Learning from interest rate implied volatilities

A relative value analysis of swaptions

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Abstract

No previous research has firmly been conducted on relative value analysis on longer expiry European swaptions. This paper conducts an empirical analysis underpinning the concept of relative value analysis for ATM European swaptions on EUR and USD market by studying time series dataset of implied volatilities. We investigate the EUR and USD market for 2010-2017 by applying a principal component analysis (PCA) framework.

The thesis investigates longer expiry swaption straddles. All with a low exposure towards changes in the underlying swap rate. This gives the opportunity to focus on hedging implied volatility level rather than delta hedging. In the analysis we have found three significant dynamics represented on the implied volatility surface. The first dynamic are interpreted as an overall implied volatility level factor. The two other dynamics captures the implied volatility curves, each specifying a dimension on the implied volatility surface. Also, we give evidence that these three dynamics on the implied volatility surface explain over 95 % of the total variance for both USD and EUR markets. This is a consistent result over the entire period from 2010-2017. Furthermore, an investigation of the linkage between principal component scores and economic variables shows a high correlation between the first principal component (PC) and the US stock market.

To put these result into perspective, an unorthodox multiple regression model for hedging PC's are introduced. The model is constructed by different very liquid asset classes as a cheap and effective hedge. Results show that this hedging strategy is only efficient during subperiods when using a rolling 30 week data window. Furthermore, an example of a relative value trade is illustrated. However relative value trading opportunities are difficult to spot as these seems to occur rarely between 2010 and 2017.

Lastly, modelling with PCA on implied volatility surface is discussed. Here we introduce the major pitfalls of a PCA setup.

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Introduction

This thesis covers both statistical and financial theory. As this is an empirical analysis, this paper will introduce concepts of mathematical finance and not dwell on advanced mathematical techniques within option pricing theory. We investigate implied volatilities for European swaptions from a statistical perspective applying PCA conceptually. This is possible by restricting the implied volatility surface to longer expiry options.

A swaption is an option on a swap, which is characterized by the expiry of the option and the underlying swap rate (also called tenor). Looking exclusively on at-the-money-forward swaptions an implied volatility surface can be produced be applying expiry and tenor of the options into a three dimensional space.

In the literature an extensive documentation on identifying Principal Components in term structure dynamics has been investigated, such as capturing swap curve dynamics. Only a few papers document similar studies on volatility dynamics for the interest rate swaption market. However this paper focuses on longer expiry swaptions with low gamma by introducing a vega segmentation of these options. Our interest lies within capturing the dynamics of the implied volatility surface of swaptions.

At-the-money-forward swaption straddles are very liquid derivatives, especially on the EUR and USD markets. These are often used for hedging and investment purposes for institutions and corporate customers as well as banks and hedge funds. They operate on the same market but mostly in different arrays of the implied volatility surface. Several different types of market participants in the fixed income markets have different regulatory jurisdictions, and as regulation differs across participants this can create relative value opportunities for perhaps some of these participants.

In September 1998, the widely known relative value hedge fund Long Term Capital Management (LTCM) collapsed. LTCM defaulted on liquidity having highly leveraged positions for example on swaptions.

Volatility can be viewed as an asset class and while volatility have a tendency to being negative correlated to asset returns, investors seeks opportunities in the volatility class to hedge their downside risk cheaply in periods of volatile markets. Political unrest in the Western part of the world such as Brexit and presidential election of Donald Trump in the United States of America has created even more uncertainty surrounding different financial markets.

When discussing volatility one should often be more specific as this can refer to different terminologies. Volatility can either be realized or implied. Implied volatility is the anticipated future volatility in a market, meanwhile realized volatility can be defined as the actual volatility observed in the market until expiry of the option. This paper concentrates on options mainly determined by the implied volatility. Changes in the implied volatility are mainly driven by supply and demand for volatility. An example could be a large pension fund actively buying swaptions as a part of a hedging strategy, which could increase volatility for the traded swaptions.

Commonly the market quotes swaptions in reference to its implied volatility instead of its price. By quoting prices in implied volatility, effects regarding parameters not related to volatility are removed, and henceforth allows for assessing prices across different swaptions. Implied volatility can be derived from the Black model which assumes lognormal distributed rates and therefore only allows rates to be positive. Although it can be argued that normal distributed rates should be applied instead, as rates are allowed to be negative. This is one of the motivations for this paper.

In the paper by Frankena (2016) log-normal volatilities tend to present jumps and variance considerably when strike level of interest rates approaches zero or negative entries, while normal volatility is observed to be more stable in the same interest rate environment.

Longstaff et al. (2001) provide empirical evidence of relative value analysis of caps and swaptions through a string market model setup. They find that the relative valuations of most swaptions in the period of 1992 to 1999 are fairly priced. However after the announcement of crash of LTCM in 1998, they find strong evidence of longer dated swaption being significantly undervalued relatively to other swaptions in a 12 weeks period.

Principal component analysis can be used as a dimension reducing tool. A PCA analysis can therefore create a great overview of changes in swaptions with different tenors and expiries for a longer period. PCA has proven to be a useful tool when applying it on interest rates, implied volatilities, swap spreads, equities and several commodities. Evidence of mean reversion has been observed for various financial variables over longer time periods, such as interest rates, butterfly spreads, spreads between gold and silver, implied volatilities etc.

In our analysis, ATMF swaption data are used to compute three principal components, where we find that during the selected period at least 95 % of total variance is explained by these three factors for the entire period. We will interpret the principal components by investigating each component's eigenvectors. Moreover we will show clusters in the data to explain a differential in classification of instruments in a vega or gamma segment controlled by the instruments expiries.

Specifically, a PCA will be performed to follow up on these research questions:

1.1. Can we explain the most important dynamics of the implied volatility surface for swaption by implementing a principal component analysis for all tenors and expiries?

1.2. Is there any mispricing in the interest rate option market when looking at relative value analysis and can a mispricing be a trading opportunity?

1.3. Are we able to construct a hedge against directional exposure on the volatility surface with cheap and liquid assets, such as equity, bonds, FX rates and commodities?

The structure of the thesis is constructed as followed:

Section 1 begin with a description of the concept of relative value analysis and discuss parallels to asset pricing theory. After this we follow up on the transition to negative rates markets. Then we will go through the fundamentals for swaps and swaptions. Section 2 will describe the Ornstein-Uhlebeck mean reversion process and the derivation of the maximum likelihood estimation for the parameters in this process. This is followed by an explanation of the methodology for a principal component analysis, as these are the ground tools for this relative value analysis. In the end of this section a segmentation of the volatility surface is styled for the purpose of restricting the analysis to the swaptions that are most influenced by vega. Section 3 describes the data input, while section 4 gives an overview of the application of the principal component analysis and how to interpret the results. Section 5 introduces a hedging strategy reducing the exposure against shift changes in the implied volatility level of the implied volatility surface. As a closing section of section 5, we explore a rare relative value opportunity but also encounter a trade with a major pitfall within the relative value analysis framework. Lastly, section 6 concludes on the analysis and the three research questions.

What is relative value?[1]

Concept

The concept of relative value is based on quantitative analysis comparing two assets on a financial market. A relative value strategy directly involves finding relatively mispriced securities caused by dislocations and anomalies in a market, which eventually will revert to its fair value. This is a strategy used by several market participants on various markets such as for equity, fixed income, credit etc.

Proposition 1

If two securities have identical payoffs in every future state of the world, then they should have identical prices today.[2]

If this statement[3] is violated the existence of arbitrage opportunities is present. Arbitrage opportunities are not consistent with the theory of equilibriums in financial markets. Today proposition 1 is an established part of financial theory. However in 1997 Myron Scholes and Robert Merton won a Nobel Prize in economics by applying this result in their work on valuating options. Myron Scholes and Fisher Black used this proposition to determine the value of options by creating a self-financing portfolio which dynamically replicated the payoff of an option. Through the valuation of the self-financing portfolio they could determine the value of a specific option.

Proposition 2

If two securities present investors with the identical risk, they should offer identical expected returns.

This is another important proposition used for defining the relative value component. If two assets states identical risks, the expected return of these should be identical. This statement is in fact more complex to prove than the first statement. However it can be proved by using the Arbitrage Pricing Theory (APT), where unobservable linear factors are drivers of return.

[1] Fixed Income Relative Analysis by Dough Huggins Christian Schaller 2013
[2] Stated in Fixed Income Relative Analysis by Dough Huggins Christian Schaller 2013
[3] The law of one price.

Our first proposition says if two assets have the same payoff in all future states they should have the same price. If this isn't the case an arbitrage opportunity prevails. An arbitrage opportunity can be viewed as a window of opportunity for a trading strategy where we have a guaranteed profit without any risk taking, also in popular terms called a free-lunch. As we stated above, the existence of an arbitrage opportunity is inconsistent with equilibrium pricing of financial assets. Nevertheless, arbitrage opportunities can be used to identify relative value opportunities.

Clearly we have two propositions that assume no-arbitrage and we use these models to identify relative value opportunities. So why do people search for these opportunities when models assume no arbitrage? This can be explained through two minor logical observations.

Firstly arbitrage opportunities rarely occur due to the fact that market participants are looking at the same opportunities. However, if we did have consistent non-arbitrage markets, no one would be searching for profitable opportunities or inconsistencies.

Secondly in practice arbitrage opportunities always carry some risk. Explaining this in a theoretical and simple manner, we observe the relation between bond prices, bond futures prices and repo rates. If a bond future is too expensive, we can exploit this by selling the contract, and then buy the bond by borrowing the funds in the repo market and setting the bond as collateral for the loan. The bond is returned at expiry by the counterparty in the repo market. This party can now deliver the bond into future contracts. This is a riskless arbitrage opportunity in theory, but these positions are associated with risk when done in practice. One of the most significant risks may be that the counterparty from the repo market fail to deliver the bond at expiry. This will make it difficult for the party to deliver the bond into future contracts. If the delivery is uncompleted this can lead to penalties that can be costly. Therefore this should be a risk factor that needs to be considered before entering this strategy in practice.

Insight

Relative value analysis can be viewed as a way of gaining insight into relationships between different financial instruments but also to develop an understanding of which market forces that drives the prices of different instruments and how different markets are interconnected. Relative value analysis has its origin in arbitrage trading but has a much wider spectrum of applications. As it can identify reason for a security priced in a certain manner, expose the source of certain relationships and compare the relative value pricing of one financial instrument against the price of other instruments.

Applications

The applications of relative value analysis scopes within numerous areas, but is mostly used for trading and hedging purposes. Within the trading aspect, one can identify rich and cheap value of securities. As securities can become even richer or cheaper a trader must understand the reasoning for why these securities are rich or cheap to form a reasonable expectation of future richening and cheapening of these securities.

From a hedging or immunization perspective, relative value analysis considers hedging or immunizing a position against several risk exposures. As an example a flow trader could improve the expected risks of a portfolio by considering hedging alternatives from a relative value analysis of German Bunds. If the trader believes cash Bunds are going to cheapen compared to other alternatives, a hedge could be created by selling cash Bunds. While if the trader expects Bund futures are going to cheapen relative to cash Bunds the hedge can be constructed through future contracts instead of using cash Bunds.

Some other applications worth mentioning is that relative value analysis can express a macro view, signal the timing of unwinding a position or help investment managers on security selection to increase alpha of a portfolio.

Risks

When applying relative value when engaging in a financial market, several risks needs consideration, such as:

- Liquidity risk - trading fixed income instruments often have different liquidity. Some trading strategies involve taking a long position in a rather illiquid security while taking a short position in a liquid security and hereby earning a liquidity premium. If financial distress should occur, it can be hard to find liquidity in the market.

- Model risk needs to be considered as many fixed income instruments are priced by models that can be misspecified or misapplied, where signaling incorrect valuation can occur.
- Event risk can be viewed as market stress situations such as financial distress or in extreme cases flight-to-quality events. These events can change market perspectives and perhaps break historical correlations between similar securities.
- Interest rate risk depends of course on the chosen strategy, but even market neutral strategies can suffer from higher financing costs if interest rate starts to rise to a higher level.
- Credit risk is also a risk factor to be considered as the counterparty can default and not be able to fulfil the contract.
- Legal/regulative risks can destroy relationships between two instruments due to perhaps new regulatory charges.

Summary

Overall, relative value analysis should be considered as a craftsmanship, as it is neither science nor art. Applying relative value analysis in a market, knowledge of statistical foundations and a less scientific related understanding of the market is needed in mastering this craft.

Negative interest rates

In classical thinking of interest rate, it is expected that a lender would receive interest of a borrowed amount. The same expectation would apply for a person depositing funds onto a bank account. However after the financial crisis in 2008, situations where lenders pay interest to borrowers and banks charging people for their deposits are the scenario of 2017 and not a theoretical debate between experts anymore on a world with negative interest rates.

Still the concept of negative interest rates is not unfamiliar. Tracing back to the 1970s, the Swiss National Bank carried out an experiment on negative interest rates for the main purpose of controlling capital inflow as an action of preventing the Swiss Franc from appreciating. Today this is common ground for several developed countries. ECB was the first to implement negative rates in June 2014 due to weak growth and inflation in Euro area. In December 2014 Switzerland moved into a negative rate environment as a result of managing upward pressure on the franc, fear of deflation and weak growth. Other countries in Europe have also turned to negative rate such as Sweden and Denmark. The central bank of Sweden, Riksbanken, adopted negative interest rates in 2009 by lowering the deposit rate to -0.25 % with the ongoing financial crisis. Bank of Japan has also implemented a negative rate policy.

Motivation for implementing negative interest rate policies differs for each central bank, but one overall significant rationale has been to improve the economy and to control inflation.

In the Euro area, Sweden, Denmark, Switzerland – negative interest rate environments affect valuation models of interest rate derivatives.

The Black model has for years been the main framework for option pricing, but the rise of negative interest rate environments has marked multiple shortcomings of the model for handling interest rate options. One key feature of the Black model is that it assumes lognormal distributed rates, which only allows rates to be positive.

Adapting to the new normal of negative interest rates has created alternative models by either constructing a shifted lognormal distributed model or a normal distributed model. Lognormal-shifted Black volatilities have also been applied by some market participants but there are no model consensuses regarding the precise value of the constant shift parameter. Throughout this paper, we will use the normal distributed model due to its properties within the negative rate environment.

Interest rate derivatives

xIBOR rates

xIBOR stands for x Interbank Offered Rate, hence x refers to a specific currency. These rates are submitted by a group of prime banks each bank/business day at 11:00 GMT and vary with an expiry ranging from one business day to 12 months. The fixing/reference rate is computed as an average the submitted bank rates. The EURIBOR fixings are set by the European Banking Federation, while the reference rate is the LIBOR for USD. The rates are should reflect the price in which prime banks can loan money to each other.

Fixing methodology differs respectively, but all xIBOR fixings are quoted using the money market convention. By this it can be concluded that interest paid on a loan at expiry is calculated as $\delta \times N \times L$ where δ is the coverage[4], N is the notional and L the xIBOR rate. First, if $D(t,T)$ is the price of a zero coupon bond at time 0, bought at time t and maturing at time T. Secondly, if $F(0,T,T+\delta\,)$ denotes the forward xIBOR rate at time t, where we can loan funding between time T and $T+\delta$. Now by using the argument of no arbitrage, we can derive the forward xIBOR rates as

$$1+\delta F(0,T,T+\delta\,)=\frac{1}{D(T,T+\delta)} <=>$$

$$F(0,T,T+\delta\,)=\frac{1}{\delta}\cdot\frac{D(0,T)-D(T,T+\delta)}{D(T,T+\delta)} <=>$$

$$F(0,T,T+\delta\,)=\frac{1}{\delta}\left(\frac{D(0,T)}{D(T,T+\delta)}-1\right)$$

Overnight index swap

The overnight index swap (OIS) is an interest rate swap, where a fixed rate is exchanged over an agreed notional given the geometric average of an overnight

[4] Coverage is seen as the year fraction expressed in years.

rate (fx. Fed Funds effective rate) for a chosen payment period. OIS discounting is used for USD and EUR swaptions.

Interest rate swaps

Interest rate swaps is today by far one of the most traded derivatives. Bank of International Settlement[5] (BIS) reported a total amount of outstanding interest rate swaps around 421 trillion USD with a estimated gross value of all contracts around 17 trillion USD. Swap contracts are traded as over-the-counter (OTC), this means that the deal is directly between counterparties and not executed from an exchange.

Valuation of an interest rate swap

A plain vanilla interest rate swap (IRS) an agreement/contract between two parties to exchange their series of payments for a pre-agreed period – one of the parties pays a fixed interest rate while the other party pays a floating interest rate, also called a fixed-for-floating swap. This common swap contract contains therefore of a fixed and a floating leg respectively. The party paying the floating rate has entered into a receiver swap, while the counterparty has entered into a payer swap.

In abstract can a swap somehow be compared to a linear combination of forward rate agreements (FRA's). Whereas FRA's are settled and fixed in advance, IRS is only fixed in advance but paid-in-arrears. Here may an IRS also have different day count conventions and payment schedules.

The floating leg is linked to some xIBOR rate fixed-in-advance and paid-in-arrears[6]. Plain vanilla IRS market conventions for both currencies can be seen below in table X.X.

				Floating leg			Fixed leg	
Currency	Index name	Sport Start	Roll	Term	Freq.	Day count	Freq	Day count
USD	USD Libor	2B	MF	3M	Q	Act/360	S	30/360
EUR	Euribor	2B	MF	6M	S	Act/360	A	30/360

[5] https://www.bis.org/statistics/dt21a21b.pdf
[6] Fixed income Derivatives. M. Linderstrom

We can find the present value of the floating leg by visualize start and end dates of an IRS, denoted from T_S to T_E, and calculate a set of coverages $\delta_{S+1}^{float}, \dots, \delta_E^{float}$. The floating leg present value can be obtained as

$$PV_t^{Float} = \sum_{i=S+1}^{E} \delta_i^{Float} F(t, T_{i-1}, T_i) N_i D(t, T_i) \qquad (1)$$

Here N_i is the notional at period i, $D(t, T_i)$ is the discount factor and $F(t, T_{i-1}, T_i)$ is the forward xIBOR rate between period T_{i-1} and T_i.

We can derive the fixed leg of the IRS in a similar fashion. From the table above, we can see the payment frequency and day count for the fixed leg. These parameters don't necessarily match the same parameters for the floating leg, so we typically create a new set of coverages and dates due to the difference in payment frequency and day count conventions. The leg still have the same start and end date, but the coverages now looks like $\delta_{S+1}^{fixed}, \dots, \delta_E^{fixed}$. By letting K represent the fixed rate paid in the swap, we can write the present value of the fixed leg as

$$PV_t^{Fixed} = \sum_{i=S+1}^{E} \delta_i^{Fixed} K D(t, T_i) \qquad (2)$$

From the equation above, it is easy to see that the fixed leg is the fixed rate payment with a discounting factor.

The present values of both legs in the swap are now obtained, hence now we are able to calculate the value of swap by combining the two equations, (1) and (2). Then referring to a swap contract, we focus on the fixed leg, therefore discussing a payer swap, will refer to a party paying the fixed rate and receives floating rate respectively. The present value of the a payer swap starting a T_S and maturing at T_E is therefore

$$PV_t^{Payer} = \sum_{i=S+1}^{E} \delta_i^{Float} F(t, T_{i-1}, T_i) N_i D(t, T_i) - \sum_{i=S+1}^{E} \delta_i^{Fixed} K D(t, T_i) \quad (3)$$

If we instead wish to obtain the price of a receiver swap, the positions are reverse, so the fixed leg can be seen as an asset and the floating leg as a liability. Hence we have that $PV^{Receiver} = -PV^{Payer}$.

