Ahead of the Curve: Forward-Looking Hedge Ratios for Cash Treasuries

December 20, 2024

Abstract

This paper introduces a framework for deriving forward-looking hedge ratios for curve and fly trades in the US Treasury (UST) market by leveraging the information embedded in the yield curve spread option (YCSO) and swaption markets respectively. Traditional approaches to hedging directional interest rate exposure often rely on historical correlations, static yield curve relationships, and/or first order approximations which can lag dynamic market conditions and specifically are prone to failure during periods of macro regime shifts. By incorporating the implied volatility of swaptions and implied correlation from the YCSO market, we can construct the covariance structure and perform every fixed income professionals' favorite linear algebra tool: Principal Components Analysis. We will first give a brief refresher of PCA in the rates market then an example of "standard" backward-looking way to calculate hedge ratios for yield curve trades. We then provide a quick primer on YCSOs and figures of deriving these hedge ratios from market quotes obtained from J.P. Morgan Markets. We will finish with a discussion of the holes in this framework and the instability of eignvectors over time.

1 Principal Components Analysis:

A sell-side researcher once told me that for every equation he wrote in a note, half of his audience would stop reading. I will take this advice soundly and keep the math to a low (good for me since I hate writing the LaTeX) but will also keep in mind that this paper is being written for a numerical analysis class so there might be an equation or two below.

First, let's start by asking the question, what the heck are we doing?

We want to take high-dimensional data i.e. something with many columns (like the term structure of interest rates) and boil it down to something more digestible without losing too much information along the way - no free lunch! This will allow us to analyze how the entire term structure of interest rates moves with respect to time, relative to specific tenors, and what has been the most important factors or principal components shaping the yield curve. This lead us to intuitive interpretations for our PCs on the UST term structure.

- PC1: Level Component ⇒ the entire curve shifts up, a parallel move
- PC2: Slope Component ⇒ 2s10s rallies, steepening/flattening moves
- PC3: Curvature Component ⇒ 2s5s10s rallies, concave/convex moves in the yield curve
- PC4, PC5, … who knows?

A little bit of math…

We start with the covariance matrix of the term structure rather than the correlation matrix since we do not want standardization between two entries but keeping the volatility observed from the entire term structure. We then construct the orthonormal basis of the eigenvectors of the covariance matrix. This is geometrically intuitive since PCs are uncorrelated relationships. The factors capturing level, slope, and curvature moves are the eigenvectors and their relative strength to the (scaled) eigenvalues. So the greatest absolute eigenvalue points to the direction of the highest variation (most important structural relationship captured in the term structure => PC1) making its corresponding eigenvector longest (with respect to in some norm).

All of this will allow us:

- Get a broad sense of how the yield curve has moved over time
- Conduct rich or cheap analysis based on maturity or even specifc CUSIPs
- Allow for superior hedge ratios for curve and fly trades

Something that we will continue to harp on throughout this paper is the instability of eignevector for every time step. Using historical data is by definition backwards-looking. We know relationships change - i.e. if the eigenvectors of said covarience matrix change after entering into a trade with backwards-looking hedge ratios computed with PCA (or regression, DV01 neutral, etc), we will become exposed to unintended factor risk. Say for example, a change of the first eigenvector results in the hedge breaking down and in directional exposure for a fly we put on - THIS IS A BIG PROBLEM. We can investigate and point to many reason for the cause of this deterioration, quoting Schaller and Huggins, “the problem is the same as in the case of correlation between factors occurring after entering into a PCA-neutral position: it loses its neutrality and becomes exposed to factors it was not intended to.” A quick fix for the perceived instability is to just pick a longer timeframe of data to run PCA but that comes with its own challenges.

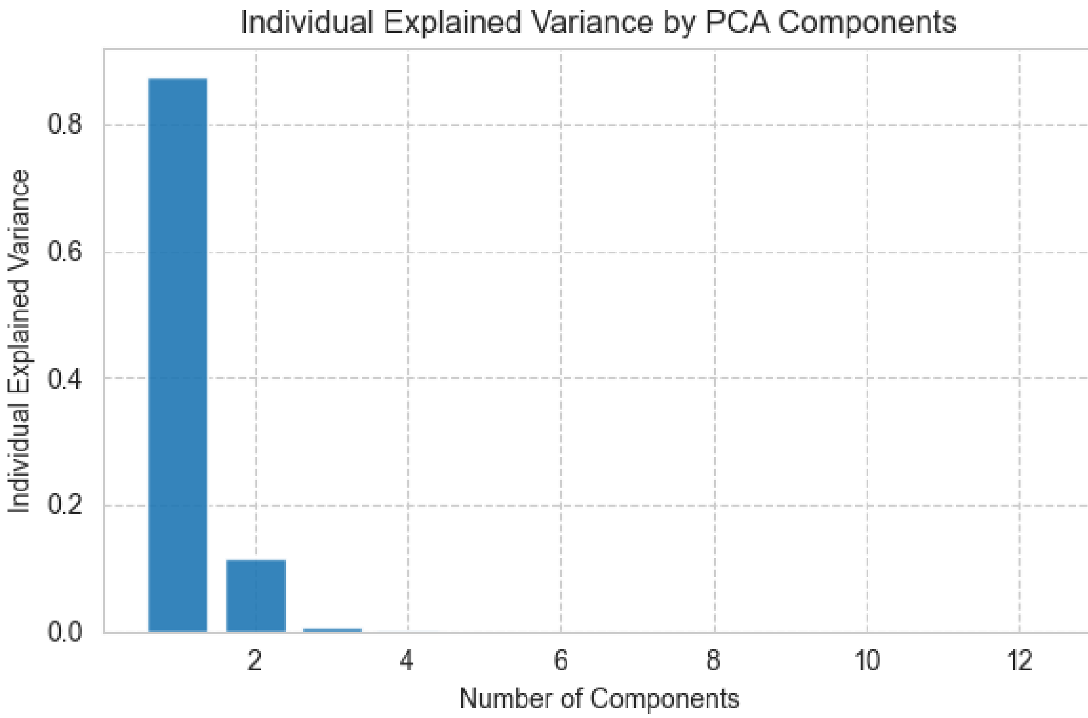

Figure 1: Individual Explained Variance of each PC. Easy to see the vast majority of the moves in the cash market can be explained by the level factor and that all we need are the first three PCs to explain all of the variability. This PCA was ran on level CMT rates from 2000 to late Q4 2024

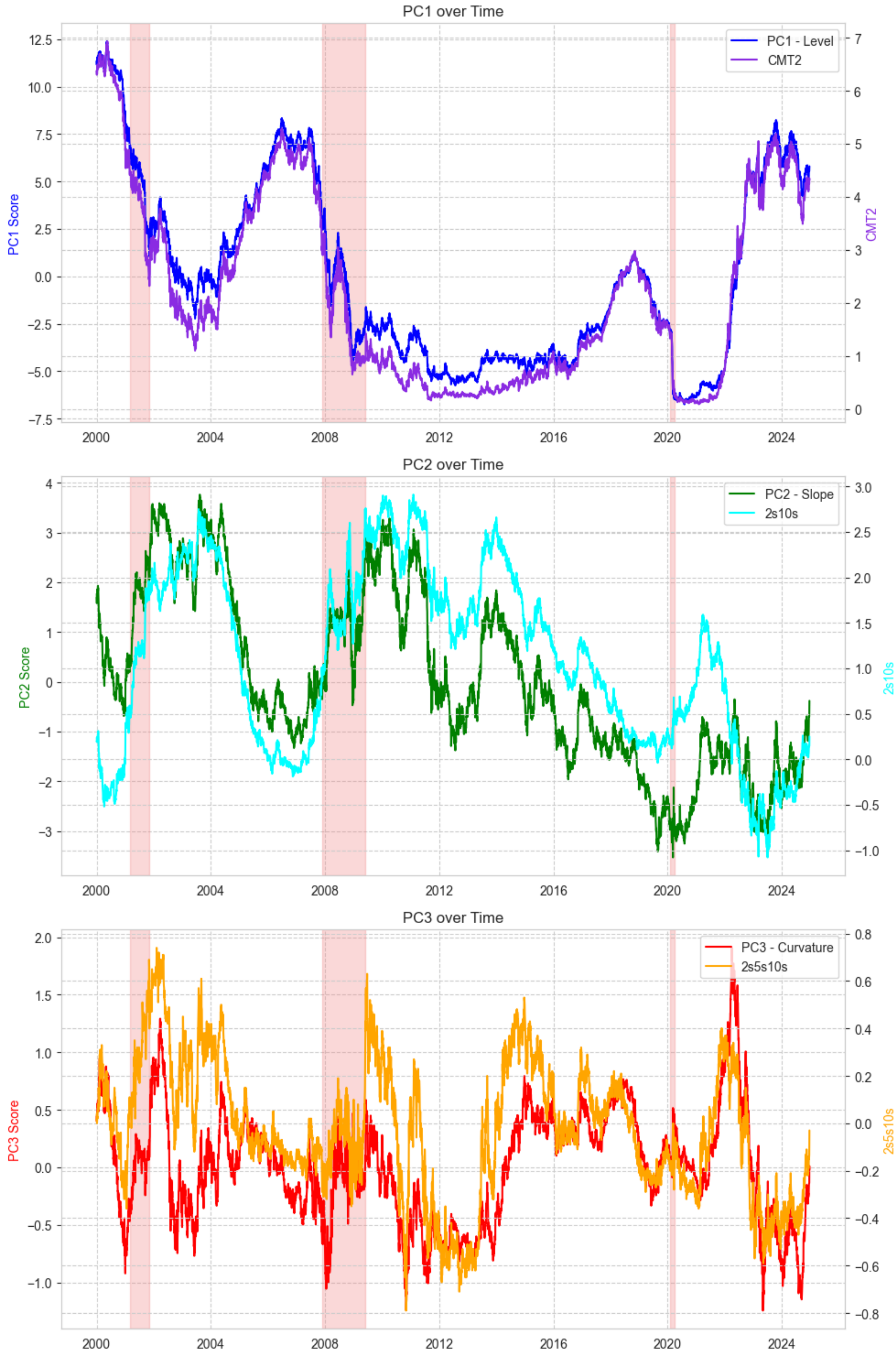
PC1 over Time
PC1 Score
CMT2
PC1 - Level
CMT2
PC2 over Time
PC2 Score
2s10s
PC2 - Slope
2s10s
PC3 over Time
PC3 Score
2s5s10s
PC3 - Curvature
2s5s10s
2000
2004
2008
2012
2016
2020
2024

