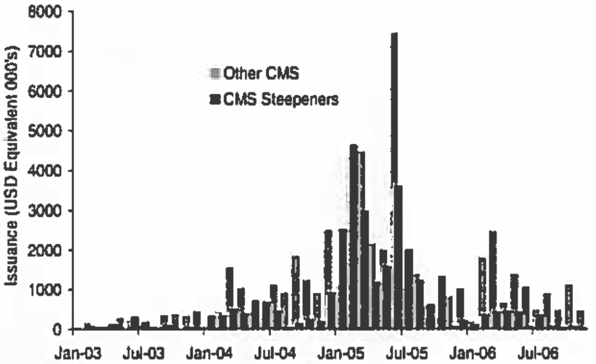
VOL COMMENT 1/25/2007

CMS and Euroland Vega

Vega becomes directional and forward slope vols as dealers rehedge

Issuance Patterns and CMS Steepeners

Over the course of 2003-2006, a notional of USD equivalent $86.4 billion worth of EUR CMS-linked products was issued in note format through MTN programs.



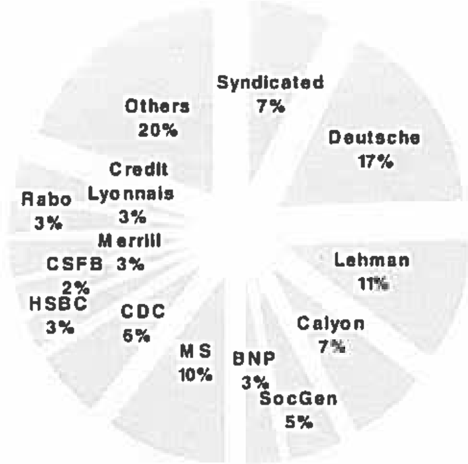
Source: MTN-I, Goldman-Sachs–5 Dec 2006

This significantly underestimates the entire CMS issuance as many were done through private placements, repackagings (which, although often listed, are not easily summarized), swaps and other OTC contracts. Of these CMS-linked products, a full $31.2bn (USD equivalent) of CMS-steepener products were done, primarily during 2005.

	Total CMS	Other CMS	Steepeners
Grand Total	86408.91	54811.33	31597.58
Deutsche Bank	9788.17	4328.54	5459.63
Lehman Brothers	8194.01	4810.86	3383.15
Morgan Stanley	4797.32	1540.74	3256.58
Syndicated	11737.95	9472.52	2265.43
Calyon	5931.12	3689.60	2241.52
IXIS CIB	4600.34	2702.20	1898.15
SG CIB	5262.28	3801.23	1461.05
BNP Paribas	4880.59	3857.14	1023.46
Merrill Lynch	2070.00	1079.88	990.12
Credit Lyonnais	1007.95	111.64	896.31
HSBC	3698.28	2802.48	895.80
Rabobank	1107.53	271.60	835.93

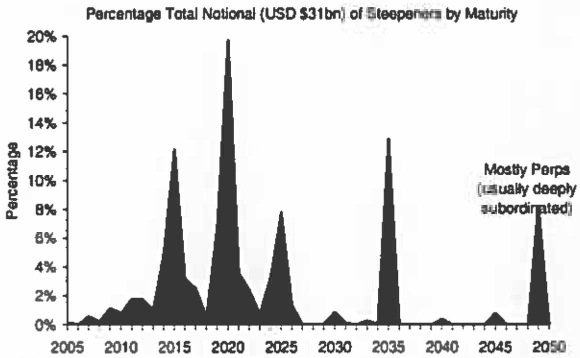
Source: MTN-I, Goldman-Sachs–5 Dec 2006

The dealers (the top 12 as ranked in MTN-I by CMS-steepener issuance listed above), placed these notes in a variety of different institutions and oftentimes ultimately in retail accounts. While many of these notes were done through regular MTN note issuance programs by third party issuers, a good many of these placements were also in the form of hybrid funding, i.e., Tier II deeply-subordinated notes.



Source: MTN-I, Goldman-Sachs–5 Dec 2006

In terms of maturities, the products were largely longer-term issues with 2020 (around 15Y maturity) making up a full 20% of the steepener issuance. The hybrid funding, mostly in the form of (callable) perpetuals made upwards of 8% of all issuance.



Source: MTN-I, Goldman-Sachs–5 Dec 2006.

Of the steepener issuance, approximately 70% was in the form of leveraged steepeners and 30% in the form of digitals or range-accruals. Typical payoffs were:

- **Levered Steepener:** Coupon = K x (10YCMS - 2Y CMS), floored at 0%, capped at X% where K is a high multiplier.
- **Digital/Range Accrual:** Coupon = N/M[30Y CMS - 10Y CMS > K] x X%, where X is a large coupon and N/M[.] indicates number of month-end observations during a coupon period.

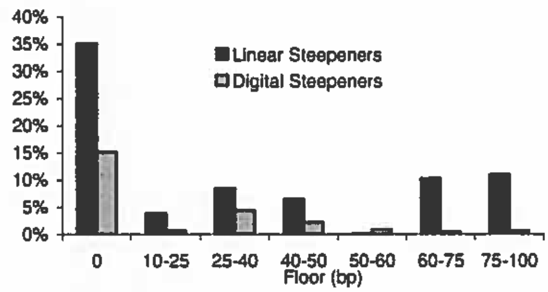
In USD-denominated structured product, the issuance pattern was quite different,[1] with Callable Range Accruals being the primary issue, while for EUR-denominated, Levered Steepeners and their other variants, (e.g., lock-in steepeners, callable

[1]Mackenzie, Michael, The Value of Non-Inversion, *Financial Times*, 29 November 2006, details some of the hedging implications of the US CMS-spread product.

snowballs or LIFTs, target redemption notes or TARNs, enhanced steepeners 10Y CMS-2Y CMS capped at 10Y CMS), CMS-Boosters (10YCMS*(10Y CMS-2Y CMS), etc), predominated. We generally saw levered steepeners with multipliers of around 4x-8x, depending on the floor (e.g. higher multipliers with lower floors) and the teaser (the initial, high, coupon). Coupons were generally paid annually with 1-5Y of fixed coupons, making the trades sensitive to a variety of forward slopes.

