A bridge between mortgage TBA options and swaptions

Shijun Liu *provides basis-point volatility option pricing formulas for swaptions and mortgage to-be-announced options that make direct comparison of volatilities easy and transparent across the two interest rate derivatives markets. Basis-point volatility models remove much of the skew observed when volatilities have been quoted in Black volatilities, as in the recent market environment*

In the interest rate derivatives markets, options are usually quoted in Black volatilities. However, Black volatilities are measured differently for different instruments. For example, they are measured by the lognormal yield[1] volatility in swaptions but by the lognormal price volatility in mortgage to-be-announced (TBA) options, which are bond options. Many dealers also quote swaptions in basis-point (BP) volatility, which is derived under the assumption that swap rates are normally distributed. BP volatility is intuitive and appealing to practitioners because the volatility measurement is the simple BP rate change each day. Moreover, direct observation suggests that the actual rate distributions, including those of swap and mortgage rates, are closer to normal than lognormal in recent times (five to 10 years) in the sense that realised BP volatilities are more stable across different levels of rates. Alternatively, if one plots the implied Black yield volatilities of swaptions at different strike rates for the same underlying swap, the skewness (negative slope) and smile (curve up at high and low strikes) of the curve are more pronounced than those in the plot of implied BP volatility against the same strike rates. We show that the BP volatility concept can easily be extended to mortgage TBA options and has the same benefits as those for swaptions. The BP volatility methodology makes relative value comparison across markets feasible and very simple. The inconsistency between rate and price distributions and rate level dependency in the different applications of the Black model are removed. Empirical results are encouraging.

Pricing formulas for swaptions and mortgage TBA options

BP volatility pricing formula for swaptions. Given that the forward swap rate is a martingale under the forward swap measure and combined with the assumption that the forward swap rate is normally distributed under the forward swap measure, the price of a European-style call (payer swaption) at time t is as follows[2]:

$$C_t\left(F,K,\sigma_{BP},\tau,A_t\right) = \left[\frac{\sigma_{BP}\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{BP}^2\tau}(F-K)^2} + (F-K)\Phi\left(\frac{F-K}{\sigma_{BP}\sqrt{\tau}}\right)\right]A_t \quad (1)$$

where F is the forward swap rate, K is the strike rate, σ_{BP} is the BP volatility for the underlying swap rate, τ is the term of the option in years, $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution, and A_t is the annuity factor at time t, which is the present value of a future cashflow stream that has payouts equal to the day count fractions at each payment date of the fixed leg of the underlying swap. See, for example, Hull (2000) for a detailed definition of the annuity factor.

The pricing formula for a European-style put (receiver swaption) is:

$$P_t = \left[\frac{\sigma_{BP}\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{BP}^2\tau}(F-K)^2} + (K-F)\Phi\left(\frac{K-F}{\sigma_{BP}\sqrt{\tau}}\right)\right]A_t \quad (2)$$

In figure 1, we plot the Black and BP volatilities for four options with different maturities on a 10-year swap rate[3] against strike prices quoted on March 12, 2007. The maturity dates of the four options are one month, three months, one year and five years. For the Black volatility plot (the upper curves), the slope of the curve is negatively skewed and the curvature of the curve exhibits a smile. The skew and smile effects are stronger for the shorter-dated options than for the longer-dated options. Specifically, they are strongest for the shortest-dated option (the one-month option) and weakest for the longest-dated option (the five-year option). For the BP volatility plots, the skew effect is significantly weaker but the smile effect remains qualitatively similar to the comparable measures in the Black volatility curves. These plots provide some evidence that the implied swap rate distributions are closer to normal rather than lognormal.

BP volatility formulas for mortgage TBA options: the reduced-form one-factor Gaussian rate model. Mortgage TBA options are forward positions in pass-through securities (bonds) backed by mortgage loan pools consistent with agency (Fannie Mae, Freddie Mac and Ginnie Mae) guidelines and stamped with the agency guarantees that make the securities AAA rated. The details (gross weighted average coupon, remaining terms, etc) of the mortgage pools are not known, hence they are referred to as TBA until 48 hours before the settlement dates. Notwithstanding that, the settlement dates of most TBA options traded in the market are within six months of the current date. The most liquid are those with settlement dates within three months of the current date. TBA options usually expire a short time (for example,

[1] *We use yield and rate interchangeably in this article*
[2] *The same formula first appeared in Zhou (2003) for payer swaptions, to the best of the author's knowledge*
[3] *Similar patterns show up for swap rates at tenors other than 10 years*

one week) before the settlement dates. The options are European-style and quoted for strikes from one point under to one point above the TBA price with half-point increments. Quotes are given in price volatility and/or percentage premium. For example, on March 12, 2007, the TBA option on the Fannie Mae 30-year 5.5 coupon mortgage for May 14, 2007 has an expiry date of May 7, 2007. The average of the price volatilities for the at-the-money strike that we obtained from three dealers was 2.52%.

Since the duration of the mortgage bond is negatively correlated with the rate levels due to the embedded short positions in the prepayment options (in figure 2, we can see the negative convexity in the price against rate shock plot), the rate of BP volatility cannot be calculated by simply dividing the price volatility by the bond duration. In the derivation of the BP volatility option pricing formulas for mortgage TBA options, our approach is similar to that in Zhou & Subramanian (2004), where we separate the modelling of prepayment option from the modelling of the option on the mortgage.

