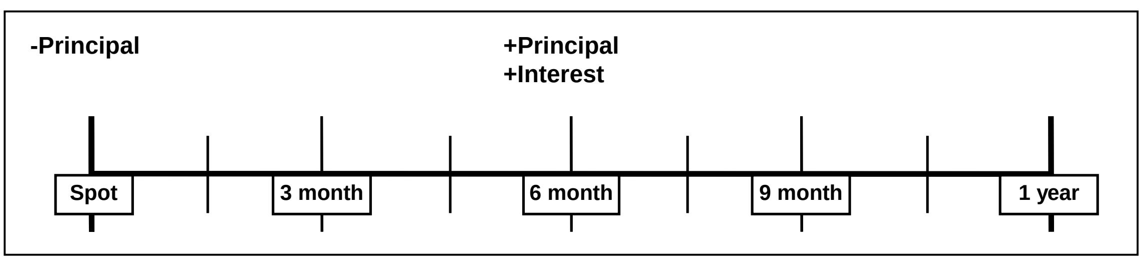

Figure 1.10: Six-Month CD from the Investor's Perspective

If the CD is sold after three months (with payment on the new spot date) at some price, the original investor gives up the opportunity to receive the principal plus interest three months later in exchange for the immediate receipt of the market price (Figure 1.11). The second investor pays the market price for the opportunity to receive the principal plus interest three months later.

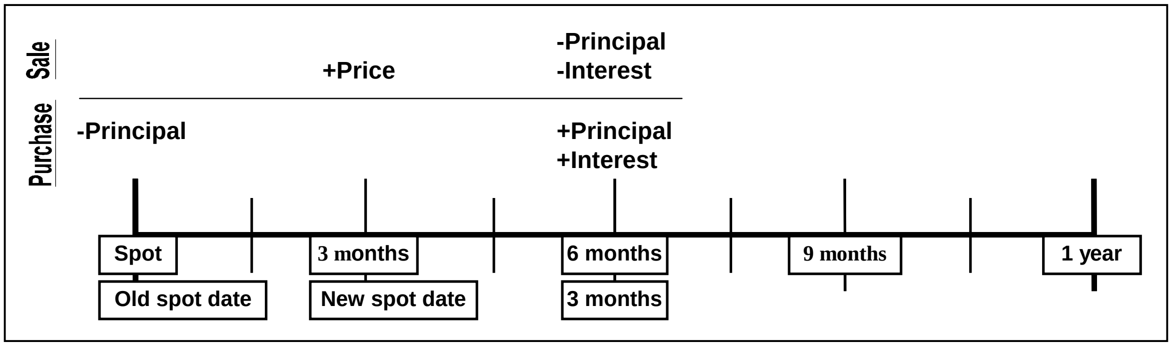

Figure 1.11: Sale of CD in Figure 1.10 from Perspective of the Original Investor

Treasury bills (T-Bill), commercial paper (CP), and bankers' acceptances (BA) have a less complicated structure. They generally quote only the face value paid at maturity. The initial price is determined in the same way the price is set on any date prior to maturity. The pricing of these securities will be discussed in Unit Two. The cash flows for these securities are illustrated in Figure 1.12.

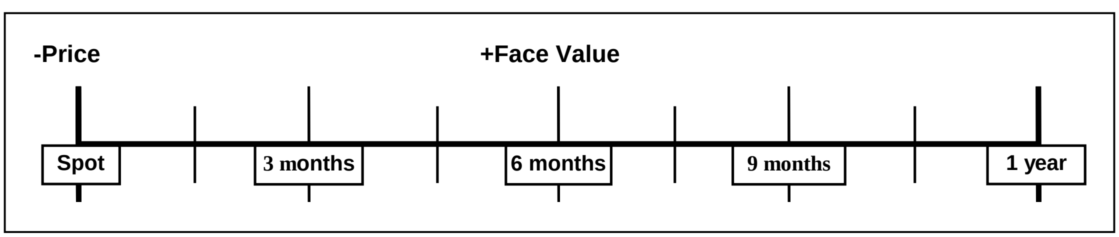

Figure 1.12: Cash Flows for 6-Month T-Bill, CP, and BA from Perspective of the Original Investor

Three months later, the original investor may sell the T-Bill, CP, or BA. At the time of sale, the investor receives the new market price and gives up the receipt of the face value in three months.

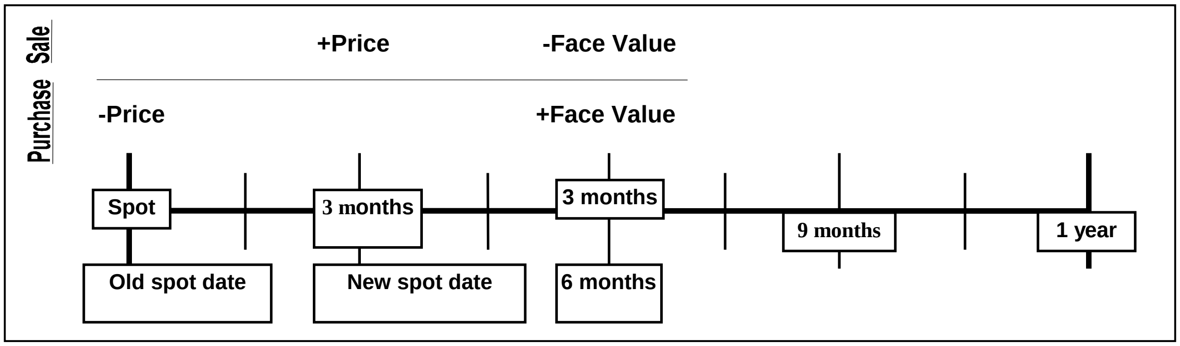

Figure 1.13: Sale of Security in Figure 1.12 from Perspective of the Original Investor

The CD market is more active for maturities up to six months. In the developed financial markets, CDs with maturities up to five years are readily available. In general, in less developed financial markets, only variable-rate term CDs are available for longer tenors.

CP, BA, and T-Bill maturities are typically less than one year. CP maturities are predominantly under one month. BAs usually mature in less than six months.

Usually rolled over

These short-term security investments are similar to bank deposits in that they are usually rolled over by the investors and borrowers. This creates longer-tenor income or expense cash flows on a variable-rate basis. Borrowers using these structures often have agreements with underwriters to issue the securities and roll over the funding continually for a period of years. The individual investors who buy the securities may change, but the underwriter assures the borrower that funds will be available even if the underwriting syndicate has to purchase the securities.

SUMMARY

The basic interest-rate products include fixed-rate and variable-rate deposits and loans with maturities ranging from *call* (which is immediate) to five years. Short-term deposits/loans with fixed rates and longer-term deposits/loans with variable rates are the most common bank interest-rate instruments. Interest is usually paid or accrued on a short-term basis even if the instrument has a long maturity. Common payment periods are one-month, three-months, and six-months.

Debt securities are common interest-rate instruments primarily used by large nonbank institutions. Short-term securities include CDs (certificates of deposit), BAs (bankers' acceptances), CP (commercial paper), and T-Bills (often government bills). Long-term securities include notes and bonds and almost always have fixed-rate payments.

Bank products refer to principal and interest, while nonbank securities most often refer to face value and coupons.

You have completed Unit One, *Basic Interest-Rate Products*. Please complete the Progress Check to test your understanding of the concepts and check your answers with the Answer Key. If you answer any questions incorrectly, please reread the corresponding text to clarify your understanding. Then, continue to Unit Two, *Time Value of Money*.

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☑ PROGRESS CHECK 1

Directions: Determine the correct answer to each question. Check your answers with the Answer Key on the next page.

Question 1: The amount paid periodically on a deposit or loan is usually called the:

_____a) principal.

_____b) interest.

_____c) face value.

_____d) coupon.

Question 2: The amount paid periodically on a bond is usually called the:

_____a) principal.

_____b) interest.

_____c) face value.

_____d) coupon.

Question 3: The amount paid at maturity of a deposit or loan is usually called the:

_____a) principal.

_____b) interest.

_____c) face value.

_____d) coupon.

ANSWER KEY

Question 1: The amount paid periodically on a deposit or loan is usually called the:

b) interest.

Question 2: The amount paid periodically on a bond is usually called the:

d) coupon.

Question 3: The amount paid at maturity of a deposit or loan is usually called the:

a) principal.

PROGRESS CHECK 1
(Continued)

Question 4: The amount paid at maturity of a debt security is usually called the:

_____a) principal.

_____b) interest.

_____c) face value.

_____d) coupon.

Question 5: Which of the following maturities is most likely for a product called a fixed-rate bank deposit?

_____a) Overnight

_____b) 6 months

_____c) 2 years

_____d) 5 years

Question 6: Which of the following maturities is most likely for a product called a corporate bond?

_____a) Overnight

_____b) 6 months

_____c) 2 years

_____d) 5 years

ANSWER KEY

Question 4: The amount paid at maturity of a debt security is usually called the:

c) face value.

Question 5: Which of the following maturities is most likely for a product called a fixed-rate bank deposit?

b) 6 months

Question 6: Which of the following maturities is most likely for a product called a corporate bond?

d) 5 years

PROGRESS CHECK 1
(Continued)

Question 7: Identify the following as “S,” “L,” or “E.”

S — Short-term, fixed rate, or revolving longer-term variable rate
L — Long-term, fixed rate
E — Either

_____a) BAs

_____b) Bank loans

_____c) CP

_____d) Bonds

_____e) Bills

_____f) Deposits

ANSWER KEY

Question 7: Identify the following as "S," "L," or "E."

S — Short-term, fixed rate, or revolving longer-term variable rate
L — Long-term, fixed rate
E — Either

__**S**__ a) BAs

__**E**__ b) Bank loans

__**S**__ c) CP

__**L**__ d) Bonds

__**S**__ e) Bills

__**E**__ f) Deposits

Unit 2

UNIT 2: TIME VALUE OF MONEY

INTRODUCTION

Financial market analysis attempts to understand the value of money as it is received or paid over time. This concept is called “time value of money” (TVM). The TVM concept is divided into three basic areas: present value (PV), future value (FV), and internal rate of return (IRR). These ideas are very important for pricing, hedging, revaluing, and marketing any financial product.

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Calculate the future value (FV) of a cash flow, given an interest earning rate
- Calculate the present value (PV) of a cash flow, given an interest discounting rate
- Calculate the internal rate of return (IRR) of a cash flow

FUTURE VALUE

Suppose you have 100 of surplus funds available and do not need the extra liquidity for a year. What should you do with the funds? You could place the funds in your desk for the year and (provided there is no theft) you will have 100 available at the end of the year. In this case, the value of 100 today is 100 in one year.

You may prefer to earn some return on surplus cash by investing in something that will return more than 100% in a year. For example, you could place the extra 100 in a bank account for one year. If the interest rate is 10%, you will have 100 + (10% of 100) = 110 at the end of the year (Figure 2.1).

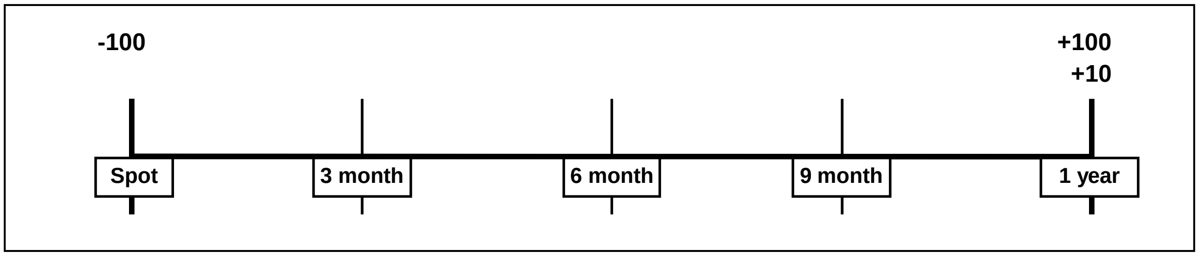

Figure 2.1: One-Year Deposit at 10%

Value of an instrument after a given period of time

In this case, the value of 100 today is 110 in one year. In other words, 110 is the one-year **future value** of 100. You may have a different future value for each day in the future, with the two-year future value likely to be higher than the one-year future value, etc. In addition, the future value depends on the earning rate — in this case, 10%. If the interest rate on the deposit is 9%, then the future value is 109.

Future Value After One Interest-Rate Period

Formula: Future value after one ir period

The formula for calculating the future value of an amount at the end of one interest-rate period is:

$$\boxed{FV = PV \times (1 + ir)}$$

Where:

FV = Future value at the end of one interest-rate period

PV = Present value: amount of money today

ir = Interest rate for one period, expressed as a decimal

For example,

If	**PV**	**= 46**
	ir	**= 5%**
Then	**FV**	**= 46 x (1 + .05) = 46 x 1.05 = 48.30**

Future Value After Multiple Fixed-Interest-Rate Periods

Future value with compounding interest

This concept of future value can be extended to situations where the future period is many interest-rate periods away. Suppose you want to know today the future value of 100 in four years. If you deposit the full amount in a 10% interest-earning account for the full four years, and earn 10% on the accruing interest, you can determine the future value in four separate steps:

1) 100.00 today is worth 110.00 at the end of year one (100.00 x 1.10)

2) 110.00 is worth 121.00 at the end of year two (110.00 x 1.10)

3) 121.00 is worth 133.10 at the end of year three (121.00 x 1.10)

4) 133.10 is worth 146.41 at the end of year four (133.10 x 1.10)

This process of calculating a future value by adding the prior period's interest to the outstanding principal balance for the next year is called **compounding** or *capitalizing* interest. In Figure 2.2, we have compounded 10% per annum interest over a period of four years.

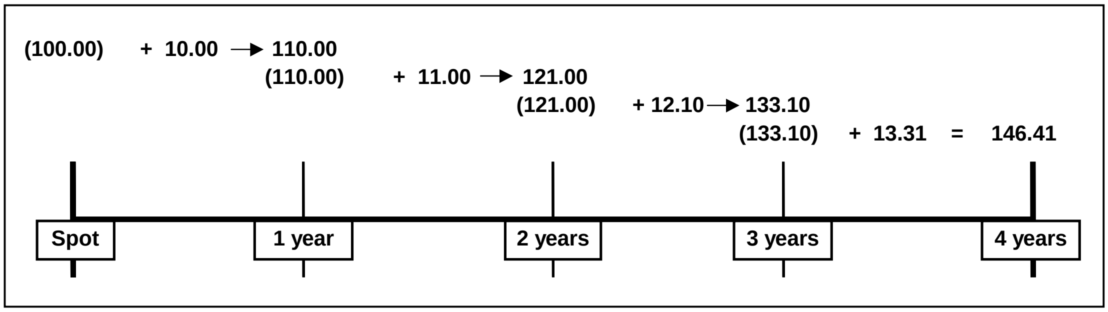

Figure 2.2: 10% Compounded for Four Years

Formula: FV over multiple fixed ir periods

The formula for calculating future value of an amount over multiple fixed-interest periods is:

$$\boxed{FV = PV \times (1 + ir)^n}$$

Where:

FV = Future value at the end of one interest-rate period
PV = Present value: amount of money today
ir = Interest rate for one period, expressed as a decimal
n = Number of interest periods

Finding FV with a financial calculator

Although we can calculate future values over multiple periods with this formula, it is easier to use a financial calculator. Financial calculators are designed to calculate future values when given a **present value**, possible periodic payments, a fixed earnings rate per period, and the number of earnings periods. Using the same example, you can enter **N (4)**, **I (10)**, **PV (-100)**, and **PMT (0)** and then press **FV** as illustrated in Figure 2.3.

N	I	PV	PMT	FV
4	10	-100	0	146.41

Figure 2.3: Compounding 10% for Four Years Using a Financial Calculator

Future Value After Multiple Variable-Interest-Rate Periods

The process of compounding interest is equally valid when the interest rate changes from period to period. Determining the future value of a floating-rate (variable-rate) income stream follows the same steps that we used to calculate a fixed-rate stream of cash flows. Unfortunately, we have to wait until after the various rates are set to perform the calculation.

Example: FV after variable ir periods

Let's look at an example. Suppose the following rates are applied to the 100 investment for four years:

Year	Rate of Return
1	6%
2	8%
3	5%
4	7%

You can determine the future value in four separate steps, just as we did for the fixed-rate investment.

1) 100.00 today is worth 106.00 at the end of year one (100.00 x 1.06)

2) 106.00 is worth 114.48 at the end of year two (106.00 x 1.08)

3) 114.48 is worth 120.20 at the end of year three (114.48 x 1.05)

4) 120.20 is worth 128.61 at the end of year four (120.20 x 1.07)

The growing principal amount is illustrated in Figure 2.4.

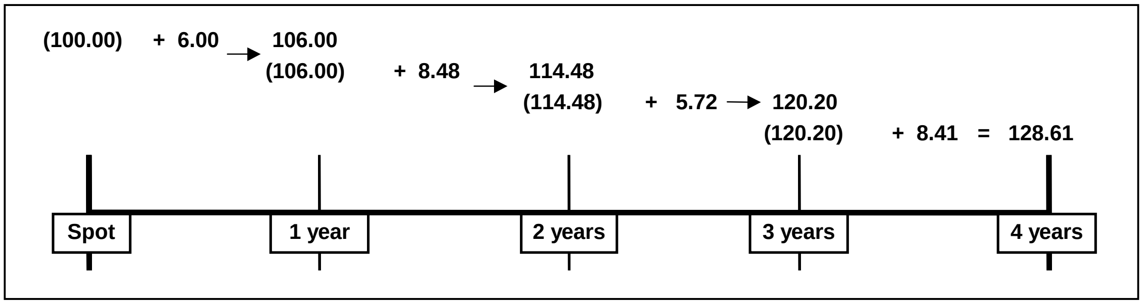

Figure 2.4: Variable Rate Compounded for Four Years

Formula: FV after variable ir periods

The formula for calculating the future value of an amount over multiple variable-interest periods is:

$$\boxed{FV = PV \times (1 + ir_1) \times (1 + ir_2) \times \bullet\bullet\bullet \times (1 + ir_n)}$$

Where:

FV = Future value at the end of one interest-rate period
PV = Present value: amount of money today
ir_j = Interest rate for period "j," expressed as a decimal
n = Number of interest periods

For example,

If $n = 4$
$PV = 100$
$ir_1 = 6\%$
$ir_2 = 8\%$
$ir_3 = 5\%$
$ir_4 = 7\%$

Then $FV = 100 \times (1 + .06) \times (1 + .08) \times (1 + .05) \times (1 + .07)$
$= 100 \times 1.06 \times 1.08 \times 1.05 \times 1.07$
$= 100 \times 1.2862 = 128.62$

This result agrees with the four-step calculation presented above, except for round-off error.

Unfortunately, most financial calculators are not designed to automate the compounding of variable rates.

Summary

We use the future value calculation to determine how much an investment will be worth after a certain period of time when the present value, number of payments, interest rate, and payment amounts are known.

Please complete Progress Check 2.1 to check your understanding of “future value” and then continue on to the section that describes “present value.”

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☑ PROGRESS CHECK 2.1

Directions: Determine the correct answer to each question. Check your answers with the Answer Key on the next page.

Question 1: If you invested 10 for three years at an interest rate of 4%, compounded annually, how much would you have at maturity?

_____a) 22.00

_____b) 11.20

_____c) 11.25

_____d) 40.00

Question 2: If you deposit 100 in a bank account earning 10% per annum, compounded annually, how much will you have after five years?

_____a) 150

_____b) 155

_____c) 161

_____d) 165

Question 3: If you borrow 100 at a variable annual interest rate of 10% during the first year, 5% during the second year, and 8% during the third year, how much will you owe after three years? (Assume that all interest is capitalized, and no payments are made until the end of the third year.)

_____a) 123

_____b) 124

_____c) 125

_____d) 126

ANSWER KEY

Question 1: If you invested 10 for three years at an interest rate of 4%, compounded annually, how much would you have at maturity?

N	I	PV	PMT	FV
3	4	-10	0	11.25

c) 11.25

Question 2: If you deposit 100 in a bank account earning 10% per annum, compounded annually, how much will you have after five years?

N	I	PV	PMT	FV
5	10	-100	0	161

c) 161

Question 3: If you borrow 100 at a variable annual interest rate of 10% during the first year, 5% during the second year, and 8% during the third year, how much will you owe after three years? (Assume that all interest is capitalized, and no payments are made until the end of the third year.)

$$
\begin{aligned}
FV &= PV \times (1 + ir_1) \times (1 + ir_2) \times \ldots \times (1 + ir_n) \\
&= 100 \times (1 + .10) \times (1 + .05) \times (1 + .08) \\
&= 100 \times 1.10 \times 1.05 \times 1.08 \\
&= 125
\end{aligned}
$$

c) 125

PRESENT VALUE

Value of instrument today

Often, we know the future value (face value) of an instrument, but need to know what that instrument is worth today. This question leads us to the second time value of money concept, which is the calculation of present value**.** The idea of future value can be *reversed* to create the concept of present value. The present value of a future payment is the amount of cash today which is equivalent in value to the amount of the payment in the future. Present value (PV) is a fundamental concept in financial analysis. It allows obligations on different dates to be compared on a reasonable, quantifiable basis.

Example: Present value

For example, suppose you are obligated to make a payment of 100 after one year, and want to set aside enough money now so that you know there will be sufficient funds to pay the obligation. How much should you set aside?

The simplest answer is to set aside 100 now — or even to pay 100 now to the intended recipient. Although this immediate payment would satisfy the obligation, it is not the best alternative. It would be wiser to place the 100 on deposit with a trustworthy bank and receive interest on the deposit for one year. After one year, the principal of 100 can be used to pay the obligation, and the interest will be available to you.

Once you start to think about the interest-earning capacity of the 100 today, you may realize that the 100 deposit is a *present value* amount and the 100 plus interest is a *future value* amount. In a similar way, the 100 payment obligation is an obligation to pay in the future — it is a future value. Due to the interest to be earned, you can set aside less than 100 today. When interest is added to the principal you set aside, the total future value must equal 100 in order to make the payment.

We can reverse the future value calculation to determine the amount of principal that we should deposit. Suppose the interest rate on deposits is 10% per year. A 100 deposit will be worth 110 in one year. How much of a deposit is necessary to be worth 100 in one year? (Figure 2.5)

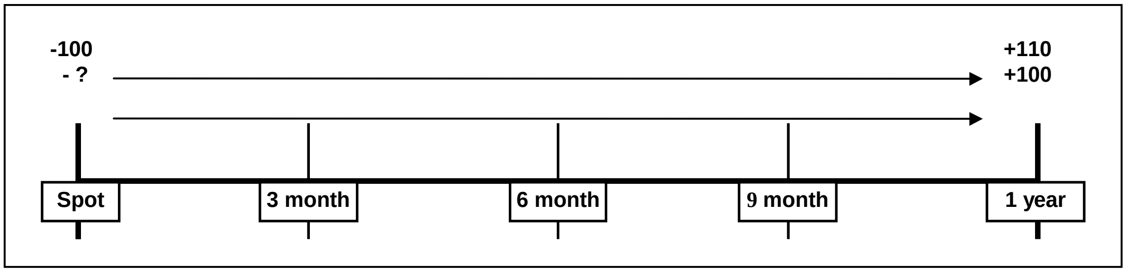

Figure 2.5: One-Year Deposit at 10% vs. One-Year Obligation of 100

Present Value of a Single-Payment Stream

The examples that follow demonstrate how to calculate a single payment in the future, given a known interest rate.

One Interest-Rate Period

Formula: Present value for one ir period

The formula for calculating the present value of an amount over one interest-rate period is:

$$PV = FV / (1 + ir)$$

Where:

FV = Future value at the end of one interest-rate period

PV = Present value: amount of money today

ir = Interest rate for one period, expressed as a decimal

Example: PV for one ir period

For example,

If	**FV**	**= 100**
	ir	**= 10%**
Then	**PV**	**= 100 / (1 + .10) = 100 / 1.10 = 90.91**

NOTE: *ir* also is known as the **discount rate**; this process also is known as **discounting cash flows**.

Multiple Fixed-Interest-Rate Periods

The PV concept applies to multiple interest-rate periods as well. For example, if the 10% interest rate is guaranteed for four years, allowing interest to be compounded, then the PV of a payment of 100 in four years can be determined by a formula or by a financial calculator.

Formula: PV over multiple fixed ir periods

The formula for the present value of an amount over multiple fixed interest periods is:

$$\boxed{PV = FV / (1 + ir)^n}$$

Where:

FV = Future value at the end of one interest-rate period
PV = Present value: amount of money today
ir = Interest rate for one period, expressed as a decimal
n = Number of interest periods

Example

In the following example, we show the calculation of present value over multiple fixed-interest-rate periods.

If	**n**	**= 4**
	FV	**= 100**
	ir	**= 10%**
Then	**PV**	**= $100 / (1 + .10)^4$ = $100 / 1.1^4$**
	PV	**= 100 / 1.4641 = 68.30**

Financial calculator

Alternatively, you can use the financial calculator. Enter **FV (-100)**, **PMT (0)**, **N (4)**, and **I (10)** in any order, then press **PV** as illustrated in Figure 2.6.

N	I	PV	PMT	FV
4	10	68.30	0	- 100

Figure 2.6: PV at 10% per Annum for Four Years

You can see that both the formula and the financial calculator produce the same result — PV = 68.30.

Multiple Variable-Interest-Rate Periods

Formula: PV over multiple variable ir periods

The PV concept also applies when the discount rate is variable. If you know the discount rate, but its known value changes from period to period, then the PV formula is modified the same way as the FV formula.

The formula for the future value of an amount over multiple variable interest periods is:

$$\boxed{PV = FV / [(1 + ir_1) \times (1 + ir_2) \times \bullet \bullet \bullet \times (1 + ir_n)]}$$

Where:

FV = Future value at the end of one interest-rate period
PV = Present value: amount of money today
ir_j = Interest rate for period “j,” expressed as a decimal
n = Number of interest periods

Example: PV over multiple variable ir periods

For example, let's assume that we know the interest rates for the next four interest-rate periods. Given the following information, we can calculate the present value.

If $n = 4$
$FV = 100$
$ir_1 = 6\%$
$ir_2 = 8\%$
$ir_3 = 5\%$
$ir_4 = 7\%$

Then

$$
\begin{aligned}
PV &= 100 / [(1 + .06) \times (1 + .08) \times (1 + .05) \times (1 + .07)] \\
&= 100 / [1.06 \times 1.08 \times 1.05 \times 1.07] \\
&= 100 / 1.2862 \\
&= 77.75
\end{aligned}
$$

To check this result, it is easier to start with a deposit of 77.75 and then add interest to it for four years to confirm that the ending balance is 100. You can determine the future value of this deposit in four separate steps:

1) 77.75 today is worth 82.42 at the end of year one (77.75 x 1.06)

2) 82.42 is worth 89.01 at the end of year two (82.42 x 1.08)

3) 89.01 is worth 93.46 at the end of year three (89.01 x 1.05)

4) 93.46 is worth 100.00 at the end of year four (93.46 x 1.07)

The growing principal amount is illustrated in Figure 2.7.