As an example, a leverage of 4x, with a cap at 10% and a floor at, say 2%, would imply that dealers are short call spreads on the slope with first strike (the "floor") at 2%/4=50bp, and the higher strike (the "cap") at 10%/4=250bp.

We generally saw caps that were typically initiated at significantly above the current slopes, somewhere on the order of 200bp or more, while many deals did not have caps. This concurs with MTN-I. But caps have recently been creeping lower as the curve has flattened and new issuance has been done at increasing levels of leverage (e.g., in the US, in 2004, it was not unusual to see 5x-10x leverage, whereas now it is possible to see 50x levered deals). These new deals effectively help to hedge floors dealers had done in the more distant past, at the same time making the unhedged lower strikes floors even more pronounced. The majority of caps captured from MTN-I are in the 160bp-250bp range. But, the floor data is much more meaningful and has the distribution as shown below.



Source: MTN-I, Goldman-Sachs-5 Dec 2006

According to MTN-I, approximately 50% (35% levered steepeners, 15% digitals) of the entire $31bn steepener issuance was done with a floor at 0%. From 2035 maturity onwards, higher strikes (60-75bp) become more predominant, and the perpetuals were almost all issued with a floor of 100bp, due to their cheaper funding costs. Nonetheless, 0% strike floors were issued at approximately three to four times as much as any other floor for maturities out to 2020, being much less significant in 2025. Approximately 95% of the entire notional of steepeners was based on 10Y-2Y spreads, 4% in 30Y-2Y, and the remainder in various other combinations.

While issuance was brisk in 2005, the flattening Euroland swap curve and subsequent low valuations lead to a significant amount of bad press for CMS steepeners, and most clients were reluctant to enter into new steepeners. This can be seen from the large drop in volumes. With curves close to flat, there is now renewed interest in highly leveraged steepeners, primarily by institutionals.

While the MTN data does not show the complete picture, it is relatively representative of the asset side. On the liability side, however, there is virtually no publicly available data. From our experience, the typical liability side trade was done with corporates banks and local authorities and was of the form:

- **Funding Cheapener:** Client Pays = X% if 10Y CMS - 2Y CMS > 0bp, or X% + Spread- K x min(10Y CMS – 2Y CMS,0%), where X% is a small coupon (typically), and Spread and K are large multipliers, making coupons rise steeply during an inversion.

The client is effectively short digital puts and regular puts on the slope. Variations on this trade have been done in large size, in various incarnations, with some quite leveraged (e.g., client becomes short a LIFT/Snowball on slope if curve inverts). According to our experience, the maturity was typically relatively short (e.g., 5 years). As is the case with the asset side trades, this funding cheapener is a carry trade, intensifying some of the hedging needs of dealers. But, unlike the asset side trades, liability trades typically hedged the low strike floors. In spite of the push from dealers, we believe that the bad press on the asset side lead many corporates to avoid liability side steepeners, and volumes have, in our view, remained temperate.

Dealer Positions and the Demand for Vanilla Hedges

Before considering the implications of this massive issuance, we need to describe how exactly CMS is hedged, and perhaps more importantly, how the hedges are altered with market conditions. We got into much more detail on the pricing and hedging of CMS in the appendix, but will summarize here.

- CMS differs from forward starting swaps due to the difference in convexity, and is actually priced using a portfolio of swaptions of all strikes (the *convexity correction*). The significant skew dependence has lead to distortions in the skew.
- CMS spread options are effectively a set of forward steepeners. A 2Y-10Y CMS Slope Note (with annual coupons) is hedged using a combination of forward steepeners, which is in total a long-dated steepener (e.g., a 10Y note would lead to a combination of 10y-15y or 10y-25y stepeners).
- CMS Spread options, due both to the spread option itself and due the underlying convexity corrections, put upward pressure on 10Y vega and may put downward

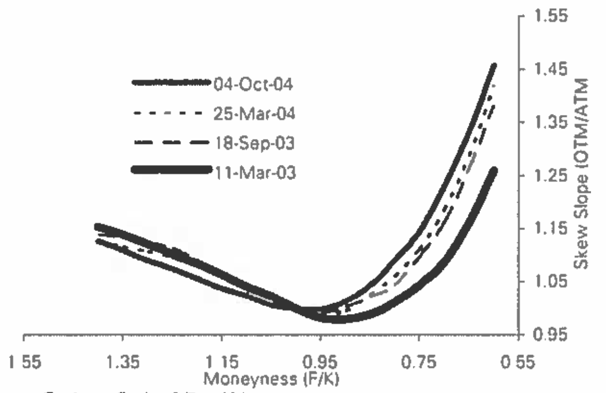
pressure on 2Y vega initially. As the floor is approached, the hedging demand for 10Y vol decreases and the hedging demand for 2Y vol increases.

- The street is effectively short spread options in the 1Y-3Y maturity range, and long in the 5Y+ range. This *correlation-vega*, however, will change rapidly with the flattening of the curve, pushing upwards the demand for medium-term hedging product.

