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Arthur M. Berd, Roy Mashal and Peili Wang

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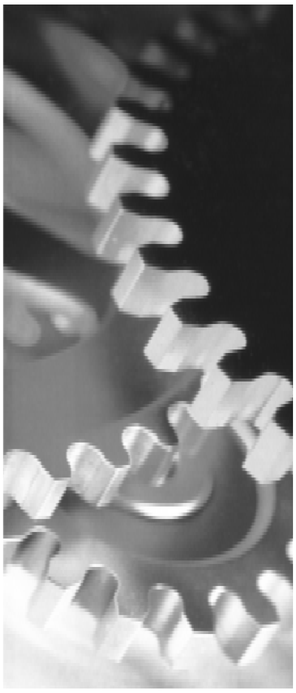

LEHMAN BROTHERS

Estimating Implied Default Probabilities from Credit Bond Prices

Arthur M. Berd
212-526-2629
arthur.berd@lehman.com

Roy Mashal
212-526-7931
rmashal@lehman.com

Peili Wang
212-526-1010
peili@lehman.com

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1. INTRODUCTION

Default probabilities implied by the prices of credit-risky bonds are an important ingredient for accurate pricing and effective management of credit risk. This article presents a new methodology for estimating these implied default probabilities. In contrast to existing methods, the estimation is executed directly on the prices of a set of given bonds rather than on the spreads of these bonds. We show that our estimation methodology is particularly useful for generating well-behaved issuer- and sector spread curves and for calculating bond-level relative value measures, especially in the case of distressed bonds.

Most of the fixed income valuation and risk methodologies are based on modeling the yield curve. The main reason is that the vast majority of debt instruments exhibit very high price return correlation. Therefore the common pricing factors encoded in the yield curve have high explanatory power. This is especially true for Treasury bonds, where the market is extremely efficient, and any deviation of individual bond valuation from the common curve is quickly arbitraged away.

For corporate bonds, the common yield curves are much less binding, even for the investment grade benchmark issuers. Starting with the most important, the driving factors for the valuation of credit-risky bonds can be listed as follows:

1. The level and shape of the underlying Treasury or swaps curves.
2. Credit quality – which is often measured by credit ratings. For certain rating agencies (Moody's), this reflects not only the probability of default, but also the recovery assumption.
3. Industry-related factors – mainly driven by portfolio managers' sector allocation decisions.
4. Issuer-specific factors – influenced by investors' security allocation decisions and by the risk management decisions of lending banks.
5. Issue-specific pricing – affected by details of the security structure (callable, puttable, amortizing) and liquidity.

For most investment grade credit bonds, the standard market pricing methodologies rely on the estimation of spread discount functions, i.e. prices of the hypothetical zero coupon credit bonds (see Monkkonen (1999)). The validity of such an approach hinges on our ability to represent the price of a credit bond as a linear weighted sum of the contractual (coupon and principal) cash flows.

[1] *Reprinted from Quantitative Credit Research Quarterly, vol. 2003-Q3.*

Whether such a representation is possible depends on the market conventions as well as the realities of the distressed bond market. Indeed, the main difference between a defaultable bond and a credit risk-free one is the possibility that the bond might default and thus have to undergo the scrutiny of the distressed bond market.

The standard approach to pricing risky bonds is the reduced-form modeling of defaultable bonds (see Litterman and Iben (1991), Jarrow and Turnbull (1995), and Duffie and Singleton (1999)). The key assumption in all of these models is the specification of the market value of a bond right after default. In other words, one must make an assumption as to the expected recovery given default (or alternatively, the loss given default).

In this paper we use the **fractional recovery of par (FRP)** recovery assumption. Under this assumption, at the time of default, a bond price would equal some fraction of its face value, regardless of maturity. This assumption is commonly used in bankruptcy workout practices, where all claims of equal seniority, regardless of their maturity or current market value, are pooled on a *pari passu* basis and allocated a weight determined by the contractual promised principal or face value of the debt. The FRP is also used by the rating agencies that collect and maintain historical recovery databases.