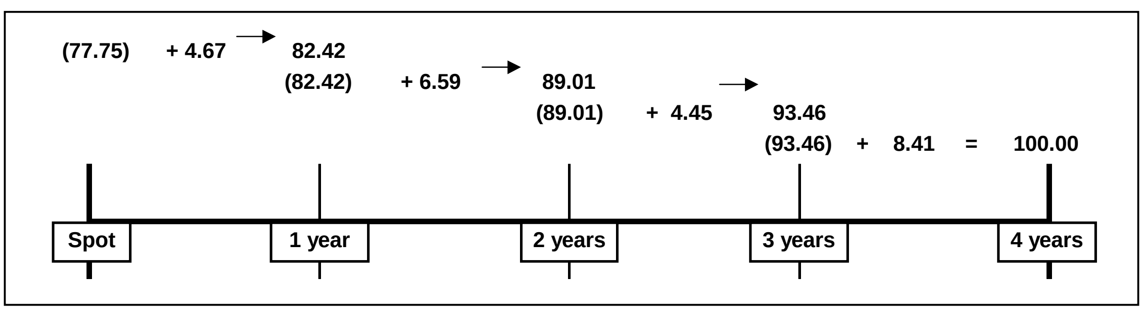

Figure 2.7: Future Value Over Multiple Variable ir Periods

The examples in this section demonstrate the present value calculation for a *single payment* in the future. Sometimes, a payment stream has *multiple payments*.

In the next section, you will see how to calculate the present value of an obligation with more than one fixed payment.

Present Value of Payment Streams with Multiple Fixed Payments

In the case where all payment streams are fixed, we can discount the cash flows and determine the present value of the complete set of cash flows using the financial calculator.

Example: PV with multiple fixed payments

For example, suppose that you have agreed to pay 100 in four years plus an additional 10 at the end of each year for four years. The PV of that obligation depends on which interest rate you use to discount the cash flows.

If you think that 10% is an appropriate discount rate, then the PV is 100, as shown in Figure 2.8. (When the price is the same as the *principal* amount of a future payment obligation, the price is called *par* in the financial markets.) This result should be obvious — if 10% is the appropriate discount rate, an obligation to 100 plus interest at 10% should be valued currently at 100.

N	I	PV	PMT	FV
4	10	100.00	-10	- 100

Figure 2.8: PV at 10% per Annum for Four Years

Identifying the proper discount rate to use is a critical issue that requires a wide range of information about market conditions. This complicated process is beyond the scope of this introduction.

Of course, the current, proper discount rate may be more or less than the contracted payment rate. For example, if you believe that you would have to pay 12% to borrow funds today, then the PV of this existing obligation is less than 100. The liability's value to you today is less than par because it is at a relatively good contract payment rate of 10%. The following calculation shows the PV:

N	I	PV	PMT	FV
4	12	93.93	-10	- 100

Figure 2.9: PV at 12% per Annum for Four Years

If the appropriate discount rate is less than 10% (e.g. 6%), then the PV will be higher than 100, since the 10% payment stream has a higher than appropriate payment rate (10%) according to the market (6%).

In Figure 2.10, we show the PV of the 10% per annum fixed cash flow when discounted at a rate of 6% per annum:

N	I	PV	PMT	FV
4	6	113.86	-10	100

Figure 2.10: PV at 6% per Annum for Four Years

You can see the importance of establishing the correct discount rate to calculate the present value of an obligation with fixed multiple payments. Let's see how this discounting process applies to variable-rate payment streams.

Present Value of Multiple Payment Streams at Variable Discount Rates

The PV concept may be applied to multiple payment cash flow streams, even if the payment streams have variable rates and the discount rate is also variable.

Example: PV with variable discount rates

For example, consider a bank that is lending money to a customer for four years and is charging an interest rate that will change annually. We will abbreviate this variable interest rate to VIR. The VIR will be set according to a formula, such as:

VIR = Market interest rate + 1%

Interest rate indices

Of course, the customer wants to know how the market interest rate is calculated. In many markets, there is an index published daily which states the level of inter-bank interest rates, or other such indices. One index is the **London Inter-Bank Offered Rate**, which is abbreviated LIBOR.

The VIR may be 6% one year, 8% the next year, 5% the next year, and 7% the last year. Of course, it can be just about any number. The question is: Can we discount a variable-rate payment stream *before* we know the actual interest rate to be used to determine the actual payment amount?

The answer is YES!

Remember, to discount a cash flow and determine its PV, we generally need to know the cash flow and a discount rate. *The only time we do not need to know these two quantities is when they are the same.* Look again at Figure 2.8 and notice that the PV is always par if the discount rate and payment rate are the same. If this is not obvious, recalculate the PV using a payment rate and discount rate of 5%. Now try the calculation with rates of 2%, 98%, or negative 16%, always keeping the payment rate and the discount rate the same.

Present value constant at par

You should conclude that whenever a cash flow grows at a rate and is discounted at the same rate, its present value stays constant at par. The same observation holds true for variable cash flows and variable discount rates, though it is not as easy to prove with the financial calculator.

Consider the cash flow from the loan described above, where:

$$L_j = \text{12-month LIBOR for period } j$$

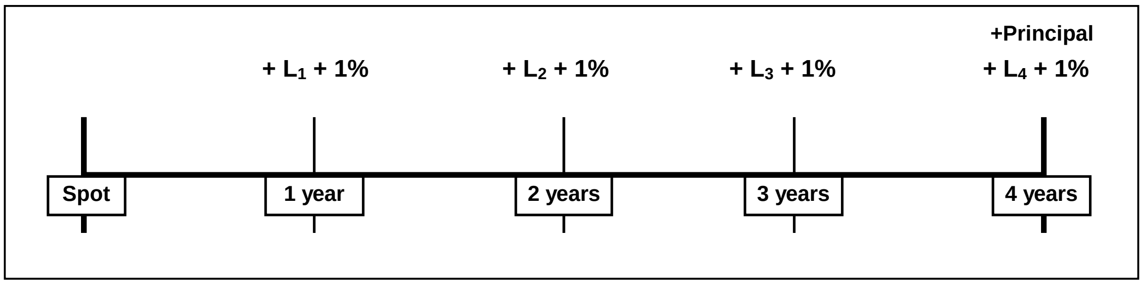

Figure 2.11: A Floating-Rate Loan from the Bank's Perspective

Discount rate equals interest rate

Suppose you need to discount this loan to determine its PV. In order to determine a discount rate, you choose the current rate for lending to this customer, which is LIBOR + 1%. At this point, you may notice that you are about to discount a cash flow at the same rate as the payment rate and you may realize that the result will be par.

We will take the PV process one step at a time to illustrate this concept. Suppose that this is the beginning of the fourth year and there is only one payment left, which is at the end of the year. What is the present value of the loan? (Think of the present value as of that date which is three years from now.)

PV over one interest-rate period

The formula for calculating the present value of an amount over one interest-rate period will help us solve this problem. Recall that the formula is:

$$\mathbf{PV = FV / (1 + ir)}$$

Where:

FV = Future value at the end of one interest-rate period
PV = Present value: amount of money today
ir = Interest rate for one period, expressed as a decimal

The difference for us is that the FV is a payment based upon principal and an interest rate.

$$\begin{aligned} \mathbf{FV} &= \mathbf{Principal + Interest} \\ &= \mathbf{Principal \times (1 + ir_4 + 1\%)} \end{aligned}$$

$$\mathbf{ir = ir_4 + 1\%}$$

Therefore, replacing FV and ir in the formula for present value:

$$\mathbf{PV = Principal \times (1 + ir_4 + 1\%) / (1 + ir_4 + 1\%)}$$

Since: $(1 + ir_4 + 1\%) / (1 + ir_4 + 1\%)$ must equal 1 (any number divided by itself is one), then PV = Principal x 1 = Principal

This is an important result. It says that *any payment stream for one period which has an FV equal to principal x (1 + ir), which is discounted at that same interest rate, yields a PV equal to the principal amount.* This is true no matter how long the period or the size of the interest rate.

Examples: PV equals principal amount

Let's look at three numerical examples to illustrate this concept:

Example 1:

If: Principal = 100 and ir = 7%, then FV = 107
PV = 107 / (1 + .07) = 107 / 1.07
= 100 (which equals principal!)

Example 2:

If: Principal = 100 and ir = 5%, then FV = 105
PV = 105 / (1 + .05) = 105 / 1.05
= 100 (which equals principal!)

Example 3:

If: Principal = 100 and ir = 8%, then FV = 108
PV = 108 / (1 + .08) = 108 / 1.08
= 100 (which equals principal!)

From the formulas, numerical examples, and basic thought, it should be clear that when the future value at the end of the period is based upon a principal amount plus interest, and the same interest rate is used to discount the cash flow, the present value is equal to the principal.

Discounting multiple-period cash flows

What may not be obvious from the examples above is that this idea for a single period can be used sequentially for a multiple-period cash flow. In each period, if the interest rate used for the payment stream is the same as the interest rate used for the discount rate, then the value of the cash flow at the beginning of the period is equal to the principal at the beginning of the period.

In Figure 2.12, this relationship is shown in equation form (with variables). The numerical examples will appear later.

ir_j = Interest rate index + Spread for period "j"

Interest payment = Principal x ir_j

Note that:

Payment at the end of each period	**=**	**Principal + Interest payment (Principal x ir)**
+ Principal + (Principal x ir)	**=**	**+ Principal x (1 + ir)**
PV	**=**	**+ $\frac{\text{Principal x (1 + ir)}}{\text{(1 + ir)}}$ = + Principal**

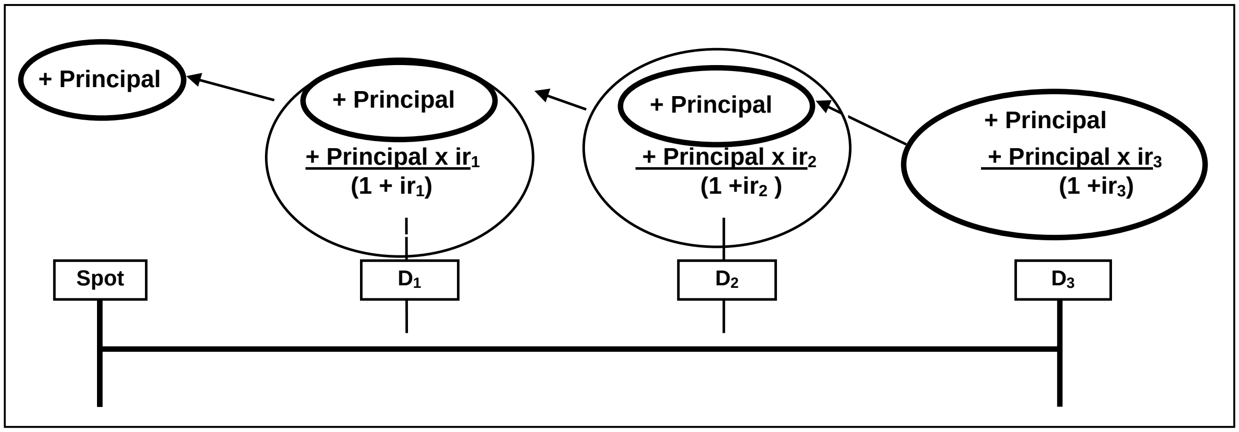

Figure 2.12: A Variable-Rate Cash Flow, Discounted

Key concepts in discounting process

Let's look at the key concepts in this discounting process more closely.

1. First, we determine the value at the end of any period that will be discounted to the beginning of the period.

2. Present value at the beginning of any period equals the value at the end of the period divided by (1 + ir).

3. For the final period only (in this case, period three), the end-of-period value = actual cash payment = principal payment + interest payment = principal x (1 + ir).

4. For the final period, PV = principal x (1 + ir_3) / (1+ir_3) = principal.

5. For the period prior to the final period (in this case, period two), the value at the end of the period equals PV of cash flow in period three (principal) + interest payment in period two.

6. Applying concept two, for period two, PV_2 =

$$\frac{\text{Principal} \times (1+ir_2)}{(1+ir_2)} = \text{Principal}$$

Steps 5 and 6 can be repeated for any number of cash flow periods. PV at the beginning of a period will always be equal to principal when the discount rate is the same as the interest rate.

Example: Discounting process with variable rates

To illustrate this point with numbers, consider this example. Suppose that you can predict the future, and you know what 12-month LIBOR will be for each of the next four years. This means you can determine the interest payments for a four-year, variable-rate loan. To make the calculation simple, we will use the four interest rates and calculations used in the prior examples and figures.

Year	LIBOR	Payment Interest Rate (Libor + 1%)
1	5%	6%
2	7%	8%
3	4%	5%
4	6%	7%

With this information, we can draw a time line of the cash flows for the four-year loan as shown in Figure 2.13.

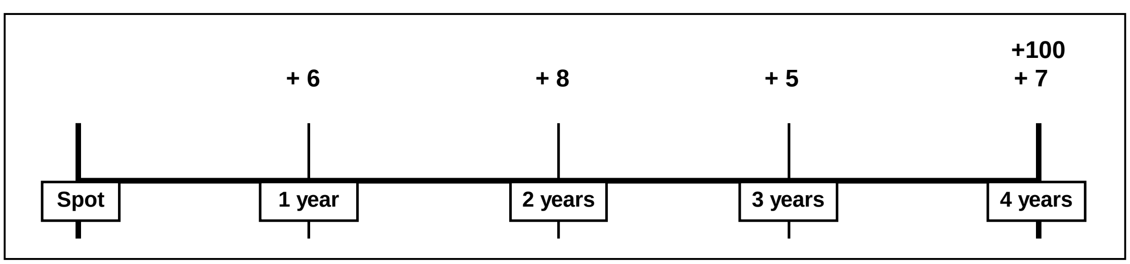

Figure 2.13: A Loan with Known Variable Payments

Of course, we need discount rates in order to determine the PV of the cash flow. Given the information we have, it is appropriate to use the known variable interest rates of LIBOR + 1%, which equal the payment interest rates:

Year	LIBOR	Discount Rate
1	5%	6%
2	7%	8%
3	4%	5%
4	6%	7%

To *discount* the four-year cash flow, we must start at the last period — the fourth year: (100 + 7) / (1 + .07) = 100. Thus, the PV for the fourth year is PV_4 = 100.

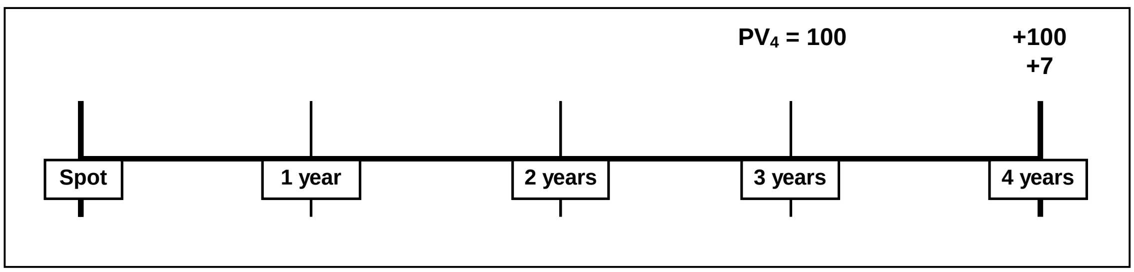

Figure 2.14: Discounting the Fourth Year at 7%

To discount the third year, we add the PV_4 of 100 to the cash flow of 5, and discount at 5%: (100 + 5) / (1 + .05) = 100. Thus, the PV for the third year is PV_3 = 100.

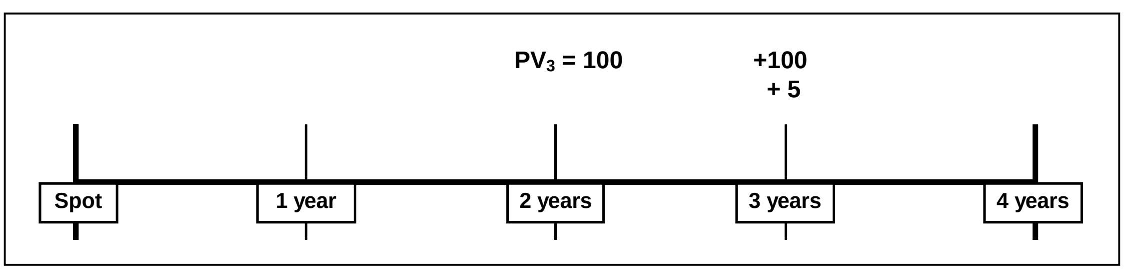

Figure 2.15: Discounting the Third Year at 5%

To discount the second year, we add the PV_3 of 100 to the cash flow of 8, and discount at 8%: (100 + 8) / (1 + .08) = 100. Thus, the PV for the second year is $PV_2 = 100$.

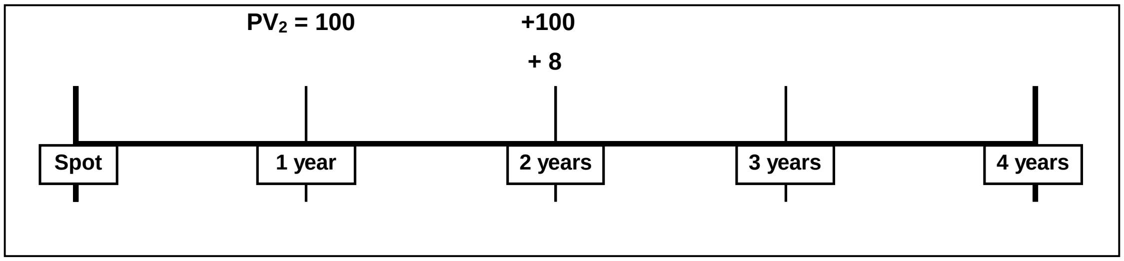

Figure 2.16: Discounting the Second Year at 8%

To discount the first year, we add the PV_2 of 100 to the cash flow of 6, and discount at 6%: (100 + 6) / (1 + .06) = 100. Thus, the PV for the first year is $PV_1 = 100$.

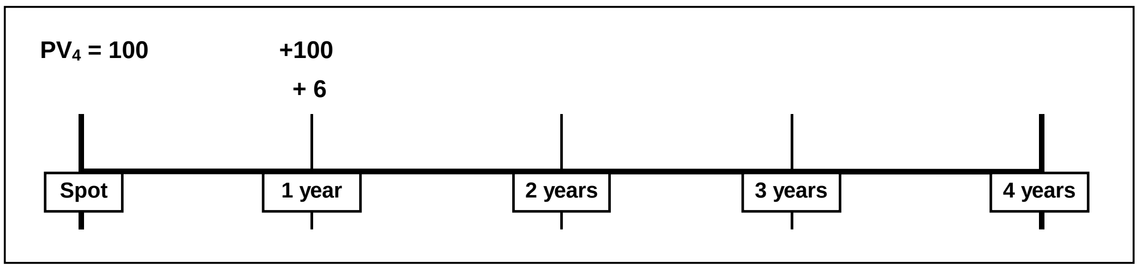

Figure 2.17: Discounting the First Year at 6%

Of course, the PV for the first year, PV_1, is the true present value for the cash flow, since the beginning of the first year is the present date. All of these examples have been presented to illustrate this point:

If a series of cash flows equals principal plus interest, and the interest rate and the discount rate are the same, then the present value of the cash flows equals the principal amount. This is true even if the interest rates are not yet set.

Collapsing the Floating-Rate Cash Flow

The process of discounting a variable-rate cash flow at the same variable rate and effectively moving the principal amount closer to the spot date is often referred to as *collapsing* the floating-rate cash flow. Unfortunately, if the discount rate and the interest payment rate are not the same, this collapsing process does not work in such a simple manner.

Example: Solving for PV using the collapsing process

Consider the bank that owns the loan at LIBOR + 1% with four years remaining. If the current market lending rate is LIBOR + 2%, then the loan is not as good for the bank as it should be. The loan is worth less than the principal amount, but how much less?

To answer this question, we need to discount a four-year income stream of LIBOR + 1% at a discount rate of LIBOR + 2%. In this case, the rate index (LIBOR) is the same for the payment stream and for the discount rate; but the spread over the index is not the same. Thus, the two rates will be different. Unfortunately, there is no way to know exactly what the rates will be and, therefore, no way to *numerically* solve for the PV!

With a little thought, however, *we can apply the collapsing concept*. The cash payment stream and discount rate will offset only if they are the same. Given our choice to lend to this customer now at LIBOR + 2%, the collapsing process will work if the loan pays LIBOR + 2% (which it does not). The “little thought” is that a loan in which the bank receives LIBOR + 1% is the same as a receipt of LIBOR + 2% added to a payment from the bank to the customer of 1%, as illustrated in the following figures. The actual loan in Figure 2.18 is the same as the combination of the loan in Figure 2.19 and the payment stream in Figure 2.20.

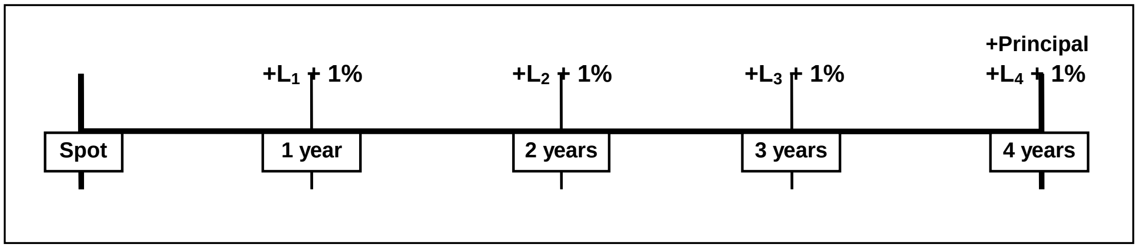

Figure 2.18: A Floating-Rate Loan at LIBOR + 1%

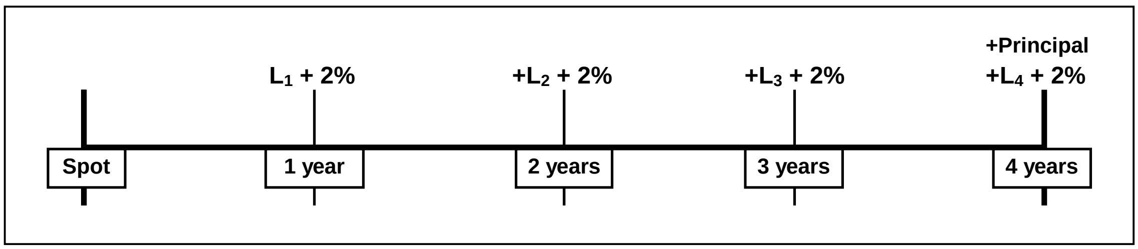

Figure 2.19: A Floating-Rate Loan at 2%

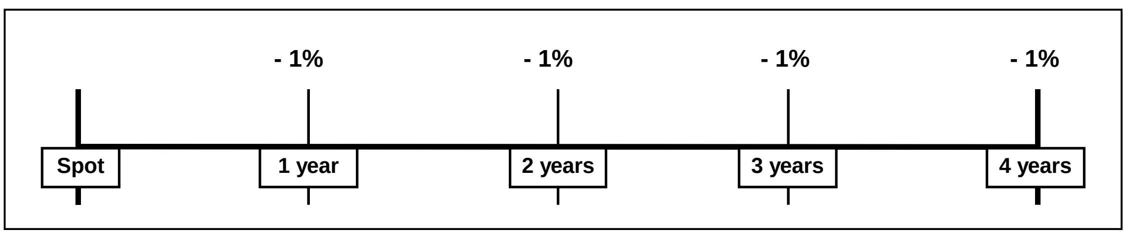

Figure 2.20: An Annuity (Annual Payment) of 1% from the Bank to the Customer

The payment stream in Figure 2.19 can be discounted at LIBOR + 2%, and will be worth the principal amount exactly. The payment stream in Figure 2.20 can be discounted if we have an appropriate interest rate for discounting equal fixed payments received at the end of each year for four years.

Discounting an Annuity

A constant cash flow like that shown in Figure 2.20 is called an annuity. It looks very similar to the cash flows of an **amortizing** loan (a loan in which principal is paid down over time, not only at maturity). If we have a fixed rate that is appropriate for an amortizing loan, we can use it to discount this annuity. For purposes of this example, suppose that the appropriate interest rate is 8%. The PV of this 1% spread can be calculated with a financial calculator as shown in Figure 2.21.

N	I	PV	PMT	FV
4	8	3.31	-1	0

Figure 2.21: PV of Four-Year Annuity at 8%

The PV of the LIBOR + 1% loan is the combination of the PV of a LIBOR + 2% loan and the PV of an outflow of 1% per annum for four years, which is 96.69. The PV is calculated as follows:

100.00	**PV of LIBOR + 2% loan at LIBOR + 2% discount rate**
- 3.31	**PV of 1% payment for four years at 8% discount rate**
96.69	**PV of LIBOR + 1% loan**

The approach just presented allows us *to discount a variable-rate cash flow using a variable discount rate — provided the payment stream and discount rate use the same index, even if the spread above or below the index differs*.

The calculations of the future value of a current cash flow and the present value of a future cash flow are important concepts in financial analysis for comparing values at a given point in time. Borrowers and investors also need to know the cost of financing and the amount of returns on investments so they can compare funding / investing opportunities. In the last section of this unit, “Internal Rate of Return,” this important time value of money concept is explained.

You have completed the sections on “Present Value.” Please complete Progress Check 2.2, then continue to the section on the “Internal Rate of Return.” As usual, if you have any difficulty with the questions, we suggest that you refer to the appropriate text before proceeding.

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☑ PROGRESS CHECK 2.2

Directions: Determine the correct answer to each question. Check your answers with the Answer Key on the next page.

Question 4: If you need 876 in two years and can earn 12% per annum (compounded annually) on a deposit, what is the minimum amount you should set aside now to ensure that you will have enough to pay the obligation in two years (rounded to the nearest 1)?

_____a) 698

_____b) 852

_____c) 1099

_____d) 876

Question 5: If you discount a floating-rate loan's cash flow at an interest rate which has the same index, but a smaller spread above the index than the cash flow, the PV will be:

_____a) equal to the principal of the cash flow.

_____b) more than the principal of the cash flow.

_____c) less than the principal of the cash flow.

ANSWER KEY

Question 4: If you need 876 in two years and can earn 12% per annum (compounded annually) on a deposit, what is the minimum amount you should set aside now to ensure that you will have enough to pay the obligation in two years (rounded to the nearest 1)?

N	I	PV	PMT	FV
2	12	698	0	-876

a) 698

Question 5: If you discount a floating-rate loan's cash flow at an interest rate which has the same index, but a smaller spread above the index than the cash flow, the PV will be:

b) more than the principal of the cash flow.