The Market Impact

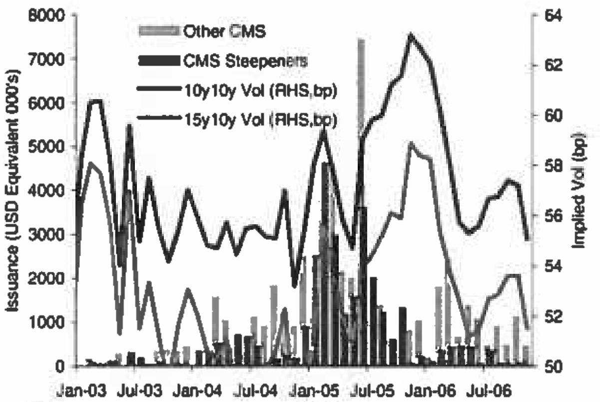
There are many who would disregard the impact of rehedging of CMS portfolios. But none would dispute the impact of CMS issuance on vega when it was initially hedged.

The large issuance of CMS in 2004-2005 (mostly linked to 10Y CMS or longer), lead to a significant steepening of the payer skew.



Source: Goldman-Sachs–5 Dec 2006

Subsequently, the market experienced a sharp rise in longer-dated ATM vols.



Source: MTN-1, Goldman-Sachs–5 Dec 2006

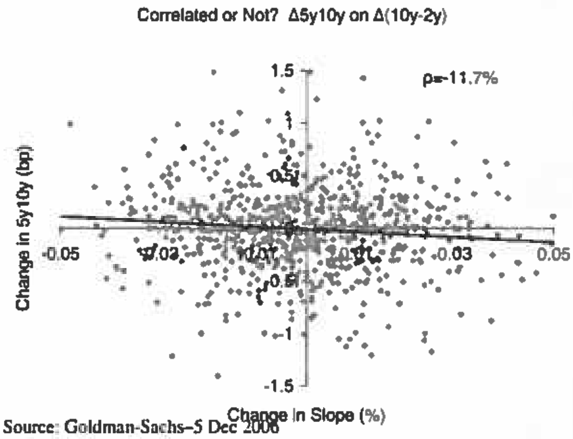
We can see this impact directly by overlaying longer-tenor vega on the issuance profile, with peaks coinciding with issuance except for the mid to late summer 2005, when pension fund hedging helped to push vol higher, and we believe that hedge funds continued the momentum.

But the impact is not limited to the time of issuance. The lack of proper hedging instruments and the consequent shifts in delta and vega exposures means that the dealers' net positions are likely to impact the market. As we mentioned above, the effects are two-fold. As the floors are approached, in a flattening, the street's delta should move so as to prompt dealers to unwind their steepeners, and this could possibly exacerbate the flattening. Moreover, in a flattening, the street's 10Y vega will decrease and 2Y vega will rise sharply, leading to the possibility of 10Y tails falling and 2Y tails rising in concert with the slope.

We visit each of these topics in turn.

10Y Vega Impact

As an example, we see below the scatter plot of 10Y vega (5y10y vol changes) on changes in slope.



Source: Goldman-Sachs–5 Dec 2006

Since 2003 there has been little net relationship between 10Y vega and slope, but if anything a modest negative correlation (i.e., if slopes rise, contemporaneously 5y10y normalized vols fall). This pattern is repeated for a variety of expiries:

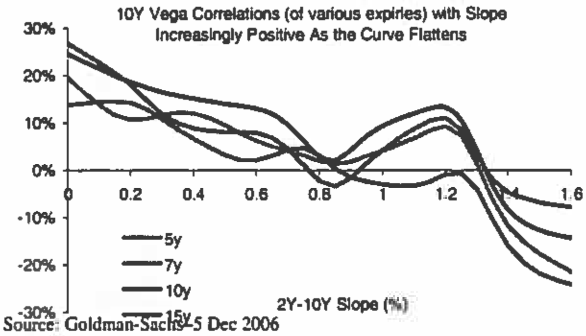
Volatility	Correlation with Slope
1y10y	-8.0%
2y10y	-9.0%
5y10y	-11.6%
7y10y	-8.4%
10y10y	-1.6%
15y10y	0.0%

But this picture is incomplete. We would expect that, as the market approached floors, 10Y vega should begin to have a positive correlation with slope, where steepening would lead to upward pressure on 10Y vol and flattening would lead to downward pressure.

This is borne out in the data if view it appropriately. Using a nonparametric correlation estimation,[2] we can condition on curve slope and compute the conditional correlations between slope change and change in vega. As we see below, the flatter the curve, the more positive the correlation between

[2] We use a Gaussian kernel with Silverman plugin bandwidth to compute conditional variances and correlations, conditioning on slope levels at the beginning of the day, to compute the correlation of daily slope and vega changes over the course of the day.

slope and vega has been, particularly for longer-expiries.



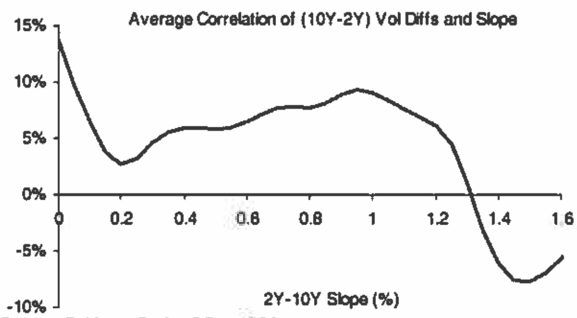
Source: Goldman-Sachs-5 Dec 2006

In other words, in spite of the modest negative correlation between slope and 10y vega over long histories, the correlation is increasingly positive with flatter curves. This is likely to be due almost entirely to rehedging.