The FRP assumption reflects the market conventions in the sense that it correctly recognizes the fact that bonds are not very likely to trade below expected par recovery, and that upon default all maturities will trade at similar dollar prices. In particular, the FRP assumption leads to a very natural explanation of one of the main features of the distressed bond market – the steeply inverted spread curves. If both near and far maturity bonds trade at a similar *dollar* price, then one must assume much higher spreads for short maturities to account for the same amount of dollar price discount compared with longer maturity bonds.

The main difficulty with this approach is that it violates the assumption of reduced-form (strippable) bond valuation, ie, a bond cannot in general be valued as a linear weighted sum of the contractual flows (coupon plus principal). However, with the advent of the credit default swap (CDS) market, the fractional recovery of par assumption has taken on a new importance. The market convention in modeling CDS spreads follows the FRP assumption, and therefore the discrepancy between CDS and bond pricing models can be large for issuers trading at very wide spreads (or what is known in the market as "trading on dollar price").

2. PRICING CREDIT BONDS WITH FRACTIONAL RECOVERY OF PAR

Consider a credit-risky bond that pays a fixed coupon with a specified frequency (usually annual or semi-annual). By standard pricing arguments, the present value of such a bond is given by the risk neutral expectation of its cash flows (we discount using the Libor curve). According to the fractional recovery of par assumption, in the scenarios where the issuer defaults, the bond recovers a fraction of the face value and possibly of the accrued interest. By considering explicitly the scenarios of survival and default, we write the price of a credit-risky bond with payment dates $t_1, t_2, \ldots, t_N$ as:

$$
\begin{aligned}
P(t_N) \quad &= \quad \sum_{i=1}^{N} CF^{tot}(t_i) \cdot Z(t_i) \cdot Q(t_i) \\
&+ \quad \sum_{i=1}^{N} \left(CF^{prin}(t_i) \cdot R^{prin} + CF^{\text{int}}(t_i) \cdot R^{\text{int}} \right) \cdot Z(t_i) \cdot D(t_{i-1}, t_i)
\end{aligned}
\tag{1}
$$

where $P(t_N)$ stands for the price today (time t_0), of a coupon bond maturing at t_N and $CF^{tot}(t)$ is the total cashflow schedule at time t (the summation of principal payments, $CF^{prin}(t)$, and coupons, $CF^{int}(t)$), $Z(t)$ is the risk-free discount factor from time t to today, $Q(t)$ is the implied (risk-neutral) survival probability to time t and $D(t_j,t_k)$, $t_j \le t_k$ is the probability of default happening in that interval.

The first term in equation [1] gives the sum of the payments at each date t_i, i=1,2,…,N weighting each payment by the probability of surviving up to this date. The second sum details the payment in scenarios where the bond defaults before maturity. In such scenarios the bond recovers a given fraction R^{prin} of the outstanding principal face value at that time, $CF^{prin}(t_i)$, plus a given fraction R^{int} of the current interest due, $CF^{int}(t_i)$. Note that $D(\cdot,\cdot)$ and $Q(\cdot)$ are related by:

[2] $$D(t_j, t_k) = Q(t_j) - Q(t_k), \quad t_j \le t_k .$$

These formulas will serve as the basis for our empirical estimation procedure.

3. ESTIMATING THE TERM STRUCTURE OF SURVIVAL PROBABILITIES

The premise of our approach is that the survival probability is a linear combination of exponentially decaying functions of time. This assumption is reminiscent of intensity-based models of default. In reduced form models of default, the survival probability is related to the default intensity, $h(t)$ (also called hazard rate), by the relationship: [2]

[3] $$Q(t) = \exp\left(-\int_0^t h(s) \cdot ds \right)$$

In the case of constant hazard rate, the survival probability term structure is exactly exponential, therefore, the exponential spline methodology (see Vasicek and Fong (1982)) naturally lends itself to estimating survival probability term structures.