Figure 2: Overlay of CMT2 and PC1, CMT 2s10s and PC2, CMT 2s5s10s and PC3 - same PCA results as Figure 1

2 "Backward Looking" Hedge Ratios:

Say that we think the yield curve is too flat and want exposure to a steeper yield curve. How do we structure the trade? We can buy some portion of the 2-year and sell some of the 10-year. Now comes the harder question, how much exactly of each? Typically, we want to take out the directional interest rate risk and thus keep ourself DV01/duration neutral. We also need enough conviction that we will realize a steeper yield curve than the forwards, overcoming the carry and rolldown of our 2s10s trade as it becomes a 1s9s in 1 year time. We haven't however mentioned that we have been assuming a yield beta of 1, which means we expect the movement between 2s and 10s to be 1:1. A simple regression of the 4.125% Oct-26s on 3.875% Aug-34s (first off-the-runs at time of writing) for the past 60 trading days (**Figure 3**) shows that a yield beta of 1 doesn't really make sense.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     91282CLS8-eod_yield   R-squared:                       0.658
Model:                             OLS   Adj. R-squared:                  0.652
Method:                  Least Squares   F-statistic:                     104.0
Date:                 Sun, 22 Dec 2024   Prob (F-statistic):           3.39e-14
Time:                         23:31:42   Log-Likelihood:                 130.75
No. Observations:                   56   AIC:                            -257.5
Df Residuals:                       54   BIC:                            -253.4
Df Model:                            1
Covariance Type:             nonrobust
===============================================================================
=======
                          coef    std err          t      P>|t|      [0.025
0.975]
-------------------------------------------------------------------------------
-------
const                  -0.0001      0.003     -0.034      0.973      -0.007
0.006
91282CLF6-eod_yield     0.6417      0.063     10.199      0.000       0.516
0.768
==============================================================================
Omnibus:                       16.346   Durbin-Watson:                   2.449
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               34.354
Skew:                          -0.796   Prob(JB):                     3.47e-08
Kurtosis:                       6.492   Cond. No.                         19.7
==============================================================================

Notes:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

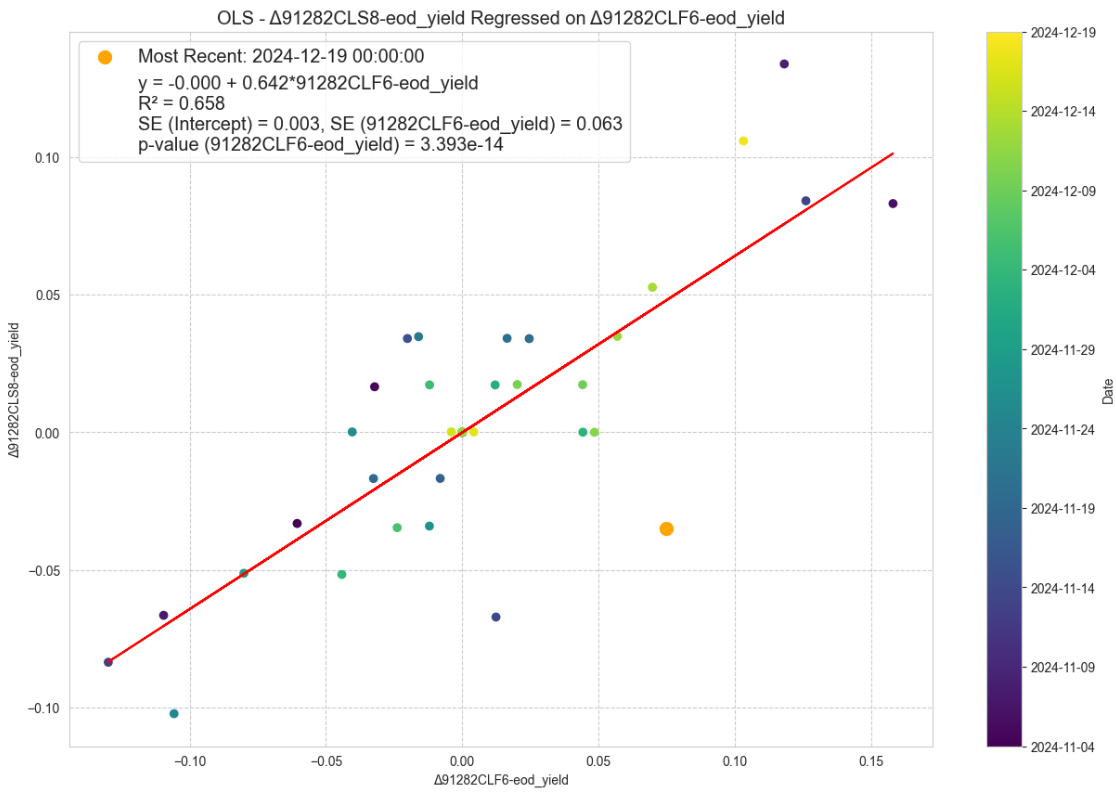

Figure 3: ~2 month OLS regression of 4.125% Oct-26s on 3.875% Aug-34s with a date color bar. Data is sourced from FedInvest's EOD markings. Perhaps call me a cherry picker because in the current macro cycle we have seen bear steepening as the Fed enters into a *front-loaded* cutting cycle so we would expect the FOMC sensitive 2-Year to somewhat diverge away from the 'tariff scared' 10-Year.

I'd say that in this current environment - *front-loaded* cutting cycle and bear-steepening - we have good reason to question the assumption of parallel yield shifts. We implicitly assume that the most relevant period for designing an empirical hedge is the recent, immediate past. This assumption is often reasonable, but there are times and situations in which some earlier period seems more relevant say like the US election. Here, we can do some middle school math to see that for every 1bp change in the 4.125% Oct-26, we expect a sixth of a bp move in the 3.875% Aug-34 - which is very different from a parallel shift. The well below 1.0 R^2 indicates that hedging in this case does not come close to eliminating all interest rate risk which is something to keep in mind. Below, we show the computation of the hedge ratios.