PROGRESS CHECK 2.2

(Continued)

Question 6: Consider a three-year loan with a variable interest rate of 4% one year, 3% the next year, and 6% the last year. For each of those years, the discount rate equals the interest rate. If the future value of the loan is 200, then the present value at the beginning of the first period is:

_____a) 100

_____b) 179

_____c) 222

_____d) 200

Question 7: Assume you have an inflow of 200 available at the end of three years and you want to borrow funds now, repaying all principal and interest in three years with the 200 receiveable. Suppose that a lender offers an unusual interest rate scheme: you would pay a variable annual interest rate of 10% during the first year, 5% during the second year, and 8% during the third year. How much could you borrow today? (Assume that all interest is capitalized, and no payments are made until the end of the third year.)

_____a) 154

_____b) 160

_____c) 165

_____d) 177

ANSWER KEY

Question 6: Consider a three-year loan with a variable interest rate of 4% one year, 3% the next year, and 6% the last year. For each of those years, the discount rate equals the interest rate. If the future value of the loan is 200, then the present value at the beginning of the first period is:

d) 200

Question 7: Assume you have an inflow of 200 available at the end of three years and you want to borrow funds now, repaying all principal and interest in three years with the 200 receiveable. Suppose that a lender offers an unusual interest rate scheme: you would pay a variable annual interest rate of 10% during the first year, 5% during the second year, and 8% during the third year. How much could you borrow today? (Assume that all interest is capitalized, and no payments are made until the end of the third year.)

$$\begin{aligned} PV &= FV / [(1 + ir_1) \times (1 + ir_2) \times \ldots \times (1 + ir_n)] \\ &= 200 / (1.10 \times 1.05 \times 1.08) \\ &= 200 / 1.2474 \\ &= 160 \end{aligned}$$

b) 160

INTERNAL RATE OF RETURN

Applied to known cash flows

The third and last basic time value of money concept is the **internal rate of return (IRR)**. IRR converts varying amounts of money paid and received over time into a single interest rate, and is used to compare cost of financing or returns on investments. It is applied when a series of cash flows is known, including the up-front cash flow.

In a fair transaction, the up-front cash inflow or outflow is the opposite of the present value of the future cash flows. For example, when a bank lends 100 principal (outflow), it receives the 100 principal plus interest in the future (inflow). Conversely, when a bank receives a deposit of 100 (inflow), it pays out the 100 plus interest in the future (outflow).

In some transactions, the up-front cash flow is not the same as the principal amount of the transaction. Consider the investment in Figure 2.22, which is the bond that we illustrated in Figure 1.9.

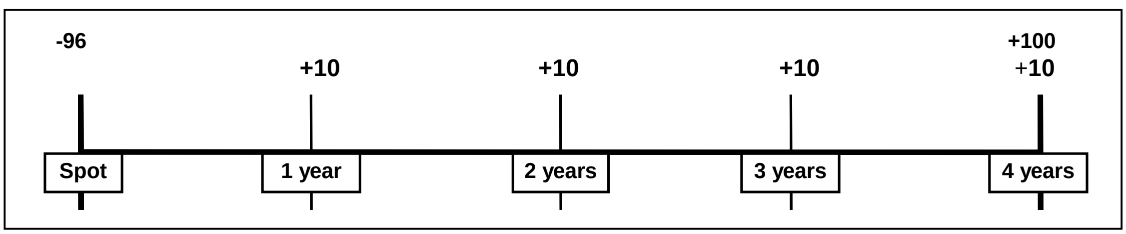

Figure 2.22: Four-Year, 10% Bond from the Investor's Perspective

IRR applied to fixed-rate cash flows

In this case, it is obvious that the payment rate and the discount rate cannot be the same because the price is not par (100). If the investor invests less than 100, and has a return of 100 at maturity plus 10% of 100 each year, the return on the 96 investment is larger than 10%. What is the investor's percentage return, including both the coupon payment of 10 and the more-than-invested final payment of 100?

One way to answer this question is to think about placing the 96 investment into a theoretical fixed-rate bank account. Each year, the balance in this theoretical bank account is increased by interest earned and decreased by the payments withdrawn. Suppose the bank guarantees a single interest rate for any amount in the account over time. What interest rate will you need from the bank to be able to withdraw exactly 10 at the end of each year and exactly 100 at maturity, leaving a zero balance in the account after that final withdrawal?

This question is difficult to answer. You can guess a specific interest rate, list the interest earned and payments withdrawn over time, and then check to see if the account has exactly 110 at maturity to allow you to withdraw the final coupon and face value amounts. If the guess is wrong, you can modify your guess and recalculate all interest amounts to see if the new guess is correct. This process can take a long time. Luckily for us, the financial calculators automate this guessing process. The result determined by the HP-19B is shown in Figure 2.23.

N	I	PV	PMT	FV
4	11.30	-96	10	100

Figure 2.23: IRR of Four-Year 10% Bond at Price of 96

IRR: Break-even interest rate

After considerable guessing and modifications of guesses, the financial calculator determines that 11.30% is the interest rate that will be needed to place 96 on deposit, withdraw 10 every year for four years, and withdraw the 100 face value at the end of the four years. The financial markets call this break-even interest rate the internal rate of return (IRR) of the cash flows.

The IRR is the interest rate which can be used to discount the future cash flows and determine a PV which is equal in amount and opposite to the up-front cash payment.

Although the process used to determine this IRR is complicated, it is relatively easy to check to see if the IRR is correct. To check the IRR, we will simulate the theoretical deposit discussed above. We will start with a deposit of 96, add interest at a rate of 11.30%, and withdraw the coupons of 10 at the end of each year and the face value of 100 at the end of the fourth year. If the deposit balance is exactly zero at the end of the process, then the IRR must be correct.

Year	Initial Balance	11.30% Interest Added	Payment Withdrawn	Ending Balance
1	96.00	+ 10.85	- 10	= 96.85
2	96.85	+ 10.94	- 10	= 97.79
3	97.79	+ 11.05	- 10	= 98.84
4	98.84	+ 11.17	- 110	= .01

Except for a round-off error of .01, the bank account has a balance of zero. This implies that the 11.30% interest rate is the rate at which an investment of 96 grows sufficiently to pay 10 each year and an additional 100 at the end of four years. The IRR of these cash flows is 11.30%.

IRR applied to floating-rate cash flows

The concept of an IRR can be applied to floating-rate cash flows as well. For example, if you purchase the LIBOR + 1% loan we showed in Figure 2.18, but pay only 95% of principal as a purchase price, what will your IRR be?

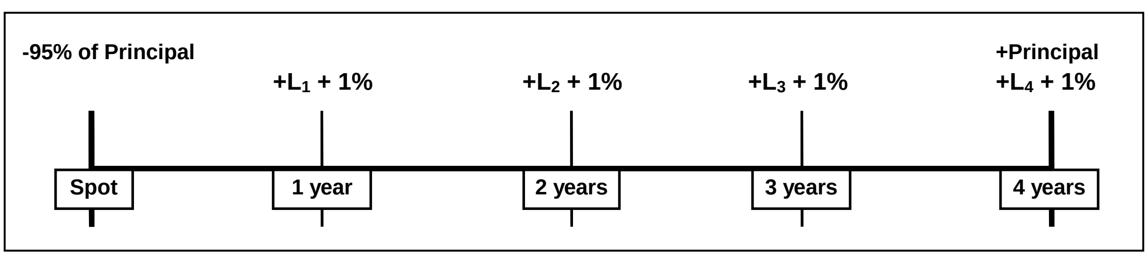

Figure 2.24: A Discounted Loan Paying LIBOR + 1%

Suppose that the principal amount is 100, and the up-front cash flow is 95. The simplest approach to determining an IRR is to amortize the 5% up-front discount in the purchase price. This calculation simulates placing on deposit an additional 5 so that the total up-front cash payment is 100. The deposit will generate returns over time, which can be added to the annual payments to determine the incremental earnings above LIBOR + 1%. The question is, how much extra money will this extra deposit generate over time?

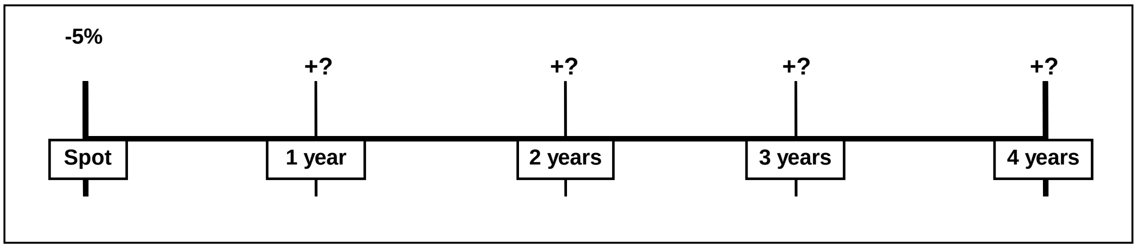

Figure 2.25: Amortizing Up-Front Discount over Four Years

Suppose that the interest rate earned on the deposit is 6% per annum. A financial calculator can determine how much payment we can withdraw each year for four years if we deposit 5 in the account up front and earn 6% per annum interest.

N	I	PV	PMT	FV
4	6	-5	1.44	0

Figure 2.26: Amortization of Up-Front Discount

Therefore, the cash flows of the deposit of 5 will be +1.44 as shown in Figure 2.27.

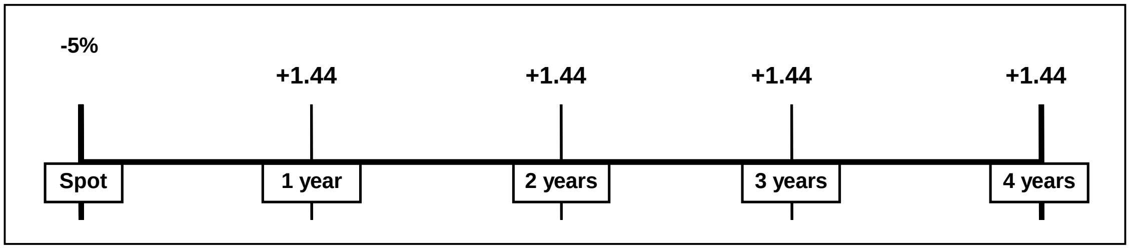

Figure 2.27: Amortizing Deposit of 5 over Four Years at 6%

These inflows of 1.44 can be expressed as 1.44% of the 100 principal amount. When these cash flows from the deposit are added to the LIBOR + 1% cash flows, the total for the bank is + LIBOR + 2.44 as shown in Figure 2.28.

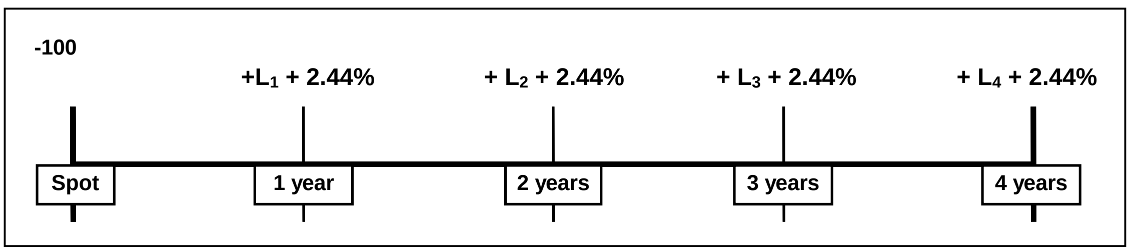

Figure 2.28: Cash Flows for a Discounted Loan Paying LIBOR + 1%

Since this cash flow is now priced at par, its IRR must equal the payment rate. The IRR is LIBOR + 2.44%.

SUMMARY

Financial markets consider *time value of money* concepts, including present value, future value, and internal rate of return.

The concept of future value recognizes that money today can often earn interest and grow to a larger amount of money in the future. Money owed today, plus interest charges, grows to larger amounts of money in the future. Interest rates provide a relationship between money now and money in the future.

The present value idea is another way to look at the future value concept. Since money today grows to a larger amount in the future, obligations in the future have proportionately smaller value today. If you want to borrow money today and repay the loan with a future receivable, you will have to borrow less than the receivable. This is because a portion of the receivable will be used to pay interest charges on the loan.

When we determine the present value of a future cash flow, we are said to *discount* the cash flow. Floating-rate cash flows can be discounted by floating discount rates, provided the interest-rate index is the same as the discount-rate index. When the cash flow payment rate is the same as the cash flow discount rate, the PV equals the principal amount, whether the interest rates are fixed or floating, known or unset.

When all of the cash flows of a transaction are defined, including the up-front cash flow, financial markets refer to the interest rate which is inherent in the cash flows. This break-even interest rate is called the internal rate of return. The IRR is the interest rate which has to be used to discount the future cash flows in order to create a PV which is equal and opposite (inflow / outflow) to the up-front cash flow. IRR is used to compare alternative uses of money from the perspective of return per period or cost per period. IRR includes all payments, thus it combines normal income ideas as well as capital gains or losses. The concept of IRR can be applied to variable-rate cash flows as well.

You have completed Unit Two, *Time Value of Money*. Please complete the Progress Check to test your understanding of the concepts and check your answers with the Answer Key. If you answer any questions incorrectly, please reread the corresponding text to clarify your understanding. Then, continue to Unit Three, *Determining the Price and Yield of Fixed-Rate Securities.*

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☑ PROGRESS CHECK 2.3

Directions: Determine the correct answer to each question. Check your answers with the Answer Key on the next page.

Question 7: If you borrow 145, then pay 12 each year for three years plus 160 at the end of three years, what is the full interest cost (IRR) on a per-annum basis?

_____a) 8.28%

_____b) 7.50%

_____c) 13.28%

_____d) 11.36%

Question 8: The internal rate of return (IRR) is the interest rate:

_____a) paid by the Internal Revenue Service (IRS) on tax overpayments.

_____b) earned by banks in internal transactions.

_____c) which discounts all cash flows of a transaction to equal the opposite of the up-front payment.

_____d) which, when used for compounding all cash flows of a transaction, creates a future value of 100.

Question 9: If you pay an up-front fee of 2% when you borrow 100 for four years at an interest rate of INDEX + 1.00% (interest paid annually, principal at maturity), what is the all-in cost of the funds (IRR) of the transaction? (The four-year amortizing interest rate is 6% per annum.)

_____a) INDEX + 1.50%

_____b) INDEX + 1.58%

_____c) INDEX + 2.50%

_____d) INDEX + 3.00%

ANSWER KEY

Question 7: If you borrow 145, then pay 12 each year for three years plus 160 at the end of three years, what is the full interest cost (IRR) on a per-annum basis?

N	I	PV	PMT	FV
3	11.36	-145	12	160

d) 11.36%

Question 8: The internal rate of return (IRR) is the interest rate:

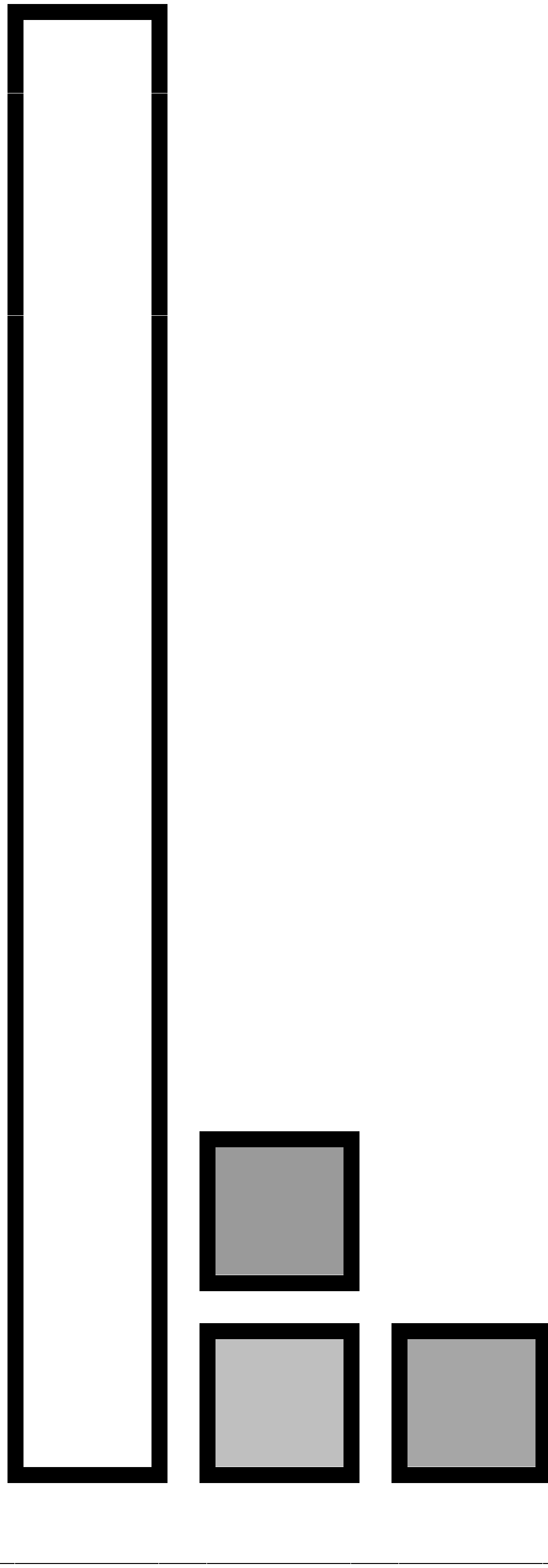
c) which discounts all cash flows of a transaction to equal the opposite of the up-front payment.

Question 9: If you pay an up-front fee of 2% when you borrow 100 for four years at an interest rate of INDEX + 1.00% (interest paid annually, principal at maturity), what is the all-in cost of the funds (IRR) of the transaction? (The four-year amortizing interest rate is 6% per annum.)

N	I	PV	PMT	FV
4	6	-2	.58	0

INDEX + 1.00%
+ .58% PMT
INDEX + 1.58%

b) INDEX + 1.58%



Unit 3

UNIT 3: DETERMINING THE PRICE AND YIELD OF FIXED-RATE SECURITIES

INTRODUCTION

The prices and yields of fixed-rate securities are related according to time value of money concepts, given specific quoting conventions. As with all financial transactions, the ultimate determinant of price is the agreement between the two parties to the transaction. In this unit, we briefly describe the factors influencing the price of a security, introduce the common quoting conventions, and illustrate the calculations of price and yield.

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize how interest-rate movements influence the price of a fixed-rate security
- Identify common quoting conventions in fixed-rate markets
- Calculate the price of a fixed-rate security, given its yield
- Calculate the yield of a fixed-rate security, given its price

PRICES OF FIXED-RATE SECURITIES

Risk concerns of investors

Fixed-rate securities, such as bonds, are issued by borrowers who need liquidity. The investors who buy the securities generally look for reasonable returns with acceptable amounts of risk. The risks involved include:

- Credit worthiness of the borrower
- Liquidity of the security before maturity
- Price risk of early liquidation due to currency and interest-rate movements

Role of underwriters

Underwriters act as intermediaries to match borrowers with investors. They advise issuers about the terms of a security offering and help provide information to the investors to assure them that the issuers are a reasonable risk. Underwriters often agree to create a secondary market for the securities in order to assure investors that they can liquidate investments at reasonable prices should they need liquidity.

In advising issuers, the underwriters consider the demands of the investors with whom they have a close relationship. Investors indicate the types of investments they will accept, and the underwriters attempt to find an issuer who meets the investors' criteria and is willing to issue the type of security the investors want to purchase. Often, the intermediary must offer a swap to the issuer to help transform the issuer's liability into acceptable terms for the issuer. (See the Swaps workbook.) Normally, the longer the term of the investment commitment, the higher the interest rate demanded by the investor.

Investor Strategies

Expectations of rising interest rates

In situations where many investors expect a rise in interest-rate levels, they likely will demand even higher returns for long-term investments. In this case, investors may prefer to invest in short-term securities or in variable-rate, longer-term securities which have periodic interest-rate payment resets. In such an environment, selling long-term, fixed-rate bonds is more difficult than usual, since bond prices tend to fall as interest rates rise.

Expectations of declining interest rates

In situations where most investors anticipate a decline in interest rates, fixed-rate bonds are easier to sell. When this expectation is combined with a normal, upward-sloping **yield** curve (long-term rates are higher than short-term rates), bonds are in high demand from investors and, therefore, easy to sell. In this case, investors will see a double benefit from investing in bonds: they will receive a higher coupon while they hold (own) the bond, and it is likely they will see a capital appreciation when they sell the bond. This appreciation is a result of increased demand for the old fixed rate after interest rates have declined.

Example: Appreciation potential

To illustrate this appreciation potential, consider an investor who wants to invest for one year and purchases an 8% annual-coupon, four-year bond at par (100%). (Figure 3.1)

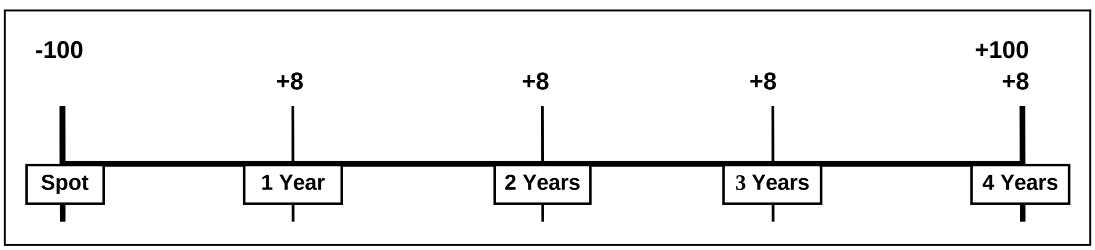


Figure 3.1: 8%, 4-Year Bond from the Investor's Perspective

Suppose the investor sells the bond in one year at a price which offers the new investor a three-year return of 7%. To determine the price of the bond at that time, we calculate its present value (PV). Using a financial calculator, you can determine that the price of the bond is 102.62 as you can see illustrated in Figure 3.2.

N	I	PV	PMT	FV
3	7	102.62	- 8	- 100

Figure 3.2: Bond Price with 3-Year Return at 7%

The original investor's one-year return is 100% of original capital plus 8% from the normal coupon plus 2.62% from the capital gain, which equals a total of 10.62% (Figure 3.3).

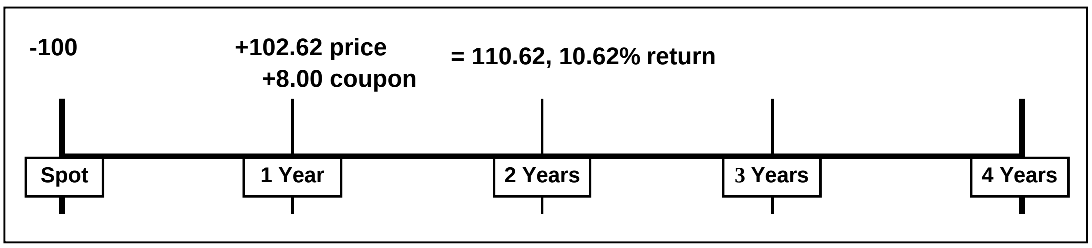


Figure 3.3: One-Year Return of Original Investor with a Gain

The one-year return of 10.62% is larger than the original one-year investment opportunities, since short-term investments at that time had lower returns than the 8% four-year bond.

This example is not intended to imply that investors only buy bonds when they expect rates to fall. In fact, even if rates had risen slightly, the investor would have realized a larger return from the high coupon on the four-year bond to compensate for a small capital loss on the sale of the bond.

Need for long-term fixed-rate securities

In addition, many large institutional investors, such as insurance funds and retirement funds, have long-term liabilities such as insurance claims or retirement payments which likely will occur many years in the future. These investors may value fixed returns for the simple reason that they are guaranteed, independent of any decline in interest rates. Individual investors saving for a specific future expenditure may value the "fixed" component of a bond, even if they give up a small amount of potential return.

As a result of these long-term liabilities, there is generally a market for long-term, fixed-rate securities. This market grows dramatically when long-term rates are especially large compared to short-term rates, and when there is the expectation of a fall in interest rates.

Example: Loss potential

Of course, investors do not always perfectly predict interest-rate movements. For example, the investor in the example above could have been wrong about interest rates. If the yield on three-year bonds at the end of the first year was 9% instead of 7%, the bond price would have been 97.47, as we can see in Figure 3.4.

N	I	PV	PMT	FV
3	9	97.47	- 8	- 100

Figure 3.4: Bond Price Calculation at 9%

The sale price of 97.47 will result in a capital loss of 2.53%. Total return will be 8% coupon plus 97.47 price = 105.47%. This is equivalent to a one-year, interest-rate return of 5.47%, as seen in Figure 3.5.

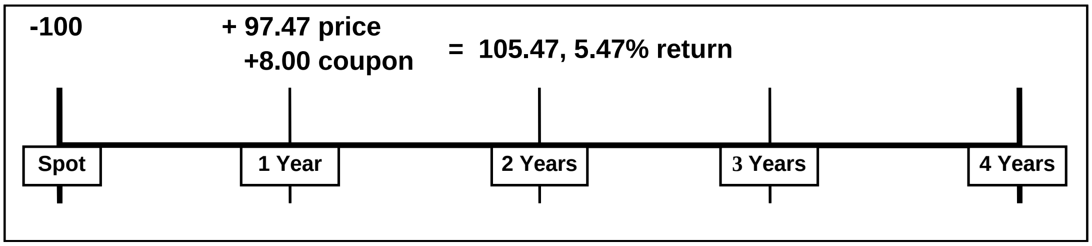


Figure 3.5: One-Year Return of Original Investor with a Loss

Holding bond to maturity

In both of these situations (rising and falling interest rates), the investor planned to sell the bond after one year. That is not always the case. If an investor plans to hold a bond to maturity, s/he is not as concerned about the fluctuating secondary-market value of the bond and does not incorporate price changes into return calculations.

QUOTING CONVENTIONS

Yield-to-maturity: Internal rate of return

Since there is no general agreement as to how long investors hold particular bonds, market participants refer to a bond's **yield-to-maturity (YTM)**. This calculation determines the internal rate of return (IRR) of a set of cash flows created by paying the purchase price of a bond and receiving the coupons plus face value. For example, if the four-year, 8% annual payment bond described above had a price of 101% of face value, its yield-to-maturity would be 7.70%, as determined with the help of a financial calculator (Figure 3.6).

N	I	PV	PMT	FV
4	7.70	-101	8	100

Figure 3.6: Bond Yield Calculation at 101% Price

Different financial markets have their own conventions for quoting yields. We will look at three rate-quoting conventions: bond equivalent yield, money market yield, and discount rate.

Bond Equivalent Yield

Bond equivalent yields are quoted according to the number of coupon payment periods. The simplest yield-quoting convention is the one used in the previous example — the annual bond equivalent yield.