We model the survival probability function as a cubic exponential spline:

[4] $$Q(t) = \sum_{k=1}^{3} \beta_k \cdot Spline_k(t)$$, where

[5] $$Spline_k(t) = e^{-k \cdot \alpha \cdot t}$$

The decay parameter α can be interpreted as the long-maturity limit of the hazard rate function, and is usually determined separately.

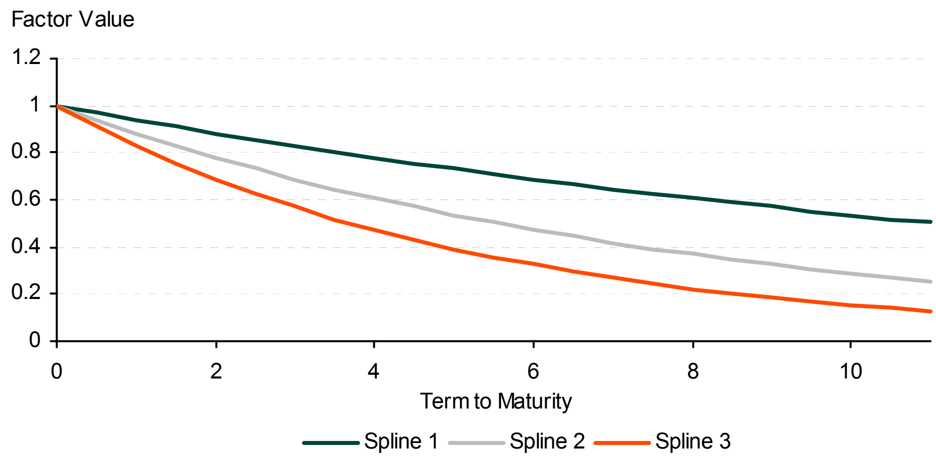
Because the survival probability at $t = 0$ must be equal to 1 (we know for sure that the issuer survived until the present date), the three spline coefficients must satisfy the constraint:

[6] $$\sum_{k=1}^{3} \beta_i = 1$$

2 The hazard rate can, in general, be stochastic. In this discussion we restrict our attention to deterministic default intensities.

Figure 1 illustrates the three components of the cubic exponential spline for α=0.06.

Figure 1. The components of a cubic exponential spline (α=0.06)



Substituting the spline equation [4] into the pricing equation [1], we obtain the cross-sectional regression setting for estimating of the survival probability term structure using observable bond prices.

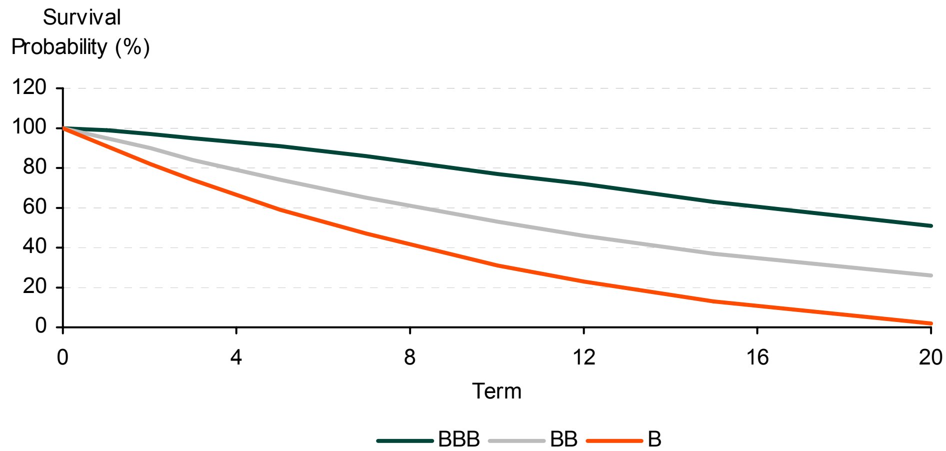
In addition to the equality constraint mentioned above, we also impose inequality constraints to make sure that the survival probabilities are positive and strictly decreasing for all times.

As a final remark, we would like to note the importance of the recovery rate assumption. Our methodology recognizes the fact that the recovery rate is an important ingredient in the market behavior of distressed bonds. Without going into great detail here, we propose using estimates of recovery rates by seniority and industry.