For DV01 neutrality, if the DV01s of bonds Oct 26s and Aug 34s are $\text{DV01}_{\text{Oct 26s}}$ and $\text{DV01}_{\text{Aug 34s}}$, then $n_{\text{Oct 26s}}$ notional of bond Oct 26s is hedged with $n_{\text{Aug 34s}}$ notional of bond Aug 34s such that,

$$\frac{n_{\text{Oct 26s}}\,\text{DV01}_{\text{Oct 26s}}}{100} + \frac{n_{\text{Aug 34s}}\,\text{DV01}_{\text{Aug 34s}}}{100} = 0$$

$$n_{\text{Aug 34s}} = -n_{\text{Oct 26s}} \frac{\text{DV01}_{\text{Oct 26s}}}{\text{DV01}_{\text{Aug 34s}}}$$

For regression hedging, we simply weight the above by the β we found.

$$\frac{n_{\text{Oct 26s}}\,\text{DV01}_{\text{Oct 26s}}}{100}\beta + \frac{n_{\text{Aug 34s}}\,\text{DV01}_{\text{Aug 34s}}}{100} = 0$$

$$n_{\text{Aug 34s}} = -n_{\text{Oct 26s}} \frac{\text{DV01}_{\text{Oct 26s}}}{\text{DV01}_{\text{Aug 34s}}}\beta$$

We can also extend the PCA framework we explore above for hedging purposes. We mentioned that PCA allows us to view and importantly quantify the driving forces of a trade into uncorrelated factors. This allows us to hedge against specific factors. For our `4.125% Oct-26s - 3.875% Aug-34s` steepener, we can simply see how changes in the first (level) factor impact the Oct 26s and Aug 34s and choose the hedge ratios in such a way that both net out. We have now exposure only to higher order PCs.

$$\frac{n_{\text{Aug 34s}}}{n_{\text{Oct 26s}}} = \frac{DV01_{\text{Oct 26s}}}{DV01_{\text{Aug 34s}}} \cdot \frac{e^1_{\text{Oct 26s}}}{e^1_{\text{Aug 34s}}}.$$

e^1 is the factor loadings (entries of the eigenvector) for PC1 of the respective bond. Factor loadings indicate how strongly each variable projects onto a principal component i.e. its the cosine of the angle between the variable's vector in the original space and the principal component's vector: a high factor loading means the variable is closely aligned with the principal component, contributing significantly to the variance captured by that component. The ratio

$$\frac{e^1_{\text{Oct 26s}}}{e^1_{\text{Aug 34s}}}$$

may look like the beta weighting we did in regression hedging however this ratio is independent of conditional expectations or the regression framework and it rather represents the relative alignment of the two variables with the direction of the first principal component. Beta measures the conditional expectation of x_1 given x_2 and thus it is directional, reversing the roles of x_1 and x_2 results in a different β.

Note: OLS is only one way of estimating the beta. There are many different variation you can make to your objective function to tweak your beta estimates. This is a bit outside the scope of this paper; however, as much as least squares is already the *best fit* considering other methods like TLS may prove to be valuable in RV-type trades.

Below are empirical results:

	91282CLF6-eod_yield	91282CLS8-eod_yield
91282CLF6-eod_yield	26.145404	16.777040
91282CLS8-eod_yield	16.777040	16.354676

Figure 4: 4.125% Oct-26s, 3.875% Aug-34s covariance matrix in bps

	PC1	PC2
91282CLF6-eod_yield	4.978674	-1.165423
91282CLS8-eod_yield	3.733565	1.554081

Figure 5: 4.125% Oct-26s, 3.875% Aug-34s factor loadings matrix in bps from a PCA using the past ~60 trading days on level changes

Method	Beta Estimate
ols	0.641682
tls	0.749901
odr	None
pc1	0.749912
box_tiao	[0.532690924628986]
johansen	[0.6947526053725753]
minimum_half_life	[0.5276945786812411]
adf_optimal	[0.2672442622001109]

Figure 6: Different *beta* (not really beta) estimations of 4.125% Oct-26s vs 3.875% Aug-34s