Entering into a swap, it is typical to trade swap with a net present value of zero. This means that setting $PV^{Fixed} = PV^{Floating}$, we can isolate the fixed rate K by setting the present value of the IRS equal to zero in equation (3) above. The isolated fixed rate can be denoted as the par swap rate and is defined as

$$R(t, T_S, T_E) = \frac{\sum_{i=S+1}^{E} \delta_i^{Float} F(t, T_{i-1}, T_i) N_i D(t, T_i)}{\sum_{i=S+1}^{E} \delta_i^{Fixed} K D(t, T_i)}$$

Let us assume we have entered into a payer swap at T_S, paying the fixed rate K and receiving a floating xIBOR rate until expiry T_E of the swap. If entering into a matching receiver swap at some time t. Here we would receive the fixed par swap rate $R(t, T_S, T_E)$ and paying some floating xIBOR rate, but the floating rates will cancel out and we will have a net cashflow of the fixed legs instead, where $R(t, T_S, T_E) - K$. Setting $A(.)$ as the time t sum of discounted fixed coverages, $A(t, T_S, T_E) = \sum_{i=S+1}^{E} \delta_i^{Fixed} K D(t, T_i)$, the present value at time t for the initial payer swap can be stated as

$$PV_t^{payer} = A(t, T_S, T_E)(R(t, T_S, T_E) - K) \qquad (4)$$

Where $A(t, T_S, T_E)$ is the so called annuity factor or level of the swap. By differentiating this equation w.r.t par swap rate, the result is precisely$A(.)$. $A(.)$ can therefore be seen as the value of receiving one basis point over the period $T_S - T_E$and hereby thought as the payers sensitivity towards the par swap rate.

The swap rate is constructed as a government bond yield plus a spread representing the increased credit risk relative to sovereign risk decided by participants in the market. We can see the swap rate as a rate including both a risk-free rate and a swap spread. Here we assume that the risk-free rate calculated on observed government bonds as they are assumed to be safe investments due to the fact that a government will never fill bankruptcy (broader speaking about highly developed countries such as USA). The swap spread reflects therefore a premium that can be explained by numerous drivers such as liquidity risk, BIS weighting, credit ratings, credit situation and demand/supply.

European swaptions

A plain vanilla interest rate option like a European swaption, labelled swaptions further on, is an option with the right (not obligated) to enter into an IRS starting on a future date T_S at predetermined fixed rate K, where the IRS is maturing at T_E. As an example if a party buys a payer swaption with $T_S = 2$ and $T_E = 7$, this means that this party has the right to pay the fixed rate K, while receiving a floating xIBOR rate at 2Y with expiry at 7Y, therefore the name 2Y7Y payer swaption. We can say this is the same as buying at put option on a bond. Therefore the party buying a payer swaption has the right to pay a fixed rate and receive floating rate, while it is vice versa for a receiver swaption. Swaptions are also traded as OTC instruments.

Before entering at swaption contract both parties must agree on the type of settlement. We have two types of settlement

- Cash settlement – here on exercise date the swap price on the underlying swap is determined by an average of prices from five predetermined banks. Both the highest and the lowest quotes are excluded in the calculation of the average price. This determines the amount of cash exchanges between the two parties.
- Physical settlement or swap settlement – here the buyer of the option will enter into the underlying swap with actual cash flow exchanges until expiry of the swap.

Swaption Pricing

As mentioned in the section above, a swaption can be settled differently. The choice of settling has impact on the cash flows but also in the valuation of swaption contract. We will price a swaption under the assumption that the holder has entered a payer swaption.

Physical settlement

For a physical settled swaption the option holder can enter into a swap contract predetermined by the conditions of the contracted option. A swap contract will only be entered if the swaption is in-the-money to the swaption holder. This reminds us of equation (4) and the ability to decide not to enter into the swap. The holder receives at T_S

$$Payer\ swaption\ PV_{T_S}^{Physical} = A(T_S, T_S, T_E)[(R(T_S, T_S, T_E) - K)]^+$$
$$where\ A(T_S, T_S, T_E) = \sum_{i=S+1}^{E} \delta_i^{Fixed} D(T_S, T_i)$$

Cash settlement

At expiry of the option the holder of a cash settled swaption will receive the present value of the underlying swap. Here we are discounting using the par swap rate. The holder receives at T_S

$$Payer\ swaption\ PV_{T_S}^{Cash} = \tilde{A}(T_S, T_S, T_E)[(R(T_S, T_S, T_E) - K)]^+$$
$$where\ \tilde{A}(T_S, T_S, T_E) = \sum_{i=S+1}^{E} \frac{\delta_i^{Fixed}}{\left(1 + \delta_i^{Fixed} R(T_S, T_S, T_E)\right)^{T_i - T_S}}$$

Overall the difference between these two settlement agreements lies in method of discounting. Following on, we will state $A(.)$ as a reference for both cases.

We are now able to apply a swaption pricing model. Several pricing models exist, varying in the assumptions regarding the behavior of the underlying swap rates. The most known is the Black model, which assumes lognormal distributed behavior. The derivation of the result behind the Black model will not be enlighten, but solitary stated by result

$$Payer\ swaption\ PV_t = A(t, T_S, T_E)\left[\left(R(t, T_S, T_E)N(d_1) - KN(d_2)\right)\right]$$

$$d_1 = \frac{\log\left(\frac{R(t, T_S, T_E)}{K}\right) - \frac{1}{2}\sigma^2(T_s - t)}{\sigma\sqrt{T_s - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T_s - t}$$

We can find the value of the swaption by determining the forward swap rate $R(t, T_S, T_E)$, the annuity factor (cash or physical) $A(t, T_S, T_E)$, the time to expiry and lastly the volatility σ.

Pricing the receiver swaption can be done easily through the put-call-parity for plain vanilla European options. If the payer and receiver option has identical strike rate K and identical timeframe (entered into a swaption at time t, start date at T_S and end date at T_E), the parity states

$$\begin{aligned} &Forward\ Starting\ Payer\ Swap(K) \\ &\qquad = Payer\ Swaption(K) - Receiver\ Swaption(K) \end{aligned}$$

Rearranging this parity, we are able to derive the price of the receiver swaption as followed

$$Receiver\ swaption\ PV_t = A(t, T_S, T_E)\big[\big(KN(-d_2) - R(t, T_S, T_E)N(-d_1)\big)\big]$$

Volatility in the pricing model can be formulated as the anticipated future volatility in an option. From Black Scholes option framework, implied volatility for swaptions are the core element in calculating premiums as implied volatility is the only unobservable parameter in this framework. Hereby the price is only influenced by the implied volatility due to the fact that the underlying, strike and expiry are all observable.

Swaptions are in the United States all almost cash settled while in Europe approximately 50 % are in fact cash settled. However all European swaptions included in this paper are cash settled.

Due to the nature behind lognormality, the Black model measures implied volatility in relative approach, i.e. relative changes of the forward swap rate, while the normal model measures the implied volatility as the absolute changes of the forward swap rate. We will now investigate the normal model.

The Normal model

The normal model is a rather simple model, but still a benchmark model when working with negative interest rates. It was introduced in 1900 by a French mathematician, named Bachelier. The evolution of the forward swap rate F_t follows the stochastic differential equation

$$dF_t = \sigma_N dW_t$$

With σ_N as the normal volatility and W_t as a Wiener process. Introducing Ito calculus[7], we are able to find a solution to this equation.

$$F_t = F_0 + \sigma_N W_t$$

Here is the forward swap rate assumed to be normally distributed or the process behaves as a standard Brownian motion.

The price of a payer swaption in the normal model can be found as

$$PV_{T_S} = A(T_S, T_S, T_E)\sigma_N\sqrt{T_S}\left(\frac{e^{-\frac{d^2}{2}}}{\sqrt{2\pi}} + dN(d)\right)$$

Where $N(\dots)$ is the probability distribution of a standard normal variate and

$$d = \frac{f - K}{\sigma_N\sqrt{T_S}}$$

By introducing the put-call parity we can easily calculate the receiver swaption as

$$PV_{T_S} = A(T_S, T_S, T_E)\sigma_N\sqrt{T_S}\left(\frac{e^{-\frac{d^2}{2}}}{\sqrt{2\pi}} - dN(-d)\right)$$

The normal volatility is not comparable with black volatility as one is quoting an absolute volatility level while the other is quoting a relative volatility level. However an approximation can be derived for converting normal volatilities into black volatilities and the other way around.

[7] Stochastic Calculus

As mentioned earlier, the normal model assumes swap rates to be normal distributed instead of log normal distributed in the Black model. Black model measures implied volatility in relatively order, as the relative changes of the forward swap rate. Conversely to the normal model, which measures implied volatility as absolute changes of the forward swap rate. It is important to note that implied volatility itself isn't the volatility of movements of the swap rate, but instead the markets opinion regarding this. Still, the linkage between these two sizes is expected to be strong.

Risk measures under the normal model

Delta

Delta is defined as the change in the value of the option when the price of the underlying asset increases.

Delta for a payer swaption is given as

$$\Delta_{PS}=\frac{\partial V}{\partial F}=A(T_S,T_S,T_E)N(d)$$

$$d=\frac{f-K}{\sigma_N\sqrt{T_S}}$$

Gamma

Gamma is defined as the sensitivity of the delta to the underlying asset.

$$\Gamma_{PS}=\Gamma_{RS}=\frac{\partial^2 V}{\partial^2 F}=\frac{A(T_S,T_S,T_E)}{\sigma_N\sqrt{T_S}}N(d)$$

Vega

Vega is defined as the change in the option price due to change in the volatility of the underlying asset.

Vega is the same for put and call options, which can be given from the put-call parity.

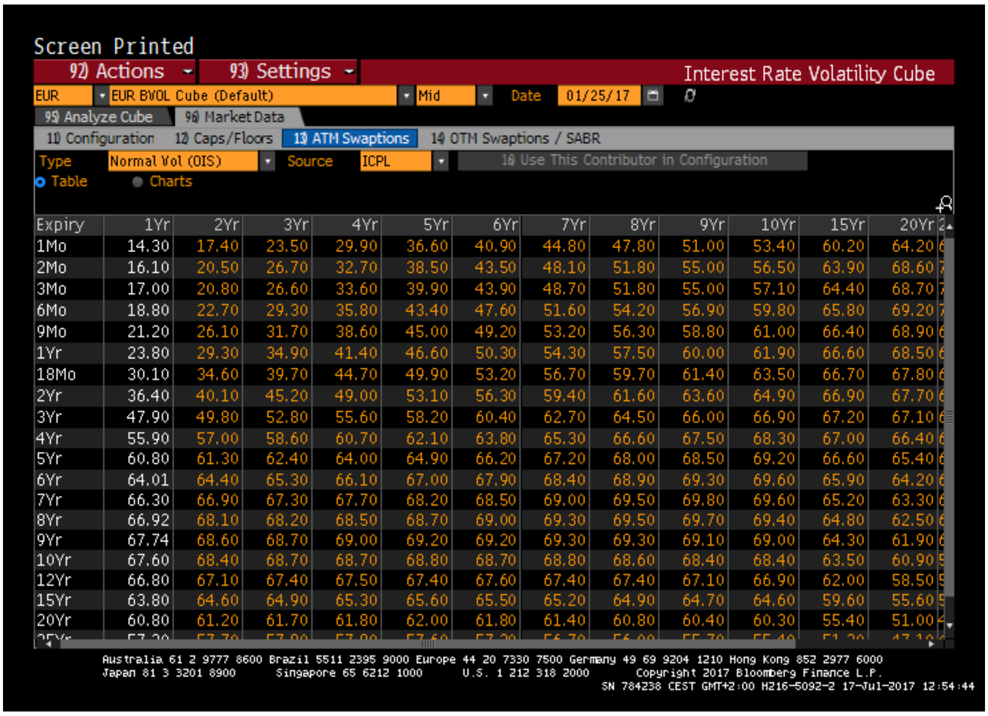
$$\nu_{PS}=\nu_{RS}=\frac{\partial V}{\partial \sigma}=A(T_S,T_S,T_E)\sqrt{T_S}N(d)$$

Theta

Theta is defined as the change in the option prices with the respect to time to maturity. A purchased option will decrease as time goes by, while holding all other parameters constant. At expiry of the option, the time value of the option is zero and the value of the option is only given by its intrinsic value.

Quoting swaptions

As mentioned earlier, swaptions are OTC products and are quoted for various currencies. They are quoted as straddle options premia, Black or Normal implied volatilities. On the figure below, we see ATM straddles quoted in normal volatility. This is what most market participants today uses when quoting prices on swaptions.



Screen Printed
92) Actions 93) Settings Interest Rate Volatility Cube
EUR | EUR BVOL Cube (Default) | Mid | Date 01/25/17
95) Analyze Cube 96) Market Data
11) Configuration 12) Caps/Floors 13) ATM Swaptions 14) OTM Swaptions / SABR
Type Normal Vol (OIS) Source ICPL 16) Use This Contributor in Configuration
Table Charts

Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	6Yr	7Yr	8Yr	9Yr	10Yr	15Yr	20Yr
1Mo	14.30	17.40	23.50	29.90	36.60	40.90	44.80	47.80	51.00	53.40	60.20	64.20
2Mo	16.10	20.50	26.70	32.70	38.50	43.50	48.10	51.80	55.00	56.50	63.90	68.60
3Mo	17.00	20.80	26.60	33.60	39.90	43.90	48.70	51.80	55.00	57.10	64.40	68.70
6Mo	18.80	22.70	29.30	35.80	43.40	47.60	51.60	54.20	56.90	59.80	65.80	69.20
9Mo	21.20	26.10	31.70	38.60	45.00	49.20	53.20	56.30	58.80	61.00	66.40	68.90
1Yr	23.80	29.30	34.90	41.40	46.60	50.30	54.30	57.50	60.00	61.90	66.60	68.50
18Mo	30.10	34.60	39.70	44.70	49.90	53.20	56.70	59.70	61.40	63.50	66.70	67.80
2Yr	36.40	40.10	45.20	49.00	53.10	56.30	59.40	61.60	63.60	64.90	66.90	67.70
3Yr	47.90	49.80	52.80	55.60	58.20	60.40	62.70	64.50	66.00	66.90	67.20	67.10
4Yr	55.90	57.00	58.60	60.70	62.10	63.80	65.30	66.60	67.50	68.30	67.00	66.40
5Yr	60.80	61.30	62.40	64.00	64.90	66.20	67.20	68.00	68.50	69.20	66.60	65.40
6Yr	64.01	64.40	65.30	66.10	67.00	67.90	68.40	68.90	69.30	69.60	65.90	64.20
7Yr	66.30	66.90	67.30	67.70	68.20	68.50	69.00	69.50	69.80	69.60	65.20	63.30
8Yr	66.92	68.10	68.20	68.50	68.70	69.00	69.30	69.50	69.70	69.40	64.80	62.50
9Yr	67.74	68.60	68.70	69.00	69.20	69.20	69.30	69.30	69.10	69.00	64.30	61.90
10Yr	67.60	68.40	68.70	68.70	68.80	68.70	68.80	68.60	68.40	68.40	63.50	60.90
12Yr	66.80	67.10	67.40	67.50	67.40	67.60	67.40	67.40	67.10	66.90	62.00	58.50
15Yr	63.80	64.60	64.90	65.30	65.60	65.50	65.20	64.90	64.70	64.60	59.60	55.60
20Yr	60.80	61.20	61.70	61.80	62.00	61.80	61.40	60.80	60.40	60.30	55.40	51.00

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Figure 1: Normal Implied Volatilities for EUR Swaption, Date 01-25-2017, Source: ICAP

A straddle strategy is a combination of buying or selling both a call option (here receiver swaption) and a put option (here payer swaption) on a swap with the same strike rate and expiry. A receiver swaption is receiving a fixed swap rate, so the holder of the swaption has the right to receive a fixed rate in a swap contract in a determined future. A payer swaption is a put option as the holder has the right to pay a fixed rate in a swap contract in a determined future.

A long position in a swaption straddle can then be expressed as

$$Straddle\ PV_{Ts} = Payer\ swaption\ PV_{Ts} + Receiver\ swaption\ PV_{Ts}$$

We can easily derive an ATMF straddle price quoted in normal volatility, since the forward rate and strike price are the same. The NPV for a straddle is then

$$Straddle\ PV_{Ts} = NA(T_S, T_S, T_E)\sigma_N \sqrt{\frac{2T_S}{\pi}}$$

Where N is the notional,$A(T_S, T_S, T_E)$ is the swap annuity factor or basis point value and σ_N is the normal implied volatility.

Straddles are often traded as to speculate how volatility in the future changes. We could look at a long short combination of two ATM straddles with different maturities. The option writer of a straddle has unlimited downside risk while having an upside limited to a premium received from the two options. Straddles are a common option strategy for investment purpose, here with the expectation of market stability/neutralization and stable or declining volatility. Hereby such a position would only be profitable before expiry decline in implied volatility or time value decay. A straddle is often applied in speculating on future changes of the volatility.

However straddle prices don't depend on forward swap rate in a directly manner, but purely on volatility. Therefore they can be viewed as a good indicator of implied volatility level.

There is no delta risk when entering at straddle, but large market volatility can cause an exposure to delta risk. ATMF straddles are also more liquid than standalone payer and receiver swaptions, due to limited dollar value of 01 basis point (DV01 risk). Often are ATMF straddles with different expiries the cheapest way to hedge gamma, vega and theta risk.

Choosing normal or log-normal

We have both introduced the the normal model and the lognormal model in the swaption framework for pricing swaptions. As European ATM swaptions are classified as a plain vanilla option, simple approaches are used on trading desks such as these two models.

As lognormality can't deal with negative interest rates and also has difficulties with low interest rates, we favor to use normal volatility.

As this paper is more interested in the changes of the implied volatility, the perspective on selecting a pricing framework isn't important for the analysis. The difference between the models are however important to understand in explaining the basis of the unit of measurement.

However an interesting issue follows when converting Black volatilities into Normal volatilities. The relationship for ATMF swaptions can be expressed as

$$\sigma_N \approx f\sigma_{Black}$$

If σ_N is constant, σ_{Black} is proportional to $\frac{1}{f}$, where f is forward rate

This can be a convenient way to approximate in either normal or black volatility. If normal volatility is constant, lognormal will be decreasing in rates. This is a so-called skew effect.

Methodology

Mean Reversion

In relative value analysis one of the most fundamental tools is the mean reversion process. This can be explained by expecting over at time period that one or several variables to follow a long-run average.

Ornstein Uhlenbeck stochastic differential equation can be found as

$$dS_t = \lambda(\mu - S_t)dt + \sigma dW_t$$

Here is dS_t the change in the value of the random variable S over time t, λ is known as the speed of mean reverting process, the long term equilibrium/average of the variable S is given by the parameter μ, σ is the instantaneous volatility of the random variable S and the dW_t is the change in the standard Wiener process W_t

Generally a stochastic differential equation is formulated as

$$dx_t = f(x_t)dt + g(x_t)dW_t$$

The first part of the formula expresses the drift process $f(x_t)$ and defines the mean of the process while the term $g(x_t)$ expresses a diffusion process where the volatility of the process is stated.

The mean reverting process can also calculate a stopping time also known as first time passage. In this paper maximum likelihood is used to calculate the parameters of the Ornstein-Uhlenbeck process. In finding these parameters, the stochastic differential equation exact solution of the OU process above is discretized and approximated as

$$\mathrm{S}_{\mathrm{t+1}} = \mathrm{S}_{\mathrm{t}}\mathbf{e}^{-\boldsymbol{\lambda}\boldsymbol{\Delta}\mathbf{t}} + \mu(1 - \mathbf{e}^{-\boldsymbol{\lambda}\boldsymbol{\Delta}\mathbf{t}}) + \sigma\sqrt{\frac{(1 - \mathbf{e}^{-\mathbf{2}\boldsymbol{\lambda}\boldsymbol{\Delta}\mathbf{t}})}{2\boldsymbol{\lambda}}}\,\boldsymbol{\Delta}\mathbf{W_t}$$

Here is $\Delta\mathbf{t}$ an infinitesimal change, while $\Delta\mathbf{W_t}$ are independent identically distributed Wiener process. This formula is useful in simulation of generating paths, we want to analyze later on.

Calibrating MR with MLE

From Calibrating the Ornstein-Uhlenbeck (Vasicek) model, Van Den Berg 2011 paper it states, S_{t+1} conditional probability density function is given as

$$P\big(N_{0,1} = x\big) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

This is a combination between the normal distribution probability density function and the solution to the stochastic differential equation above.