In practice, we ignore the recovery of accrued interest and set R^{int} to zero as we believe that the market does not efficiently price the coupon recovery (we do not have any reasonable way to estimate this parameter in the first place). In what follows, we denote the principal recovery as R.

Figure 2 shows the survival curves for Consumer Cyclicals BBB, BB and B sectors. For all three sectors the survival probability term structures are indeed monotonically decaying with growing maturity, with the riskier sectors exhibiting faster drops in survival probability.

Figure 2. Fitted implied survival probability, Consumer Cyclicals BBB, BB and B Sectors (as of 30 June 2003)



4. FITTED DEFAULT, SURVIVAL AND SPREAD TERM STRUCTURES

The estimated exponential spline coefficients β_k provide us with a smooth term structure of survival probabilities. We can derive other important valuation and relative value measures directly from this term structure.

4.1. Implied survival and default probability term structures

The survival probability term structure is given by the definition from which we started.

$$Q(t) = \sum_{k=1}^{3} \beta_k \cdot e^{-k \cdot \alpha \cdot t} \qquad [7]$$

The cumulative default probability term structure is determined via the relation:

$$D_{cumulative}(t) = 1 - Q(t) \qquad [8]$$

Once we obtain the survival probability term structure in a continuous form provided by the exponential spline, the derivation of the corresponding hazard rate term structure is straightforward. From equation [7] we get the formal definition:

$$h(t) = -\frac{1}{Q(t)} \frac{dQ(t)}{dt} = \frac{\sum_{k=1}^{3} k \cdot \alpha \cdot \beta_k \cdot e^{-k \cdot \alpha^{surv} \cdot t}}{\sum_{k=1}^{3} \beta_k \cdot e^{-k \cdot \alpha^{surv} \cdot t}} \qquad [9]$$

The fitted hazard rate is a smooth function of maturity, in contrast to piece-wise linear hazard rate functions that are usually obtained using bootstrapping techniques.

Figure 3. Fitted implied hazard rate term structure, AT&T vs. BBB Communications and Technology Sector (as of 30 June 2003)

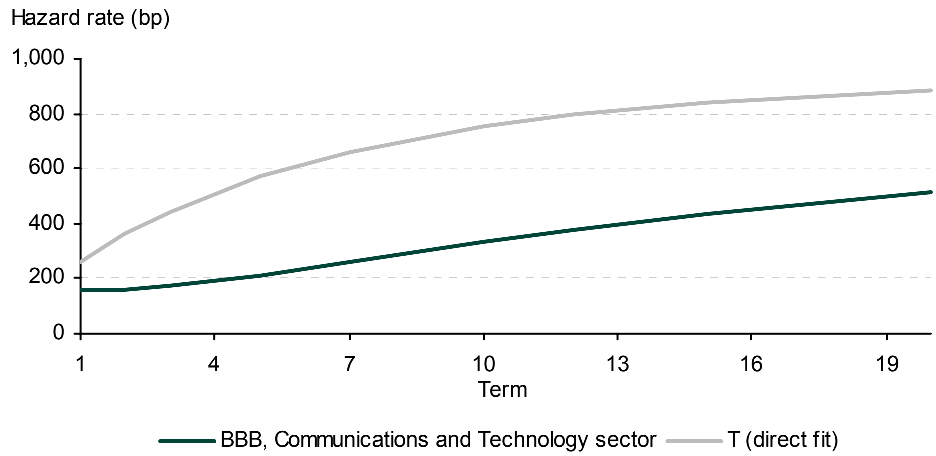


Figure 3 illustrates one of the applications of our methodology – a comparison of the implied hazard rate term structures of a given issuer, AT&T, with the broader industry/rating sector – in this case the BBB Communications and Technology sector. As we can see, not only are the levels of the fitted implied hazard rates different, but there is also a significant difference in the shapes of these two term structures. Such information can be of particular interest to portfolio managers who have a view on AT&T credit compared with its peer group, and need to optimize their maturity selection for maximum risk-adjusted return.