Below, I pulled the output of my Python script that calculates different hedge ratios based on a passed in beta adjustment. I set the front-leg notional amount to 100mm of the 4.125% Oct-26.

```
4.125% Oct-26 / 3.875% Aug-34
BPV Neutral Hedge Ratio: 4.257645882702951
Front Leg: 4.125% Oct-26 (OST 2-Year, TTM = 1.865753) Par Amount = 100_000_000
Back Leg: 3.875% Aug-34 (OST 10-Year, TTM = 9.660274) Par Amount = 23_487_158
Total Trade Par Amount: 123_487_157.63475269
-------------------------------------------------------------------------------

4.125% Oct-26 / 3.875% Aug-34
OLS Beta Weighted Hedge Ratio: 2.732056
Front Leg: 4.125% Oct-26 (OST 2-Year, TTM = 1.865753) Par Amount = 100_000_000
Back Leg: 3.875% Aug-34 (OST 10-Year, TTM = 9.660274) Par Amount = 36_602_477
Total Trade Par Amount: 136_602_476.796359
-------------------------------------------------------------------------------

4.125% Oct-26 / 3.875% Aug-34
PC1 Neutral Hedge Ratio: 3.192858
Front Leg: 4.125% Oct-26 (OST 2-Year, TTM = 1.865753) Par Amount = 100_000_000
Back Leg: 3.875% Aug-34 (OST 10-Year, TTM = 9.660274) Par Amount = 31_319_903
Total Trade Par Amount: 131_319_902.65392204
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```

Using the 3.733565/4.978674 PC1 factor loadings ratio, we can confirm that we are hedged from PC1 moves using the calculated hedge ratios above

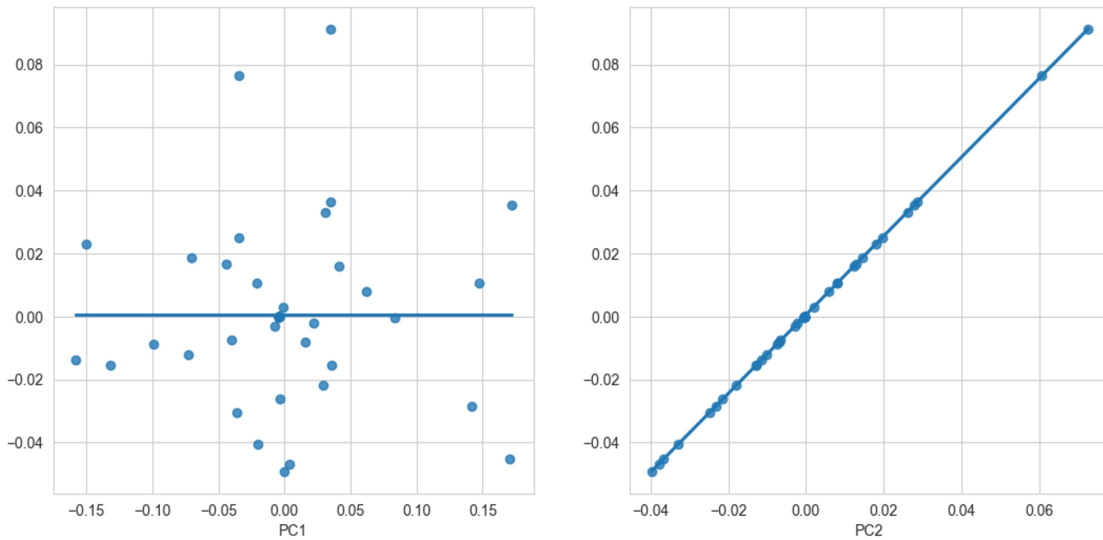

Figure 7: Regression plots of PCA weighted 4.125% Oct-26 / 3.875% Aug-34 steepener on PC1 scores and PC2 scores

There's an infinite number of ways to tweak the way you run PCA but we largely get the same results between our two "backward looking" hedge ratios. Give that the 3.875% Aug-34 are less volatile than previously assumed in the DV01 neutral case - we overweight them. While these hedge ratios may perform well in an in-sample backtest, there is a possibility that we will become unhedged as we experience a shift in the upcoming regime change. We know that historical betas between two different points on the curve can be wildly different from ex-post betas in periods of regime change. This makes us wonder whether we could use information from forward-looking markets to derive forward looking hedge ratios - using only forward-looking implied vol and implied correlation information which then allows our trade to be subject to no lags whatsoever. This framework was explored by Srini Ramaswamy et al (2022) in their note "Interest Rate Derivatives: Implied Principal Components Analysis" *JP Morgan North America Fixed Income Strategy* which they coined the "i-PCA: Implied Principal Component Analysis". We will explore the extension of this i-PCA framework into the cash markets for curve and fly trades.

Part of the i-PCA framework includes observing forward looking implied correlations from the YCSO market. But before we continue with the framework, let's briefly explore YCSOs.

3 Yield curve spread options (YCSOs):

Yield curve spread options (YCSO) are options on the yield curve which is defined as the spread of two swap rates. These options are typically traded in the OTC market and the underlying is the spread between two swaps with different maturities. We consider two different payoff cases:

Payoff of a curve cap (steepener):

$$\max((r_10 - r_2) - \text{Strike}, 0) \cdot \text{Notional}$$

Payoff of a curve floor (flattener):

$$\max(\text{Strike} - (r_10 - r_2), 0) \cdot \text{Notional}$$

where r_10 is the longer maturity 10-year swap rate and r_2 is the shorter maturity 2-year swap rate.

The performance of an YCSO is independent of the absolute level of interest rates and is entirely a function of the spread between these two rates. It's easy to see that then its performance is dependent on the correlation (and the varience) between these two rates. The variance of the yield curve can be expressed as:

$$\sigma^2_{r_10-r_2} = \sigma^2_{r_10} + \sigma^2_{r_2} - 2\rho\sigma_{r_2}\sigma_{r_10}$$

where σ^2 is the variance of the respective swap rate and ρ is the correlation between the two rates.

This is about all we need to know right now to continue with the i-PCA framework.

We explained above in the regression hedging section that betas depend on volatility as well as correlation. This is exactly why say the ratio between 6Mx2Y implied bpvol and 6Mx10Y implied bpvol won't provide the necessary forward-looking beta adjustment given that it has no information about the correlation (implictly assumes a correlation of 1). We just saw how YCSO can be expressed in terms of volatility and correlation - just what we're looking for! A small reminder that beta can be expressed in terms of the correlation coefficient between some variable X and Y multiplied by the volatility of Y and divided by the volatility of X. This allows us to derive an implied beta as simply the product of the implied vol ratio and the implied correlation from YCSOs.

Our beta can be written as:

$$\beta = \frac{\text{Cov}(\Delta r_{10}, \Delta r_2)}{\text{Var}(\Delta r_{10})}.$$

ρ is the correlation coefficient and can be written as:

$$\rho = \frac{\text{Cov}(\Delta r_{10}, \Delta r_2)}{\sigma_{r_{10}}\sigma_{r_2}}.$$

Rearranging, we then get:

$$\beta = \rho \cdot \frac{\sigma_{r_2}}{\sigma_{r_{10}}}.$$

This expression allows us to write beta in terms of the correlation coefficients and volatilities. We can use information from the option market as the forward looking inputs. Say as an example, given the premium for an 6M option on the 2s/10s curve, we can back out an implied curve volatility $\sigma_{r_{10}-r_2}$ and then implied correlation can then be inferred from the 6Mx2Y and 6Mx10Y swaptions implied volatilities and the backed out implied curve volatility.

We can plug those input into the β equation to derive our 6M forward looking β

$$\beta_{\text{implied, 6M}} = \rho_{\text{6M}} \cdot \frac{\sigma_{r_{2,\ \text{6M}}}}{\sigma_{r_{10,\ \text{6M}}}} = \frac{\sigma^2_{r_{2,\ \text{6M}}} + \sigma^2_{r_{10,\ \text{6M}}} - \sigma^2_{r_{10,\ \text{6M}} - r_{2,\ \text{6M}}}}{2\sigma_{r_{10,\ \text{6M}}}\sigma_{r_{2,\ \text{6M}}}}.$$

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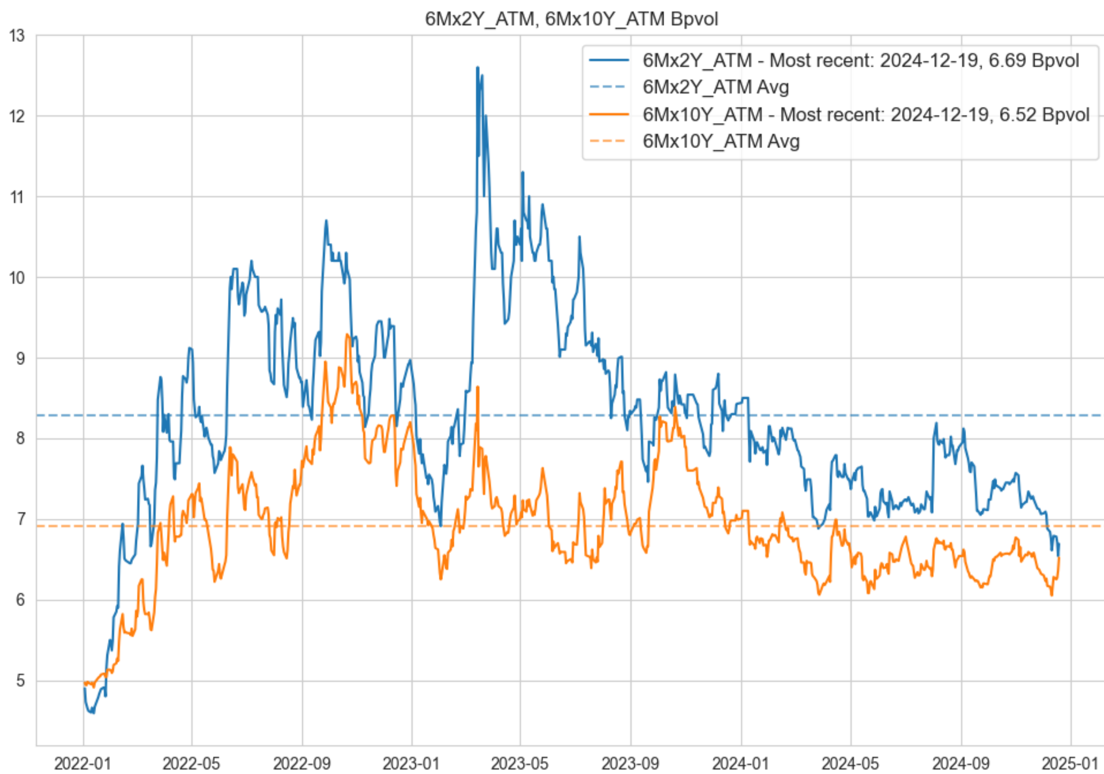

Figure 8: Overlay plot of 6Mx2Y and 6Mx10Y Bpvol from 2022 to late Q4 2024

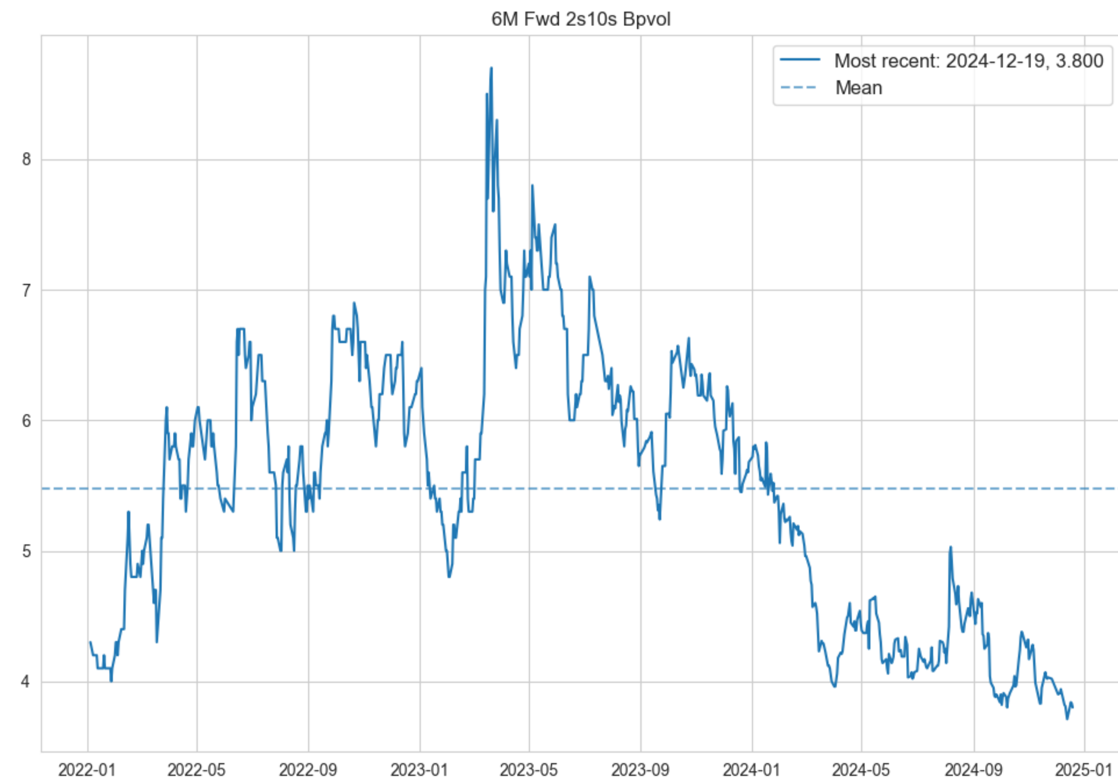

Figure 9: 6m 2s10s ATM YCSO Bpvol from 2022 to Late Q4 2024

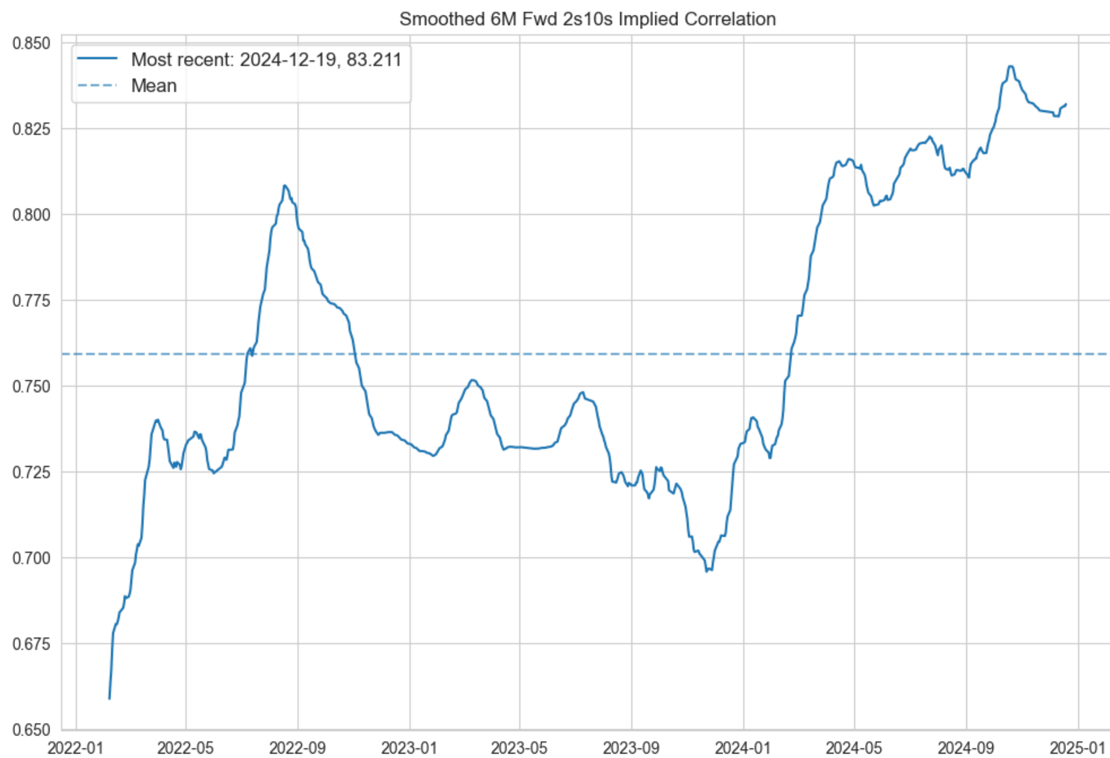

Figure 10: Smoothed 6M Fwd 2s10s Implied Correlation from 2022 to late Q4 2024

6M Fwd 2s10s Implied Correlation gives a good summary of the macro backdrop as the world came out of the COVID slump and experienced decades high inflation and the Fed's response to it. In early 2022, U.S. inflation data starts coming in hotter, the Fed shifts from a dovish stance (QE, near-zero rates) to telegraphing a series of rate hikes. The short end (2s) reprices higher very quickly on the Fed's guidance for front-loaded hikes. The long end (10s) also moves up but is partially anchored by longer-run growth/inflation expectations. Initially, the two ends of the curve often rise or fall together in response to a single dominant shock—namely, the Fed pivot. That can boost implied correlation in early phases of a hiking cycle, because both ends of the curve are reacting to the same macro signal. By the summer of 2022, markets are debating whether inflation has peaked and whether recession risk is looming. There's increasing talk of an inverted yield curve. Rate volatility is extremely high. The front end prices ever-higher Fed terminal rates, but the long end starts flirting with the notion of a recession, which caps 10-year yields somewhat. As markets struggle with the "stagflation vs. recession" debate, the short end's sensitivity to each new CPI print or Fed statement can diverge from the long end's focus on growth risks. That divergence can reduce the day-to-day correlation between 2y and 10y yields. Heading into 2023, inflation does indeed moderate somewhat—but not enough for the Fed to sound dovish. The "higher for longer" mantra emerges. Meanwhile, growth doesn't crater as quickly as some had feared, fueling a soft-landing narrative. 