Now looking at the conditional probability density function of an observation S_{t+1} given a previous observation S_t, here with δ as the time interval between the two observations. The equation of the conditional probability density function is therefore given as

$$f(S_{t+1}|S_t; \mu, \lambda, \widehat{\sigma}) = \frac{1}{\sqrt{2\pi\widehat{\sigma}}} \exp[- \frac{\left(S_t - S_{t-1}e^{-\lambda\delta} - \mu\left(1 - e^{-\lambda\delta}\right)\right)^2}{2\widehat{\sigma}^2}$$

Now we can derive the log likelihood function of a set of observations $(S_0, S_1, \dots, S_n)$ from the conditional probability density function. Showed here as

$$\mathcal{L}(\mu, \lambda, \widehat{\sigma}) = \sum_{t=1}^{n} \ln f(S_t S_{t-1}; \mu, \lambda, \hat{\sigma})$$

$$= -\frac{n}{2}\ln(2\pi) - nln(\widehat{\sigma}) - \frac{1}{2\widehat{\sigma}} \sum_{t=1}^{n} \left[S_t - S_{t-1}e^{-\lambda\delta} - \mu\left(1 - e^{-\lambda\delta}\right)\right]^2$$

We then find the first order conditions for the maximum likelihood estimation by setting these equal to zero

$$\frac{\partial\mathcal{L}(\mu, \lambda, \widehat{\sigma})}{\partial\mu} = 0$$

$$= \frac{1}{\hat{\sigma}^2} \sum_{t=1}^{n} \left[S_t - S_{t-1} e^{-\lambda\delta} - \mu\left(1 - e^{-\lambda\delta}\right) \right]^2$$

$$\mu = \frac{\sum_{t=1}^{n} \left[S_t - S_{t-1} e^{-\lambda\delta} \right]}{n(1 - e^{-\lambda\delta})}$$

$$\frac{\partial \mathcal{L}(\mu, \lambda, \hat{\sigma})}{\partial \lambda} = 0$$

$$= -\frac{\delta e^{-\lambda\delta}}{\hat{\sigma}^2} \sum_{t=1}^{n} \left[(S_t - \mu)(S_{t-1} - \mu) - e^{-\lambda\delta}(S_{t-1} - \mu)^2 \right]$$

$$\lambda = -\frac{1}{\delta} \ln \frac{\sum_{t=1}^{n} (S_t - \mu)(S_{t-1} - \mu)}{\sum_{t=1}^{n} (S_{t-1} - \mu)^2}$$

$$\frac{\partial \mathcal{L}(\mu, \lambda, \hat{\sigma})}{\partial \hat{\sigma}} = 0$$

$$= \frac{n}{\hat{\sigma}} - \frac{1}{\hat{\sigma}^3} \sum_{t=1}^{n} \left[S_t - \mu - e^{-\lambda\delta}(S_{t-1} - \mu) \right]^2$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{t=1}^{n} \left[S_t - \mu - e^{-\lambda\delta}(S_{t-1} - \mu) \right]^2$$

The three solutions depends on each other but as seen on the equations above λ and μ are independent of $\hat{\sigma}$, wherefore if we either have the value of λ or μ, the other parameter can be derived. Lastly, we are able to derive $\hat{\sigma}$ when both λ and μ are known. For solving the equations, finding either λ or μ will be sufficient.

A derivation of the following results can be found in the appendix. Solving these equations gives following maximum likelihood estimators

$$\mu = \frac{S_y S_{xx} - S_x S_y}{n\left(S_{xx} - S_{xy}\right) - (S_x^2 - S_x S_y)}$$

And for speed mean reversion rate

$$\lambda = -\frac{1}{\delta} \ln \frac{S_{xy} - \mu S_x - \mu S_y + n\mu^2}{S_{xx} - 2\mu S_x + n\mu^2}$$

Lastly we get the variance as

$$\sigma^2 = \hat{\sigma}^2 \frac{2\lambda}{1-\alpha^2}$$
$$\alpha = e^{-\lambda\delta}$$

Where

$$\hat{\sigma}^2 = \frac{1}{n}\left[S_{yy} - 2\alpha S_{xy} + \alpha^2 S_{xx} - 2\mu(1-\alpha)\left(S_y - \alpha S_x\right) + n\mu^2(1-\alpha)^2\right]$$

These results are used to compute the expected future path and the standard deviations.

Principal component analysis[8]

When looking at financial data such as swaptions quotes, we sometimes deal with large data sets that are driven by only a couple of factors. These factors can be found through reducing the dimensionality of variables and still preserving the variables explaining most of the variance in the data set. Principal component analysis (PCA) is a statistical tool used for the purpose of identifying patterns and expressing the data in formal approach to expose their differences and similarities. This can help us in analyzing and identifying relative value opportunities, where the relative value of one or several instruments are independent of market direction or hedging solutions etc.

The main assumption for PCA is that factors/principal components as Y_i are uncorrelated. This is done by finding a rotation of the variables. This is a rather weak assumption and therefore gives the market the ability to add additional information regard about shape and strength of each factor.

We see the principal components as linear combinations of random variables in a p-dimensional space $X_1,\ X_2, \dots, X_p$. By rotating the original system of the random variables, a new coordinate system is created as a representation of the linear combinations. These new coordinate axes characterize the directions with maximum variability and gives a stricter but also simpler portrayal of the covariance structure. As a matter of fact the principle components focus exclusively on a covariance matrix Σ (or correlation matrix) and don't require any assumptions regarding multivariate normality.

If we have a random vector $\boldsymbol{X}' = [X_1,\ X_2, \dots, X_p]$ with the covariance matrix $\boldsymbol{\Sigma}$ and eigenvalues $\lambda_1 \geq \ \dots \geq \ \lambda_p \geq \ 0$.

Let us consider the linear combinations

$$
\begin{aligned}
Y_1 &= \boldsymbol{e}_1^T \boldsymbol{X} = e_{1,1} X_1 + \cdots + e_{1,p} X_p \\
Y_2 &= \boldsymbol{e}_2^T \boldsymbol{X} = e_{i,1} X_1 + \cdots + e_{2,p} X_p \\
\vdots &= \vdots \\
Y_p &= \boldsymbol{e}_p^T \boldsymbol{X} = e_{p,1} X_1 + \cdots + e_{p,p} X_p
\end{aligned}
$$

[8] Richard A. Johnson & Dean W. Wichern, "Applied Multivariate Statistical "

$Y_1, Y_2, .., Y_p$ form the principal components of $\boldsymbol{X}$. The principal component Y_1 explains most of the variance in the used dataset, therenext Y_2 and so on. Hereby we are interested in choosing the fewest number of factors while keeping most variance, as these factors gives us information about movements in the data set.

From linear algebra we know that linear combinations we can find

$$Var[Y_i] = \boldsymbol{e}_i^T \boldsymbol{\Sigma} \boldsymbol{e}_i = \lambda_i \qquad i = 1, \dots, p$$

$$Cov[Y_i, Y_k] = \boldsymbol{e}_i^T \boldsymbol{\Sigma} \boldsymbol{e}_k = 0 \quad for\ i \neq k, \qquad i, k = 1, \dots, p$$

And Σ has the eigenvalue-eigenvector pair $(\lambda_1, e_1) \dots, (\lambda_p, e_p)$

Further we have that

$$E(Y_i) = 0$$

$$\sum_{i=1}^{p} \mathrm{Var}[Y_i] = \sum_{i=1}^{p} \mathrm{Var}[X_i]$$

We have the following linear combination of X

$$\boldsymbol{Y_i} = \boldsymbol{a}_i^T \boldsymbol{X} \text{ where } Var(\boldsymbol{Y_i}) = \boldsymbol{a_i^T \Sigma a_i}$$

The first principal component is a linear combination with most variance, where we maximize $Var(\boldsymbol{Y_i})$

$$Max_{a \neq 0} \left[\frac{a^T \Sigma\, a}{a^T a} = \lambda_1 \right]$$

As $e_1^T e_1 = 1$, we get that

$$\lambda_1 = \frac{e_1^T \Sigma\, e_1}{e_1^T e_1} = e_1^T \Sigma\, e_1 = Var[Y_1]$$

The next principal components are linear combinations which maximize their variance and are uncorrelated to previous k components

$$Max_{a \perp e_1, \dots, e_k} \left[\frac{a^T \Sigma\, a}{a^T a} = \lambda_{k+1} \right], \; k = 1,2, \dots, p-1$$

When $e_{k+1}\ and\ e_{k+1}^t e_{k+1} = 1$

Then

$$\lambda_{k+1} = \frac{e_{k+1}^T \Sigma e_{k+1}}{e_{k+1}^T e_{k+1}} = e_{k+1}^T \Sigma e_{k+1} = Var[Y_{k+1}]$$

Where we have $Var[Y_i] = \lambda_i$

The sum of all eigenvalues are equal to the sum of the variance for all variables, here the i-th principal component explains a proportion of the total variance

$$\frac{\lambda_k}{\lambda_1 + \cdots + \lambda_p} \quad k = 1,2, \dots , p$$

What does the scores represents? As the relationship between each eigenvector of the covariance matrix in a PCA is orthogonal, the eigenvectors are used to project the data from its original axes into the ones characterized by the computed principal components. The factor scores is a rebasing of the original coordinate system of the entire dataset into a space given by new axes defined by the eigenvectors with the greatest variance.

In deriving a PCA, correlation or covariance matrix can be used for the derivation. Using the covariance matrix, the computed principal component scores will be presented in the original units, while choosing the correlation matrix instead will produce unitless results. This is due to the fact that the correlation matrix standardizes all dimensions of the dataset and this paper chooses to operate with the covariance matrix.

The data used for a PCA is not required to follow a Gaussian distribution, but assumes linearity as PCA computes a projection of the data on dimension reduced linear subspace. Any non-linear relationships between variables are therefore not regarded in this process.

For hedging purposes these uncorrelated factors can give strong hedging opportunities. The factors can be viewed as risk factors. Here we use the weights and loadings from our PCA in calculating hedge ratios that immunize a portfolio of securities against changes in factors. Remember this is a linear combination.

Using PCA in R

All calibrations are done in the statistical program R. Here we have used the function Prcomp(), in the calibration of PCA tool, from a library called PCA in the statistical program R. Prcomp executes a PCA by a singular value decomposition method of the data matrix, which is calculated from the covariance matrix. Other methods are a possibility but won't be regarded in this paper. The singular value decomposition is given as

$$X = UDV^T$$

U is a $m\ x\ m$ where the columns are orthonormal vectors

V is $n\ x\ n$ where the columns are orthonormal vectors

D is $m\ x\ n$ diagonal with diagonal elements called singular values of X.

We are especially looking at the arguments $rotation and $x in R function prcomp. The argument $rotation gives a matrix of the eigenvectors or factor loadings which shows the weighting of variables/instruments meanwhile argument $x gives the PC or scores/factors as the new variables.

Algorithm for PCA based on SVD

1) Collect the p observed data samples into a matrix $\boldsymbol{X} = [X_1,\ X_2, \ldots, X_p]$
2) Compute the singular value decomposition of the matrix by
$$X = UDV^T$$
3) Find the principal directions in the columns U
4) Find the principal components that are stored in the columns of matrix $Z = DV^T$

Distribution of the volatility surface

We can divide ATM options into two different types of options by their expiry. Looking at ATM options with relatively short time to expiry, prices are mainly depended on moves in the underlying asset. These options have high gamma and therefore will delta changes significantly when the underlying asset moves. Whereas ATM options with longer maturities are mainly depended on moves in the implied volatility level. To summarize this, options with a shorter expiry are greatly affected by changes in the underlying and therefore can be classified to be exposed to changes in the realized volatility. Here we can state the realized volatility as the movement in the underlying asset. We define this as the gamma sector of the volatility surface.

However, options with a longer expiry horizon are less impacted by changes in the underlying as but instead have a significant exposure to changes in the implied volatility level. We define these options as vega sector options on the implied volatility surface.

We can therefore differentiate between these two markets by the time to expiry. There is no conclusive line between the classifications of these two sectors on the volatility surface. Options with around two years to expiry can't be distinctly classified and falls into a gray area between the gamma and vega sector. These 'gray area' options are considered in each particular analysis of the volatility surface whenever they belong in the gamma or vega sector. This will of course be followed up on in the analysis. The expiry of the underlying is irrelevant in this discussion. The classifications of the two segments are illustrated in figure 2 below.

Options classified as in the gamma sector can be traded actively within a Black-Scholes or Bachelier valuation framework using delta hedging. By focusing on the vega sector instead, this gives rise for another mindset that is unlike the well-known delta hedging strategy. Introducing a statistical analysis on the implied volatility surface can be a powerful tool in the decision making of relative value opportunities, hedging etc. A common tool such as a PCA can identify the drivers of the implied volatility that affects the whole surface for vega sector options and use this information in the decision making of trading on the implied volatility or in hedging a portfolio of swaptions. As implied volatility is the overarching parameter, it makes little sense to search for a superior valuation model that can undermine the Black

Scholes framework but instead use this statistical tool to identify value in the vega sector. Realized volatility in the underlying are of a little concern for pricing straddles in the vega sector, as these are more or less strictly exposed to changes in the implied volatility level. Given this characteristic, we can discard linkages to external variables such as movement in the underlying swap rate, and instead concentrate on internal statistical relationships between instruments on the implied volatility surface by considering these as time series used for statistical analysis.

Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	6Yr
1Mo	35.5	53.8	63.5	69	73.7	73.3
3Mo	46.6	61	71.3	76.2	80.2	79.9
6Mo	51.4	65	73.5	77.2	80.8	81.2
1Yr	61.5	70.1	76.2	79.3	82.3	82.6
2Yr	76.5	80.7	[illegible]	[illegible]	84.2	84.2
3Yr	85.6	85.4	85.4	85.5	85.6	85.4
4Yr	89	88.1	87.7	87.2	86.8	86.3
5Yr	90.1	89.6	88.9	88.2	87.5	86.8
7Yr	88.5	88.5	87.6	86.7	85.7	84.8
10Yr	83.9	82.6	81.7	80.8	79.9	79.1
15Yr	72.9	71.3	70.8	70.3	69.8	69.2

GAMMA

GRAY AREA

VEGA

Figure 2 - Classification of ATM Swaption volatility matrix for USD, date 5-18-2016

This paper focuses on volatility surface of at-the-money-forward (ATMF) swaptions and does not take different strikes levels into consideration. Adding the feature would create a cube instead of a surface, where we would need to account skew as an extra dimension. It is however a possibility to add this dimension in the vega sector without implementing an option price model as for the gamma sector. However given limited liquidity for out-the-money-forward swaptions, data don't necessarily reflect actual tradable prices but more often constructed prices from brokers. Therefore will a PCA with this extra dimension not represent the actual movements in the implied volatility market but an artificial reality decided by models quoting the OTMF swaptions volatilities.

Data

Description

Our concentration lies on EUR and USD European Swaptions. Here is implied normal volatility data on both markets extracted. Data includes swaptions with expiry ranging from 3 months to 30 years, while tenors are ranging from 1 year to 30 years. These options can be exercised at expiry as an X year swap. If we have 5y10y swaption, this is an option to pay or receive fixed for five years in 10 years' time.

The data for the swaption volatility matrix is extracted from a Bloomberg terminal with a data license to ICAP. The ICAP broker is considered by several market makers to be a valid distributer of rather accurate ATMF swaption quotes, reflecting actual tradable implied volatilities. The volatility quotes extracted are all mid quotes. The following Bloomberg function VCUB was used for finding ATMF implied volatilities for the EUR and USD markets. Both markets trades European swaptions actively and are considered to be two of the largest and most liquid markets trading European swaptions. As mentioned earlier, OTM swaptions can be very illiquid, which is one of the main reasons that this thesis only focuses on ATM swaptions.

The underlying of a swaption is the forward swap rate, which are the cornerstone when pricing these instruments, but this rate are not directly traded on the OTC market. The rates are actually computed by a yield curve model and the computed forward swap rates will not always represent the actual market forward swap rate. But such brokers such as ICAP supply both implied forward swap rates with quoted volatilities.

When modelling the estimated parameters in a PCA with high accuracy, it is of great importance that the chosen market data has an adequate lifespan. There was no data available for EUR swaption from ICAP or other brokers before the 1. November 2010 and this is therefore the starting point for EUR swaptions. Starting date for USD swaptions is the 1st of January 2010 with ending date at the 25th of January 2017. Data is collected daily and the chosen timestamp for the extracting is at end-of-day. We start off with investigating implied volatility level data in basis points units.

Stock indexes, currency crosses, fixed income data and commodity data are collected on a daily basis from 2010 to 2017. Some equity data are not available for all days considered for swaptions. Therefore will days without data, be given the same value as the latest former trading day. This should not conflict with the collected swaption data as many of these holidays; we see no change in normal volatility quotes. This is not a problem for the currency crosses as these are traded 7 days a week.

Empirical Results

Analysis

PCA is run for a 2.5 year timespan starting by taking data from the 1st of January 2010 and running each single day until the 25th of January 2017 to avoid biases when evaluating trading signals or for the interpretation of the results constructed from a PCA. Longer or shorter period could be chosen. Mainly focusing on capturing structural changes in level data, a longer period is preferable. Fewer instruments can be regarded depending on the area of interest. We want to construct a consistent framework, which can identify and extract value on the USD and EUR ATMF swaption markets. This framework is projected to help develop informal decisions regarding relative value analysis for purposes such as trading strategies, asset allocation or hedging risk factors.

As the swaption matrix are a two dimensional surface defined by expiry and tenor, data must be transformed into a one dimensional vector, as the loadings or the covariance matrix in a PC analysis otherwise would compute a multi-dimensional covariance matrix, which would be impossible to work with. The solution is to compile all instruments into one dimensional vector by combining expiry and tenor for all possible outcomes and stack them together.

$$1xN\ vector = \begin{pmatrix} \sigma_N^{1M1Y} \\ \sigma_N^{3M1Y} \\ \vdots \\ \sigma_N^{1Y2Y} \\ \vdots \\ \sigma_N^{30Y30Y} \end{pmatrix}$$

After this is obtained, a two dimensional space can be introduced as all instruments is set in a one dimensional vector and including time as the new dimension. This enables us to visualize each principal component eigenvectors of all instruments as a 1xN vector, where n is the number of instruments.

There is an important choice to consider before running a PCA. Thus is deciding if data for each variable should be standardized. This is mentioned as PCA is not scale invariant, because the calculated eigenvectors are not scale invariant. Notably, this can give instruments with higher implied normal volatility larger weighting in the analysis. However this is not conclusive since the difference between the lowest and highest volatilities across instrument isn't more divided. On figure 4 and 5, the scaling of all swaption with at least 2Y to expiry for USD and EUR are shown as boxplots. We see implied volatilities varies most for short expiry options, but all instruments moves within the same range. As the normal implied volatility across the whole surface regarding different maturities and underlying swap rates gives values in a comfortable range, standardization are not acknowledged. Moreover, constructing a PCA with a covariance matrix is preferable when dealing with variables stated in the same unit of measurement and same scaling as the computed factor scores can be interpreted in the same unit as the input data.