2Y Vega Impact

Unfortunately, the impact on 2Y vega is not as conclusive. While 2Y vega should in general be even more directional for spread options (due to the fact that 10Y vega is primarily driven by the convexity correction while 2Y vega has little convexity so is more sensitive to the relative "moneyness" of the spread option), we have found little evidence of this. We should expect strong negative correlations between slope and 2Y vega close to floors.

It is possible that 2Y tails have been affected by quite a number of other deal flow over the course of the flattening. Instead we have seen modestly positive correlations. However, the correlation of 2Y-10Y vol spreads (averaged over 5,7,10,15y expiries below) with slope is positive for flatter slopes, as we should expect. This shows that movements in 10Y vega have tended to dominate any movement in 2Y vega as the market has approached the floors.



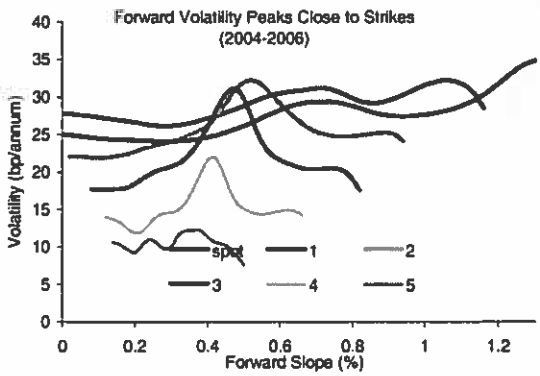
Source: Goldman-Sachs-5 Dec 2006

In general, this would mean seemingly innocuous vega trades are directional. For instance, going Long 5y2y straddles, short 5y10y straddles vega neutral, is actually a flattener in the current flat slope environment, due entirely to the positive correlation

Slope Impact

The Euroland swaps market is a particularly deep market and consequently, our expectation would be that there should be no impact from hedging. In particular, the slope of the swap curve is driven by a number of underlying economic factors, particularly, the output gap (or the market's perception of this), expected deficit/gdp, overall levels of volatility, and, less importantly, the difference between short-run inflation expectations and the long-term target. Our expectation is that this dependence can only be affected imperceptibly by the hedging activities of issuers. Yet, it may be possible that less liquid areas of the curve or more subtle relationships should be impacted.

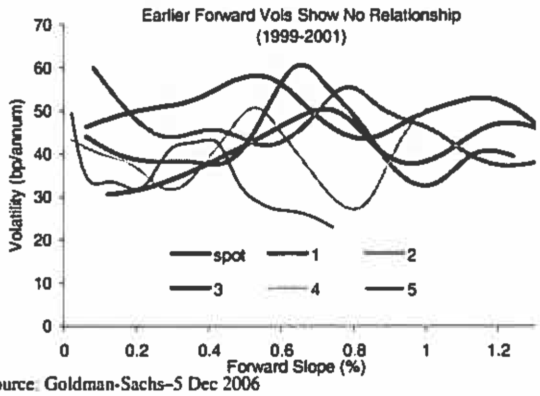
While spot slope realized volatility is generally positively correlated with slope (i.e., slope vol is high when slopes are steep and is low when slopes are flat), this is apparently not the case with forward slopes. If we look at forward slope volatility from Dec-2004-onwards (2Y of data), conditioning again on the level of each forward slope,[3] a curious picture develops with various of the shorter dated slopes' conditional volatility peaking at what we believe to be strikes.



Source: Goldman-Sachs-5 Dec 2006

If we plot the same conditional volatilities over the period of 1999-2001, 2Y of data which were chosen to have roughly the same span (i.e., the curve inverted briefly during this period), we see no such alignment of peaks in volatilities, and rather they remain erratic.

[3] Utilizing the same nonparametric conditioning methodology as above to produce a conditional standard deviation.



Source: Goldman-Sachs–5 Dec 2006

Although this is in no way conclusive, it suggests that forward volatilities should continue to peak as forwards pass through lower and lower strikes.

Benefiting from Dealer Positioning

While directionality of vega trades is something that is not easy to monetize (i.e., it is unlikely that anyone should adjust their vega based on the previous day's change of slope, it is a relationship which can be monetized through certain structured product.

One product that takes advantage of the recent positive correlation between vol and slope would be a play on forward vol.

Example 1: Switcher Straddles

Client Rcvs Xy2y Straddles
ATM ForwardStrike
If (CMS10-CMS2)<Kbp in TY time

Otherwise Xy10y Straddles
ATM ForwardStrike
If (CMS10-CMS2)>Kbp in TY time

Where strikes are all set as ATMF in TY time.
This trade benefits from the fact that steeper slopes tend to result in higher 10Y implieds, in the margin, while flatter slopes should result in higher 2Y implieds.

Example 2: Variable Strike Swaptions

Another sort of trade which could similarly benefit is the following switch, with client paying a variable strike receiver and receiving a fixed strike receiver in exchange.

Client Pays Xy10y receivers, struck at
K = max(Fixed - Mult*(CMS10-CMS2),0).

Client Rcvs Xy10y receivers struck at K*.

This could be structured so as to monetize an unlikely state of the world as we will see below. But this trade would also monetize the positive correlation between slope and 10Y vol. If levels are unchanged, as the curve steepens, the short-receiver becomes more OTM and its vega decreases. Thus the client net vega increases in a steepening, and decreases in a flattening. Looking to P&L from vega, it should essentially

$$VegaP\&L = \int Vega(Slope(t))\Delta 10yVol(t)dt$$

$$= \int (Vega(t-1) + Vega' \cdot \Delta Slope(t))\Delta 10yVol(t)dt$$

With Vega'(Slope)>0, the client benefits from all market moves with a positive realized correlation between slope and vega.