2s yield keeps climbing on hawkish Fed rhetoric, while the 10y yield moves somewhat sideways, torn between sticky inflation and a still-resilient labor market. Implied correlations move in a relatively moderate band—there's enough commonality (the Fed remains in tightening mode) to keep a floor under correlation, but enough uncertainty about the long end's reaction to keep it below peak levels. As 2023 progresses, central banks globally face diverging data. Some see inflation starting to roll over, while pockets of persistent pricing pressure remain. Markets also watch for cracks in growth in China, Europe, and the U.S. The front end starts to price in "the Fed might pause soon," but the 10y yield is pulled in multiple directions—on one hand, if the Fed is done, that's bullish for duration; on the other hand, if inflation remains sticky, then 10y yields might rise. The interplay of contradictory forces can temporarily depress 2s10s correlation. Day-to-day yield moves become more idiosyncratic, depending on the economic data release du jour. By 2024, the Fed either pauses or moves to a gentler path of rate hikes—still "higher for longer," but the pace is less aggressive than in 2022–23. Meanwhile, recession chatter may either come to fruition or continue to get pushed out. If growth disappoints, the 10y yield could drop and the curve might steepen from the front end. If growth stays resilient, the 10y yield could move higher in tandem with 2s. As the big macro question flips from "when does the Fed tighten" to "when does the Fed cut," both ends of the curve can start trading off the same eventual policy pivot (timing + magnitude), pushing correlations higher. By late 2024, the market narrative centers on whether the Fed (and other central banks) will begin an easing cycle in 2025, or if inflation worries might flare up again. Yields at both ends of the curve often get whipsawed by the same fundamental narrative: is the economy strong enough to justify higher real yields at the long end, and is the Fed truly done tightening at the short end? The chart suggests that by Q4 2024, the 2y and 10y yields are largely moving in sync, responding to the same big macro factors—Fed policy, inflation outlook, and real economy signals—leading to elevated implied correlation.

From a macro perspective, implied correlation often spikes when the market sees a single dominant driver (e.g., a major Fed pivot, crisis fears, or a unifying inflation narrative) that affects both short- and long-term yields in broadly the same direction. It tends to fall when the market is pulled in two directions—say, near-term policy uncertainty in the front end, versus longer-run growth and inflation expectations shaping the back end. Across 2022–2024, the Fed's turbocharged hiking in 2022 initially lifted correlation, as everything priced off inflation prints and the Fed's hawkish pivot. Mid-2022 into 2023 saw more divergence between short-end and long-end drivers as growth concerns and inflation debates churned. By late 2023 and into 2024, markets increasingly shift toward consensus about the forward path of policy (whether that's an eventual cut or an even higher plateau), which again drives up correlation as the "one big theme" reemerges.

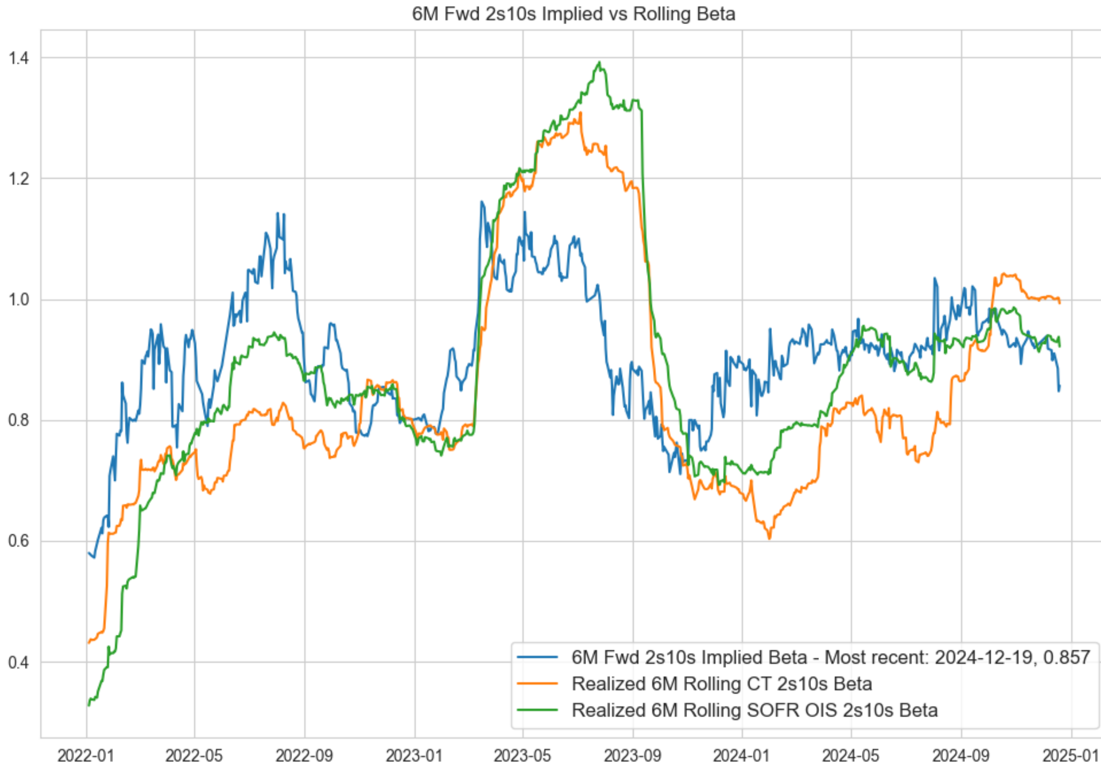

Figure 11: An overlay plot of 6M Fwd 2s10s Implied Betas and 6M Rolling Realized Betas from CT yields and SOFR OIS rates from 2022 to late Q4 2024

Similar to implied correlations, implied betas also give a good summary of the past two years in the US rates market. In early to mid-2022, we see that rate volatility surges, the market begins pricing in strong co-movement (i.e., a high slope coefficient) between 2s and 10s for the next 6 months. Traders see both ends of the curve responding to the same dominant driver—namely, the Fed's near-term hikes. In effect, the implied beta is a forecast of "if the 10y moves 1bp, how much will 2y move?" The Fed pivot fosters strong expectations of a "together" move. The CT yields (orange) may initially lag behind the implied forecast. Realized co-movement is more idiosyncratic: each day's short-end moves can diverge from the long end depending on data surprises. The SOFR OIS (green) curve's realized beta also climbs, though from a lower base. OIS rates reflect a more "pure" interest rate view with fewer term premium and credit components, so the realized co-movement with 2s might differ from the CT yields. Still, both realized betas are generally trending higher through mid-2022, converging toward the implied. By summer/fall 2022, rate-hike expectations are near their peak. The 2s10s curve is flirting with or fully in inversion. Market participants debate if the Fed will overshoot and push the economy into recession. Because forward markets anticipate both ends of the curve moving in reaction to each new inflation print or Fed statement, the 6M forward implied beta remains high. The actual day-to-day correlation can sometimes weaken when the front end trades off immediate Fed expectations, whereas the 10y yield becomes more sensitive to growth and recession risks. As a result, the realized betas (orange and green) might be more volatile, sometimes dipping below the implied measure when near-term data triggers short-end movement but less so in the long end. From late 2022 to early 2023, the Fed remains hawkish—signaling higher for longer rather than quick rate cuts. The implied beta may remain at a relatively high plateau—markets are pricing that 2s and 10s will move together if conditions suddenly change (e.g., if inflation either re-accelerates or if data confirm a slowdown). Meanwhile, the realized betas can lag or deviate. If the data prints remain mixed, sometimes the front end re-prices significantly while the 10y remains more rangebound—and vice versa. The gap between implied (forward) and realized betas can become noticeable in periods of repeated "data whiplash." By mid-2023, some central banks pause rate hikes, and markets increasingly speculate about an eventual Fed pivot. Meanwhile, concerns over global growth, China's reopening, or Europe's energy issues add cross-currents. As markets see a potential turning point for the Fed, the forward 2s10s beta can drop—if participants believe the short end's future path will be dominated by a Fed pivot (potential cuts) while the 10y might respond more slowly to sustained growth or inflation surprises. Alternatively, if the entire yield curve is expected to rally or sell off together upon a pivot, the implied beta can remain high. One can see in the chart that it eventually does come down from the peak, but perhaps not drastically, as a single macro driver (Fed cuts on the horizon) can keep correlation high. We see that realized betas show distinct patterns: The CT yields line (orange) might respond more strongly to supply/demand and risk sentiment, so it can diverge from the simpler OIS measure. The SOFR OIS measure (green) is more about pure rates, so it might track the implied measure better or remain more stable if credit risk or supply factors in CTs deviate. From late 2023 to early 2024, see that the implied betas tends to fall, then stabilize. The 6M forward 2s10s beta might come off earlier highs as the market sees that the short end could drop significantly if/when the Fed eases, whereas the 10y might not rally quite as strongly if longer-term inflation risk remains. However, it won't necessarily collapse if markets foresee both ends of the curve moving in tandem once a cut cycle starts (both short and long yields might rally). Compare this to realized where we often see in the chart that the realized betas can eventually converge more with the implied measure as the overarching policy narrative becomes more singular: "the Fed shift." By the end of 2024, the market is looking ahead to 2025, debating whether inflation could resurge or if deeper cuts are warranted. As the major drivers become more uniform (e.g., either "Fed on hold" or "Fed easing cycle"), the short and long ends can indeed trade more in unison, boosting realized betas closer to the implied measure.

Let's see what our `4.125% Oct-26 / 3.875% Aug-34` steepener hedge ratio looks like with the 6M Fwd 2s10s implied beta:

```
4.125% Oct-26 / 3.875% Aug-34
6M Fwd 2s10s Implied Beta Weighted Hedge Ratio: 3.646983
Front Leg: 4.125% Oct-26 (OST 2-Year, TTM = 1.865753) Par Amount = 100_000_000
Back Leg: 3.875% Aug-34 (OST 10-Year, TTM = 9.660274) Par Amount = 27_419_922
Total Trade Par Amount: 127_419_921.7569536
------------------------------------------------------------------------------
```

Recall that our backwards looking beta was `0.641682`. The 6M Fwd 2s10s implied beta is `0.857`. This means that the forward looking information from the option market is signaling higher volatility for 10s (so we can sell less of it if we're buying 100mm of 2s) compared to historical information, which makes reasonable sense given the current macro backdrop as fiscal uncertainity remains high as the Trump administration enters back into Washington and confidence of the progress of inflation toward the FOMC 2 percent objective has wavered.

4 Implied Principal Component Analysis:

We have already described the process of backing out implied correlations from YCSOs. Let's explore how we can construct the implied covariance matrix so that we can conduct eigendecomposition on this to derive a hedge ratio that elimiates PC1 (directional) exposure on a forward-looking basis. We first multiply the implied 2s10s correlation from YCSOs by the product of implied volatilities from corresponding swaptions (same expiry as YCSOs and 2s, 10s tails) to get the implied covariance between 2s and 10s. We can repeat this for a number of different tenors yielding us an implied covariance matrix. We describe the factorization/eignedecomposition process below:

I have limited this example to only two tenors (2s, 10s) because I hate writing the LaTeX.

Let's first revisit the process of going from spread vols to pairwise covariances:

Let r_i and r_j denote two rates with implied volatilities σ_{r_i} and σ_{r_j}. We also have an implied volatility for the spread $r_j - r_i$, denoted $\sigma_{r_j - r_i}$. By definition, we know:

$$\sigma^2_{r_j - r_i} = \mathrm{Var}(r_j - r_i) = \mathrm{Var}(r_j) + \mathrm{Var}(r_i) - 2 \cdot \mathrm{Cov}(r_j, r_i).$$

In symbols:

$$\sigma^2_{r_j - r_i} = \sigma^2_{r_j} + \sigma^2_{r_i} - 2 \cdot \mathrm{Cov}(r_j, r_i).$$

Rearrange to solve for $\mathrm{Cov}(r_j, r_i)$:

$$\mathrm{Cov}(r_j, r_i) = \frac{\sigma^2_{r_j} + \sigma^2_{r_i} - \sigma^2_{r_j - r_i}}{2}.$$

Equivalently, if we let ρ_{ij} be the correlation between r_i and r_j, then:

$$\mathrm{Cov}(r_j, r_i) = \rho_{ij} \cdot \sigma_{r_j} \cdot \sigma_{r_i},$$

and thus:

$$\rho_{ij} = \frac{\mathrm{Cov}(r_j, r_i)}{\sigma_{r_j} \cdot \sigma_{r_i}} = \frac{\sigma^2_{r_j} + \sigma^2_{r_i} - \sigma^2_{r_j - r_i}}{2 \cdot \sigma_{r_j} \cdot \sigma_{r_i}}.$$

Hence, if one is given σ_{r_i}, σ_{r_j}, and $\sigma_{r_j - r_i}$, one obtains both the covariance and correlation of r_i and r_j.

Now on the construction of the Implied Covariance Matrix:

Suppose we have n rates $r_1, \ldots, r_n$. Their implied volatilities form:

$$\sigma_{r_1}, \sigma_{r_2}, \ldots, \sigma_{r_n}.$$

We may also have implied spread volatilities $\sigma_{r_j - r_i}$ for selected pairs (i, j). For each pair (i, j), the covariance is:

$$C_{\mathrm{imp}}(i, j) = \mathrm{Cov}(r_i, r_j) = \frac{1}{2}\left(\sigma^2_{r_i} + \sigma^2_{r_j} - \sigma^2_{r_j - r_i}\right).$$

On the diagonal, $C_{\mathrm{imp}}(i, i) = \sigma^2_{r_i}$. Thus the implied covariance matrix C_{imp} can be written in block form. For instance, with (2Y, 10Y), the (i, j) diagonal entries are $\sigma^2_{r_{2Y}}, \sigma^2_{r_{10Y}}$; the off-diagonal entries use the above formula.

Formally, the matrix is:

$$C_{\text{imp}} = \begin{bmatrix} \sigma^2_{r_{2Y}} & \frac{1}{2}\left(\sigma^2_{r_{2Y}} + \sigma^2_{r_{10Y}} - \sigma^2_{r_{10Y}-r_{2Y}}\right) \\ \frac{1}{2}\left(\sigma^2_{r_{2Y}} + \sigma^2_{r_{10Y}} - \sigma^2_{r_{10Y}-r_{2Y}}\right) & \sigma^2_{r_{10Y}} \end{bmatrix}.$$

Provided these inputs are consistent (i.e., no arbitrage constraints on spread vols), C_{imp} will be a positive semidefinite matrix.

After brushing off my linear algebra textbook and once we have the implied covariance matrix C_{imp}, we can decompose it via a standard eigendecomposition:

$$C_{\text{imp}} = PDP^T,$$

where:

- $D = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$ is the diagonal matrix of eigenvalues.
- $P = [\mathbf{p}_1\, \mathbf{p}_2\, \dots\, \mathbf{p}_n]$ is the orthogonal (or orthonormal) matrix of eigenvectors, so $P^T P = I$.

By convention, we sort $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$, so the first eigenvalue is the largest.

Some reminders:

- Each eigenvalue λ_i measures the variance explained by the i-th principal component (PC).
- Each eigenvector $\mathbf{p}_i$ is an n-dimensional vector that describes how that PC's "shock" loads onto each of the n rates (2Y, 10Y in our case above but is easily extended to any number of tenors).
- Orthogonality means the principal components are uncorrelated with one another.

We need to get a sense of comparable magnitude if we want to get a use of the eigenvalues.

After sorting the eigenvalues, we define:

$$V = P\sqrt{D},$$

where $\sqrt{D} = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_n})$. The columns of V are then:

$$\mathbf{v}_i = \sqrt{\lambda_i}\, \mathbf{p}_i, \quad i = 1, \dots, n.$$

Mathematically, $\mathbf{v}_i$ is the direction of the i-th PC scaled by $\sqrt{\lambda_i}$. Interpreting $\mathbf{v}_i$ in yield-curve analysis:

- A 1-sigma move in the i-th factor (i.e., a $\pm\sqrt{\lambda_i}$ shift) will change each tenor's rate r_j by the corresponding component of $\mathbf{v}_i$.
- Typically for yield curves:
  - PC1 (largest eigenvalue) is a **"level" factor** (all tenors moving in the same direction).
  - PC2 is a **"slope" factor** (short vs. long moving with opposite signs).
  - PC3 is a **"curvature" factor**, etc.

Formally, if $\Delta r = \mathbf{v}_i$, the covariance of that move is:

$$\mathbf{v}_i^T \mathbf{v}_i = \lambda_i,$$

meaning that shock has variance λ_i.

The ratio:

$$\frac{\lambda_i}{\sum_{k=1}^{n} \lambda_k}$$

gives the fraction of the total variance in C_{imp} explained by the i-th principal component.

In practice:

- If $\frac{\lambda_1}{\sum_{k=1}^{n} \lambda_k} \approx 80\%$, the first PC alone captures ~80% of the yield curve's movement.
- The second and third PCs often capture the next largest contributions, and so on.

From a more rigorous standpoint, for any vector $\mathbf{x}$:

$$\mathbf{x}^T C_{\text{imp}} \mathbf{x} = \sum_{i=1}^{n} \lambda_i (\mathbf{x}^T \mathbf{p}_i)^2,$$

so the decomposition partitions the total quadratic form (variance measure) into mutually orthogonal directions.

This I-PCA framework provides a factorized view of how the curve *might* move in the forward based on current expectation, under the assumption that the input vol/spread-vol data are internally consistent and reflective of the market's forward-looking views.

Below is a Python implementation of I-PCA:

```python
def implied_covar_matrix(
    implied_corr_matrix: np.ndarray,
    implied_vol_vect: np.ndarray
) -> np.ndarray:
    D = np.diag(implied_vol_vect)
    return D @ implied_corr_matrix @ D

# returns tuple of size 3:
# (sorted eigenvalues, eigenvectors, scaled factor loading)
def implied_principal_components(
    covar_matrix: np.ndarray
) -> Tuple[np.ndarray, np.ndarray, np.ndarray]:
    # 1) Eigen-decomposition:
    w, P = np.linalg.eigh(covar_matrix)

    # 2) Sort eigenvalues & eigenvectors in descending order of eigenvalues
    idx = np.argsort(w)[::-1]  # indices for sorting from largest to smallest
    w = w[idx]  # sorted eigenvalues
    P = P[:, idx]  # reorder eigenvectors accordingly

    # 3) Construct the V matrix = P * sqrt(D)
    #    If w[i] < 0 (shouldn't happen for a well-defined cov),
    #    watch out for numerical issues
    D_sqrt = np.sqrt(np.diag(w))
    V = P @ D_sqrt
    return w, P, V
```

Let's plug in our implied correlation matrix from 6M 2s10s YCSOs and bpvols from 6Mx2Y, 6Mx10Y swaptions into the I-PCA framework.

	2Y	10Y
2Y	1.000000	0.832106
10Y	0.832106	1.000000

Figure 12: Implied Correlation Matrix backed out from 6M 2s10s YCSOs

6Mx2Y Bpvol	6Mx10Y Bpvol
6.69	6.52

Figure 13: 6Mx2Y, 6Mx10Y Bpvol vector

From Figure 12 and Figure 13 along with the I-PCA framework, this yields:

	2Y	10Y
2Y	44.756100	36.295454
10Y	36.295454	42.510400

Figure 14: Implied Covariance Matrix

	I-PC1	I-PC2
2Y	-6.419429	1.883358
10Y	-6.223906	-1.942523

Figure 15: Scaled Implied Factor Loading Matrix

The ratio

$$\frac{e_{\text{2Y}}^{\text{I-PC1}}}{e_{\text{10Y}}^{\text{I-PC1}}}$$

is the forward looking (not) beta adjustment to hedge against PC1 moves. This is comparable to the backward looking (not) beta adjustment

$$\frac{e_{\text{Oct 26s}}^{1}}{e_{\text{Aug 34s}}^{1}}$$

we explored above in regression hedging. In a similar manner, the forward looking ratio is independent of conditional expectations or the regression framework and it rather represents the relative alignment of the two variables with the direction of the *implied* first principal component. While implied beta measures the forward conditional expectation of x_1 given x_2 and thus it is directional, reversing the roles of x_1 and x_2 results in a different implied β. Using I-PCA to construct our hedge ratios, we can - in principal (pun intended) - be truly to isolate a "slope alpha" in the next few months, hedging out whatever the market sees as the level factor from an implied perspective, not from last year's realized data nor from a naive DV01 perspective.

Let's see what our `4.125% Oct-26 / 3.875% Aug-34` steepener hedge ratio looks like with the 6M Fwd 2s10s implied PCA results:

```
4.125% Oct-26 / 3.875% Aug-34
I-PC1 Neutral Hedge Ratio: 4.390960
Front Leg: 4.125% Oct-26 (OST 2-Year, TTM = 1.865753) Par Amount = 100_000_000
Back Leg: 3.875% Aug-34 (OST 10-Year, TTM = 9.660274) Par Amount = 22_774_065
Total Trade Par Amount: 122_774_064.58426127
--------------------------------------------------------------------------------
```

Recall that our backwards looking PC1 factor loadings ratio was 0.749912. The forward looking implied PC1 factor loadings ratio is 6.419429 / 6.223906 = 1.03131. This means that the forward looking information from the option market is signaling higher volatility for 10s compared to historical information, which makes reasonable sense given the current macro backdrop as fiscal uncertainity remains high as the Trump administration enters back into Washington and confidence of the progress of inflation toward the FOMC 2 percent objective has wavered.

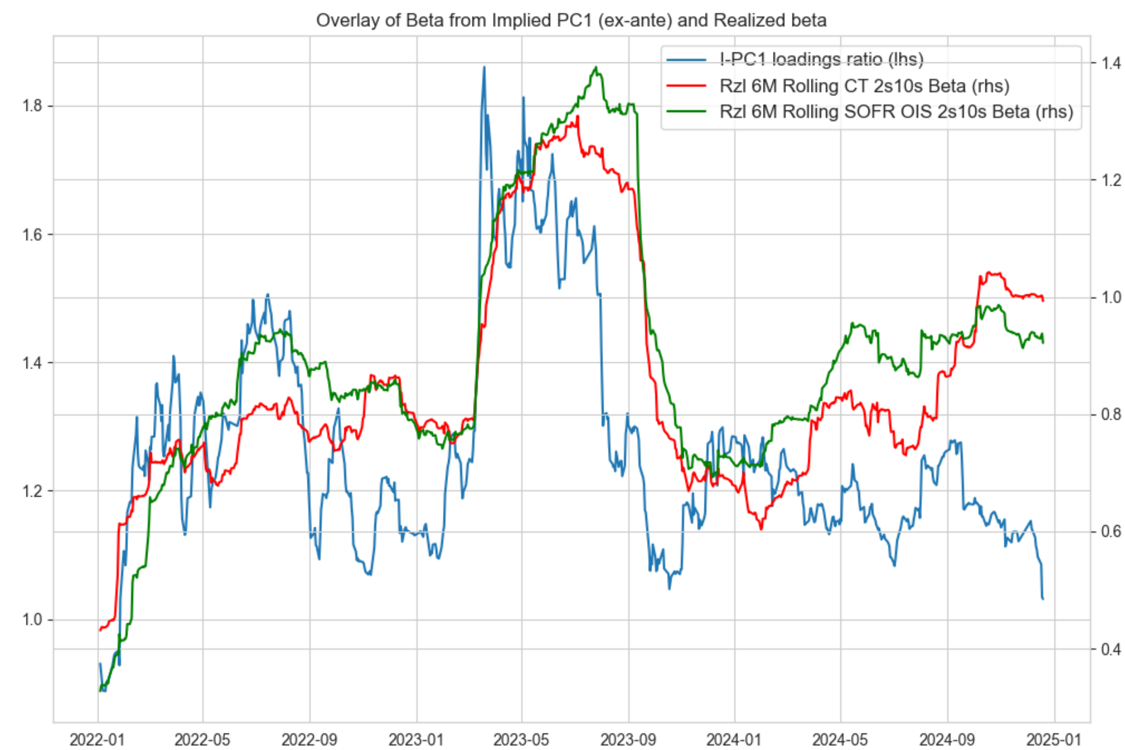

Figure 16: Overlay of Beta from Implied PC1 (ex-ante) and Realized betaas from CT and Swap yields

5 Eigenvector Stability in the Implied PCA (I-PCA)

While many of the pitfalls are similar to those in "historical" PCA, certain nuances arise because i-PCA relies on forward-looking implied volatilities and correlations, which can shift rapidly if central bank policy (or broader market expectations) changes in unexpected ways. This can cause the first (directional) eigenvector or other factors to drift over time—thereby breaking hedge ratios that were originally designed to be neutral to certain factors.

First let revisit the fundamental problem:

In a typical PCA-based hedge, you identify which principal components you want to be exposed to (e.g., a slope or curvature trade) and which ones you want to hedge out (often factor 1, the "directional" factor). You set your notionals according to the shape of those eigenvectors at the time you enter the trade. If the eigenvector for factor 1 changes shape during your holding period, you are no longer hedged against the "new" factor 1. You can end up with unintentional directional exposure—or exposure to other unintended factors.

One might ask: Why can the first eigenvector change?

- Shifts in central bank policy (normal policy $\to$ ZIRP $\to$ normal policy again).
- Regime changes in market structure (e.g., QE, yield curve control, or significant supply/demand changes).
- Short sample windows or insufficient data: If you're estimating the implied covariance matrix over a narrow time window (and implied vol data can be noisier than realized vol data), the eigenvectors may be overly sensitive to daily fluctuations or short-term anomalies.

In implied PCA, the eigenvectors are derived from an implied covariance matrix built from implied volatilities and (ideally) implied spread volatilities. Because implied vol surfaces can jump when the market's forward expectations shift (for instance, in response to a big Fed surprise), the resulting eigenvectors can realign more frequently than you might see in a stable historical dataset.

Changes in factor levels: The factor itself can be highly volatile. For instance, factor 1 might move from a big rally (+100bps) to a big sell-off (–100bps). That, by itself, does not break your hedge, provided the shape of the eigenvector remains the same.

Changes in factor loadings (the eigenvector shape): This does break your hedge. For example, if short-end rates become pinned at zero, the relative volatility of the short end vs. the long end changes. The first eigenvector may tilt more toward the belly and long end if the short end is no longer expected to fluctuate meaningfully.

Let go through a specific senarios:

1. Initial Hedge Ratios: You perform i-PCA at time t_0. The first eigenvector $\mathbf{p}_1^{(t_0)}$ strongly weights the short end, so you design a 5s–10s steepener that is "factor 1 neutral."
2. Policy Shift: Six months later, the central bank announces a new policy pegging short rates near zero for an extended period. The implied volatility on short-dated instruments collapses, and the new i-PCA at time t_1 yields a first eigenvector $\mathbf{p}_1^{(t_1)}$ that underweights the short end.
3. Hedge Breakdown: Your trade is still hedged against $\mathbf{p}_1^{(t_0)}$, not $\mathbf{p}_1^{(t_1)}$. The new eigenvector means the short end barely moves with directional shifts, while the belly or long end moves more. Therefore, you've effectively acquired an unintentional directional exposure, exactly what you tried to hedge out.

5.1 Why Instability Can Be Even More Pronounced in I-PCA?

Implied vol surfaces are highly sensitive to events like central bank meetings, macro data surprises, or even changes in risk sentiment. Large forward shifts can cause recalibration of the entire implied correlation structure—hence the factor shapes can jump. Furthermore, if near-term or short-end implied vols become pinned (or spike drastically), the cross-tenor correlation structure may rearrange more dramatically than in a historical average sense. Also important to note, the implied matrix is built from traded option prices, which can have liquidity constraints at certain tenors/spreads. If liquidity temporarily dries up or trading flows shift, the estimated implied covariance can become "patchy," causing abrupt changes in PCA results.

The importance of this problem in your trade depends on your goals, say for short-term relative value (RV) trades, eigenvector instability is typically a smaller problem since factor shapes rarely shift that dramatically in a matter of weeks. The excerpt's example of "factor 3" (curvature) having an average mean-reversion horizon of ~84 days means you're less likely to face a major central bank regime shift within that time. Long-term macro trades (e.g., multi-year bets on factor 1) face more danger. A 5-year or 10-year horizon can easily encompass multiple Fed cycles, zero-rate policies, yield-curve controls, and so forth—each potentially reshaping the first eigenvector.

The biggest takeaway is that all of the extra math done for implied PCA does not make it into a panacea - It's forward-looking, but can be more sensitive to changes in market sentiment or policy expectations.

6 A Glaring Pitfall:

When we talk about "implied PCA," we generally mean building an implied covariance matrix out of Implied volatilities on swap rates (2y, 5y, 10y, etc.) and implied spread volatilities between those swap tenors (e.g., 5y–10y spread vol). This approach reflects the forward-looking market consensus about how different swap rates co-move. By performing an eigen-decomposition of that matrix, you get principal components that describe the major factors (level, slope, curvature, etc.)—but in the swap space, not necessarily in the cash-bond space. If our I-PCA analysis is purely on swap implied data, you're effectively building factor hedges that are correct for how swaps are expected to move. But if you then implement a cash-bond trade (e.g., 5y–10y steepener on treasuries, or a butterfly using bund cash bonds), part of the P&L may come from changes in swap spreads that are not captured in the swap-based I-PCA.

Suppose you set up a "PCA-neutral" steepener in the swap space (using I-PCA on swaps) but then implement it using cash bonds. The swap spreads (5y spread vs. 10y spread) might shift in a way that gives you unexpected exposure. Say if 5y swap spreads widen significantly while 10y swap spreads stay put, your cash bond yields vs. swap yields might deviate from your original "hedge design," suddenly giving you a net directional exposure.

6.1 Specific events that drive swap spreads and affect our trade:

- Front End vs. Long End Differences:
  - Front-end swap spreads react strongly to bank funding condition. Meanwhile, the long end may respond more to duration hedging flows or pension demand for long govies.
  - If your I-PCA analysis lumps 2y–5y–10y swaps together, the resulting factor loadings may not reflect what actually happens in 2y–5y–10y cash yields, especially in times of stress.
- Risk-Off events:
  - In a flight-to-quality scenario, treasury yields might rally more aggressively than swap rates, so swap spreads typically widen. Your I-PCA hedge might assume that 10y "level factor" moves 1:1 in swaps and govies, but in reality, the treasury yield could decline more than the swap rate.

- The cash-bond trade that was “hedge-neutral” in an I-PCA sense ends up having an unintentional bullish (or bearish) exposure as spreads blow out.

- Central bank operations (QE or QT) can disproportionately affect government bond yields relative to swap rates. If the market expects large Fed (or ECB, BoJ) purchases in certain maturities, the “cash” yield might be more influenced than the corresponding swap. The I-PCA hedge ratio based on swap data may fail to capture these supply/demand distortions in the cash market.

6.2 Potential Solution: Hedge Them Explicitly

Implement the trade in cash bonds but simultaneously enter a swap spread hedge. For example, if your I-PCA steepener is long 5y bonds, short 10y bonds, you might also buy or sell the corresponding 5y–10y swap spread (receiving in 5y, paying in 10y, or vice versa) to neutralize the risk that spreads blow out. The net result is that you remain exposed to the intended “curve shape factor” (as predicted by I-PCA on swaps) but you’re less exposed to a pure “swap spread blowout” scenario.

There’s still a good amount of stuff needed to be explored before using forward looking information from the derivatives land in cash space.

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