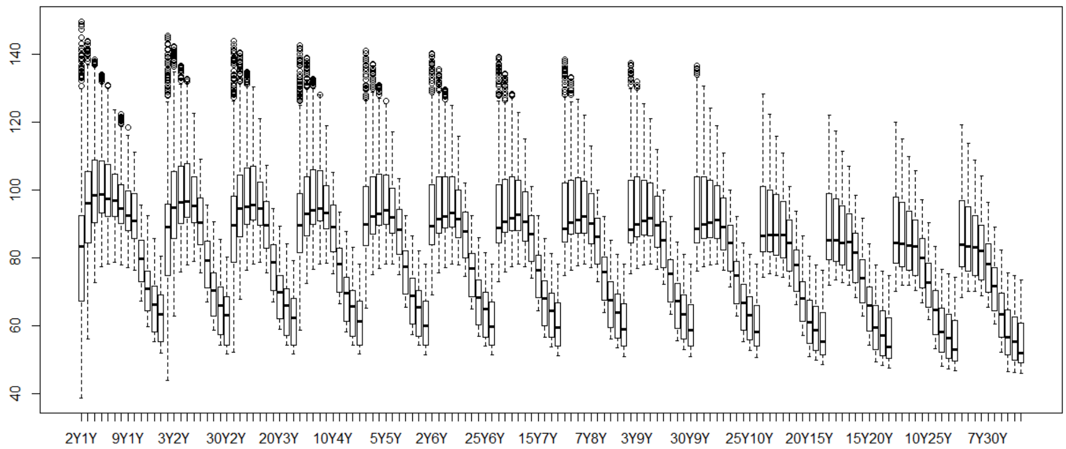


Figure 3 – USD boxplots of the scaling of normal implied volatility values for vega sector

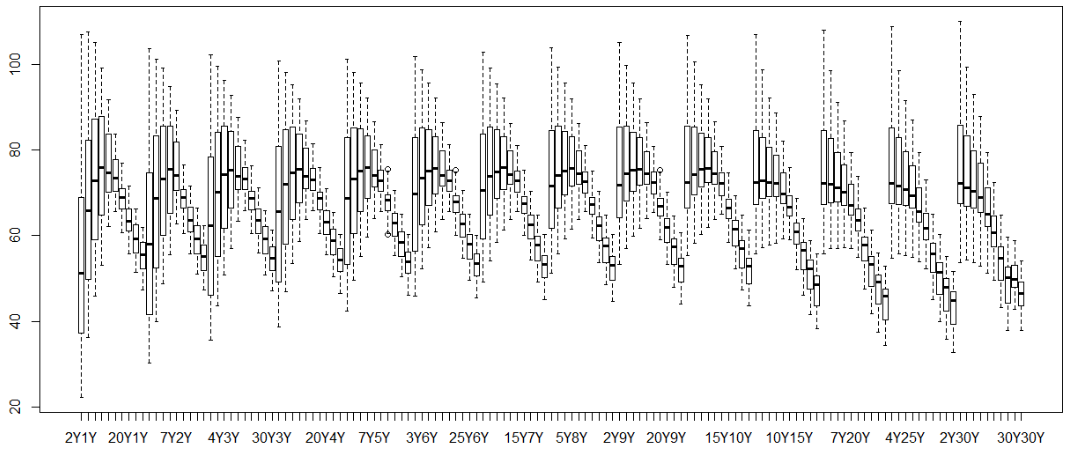


Figure 4 - EUR boxplots of the scaling of normal implied volatility values for vega sector

The USD normal volatility surface for ATM swaption are enlighten in a three dimensional surface on figure 6. The y-axis presents the underlying swap rate if the option and the x-axis describe the expiry of the option. Clearly, implied normal volatility decreases as expiry increases. Volatility is low in the very short end for a 1 year tenor and only one month to expiry. But all other short expiry swaptions offers higher implied normal volatilities as a volatility risk premium. The implied normal volatilities are shown in basis points (bps).

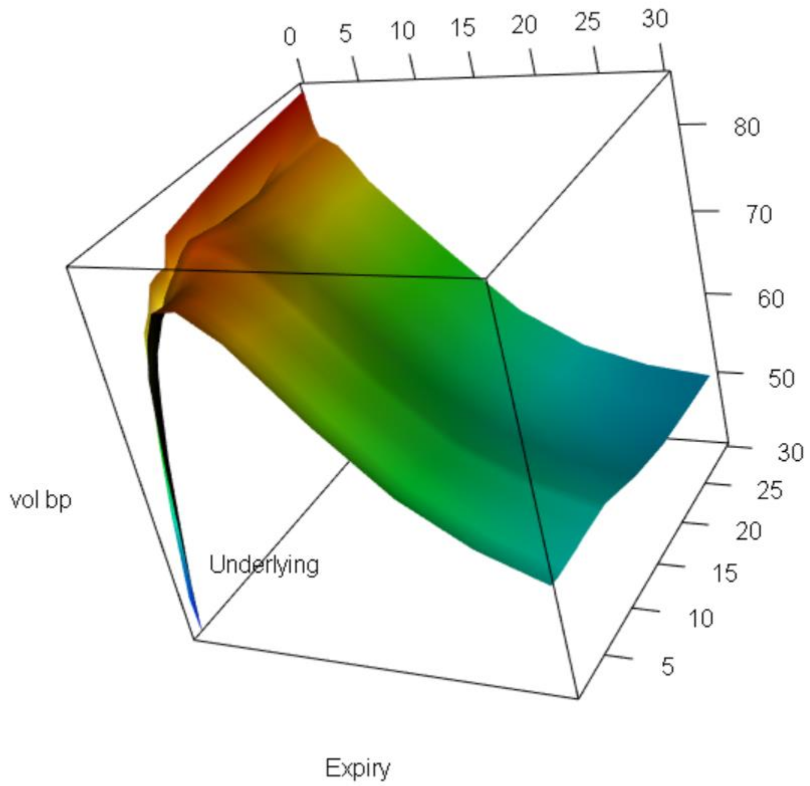


Figure 5 Normal volatility surface USD date 9 November 2016

The surface features a downward slope, decreasing as expiry rises. Focusing only on changes in tenor, we see a spike at 10Y tenor. If we exclude expiries less than 2Y only, we only see smaller variations on the implied volatility surface.

Interpretation of the eigenvectors

Firstly, by running a daily 630 days PCA for the whole implied volatility surface for USD, no instruments are excluded from the analysis. Here the purpose is to illustrate the dynamics of the first eigenvector and give an interpretation of this. This will be followed up by an interpretation of the eigenvectors for the three most influential principal components for the restricted vega sector. The interpretation is done by evaluating the weights, also called sensitivities, for each instrument. The weights illustrated are for that specific day, as weights for all instruments changes over time while staying in the same pattern.

First eigenvector

From figure 7 below, the weights of the first eigenvector of a PCA on the whole surface USD are shown. The instruments are grouped and colored by their tenors. For every tenor, instruments are arranged in ascending expiry order starting with 1M.

We observe that sensitivities of the eigenvector decreases with expiry and negative entries are introduced for 10Y to 30Y expiry options. As the entries for input variables has opposite signs this can be seen as a strong indication for introducing a segmentation of the gamma and vega sector, as we explained in the section 'Distribution of the volatility surface'.

Restricting the input data to options with at least 2 years expiry resolves the problem as seen in figure 8 below, where we have illustrated the weights of first eigenvector, which is generally calculating negative weights. The interpretation of the negative weights for the first principal component tells us that all instruments are affected in the same direction. However some instruments are more affected than others, e.g. the first principal can be understood as a level factor. This level factor describes the dynamics in the overall implied volatility level.

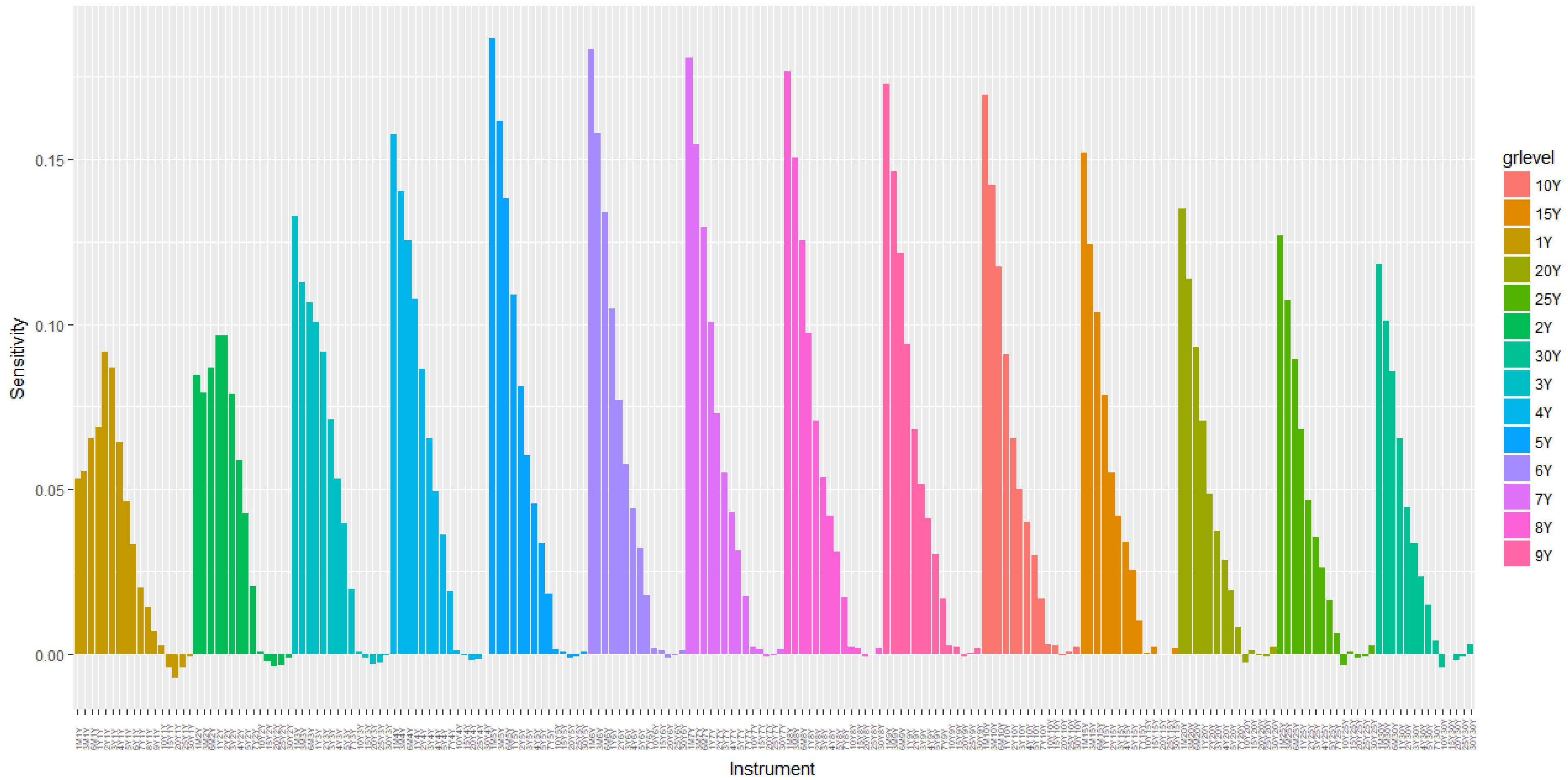


Figure 6 - First eigenvector of the whole USD volatility surface, Date: 2015-07-03

Analogously, we see that almost every 3Y expiry options are undoubtedly a part of the vega sector as the sensitivity values all are negative. However we observe that 2Y1Y, 3Y1Y, 2Y2Y and 2Y3Y swaptions all have positive entries. This can be explained by these shorter expiry options still must have some gamma influence during this period. But the positive sensitivities of the 3Y1Y and 2Y3Y are quite close to zero, giving us no reason not to include these instruments as a part of the vega sector, which a cluster analysis, described later on, will validate.

While the rest of the 2Y expiry options all have a negative sensitivity, the figure below gives a pattern of the attributes of the first eigenvector. The longer the expiry of the option (more or less), it gets less important what swap rate is the underlying, when being certain of an option could be classified as part of the vega segment. Especially for swap rates 1Y to 7Y, sensitivity becomes more negative due to longer option expiry. We can say that the longer the tenor is, the shorter the expiry needs to be in the extent of an option being a part of the vega sector.

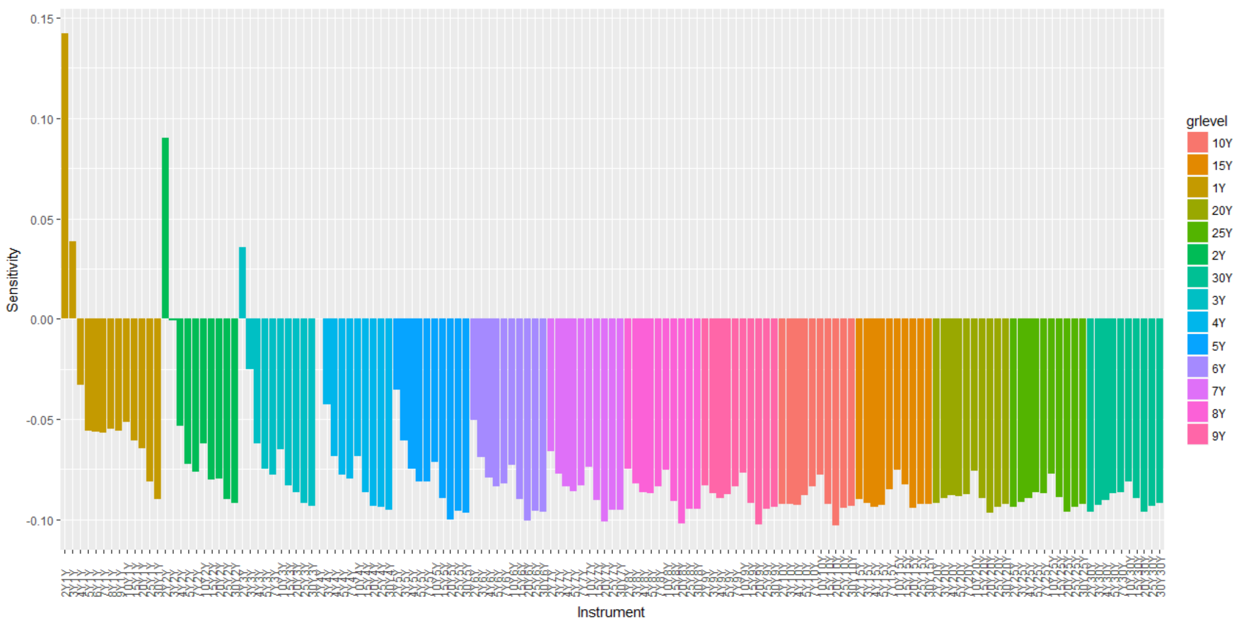


Figure 7 First eigenvector of vega sector, USD surface, Date: 2015-07-03

The sensitivities are roughly uniform distributed, which in essence gives associations towards empirical evidence of swap curve shifts from PCA. This support our interpretation on how different ATMF swaptions are affected by shifts on the implied volatility surface. PCA is often used to show how a shift in a curve empirically differs from theory. In theory a parallel shift occurs when expectation changes at once on a curve for all tenors by the same amount, but in empirically the shift differs for short-term and long-term tenors. In this example, the result is the similar but instead with a combination of short expiry-tenor and long expiry-tenor. This is the difference between the most volatile and least volatile implied volatility swaptions. The sensitivities for 4Y3Y and 7Y3Y are -0.0619 and -0.0778. This means that if 4Y3Y falls with one basis point, the 7Y3Y swaption is expected to fall with 1.26 basis points calculated from -0.0778/-0.0619=1.26.

We actually find that 3Y1Y, 2Y2Y and 2Y3Y swaptions all have negative sensitivities as all the other instruments except for shorter periods. We are illustrating the positive values in the figure above in as a statement of determine which instruments in the 'grey' area there should be included into the vega sector. As only one of the 2Y expiry options has a positive sensitivity in smaller subperiods, we will remark the 2Y expiry as a decisive part of the vega sector for the rest the analysis.

We are now able to identify and analyze how the second and third eigenvectors impacts the implied volatility surface.

Second eigenvector

The interpretation of the second principal component revealed from a PCA, describes the steepness of the volatility curves. Here with different option expiries but with the same underlying swap rate. This eigenvector captures the dynamics of the term structure of each tenor for instrument in the vega sector. An increase in the second factor will lead to an increase in options with a longer swap rate such as 7, 10 or 15 years relatively to shorter swap rates such as 1, 2 or 3 years. These behaviors are also shown on figure below, where we observe the second factors sensitivities for all input variables.

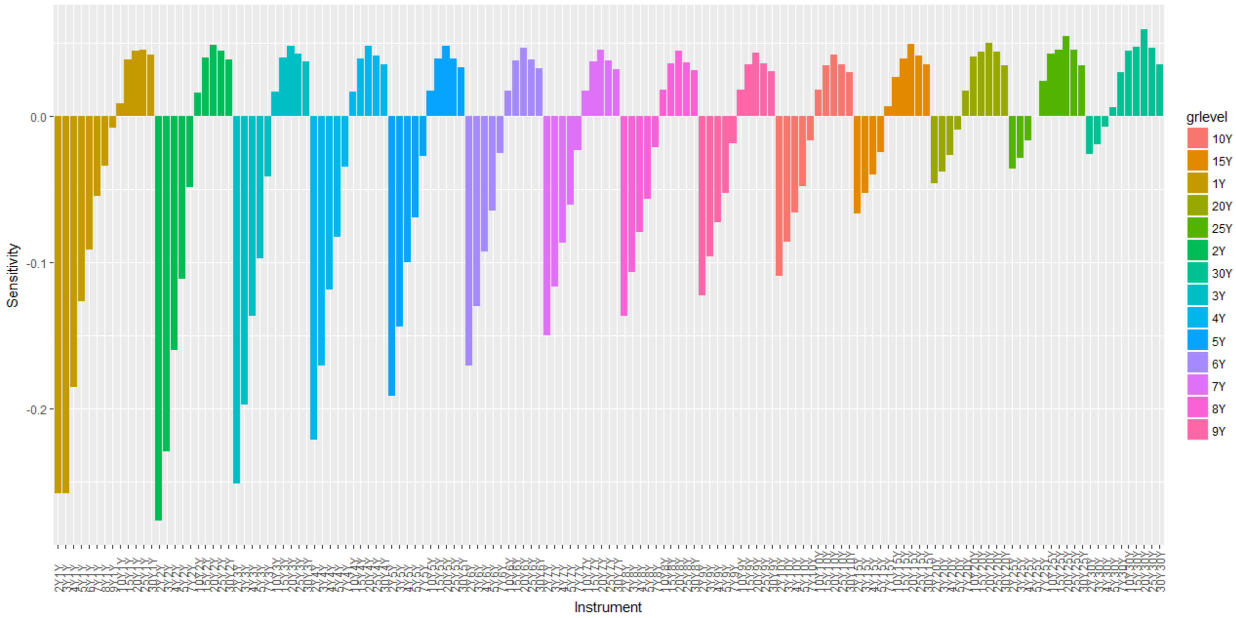


Figure 8 Second eigenvector of vega sector, USD surface, Date: 2015-07-03

Here we see that the expiry dimension determines the level of the sensitivity for all instruments. As an illustration look at all instruments with a 15 year expiry regardless of the underlying swap rate. All same expiry instruments surfaces around the same sensitivity more or less. Still the shorter expiry options, the more decrease the sensitivity for all long end tenors. As the tenor increases, options tend to move in a positive direction. Here could a curve trade involve a short expiry swaption with a large negative weighting and a long expiry swaption with the same tenor, but with a positive weighting as we expect these two positons to go in opposite directions. The largest movements are given by the largest absolute weights. This means that the most interesting curve trades is given for shorter tenors as these volatility curves tend to change the most. As an example, if the sensitivities for 4Y3Y and 15Y3Y are -0.1367 and 0.0401, this means that if 4Y3Y falls with one basis point, the 15Y3Y swaption implied volatility is expected to increase with 0.29 basis points by dividing the two numbers.

Third eigenvector

The third factor describes the steepness of the implied volatility curves, likewise to the second principal component but with a focal point on different tenors and same option expiries instead. The instruments are now grouped and colored by their expiries, while sorted by tenor in ascending order, as illustrated on figure 10.

The coefficients for third eigenvectors illustrate that same expiry options with a 2Y tenor starting point has a negative entry but increases to a positive value when finishing at the 30Y tenor.

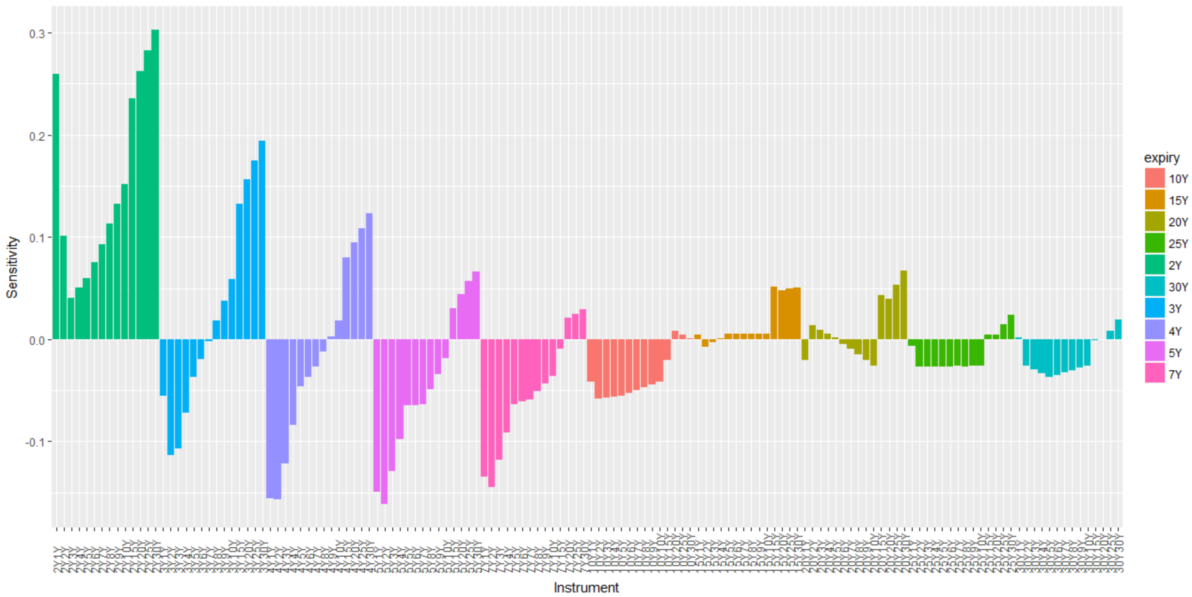


Figure 9 Third eigenvector of vega sector, USD surface, Date: 2015-07-03

The value of the weights is highest for shorter expiry options while long expiry options seems to less affected by changes in the steepness than their counterparties. A steepener trade must involve a short expiry swaption with a large negative weighting and a long expiry swaption with the same tenor, but with a positive weighting as we expect these two positons to go in opposite directions. It is a less structural pattern that can be observed compared to the previous eigenvectors, and 2Y expiry options only all have positive entries for all tenors. The 2Y expiries options doesn't capture the same dynamic as the rest of the expiry options and cannot be used in a relative value trade on the third principal component here. But as mentioned earlier, sensitivities changes over time and we find days where 2Y expiry options can be used in a relative value trade on the third principal component.