4.2. Par coupon and par spread term structures

For any integer number of payment periods $t_N = N \cdot \frac{1}{f}$ (here f refers to the bond's coupon frequency, usually annual or semi-annual), we define the par coupon term structure $C(t_N)$ as the coupon on a hypothesized t_N-maturity bond for which $P(t_N)$=100. We can calculate it using the fitted term structure of survival probabilities and discount function (the same one that was used to calculate the survival probability):

[10] $$C(t_N) = f \cdot \frac{1 - Q(t_N) \cdot Z(t_N) - R \cdot \sum_{i=1}^{N} \left(Q(t_{i-1}) - Q(t_i) \right) \cdot Z(t_i)}{\sum_{i=1}^{N} Q(t_i) \cdot Z(t_i)}$$

Par coupons are defined as fractions of 100 face, ie, 5% coupon appears as C=0.05.

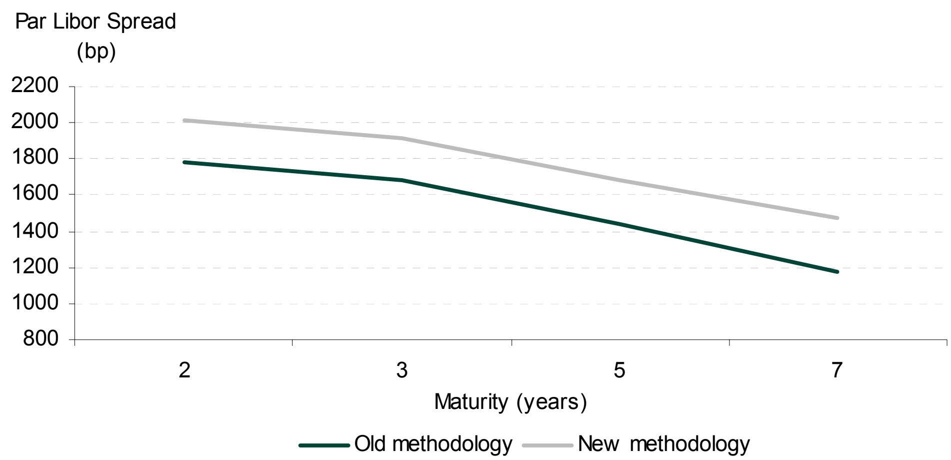
The term structure of par spreads to Treasury (swap) can be derived by subtracting the par Treasury (swap) yields of the same maturities from the same compounding frequency. We define:

[11] $$S_{treas}(t_N) = C_{treas}(t_N) - y_{treas}(t_N),$$

[12] $$S_{LIBOR}(t_N) = C_{LIBOR}(t_N) - y_{swap}(t_N),$$

where, $C_{treas}(t_N)$ and $C_{Libor}(t_N)$ correspond to the par coupon based on the Treasury and Libor discounting curves respectively, and $y_{treas}(t_N)$ and $y_{swap}(t_N)$ are the par Treasuries and swaps rates, respectively.

Figure 4. Fitted Libor spread term structure in old and new methodology, Calpine (as of 30 June 2003)



As we can see from Figure 4, the differences between the old and new methodology can be substantial in estimating the par spreads for Calpine (B3/CCC utility company) bonds that trade at deep discounts, in the range of $70-80. For names that trade "on spread" rather than "on dollar price", and whose bonds are priced close to or above par, the difference between the two methodologies becomes much smaller.