Depending on the choice of Multiplier, it is possible to structure the trade with a negative 2-10 correlation vega (i.e., it benefits the street, much like selling low-strike floors would), but also so that it has a positive payoff except in unlikely situations such as bull-flattenings.

Conclusions

The impact of the continuing rehedging of steepener issuance is very real, and the story continues to evolve. As the Eurozone swap curve flattens through 0, we expect continuing hedging to place downward pressure on 10Y vega, to make realized forward slopes peak in volatility and to possibly make 2Y vega rally. While these phenomena are significant, they may also only be temporary. If the slope steepens sufficiently, the need to rehedge will be obviated, lowering any impact. Meanwhile, at the currently low valuations, there are likely to be many retail-based lawsuits that could lead to unwinds, again reducing the need for rehedging. Yet, in the current flat environment, we expect this behaviour to continue for the coming year.

Appendix: CMS Pricing and Hedging

CMS, unlike most other interest rate product, provides a linear payoff to clients. It is used by retail and, typically, by less sophisticated institutionals due to the ease in understanding the payoff, compared to most other fixed income products. In addition, it is used by more sophisticated institutionals in order to hedge issuance programs, helping lower cost of funding, etc.

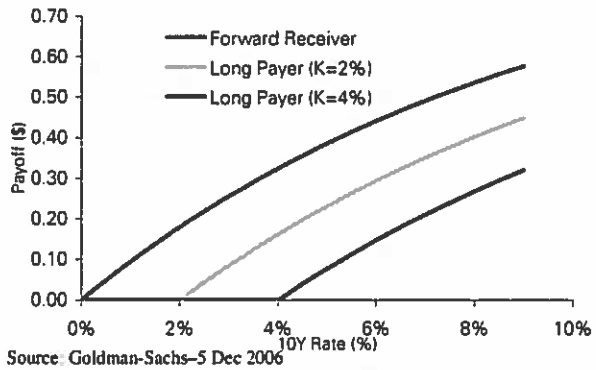
Plain CMS Pricing and Convexity Corrections

A CMS cashflow is linear in rates, unlike virtually any other fixed income instrument. Previously, CMS cashflows were hedged with forward starting swaps. This mismatch, the fact that the short payer swap is negatively convex while the CMS is linear, leads to a convexity adjustment. One such version of the convexity adjustment is the following:

$$E[CMS10] - FwdSwap = -\tfrac{1}{2}F^2\sigma^2 T\frac{G'(F)}{G(F)}$$ where

G(F) is the dv01 of the swap and G'(F) is the convexity. This formula leads to convexity-adjusted forwards, and can be seen to lead to larger corrections for higher rates and for higher vols and as we would expect, larger corrections for longer dated rates.

CMS can be more accurately replicated using the entire skew (in particular, payer swaptions of all strikes).[4] Payer swaptions, upon expiry, pay the dv01 of the underlying swap rate (the *annuity*) multiplied by the rate differential, $(S-K)^{+}dv01(S)$. The payoff is concave in rates.

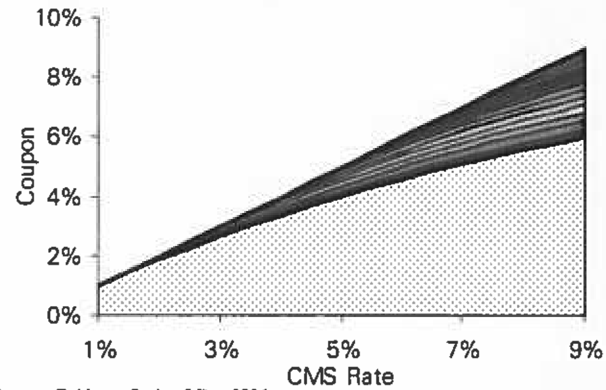


Source: Goldman-Sachs–5 Dec 2006

We can use combinations of these payer swaptions to piece together a CMS payoff. Consider a CMS cap struck at K (e.g., payoff $(S-K)^{+}$) where we replicate the payoff at N points using N payer swaptions: $CMS(S,K)=\sum w_i\, P(S,K_i)$ where P(S,K) is the payoff of a payer struck at K. Here $K_0=K$ and $K_{i+1}=K_i+\Delta K$. Then equating the payoff with the replicating portfolio at the points K_i, we get

$$\underbrace{\begin{bmatrix} CMS(K_1,K) \\ CMS(K_2,K) \\ \vdots \\ CMS(K_N,K) \end{bmatrix}}_{C} = \underbrace{\begin{bmatrix} P(K_1,K) & 0 & \cdots & 0 \\ P(K_2,K) & P(K_2,K_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ P(K_N,K) & P(K_N,K_1) & \cdots & P(K_N,K_{N-1}) \end{bmatrix}}_{P} \times \underbrace{\begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{N-1} \end{bmatrix}}_{w}$$

Or in matrix notation, C = Pw. This implies that the vector of weights $w = P^{-1}C$ and the CMS cashflow is exactly replicated at points K_i. By using infinitely many payer swaptions ($N\rightarrow\infty$) at a continuum of strikes ($\Delta K \rightarrow 0$), we can statically replicate the CMS cashflow. We show this below with the weighted payoffs of the underlying payer swaptions, the sum of which combines to form the CMS cashflow.