The 2Y1Y and 2Y2Y have extra high positive entries; this could be related to the gamma influence we mentioned earlier. If a relative value trade was interesting for the third principal component on the 3rd of July 2015, a position would eventually be taken on either 3Y, 4Y, 5Y or 7Y expiry options as these seems most suitable for a trade on the steepness of the volatility curves with same expiry.

As an example, if the sensitivities for 4Y3Y and 4Y15Y are 0.1219 and -0.0795, this means that if 4Y3Y falls with one basis point, the 4Y15Y swaption is expected to increase with 0.6525 basis points by dividing the two numbers.

Summary

The eigenvectors of three principal components of the vega sector for EUR market are exemplified in the appendix. They depict the same dynamics as pinpointed for the USD market.

Given this knowledge, we can conclude that the first principal component uses both dimensions, expiry and underlying, meanwhile the second and third principal components are dimension specific factors, explaining each a facet of curve movements. By diversifying the different dynamics, these finding can now be used to identifying trading opportunities but also for risk managers interested in hedging against specific risk exposures.

Scaled eigenvalues

The market mechanism of the implied normal volatilities during the sample period of 2010 to 2017 has shown significant commonality. Throughout the sample period, the first principal component explains over 81 % on average for EUR and on average 73 % for USD swaption matrix after introducing the two year expiry restriction. Interestingly, the first three principal components together explain around 98 % of the vega sector for the entire period investigated for both USD and EUR. Nevertheless, the three principal components for both USD and EUR appear to consistently explain practically all of the variance found in the implied volatility surface data.

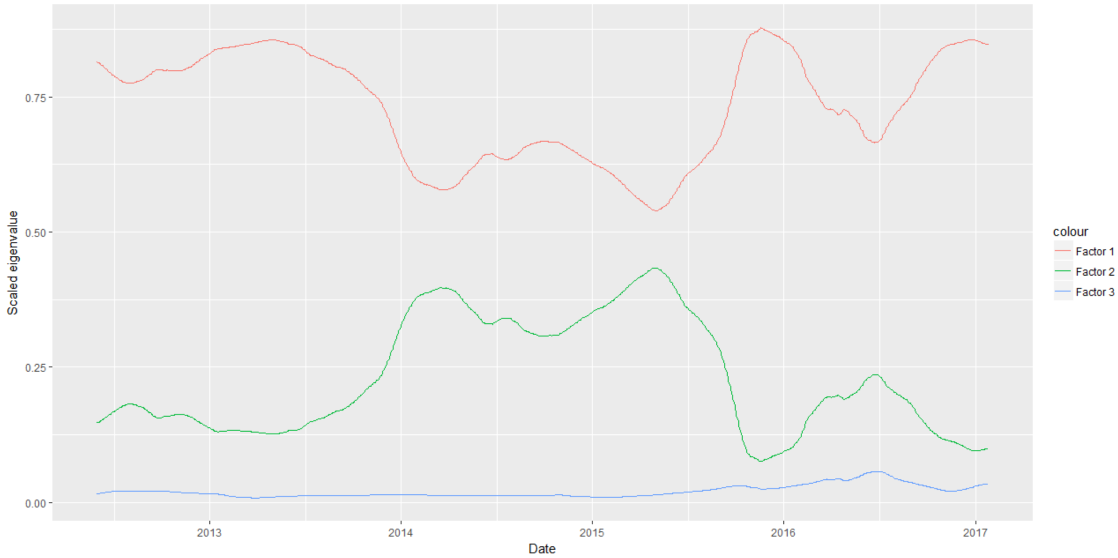


Figure 10 - Scaled eigenvalue on a PCA for vega sector USD

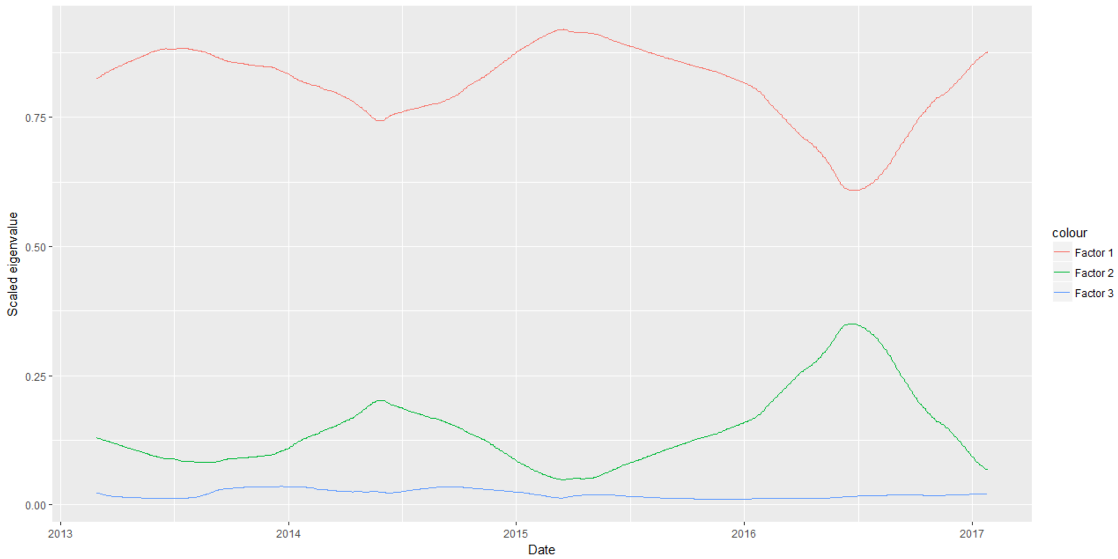


Figure 11 - Scaled eigenvalue on a PCA for vega sector EUR

The figures above reveals the structure of the implied volatility surface, sorted by scaled eigenvalue summing to nearly one each day. The scaled eigenvalue of the first principal component fluctuates much more for USD than EUR, but we see a decrease on both market around primo 2014 and again primo 2016 followed by an increase in Medio 2016. The distribution of scaled eigenvalue between the three first components often varies over time due to changes in the eigenvectors. These changes often occur due to macroeconomic events such as changing market regimes.

It is by far the first two principal components that explain almost all of the movements in the two datasets. Given the interpretation of the eigenvectors earlier, it must imply that most of the exposure for vega sector swaptions on the volatility surface is given by changes in the overall implied volatility level. Also, there is an inverse relationship between the first and second scaled eigenvalues for both EUR and USD, where a decrease in the first scaled eigenvalue will result in a similar increase in the scaled second eigenvalue. The third eigenvector are quite constant during the whole period for both markets. The second principal component explains around 7.9 % of the variance while the third component explains almost 3.5 % of the variance. It is difficult to interpret the fourth principal component and further as these often are difficult to identify as there percentage of the overall variance are so low. We are limiting ourselves to only three principal components as these accounts for most of the variance explained.

As the numbers, shown on the two figures above, are computed from a daily 630 days PC analysis, deviations in the explaining ratio are an excellent and robust test for warnings about model issues or even sudden changes in the behavior of the volatility surface.

From observing the sensitivities of the first eigenvector of the USD volatility surface from 1. June 2012 (first day of the 630 day period) and six month forward the interpretation of the first two eigenvectors are not as given in the section before. Here is the interpretation of the first eigenvector expressed as the steepness of volatility curves, with same tenor but different expiries, while the second eigenvector expresses the overall implied volatility level. This demonstrates a switching tendency between the interpretation of the first and second eigenvector. The exact same condition applies for the two eigenvectors during 2014. This is a clear signal of the eigenvectors being unstable in subperiods. Interestingly, this instability only occurs for the USD market, while all three eigenvectors are stable for EUR market.

The instability of eigenvectors is somehow understandable when dealing with instruments driven by different dynamics. If we divide between a mix of short expiry and long expiry options where longer expiry options volatilities not always are not affected in the same direction as the short end options that have larger dispersion. If the long end expiry and tenor instruments were excluded, eigenvectors would become more stable due to the similarity in the dynamics. This means an investigation of only long expiry options could also be applied, depending on the desired specifications. For example are pension funds more concerned with buying long term protection on their portfolio and participate in the long tenor and long expiry swaption market.

Clustering

As we discussed, the first and second have by far the largest scaled eigenvalues compared to the third eigenvalue for the vega sector. An interesting discovery can be found by comparing options sensitivities of the first two eigenvectors for the whole volatility surface. We found that the first two eigenvectors determines the majority of the shape for all instruments on the implied volatility surface. The eigenvectors given by a PCA can be used as a visualization tool for detecting clusters in the dataset. By investigating the positioning of the first two eigenvectors of the whole implied volatility surface and not only the vega sector, we discover clusters for both groups - short and long expiry options, as seen on figure 13. Notice how instruments with either long or short expiry tend to group together on the plot. These clusters support the decision of dividing options into a gamma and a vega segment, as we discussed in the section 'Distribution of the volatility surface'. PCA can commonly be used for identifying clusters within datasets. This is a powerful tool if we only need to consider a few principal components, as in this situation. A comparison with the sensitivity of the third eigenvector can also be applied for better understanding of the behavior of the different instruments, but seems less interesting as this component only describes a little part of the total variance in the data.

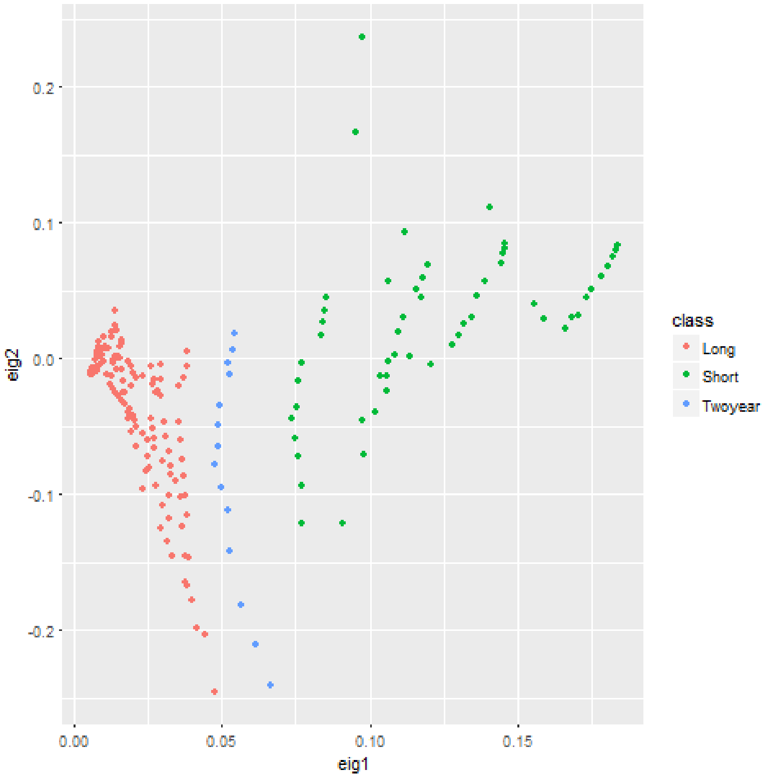


Figure 12 Cluster analysis of the whole USD volatility surface, Date 18 Oct. 2016

As seen on the figure above, a clear picture can be observed, where we introduces three classifications. The sensitives of the two eigenvectors are displayed on the axes. The green marks indicate swaptions with an expiry under two years, the blue ones are options with precise two year expiry and the red marks are options maturing in three years or more. On the right side of the figure are all short options placed, while a compact clusters for the long expiry options are shown on the left side. Two years options are placed in the middle of the two clusters but are more identifiable with the long expiry options cluster. The clusters appear clearly in each daily PCA of the whole surface for both USD and EUR.

Factor Residuals

The interpretation of the three first factors has now been considered and we will now focus on looking on the residuals by redefining factors as residuals.

From PCA, we start with n input variables/dimensions, and modelling this n factors will appear (k=n). No residual is outlined in the n factor PCA, however the unused factors as we only choose three factors, can be expressed as residual. Here

$$\begin{pmatrix} \varepsilon_1^t \\ \vdots \\ \varepsilon_n^t \end{pmatrix} = \sum_{i=k+1}^{n} \alpha_i^t \cdot \begin{pmatrix} e_{i1} \\ \vdots \\ e_{in} \end{pmatrix}$$

Our first look at a PCA looks like

$$\begin{pmatrix} y_1^t \\ \vdots \\ y_n^t \end{pmatrix} = \sum_{i=1}^{n} \alpha_i^t \cdot \begin{pmatrix} e_{i1} \\ \vdots \\ e_{in} \end{pmatrix}$$

But after adding the residuals it converts into

$$\begin{pmatrix} y_1^t \\ \vdots \\ y_n^t \end{pmatrix} = \sum_{i=1}^{k} \alpha_i^t \cdot \begin{pmatrix} e_{i1} \\ \vdots \\ e_{in} \end{pmatrix} + \begin{pmatrix} \varepsilon_1^t \\ \vdots \\ \varepsilon_n^t \end{pmatrix}$$

When investigating the factor residual, we are interested in one of the most fundamental part of relative value analysis, the indication of relatively rich and cheap instruments. If an instrument has high negative residual value, measured in basis points, this indicates that this specific instrument is relatively cheaper than the rest of the market. Meanwhile if an instrument has a high positive residual, this instrument is rich and therefore relatively more expensive than the rest of the market. Applying the factor residuals in this paper, we are able in identifying one or several instruments appropriate candidates for a chosen relative value trading strategy.

After moving rearrange the equation above, a one factor residual is calculated as

$$residual = Last\ observed\ value - Mean - Loading\ 1\ for\ each\ variable * Score\ 1$$

As the equation shows, the residuals can basically be regarded as the original projected data points minus the fitted data points. First we subtract the mean from the original data. By multiplying each coefficient in the factor loading with the equivalent factor score, we are able construct new coordinates as a fitted projection given the original coordinate system. This leaves us with the residual from a one factor model.

A two factor residual is calculated from the same principal as the first factor residual, here adding our projection from the second factor as

$$residual = Last\ observed\ value - ColMean - Loading\ 1\ for\ each\ variable * Score\ 1 - Loading\ 2\ for\ each\ variable * Score\ 2$$

A three factor residual is calculated as

$$residual = Last\ observed\ value - ColMean - Loading\ 1\ for\ each\ variable * Score\ 1 - Loading\ 2\ for\ each\ variable * Score\ 2 - Loading\ 3\ for\ each\ variable * Score\ 3$$

Factor residual can also be used to identify cheap trades expressed as a macro view. Residuals can pinpoint a cheap entry position for an investor in a specific instrument. As an example, if implied volatility falls, which instruments has fallen more relatively to others and therefore best to take a short position in.

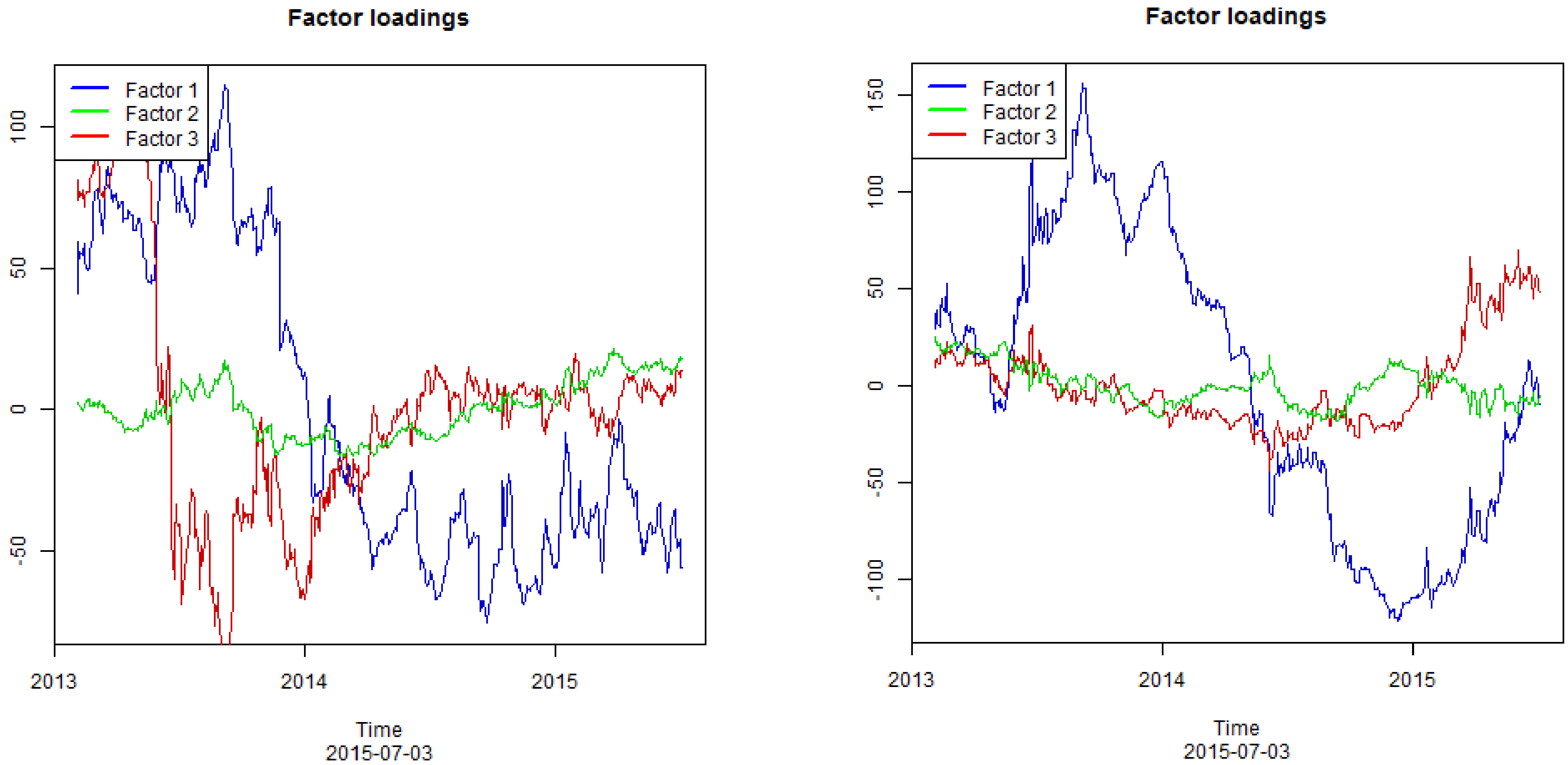
Correlation with market indicators

We can further explore the interpretation of the three principal components by creating a time series for these. This is done by using the factor scores of each component. A statistical tool such as PCA assimilates market dynamics by designing a backward view, but incorporating fundamental variables will enlighten us to visualize forward looking expectations. Let us here define fundamental variables as economic variables. As mentioned earlier, PCA composes mean-reverting uncorrelated factors which often can be interpreted though economic viewpoints such as business cycles etc. This is a strong property of PCA which enables us to investigate and understand the ever-changing dynamics of the financial markets by gaining a symbiotic relationship between economic and statistical analysis. The viewpoints are constructed by analyzing the correlation between the fundamental variables and the uncorrelated factor scores. We are therefore able to pinpoint potential future risks.