4.3. Constant Coupon Price (CCP) term structure

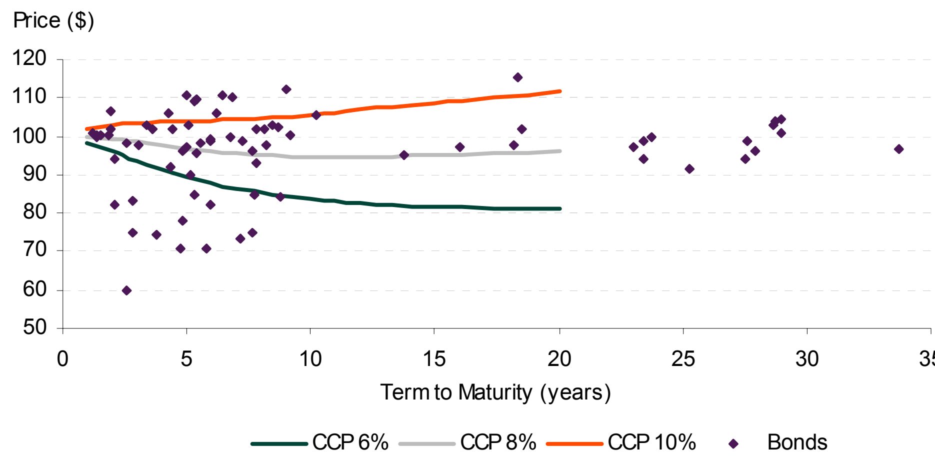
We next define the constant coupon price (CCP) term structure as the prices of a set of bonds with varying maturities and a common coupon. We compute CCP term structures for three levels of coupon (6%, 8%, 10%) using the fitted survival curve *Q(t)* obtained earlier.

$$P^{CCP}(t_N) = \left[\sum_{i=1}^{N} \frac{C}{f} \cdot Z(t_i) \cdot Q(t_i) + Z(t_N) \cdot Q(t_N) + \sum_{i=1}^{N} R \cdot Z(t_i) \cdot \left(Q(t_{i-1}) - Q(t_i) \right) \right] \quad [13]$$

Prices are calculated as fractions of a 100 face, ie, par price 100% appears as 1.

Figure 5 demonstrates the usefulness of the constant coupon price term structure concept. We show the individual bond prices that were used for fitting the survival spline, and the 6%, 8% and 10% constant coupon price term structures derived from that survival probability. The majority of bonds fall within the range delineated by the CCP curves. The ones lying below are those that trade at a significant price discount to the rest of this sector. Depending on the respective actual bond coupons, one can estimate from Figure 5 the potential amount in dollars by which the corresponding bonds are undervalued with respect to their peer group. If, for example, the credit outlook for a given 6% coupon bearing bond is such that its fair price should be similar to the sector average, then the price difference between that bond and the corresponding 6% CCP curve for the same maturity represents the relative value an investor can capture by buying this bond.

Figure 5. Constant coupon price term structures, Basic Industries B (as of 30 June 2003)



The case of deeply discounted bonds is illustrated in Figure 6, using the example of the issuer curves for Calpine. As we can see, our methodology correctly captures the phenomenon of nearly flat price curves, ie, bonds of various maturities trading at a similar dollar price. The level of that dollar price depends, of course, on the coupon of the bond.

Figure 6. Constant coupon price term structures, Calpine (as of 30 June 2003)