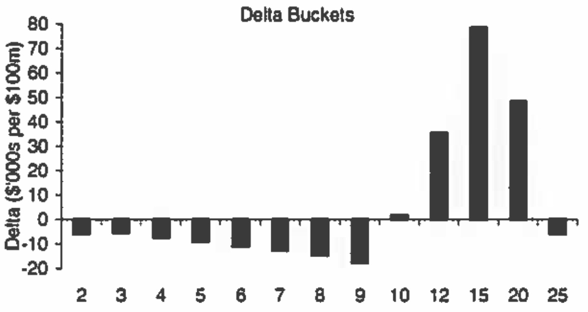
Source: Goldman-Sachs–5 Dec 2006

In the case shown above, we have a 10Y CMS rate, and the replication was done with payers of strikes between 0% and 8.5% with a step of 0.50%. Approximately 48% of the notional of the cashflow must be hedged with 0% strike payers, with the remainder spread very evenly across all other strikes, with a full 3.30%-3.40% of notional in each strike above 7%. The hedge ratios do not vary with the timing of the cashflow. In general the hedge that is traded will be through forward swaps, and ATM vol and vol of other various strikes, depending on the trader.

Plain CMS Delta and Gamma

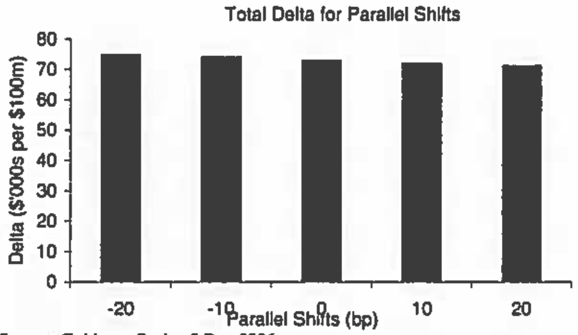
The CMS delta, if we consider a single expiry, is quite simple. For instance, a 2Y expiry 10Y CMS is hedged with a 2x10 forward. A 10y CMS-linked note, with its annual cashflows, is hedged with a series of 1x10, 2x10, 3x10, etc forwards, creating a long exposure in the 10Y-20Y swaps, and shorts in the 1Y to 10Y swaps. In essence, CMS by itself is partly a steepener. (Note that we only show the deltas of the CMS leg below).

[4] See for example, Fabio Mercurio and Andrea Pallavicini, Swaption skews and convexity adjustments, *online manuscript*, July 2006, or Pat Hagan, Convexity Conundrums: Pricing CMS Swaps, Caps, and Floors, *Wilmott Magazine*, (Mar 2003) 38-44.



Source: Goldman-Sachs-5 Dec 2006

This Delta changes only modestly with changes in the level of rates and consequently, we expect little to no impact.



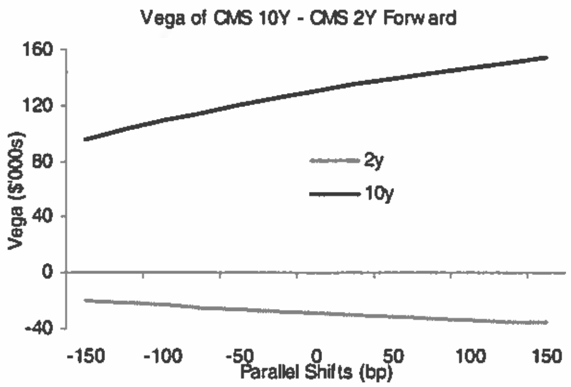
Source: Goldman-Sachs-5 Dec 2006

While there are no products that exactly replicate CMS structures (since there is no market for very high strike swaptions) much of the plain CMS is largely hedged for most reasonable rate scenarios.

Plain CMS Vega and Vanna

We can see from the formula for the convexity correction that the level of rates and volatilities enter explicitly. Consequently, we should expect that the vega should also depend on the level of rates. If we consider the replicating portfolio, payer swaptions move on average ITM or OTM depending on a shift in rates, thus varying the vega.

We see this for a $100m notional 10Y CMS - 2Y CMS forward below. Note that the vega is increasingly positive for 10Y and increasingly negative for 2Y with any rise in rates (i.e., it has a 10Y positive *vanna* or d^2Price/[d(vol)d(Rates)] and a negative 2Y *vanna*). The 2Y has a lower convexity correction so lower absolute vega.



Source: Goldman-Sachs-5 Dec 2006

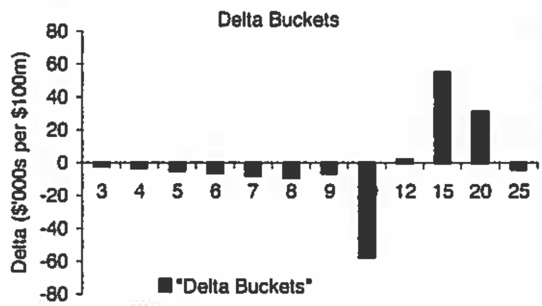
CMS Steepener Hedging

But unlike plain CMS, CMS-steepeners are significantly more complicated. In particular, while a bank paying 10YCMS-2YCMS will naturally initiate a forward steepener in rates (e.g., a 7y-15y steepener or longer depending on the expiry) and will buy 10Y vega and sell 2Y vega, pushing up 10Y tails and depressing 2Y tails, this is effectively only a delta and a vega hedge. Without the cap and the floor, the delta and vega of the position will make large swings depending on market conditions.

Since dealers are short call spreads on the slope, the net dealer position is short slope gamma close to the floor (and long gamma at the cap). This means, without products to hedge, dealers would be forced to delta hedge and adjust it over time, thereby possibly accelerating the flattening or steepening. This importance of delta rehedging depends entirely on the size of the net-gamma. Recall that gamma risk is spread closely around the strike of any option, with the peak in gamma being more pronounced the shorter the expiry.