The two figures below illustrate the evolution of factor scores over the 630 day period for USD and EUR. The factors scores can be seen as volatility in bps as mentioned in the method section. The first principal components behavior is cyclical and moves within the same level in longer periods eventually reverting back to its mean. In the section in scaled eigenvalues, in Medio 2015 the second PC for USD percentage of total variance explained was around 40 % and almost explaining the same amount of total variance as the first principal component. We can also observe correlation between the first and second PC for EUR during subperiods. The third PC's fluctuates around its mean and moves smoothly throughout this period. All principal components are mean reverting but at a different pace.

From the figure below, we see the three factors scores over time. The interesting part is to determine if any of these factors should revert to its long term mean in the nearest future. The first factor often produces a low speed of mean reverting, which makes sense since this can be considered as a macroeconomic determined factor.

Figure 13 –Movement in factor scores for USD and EUR



We are now interested in exploring the principal components further, by comparing the scores with fundamental economic variables. As an example we will look at the correlation between principal components and fundamental variables. Firstly we consider a 630 days correlation on the 3rd of July 2015. Secondly we regard the absolute average of the rolling 630 days correlation for the entire period as well to identify consistent market interactions with the principal components. Absolute average is chosen as some indicator varies in being negatively or positively correlated to the components. Calculating with a regular average can compose result with small correlation in these situations

One of the chosen fundamental variables is the VIX index, which is regarded as an investors 'Fear gauge'. It is an index that measures the implied volatility for the current stock market index of S&P 500. Given this index, we can investigate if there is any relationship between equity market volatility risk and the principal components of the implied volatility surface. The following stock markets are included as fundamental variables; DAX, NASDAQ, S&P 500 (SPX) and Euro Stoxx 50 (SX5E).

The generic MOVE index is also introduces. This is constructed by Merrill Lynch and measures a weighted average[9] of ATM normalized implied volatility of the yields to expiry of long term (maturities of two, five, ten and 30 years) Treasury options over a one month period. Other notable fundamental variables are Crude Oil, German and American government bonds, 5Y swap rates on EUR and USD. We would also like to compare the dynamics to two other huge swaptions markets. Here Japan and United Kingdom, so 5Y5Y swaptions volatilities are also included for JPY and GBP.

If the third principal component is uncorrelated with chosen macroeconomic variables this could indicate that this is a "pure" relative value factor, meaning it is dictated by a statistical component rather than macroeconomic changes. Equally factors are often more uncorrelated to these variables due to the fact that higher number factors are more driven by statistical analysis than by macroeconomic events. So if we could give a reasonable interpretation of the fourth principal component, this could be used as a relative value factor.

The table below shows that the first USD principal component is highly correlated with several economic variables on the 3rd of July 2015. Seemingly, there is a negative linkage with all stock indexes used, which we mentioned above. This can

[9] The average weighting is set as 20 % two-year, 20 % five-year, 40 % ten-year and 20 % 30-year bond. This weighting is chosen by estimates of option trading volumes for each of the different maturities.

be compiled to a high negative correlation with USD stock markets and EUR stock markets, where correlation is most significant with the USD stock market. The situation is actually the same for the first principal component for EUR. In the case of absolute average rolling correlation, USD stock markets tends to be better at explaining the most dominate dynamic on the volatility surface for both EUR and USD swaptions compared to EUR stock markets. This states that these two components have a high linkage toward essential risk variables.

Importantly, the first principal component for both EUR and USD are highly positive correlated to the overall level of underlying swap rates in their denominated currency. We see that the first principal component for USD is correlated the overall level of the implied volatility for JPY swaptions.

We see that the second USD principal component is highly correlated to the US Generic Government 5Y and 10Y bonds on this day, but less correlated over the entire period. Moreover there is a high correlation between the second principal component on EUR and the foreign exchange rate between EUR/USD. The second factor for USD can be considered as a less definite macroeconomic factor as correlation is high at the current date, but is overall at a lower level on average for USD, while the second principal component for EUR is clearly more affected by market indicators consistently.

Interestingly, crude oil future prices and PC3 are highly negatively correlated on the observed date, but in general the differential between tenors are unaffected by fundamental variables. The MOVE index signals interest rate movements as uncertainty change in the bond markets, and are overall fairly correlation to the three principal components for USD. The index is however not an independent fundamental variable when comparing to the dynamics of the implied volatility surface. The MOVE index is widely used as an indicator to proxy tensions in fixed income markets (inflation, deflation and concerns regarding government debt) and could be loosely considered as a liquidity indicator, which affects all dynamics found for USD.

Looking at the 630 day rolling correlation for the entire period, taking the mean of the absolute correlation for each market indicator towards the third principal component, we observe an overall lower level of correlation for all outcomes. Some of the crosses have a significant correlation to EUR or USD for the whole period, but all at level lower than 50 %. We therefore roughly postulate that the third principal components as a purely statistical component. This gives some possibilities in constructing relative value trades, as this component have relatively little exposure towards market indicators and therefore market directions. This is nice feature in a trading perspective, as this shows that taking a position on this component, performance will not be affected by macroeconomic trends or changes. Statistically, macroeconomic exposure will rises by trading on the second principal component and even more if a position is made on the first principal component.

We see the first principal components for USD and EUR are highly correlated with a value of 0.737 measured on 03-07-2015. Overall for the entire period, for these components are not as correlated compared to this date. The third principal component for USD and EUR are not significantly correlated to any other principal components, which isn't surprising.

By using the times series of the factor scores on the daily PCA as input for an Ornstein-Uhlebeck process we are able to assess the speed of mean reversion for each factor. Here, a much higher level of speed of mean reversion is observed for the third factor than for the second and first factor. The specifications for speed of mean reversion, variance, days to converge 50 % and 90 % to the mean can be found in the appendix. This will be introduced further in the strategy section on trading signals.

Table 1 - 630 days rolling correlation for USD and EUR factor on 03-07-2015 and absolute mean for entire period

Correlation on 03-07-2015

USD	VIX	SPX	OIL	5Y US Gov	10Y US Gov	DAX	SX5E	MOVE	10Y GER Gov	5y5y GBP	5y5y JPY	EURUSD	USD 5Y swap rate	EUR 5Y swap rate	NASDAQ
Factor 1	0.034	-0.863	-0.432	0.626	-0.001	-0.690	-0.728	0.199	0.599	0.245	0.652	0.315	0.605	0.718	-0.830
Factor 2	-0.126	-0.176	-0.178	-0.716	-0.799	-0.156	-0.155	-0.482	-0.317	-0.758	-0.141	-0.186	-0.721	-0.365	-0.213
Factor 3	0.223	0.340	0.680	-0.004	-0.452	0.435	0.369	0.732	-0.614	0.169	-0.429	-0.841	0.070	-0.468	0.378

Absolute average rolling correlation

USD	VIX	SPX	OIL	5Y US Gov	10Y US Gov	DAX	SX5E	MOVE	10Y GER Gov	5y5y GBP	5y5y JPY	EURUSD	USD 5Y swap rate	EUR 5Y swap rate	NASDAQ
Factor 1	0.203	0.596	0.408	0.731	0.743	0.398	0.526	0.423	0.658	0.543	0.621	0.531	0.762	0.646	0.608
Factor 2	0.393	0.324	0.337	0.432	0.434	0.148	0.392	0.472	0.403	0.426	0.273	0.279	0.380	0.415	0.355
Factor 3	0.182	0.309	0.283	0.081	0.167	0.426	0.338	0.435	0.217	0.169	0.171	0.432	0.109	0.202	0.305

Correlation on 03-07-2015

EUR	VIX	SPX	OIL	5Y US Gov	10Y US Gov	DAX	SX5E	MOVE	10Y GER Gov	5y5y GBP	5y5y JPY	EURUSD	USD 5Y swap rate	EUR 5Y swap rate	NASDAQ
Factor 1	-0.105	-0.715	0.669	0.155	0.564	-0.536	-0.546	0.083	0.881	0.534	0.809	0.593	-0.174	0.948	-0.657
Factor 2	0.062	0.117	-0.562	-0.292	-0.472	0.388	0.309	0.474	-0.399	0.203	-0.113	-0.711	-0.259	-0.254	0.190
Factor 3	0.074	-0.544	0.035	-0.703	-0.511	-0.510	-0.547	-0.074	0.086	-0.377	0.281	0.041	-0.694	0.130	-0.574

Absolute average rolling correlation

EUR	VIX	SPX	OIL	5Y US Gov	10Y US Gov	DAX	SX5E	MOVE	10Y GER Gov	5y5y GBP	5y5y JPY	EURUSD	USD 5Y swap rate	EUR 5Y swap rate	NASDAQ
Factor 1	0.372	0.490	0.267	0.181	0.318	0.507	0.414	0.428	0.609	0.556	0.491	0.425	0.253	0.729	0.539
Factor 2	0.575	0.379	0.371	0.622	0.754	0.534	0.501	0.355	0.579	0.417	0.386	0.739	0.622	0.415	0.537
Factor 3	0.257	0.257	0.298	0.261	0.186	0.304	0.245	0.282	0.133	0.273	0.262	0.162	0.233	0.159	0.324

Correlation on 03-07-2015

USD\EUR	Factor 1	Factor 2	Factor 3
Factor 1	0.737	0.159	0.456
Factor 2	0.396	0.219	0.554
Factor 3	-0.326	0.696	0.038

Average correlation for entire period

USD\EUR	Factor 1	Factor 2	Factor 3
Factor 1	0.450	0.575	0.257
Factor 2	0.490	0.380	0.256
Factor 3	0.267	0.371	0.298

The correlation between the external variables and the principal components evolves as the relationship changes. Monitoring rolling correlation is therefore needed to be considered before trading on any of the principal components.

From the start of the investigated period, we are able to register an overall very high negative correlation between the first factor and SPX index but this correlation decreases over time to a moderate level. Meanwhile the commodity Oil moves with a small margin during this period at a moderate level of correlation. The graph below helps in identifying the first factor as highly correlated to risk variables such as commodity and stock prices denominated in USD. By following up on this, factor 1 - stated as the overall volatility level behaviors in both dimensions are regarded in the same manner as some very risky assets. Rolling correlation between the first principal component and SPX fluctuates highly and when correlation is low in 2014, the second principal component is highly correlated with the SPX index. The transition happens at the exact same time we encounter instability between the first and second principal component.

Figure 14 - Rolling correlation between USD PC's against SPX and OIL

At most, we illustrate that external data regarding macroeconomic trends and events are helpful in explaining movements in the principal components and also gaining valuable forward looking information as part of the analysis. This information can be used to forecast future behaviors of the first and second principal components and visualize future expected risks.

Applying the setup

Hedging with very liquid assets

With the discovery of high correlation between factors and several fundamental variables interesting possibilities arises. Typically, an investor would hedge a portfolio of swaptions with swaptions. This can be a quite expensive hedge. Instead if we could use fundamental variables to reduce exposure against the dynamics of the first principal component, as this would definitely be a cheaper alternative. The most ideal hedging assets have the following properties; they are cheap to take a long or short position, easy access and liquid market for the security and highly correlated to the risk exposure (here the three factors).

Here could one of the USD stock market indexes, SPX or NASDAQ, be used as hedging asset. We witnessed negative correlation between the two indexes and the first principal component and would therefore buy one of the indexes as a hedge.

We will now introduce a multiple regression on principal components for hedging different exposures by using liquid assets such as equities, currency crosses and fixed income bonds. Testing different combinations, only a few variables are considered and our goal is to find a solution with a high adjusted R squared value. A bias on finding variables denominated in same currency is created for not introducing currency risk. As a setup, we are interested in a dynamic hedge, where we are rebalancing the hedge portfolio over time. As principal components are uncorrelated, due to the orthogonal eigenvectors, any position can be regressed directly to each of the found risk factors on the implied volatility surface. The factor scores for the first principal component are carried out as the dependent variable in the regression analysis.

So far we have only worked with level data, but differenced data is preferred for risk management purposes, as focus here lies in capturing short term dynamics and rebalancing the hedge over a period. Level data are however better at capturing long term structural behavior of relationship between the implied volatilities. Statistically, there is a loss in information regarding level impact if differenced data is used. But non stationarity are a concern when choosing level data, especially for the first principal component, where we will apply the factor score to a multiple regression analysis. Here is differenced data naturally more stationary and not a concern. Therefore is differenced data applied instead, as this gives the craved prerequisites for the hedge. Applying a hedge on differenced data on a short dataset is useful for short term hedging purposes. However for long term hedging circumstances, level data on implied volatilities could be an option.

We are only interested in hedging the first principal component as this explains on average 75 % of the total variance in the data for USD. Given the scaled eigenvalues during the investigated period from 2010 to 2017 we can see that the first factor is a dominate parameter in explaining the majority of the movements in the implied volatility surface. A hedge against the first principal component will give a non-directional position against shifts in the overall implied volatility surface.

Weekly changes are composed from Wednesday to Wednesday as prior to avoid effects from Monday opening and Friday close. These changes can be expressed as linear combinations of the form

$$xxYxxY(\sigma_N(T) - \sigma_N(T-1)) = \boldsymbol{e}_p^T \boldsymbol{X} = e_{p,1}X_1 + \cdots + e_{p,p}X_p$$

where $xxYxxY(\sigma_N(T) - \sigma_N(T-1))$ is the absolute difference in implied volatility for a specific instrument $xxYxxY$ from time $T-1$ to T. Here are $\boldsymbol{X}$ the uncorrelated principal components with the factor loadings or weights $\boldsymbol{e}_p^T$.

When testing the duration of the length of weekly changes, by increasing the interval, our attempt to regress the first principal component against fundamental variables results in a lower degree of rolling adjusted R-squared value. As the weekly changes are differenced data, PCA performs better for short datasets when regressed against several fundamental variables. This corresponds to similar observations from "Market Musings"[10] report on Relative value across the US swap surface. Intuitively this makes sense as differenced data capture short term changes in implied volatility dynamics and short datasets provide such information. From a hedging perspective, traders often want to eliminate some exposure on their trading book against short term changes in the implied volatility dynamics, so we are using weekly change data and only concentrating on a short historical view.

The computed factor scores, used for the rolling multiple regression analysis, are settled from a PCA on weekly changes for a 120 days period. The rolling correlation between the factors scores of the first principal component and the fundamental

[10] TD Securities (2015), "Market Musings — Relative Value Across the U.S. Swap Surface: A PCA Approach"

variables discussed earlier, are not as significant as with level data. However several of the fundamental variables have been transformed into differenced data as well. As stated below, NASDAQ, 5 year USD swap rate and VIX index.

$$y = \beta_1 \Delta(USSW5) + \beta_2 \Delta(VIX) + \beta_3 \Delta(NASDAQ)$$

where y represents PC 1 scores, β_1 is the amount of 5y USD payer interest swap , β_2 is the amount of VIX futures and β_3 is the amount of NASDAQ index stocks used to produce a linear hedge on changes in the overall implied volatility level, which is the PC 1 scores. NASDAQ and SPX are almost similar, but NASDAQ was a slightly better fit here. Important to know is that the different securities can be traded in both a long and a short position.

After converting from level data to differenced data and reducing the period to 30 weeks instead, the first eigenvector for USD is now stable throughout the entire period. This tell us that, over a short term, risk managers main focus should be in reducing the exposure toward the overall implied volatility level.

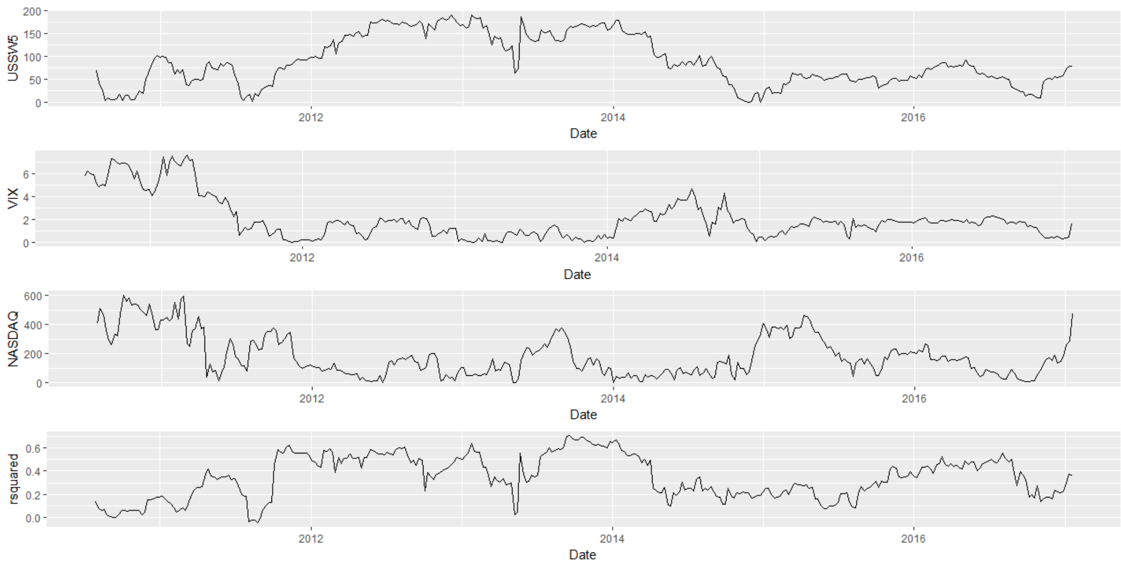


Figure 15 - Rolling beta coefficients on 5y swap rate, VIX and NASDAQ to first principal component scores. Last figure shows rolling adjusted r squared.

The beta coefficients are not stable during the whole period and expresses against scores in bps, therefore all coefficients should be divided by 100. Some subperiods however are the hedge nonexistence as the r squared value close to zero. By converting from level data to differenced data and only investigating a 30 week period, eigenvectors are stable during the entire period, where the three eigenvectors clearly depicts the interpretation given earlier.

Finding several numbers of liquid hedging assets in the market with adequately significant correlations to the first principal component - implied volatility level risk can be reduced additionally with a multiple regression. From a statistical point of view, selecting more hedging assets will boost the coefficient of determination (r-squared) closer to one.

The simplicity of the method using r-squared should be considered in a multiple regression when choosing more than one hedging asset. Choosing more than one will introduce a risk of multicollinearity. This means that hedging assets or independent variables are in risk of not being entirely independent. As it is commonly known markets can be more or less correlated. The risks of introducing multicollinearity can therefore be quite substantial when applying several hedging assets in one regression. To reduce this problem we can focus on maximizing the adjusted R squared when applying different hedging assets. The adjusted R squared differs from the simple R squared, because it adjusts for degrees of freedom. By focusing on maximizing the adjusted R squared hedging assets will only be allowed in a multiple regression if it makes a noteworthy contribution to the value of the adjusted R squared.

The coefficient of determination expresses the percentage of risk possible to eliminate by trading with the hedging assets. So this indicates how effective the linear hedge is by using VIX futures, NASDAQ index and 5 y USD swap rates.

The adjusted R squared value range around 0.2 and 0.51 75% of the time, however the distance between minimum and maximum shows great inconsistent for a linear hedge against the changes in the overall implied volatility if a weekly hedged is chosen.

Min	1st Qu.	Median	Mean	3rd Qu.	Max.
0.00099	0.2015	0.3501	0.3476	0.5113	0.7033

This shows us that some of the directional exposure can be dynamically hedged cheaply with stocks, VIX futures and swaps. However the fluctuations in the r squared value varies much and this indicates the multiple regression don't generate the presumed hedge for the entire period. Other hedging intervals can be used to perhaps find a more significant result.

We need however to acknowledge a 30 weeks factor scores presented as the independent variable, possibly aren't extremely robust. PCA is sensitive to outliers since the principal components are constructed from a covariance matrix, which is highly sensitive to outliers. Sudden jumps in the implied volatility surface could leave us with a rigid result.

Turning the regression around between explanatory and response variables, we have an asset portfolio consisting of equity and bonds instead, where we could hedge downside risk by taking a position in a swaption. Here as a hedge spanning over multiple asset classes. This can be an efficient way to hedge risk exposure such as volatility risk during volatile periods.

A portfolio of swaption exposure towards changes in the implied volatility level will be reduced to some extent, by dynamically hedging against the first principal component scores with the dependent variables used in the regression. As these variables are asset that are considered very liquid and can be seen as cheaper hedge alternative relatively to hedging with swaptions. The hedge is however non-existent in subperiods.