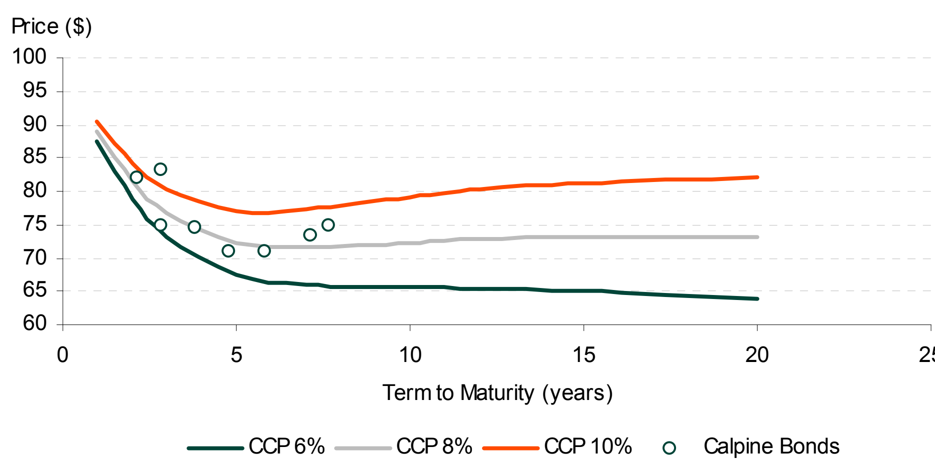
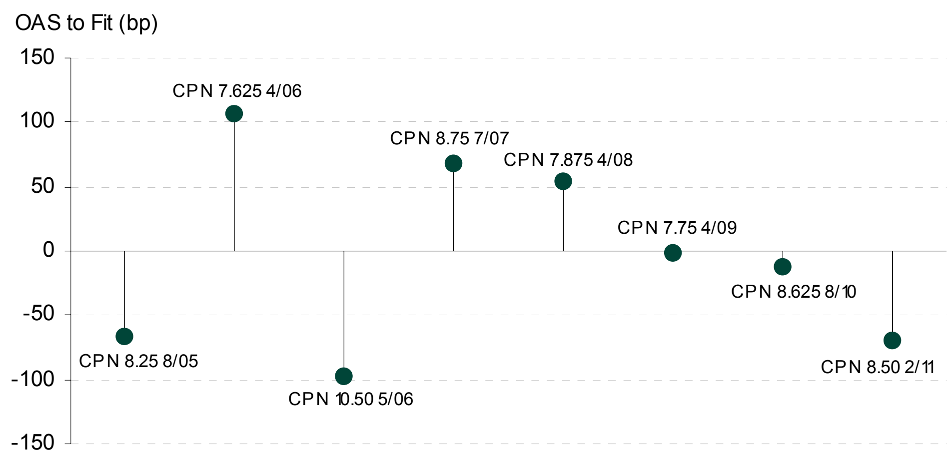


Figure 7: OAS-to-Fit, Calpine (as of 30 June 2003)



4.4. OAS to Fitted Curve

Because the estimation methodology is based on cross-sectional regressions, the individual bond prices are fitted with some residual error. As an estimate of the pricing error (and potentially useful relative value information), we introduce a constant spread (OAS-to-Fitted Curve) such that a given bond's price is obtained precisely when the additional discount term is applied to all future cash flows in all default scenarios.

$$
\begin{aligned}
P(t_N) \;&=\; \sum_{i=1}^{N} \frac{C}{f} \cdot Z(t_i) \cdot Q(t_i) \cdot e^{-OAS \cdot t_i} + Z(t_N) \cdot Q(t_N) \cdot e^{-OAS \cdot t_N} \\
&+\; \sum_{i=1}^{N} R \cdot Z(t_i) \cdot \big(Q(t_{i-1}) - Q(t_i)\big) \cdot e^{-OAS \cdot t_i}
\end{aligned} \qquad [14]
$$

OAS-to-Fit can be either positive or negative. It represents the issue-specific pricing deviations from the fair value given by the fitted price of the bond.

An example of the OAS-to-Fit calculation for the Calpine bonds that were shown in Figure 6 is presented in Figure 7. As we can see, according to our model, CPN 7.625 4/06, priced at \$75, appears almost 100bp cheap to fitted value, whereas a similar maturity CPN 10.50 5/06, priced at \$83.3 appears almost 100bp rich to fitted value. Because both bonds have similar amounts outstanding (\$250m and \$180m, respectively) the difference in pricing is probably due to the fact that the higher coupon bond has a higher dollar price and thus represents a greater apparent loss risk to portfolio managers.

5. CONCLUSIONS

This article presents a new methodology for estimating survival probabilities. In contrast to existing methods, the estimation is executed directly on the prices of a set of credit-risky bonds rather than on their spreads. We believe that this estimation methodology is more robust, especially for distressed bonds. We demonstrated this using both issuer and sector spread curves.

There is much information that can be extracted from the estimated survival curves, some of which is presented throughout this paper. We are currently backtesting and extending the applications of this methodology, and will publish the results of that study in a forthcoming paper. Overall, we feel that the new tools allow for a better quantitative analysis of the risky bond market/sector/issuer, and provide a sounder foundation for security selection and portfolio construction.

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