Annual Bond Equivalent Yield

One coupon per year

In the previous examples, the convention for quoting yield is identical to the one used in the international bond market and the domestic bond markets of most European countries. It is called the annual **bond equivalent yield** (**BEY**). The bonds in these markets have one payment per year, and the annual BEY represents the IRR of the cash flows. As a matter of review, the yield-to-maturity (expressed on an annual BEY basis) of a four-year, 8% bond, when priced at 101% of face value, is 7.70% (Figure 3.6).

Semi-Annual Bond Equivalent Yield

Two coupons per year

Many bonds pay two coupons per year. These semi-annual payment bonds express yields on a semi-annual, bond-equivalent-yield basis. For example, if a four-year bond pays 8% per year, but in equal payments every six months, the cash flows will be as shown in Figure 3.7.

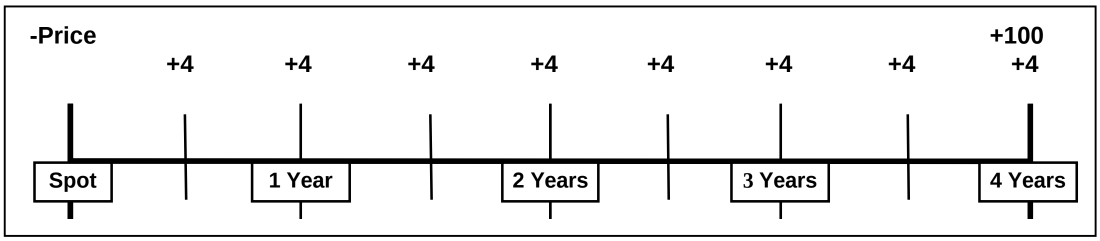


Figure 3.7: 10%, 4-Year Semi-Annual Bond

If the price of the bond is 101% of the face value, the YTM of the bond (on a semi-annual BEY basis) is determined through the standard IRR calculation. The calculators, however, expect as input the number of payments (in this case, 8), and express yield on a per-period basis (in this case, semi-annual).

N	I	PV	PMT	FV
8	3.85	-101	4	100

Figure 3.8: Semi-Annual IRR Calculation at 101% Price

The result of this calculation is the IRR of 3.85% per six-month period. Since few people express interest rates on a six-month basis, the market convention is to express the interest rate or yield for a one-year period. The simple approach with semi-annual bonds is to multiply the semi-annual IRR by two to express an annual quote. This annual quote is the yield-to-maturity on a semi-annual basis. In the case above, the YTM is:

IRR 3.85 x 2 = 7.70% YTM on a semi-annual BEY basis

This semi-annual BEY convention is used by some of the largest bond issuers in the world (governments and corporate issuers in the domestic bond markets of the US, UK, and Japan).

Other Bond Equivalent Yields

Quarterly or monthly basis

In a few markets, bond coupons are paid on a quarterly basis, or possibly even on a monthly basis. In theory, the idea of a bond equivalent yield can be applied to any payment frequency (even daily BEYs). The basic fact about bond equivalent yields is that the annual quote is simply the number of payments per year times the actual payment on each payment date. For example, if a four-year bond pays 3% coupons every quarter, then its coupon rate is 12% *per annum*. A bond with a price that is 95% of face value has a quarterly internal rate of return as calculated in Figure 3.9.

N	I	PV	PMT	FV
16	3.41	-95	3	100

Figure 3.9: Quarterly IRR Calculation at 95% Price

The YTM of this bond on a quarterly BEY basis is:

IRR 3.41% x 4 = 13.64% YTM on a quarterly BEY basis

Although the bond market convention for quoting interest rates and yields is straightforward, it does not easily handle interest periods which are not a fixed number per year. For example, if a borrower borrows principal under a credit line, and repays principal plus interest 19 days later, how is the interest payment determined? The second rate-quoting convention, money market yields, provides a solution.

Money Market Yields

Payment amounts modified by number of days

Lenders offering interest periods which can be any number of days have created their own convention. They express interest rates on an annual basis, but they modify the payment amount according to the number of days in the payment period.

Annual

The interest payment formula for annual interest payments is simple:

Payment = Principal x Interest rate

Fraction of a year

The interest payment formula for a fraction of a year is:

Payment = Principal x Annual interest rate x (Fraction of a year)

In general, for “t” payments per year, the formula used to determine each payment is:

Payment = Principal x Interest rate x 1 / t

Semi-annual

For example, when interest is paid twice a year, the formula is modified accordingly:

Payment = Principal x Interest rate x 1 / 2

Monthly

For lenders who are paid every month, the fraction of a year is (1 / 12), and the formula is:

Payment = Principal x Interest rate x 1 / 12

Adjustment for number of months

Many lenders have kept this idea and modified it according to how many months they allow the borrower to go before paying interest. Thus, a borrower who pays interest every three months has a fraction of a year of (3 / 12), and pays according to the following formula:

Payment = Principal x Interest rate x 3 / 12

In general, the fraction of the year is (number of months / 12), and the formula is:

Payment = Principal x Interest rate x Number of months / 12

Adjustment for extra number of days

Of course, if a borrower repays the loan on a date which is not exactly a number of months since the last interest payment, the determination of the interest payment will have to include the extra number of days (in addition to the number of full months). These lenders assume 30 days in the average month to determine the fraction of a month represented by the extra days. The fraction of the year becomes:

(Number of months + Fraction of a month) / 12

This is the same as:

(Number of months + Number of extra days / 30) / 12

An easier way to write this formula is:

[(30 x Number of months) + Number of extra days] / (12 x 30)

To make it even easier, the formula may be written as:

[(30 x Number of months) + Number of extra days] / (360)

30 / 360 convention

This is called the "**30 / 360**" convention. Interest-payment formulas using this convention have the following form:

PMT = Principal x IR x [(30 x Number of months) + Number of extra days] / 360

As long as there are no extra days, this 30 / 360 convention is the same as the monthly interest calculation or the BEY as shown above. The advantage of this convention is that it can handle interest payment periods which are not simple multiples of months.

This formula is used for many interest products which normally have periodic payments, but occasionally may have an odd extra number of days. It is commonly used for accrued interest calculations on bond purchases and sales (e.g. US domestic corporate bonds and international bonds).

Actual / 360 convention

Of course, for payment periods with a small actual number of days (such as weekly payment periods or overnight), the 30 / 360 convention doesn't have any "number of months." In this case, the interest-payment formula is:

PMT = Principal x IR x Actual number of days / 360

The "fraction of a year" is simply **actual number of days / 360**.

This convention has become known as the **actual / 360** convention and is used in many large short-term, interest-rate markets. In fact, it is used for all domestic US$ interest-rate products and most international (EURO) inter-bank, interest-rate products with original maturities of less than one year.

The unusual result of this convention is that if the payment period is one *calendar year*, the fraction of a year will be 365 / 360 because there are 365 days in a year!

Fraction of a year = 365 / 360 = 1.0139

This creates an effective interest rate which exceeds the quoted rate. For example, if you borrow 100 at 10% interest rate for one year, your interest payment will be:

Interest payment = 100 x .10 x 365 / 360 = 10.14

Actual / 365 and actual / actual conventions

Many other market developers have looked at this unusual result and made a simple modification to their formulas which removes the problem. They remove the "360" and replace it with "365" — or with the actual number of days in a year (which is 366 during every leap year). Their modifications have resulted in two conventions:

Actual / 365

Actual number of days in period / 365

Actual / Actual

Actual number of days in period /
Actual number of days in calendar year (365 or 366)

These conventions are used by many short-term, interest-rate markets, including the UK, the Commonwealth countries, and the most recently-developed interest-rate markets. It is possible to have the domestic market use the 365-day convention while the international (Euro or offshore) market for the same currency uses the 360-day convention. The important point here is to ensure that *you and everyone else involved in a transaction agree on which convention is to be used to calculate the interest payment.*

These conventions are called **money market yield*s*** (**MMY**). For example, if a six-month deposit is accepted by a bank at 10%, the 10% is called a semi-annual, money market yield. The difference between this 10% and the 10% semi-annual BEY is that the semi-annual BEY means exactly 5% on each payment date, independent of the number of days in each six-month period. In Figure 3.10, we see the cash-flow differences resulting from the four interest-rate conventions (BEY, actual / 360, actual / 365, and actual / 366), for the period from January 1, 1996 to January 1, 1997.

Total: 10.00%	=	.10 x 1/2	= .0500	+	.10 x 1/2	= .0500
Total: 10.17%	=	.10 x 182 / 360	= .0506	+	.10 x 184 / 360	= .0511
Total: 10.03%	=	.10 x 182 / 365	= .0499	+	.10 x 184 / 365	= .0504
Total: 10.00%	=	.10 x 182 / 366	= .0497	+	.10 x 184 / 366	= .0503

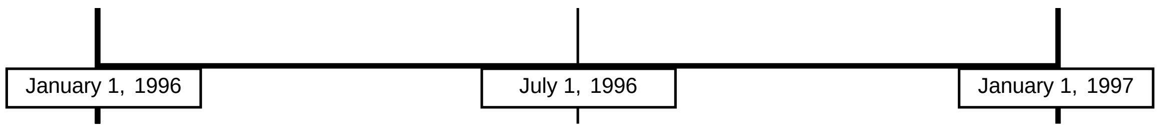


Figure 3.10: Comparing 10% Interest Quote on the Basis of Four MMY Quoting Conventions

Now that you know how to calculate bond equivalent yields and money market yields, we will look at the last major form of interest-rate quotation — the discount rate.

Discount Rate

Example: Discounted loan amount

Consider a farmer who has a firm contract to sell a reasonable amount of a crop to a well-known buyer with good credit. The contract states that the buyer will pay a total of 100 upon receipt of the crop. Unfortunately, this farmer needs to borrow money now to buy seed, plant a crop, and harvest the crop. He feels comfortable borrowing the money now, knowing that he has a 100 receivable which can be used to repay the loan with interest.

Suppose this farmer approaches lending institutions and asks to borrow money now, pledging as collateral the receivable of 100 from the buyer. How much will they lend him?

Many banks will not lend the full 100, since the 100 coming in is needed to repay the principal and the interest. To make the calculation simple, assume that the receivable is expected in one year. The banks discount the 100 receivable, lending less than the 100.

Now, let's look at how a bank may discount the farmer's 100 receivable by 10%. To the lender, this means:

Discount = Amount at back-end x Discount rate

Discount = 100 x 10% = 10

The amount of the loan to the farmer will be:

Amount up front = Amount at back-end - Discount

Amount up front = 100 - 10 = 90

The lender will lend 90 now and will be repaid with 100 at the end of the year.

Effective interest rate

Of course, the effective interest rate being charged is larger than 10%, since 10 is paid on a loan principal of 90. The effective interest rate is calculated as follows:

Effective interest rate = Interest amount paid / Amount borrowed

Effective interest rate = 10 / 90 = 11.11% of amount borrowed

Quoted interest rate

The 10% quoted rate is a rate on the amount paid at the end:

Quoted discount rate = Interest amount paid / Total amount paid at end

This quoting convention is used not only in agricultural trade, but in many forms of trade. It became so common hundreds of years ago that it became the normal way to quote amounts paid up front against future receivables. As a result, this is the quoting convention used by the:

- US treasury for all instruments with an original maturity under one year (US Treasury Bills, called T-Bills)
- US commercial paper (CP) market (maturities averaging from less than one month up to 9 months)
- US banker's acceptance (BA) market (maturities up to 9 months, but averaging closer to 3 months)

Each of these three markets is large and liquid. As a result, the discount quotation convention is used quite regularly in the financial markets.

This discount quoting convention should not be confused with the idea of a *discount instrument*. Many financial instruments trade in the secondary market at a price below par (100%). This often is called *trading at a discount*, and people often quote the discount. Some instruments, such as zero-coupon bonds, are called *discount securities*. None of these uses of the word "discount" mean that the product is quoted on a discount basis as described above. In fact, many bank bills and other instruments in money markets outside of the US are quoted as discount securities, but the rate quoted is an interest rate that can be used to discount (present value) the cash flow to determine its price. This type of "discount rate" is not the same as the idea of a discount quote.

Actual / 360 convention for discount quotes

The day count conventions described in the money market yield section must be applied here as well because the true discount quote is used for short-term securities. Since the discount-quote markets listed above are domestic US markets, they use the actual / 360 convention common in the US short-term, interest-rate market.

Example: Discount for actual number of days

For example, if a US T-Bill with a face value of $100,000 maturing in 87 days is purchased at a discount of 10%, the discount and price is determined as follows:

Discount = Face value x Discount rate x Actual days / 360
Discount = $100,000 x .10 x 87 / 360 = $2,416.66
Purchase price = Face value - Discount
Purchase price = $100,000.00 - $2,416.66 = $97,583.34

If a $100,000 CP maturing in 19 days is purchased for $99,500, the discount is:

Discount = Face value - Purchase price
Discount = $100,000 - $99,500 = $500

The quoted discount rate is:

Discount rate quoted = Discount / Face value x 360 / Actual days
Discount rate quoted = $500 / $100,000 x 360 / 19 = 9.47%

SUMMARY

Time value of money concepts such as PV and IRR are used to discuss the price and yield of fixed-rate securities.

Borrowers, including governments and corporations, issue fixed-rate securities to generate liquidity. Investors, including pension funds, insurance funds, mutual funds, corporations, governments, and individuals, purchase fixed-rate securities looking for reasonable returns given credit risks, liquidity risks, and rate risks. Underwriters act as intermediaries between borrowers and investors. Underwriters consider investor demands when advising borrowers on the terms of debt offerings.

Fixed-rate securities generally appreciate in value as interest rates fall. Therefore, it is usually much easier for underwriters to place fixed-rate securities when interest rates are expected to fall than when they are expected to rise. Investors who purchase fixed-rate securities, such as bonds, receive periodic interest payments in the form of coupons plus a potential increase in the price of the security if they sell it prior to maturity. Of course, they face the risk of a price decline before maturity as well.

Basically, there are three rate-quoting conventions:

- Bond equivalent yield
- Money market yield
- Discount rate

When you know a bond's price, you can determine its yield-to-maturity (YTM). The YTM of a bond is the internal rate of return (IRR) of its cash flows, given its price. The YTM of annual payment bonds is the annual IRR of the bond's cash flows. It is called an annual bond equivalent yield, or annual BEY. This convention is used for most international bonds and the domestic bonds of many European countries. The YTM of semi-annual payment bonds is expressed on a semi-annual payment basis. It is the semi-annual IRR multiplied by two. This is called a semi-annual BEY. This convention is common in the domestic and government bonds throughout the US, UK, Japan, and other issuers. Similar concepts apply for quarterly-payment bonds, and for any other payment frequency on a bond.

The bond market yield quoting conventions are not generally used in the bank loan market, even if the loan has a fixed rate for a long tenor. The bank market usually uses a quoting convention called the money market yield (MMY). Money market yields include the number of days in the payment period in the calculation of interest payments. The primary MMY conventions used for determining the portion of an annual interest rate applicable to a specific period are 30 / 360, actual / 360, and actual / 365. The formulas are:

30 / 360:	**[30 x (Number of months + Extra number of days)] / 360**
Actual / 360:	**Actual number of days / 360**
Actual / 365:	**Actual number of days / 365**

These conventions are used for determining interest payments as well as for calculating interest accruals on many fixed-rate securities.

- Actual / 360 is used throughout most of the US money markets, corporate loans, and for many Euro-markets (offshore markets).
- 30 / 360 is used for many corporate and international bond coupon accruals, and for many corporate bank loans.
- Actual / 365 is used throughout Commonwealth countries and most of the newly-developed financial markets.

The third primary form of rate quoting convention is the discount rate convention. The term “discount rate” used here is not the same as the idea of discounting cash flows with a discount rate. This third quoting convention’s name comes from the process of lending money on trade-related transactions where the seller has a contract with the buyer which stipulates a future transaction price. The lenders discount the amount in the contract and loan less than the contract amount. The difference between the up-front amount of the loan and the future value repaid is called a discount (similar to interest).

The discount divided by the future amount in the contract is called the discount rate. This is different from normal interest rates which divide the interest amount by the up-front payment amount in order to determine a rate. Although this convention is confusing, discount rates are used in a number of large US$ short-term investment markets and, thus, are a normal part of business for much of the financial world.

Discount rates can be expressed using any day-count basis found in money market yields or bond equivalent yields.

You have completed Unit Three, *Determining the Price and Yield of Fixed-Rate Securities*. Please complete the Progress Check to test your understanding of the concepts and check your answers with the Answer Key. If you answer any questions incorrectly, please reread the corresponding text to clarify your understanding. Then, continue to Unit Four, *Converting Between Quoting Conventions*.

☑ PROGRESS CHECK 3

Directions: Determine the correct answer to each question. Check your answers with the Answer Key on the next page.

Question 1: It is easiest for an underwriter to place fixed-rate bonds in a:

_____a) stable yield environment.

_____b) rising yield environment.

_____c) declining yield environment.

Question 2: What is the yield-to-maturity of a 10-year bond priced at 104 with an annual coupon rate of 8% and semi-annual payments?

_____a) 7.04%

_____b) 8.00%

_____c) 7.69%

_____d) 7.43%

Question 3: Suppose you borrow 100 and agree to repay the 100 at the end of 5 years plus 1 every month until maturity. If you are told that this represents a 12% per annum cost of funds, what quoting convention is being used?

_____a) Money market basis

_____b) Bond basis

_____c) Discount basis

_____d) Annual percentage rate basis

ANSWER KEY

Question 1: It is easiest for an underwriter to place fixed-rate bonds in a:

c) declining yield environment.

Question 2: What is the yield-to-maturity of a 10-year bond priced at 104 with an annual coupon rate of 8% and semi-annual payments?

N	I	PV	PMT	FV
20	3.713	-104	4	100

3.713 (Semi-annual YTM) x 2 = 7.43%

d) 7.43%

Question 3: Suppose you borrow 100 and agree to repay the 100 at the end of 5 years and 1 every month until maturity. If you are told that this represents a 12% per annum cost of funds, what quoting convention is being used?

b) Bond basis

Bond-based quoting determines the IRR of a set of cash flows created by paying the purchase price of a bond and receiving coupons plus face value.

PROGRESS CHECK 3
(Continued)

Question 4: If you purchase a $1,000,000 US Treasury Bill which matures in 147 days, and it is priced at a discount of 5%, what do you pay for the security?

_____a) $1,000,000

_____b) $979,863

_____c) $979,583

_____d) $950,000

Question 5: If you borrow 100 and agree to repay principal plus interest in 25 days, the payment will be quoted on a:

_____a) Money market basis

_____b) Bond basis

_____c) Discount basis

_____d) Annual percentage rate basis

Question 6: A manufacturer has a firm contract to deliver goods in one year at a price of $500,000. The company needs to borrow money now to manufacture the merchandise. It will repay the loan with the $500,000 receivable. The bank will probably quote the loan on a:

_____a) Money market basis

_____b) Bond basis

_____c) Discount basis

_____d) Annual percentage rate basis

ANSWER KEY

Question 4: If you purchase a $1,000,000 US Treasury Bill which matures in 147 days, and it is priced at a discount of 5%, what do you pay for the security?

Discount	***=***	***FV x DR x Actual days / 360***
		= 1,000,000 x .05 x 147 / 360
		= 20,416.66
Purchase Price	***=***	***FV – Discount***
		= 1,000,000 – 20,416.66
		= 979,583

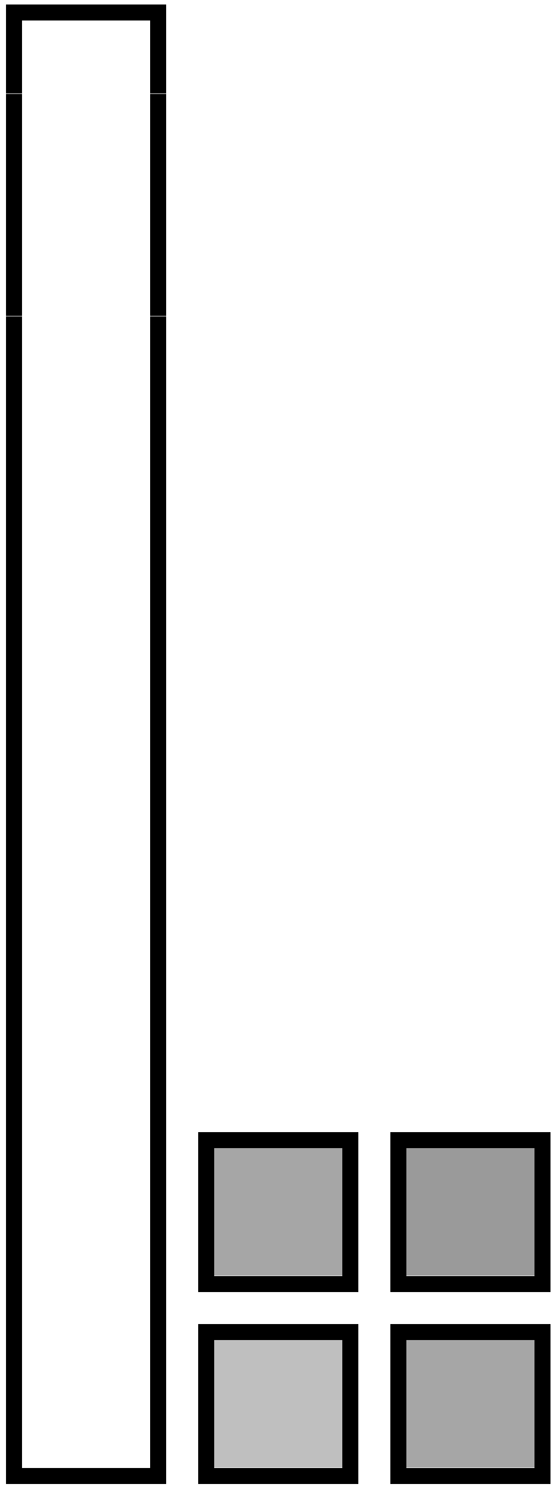
c) $979,583

Question 5: If you borrow 100 and agree to repay principal plus interest in 25 days, the payment will be quoted on a:

a) Money market basis

Question 6: A manufacturer has a firm contract to deliver goods in one year at a price of $500,000. The company needs to borrow money now to manufacture the merchandise. It will repay the loan with the $500,000 receivable. The bank will probably quote the loan on a:

c) Discount basis



Unit 4

UNIT 4: CONVERTING BETWEEN QUOTING CONVENTIONS

INTRODUCTION

In Lesson Three, you learned many of the conventions used throughout the world for quoting interest rates. Often, two competing transactions need to be compared, but the interest rates are quoted based on different conventions. In this unit, we explain how to convert rates quoted using “one convention to another” basis. Once rates are quoted on a common basis, comparing them is easy.

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Compound or decompound bond equivalent yields
- Convert discount rates to interest rates
- Convert money market yields to bond equivalent yields and vice-versa

Before You Begin This Unit —

These abbreviations are used throughout this unit. Use this list as a reference when you study the conversion calculations.

DR = Discount Rate

MMY = Money Market Yield

BEY = Bond Equivalent Yield

DP = Days in Period (actual)

DY = Days in Year (convention)

ADY = Actual number of Days in a Year (365 or 366)

fn = ‘From’ number of periods per annum

tn	=	‘To’ number of periods per annum
PV	=	Present Value
FV	=	Future Value

As an additional help, you may want to preview the sections “Basic Conversion Guidelines” and “Summary of Conversion Formulas” on pp. 4-21 and 4-22. These will serve as useful references both during and after the Interest Rates course.

COMPOUNDING AND DECOMPOUNDING BOND EQUIVALENT YIELDS

Annual bond equivalent yield

As you learned in Unit Three, annual payment bonds have the simplest rate quoting convention. This convention, generically called the annual bond equivalent yield (BEY), expresses yields based on the assumption that interest is paid only once per year at the end of the year, and that the interest payment should be divided by the principal amount to determine a rate. The annual BEY is the internal rate of return (IRR) of the annual cash flows of a transaction. This rate quoting convention is used by the Association of International Bond Dealers (AIBD), and is sometimes referred to as the “AIBD convention.”

Example: Annual BEY

If you borrow 100 for one year and pay an annual BEY of 10%, then your payment at the end of the year is simple to calculate:

Payment = 100 +(.10 x 100) = 110

The cash flows of the 10% annual BEY loan are illustrated in Figure 4.1.

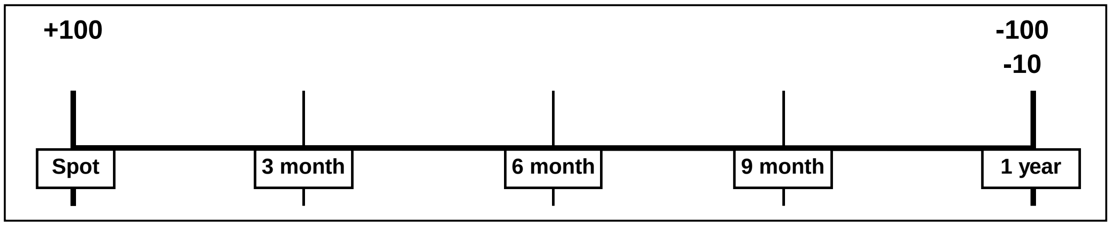


Figure 4.1: One-Year Loan – 10% Annual BEY

This simple idea of an annual interest rate will serve as the basis for comparison throughout the unit.

Compounding Bond Equivalent Yields

Semi-annual BEY

The complication with bond equivalent yields, in general, is the problem of multiple payments per year. If the 10% loan requires payments at the end of each six-month period, with a total of 10% paid throughout the year, it will still be called a 10% BEY. However, in this case, it will be a 10% semi-annual BEY. In Figure 4.2, we illustrate the cash flows of a semi-annual BEY loan.

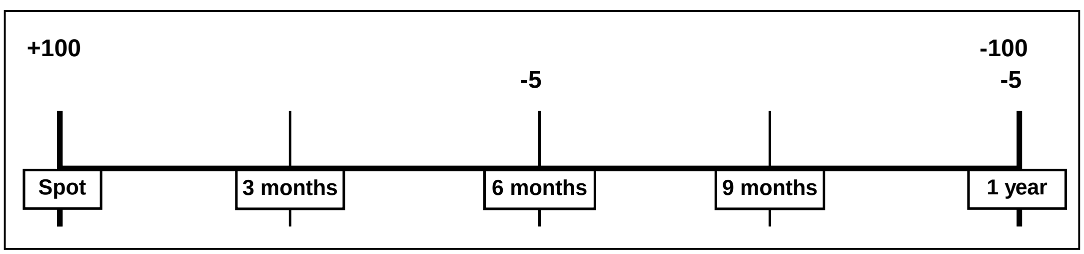


Figure 4.2: One-Year Loan – 10% Semi-Annual BEY

Annual rate better than semi-annual rate

If you could choose between borrowing with the loan in Figure 4.1 or the loan in Figure 4.2, you would probably choose the loan in Figure 4.1. When you are required to pay interest more than once per year, the effective cost of the loan rises. This increase is due to the added interest expense created if you borrow the money needed to pay the intermediate interest. Alternatively, if surplus cash is available to pay the interest, the added cost is the loss of potential interest that may be earned on surplus cash because you had to pay it to the bank. From either perspective, the cost of the loan in Figure 4.2 is larger than the cost of the loan in Figure 4.1.