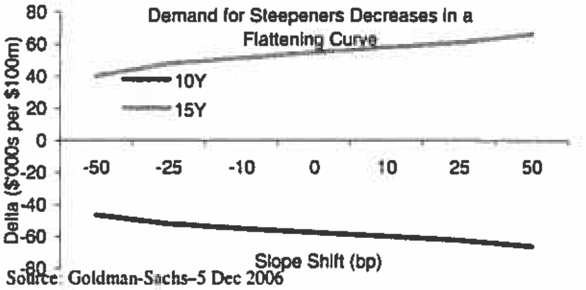
CMS Steepener Delta and Gamma

The spread option delta, again, in the case of a single expiry is trivial. For instance, a 2Y expiry simple call on slope should be hedged with 2Y forward 10s-2s steepeners. But a leveraged steepener typically has annual cashflows, so initiating a combination of 1Y, 2Y, 3Y, etc forward steepeners simultaneously amounts to being long a set of longer-dated forwards and short a set of extremely long-dated forwards. For example, 10Y maturity leveraged steepener on 2Y-10Y may actually be delta hedged as a 10Y-15Y+ steepener. As seen below, negative 10Y deltas are hedge through buying 10Y and positive 15Y through selling 15Y+.



Source: Goldman-Sachs-5 Dec 2006

We can see the impact of rehedging curve delta through recomputing the delta over a variety of slope scenarios. In general the short-curve-gamma position means that flattening should lead to unwinding the delta hedge (i.e., unwinding the steepener), and steepening would lead to adding to it.

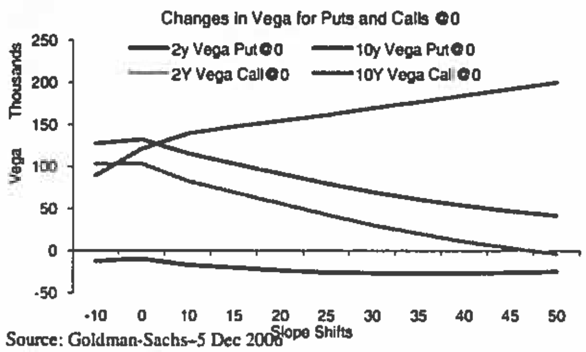


Source: Goldman-Sachs-5 Dec 2006

So, as the curve flattens the demand for steepeners decreases, and the short-gamma position has the possibility of amplifying moves in slopes. Although realized slope volatility is generally positively correlated with the level of slopes, this short gamma has the possibility of increasing slope volatility in low rate scenarios as well.

CMS Steepener Vega and Vanna

But perhaps more importantly, rehedging also puts a strain on vega. For a simple call on slope and a simple put on slope, we have a different vega impact for different curve shapes. As we see below, as the curve flattens 10Y vega falls for the call while 2Y vega rises even more strongly. For a put, the 10Y vega is mostly constant, while the 2Y vega rises.



Source: Goldman-Sachs-5 Dec 2006

We can see this approximately by combining (Normal) Black Scholes or Bachelier and the static replication formulas.[5] Calculating the vega with the σ_i for i=2,10:

$$Vega = \frac{d}{d\sigma_i} E[(CMS10(T) - CMS2(T) - K)^+ DF(t,T)]$$

$$\approx Vega_{BS}(\sigma_{spd}, S-K)(d\sigma_{spd} / d\sigma_i)$$

$$+ \Delta_{BS}(\sigma_{spd}, S-K)(dFwdSpread / d\sigma_i)$$

Where $\sigma_{spd}^2 = \sigma_{10}^2 + \sigma_2^2 - 2\rho\sigma_2\sigma_{10}$. While the Δ_{BS} term allows for contributions to vega of different sign for 10Y vs 2Y tails, the $Vega_{BS}$ term adds to the vega of the 2Y (as long as 2Y tails are rich to 10Y tails and correlation is high enough) and correspondingly, adds much less to the vega of the 10Y. This set of relationships holds so long as $\sigma_2 - \rho\sigma_{10} > 0$ and $\sigma_{10} - \rho\sigma_2 > 0$. Note, though, that in the current environment, $(\sigma_2 - \rho\sigma_{10})/(\sigma_{10} - \rho\sigma_2)$ is approximately a factor of 20:1, or in other words, the proximity of the strike has significantly more impact on 2Y vega than it does on 10Y vega.

Based on our analysis of the asset side position and our assumptions on the liability side, most of the street is short calls on the slope at low strikes. Consequently, we should have expected large upward pressure on 10Y vega and minor upward pressure on 2Y vega when deals were first printed, and we would expect to see 10Y vega fall and 2Y vega rise as slopes flatten. This movement in vega should be that much more pronounced as the market nears the underlying strike.

Spread Options and Correlation

Until recently no products which could hedge the correlation risk between CMS legs. This changed with the introduction of the CMS Spread Option. CMS-Spread Options are the following structures:

- **CMS Call:** Coupon = max(10Y CMS - 2Y CMS, K) x Notional, Quarterly pay.
- **CMS Put:** Coupon = min(10Y CMS- 2Y CMS, K) x Notional, Quarterly Pay

The CMS Spread option, except for modest differences in coupon payments, almost perfectly hedges the cap and floor risk that the dealer community is short.