Strategies for relative value trades

Overall, there is a great existence of different volatility trades. The most common strategy is by selling options and dynamically delta hedging the exposure towards the underlying asset. This is a play for an investor to profit from market known feature, where implied volatility tends to exceed realized volatility.

In relation to the vega sector for swaptions, constructing a relative value trade is recognized as vega trading. The pricing of ATM options mainly depended on changes in the implied volatility as expiry increases. As vega increases with expiry, decreases gamma. If gamma is low for an option, changes in the underlying have no significant effect on delta. This implies that delta hedging has no impact on the P&L for vega sector swaptions.

Due to the low exposure to changes in the underlying for straddles in the vega sector, this allows for conceptual base the volatility surface with a statistic tool as PCA and therefore hedge against the implied volatility level, from the interpretation of the first principal component, instead of delta hedging. Given the knowledge of the dynamics of the implied volatility surface after an interpretation of the eigenvectors for the three principal components, three different kinds of trades can be constructed.

Let us summarize the different exposures from different types of option trades. Firstly, trading with one instrument gives an exposure to the absolute level of the implied volatility. Secondly, trading with at least two instruments gives a hedge against the exposure to the absolute level of the implied volatility and an exposure on an implied volatility curve.

Daily run of mean reverting process for the three principal components to find opportunities by evaluating the speed of mean reversion. After choosing the desired position on one of the principal components, we display the relevant residuals. The residuals expresses if an option is relatively cheap or rich and therefore helps identifying assets from a relative value viewpoint. A larger difference between the rich and cheap instrument will signify a profitable opportunity. If the trade consists of two instruments, a relevant hedging ratio is calculated. Lastly, the position will be unwound after a shorter holding period.

Salomon Smith Barney (2000) shows that trading signals differs for level data and change data. The timing of trading signals is different for the two approaches. This is however applied by tracking the P&L for butterfly trades on swap rates and not for swaptions. Given these observations, they choose level data for timing of trades as these produces generally higher Sharpe ratios. We will however only focus on level data for trading purposes.

First PC

Given a position in a single instrument is susceptible to a shift in volatility surface level. If high speed of mean reversion was observed for the first principal component and this is far away from the mean, we can trade on a change in the direction of the implied volatility surface. Such a situation is however rather unlikely given the outlined properties, mainly driven by macroeconomic fundamentals. But perhaps this relationship can be exploited. If a high correlation between the factor score and 5Y swap rate is observed and we anticipate the swap rate to move in favor of the mean reversion of the principal component very soon this could be an opportunity of a profitable trade. As directional positions normally have a low speed of the mean reversion, investors need to be patient and expect longer holding periods. Or else attractive relative value trade on the first principal component seems diluted.

The first principal component is the main interest factor from a risk management perspective; meanwhile the second and third principal component is the main interest for relative value trading opportunities as they serve a higher speed of mean reversion.

Second PC

As the second principal component represents the dynamics of the implied volatility curves for dimensional equivalent tenors, a positioning for a change in the slope could be considered. Here either by anticipating a curve steepener or flattener. If a relative value opportunity suddenly appears, an investor could take a position by selling a rich and buying a cheap swaption with same tenor and different expiry. The rich and cheap assets can be identified for the one factor residuals. Before an investor trade, the second eigenvector sensitivities of the two instruments have to be moving in different directions when trading on changes in the slope. The investor isn't exposed the direction of the implied volatility level, but are exposed to the dynamics of the third principal component. The exposure should be hedged if the scaled eigenvalue of the third principal component is high.

Third PC

Last strategy takes a view on the implied volatility curves, with the same expiry options but with different tenors. Trading on the tenor dimension is also characterized as a position on the slope. So here is a curve steepener or flattener also considered. Here rich and cheap instruments are selected from a two factor residual. For example, buying a 10Y2Y straddle and selling a 10Y10Y straddle and perhaps unwind after one month, the position would be a 10Y2Y minus 10Y10Y implied volatility position, with almost none exposure towards curve steepness and towards realized volatility. The weighting of the eigenvector for the third principal components should always be checked before entering a position, in the sense as trading on the second principal component.

Implied volatility curves set on the same tenor tend to be stable in the longer tenors. This is captured by the sensitivities for the second eigenvector. Here were weights close to zero for the longest tenors. This gives a picture of trading on long tenor curves doesn't seem to be an attractive choice.

Optimal holding period

The upper limit of the holding period is set to one month. A short holding period is important as we can postulate that time decay of theta and changes in the underlying swap rate will not influence the P&L for a chosen position. By only trading with straddles a non-directional position is taken on movements in the underlying tenor. We are able to hedge against the first principal component while not being concerned with delta hedging as this is not required. This allows a relative value trader, maneuvering in the vega sector, to focus on hedging exposures on principal components instead.

For longer holding periods, large changes in the underlying can introduce an unhedged jump risk in the P&L. The holding period should therefore be short so straddle returns isn't affected by both volatility and jump risk.

As mentioned before, theta is low for swaptions in the vega sector, but if holding periods becomes very long, theta will begin to matter. This is primarily the case for positions on the second principal component where theta is very different for the options with same tenor but different expiries.

We are only able to calculate returns on relative value trades as P&L for changes in implied volatility for any strategies. The implied volatility for ATMF swaptions on a traded date will be compared to implied volatility for new ATMF swaptions at the end of the holding period. We are aware that the underlying swap rates changes but is assumed to be ignored due to the very short holding period and no precise way to determine the value of a "old" non ATMF swaption straddle.

Hedging of factors

As the principal components can be viewed as risk factors, hedging ratios can be calculated to reduce exposure towards these risk factors. This can be done by using linear algebra by applying both eigenvalues and eigenvectors.

We want an optimal hedge ratio between two instruments. Whereas x and y are the two instruments traded, being long in x and short in y. Here can x and y be expressed as linear combinations of factor loadings and scores in a PCA.

$$x_t = c_t^1 e_{x1} + c_t^2 e_{x2}$$
$$y_t = c_t^1 e_{y1} + c_t^2 e_{y2}$$

Here are x_t and y_t implied volatility levels of our two chosen instruments. These are vectors as they are time series. c_t^1 and c_t^2 are the factor scores for the first and second principal component. Both scores are given as time series vectors. $e_{(x,y),(1,2)}$ is the factor loadings for the two instruments for each direction and is given as numbers.

A solution for the minimum variance function[11] w.r.t γ can be found as

$$\min_{\gamma \in \mathbb{R}} Var(x - \gamma y)$$

Where γ is defined as the hedge ratio. So hedging against the first principal component, the sensitivities for the first eigenvector of two instruments are used. We find the hedge ratio as

$$\gamma = \frac{e_{x1}}{e_{y1}}$$

Where γ is the amount of instrument x while the amount of y is one, e_1 is the first factor sensitivity for the specific instrument.

While hedging against both the first and second principal components, we find the hedge ratio as

$$\gamma = \frac{e_{x1} e_{y1} \lambda_1^2 + e_{x2} e_{y2} \lambda_2^2}{e_{x1}^2 \lambda_1^2 + e_{y1}^2 \lambda_2^2}$$

Where λ_1^2 is the variance for the first principal component and λ_2^2 is the variance for the second principal component. γ is the amount of instrument x while the

[11] PCA unleashed

amount of y is one, $e_{\#}$ is either the first or second eigenvectors sensitivity for the specific instrument.

An example of a trading opportunity

For our construction of trade ideas, we have used the time series of all three principal components scores for EUR and USD on each day from 2012 to 2017 created from our daily PCA, to identify days with highest speed of mean reversion. A maximum likelihood estimation method is used for producing the parameters of an Ornstein-Uhlebeck mean reverting process. This gives a possibility signaling trading opportunities by calculating speed of mean reversion of the computed principal components scores. This gives us a statistical reason in deciding which principal component we should investigate further on a specific date. This is illustrated by the trading signal tables given in the appendix. The 06-10-2015 is revealed as a day with a relatively high speed of mean reversion on the second principal components scores for USD and the current factor score point is also far away from its mean. This day is chosen as this is the first day in a six day period with high speed of mean reversion. Moreover the expected count of days for the process to converge to its mean is 19.4 days[12] with a 50 % probability. This seems like a reasonable opportunity given the 21 days holding period setup explained earlier.

Taking a position on the second principal component, a long and short position in two instrument with the same tenor but different expiries will be considered. As described in the interpretation of the second eigenvector, slope trades are only interesting for short tenors because instrument sensitivities for longer tenors all moves in the same direction. Swaptions with a 2Y tenor is listed in the table below as this implied volatility curve reveals the largest differences for the one factor residuals. The table also includes the sensitivities for each instrument towards the first and second eigenvector. Lastly, implied hedge ratios against the first principal component are calculated from the minimum variance framework for each instrument.

[12] See trade signals in the appendix.

Table 2 – Properties of the PCA for trade date 06-10-2015 on USD volatility surface.

	2Y2Y	3Y2Y	4Y2Y	5Y2Y	7Y2Y	10Y2Y	15Y2Y	20Y2Y	25Y2Y	30Y2Y
					One Factor Residual for USD					
Bp	-9.037	-6.487	-3.814	-3.050	-2.197	-1.469	-0.083	1.491	0.545	0.090
					Sensitivity of the First Eigenvector					
Weight	-0.013	-0.084	-0.109	-0.110	-0.091	-0.055	-0.061	-0.056	-0.069	-0.073
					Sensitivity of the Second Eigenvector					
Weight	-0.282	-0.217	-0.141	-0.093	-0.042	0.020	0.061	0.078	0.071	0.066

Implied Hedge Ratio against First PC

	2Y2Y	3Y2Y	4Y2Y	5Y2Y	7Y2Y	10Y2Y	15Y2Y	20Y2Y	25Y2Y	30Y2Y
2Y2Y	1.000	0.152	0.117	0.117	0.140	0.232	0.208	0.227	0.186	0.175
3Y2Y	6.587	1.000	0.770	0.768	0.925	1.526	1.372	1.493	1.223	1.154
4Y2Y	8.558	1.299	1.000	0.998	1.202	1.983	1.783	1.940	1.590	1.499
5Y2Y	8.579	1.302	1.002	1.000	1.205	1.988	1.787	1.945	1.593	1.502
7Y2Y	7.118	1.081	0.832	0.830	1.000	1.649	1.483	1.613	1.322	1.247
10Y2Y	4.316	0.655	0.504	0.503	0.606	1.000	0.899	0.978	0.802	0.756
15Y2Y	4.801	0.729	0.561	0.560	0.674	1.112	1.000	1.088	0.892	0.841
20Y2Y	4.412	0.670	0.516	0.514	0.620	1.022	0.919	1.000	0.819	0.773
25Y2Y	5.384	0.817	0.629	0.628	0.756	1.247	1.121	1.220	1.000	0.943
30Y2Y	5.710	0.867	0.667	0.666	0.802	1.323	1.189	1.294	1.061	1.000

The largest residual difference is between the two instruments 2Y2Y straddle and 20Y2Y straddle. 20Y2Y has the highest positive residual indicating a cheap trade. There are several possible rich trades with high negative residual such as 2Y2Y, 3Y2Y, 4Y2Y etc. From the sensitivities of the second eigenvector we expect the 2Y-7Y expiry options to decrease and 10Y-30Y expiry options to increase when trading on the slope of the 2Y implied volatility curve. Buying a 20Y2Y straddle and selling a rich trade straddle, with a hedge against the first principal component, will give the exposure of implied 20Y2Y volatility minus an implied xY2Y volatility position with very little exposure to realized volatility.

Importantly, the second principal is only significantly correlated with the 5Y USD government bond on the trading day, which nevertheless isn't an indicator of risk. This means that the factor can currently be characterized as a statistical component and will not include any macroeconomic exposure in this situation, however over time, the current state would most likely change. If we reflect on our earlier observations on rolling correlation with fundamental variables for the two first principal components, a change seems rather plausible.

From the figure below we see the implied volatility curve for 2Y tenor, for three different days; on trading day, 21 days before and day of unwinding (21 days after trade day). It appears that short expiry options implied volatility decreases over time.

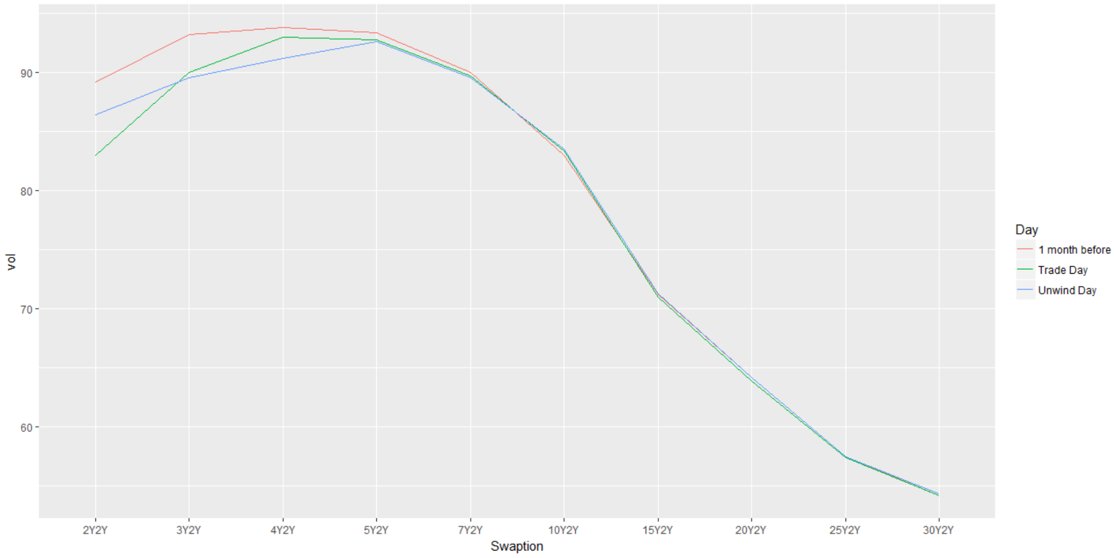


Figure 16 – Changes in the term structure of the 2Y tenor in a two month spectrum

20Y2Y has gained 0.3 bps 21 days after the trade, while all volatilities for the shorted positions except for 2Y2Y have decreased.

For the position of selling 3Y2Y and buying 20Y2Y, the hedge ratio can be found in the table above. Columns are considered as one unity, while lines are considered as beta unity. By buying 20Y2Y and hedging against the overall implied volatility level, 1.493 units 3Y2Y are needed. If we unwind the position after a holding period of 21 days (one month), we would profit 0.74 bps.

P&L can be measured as the change in the implied volatility position as stated just above. Given the different combinations, P&L can be found as

Buying			20Y2Y		
Selling	**2Y2Y**	**3Y2Y**	**4Y2Y**	**5Y2Y**	**7Y2Y**
P&L in bp	-0.347	0.740	2.848	0.583	0.417

We see all positions profits except for a short 2Y2Y straddle. Notice that the weight of the first principal component for 2Y2Y is -0.013, which is close to zero, could possibly be influenced by gamma. Given this information, one could consider to exclude 2Y2Y as this position bears a lot uncounted risk.

Pitfalls of a PCA trade

In this section, we will illustrate an example of a relative value trading opportunity, which eventually can't be considered to be an opportunity due to an unhedgeable first principal component.

From the figure below, we see that the first factor scores for USD in a PC analysis have decreased in 2016 from late summer until late autumn. This decrease can be explained by an overall fall in the implied volatility level for this period. By comparing implied volatility values for all instruments from the entire period to the levels in autumn 2016, implied volatility for all instruments are at its lowest level since mid-2013. However this changes in mid-November 2016, where we observe a turnaround in the trend of the first factor. Here increases implied volatility for all swaptions substantially. This change is happening exactly the days after the presidential election in USA.

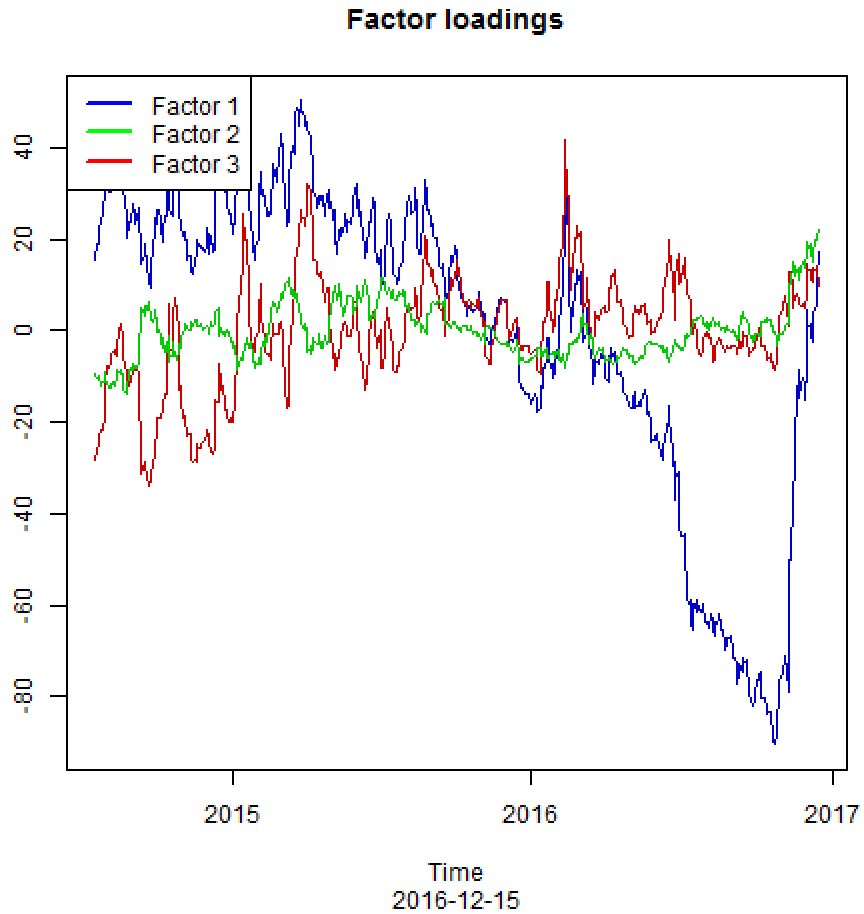


Figure 17 – Time series of factor scores for USD

We find speed of mean reversion relative high for the second principal component on 15th of December 2016. However significant correlations between all three principal components are observed since late November 2016, which is also observable on figure 19. This is a problem for relative value trades on either the second or third principal component. Hedges are very likely to break down during this subperiod. A trade on the second PC with a hedge against first PC would not work and therefore be exposed as well to the first PC during this period. So performance of this trade would be driven by direction. Generally correlations between principal components are zero for the whole analyzed timespan, but sometime correlation can occur during subperiods. This is a major pitfall within the PCA framework on relative value trading. Therefore should relative value traders consistently monitoring principal component correlation before entering a trade. This will reduce the risk of setting up a trade with a directional exposure or else can such trades eventually lead to significant loses over time.

Correlation between principal components is not an issue from a risk management perspective. Firstly, a manager wants to reduce the risk in first principal component as this exposure is the most dominate risk. If this is done with other asset classes, as we modelled earlier, a positive correlation between the first principal component and another principal component will automatically also be a hedge against the exposure of other risk factors in these subperiods. This will overall reduce the exposure against multiple dynamics of the implied volatility surface unintentionally.

The main pitfalls of trading options with the assistance of a statistical tool such as PCA, is if we compare this to a one dimensional yield curve PCA setup, the structure of the principal components has a more clear structure for the yield curve. Both the second and third principal component of a PCA on the EUR and USD volatility surface signifies curve steepness, but within different dimensions. Going back to the one dimensional yield curve, the second principal component represents all information regarding curve steepness, while the third principal component is less influential and denotes all information on curvature in the exact same dimension. This can result in that higher order eigenvectors on the volatility surface tends to be less stable compared to the one dimensional yield curve. The instability of the first and second eigenvectors for USD is somewhat a perfect example of this.