The amount of increased cost is directly related to the interest rate available in the second semi-annual period. Market practice is to use the semi-annual BEY rate as the interest rate to compound the intermediate payment and determine a comparable annual cost.

Example: Compounding semi-annual BEY

Suppose that we compound the payment of 5 after six months at the semi-annual BEY rate available (10% per annum = 5% for a six-month period). The compounded interest, shown in Figure 4.3 is:

5 payment x .05 interest rate = 0.25 additional interest

The cost at the end of the year is 100 principal plus 10.25 interest, as shown in Figure 4.3. This assumes that the 5 borrowed to make the interest payment after the first six months will be repaid after one year plus .0025% interest.

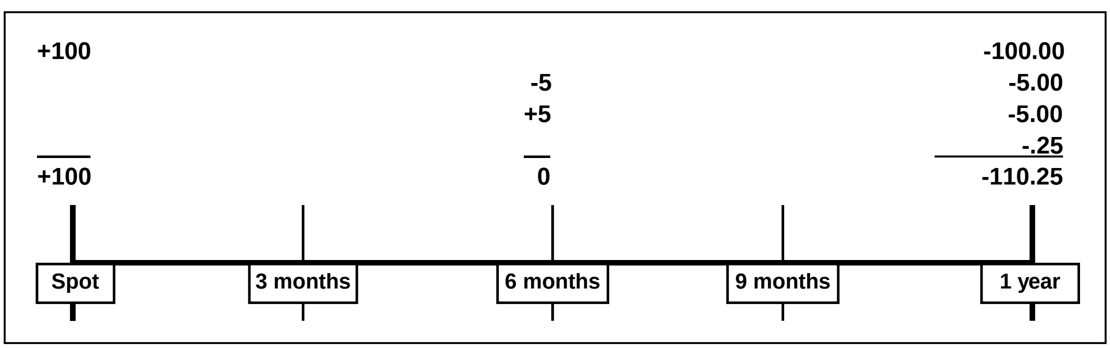


Figure 4.3: 10% Semi-Annual BEY = 10.25% Annual BEY

This process illustrates that, when interest is paid more frequently than once a year, the effective cost is raised. In this case, 10% semi-annual BEY = 10.25% annual BEY. The conversion may be handled easily with a financial calculator. To compound the interest rate, enter the number of compounding periods into **N**, the annual interest rate divided by **N** into **I**, -100 into **PV**, zero into **PMT**, and press **FV**. Then subtract 100 from **FV** (Figure 4.4).

N	I	PV	PMT	FV
2	5	-100	0	110.25

Figure 4.4: Compounding Using a Calculator

In this case, the calculation shows the same result as shown in Figure 4.3: the annual BEY of 10.25% is equivalent to 10% semi-annual BEY. This process also can be used for quarterly, monthly, weekly, or daily compounding.

Decompounding Bond Equivalent Yields

Now let's look at an opposite idea — **decompounding** — which is similar to compounding. Imagine that you are used to paying interest on a semi-annual basis, and someone approaches you with the idea of paying interest on an annual basis. Obviously, if the interest rate is the same in either case, you will prefer to pay interest annually, but how much better is that? How can you convert the annual interest quote to a semi-annual interest quote basis to compare the two? Basically, we need to know the payment we can make each six months which, when compounded, gives a cost equal to the annual quote we are given.

Example: Decompounding annual rate

With a financial calculator, this is easy to solve. To decompound the annual 10% interest rate, enter the number of compounding periods into **N**, the annual interest rate plus 100 into **FV**, -100 into **PV**, zero into **PMT**, and press **I**, then multiply **I** by **N**:

N	I	PV	PMT	FV
2	4.88	-100	0	110.0

Figure 4.5: Decompounding Using a Financial Calculator

When we multiply the result by **N**, we get the semi-annual bond equivalent yield.

4.88 x 2 = 9.76

In this case, the calculation shows that 9.76% is the semi-annual BEY equivalent to 10% annual BEY. This process also can be used for quarterly, monthly, weekly, or daily decompounding.

This result is relatively simple to check. We can determine the interest-on-interest owing after the second six-month period and ensure that all payments total 110. Suppose that we compound the payment of 4.88 after six months at the available semi-annual BEY rate (9.76% per annum = 4.88% for a six-month period). The compounded interest is:

4.88 payment x .0488 interest rate = 0.24 additional interest

The cost at the end of the year is 100 principal plus 10 interest. The compounding process is illustrated in Figure 4.6.

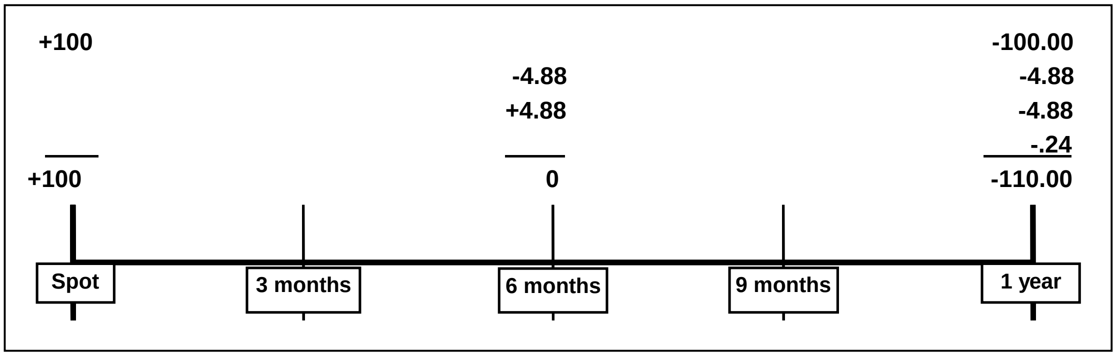


Figure 4.6: 10% Annual BEY = 9.76% Semi-Annual BEY

Formula for compounding and de-compounding BEYs

These calculations may be written in a formula which handles both compounding and decompounding equally well.

$$\mathbf{BEY_{tn} = [(1 + BEY_{fn} / fn)^{\{fn/tn\}} - 1] \times tn}$$

The following examples demonstrate how to use this formula. If you have difficulty with the mathematics, remember to always use decimals for interest rates and follow the steps in the order listed below:

1) Divide the BEY_{fn} by the “fn.”
2) Add 1 to the result of step 1.
3) Divide the “fn” by the “tn.”
4) Exponentiate (raise to a power) the result of step 2 by the result of step 3. (The key usually looks like “y^x.”)
5) Subtract 1 from the result of step 4.
6) Multiply the result of step 5 by “tn.”

Example: Semi-annual equivalent of annual BEY

We have already done this calculation with the financial calculator, but sometimes it is helpful to see the calculation for the formula. The semi-annual BEY equivalent of 10% annual BEY is calculated as follows:

tn = 2 **fn = 1** **BEY_1 = .10**

$$
\begin{aligned}
BEY_2 &= [(1 + .10 / 1)^{\{1/2\}} - 1] \times 2 \\
&= [(1.10)^{0.5} - 1] \times 2 \\
&= [1.0488 - 1] \times 2 \\
&= [0.0488] \times 2 \\
&= 0.0976 \text{ (which can be expressed as 9.76\%)}
\end{aligned}
$$

This example shows that the formula is consistent with the prior explanation for decompounding.

Example: Quarterly BEY equivalent of monthly BEY

The quarterly BEY equivalent of 9.43% monthly BEY is calculated using the same formula.

tn = 4 **fn = 12** **BEY_{12} = .0943**

$$
\begin{aligned}
BEY_4 &= [(1 + .0943 / 12)^{\{12/4\}} - 1] \times 4 \\
&= [(1.0078583)^{3} - 1] \times 4 \\
&= [1.02376 - 1] \times 4 \\
&= [0.02376] \times 4 \\
&= 0.09504 \text{ (which can be expressed as 9.504\%)}
\end{aligned}
$$

The compounding effect over three months raises the quoted rate to 9.50%.

If you are not comfortable with formulas such as this one, you can check this result using a financial calculator as before, but you will need two steps. First, compound the monthly BEY to an annual BEY, then decompound the annual BEY into a quarterly BEY. First, though, you need to divide 9.43 by 12 to determine the actual monthly interest rate: **9.43 / 12 = 0.78583**. The result is shown in Figure 4.7.

N	I	PV	PMT	FV
12	0.78583	-100	0	109.85

Figure 4.7: Compounding Monthly BEY to Annual BEY Using a Financial Calculator

The result of this calculation compounds the monthly BEY to an annual BEY of 9.85%. To decompound to the quarterly BEY, do not change **PV**, **PMT**, or **FV**. Change **N** to 4 (for 4 quarters), and press **I**. The result is shown in Figure 4.8.

N	I	PV	PMT	FV
4	2.376	-100	0	109.85

Figure 4.8: Decompounding Using a Financial Calculator

Now multiply the 2.376 result by 4 (quarters): 2.376 x 4 = 9.504. The result is the same as the result derived by using the formula.

In this section you have seen how to compare bond equivalent yields with different payment intervals. Next, we will see how to compare bond equivalent yields with money market yields. First, however, check your understanding of compounding and decompounding bond equivalent yields by answering the Progress Check questions which follow. If you answer any question incorrectly, please reread the corresponding text.

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☑ PROGRESS CHECK 4.1

Directions: Determine the correct answer to each question. Check your answers with the Answer Key on the next page.

Question 1: Convert a quarterly BEY of 8.76% to an annual BEY equivalent.

_____a) 8.85%

_____b) 8.95%

_____c) 9.05%

_____d) 9.15%

Question 2: Convert an annual BEY of 12.34% to a semi-annual BEY equivalent.

_____a) 11.86%

_____b) 11.98%

_____c) 12.05%

_____d) 12.11%

ANSWER KEY

Question 1: Convert a quarterly BEY of 8.76% to an annual BEY equivalent.

N	I	PV	PMT	FV
4	2.19	-100	0	109.05

109.05 – 100 = 9.05%

c) 9.05%

Question 2: Convert an annual BEY of 12.34% to a semi-annual BEY equivalent.

N	I	PV	PMT	FV
20	5.99	-100	0	112.34

5.99 x 2 = 11.98%

b) 11.98%

CONVERTING BETWEEN DAY-COUNT CONVENTIONS: MMY TO BEY

If all compounding and decompounding uses and creates bond equivalent yield (BEY), then we must find a method for converting money market yield (MMY) to BEY, and BEY to MMY. The timing of interest payments is the same for BEY and MMY; the difference is the amount of each payment on each date. This difference can be split into two components:

1) Amount of interest payments throughout a year

2) Distribution of those payments within the year

The second issue is insignificant and will not be addressed at this time. The first issue is the important one on which we focus in this section.

Payment amount depends on day-count and convention

When a bond states that 10% interest will be paid in two equal installments, it means that each payment will equal 5%. When a MMY is quoted at 10%, but paid semi-annually, the amount of the payment depends on the days involved and the day-count convention that is applied. If the day-count convention uses the actual number of days in a year, there is little distortion.

Example: 365 convention

Consider the one year period from January 10, 1995 to January 10, 1996. Imagine that interest payments on the 100 principal are to be made on July 10, 1995 and January 10, 1996, and the interest rate is 10%. If the day-count convention is “actual” or “365,” the basis of the calculation will be 365 days. Interest payments will be:

First period (181 days): **Interest = 100 x .10 x 181 / 365 = 4.96**

Second period (184 days): **Interest = 100 x .10 x 184 / 365 = 5.04**

Example: 360 convention

Considering the same situation with a day-count convention of 360 days per year, interest payments will be:

First period (181 days): **Interest = 100 x .10 x 181 / 360 = 5.03**

Second period (184 days): **Interest = 100 x .10 x 184 / 360 = 5.11**

These three different possibilities are displayed in Figure 4.9. You can see that when the day-count convention is “365,” the total interest for the year is 10 (100 x 10% x 365 / 365). If the day-count convention is “360,” the total interest for a year is 10.14 (100 x 10% x 365 / 360). The 360-day convention significantly distorts interest cost because it states the interest paid for a 360-day period. Interest paid per annum will be more.

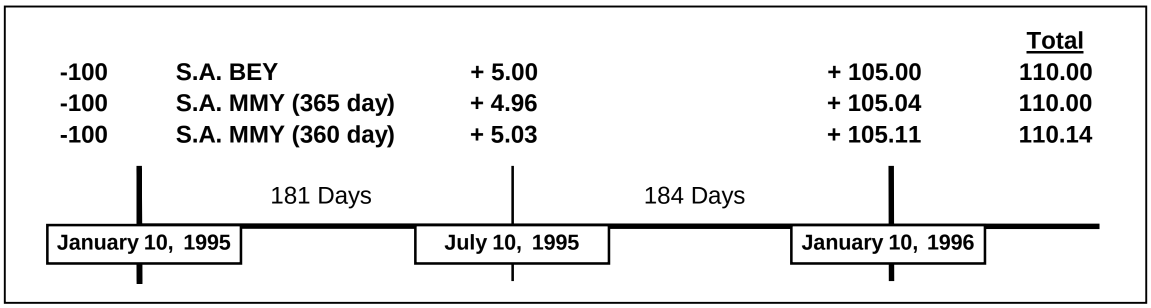


Figure 4.9: 10% Semi-Annual BEY and 10% Semi-Annual MMY Quotes

Formula: Converting from MMY to BEY

In order to convert between the 360-day MMY convention and BEYs, we must adjust the quotes for this different day-count basis. Yields quoted on any MMY basis must be scaled up to the actual number of days in a year using this formula:

BEY = MMY x ADY / DY

Yields quoted on the 360-day MMY basis must be scaled up to a full 365-day or 366-day quote. Yields quoted on a 365-day convention need be converted only when the actual number of days in the year is 366 (leap year). MMYs quoted on an *actual* basis include 366 days in the calculation during a leap year; thus, no conversion is necessary. This conversion creates the following adjustments:

MMY Basis	During Normal Years	During Leap Years
360	365 / 360	366 / 360
365	365 / 365	366 / 365
Actual	365 / 365	366 / 366

Financial market participants rarely refer to leap year conditions, even though the contractual difference does exist. These individuals err in quoting rates by not converting 365-day quotes in a leap year and by always converting 360-day quotes to a 365-day basis.

Example: Converting MMY to BEY

Suppose a counterparty quotes an 8.15% annual interest rate for US$ 3-month LIBOR, and you know that the US$ LIBOR day-count convention is 360. If you want to compare the cost to a US$ bond with quarterly payments, you can convert the money market quote to a quarterly BEY basis for a normal year as follows:

Quarterly BEY = 8.15% x 365 / 360
Quarterly BEY = 8.26%

Formula: Converting from BEY to MMY

For those parties more comfortable with the MMY basis of quoting interest rates, we can convert BEYs to MMYs by reversing the process.

MMY = BEY x DY / ADY

Example: Converting BEY to MMY

Suppose you are considering investments and a counterparty quotes a 9% yield-to-maturity on a US$ bond. You have been a conservative investor and normally deposit surplus funds in money market bank accounts quoting interest on a 360-day basis. If you want to compare the return from the bond with six-month deposit rates, you can convert the quote to a semi-annual MMY basis (for a normal year) as follows:

Semi-annual MMY = 9.00% x 360 / 365
Semi-annual MMY = 8.88%

If the return on a 9% bond is equivalent to an 8.88% return on deposits, this implies that a return of 8.95% on a deposit will exceed the return from the 9% bond by 8.95% - 8.88% = 0.07% per annum. Let’s check this conclusion. If we invest 100 in a deposit earning an 8.95% 360-day MMY, do the cash flows exceed a 9% bond? In Figure 4.10, you’ll see that the deposit pays .07% more than the bond.

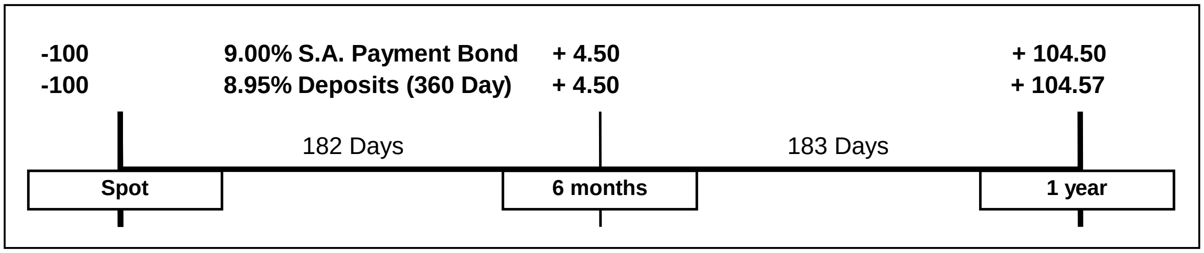


Figure 4.10: 9% Semi-Annual BEY and 8.95% Semi-Annual MMY Quotes

So, now we know how to compare the yields on bonds that are quoted based on different payment intervals and how to compare money market yields with bond equivalent yields. Let's look at one more type of comparison — between discount rates and interest rates.

CONVERTING DISCOUNT RATES TO INTEREST RATES

In Unit Three, you learned that some financial markets quote discount rates instead of quoting true interest rates. Unfortunately, we cannot directly compare discount rates with interest rates. Discount rates divide the interest amount by the future payment (which includes principal plus interest). Interest rates divide the interest amount by the present payment (which is principal only, no interest).

In Figure 4.11, we show a simple example of the difference between a *discount rate* and an *interest rate*. If you purchase a security which pays 100 at maturity in one year, and is priced at a 10% discount, the discount will be 100 x .10 = 10. The purchase price will be 100 – 10 = 90. In Figure 4.11, you see the cash flows of this investment, as well as the cash flows of an investment of 90 at 10% interest, and an investment of 90 at 11.11% interest.

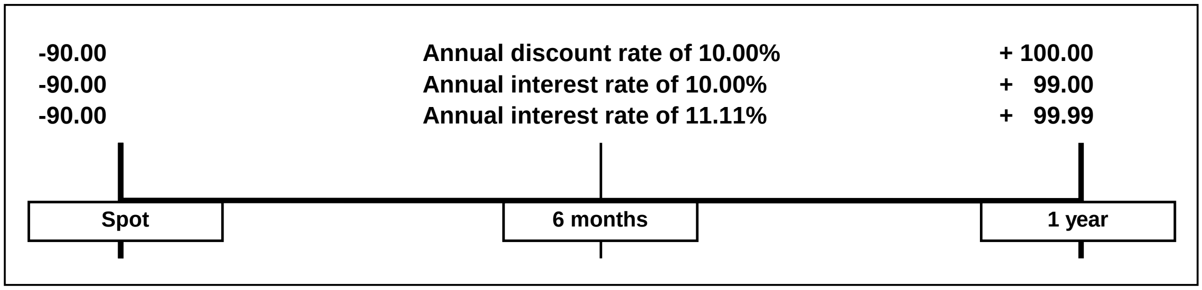


Figure 4.11: 10% Annual Discount Rate Versus Interest Rates

The figure shows that a *discount rate* of 10% generates more return than an *interest rate* of 10%, and slightly more than the return from an *interest rate* of 11.11%. The 11.11% is not a random number.

Simple conversion calculation

To convert discount rates to interest rates, divide the interest amount (which equals the discount amount) by the up-front amount (which represents the idea of *principal*). In the case of a discount-rate security, the up-front amount is the *purchase price*, which equals face value minus discount. In our example, the price is 90, and the discount or interest amount is 10. The effective interest rate is 10 / 90 = 0.1111 = 11.11%.

Complicated by day-count calculation

Unfortunately, this simple calculation becomes more complicated in actual situations. Most securities quoted on a discount basis have maturities under one year. These quotes must have a convention for day-count calculation just like MMYs; and most of these discount rates are quoted with the 360-day convention, which slightly complicates the formulas.

Example: Conversion based on day-count calculation

Consider a US Treasury bill issued with a maturity of 182 days, settled on February 15, 1996, and maturing on August 15, 1996. If the face value is US$100,000, and the discount rate is 9%, then:

Discount = US$100,000 x .09 x 182 / 360 = US$4,550

Purchase price = US$100,000 - US$4,550 = US$95,450

The effective 182-day interest rate is **US$4,550 / US$95,450 = 0.04767 = 4.767%.** To express this interest rate on an annual basis, with the same conventions as the US deposit market (360-day MMY), we need to scale it up to 360 days. The result is: **4.767% x 360 / 182 = 9.43%**.

To quickly check this result, suppose that you deposit US$95,450 in an account paying 9.43% on a 360-day MMY. How much interest will you earn in 182 days?

Interest = 95,450 x .0943 x 182 / 360 = 4,550

How much principal plus interest will you receive after 182 days?

Principal plus interest = 95,450 + 4,550 = 100,000

This shows that the 9.43% interest rate is equivalent to the 9% discount rate. The cash flows of the investment are shown in Figure 4.12.

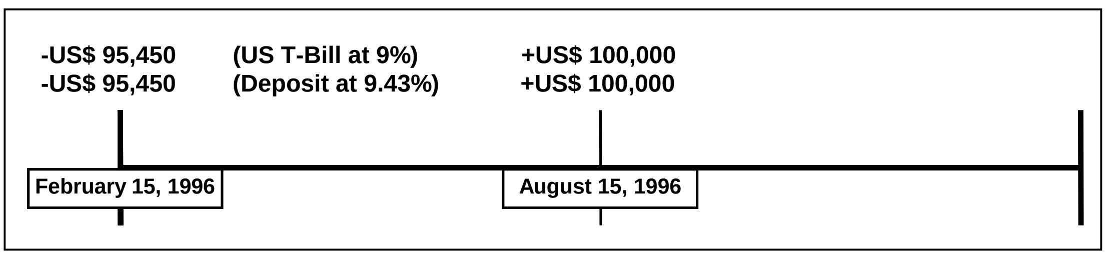


Figure 4.12: Purchase of 182-day 9% US Treasury Bill

Formula: Converting discount rates to MMYs

The process used to convert a discount quote to a MMY can be written with a formula. The conversion divides the discount rate by the up-front amount of money, which is the face value of 100% minus the discount.

MMY = DR / [1 - (DR x DP / DY)]

If you have any difficulty with the mathematics, remember to *always 0enter interest rates as decimals* and not percentages, and to follow these steps in the order listed below:

1) Multiply DR by DP and divide by DY.

2) Change the sign on the result from step 1 (“+/-” key or “CHS” key).

3) Add 1 to the result of step 2.

4) Invert the result of step 3 (use “1/x” key).

5) Multiply the result of step 4 by the DR.

6) Multiply by 100 if you want the answer in percentage form.

Example: Discount rate to MMY

If we have a discount rate of 9% for 182 days (with the 360-day convention), the comparable MMY is:

MMY = .09 / [1 - (.09 x 182 / 360)]
= .09 / (1 - .0455)
= .09 / .9545
= .0943 (which can be expressed as 9.43%)

The 9% discount rate for 182 days on a 360-day basis equals a money market yield of 9.43%.

Example

If we have a discount rate of 6% for 76 days (360-day basis), the MMY equivalent is:

MMY = .06 / [1 - (.06 x 76 / 360)]
= .06 / (1 - .012667)
= .06 / .987333
= .0608 (which can be expressed as 6.08%)

The 6% discount rate for 76 days on a 360-day basis is the equivalent of a 6.08% money market yield.

Formula: Converting MMYs to discount rates

Although it is relatively rare, there are some investors who are more comfortable with discount rate quotes than with MMY rate quotes. To help them compare MMYs to discount quotes, we can convert MMYs to a discount basis. In this case, we need to divide the interest by the future payment of principal plus interest. The formula is:

DR = MMY / [1 + (MMY x DP / DY)]

Remember to always enter interest rates as decimals, not percentages, and follow these steps in the order listed below:

1) Multiply MMY by DP and divide by DY.
2) Add 1 to the result of step 1.
3) Invert the result of step 2 (“1/x” key).
4) Multiply the result of step 3 by MMY.
5) Multiply by 100 if you want the answer in percentage form.

Example: MMY to discount rate

Suppose you are considering switching to a US$ bank deposit instead of purchasing a 91-day banker’s acceptance (BA). If the deposit has a MMY (360-day basis) of 8%, will it provide a higher return than a BA rate (360-day discount basis) of 7.90%?

We can convert the 8% MMY to a DR basis as follows:

DR = .08 / [1 + (.08 x 91 / 360)]
DR = .08 / [1 + .020222]
DR = .08 / 1.020222
DR = 0.078414 (which can be expressed as 7.8414%)

The 8% money market yield equals a 7.84% return on a discount basis. This implies that the deposit’s return is less than the return on the 7.90% banker’s acceptance.

CONVERTING FROM ANY QUOTING CONVENTION TO ANY OTHER BASIS

As you have seen in the previous sections, when attempting to compare two rate quotes, we may have to convert one to the convention used by the other. In other words, we may have to compound or decompound the rates, adjust for day-count convention differences, or adjust for discount versus interest-bearing conventions. The order in which we make these adjustments matters, and the following guide may help you with this process.

Basic Conversion Guidelines

Whenever you convert a rate quote from one basis to another, begin by listing the following:

- Rate in decimal (such as ".10" for 10%)
- Maturity and day-count basis (such as 91/360 or 1/2 of a year)
- Type of quote you have (DR, MMY, BEY)
- Day-count basis and type of quote to which you would like to convert

Start your conversion with the line that starts with the type of quote you have. Follow the lines down from there, ending with the line that ends with the type of quote you want — compounded or decompounded as necessary. Remember to use decimals for the interest rates (not percentages), and to follow the standard order of operations as follows:

First, do calculations in brackets — ( ), { }, or [ ]. Within the ( ), { } or [ ], follow this order:

- Exponentiate
- Multiply or divide in order, left to right
- Add or subtract in order, left to right

Summary of Conversion Formulas

In the table below, you can see a summary of the conversion formulas for changing one rate quote into its equivalent for another type of rate quote. We have explained all of these formulas in this unit.

Starting with	Conversion	Ends With
BEY	$BEY_{tn} = [(1 + BEY_{fn} / fn)^{\{fn/tn\}} - 1] \times tn$ compounds or decompounds	BEY
BEY	MMY = BEY x DY / ADY converts to a day-count basis from actual days	MMY
MMY	BEY = MMY x ADY / DY converts from a day-count basis to actual days	BEY
DR	MMY = DR / [1 - (DR x DP / DY)] converts from a discount basis to an interest basis	MMY
MMY	DR = MMY / [1 + (MMY x DP / DY)] converts from an interest basis to a discount basis	DR

Other Conversions

Sometimes a convenient formula doesn't exist and the conversion of one rate into its equivalent in another type of rate requires a combination of these formulas. We illustrate the conversion of a discount rate to a bond equivalent rate in the following example.