And, after its introduction, CMS Spread Options became very active. According to our own estimation and that of several brokers, in 2005, approximately $50bn notional in EUR denominated spread options traded hands between dealers, and in 2006, it is expected to be more. To put this in context, if the entire universe of leveraged CMS spreads issued (about $22bn) had a leverage of 5, $110bn notional of spread options would be needed to hedge the floors, let alone the caps. Moreover, although active, spread options are almost exclusively traded between dealers, with hedge funds entering only recently and warehousing relatively minor quantities of the risk that needs to be hedged.

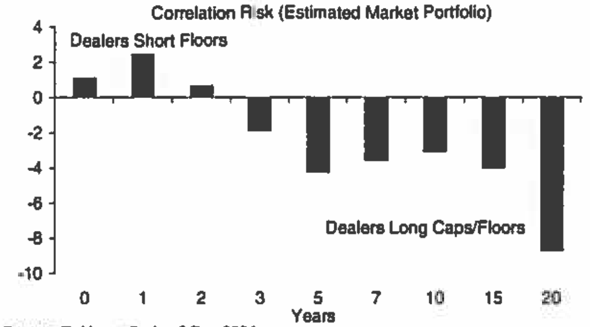
In our estimation, the dealer community remains net short-gamma on a range of strikes but largely at lower ones. Interestingly enough, as the curve has flattened, the 0% strike floors, with the largest demand, have had no offers on the broker market for the past several weeks.

As we can see from our BS Spread option approximation formula,

[5] See for example, René Carmona and Valdo Durrleman, Pricing and Hedging Spread Options, SIAM Review, 45 (2003) 627-685, for a more thorough explanation of the Greeks.

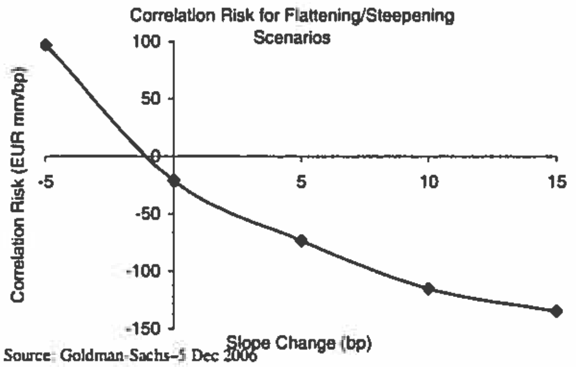
$$CorrVega = \frac{d}{d\rho} E[(CMS10(T) - CMS2(T) - K)^{+} DF(t,T)]$$
$$= Vega_{BS}(\sigma_{spd}, S-K)(d\sigma_{spd} / d\rho)$$
$$= -Vega_{BS}(\sigma_{spd}, S-K)(\sigma_{10}\sigma_2)/\sigma_{spd}$$

And consequently, long calls (and long puts) have a negative correlation vega. Note as well that the correlation risk changes with the $Vega_{BS}$, so the correlation-vega becomes even more negative as the strike is approached.



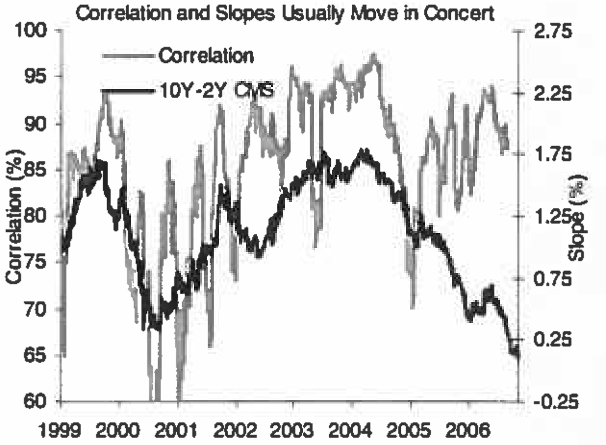
Source: Goldman Sachs–5 Dec 2006

In our estimation, the market portfolio (combination of MTN-I and our own estimation of the liability side trades), is effectively long slightly more than EUR 2mm per bp correlation in the 1Y sector, and short correlation in the 3Y-20Y sector. Effectively, caps have a more pronounced effect for longer-dated risks. This corroborates the healthy two way flow for CMS spread product in 5Y-10Y CMS spreads, with no offers for floors in the 1Y-5Y sector. In total, our estimation is that dealers are long approximately EUR 21m per bp correlation across all maturities. Perhaps more importantly, the correlation risk will alter drastically as the curve flattens or steepens.



Source: Goldman Sachs–5 Dec 2006

In the current situation, if net correlations fall, unhedged dealers' P&L is effectively positive, but with minor flattenings from the current environment, the market becomes short correlation. This may increase the demand for spread options if correlations fall in a flattening environment. Typically correlations and slopes do move in concert (see below for correlation for a 1Y expiry 10Y-2Y CMS, with rolling 2M window).



Source: Goldman-Sachs–5 Dec 2006

General Dependencies and Correlation

In general, options whose payoffs depend on 2Y and 10Y CMS alone with weights w_2 and w_{10} respectively, $F(w_2$ CMS 2Y + w_{10} CMS 10Y) have a 2Y-10Y correlation-vega

$dP/d\rho_{2\text{-}10} = w_2 w_{10} Vega_{BS}\sigma_2\sigma_{10}$.

Thus by having w_2 and w_{10} of different sign, the trade effectively sells the correlation exposure that the street needs for hedging, just like selling spread options. But, if the weights are $w_2 \neq - w_{10}$, the payoff can be seen to benefit from much more interesting yield curve movement, unlike the case of simple spread options. For instance, by varying the weights, it is possible to construct a trade which only takes downside risk to large bull-flattenings, or to large bear-steepenings. The end-product is a trade that helps the street hedge with downside potential being very unlikely.

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