Conclusion

Throughout this paper, we have investigated the relative valuation in the interest rate options markets. We have focused on implied volatility dynamics for long expiries, with at least two year. The result from our empirical analysis of the EUR and USD market from 2010 to 2017 identifies three different dynamics offsetting the movements on the implied volatility surface. We have found strong evidence that the implied volatility surface for EUR is mostly influenced by a directional market mechanism. This is however not the case for the USD market, where the most influencing mechanism varies between a directional dynamic and a steepness of the volatility curves for same tenor's swaptions. Overall we have evidence of three significant dynamics on the implied volatility surface explaining over 95 % of the total variance for both USD and EUR markets over the entire period. This is a consistent result over the entire period from 2010-2017. In the analysis we have found three significant dynamics represented on the implied volatility surface. The first dynamic are interpreted as an overall implied volatility level factor. The two other dynamics captures the implied volatility curves, each specifying a dimension on the implied volatility surface.

By linking the principal components with economic and fundamental variables, we have gained valuable insight into identifying important macroeconomic drivers for implied volatility, which can help predict future movements of each dynamic. This is a strong property to PCA, which is based on historical data and can therefore normally be viewed as a backward-looking method. We found that the movements of the first principal component for both USD and EUR are highly correlated with the level of the US stock markets.

The paper has provided evidence of specifying a hedge against the first principal component for USD using a multiple linear regression. Here we repeat a PCA on differenced data and can conclude that the hedging strategy with NASDAQ, VIX and the 5y USD swap rate has been successful in some subperiods. While through other periods the hedging portfolio has shown little or no effect on reducing the exposure toward the dynamics of the overall level on the implied volatility surface. Three hedging assets were most significant in reducing the exposure. However, there is a trade-off between increasing or decreasing the number of independent variables in the regression. Adding hedging assets can increase the adjusted R-squared significantly. This means that the percentage of hedging performance will rise, however fewer variables will reduce transaction costs on the weekly rebalancing.

If traded swaptions suddenly stops to covary, perhaps due to market distress or structural changes, a hedging or trading strategy within the relative value framework will be pointless due to future expectation of co-integrated movements, as the principal components are designed from a covariance or correlation matrix. As we demonstrated in the methodology section concerning PCA, this tool assumes that the directions with the most variance are the most interesting.

Furthermore, we introduced a trading strategy to identify 'true' relative value opportunities in the EUR and USD markets. We found out that investors tend to trade on the second or third principal component due to the higher level of mean reversion. However only very few opportunities were found and we are therefore not able to validate the strategy as a viable trading opportunity. However it seems that these methods have been used by practitioners for ages. If an investor wants higher return on the implied volatility position, they should exclude the long expiry and the long tenor options as variation in implied volatility is relatively low. But the exclusion comes with a risk, as theta and gamma are more likely to influence your position.

Finally we can acknowledge PCA as a practical tool to identify risk factors, macroeconomic co-tendencies and trading opportunities. It must however be treated with caution as there are several pitfalls when applying PCA to a relative value analysis. A PCA on an implied volatility surface extracts higher order eigenvectors, which tends to be less stable compared to the eigenvectors in a one dimensional yield curve setup. The discovered instability of the first and second eigenvectors for USD depicts this issue when applying PCA on an implied volatility surface.

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Appendix

Derivation of solution for maximum likelihood estimates

We are able to find μ by substituting the derived λ equation into the derived μ equation. By changing the notation for λ and μ, this will be helpful for the derivation. We have that

$$S_x = \sum_{t=1}^{n} S_{t-1}$$

$$S_y = \sum_{t=1}^{n} S_t$$

$$S_{xx} = \sum_{t=1}^{n} S_{t-1}^2$$

$$S_{xy} = \sum_{t=1}^{n} S_{t-1} S_t$$

$$S_{yy} = \sum_{t=1}^{n} S_t^2$$

This leads us to

$$\mu = \frac{S_y - e^{-\lambda\delta} S_x}{n(1 - e^{-\lambda\delta})}$$

$$\lambda = -\frac{1}{\delta} \ln \frac{S_{xy} - \mu S_x - \mu S_y + n\mu^2}{S_{xx} - 2\mu S_x + n\mu^2}$$

Here we substitute λ into μ

$$n\mu = \frac{S_y - \left(\frac{S_{xy} - \mu S_x - \mu S_y + n\mu^2}{S_{xx} - 2\mu S_x + n\mu^2}\right) S_x}{1 - \left(\frac{S_{xy} - \mu S_x - \mu S_y + n\mu^2}{S_{xx} - 2\mu S_x + n\mu^2}\right)}$$

Then removing the denominators

$$n\mu = \frac{S_y(S_{xx} - 2\mu S_x + n\mu^2) - \left(S_{xy} - \mu S_x - \mu S_y + n\mu^2\right)S_x}{(S_{xx} - 2\mu S_x + n\mu^2) - \left(S_{xy} - \mu S_x - \mu S_y + n\mu^2\right)}$$

Now we collect the terms

$$n\mu = \frac{\left(S_y S_{xx} - S_x S_{xy}\right) - \mu\left(S_x^2 - S_x S_y\right) + \mu^2 n\left(S_y - S_x\right)}{(S_{xx} - S_{xy}) - \mu\left(S_y - S_x\right)}$$

By isolating μ on the left side of the equation

$$n\mu(S_{xx} - S_{xy}) - \mu\left(S_x^2 - S_x S_y\right) = S_y S_{xx} - S_x S_{xy}$$

And finally we find a solution

$$\mu = \frac{S_y S_{xx} - S_x S_y}{n\left(S_{xx} - S_{xy}\right) - (S_x^2 - S_x S_y)}$$

And for speed mean reversion rate

$$\lambda = -\frac{1}{\delta}\ln\frac{S_{xy} - \mu S_x - \mu S_y + n\mu^2}{S_{xx} - 2\mu S_x + n\mu^2}$$

Lastly we get the variance as

$$\sigma^2 = \hat{\sigma}^2 \frac{2\lambda}{1 - \alpha^2}$$
$$\alpha = e^{-\lambda\delta}$$

Where

$$\hat{\sigma}^2 = \frac{1}{n}\left[S_{yy} - 2\alpha S_{xy} + \alpha^2 S_{xx} - 2\mu(1 - \alpha)\left(S_y - \alpha S_x\right) + n\mu^2(1 - \alpha)^2\right]$$

Trading signals for USD and EUR

Date	Last Point	Mean	Lambda	Sigma2	Exphalflife	Exp90pct
10/14/2015	24.362	-3.793	0.037	11.585	18.707	62.144
10/7/2015	23.236	-4.602	0.037	12.393	18.748	62.281
10/19/2015	23.964	-3.420	0.037	10.969	18.887	62.741
10/16/2015	24.277	-3.530	0.036	11.174	19.275	64.030
10/6/2015	24.365	-4.760	0.036	12.551	19.400	64.444
10/13/2015	22.523	-4.000	0.035	11.757	19.532	64.886
10/15/2015	24.080	-3.661	0.035	11.364	19.632	65.216
10/9/2015	22.060	-4.325	0.035	12.049	19.644	65.257
10/8/2015	22.455	-4.477	0.035	12.194	19.694	65.420
10/12/2015	22.244	-4.163	0.035	11.899	19.837	65.896
12/15/2016	9.882	1.769	0.035	8.595	20.091	66.742
12/21/2016	4.649	1.326	0.034	8.565	20.139	66.899
12/20/2016	6.835	1.483	0.034	8.565	20.152	66.944
10/20/2015	23.251	-3.312	0.034	10.693	20.171	67.007
12/23/2016	2.615	1.145	0.034	8.554	20.188	67.062
12/13/2016	14.166	2.094	0.034	8.592	20.295	67.420

Figure 18 - Mean reversion on daily PCA for USD PC 2 scores. Sorted by - speed of mean reversion (Lambda).

Date	Last Point	Mean	Lambda	Sigma2	Exphalflife	Exp90pct
11/16/2015	-0.942	1.163	0.051	5.957	13.609	45.207
11/13/2015	-0.543	1.242	0.048	5.750	14.312	47.544
11/12/2015	-0.238	1.340	0.047	5.533	14.858	49.357
4/22/2016	6.259	-0.862	0.047	2.794	14.871	49.399
4/21/2016	-6.188	0.892	0.045	2.794	15.340	50.959
11/17/2015	-1.351	0.877	0.044	5.882	15.845	52.636
4/20/2016	-5.668	0.927	0.043	2.787	16.035	53.266
4/19/2016	-6.658	0.942	0.043	2.782	16.218	53.874
4/18/2016	-7.125	0.960	0.042	2.776	16.692	55.448
4/15/2016	-7.242	1.001	0.041	2.772	16.866	56.027
11/18/2015	-2.074	0.585	0.040	5.850	17.175	57.056
11/11/2015	-0.430	1.185	0.040	5.516	17.433	57.912
11/10/2015	-0.244	1.086	0.036	5.440	19.025	63.199
2/8/2016	-0.728	-1.331	0.036	3.695	19.117	63.507
2/3/2016	4.055	1.580	0.036	3.732	19.299	64.109
11/19/2015	-2.876	0.005	0.036	5.567	19.323	64.190

Figure 19 - Mean reversion on daily PCA for USD PC 3 scores. Sorted by - speed of mean reversion (Lambda).

Date	Last Point	Mean	Lambda	Sigma2	Exphalflife	Exp90pct
1.10.2017	2.546	-4.585	0.017	1.658	41.975	139.437
12.28.2016	0.515	-5.488	0.016	1.708	42.482	141.121
1.4.2017	1.302	-5.027	0.016	1.680	42.842	142.317
1.9.2017	1.759	-4.768	0.016	1.663	43.223	143.585
12.26.2016	-0.425	-5.797	0.016	1.719	43.502	144.510
12.27.2016	-0.334	-5.704	0.016	1.713	43.584	144.782
1.6.2017	1.660	-4.866	0.016	1.667	43.662	145.041
1.3.2017	1.198	-5.149	0.016	1.685	43.811	145.537
12.23.2016	-0.532	-5.918	0.016	1.726	43.992	146.140
12.29.2016	0.767	-5.445	0.016	1.700	44.185	146.778
1.5.2017	1.729	-4.960	0.016	1.672	44.470	147.725
1.2.2017	1.164	-5.268	0.015	1.691	44.927	149.244
12.30.2016	0.992	-5.367	0.015	1.695	44.932	149.261
1.11.2017	2.766	-4.578	0.015	1.640	45.289	150.445
12.22.2016	-0.864	-6.110	0.015	1.735	45.578	151.407
12.21.2016	-1.030	-6.260	0.015	1.742	46.564	154.682

Figure 20 - Mean reversion on daily PCA for EUR PC 2 scores. Sorted by - speed of mean reversion (Lambda).

Date	Last Point	Mean	Lambda	Sigma2	Exphalflife	Exp90pct
3.10.2015	12.560	-0.389	0.043	5.988	16.216	53.868
3.9.2015	11.788	-0.466	0.043	5.901	16.259	54.010
3.5.2015	7.003	-0.711	0.042	5.699	16.405	54.497
3.6.2015	9.848	-0.548	0.042	5.802	16.465	54.695
3.4.2015	6.870	-0.768	0.042	5.610	16.592	55.118
3.11.2015	13.672	-0.227	0.042	6.057	16.629	55.239
6.28.2013	-7.135	0.165	0.041	5.191	16.865	56.025
6.27.2013	-8.304	0.144	0.041	5.185	16.937	56.265
6.26.2013	-7.319	0.096	0.041	5.188	17.106	56.824
3.26.2013	0.200	0.912	0.040	4.866	17.119	56.868
2.27.2015	4.651	-0.993	0.040	5.360	17.147	56.962
3.16.2015	17.178	0.176	0.040	6.455	17.148	56.963
3.22.2013	-1.747	-1.073	0.040	4.804	17.152	56.976
3.3.2015	5.562	-0.811	0.040	5.517	17.153	56.982
6.25.2013	8.200	-0.078	0.040	5.183	17.172	57.044
7.1.2013	-8.278	0.010	0.040	5.180	17.258	57.331

Figure 21- Mean reversion on daily PCA for EUR PC 3 scores. Sorted by - speed of mean reversion (Lambda).

Selected R code

Code: MLE of OU process for third principal component

```
for (j in 630:dim(usdnew)[1]){
PC<-prcomp(usdnew[(j-629):j,2:dim(usdnew)[2]],scale.=FALSE)

p<-vector(mode="numeric",length=nrow(PC$x))
d<-vector(mode="numeric",length=(nrow(PC$x)-1))
for (i in 1:nrow(PC$x)){
p[i]<-
PC$x[i,3]*PC$x[i,3]
for (l in 2:nrow(PC$x)){
d[l]<-PC$x[l-1,3]*PC$x[l,3]
}
}

n<-nrow(PC$x)
Sx<-sum(PC$x[,3])-PC$x[nrow(PC$x),3]#last point
Sy<-sum(PC$x[,3])-PC$x[1,3]#first point
Sxx<-sum(p[1:nrow(PC$x)])-p[i]
Syy<-sum(p[1:nrow(PC$x)])-p[1]
Sxy<-sum(d[])

lastpoint[j]<-PC$x[nrow(PC$x),3]
middel[j]<-(Sy*Sxx-Sx*Sxy)/(n*(Sxx-Sxy)-(Sx*Sx-Sx*Sy))
deviation[j]<-(lastpoint[j]-middel[j])/middel[j]*10000
lambda[j]<-(-log((Sxy-middel[j]*Sx-middel[j]*Sy+n*middel[j]*middel[j])/(Sxx-
2*middel[j]*Sx+n*middel[j]*middel[j])))
alpha[j]<-exp(lambda[j]*-1)
sigmaquer[j]<-(Syy-2*alpha[j]*Sxy+alpha[j]*alpha[j]*Sxx-2*middel[j]*(1-
alpha[j])*(Sy-alpha[j]*Sx)+n*middel[j]*middel[j]*(1-alpha[j])*(1-alpha[j]))/n
sigma2[j]<-sigmaquer[j]*2*lambda[j]/(1-alpha[j]*alpha[j])
sigma[j]<-sqrt(sigma2[j])
exphalflife[j]<-log(2)/lambda[j]
exp90pct[j]<-log(10)/lambda[j]
}
```

Eigenvectors on vega sector for EUR

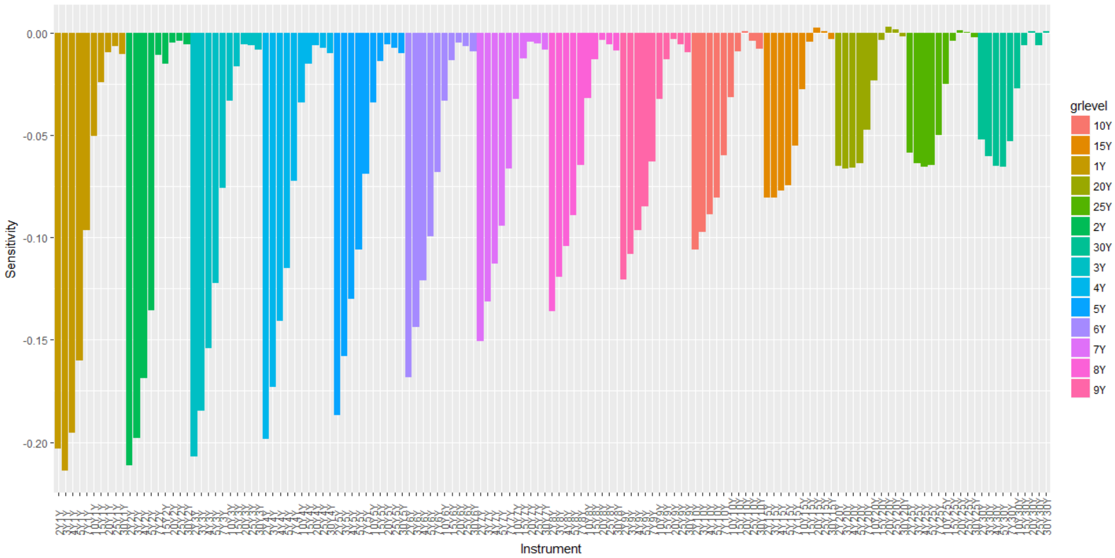

Figure 22 – First eigenvector of vega sector, EUR surface, Date: 2015-07-03

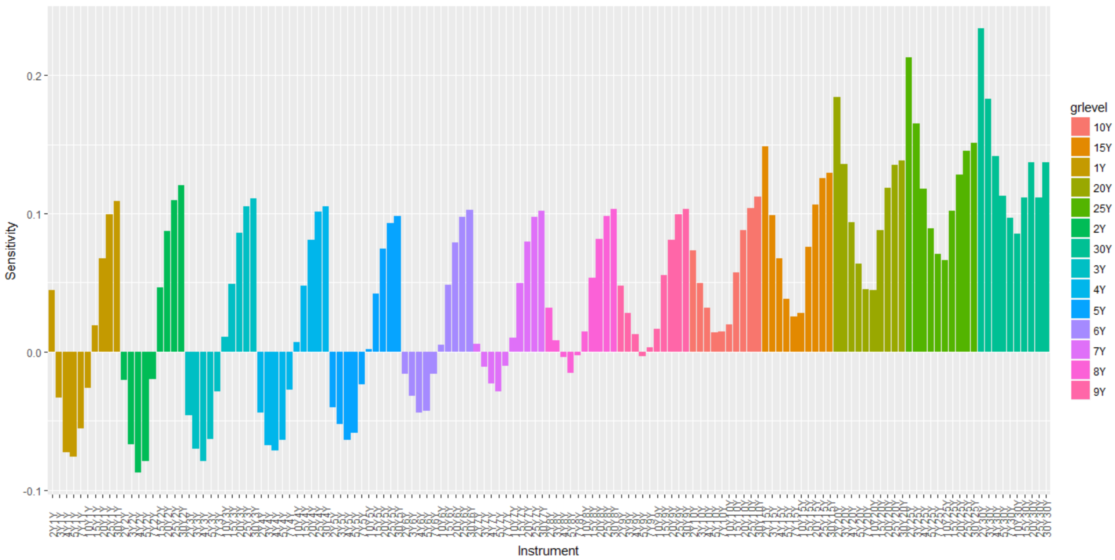

Figure 23 – Second eigenvector of vega sector, EUR surface, Date: 2015-07-03

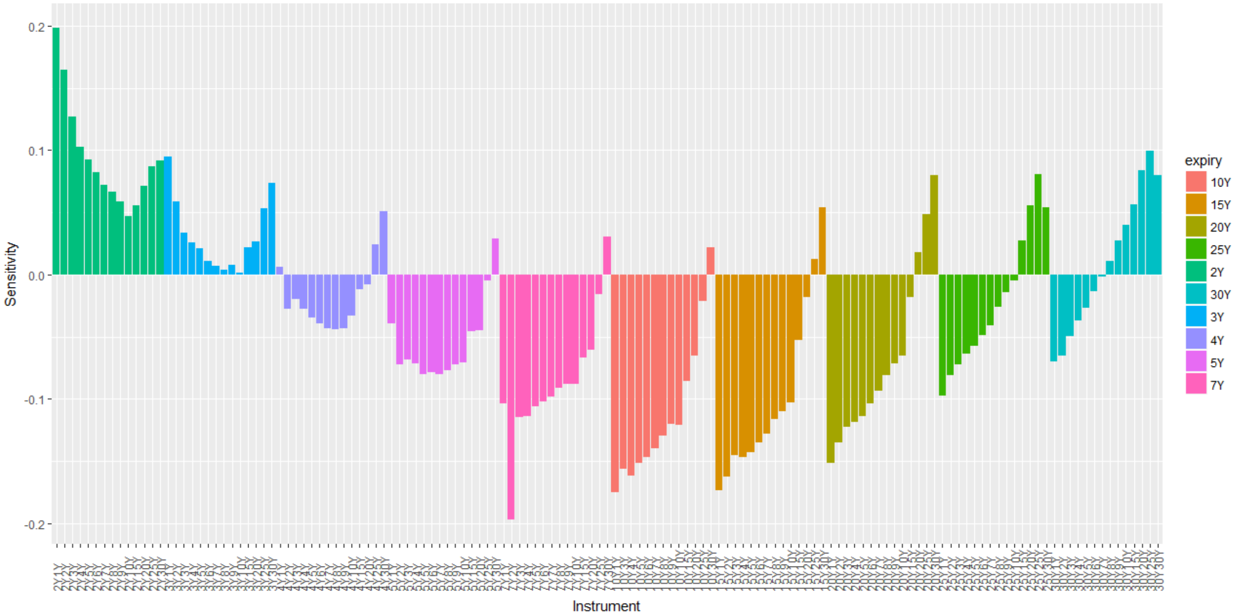

Figure 24 – Third eigenvector of vega sector, EUR surface, Date: 2015-07-03