Example: Converting DR to BEY

Suppose you are an investor who normally purchases semi-annual payment bonds, but are considering purchasing 91-day discount securities such as banker's acceptances. The BAs are quoted at 10%, but you know that you will have to convert this rate to compare it to typical bond rates. Let's see how the steps we have discussed apply in this situation.

Step 1: List the type of rate quote you have and the type of quote you want:

Rate = 10% = .10

Rate reset or maturity = 91-day

Day-count convention = 360-day

Type of quote you have = DR

Type of quote you want = BEY

Step 2: Start with 91-day DR to 91-day MMY conversion:

$$
\begin{aligned}
\text{MMY} &= \text{DR} / [1 - (\text{DR} \times \text{DP} / \text{DY})] \\
&= .10 / [1 - (0.10 \times 91 / 360)] \\
&= .10 / [1 - 0.025278] \\
&= .10 / 0.974722 \\
&= .102593
\end{aligned}
$$

Step 3: Convert 91-day MMY to 91-day BEY:

$$
\begin{aligned}
\text{BEY} &= \text{MMY} \times \text{ADY} / \text{DY} \\
&= .102593 \times 365 / 360 \\
&= .104018
\end{aligned}
$$

Step 4: Compound 91-day BEY to semi-annual BEY:

$$
\begin{aligned}
\text{BEY}_{tn} &= [(1 + \text{BEY}_{fn} / fn)^{\{fn/tn\}} - 1] \times tn \\
fn &= 365 \text{ day} / 91 \text{ day} \quad tn = 2 \\
\text{BEY}_2 &= [(1 + .104018 / (365/91))^{\{(365/91)/2\}} - 1] \times 2 \\
&= [(1 + .025933)^{2.005495} - 1] \times 2 \\
&= [(1.025933)^{2.005495} - 1] \times 2 \\
&= [1.052687 - 1] \times 2 \\
&= .052687 \times 2 \\
&= .105374 \text{ (which could be expressed as 10.54\%)}
\end{aligned}
$$

Thus, a 10% 91-day discount rate equals a 10.54% semi-annual BEY.

As you can see, we followed the basic conversion guidelines to convert the discount rate to a bond equivalent yield. Using this concept as a model, you should be prepared to convert any type of rate into its equivalent in another type of rate.

SUMMARY

Interest rates are quoted using different conventions in different markets. The primary conventions are those used in the:

1) Bond markets which are called bond equivalent yields (BEY).

2) Money markets called money market yields (MMY).

3) Short-term, discount-instrument markets called discount rates (DR).

Rates quoted using these different conventions are not directly comparable. To compare them, we must convert between one convention and another. To convert between rate-quotation conventions, we consider compounding, decompounding, day-count conventions, and interest-bearing versus discount conventions.

The most straightforward convention is that used by the Association of International Bond Dealers. This convention uses annual payment bonds as its basis.

The next most accurate convention is that used for bonds with payments that occur more frequently than once per year, such as semi-annual payment bonds or quarterly-payment bonds. To accurately compare these rates to the AIBD convention, we must compound the interest.

The next most accurate convention is the one used by most money markets, which multiplies a quoted rate by the fraction of a year in the payment period. When the fraction is based upon actual days in the payment period and actual days in the year, this convention is almost identical to the bond conventions.

Often, though, markets use something other than the actual number of days in the payment period or the actual number of days in the year. Common substitutes include 365 days or 360 days instead of the actual number of days in the year, and 30 days per month instead of the actual number of days in the month. In order to compare these MMYs to BEYs, we must reverse any improper day-count conventions.

The least accurate and most confusing convention is that of discount rates. Discount rates are used primarily in trade-related transactions and short-dated US$ transactions such as US Treasury bills, US commercial paper, and US bankers acceptances. Discount rates take the discount (similar to interest) and divide it by the future payment of face value (similar to principal plus interest). Since these calculations always divide by more than principal, the rate quoted is always less than a true interest rate.

To compare these discount rates with interest rates such as MMYs and BEYs, we must reverse this consistent under-reporting in the rate calculation process. The largest discount rate markets also use the 360-day convention, and most often are for short-term securities; thus, attempts to determine annual effective rates such as the AIBD convention require a triple conversion.

The formulas used to convert between the various rate conventions are:

- **$BEY_{tn} = [(1 + BEY_{fn} / fn)^{fn/tn} - 1] \times tn$**
- **MMY = BEY x DY / ADY**
- **BEY = MMY x ADY / DY**
- **MMY = DR / (1 - DR x DP / DY)**
- **DR = MMY / [1 + (MMY x DP / DY)]**

You have completed Unit Four, *Converting Between Quoting Conventions*. Complete the Progress Check to test your understanding of the concepts and check your answers with the Answer Key. If you answer any questions incorrectly, we suggest that you reread the corresponding text to clarify your understanding. Then, continue to Unit Five, *Fundamentals of Interest-Rate Forwards.*

☑ PROGRESS CHECK 4.2

Question 3: The expectation for a one-year deposit of 100 on July 1, 1995 is that it will grow to a value of 112 by maturity (July 1, 1996). What is the return expressed on a 360-day MMY basis?

_____a) 11.80%

_____b) 11.85%

_____c) 11.92%

_____d) 12.00%

Question 4: A US Treasury bill maturing in 67 days has a discount rate quoted at 6.13%. What is the equivalent MMY?

_____a) 6.18%

_____b) 6.20%

_____c) 6.22%

_____d) 6.25%

Question 5: Assuming a 6-month period with 182 days, when you convert a semi-annual MMY of 8% to a discount rate, the result is:

_____a) 7.6%

_____b) 8%

_____c) 8.32%

_____d) 7.70%

ANSWER KEY

Question 3: The expectation for a one-year deposit of 100 on July 1, 1995 is that it will grow to a value of 112 by maturity (July 1, 1996). What is the return expressed on a 360-day MMY basis?

MMY = BEY x DY / ADY
= .12 x 360 / 365
= .118

a) 11.80%

Question 4: A US Treasury bill maturing in 67 days has a discount rate quoted at 6.13%. What is the equivalent MMY?

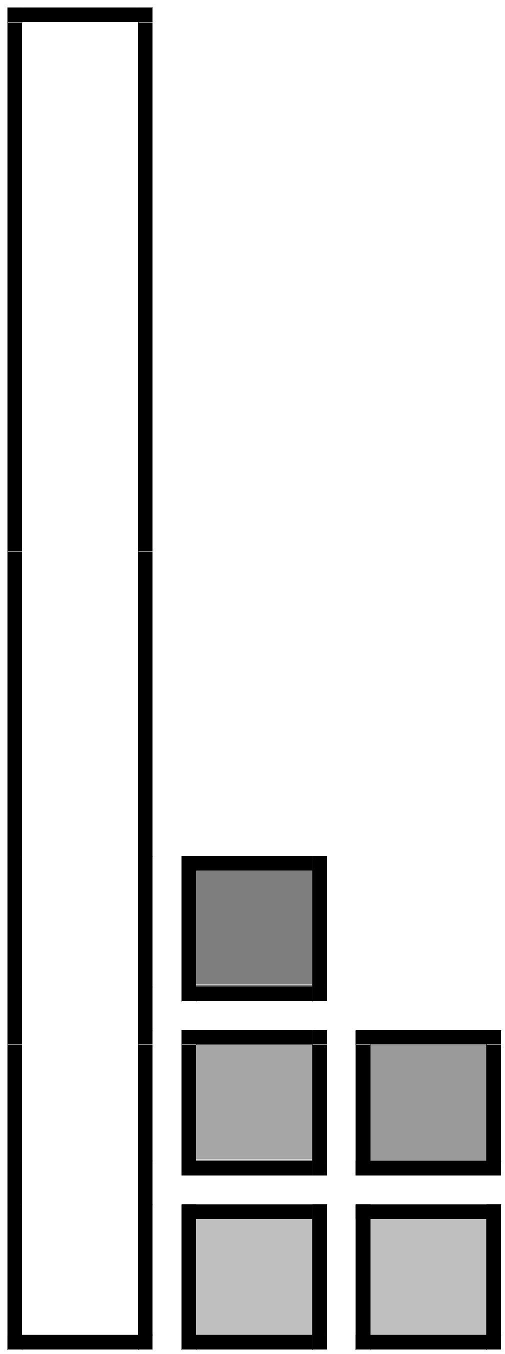
MMY = DR / [1 - (DR x DP / DY)]
= .0613 / [1 - (.0613 x 67 / 360)]
= .0613 / (1 - .011409)
= .0613 / .988591
= .0620

b) 6.20%

Question 5: Assuming a 6-month period with 182 days, when you convert a semi-annual MMY of 8% to a discount rate, the result is:

DR = MMY / [1 + (MMY x DP / DY)]
= .08 / [1 + (.08 x 182 / 360)]
= .08 / 1.0404444
= .077

d) 7.70%



Unit 5

UNIT 5: FUNDAMENTALS OF INTEREST-RATE FORWARDS

INTRODUCTION

Now that you are familiar with the fundamental concepts associated with basic interest-rate products, we will begin our discussion of the forward interest-rate products.

Although forward agreements have a long history, their most significant development has occurred in the past 20 years. In this unit, we will trace that history, and follow the rise of the interest-rate forwards market. In addition, you will learn about the structure and use of forward rate agreements (FRAs).

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the conditions which led to the development of an interest-rate forward market
- Identify a forward rate agreement (FRA)
- Recognize the reasons for the FRA structure
- Recognize the settlement procedure for an FRA
- Understand how FRAs can be used to hedge interest-rate risk

WHAT IS A FORWARD AGREEMENT?

A **forward agreement** (or forward contract) is a legally binding commitment to either:

1. Buy or sell an asset in the future at a predetermined price or

2. Pay / receive in cash the amount of loss / profit caused by the change in price of an asset from the pre-determined level.

History

Forward contracts for commodities

Forward contracts have been used for centuries to facilitate future delivery of many commodities around the world. These contracts are legally binding commitments to buy or sell a specific quantity of a well-defined asset at a well-defined time in the future. The delivery of the asset may be specified either for a particular date or within a particular short period of time.

Agricultural products have the longest history of any forward agreement. Rice, wheat, barley, oats, coffee, soybeans, meats, milk, cheese, wine, tulips, precious metals, oil products, and much more have been traded in large volumes for forward delivery.

Development of financial forward contracts

The increased use of these trade-related forward agreements led to the development of financial forward contracts. Until the early 1970s, international financial markets were either poorly developed or relatively stable. However, in the 1970s, major political shocks to the financial world caused interest rates and currency rates to fluctuate wildly. US dollar interest rates rose dramatically to above 20% in 1981 and 1982, primarily due to four factors:

- Collapse of fixed currency exchange rates
- High inflation
- Changes in US monetary and fiscal policy
- Deregulation of the US financial industry

With the large number of institutions funding in the US dollar market, the effect of these rate fluctuations was felt around the world.

The banking community historically had funded itself short-term with deposits, and had lent the funds on a longer-term (fixed-rate) basis to business and consumers. By the end of the 1970s, the banks were paying more for their funding than they were earning on fixed-rate loans made years earlier.

Variable rate basis

Since it was not feasible to fix their cost of funds through long-term deposits, banks decided to change their return on assets to meet their cost of funds. They decided to lend on a variable-rate basis, even for long-term loans. As a result, borrowers were forced to pay variable rates for their funds, which represented risk for them. From the perspective of a corporation preparing to produce a product or begin a project, it was important to know the cost of financing. After the mid 1970s, financing cost became increasingly more difficult to predict.

Example: Risk of variable rates

Let's see how variable rates can affect a corporation. We will use the same corporation in all of the following situations.

A large corporation was funded with US$0.5 billion equity, $0.5 billion fixed-rate debt, and $1 billion on a variable-rate basis. Total assets equaled $2 billion. The variable interest cost was LIBOR + 2%, and LIBOR was 9%. (LIBOR is the London Inter-Bank Offer Rate.) Their annual variable interest charge was:

9% + 2% = 11%

$1 billion (variable rate funding) x .11 = $110 million

The company's return on the $2 billion of assets (after all financing costs) was 5% ($100 million). If a 1% increase in the market interest rate (from 9% to 10%) occurred, how would that affect the company's profitability?

The increased cost was:

10% + 2% = 12%

$1 billion x .12 = $120 million

This represents an increased funding cost of $10 million.

The effect on the company's profitability was:

$$\frac{\text{\$10 million (increased cost)}}{\text{\$100 million (return on assets)}} = \text{10\% decrease in profit}$$

This additional cost represented 10% of the company's profit for the year and decreased the value of the company proportionately. During one twelve-month period beginning in 1981, the short-term LIBOR rose from around 9% to 18%.

For this company, the additional cost was:

$1 billion x .9 = $90 million

The effect on profitability was:

$$\frac{\$90 \text{ million}}{\$100 \text{ million}} = 90\% \text{ decrease in profit}$$

In some cases, this 9% per annum increase in funding cost could consume the full profit of a company and wipe out what was a previously healthy company.

Example: Effect of variable rates

Suppose that interest rates were set for annual periods and then paid at the end of the year (a simplification for example purposes). If the funding rate was LIBOR + 2% and LIBOR moved to 18%, then the variable cost of funds was:

18% + 2% = 20%

On a principal amount of $1 billion, the increase in LIBOR created a real cost of:

$1 billion x 0.20 = $200 million

Risk of lending at a fixed rate

Once the rate was set for a year, on a date agreed upon in the contract, it was not a risk for that year. However, there was a risk that the rate for the next year could be higher. The corporation may have wanted to "fix" the loan cost for the next year at 20%, but commercial banks could not risk lending at fixed rates.

The key issue was that, from the corporation's perspective, even if they could survive with a net cost of $200 million on their variable-rate financing, banks wouldn't commit to that. The lending bank would only commit to a spread of 2% above LIBOR.

As interest rates soared, many companies realized that they had to take some action. Fearing future movements, they sought some way to remove the interest-rate risk and, if possible, reduce their net funding cost as well.

Question:

How could this problem of interest rate instability be solved to the satisfaction of all parties? How could a "win-win" answer be created?

Solution:

The answer came from investment banks that began to develop a solution for this growing market of dissatisfied variable-rate borrowers.

Reducing the Company's Rate Risk

Fixing the borrowing cost

These investment banks agreed to help the borrower "fix" the cost for the next year at $200 million. The spread charged by the funding bank was already fixed at $20 million (2%), so the variable cost was the $180 million (18%) due to LIBOR.

Suppose an investment bank agreed to receive 18% from the corporation and to pay the corporation next year's interest at LIBOR.

Spread	**(2%)**	**$ 20 million**
Net Variable Cost	**(18%)**	**180 million**
Total Loan Cost	**(20%)**	**$200 million**

Since the LIBOR received from the investment bank and paid to the lending bank would cancel, the full loan cost to the borrower was 18% + 2% = 20%.

Reducing the Contract's Settlement Risk

Netting the payments

Since defaults at settlement were possible, and because withholding taxes were often due on gross payments, it didn't make sense to have the investment bank pay LIBOR to the corporation and the corporation pay 18% to the investment bank. Both parties to the agreement were exposed to less risk and cost if the payments were *netted*. Therefore, the investment bank offered to pay the corporation any *increase* in cost due to LIBOR rising above 18% if the corporation agreed to pay the investment bank any savings generated from a *decline* in LIBOR below 18%.

Consider the possibility of an increase in LIBOR versus a decrease in LIBOR.

Increase in LIBOR:

If LIBOR increased to 30%, the corporate borrower paid:

30% (LIBOR) + 2% (spread) = 32% interest to the bank

$1 billion x .32 = $320 million

The corporate borrower received 30% (LIBOR) - 18% from the investment bank.

$1 billion x (.30 - .18) = $120 million

Netting the receipt from the interest cost yielded a net cost of $200 million.

$320 million	**(Interest on loan)**
- 120 million	**(Due to borrower from bank)**
$200 million	**(Borrower's net cost)**

This net cost was the same as a 20% interest expense on $1 billion.

Decrease in LIBOR

If LIBOR dropped to 15%, then the corporation paid 15% (LIBOR) + 2% (Spread) = 17% (interest on the loan).

$1 billion x .17 = $170 million

The corporate borrower paid 18% - 15% to the investment bank.

$1 billion x (.18 - .15) = $30 million

Adding the payment to the interest cost yielded a net cost of $200 million.

$170 million
+ 30 million

$200 million

This net cost was the same as a 20% interest expense on $1 billion.

Decreasing the Contract's Credit Risk

In the process of reducing rate risk, a companion **credit risk** might be created. The question, then, was how could this credit risk be reduced?

Example: Reducing credit risk

Suppose the corporate loan's next rate-setting date was January 14, 1981 (value date of January 16, 1981), and an agreement was reached on July 1, 1980 to fix the rate for that setting date. The corporate loan interest payment corresponding to the January 14, 1981 rate setting would occur on January 16, 1982 (one year later).

Since the rate was set in 1981, by January 14, 1981 the investment bank and the corporation knew which was to make a payment to the other — and they knew the size of the payment. Using the last example above, on January 16, 1982 the corporation would pay the investment bank according to the following:

LIBOR 15%

Payment ....3% x $1 billion or $30 million

Pay present value on spot value date

Most investment banks, however, did not normally engage in lending money — and normally did not have one-year receivables from corporations. They suggested that, once the payment was determined (and, therefore, the rate risk eliminated), there was no reason for the contract to remain for another year. They thought it would be better to discount the payment and pay the present value of the payment on the spot value date.

In this example, the discounted payment would be made on January 16, 1981. If the payer didn't have the cash available at the beginning of the year, s/he could easily borrow the amount of the payment for one year at a market interest rate, such as LIBOR plus a spread. For this reason, market LIBOR was chosen as the discount rate used to determine present value (the value of a future cash flow discounted to current value).

The effect of the added cost or decreased return due to the corporation's spread above / below LIBOR was considered small compared to the benefits of removing credit risk earlier.

Question:

In the above example, what solution could investment banks offer the corporation?

Solution:

They could offer the following contractual agreement.

The agreement was made on July 1, 1980. The Contract Rate was set at 18% and the Contract Amount was set at $1 billion. On January 14, 1981, the annual LIBOR, as quoted in the financial markets, was the **settlement rate** of the contract. Payments under the contract occurred on January 16, 1981.

Under this agreement, there were two market possibilities.

A. If the settlement rate was greater than the contract rate, the investment bank paid the corporation:

$$\frac{\text{Contract amount x (Settlement rate - Contract rate)}}{\text{(1 + Settlement rate)}}$$

B. If the settlement rate was less than the contract rate, the corporate borrower paid the investment bank:

$$\frac{\text{Contract amount x (Contract rate - Settlement rate)}}{\text{(1 + Settlement rate)}}$$

Forward rate agreement

This contract was called a **forward rate agreement (FRA).**

Let's evaluate the solution by looking at examples of both possibilities.

Increase in LIBOR

If LIBOR increased to 30%, the investment bank paid the corporation:

$$\frac{\text{\$1 billion x (.30 - .18)}}{(1 + .30)} = \text{\$92,307,692.31}$$

If the corporation did not want the cash up front, but needed it at the end of the year to pay the added interest expense, it could deposit the payment with a bank and receive approximately 30% interest.

- Interest would equal $92,307,692.31 x 0.30 or $27,692,307.69.
- Payment plus interest would equal $120 million, or 12% of the principal amount.
- Loan cost would be 30% + 2% = 32%.
- Net expense would be 32% - 12% = 20%.

Decrease in LIBOR

If LIBOR decreased to 15%, the corporate borrower would pay to the investment bank:

$$\frac{\$1 \text{ billion} \times (.18 - .15)}{(1 + .15)} = \$26{,}086{,}956.52$$

If the corporation did not have the cash up front, but would have a lower interest payment to the lending bank at the end of the year, it could borrow the payment from a bank and pay approximately 15% interest. Then the following would apply:

- Interest would equal $26,086,956.52 x 0.15 or $3,913,043.48.
- Payment plus interest would equal $30,000,000, or 3% of the principal amount.
- Loan cost would be 15% + 2% = 17%.
- Total expense would be 17% + 3% = 20%.

Independent of the direction and magnitude of the LIBOR move, the total cost of funds would be 20%.

The contract could be modified for other rate quotations or periods of time. If the interest payment in a given period was determined by a formula, that formula would be included in the contract. For example, if the FRA covered the period from January 16, 1981 to July 16, 1981, and if the "actual / 360" day-count basis were used (as it is for LIBOR), the actual FRA equation would be:

$$\frac{\textbf{Cont. amt. x (Cont. rate x Days / 360 - Settlement rate x Days / 360)}}{\textbf{(1 + Settlement rate x Days / 360)}}$$

And "181 days" would be used in the place of "Days" because that is the actual number of days in the period.

The days covered by an FRA can be quite short-term, such as one week or one day. In this case, the settlement rate would be a one-week or one-day interest rate in the market, and the “days” would be seven or one.

An interesting variation occurs when the FRA covers a period that is different from the settlement rate periods. For example, an FRA covering a contract period of three months may have a series of settlement rates, each covering one day. In this case, the FRA payment cannot be determined at the beginning of the period and, therefore, must be paid at maturity. The “settlement rate” to be used in the formula above is the compounded rate determined from each of the daily settlement rates.

A less likely occurrence is to have a settlement rate corresponding to a period longer than the FRA contract period, such as a six-month LIBOR rate used as a settlement rate for a three-month contract.

In theory, the contract period, settlement rate period, and frequency of settlement rate determination may all be different. For example, a six-month LIBOR quote determined every week may be compounded weekly to settle a three-month FRA against a predetermined contract rate.

SUMMARY

Forward agreements and contracts have existed for centuries, starting with agricultural products. The high volatility of interest rates and exchange rates throughout the 1970s created demand for financial forwards. When commercial banks reduced long-term, fixed-rate lending, borrowers were not prepared to accept significant interest-rate risk.

Investment banks have solved the problem by creating a financial product to reduce rate risk inherent in existing funding vehicles. The generic name for this product is a forward rate agreement (FRA).

An FRA is an agreement to pay the difference between an interest payment calculated using a market rate and one calculated using an agreed contract rate. The agreement specifies a net payment to decrease settlement and tax risks. The net payment is usually discounted and paid up front to reduce credit risk.

Now you should be familiar with the basic fundamentals of Interest-Rate Forwards. Before you continue to Unit Six: *Forward Rate Agreement (FRA) Transactions*, let's see how well you can do on the Progress Check that follows. Remember to review the section on any question that you answer incorrectly. Good luck!

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☑ PROGRESS CHECK 5

Directions: Read the following questions and then mark the correct answer in the space provided. Check your solutions with the Answer Key on the next page.

Question 1: How long have forward markets existed?

____ a) Since the early 1980s

____ b) For 20 to 30 years

____ c) Since World War II

____ d) For centuries

Question 2: The highly volatile interest rate markets and currency exchange markets in the 1970s caused the growth and development of which market?

____ a) New York Stock Exchange

____ b) Corporate loan market

____ c) Financial forward market

____ d) Commodity market

Question 3: Which definition best describes an FRA?

____ a) An agreement to deliver a predefined interest rate to a prespecified counterparty on a specific date, depending on how market interest rates have changed

____ b) An agreement to pay the difference between an interest payment as calculated using a market rate and one calculated using an agreed upon contract rate

____ c) An agreement to lend a specific principal amount of money at a fixed rate for a fixed period (usually 3 or 6 months)

____ d) An agreement to set in advance ("fix") the rate on an existing variable-rate loan prior to its normal forward rate-setting date

ANSWER KEY

Question 1: How long have forward markets existed?

d) For centuries

Question 2: The highly volatile interest rate markets and currency exchange markets in the 1970s caused the growth and development of which market?

c) Financial forward market

Question 3: Which definition best describes an FRA?

b) An agreement to pay the difference between an interest payment as calculated using a market rate and one calculated using an agreed upon contract rate

PROGRESS CHECK 5
(Continued)

Question 4: In order to reduce settlement and tax risks in forward rate agreements, banks defined the contract to:

____ a) make net payments only.

____ b) be for short maturities only.

____ c) pick a market rate for settlement purposes.

____ d) make payments of present value only.

Question 5: If a borrower is paying LIBOR + 1.5% on a loan and enters into an FRA with a contract rate of 10%, what is the borrower's fixed cost of funds?

____ a) 11.5%

____ b) 10.0%

____ c) 8.5%

____ d) 15.0%

Question 6: How much would the borrower in Question 5 pay to his/her lending bank if LIBOR equaled 6% on the rate-setting date?

____ a) 10.0%

____ b) 11.5%

____ c) 6.0%

____ d) 7.5%

ANSWER KEY

Question 4: In order to reduce settlement and tax risks in forward rate agreements, banks defined the contract to:

a) make net payments only.

Question 5: If a borrower is paying LIBOR + 1.5% on a loan and enters into an FRA with a contract rate of 10%, what is the borrower's fixed cost of funds?

LIBOR + 1.5%
= .10 + .015
= .115
= 11.5%

a) 11.5%

Question 6: How much would the borrower in Question 5 pay to his/her lending bank if LIBOR equaled 6% on the rate-setting date?

LIBOR + 1.5%
= .06 + .015
= .075
= 7.5%

d) 7.5%

PROGRESS CHECK 5
(Continued)

Question 7: How much would the borrower pay the investment bank in Questions 5 and 6?

____ a) 1.5%

____ b) 4.0%

____ c) 0.0%

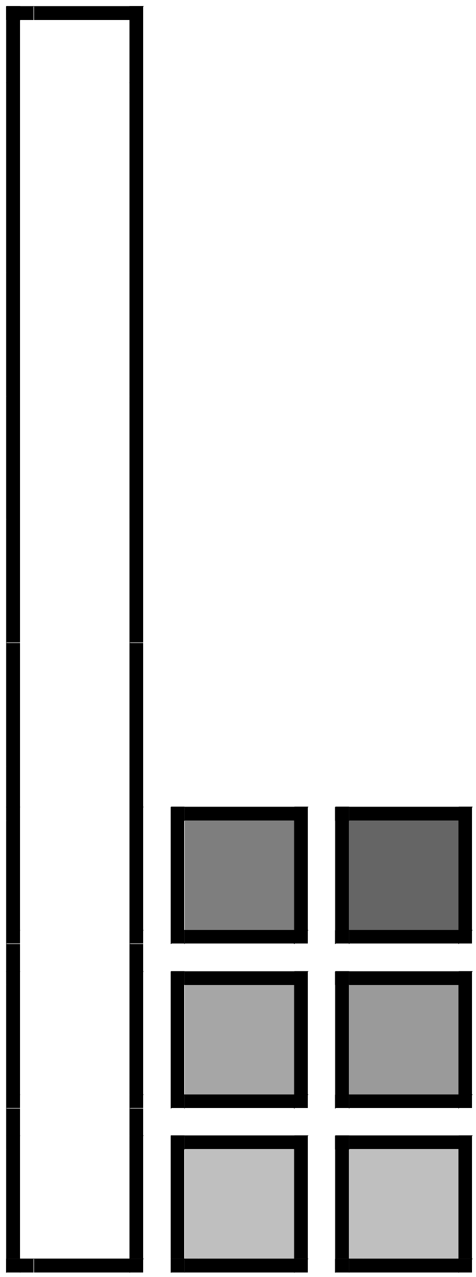
____ d) 2.5%

ANSWER KEY

Question 7: How much would the borrower pay the investment bank in Questions 5 and 6?

11.5% - 7.5% = 4.0%

b) 4.0%



Unit 6

UNIT 6: FORWARD RATE AGREEMENTS (FRA)

INTRODUCTION

The development of forward rate agreements (FRAs) and their use was described in Unit Five. Today, they are a major factor in the economies of all developed countries. In this unit, you will learn what constitutes an FRA, and how FRAs benefit both banks and their customers.

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the elements which must be defined in every FRA
- Understand how FRAs are used to meet customer needs
- Understand how FRAs are used internally by banks

SIZE AND SCOPE OF THE FRA MARKET

Major economic factor

Though FRAs barely existed 20 years ago, they are now a major economic factor in all developed economies. In the US$ FRA market, it is easy to deal quickly in amounts of $100,000,000 and up to billions of US$ in a short time period.

Popular rate-setting maturities include 1-month, 3-month and 6-month, but competitive quotes can be obtained for almost any maturity under two years. FRAs are often quoted out to four years. Longer-dated FRAs are often included in interest-rate swap books and, thus, are hedged in the same way as interest-rate swaps. (See Swaps workbook). **Interest-rate swaps** are essentially multi-period FRAs.

FRA contract amounts of hundreds of billions of US$ are negotiated each year. The exact numbers are increasingly difficult to extract from the overall swap volume, which has reached trillions of US$ per year.

Maturities match cash money market

The standard quoting practice in the FRA market is to quote FRAs for maturities which match the cash money market. For example, a typical FRA is the "3 by 6" FRA (written 3 x 6) which covers the gap from the end of the 3-month cash market period to the end of the 6-month cash market period. This is illustrated in Figure 6.1.

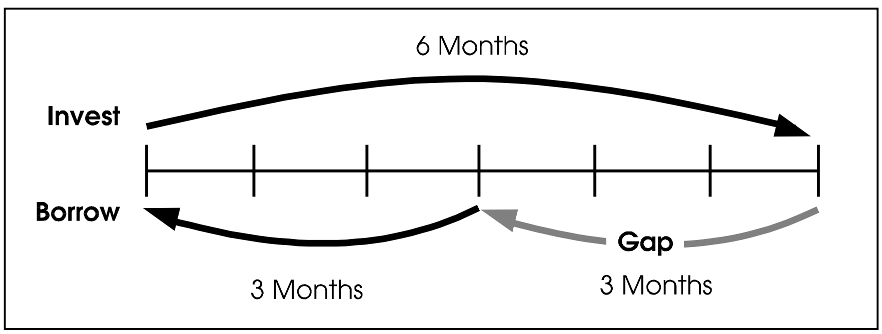


Figure 6.1: 3 x 6 Money Market Position

Similar 6x9, 9x12, 1x4, 2x5, 3x9, 6x12 maturities are popular. When a customer request is for a period which does not match these cash market dates, the FRA dealer will adjust the quote accordingly. FRAs can, and are, written for almost any date and any time frame.

Market settlement rate

FRAs may be written using many different market rate indices as the settlement rate. The US$ market has the largest number of these market settlement rates, including LIBOR, Federal Reserve Funds, PRIME, Treasury Bills, Commercial Paper (CP), Cost-of-Funds in a particular Reserve District, etc. Provided a rate index can be established and referenced without risk of manipulation, it can be properly used in an FRA.

FRA elements

Every FRA must define the following elements:

- The counterparties
- How the settlement rate will be determined (market index)
- The date on which the settlement rate will be determined
- The contract rate to be compared to the settlement rate
- The formulas used to determine the payment to be made, including the:
  - Payment currency
  - Contract amount (notional)
  - Day-count or other short-period convention
  - Discounting process
- Who pays when the settlement rate is higher than the contract rate
- Who pays when the settlement rate is lower than the contract rate
- The date on which the payment is to be made

FRA price

Given all other terms of the FRA, the *contract rate* is considered to be the **FRA price**. The party which will effectively pay this price is called the **FRA buyer**; the party which effectively receives this price is called the **FRA seller**.

Now that we have defined the scope of the FRA market and identified the contract requirements, let's look at the various uses for FRAs. We'll begin with those that apply to the customer.

CUSTOMER USES FOR FORWARD RATE AGREEMENTS

Borrowers

Fix interest cost

One of the first customer uses of FRAs was to fix interest cost on a variable-rate loan. The example in Unit Five shows how this developed. Fixing interest cost provides two distinct benefits:

1. By determining a fixed rate for funding, capital projects can be more accurately forecasted and compared. Independent of the level of the fixed rate, the fact that the rate is fixed helps in the planning process.

2. Fixing the cost of funds helps the process of setting prices for products. Since funding costs are a key expense which eventually must be recovered through product sales, knowing the cost of funds helps to identify a break-even product price and, thus, removes one variable from the complicated process of running a business.

Investors

Investors also may benefit from fixing rates. Sometimes they are concerned with the prospect of falling interest rates and how their returns on investments will vary. Investors in variable-rate or short-term securities may be eager to take the opposite side of an FRA contract (compared to the borrowers above). Two situations are possible:

1. If the market rates drop below the contract rate, the investor will want the investment bank to pay.

2. If market rates rise above the contract rate, the investor may be willing to pay.

Attractive Returns or Costs

Sometimes, borrowers or investors can lock in what they consider to be attractive rates — and the rate level is more important than the fact that it is fixed.

Example: Fixed cost

When US$ interest rates were near 20% in the early 1980s, investment banks were able to offer FRAs at a fixed cost of 17%. (This was 3% below the market rate!) Even though borrowers were afraid of risking a rise in variable rates, an inverted yield curve (long-term rates are lower than short-term rates) allowed FRA rates to be dramatically less than current levels. As a result, some borrowers were happy to fix a cost of funds below current rates.

Opportunity cost

It is true that, if market rates fell to 10% by the rate-setting date, the borrower would pay on the FRA contract in addition to the loan and a true "cost" would exist. These borrowers considered that to be an "**opportunity cost**," since they had lost the opportunity to pay the lower rate. However, given that they had considerably more to lose if rates went up, they were willing to take the risk of an opportunity cost.

What about those investors who were expecting rates to fall? They were happy to fix their returns at levels which were much higher than "normal" for them. They did not compare their contract rate to the current levels because they considered those to be aberrations. Since the investors didn't have much to lose if rates did rise, they were happy to lock in abnormally high returns — even if that meant they might give up some potential profits.

As we've seen, customers have a variety of uses for FRAs. There are also a number of applications for FRAs that banks can employ. This next section covers those uses.

BANK USES FOR FORWARD RATE AGREEMENTS

Asset / Liability Mismatches

Hedge gaps in money market books

A primary use for FRAs in a smaller bank portfolio is to offset (hedge) gaps in money-market books. A gap occurs when banks fund for shorter periods of time than they lend. Each time the funding rate is reset, the bank has an exposure to increased market rates. This risk is similar to that of the corporate borrower described in Unit Five.

Consider the following scenarios:

Example: Hedging a gap

Suppose the current three-month LIBOR is 8% and the current six-month LIBOR is 8.5%. A bank charges 1% above LIBOR on a customer loan. If the customer chooses to borrow for six months at 9.5%, the bank can lock in its spread by funding for six months at 8.5%.

Running a gap position

Sometimes, though, the bank expects a decline in rates and takes a risk by funding for only three months at 8%. If rates stay constant, the refinancing cost will be 8% in three months and the bank will have an extra 0.50% profit. If rates decline as expected, the bank may enjoy a larger profit. This mismatched funding strategy is called "running a gap position" and, in this case, the gap is called a 3 x 6 gap (see Figure 6.2).

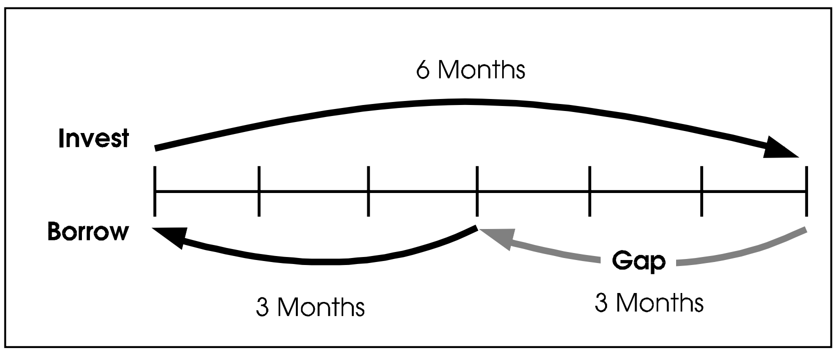


Figure 6.2: Running a Gap Position

Decreasing LIBOR rates

Suppose the bank’s view is accurate and within one month all LIBOR rates with maturities less than six months decline to 7.75%. The bank may continue to leave the gap unhedged or it may wish to "close" the gap. The gap will be a 2 x 5 gap, since one month has already elapsed (see Figure 6.3).

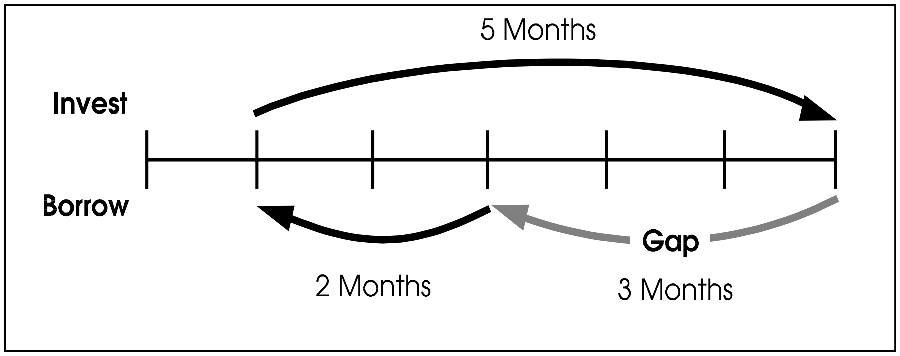


Figure 6.3: 2 x 5 Gap

Let's look at two approaches the bank may use to close the 2 x 5 gap.

1. An expensive approach to closing the gap is to fund for five months and invest the funds for two months at 7.75%. In two months, the newly invested funds may be used to repay the original three-month funding. In five months, the newly borrowed funds may be repaid from the proceeds of the customer loan repayment. This "cash market" approach probably will entail paying spreads in the cash market and increase the balance sheet of the bank. New full-principal credit risk will be established for two months as well (see Figure 6.4).

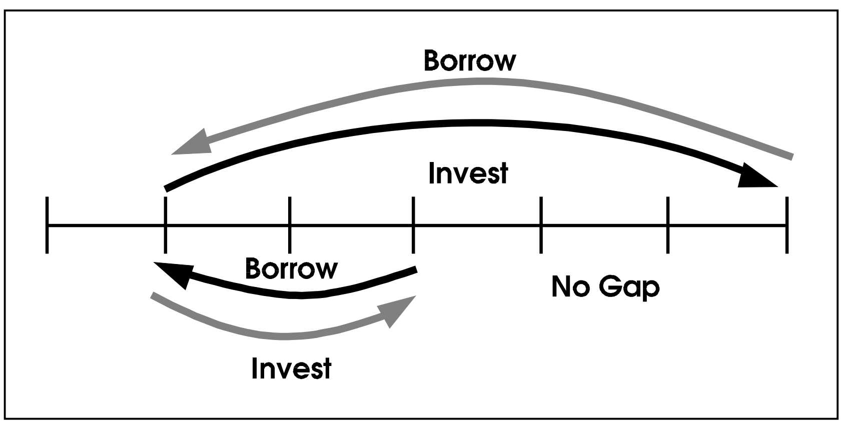


Figure 6.4: Cash Market Approach to Closing the Gap

2. A more efficient method for closing the gap is to agree to a 2 x 5 FRA, probably at 7.75% (ignoring the effect of compounding). The bank agrees to "pay" the FRA contract rate and "receive" the variable settlement rate in two months. Of course, the payments will be netted, discounted, and paid up front. The payment to / from the FRA counterparty will lock in a 7.75% funding cost for the rollover in two months and the bank will have guaranteed its additional profit. (see Figure 6.5).

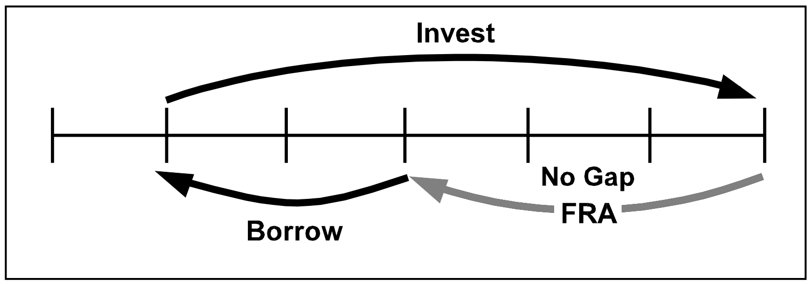


Figure 6.5: Using an FRA to Close the Gap

Increasing LIBOR rates

Of course, the bank's view may not be accurate. If LIBOR rates increase over the first month of the loan, the bank may be forced to consider stopping the potential refinancing losses. Again, the FRA will be the more efficient market for closing the gap. The bank will take the same position in the FRA market as described above, but the FRA contract rate probably will be higher than 8.5%. In this way, the bank will still remove its exposure to increasing LIBOR rates, but will lock in a limited loss instead of a profit.

Citibank dealers

For money-center banks such as Citibank, the deposit and loan (money market) books are not hedged or funded on a case-by-case basis. Due to customer transactions, which are not necessarily controlled by Citibank, the funding desk may find itself with longer-maturity assets or longer-maturity liabilities. If the Citibank dealer's view does not correspond to the positions given to the bank by its customers, the dealer may engage in an FRA not only to offset the net risk of the portfolio, but to reverse the net position to correspond to the dealer's view of rate movement.

In developing FRA markets, this may not be possible since the liquidity of the FRA market bid and offer is directly correlated to the underlying customer deposit and loan demand. In developed financial markets, though, this deposit / loan portfolio rebalancing is a way of modifying risk based upon objectives. It is a primary use of the FRA market.

This concludes our discussion of how both customers and banks can use FRAs. In the next section, we will see how the approach we've just covered with deposit and loan portfolio hedging can be applied to other derivative products.

Hedging Other Derivative Products

The approach to deposit and loan portfolio hedging can be directly applied to the hedging of other derivative products. From a rate-risk management perspective, Citibank's interest-rate swap book and currency swap book can be considered identical to a portfolio of variable-rate and fixed-rate deposits and loans.

Swaps

Though interest-rate swaps are not discussed in this workbook, it is important to note that they can be directly hedged with FRAs. Likewise, FRAs can be hedged with swaps; therefore, the pricing of FRAs is related to the pricing of the swap market.

> ***NOTE:***
>
> A combination of identical-maturity FRAs in two currencies creates the same rate-risk as Foreign Exchange forward-forward swaps with the same maturity (see Foreign Exchange workbook). Because of this relationship, the FRA markets can be used to hedge **FX swap** portfolios, and vice versa.

Futures

There is a direct relationship between interest-rate futures and FRAs and, therefore, they are used to hedge each other. The pricing of the futures market has a direct impact on the pricing in the FRA market.

Options

"Ceilings" and "floors" are sometimes referred to as options on FRAs. Option products are more complicated than FRAs. The strategies for hedging ceilings and floors most often determine the amount of FRAs which bear the same price risk as the ceiling or floor. These strategies continually vary the amounts of FRAs necessary to offset the bank's price risk in an interest-rate option product.

This brief discussion of other derivative products shows that FRAs are integrally related to forwards, futures, swaps, and option products. For this reason, banks often use FRAs in hedging strategies for their derivative portfolios.

Pricing of derivatives

The pricing of FRAs, and any derivative product, comes from the costs or returns from its hedge. In developed financial markets, the availability of hedges creates pricing which is dependent on the forward rates currently implied in the spot yield curves. In less developed markets, the forward rates are largely influenced by expectations of future spot movements. Even in developed markets, spot market movement can be largely influenced by expectations (for example, the influence of Eurodollar futures on the cash LIBOR market).

As a result, the pricing of FRAs, which is dependent upon currently available hedges and market expectations, can be more of an art than a science.

One important result of the hedging and pricing relationships in developed, liquid markets is that FRAs have smaller bid / offer spreads than do the cash markets. Also, they generally have lower transaction costs. This is largely due to the significantly lower risks for the banks in FRAs compared to the cash markets.

SUMMARY

Changing political and economic considerations in the 1970s resulted in wildly fluctuating US interest rates. FRAs developed from a need to stabilize the resulting interest expense. Now, FRAs are offered in all developed industrial economies and in many other financial currencies as well. Volumes in these markets total billions of US$ per year and individual transactions up to $100 million are reasonable.

FRAs may be written with any market index of rates in any currency, provided the market index is not subject to manipulation by either party to the FRA. Rate setting maturities of FRAs usually conform to the maturity structure of the local money market or FX swap market.

FRA elements

Every FRA must define:

- The counterparties
- How the settlement rate will be determined (market index)
- The date on which the settlement rate will be determined
- The contract rate to be compared to the settlement rate
- The formulas used to determine the payment to be made
- The date on which the payment is to be made

Given all other terms of the FRA, the contract rate is considered the *price* of the FRA. The party which effectively pays this price is called the FRA *buyer*; the party which effectively receives this price is called the FRA *seller*.

Customer use of FRAs

A primary customer use of FRAs is to lock in the cost of funds on a variable-rate liability. They also may be used to fix the return for an investor in variable-rate assets or to transform fixed-rate assets and liabilities into variable-rate assets and liabilities. The rates in the FRA market may appear more attractive than those in the spot cash markets, depending upon the shape of the yield curve in that currency. To bank customers, the value of FRAs is primarily the simplicity of administration and the accuracy of exactly offsetting rate risk — while not disturbing underlying sources of liquidity or investment return.

Bank use of FRAs

Banks use the FRA primarily as a portfolio management tool to offset portfolio risk or to transform it to the opposite type of risk. FRAs can be used by banks to hedge the risk of deposit and loan books, FX swaps, interest-rate swaps, currency swaps, financial futures, and many option products. For banks, FRAs are useful because they are economically accurate contracts which are easy to define and administer, are settled only once, and create relatively little credit risk or use of capital reserves.

You have completed this unit on *Forward Rate Agreements (FRA).* Before continuing to Unit Seven: *A Bond Forward Agreement – The REPO*, please turn to the Progress Check that follows. If you answer any questions incorrectly, please review the appropriate text.

☑ PROGRESS CHECK 6

Directions: Read the following questions and mark the correct answer in the space provided. Check your solutions with the Answer Key on the next page.

Question 1: FRA markets developed in the 1970s because:

____ a) speculators needed a new product to gamble on interest-rate movement and the cash markets were too expensive.

____ b) interest rates started fluctuating; neither commercial banks nor corporate borrowers wanted to accept the rate risk.

____ c) the US government wanted to fix interest rates to help control inflation.

____ d) investment banks wanted to take business away from commercial banks.

Question 2: The "price" of an FRA is:

____ a) its settlement rate.

____ b) its contract rate.

____ c) paid from the buyer to the seller when the contract is signed.

____ d) paid from the buyer to the seller when the contract is settled.

Question 3: Which of the following are known when an FRA is negotiated?

____ a) Its contract rate, which party will pay the other, and when payment will be made

____ b) The formulas for determining payment, its contract rate, and which party will pay the other

____ c) Its contract rate, the formulas for determining payment, and when payment will be made

____ d) The formulas for determining payment, which party will pay the other, and when payment will be made

ANSWER KEY

Question 1: FRA markets developed in the 1970s because:

b) interest rates started fluctuating; neither commercial banks nor corporate borrowers wanted to accept the rate risk.

Question 2: The "price" of an FRA is:

b) its contract rate.

Question 3: Which of the following are known when an FRA is negotiated?

c) Its contract rate, the formulas for determining payment, and when payment will be made

PROGRESS CHECK 6

(Continued)

Question 4: If a variable-rate borrower is afraid of interest rates rising by the time of the next loan rate reset, the simplest and most likely strategy to remove the rate risk is to:

____ a) repay the loan.

____ b) negotiate with the bank to extend the rate reset.

____ c) buy an FRA.

____ d) sell an FRA.

Question 5: Large money-center banks most likely would use FRAs to:

____ a) hedge each loan they write.

____ b) control interest rates.

____ c) provide cheaper financing to their customers.

____ d) transform rate risk in portfolios.

Question 6: Which of the following are advantages of the FRA market when compared to the cash deposit and loan markets?

____ a) Liquidity available, ease of administration, and credit risk taken

____ b) Ease of administration, credit risk taken, and capital reserves necessary

____ c) Liquidity available, credit risk taken, and capital reserves necessary

____ d) Liquidity available, ease of administration, and capital reserves necessary

ANSWER KEY

Question 4: If a variable-rate borrower is afraid of interest rates rising by the time of the next loan rate reset, the simplest and most likely strategy to remove the rate risk is to:

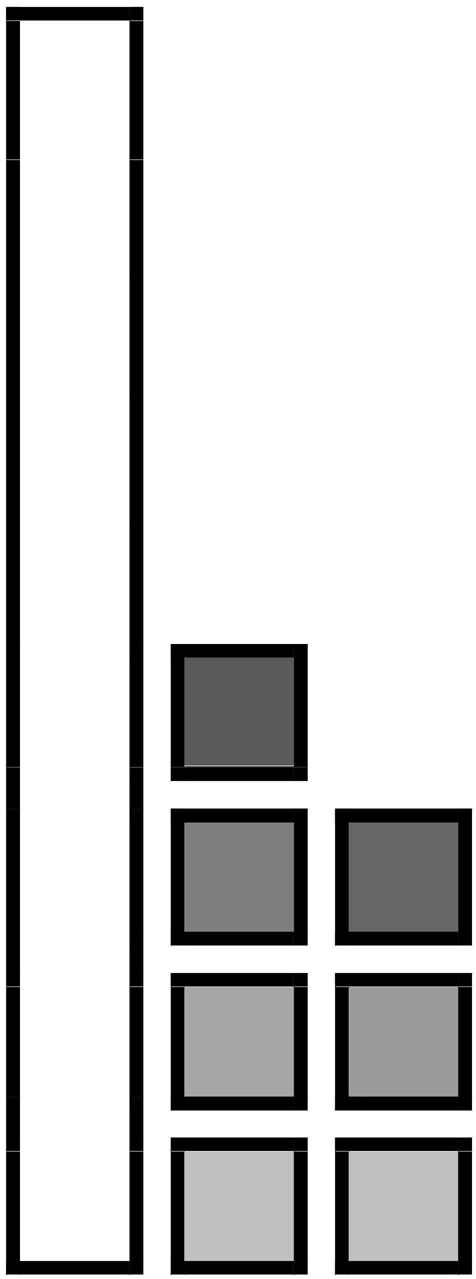
c) buy an FRA.

Question 5: Large money-center banks most likely would use FRAs to:

d) transform rate risk in portfolios.

Question 6: Which of the following are advantages of the FRA market when compared to the cash deposit and loan markets?

b) Ease of administration, credit risk taken, and capital reserves necessary



Unit 7

UNIT 7: A BOND FORWARD AGREEMENT – THE REPO

INTRODUCTION

In Units Five and Six, we concentrated on the FRA, its structure, and how FRAs can be used to meet the bank's internal needs and the needs of its customers. In this unit, we cover another interest-rate forward product that has specific application to the bond market, namely the repurchase agreement, or REPO. You will learn how it differs from other interest-rate forward products and how and why it is used by bond dealers.

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Identify the uses for repurchase agreements (REPOs)
- Recognize how REPO prices are derived

USES FOR REPURCHASE AGREEMENTS

Actual delivery of asset

Besides FRAs, other interest-rate forward products involve the actual delivery of an interest-rate-sensitive asset. These transactions are generally not used for purely speculative reasons.

One such product is the "repurchase" agreement, commonly called **REPO**. REPOs are mostly overnight transactions, but **term REPOs** may be used as longer-term solutions for other bank needs. Actually a bond forward agreement, the REPO is used for financing debt portfolios and making two-way markets.

Let's look at these uses individually.

Financing Debt Portfolios

In many countries, dealers are allowed to buy government debt for forward settlement as well as current (spot) settlement.

Bond dealers finance inventories overnight

Bond (long-term debt) dealers who need to finance inventories overnight often effectively finance their portfolios by simultaneous agreements to sell the asset for current delivery and repurchase it the following day. The price for the current sale is the market price. This price (plus accrued interest) effectively is a loan to the bond dealer from the bond purchaser. The price for the repurchase includes the loan amount (price plus accrued interest), which is the principal, and some interest on the amount "lent" to the dealer. The overnight financing of debt portfolios is illustrated in Figure 7.1.

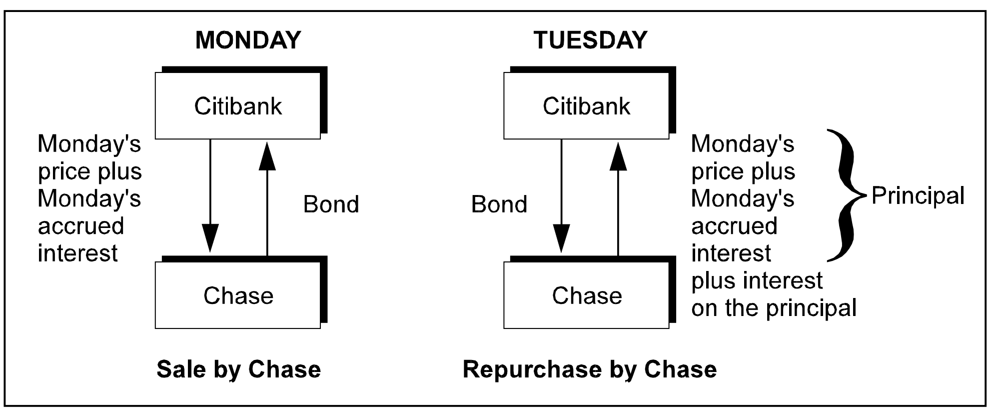


Figure 7.1: Financing Debt Portfolios

Example: Bond REPO

Suppose a trader needs to finance, for one night, a bond that has the following characteristics:

	Current price	**96.250% of face value**
+	**Current accrued interest**	**+ 00.190% of face value**
	Current value	**96.440% of face value**

Suppose the bond is pledged as collateral and the trader finds counterparties willing to lend at an interest rate of 3.36% per annum with daily interest calculated on a 360-day basis.

If the bond dealer can borrow the full value of the collateral, the loan principal will be 96.44% of face value.

Interest on the loan will be:

$$\frac{\textbf{96.44\% of face value x 0.0336}}{\textbf{360}} = \textbf{0.009\%}$$

Loan principal plus loan interest due the next day equals:

	Loan principal	**94.440% of face value**
+	**Loan interest**	**+ 00.009% of face value**
	Total due	**96.449% of face value**

The accrual of interest on the bond occurs with the passage of time and is added automatically to any quoted bond price by settlement systems. Suppose that on the next day, the bond's accrued interest will be:

Forward accrued interest = 00.217% of face value

Quoting a forward bond price

To quote a forward bond price which will result in payment of the total due, the forward accrued interest must be subtracted from the total due:

	Total due	**96.449% of face value**
–	**Forward accrued interest**	**– 00.217% of face value**
	Forward price	**96.232% of face value**

To fund the bond overnight, the dealer must agree to sell the bond at the current price of 96.250% of face value, then agree in advance to repurchase (REPO) the bond one day later at a forward price of 96.232%.

Bill (short-term debt) dealers engage in similar REPO transactions. The simpler pricing convention for that market does not involve accrued interest in the settlement procedure, thus making simpler calculations for the price of the forward bill.

Making Two-Way Markets

Two-way quotes

In developed government debt markets, such as the US Treasury market, primary dealers quote two-way prices. This means that they provide both a bid and an offer to customers who call. Any customer may "take the offer" and buy a reasonable amount of any US Treasury security.

Obviously, each dealer does not own an infinite amount of each security quoted. Therefore, dealers may agree to sell securities they do not own. If this occurs, the dealer must find another customer who is willing to sell enough of the security to cover the shortage. Otherwise, the dealer will not be able to deliver the security that was sold.

How do the dealers solve this problem? They call other professional bond dealers and ask if they have any of the needed bond. Let’s say that Chase has the bonds and needs to finance that position overnight. The Citibank dealer offers to buy the needed bond at the current market price and then sell it back to Chase the next day, financing Chase through the REPO structure explained above. In the process, Citibank will have the bond to deliver to its customer and will have "rolled-forward" its delivery risk.

The next day, Citibank faces the risk of delivery to Chase. There's always a possibility that, on the next day, Chase may need to continue funding the bond position and will be happy to "roll-over" the REPO. This roll-over process can continue for a long time. Meanwhile, the Citibank bond dealers may raise their bid for the bond, attempting to attract any potential seller. This allows Citibank to deliver the bond to Chase without continuing the roll-over, as illustrated in Figure 7.2.

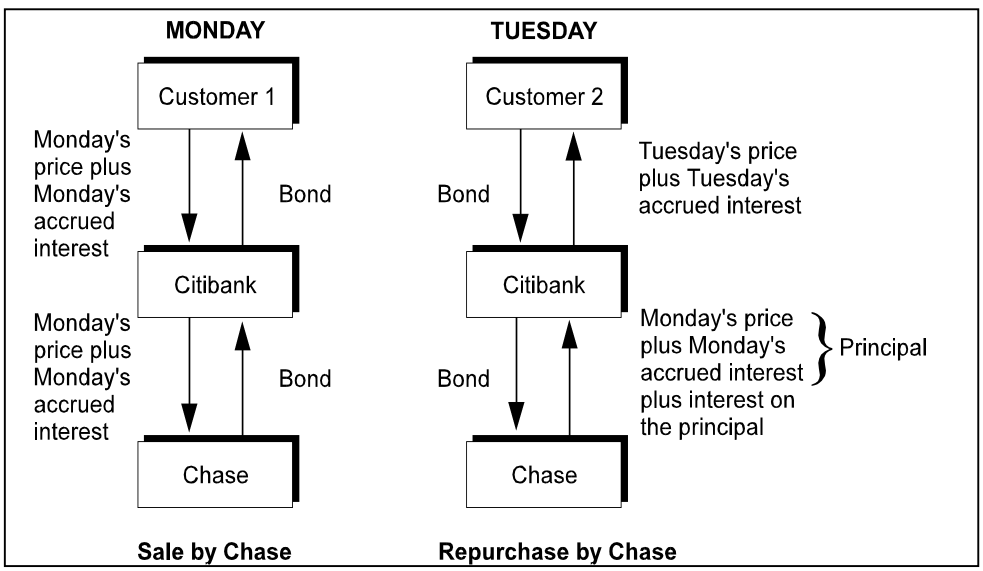


Figure 7.2: Making Two-Way Markets

Does this seem like a risky strategy? Let's look at the possibilities.

Longer-term REPOs

Let’s say that Chase doesn’t agree to the REPO and the Citibank dealer cannot "borrow" the bond from the professional bank market. In this case, the Bank attempts to engage in a longer-term REPO with the investor who buys the bond from Citibank. (Remember, *someone* owns the bond; it does not disappear.) Pension and insurance fund managers who own the bond may be happy to agree to a REPO with Citibank on a six-month basis, always receiving the Treasury coupon payment and also some profit directly from Citibank.

In the worst case, a primary dealer can engage in a REPO transaction with the US Treasury itself. This is one of the significant benefits to primary dealers and is necessary to allow them to make competitive, two-way markets.

In some less-developed financial markets, this "reverse REPO" is not allowed by the government if the bond is subsequently sold, which is an obstacle to further development of the financial market.

Term REPOs

Term REPO is a special use of the repurchase agreement. REPO transactions are usually short-term (mostly overnight), but they can be arranged for longer terms such as three or six months.

Uses for banks

They are used either as longer-term solutions for banks attempting to balance supply and demand or for rate-risk management. Term REPO of government *bills* can be perfect rate-risk hedges for certain FRAs. Term REPO of government *bonds* can be perfect rate-risk hedges for long maturity interest-rate swaps.

SUMMARY

Besides FRAs, other interest-rate forward products involve the forward purchase or sale of debt instruments. These forward transactions are normally called repurchase agreements, or REPOs.

Repurchase agreements are useful for financing debt portfolios and facilitating two-way dealing. Term REPOs are used for hedging FRAs or swaps.

This completes Unit Seven, *A Bond Forward Agreement – The REPO*. Now it's time to check your progress before continuing to the final unit, *Risks of Interest-Rate Forward Transactions*.

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☑ PROGRESS CHECK 7

Directions: Read the following questions and mark the correct answer(s) in the space(s) provided. Check your solutions with the Answer Key on the next page.

Question 1: **Three** primary uses for repurchase agreements (REPOs) are to:

____ a) hedge an interest-rate reset on a bank loan.

____ b) facilitate two-way quoting by dealers.

____ c) speculate on future interest-rate movements.

____ d) manipulate the government bond markets in developing countries.

____ e) cheaply finance a dealer's excess inventory.

____ f) hedge derivative transactions.

Question 2: In determining the forward price of a bond, which **one** of the following has the exact necessary information?

____ a) The current price for the bond, the time until the forward delivery, and an interest rate for the period of time until the forward delivery

____ b) The current price plus accrued interest for the bond, the time until the forward delivery, an interest rate for the period of time until the forward delivery, and the accrued interest for the forward delivery date

____ c) The current price for the bond, the time until the forward delivery, an interest rate for the period of time until the forward delivery, the bond's coupon rate, and the bond's maturity

____ d) The current price plus accrued interest for the bond, the time until the forward delivery, an interest rate for the period of time until the forward delivery, the accrued interest for the forward delivery date, the bond's coupon rate, and the bond's maturity

ANSWER KEY

Question 1: **Three** primary uses for repurchase agreements (REPOs) are to:

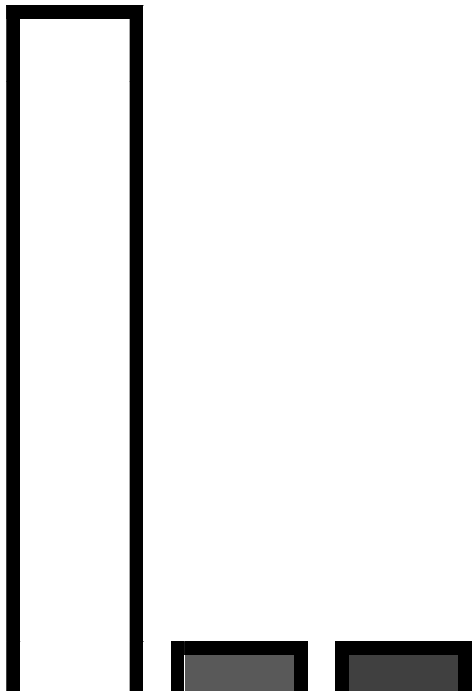
b) facilitate two-way quoting by dealers.

e) cheaply finance a dealer's excess inventory.

f) hedge derivative transactions.

Question 2: In determining the forward price of a bond, which **one** of the following has the exact necessary information?

b) The current price plus accrued interest for the bond, the time until the forward delivery, an interest rate for the period of time until the forward delivery, and the accrued interest for the forward delivery date



Unit 8

UNIT 8: RISKS OF INTEREST-RATE FORWARD TRANSACTIONS

INTRODUCTION

In the previous units, we concentrated on the benefits of interest-rate forward products. There are, however, a number of risks that are associated with interest-rate forward transactions. In this unit, we describe the risks that banks and their customers assume when they use forward transactions.

UNIT OBJECTIVES

When you complete this unit, you will be able to:

- Recognize the risks involved in interest-rate forward transactions from the perspective of the bank's customers

- Recognize the risks involved in interest-rate forward transactions from the perspective of the bank

- Identify reasons for the large visibility risk associated with interest-rate forward transactions

RISKS FROM THE CUSTOMER'S PERSPECTIVE

Bank customers use interest-rate forward transactions to reduce interest-rate risk. In the process, however, this use of an FRA creates additional risks. They are:

- Credit risk (including settlement risk)
- Opportunity risk
- Visibility risk

Credit Risk

In order to achieve the desired fixed rate with an interest-rate forward, the customer (borrower or investor) depends on the bank counterparty to make payments either to a variable-rate borrower when rates rise, or to a variable-rate investor when rates fall. This represents a credit risk.

Risk that bank will default

If the bank defaults (which is always a possibility), the customer has no hedge against a potential adverse interest-rate movement. In this situation, an investor does not receive compensation for falling interest rates and a borrower is not compensated for increased rates.

Risk that bank will fail to settle

One form of credit risk is the risk that the bank will not perform on the settlement date. This **settlement risk** is especially important in REPO transactions, since full principal and interest are exchanged.

Opportunity Risk

By fixing an interest rate with an interest-rate forward, the bank customer relinquishes any benefit of rates moving in a direction that is favorable to an unhedged position.

Example: Opportunity cost

For a corporate borrower who has locked in at a 17% fixed rate, a rate decline to 10% provides no benefit. This is often called an opportunity cost — the loss of the opportunity to benefit from an otherwise favorable rate movement. For example, if competitors remain unhedged, the hedged company may find itself at a disadvantage in pricing its product or service due to its comparatively high cost of debt.

Visibility Risk

When **opportunity risk** is felt, a corresponding **visibility risk** arises.

Example: Visibility risk

If a corporate treasurer uses an FRA to fix an interest rate at 17%, this does not provide the same position as having a loan with a fixed rate of 17% (before spreads). Why?

A fixed-rate loan at 17% demands one payment to the bank that is providing financing, and that payment generally requires no special monitoring or approval. Even if variable rates fall to 10%, no noticeable change is observed in the loan payments.

A variable-rate loan plus a 17% FRA, however, has a different payment structure. In this case, the following occurs:

- The loan rate declines.
- The loan payment becomes noticeably smaller.

- The customer pays a large payment to the FRA bank counterparty in addition to the small loan payment. This separate payment makes it obvious how much additional cost the company pays due to the decision to fix the cost at 17%.

Though the total cost is the same, the visibility of the benefit or cost of the hedge decision is much larger for the FRA transaction than it is for the fixed-rate loan transaction. Since judging with the advantage of hindsight is a relatively common occurrence in the corporate world, many treasurers prefer to avoid this increased visibility.

Risks from the Bank's Perspective

When banks offer FRAs to customers to help them reduce their interest-rate risk, the bank accepts a new set of risks for itself. The most important are:

- Credit risk (including settlement risk)
- Rate risk
- Visibility risk
- Documentation risk

The following sections describe each of these risks.

Credit Risk

If the market rate movement is in favor of the customer's unhedged position, the customer is required to make a payment to the bank. The bank is exposed to credit risk until that payment occurs.

If the customer defaults prior to the rate settling date, the bank will still lose because it must replace the unmatured contract to maintain balanced books. However, another customer will not enter into a contract at the same rates as the original contract, which were more favorable to the bank's position.

Another form of credit risk for the bank is the risk that the customer will not perform on the settlement date.This settlement risk is especially important in REPO transactions, since full principal and interest are exchanged.

Rate Risk

By fixing an interest rate with an interest-rate forward, the bank creates rate risk for itself. Though the customer's underlying position is offset with the interest-rate forward, the bank usually has no such position. In this case, the bank needs to hedge the unwanted rate risk. If it does not, or cannot, it bears the risk of loss from an unfavorable rate movement.

Visibility Risk

The bank has two forms of visibility risk resulting from customer payments.

Customer perception of bank's profit

First, when the customer makes a payment to the bank, the customer often views the amount as pure profit for the bank. The customer does not always understand that the bank has hedged the position and is paying the bulk of the receipt to another counterparty. If a significant rate movement occurs, the customer may feel that the payments are excessive and, consequently, the relationship between the bank and the customer may suffer.

Corporate relationships

A second form of visibility risk may result from the mechanics of transactions. The bank often has a corporate customer because of the relationship built with a key employee of that corporation. If that employee's position is jeopardized, the bank's relationship with the corporation also may be in trouble.

For example, suppose the Treasurer of the corporation has a good working relationship with Citibank, listens to a presentation on FRAs, recommends the deal to senior corporate officers, and does the deal. As a result, the corporation is making large payments to the bank. This causes senior management to put considerable pressure on the Treasurer.

It is common for treasurers or chief financial officers to suffer from less-than-perfect understanding of derivatives on the part of senior management (which may have changed since the original decision was made). If the bank's contact in the corporation suffers, so does the bank. The new treasurer will be very reluctant to deal with the bank in FRAs or to consider any new idea brought to the corporation by that bank. As a result, the bank may lose a customer.

Documentation Risk

Since the FRA contract is unusual compared to the cash-market contracts used for years, proper documentation is very important. In addition to creating legal, accounting, tax, and other regulatory risks, improper documentation can create a problem for a bank. This occurs when customers complain after the fact that they did not fully understand the contract or did not have full authority to commit their organization to the deal. Losses from such claims have cost banks significant sums in the past ten years and, thus, this risk must be addressed.

SUMMARY

From the perspective of a bank's customers, the risks of interest-rate forwards are limited to credit risk (including settlement risk), opportunity risk, and visibility risk. The customer accepts these risks:

- Bank fails to perform under the contract
- Market conditions move against the hedge
- Payment to the FRA bank is an explicit and highly visible cost of a decision to hedge

From the perspective of a bank, the risks of interest-rate forwards are limited to credit risk (including settlement risk), rate risk, visibility risk, and documentation risk. The bank accepts these risks:

- Customer fails to perform under the contract
- Market conditions move against the interest-rate forwards before the bank hedges
- Payment to the bank is an explicit and highly visible cost of a decision to hedge, which may harm the relationship between the bank and the customer
- Documentation of the contract is not proper when considering the counterparty's:
  - Authorization to deal
  - Knowledge of the risks of the transaction
  - Tax, legal, accounting, and regulatory issues

This completes the final unit, *Risks of Interest-Rate Forward Transactions*. After you have satisfactorily completed the Progress Check, you will have completed this course, Interest Rates. Congratulations!

☑ PROGRESS CHECK 8

Directions: Read the following questions and mark the correct answer in the space provided. Check your solutions with the Answer Key on the next page.

Question 1: Following an FRA agreement, which **one** of the following risks of interest-rate forwards is usually faced by the customer and usually not by the bank?

____ a) Credit risk

____ b) Visibility risk

____ c) Rate risk

____ d) Opportunity risk

Question 2: After agreeing to an FRA, which of the following risks of interest-rate forwards is usually faced by the bank and usually not by the customer?

____ a) Credit risk

____ b) Visibility risk

____ c) Rate risk

____ d) Opportunity risk

Question 3: Why do FRAs have more visibility risk for customers than fixed-rate assets and liabilities?

____ a) FRA rates are often higher than market interest rates.

____ b) Fixed-rate assets and liabilities do not indicate how much more return or less cost would have occurred if the decision to fix rates had not been made.

____ c) FRAs are negotiated by treasurers who do not explain the transaction well to their senior management.

____ d) Senior management is constantly changing.

ANSWER KEY

Question 1: Following an FRA agreement, which **one** of the following risks of interest-rate forwards is usually faced by the customer and usually not by the bank?

d) Opportunity risk

Question 2: After agreeing to an FRA, which of the following risks of interest-rate forwards is usually faced by the bank and usually not by the customer?

c) Rate risk

Question 3: Why do FRAs have more visibility risk for customers than fixed-rate assets and liabilities?

b) Fixed-rate assets and liabilities do not indicate how much more return or less cost would have occurred if the decision to fix rates had not been made.

APPENDIX

GLOSSARY

Amortizing	The process of spreading an up-front cost over multiple time periods, usually on the basis of a constant payment per period
Bond Equivalent Yield (BEY)	Yield as quoted in the bond market — assuming a number of equal payments throughout the year
Compounding	The process of adding interest to principal on a period-by-period basis, always applying the new-period interest rate to the sum of the principal and any prior-period interest
Credit Risk	The risk involved when one counterparty depends on the other to make payments to settle a contract
Decompounding	The opposite of compounding
Discounting Cash Flows	The process of determining the value of a future cash flow at the beginning of the period in question by dividing the cash flow by the sum of 1 and the appropriate interest (discount) rate
Discount Rate (DR)	Rate as quoted in security markets which determines the price of a security by subtracting a discount based upon a quoted rate, time to maturity, and face value amount — used primarily in the US Treasury bill (T-Bill), banker's acceptance (BA), and commercial paper (CP) markets
Forward Agreement (or Forward Contract)	A legally binding commitment to either: 1. Buy or sell an asset in the future at a pre-determined price or 2. Pay / receive in cash the amount of loss / profit caused by the change in price of an asset from a predetermined level

Forward Rate Agreement (FRA)	An agreement to pay the difference between an interest payment as calculated using a market rate and one calculated using an agreed-upon contract rate. Most often, the net payment is discounted and paid up front to reduce credit risk.
FRA	(See: Forward Rate Agreement)
FRA Buyer	The party which effectively pays the FRA price and pays when the settlement rate is lower than the FRA contract rate
FRA Price	The FRA contract rate
FRA Seller	The party which effectively receives the FRA price and pays when the settlement rate is higher than the FRA contract rate
Future Value (FV)	The value a series of cash flows will have on some date in the future, usually determined by adding interest to the series of cash flows
FX Swap	A transaction which involves the simultaneous purchase and forward sale, or spot sale, and forward purchase of the same currency. "FX" stands for foreign exchange.
Interest-Rate Swaps	Essentially, multi-period FRAs. Refer to the Swaps Workbook for a complete discussion of interest-rate swaps.
Internal Rate of Return (IRR)	The effective interest rate inherent in a series of cash flows, usually determined by guessing a discount rate then discounting the future cash flows at that rate, and adjusting the guess until the present value of the cash flows is opposite of the current cash payment. The internal rate of return is the rate which, when used to discount a series of cash flows, determines a net present value of zero.
LIBOR	The London Inter-Bank Offer Rate

Money Market Yield (MMY) Yield as quoted in money markets for short-term, interest-rate obligations and is calculated based on various day-count assumptions for the number of days in a payment period and number of days in a year

Opportunity Cost The amount of additional cost or decreased return due to a decision to hedge. For borrowers or investors using FRAs, this equals the FRA payment which they make.

Opportunity Risk The risk of losing an opportunity for lower costs or higher returns

Present Value (PV) The current value of a series of future payments, usually determined by discounting the cash flows using an appropriate interest rate

Price The current value of a security which is the equivalent of the present value of the cash flows of the security

REPO An agreement to currently sell a security and then repurchase it in the future. In the debt markets, REPO agreements are used to finance portfolios and to assist in making two-way markets (borrowing securities to deliver to customers). REPOs are usually short-term (under one week).

Settlement Rate The market rate in effect on the settlement date which is used to determine payments

Settlement Risk The form of credit risk that a counterparty does not perform on the settlement date of the contract

Term REPO REPO transactions which have maturities longer than one week. These are used to cover short positions for long periods, or to effect interest-rate hedges similar to interest-rate swaps.

Visibility Risk The risk that arises when a specific payment is made to settle a risk-rate management product. The payment makes the cost of the decision to hedge very visible.

Yield	The IRR of a security's cash flow
Yield-to-Maturity (YTM)	The yield or IRR of a bond using all cash flows to the maturity of the bond

APENDICE

GLOSARIO

Amortizing/ *Amortizar*	El proceso de repartir un costo inicial a través de periodos múltiples. Generalmente sobre la base de un pago constante por periodo.
Bond Equivalent Yield (BEY)/ Rendimiento Equivalente de Bono	Rendimiento según cotización en el mercado de bonos - asumiendo un número de pagos iguales en el año.
Compounding/ Compuesto	El proceso de agregar intereses al principal sobre una base de período-por-período, aplicando siempre la tasa de interés del nuevo período a la suma principal y cualquier interés de períodos previos.
Credit Risk/ Riesgo de Crédito	El riesgo presente cuando una contraparte depende de la otra para hacer pagos para la liquidación de un contrato
Decompounding/ Descompuesto	***Lo opuesto de compuesto.***
Discounting Cash Flows/ Flujos de Caja de Descuento	El proceso de determinar el valor de un flujo de caja futuro al inicio del periodo en cuestión por medio de dividir el flujo de caja por la suma de 1 y la tasa de interés (descuento) apropiada.

Discount Rate (DR)/
Tasa de Descuento

La tasa según la cotización de los mercados de valores, la cual determina el precio de un valor al restarle un descuento con base a una tasa cotizada, tiempo para el vencimiento y el valor nominal - usada principalmente en los mercados de la Tesorería de los EE.UU. (T-Bill), de aceptaciones bancarias (BA) y de papeles comerciales (CP).

Forward Agreement (o Forward Contract)/
Acuerdo o Contrato Futuro

Un compromiso obligatorio legal para:

1. Comprar o vender un activo en el futuro a un precio determinado o

2. Pagar / recibir en efectivo la suma de pérdida / ganancia causada por el cambio en el precio de un activo desde un nivel predeterminado

Forward Rate Agreement (FRA)/
Acuerdo de Tasa a Futuro

Un acuerdo para pagar la diferencia entre un pago de interés según su cálculo utilizando un tasa de mercado y uno calculado utilizando una tasa acordada de contrato. Con frecuencia, el pago neto se descuenta y se paga de forma anticipada para reducir el riesgo de crédito.

FRA/
FRA

(Ver: Forward Rate Agreement - Acuerdo de Tasa a Futuro)

FRA Buyer/
Comprador FRA

La parte que paga de forma efectiva el precio FRA y paga cuando la tasa de liquidación es menor que la tasa del contrato FRA.

FRA Price/ Precio FRA

La tasa del contrato FRA

FRA Seller/ Vendedor FRA

La parte que de forma efectiva recibe el precio FRA y paga cuando la tasa de liquidación es mayor que la tasa del contrato FRA.

Future Value (FV)/ Valor Futuro

El valor que una serie de flujos de caja tendrán en alguna fecha en el futuro, generalmente determinada al adicionar intereses a la serie de flujos de caja.

FX Swap/ Swap FX

Una transacción que involucra la compra y venta futura simultánea, o la venta de contado, y la compra futura de la misma moneda. FX quiere decir cambio extranjero

Interest-Rate Swap/ Swap con Tasa de Interés

Esencialmente, FRAs con periodos múltiples. Consulte el Cuaderno de Ejercicios de Swaps para una explicación completa de swaps con tasa de interés.

Internal Rate of Return (IRR)/ Tasa Interna de Retorno

La tasa de interés efectiva inherente en una serie de flujos de caja, generalmente determinada calculando una tasa de descuento y luego descontando los flujos de caja futuros a esa tasa, y ajustando el cálculo hasta que el valor presente de los flujos de caja sea opuesto al pago efectivo actual. La tasa interna de retorno es la tasa que, cuando se utiliza para descontar una serie de flujos de caja, determina un valor presente neto de cero.

LIBOR/ LIBOR

La Tasa de Oferta Interbancaria de Londres.

Money Market Yield (MMY)/ Rendimiento del Mercado Monetario

El rendimiento según la cotización en los mercados monetarios para obligaciones de tasa de interés a corto plazo y se calcula con base en varias suposiciones de conteo diario por el número de días en un periodo de pago y número de días en un año.

Opportunity Cost/ Costo de Oportunidad

La suma de costo adicional o de retorno disminuido debido a una decisión de cobertura. Para prestatarios o inversionistas que utilizan FRAs, esto es igual al pago FRA que ellos hacen.

Opportunity Risk/ Riesgo de Oportunidad

El riesgo de perder una oportunidad de costos menores o retornos mayores.

Present Value (PV)/ Valor Presente

El valor presente de una serie de pagos futuros, generalmente determinada al descontar los flujos de caja utilizando una tasa de interés apropiada.

Price/ Precio

El valor actual de un valor el cual es el equivalente del valor presente de los flujos de caja del valor

REPO/ REPO

Un acuerdo para vender actualmente un valor y luego recomprarlo en el futuro. En los mercados de deudas, los acuerdos REPO se utilizan para financiar portafolios y ayudar a hacer mercados dobles (préstamo de valores para entregar a clientes). Los REPO son generalmente a corto plazo (menos de una semana).

Settlement Risk/ Riesgo de Liquidación	La tasa del mercado vigente en la fecha de liquidación, la cual es utilizada para determinar pagos
Settlement Risk/ Riesgo de Liquidación	La forma de riesgo de crédito de que una contraparte no cumpla en la fecha de liquidación del contrato
Term REPO/ Término REPO	Transacciones REPO que tienen vencimientos mayores a una semana. Estos son utilizados para cubrir posiciones cortas para periodos largos, o para efectuar coberturas de tasas de interés similares a los swap con tasa de interés.
Visibility Risk/ Riesgo de Visibilidad	El riesgo que surge cuando un pago específico se hace para liquidar un producto de manejo de tasa de riesgo. El pago hace el costo de la decisión para cubrir bastante visible.
Yield/ Rendimiento	La IRR del flujo de caja de un valor
Yield-to-Maturity (YTM)/ Rendimiento al Vencimiento	El rendimiento o IRR de un bono utilizando todos los flujos de caja al vencimiento del bono

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