Our model differs from Zhou & Subramanian's in two ways. First, we maintain the highest degree of flexibility in fitting the mortgage yield/rate level to price function mapping. We allow the rate-to-price function to be given either analytically or numerically. Zhou & Subramanian fit an analytical function for the mortgage yield-to-price profile. Second and most importantly, we discuss explicitly the effect of the financial constraints – the option price as the present value of the discounted payouts, the put call parity and the iterated conditional expectation property of the TBA price as a forward price on the joint determination of the parameters of the rate distribution, which we assume to be normal (Gaussian). In our model, put call parity holds by construction. Thus, the BP volatilities are identical for the put and call options as long as they have the same strike price.

From the pricing equation of an option and the definition of a TBA option, we can construct a system of two equations:

1. A pricing equation for a TBA option (call or put) for a given strike price.

2. A pricing equation for a TBA option under iterated conditional expectation.

For item 1, the pricing equation of the TBA call option can be written as:

$$C\left(TBA(t),\tau,T,K,\mu,\sigma_{BP}\right) = P(t,t+T)E_t^{Q^P}\left[\left(TBA(t+\tau)-K\right)1_{TBA(t+\tau)\geq K}\right] \quad (3)$$

where $C(\cdot)$ is the call option price, $TBA(t)$ is the TBA price at time t, $\tau \leq T$ is the option term, K is the strike price, $P(t, t+T)$ is the time t price of the discount bond (one unit face value) that matures at time $t+T$, which is the settlement date of the TBA, $1_{TBA(t+\tau)\geq K}$ is an indicator function that takes the value one if $TBA(t+\tau) \geq K$ and zero otherwise, $E_t^{Q^P}$ is the expectation operator taken under the 'forward risk-neutral' measure for which the discount bond (maturing at time $t+T$) P is the numeraire.[4] Later on the normal distribution assumption is assumed under this specific probability measure.

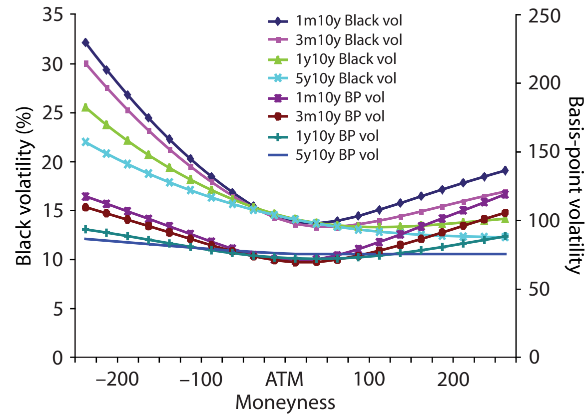
For item 2, given that the TBA option price is a forward price, by the law of iterated conditional expectation, we have:

$$TBA(t) = E_t^{Q^P}\left[TBA(t+\tau)\right] \quad (4)$$

An equation similar to (3) can be written for the put option:

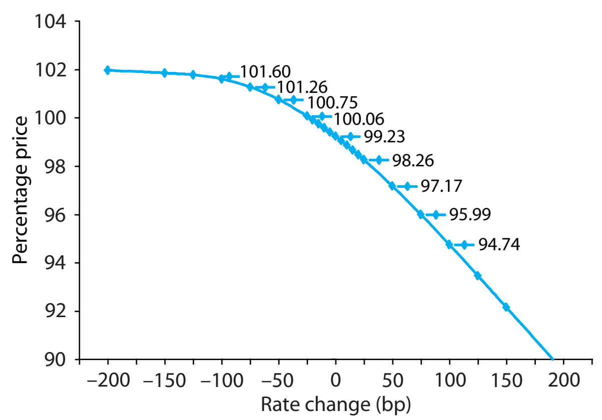
$$P\left(TBA(t),\tau,T,K,\mu,\sigma_{BP}\right) = P(t,t+T)E_t^{Q^P}\left[\left(K-TBA(t+\tau)\right)1_{TBA(t+\tau)\leq K}\right] \quad (5)$$

1 Swaption Black and BP volatility: Mar 12, 2007



Source: dealer survey

2 Sample TBA price profile against rate shocks for Fannie Mae 30Y 5.5 coupon May 2007 settlement: Mar 12, 2007



Note: profile generated using parallel rate shocks to the benchmark yield curve – Libor/swap curve. Prices for –100 to +100 basis points are listed for each 25bp increment in rate shock

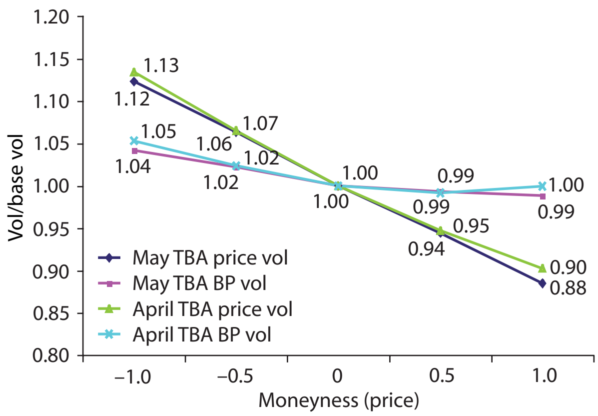
It is easily seen that equations (3)–(5) confirm that put call parity holds. One should also note that the parity does not depend on the distribution assumption.

The yield of the underlying mortgage (alternatively, under the term of yield curves, one can think of it in terms of general rate level) of the option on the expiry date is assumed to be normally distributed under the forward risk-neutral probability measure. We work with the changes in the forward terminal date $t+T$ mortgage yield/rate level from the option pricing date t to the option expiry date $t+\tau$. More specifically, if the option pricing date is time t and the option maturity date is $t+\tau$, then we assume that $R(T|t+\tau) - R(T|t) = Y \sim N(\mu, \sigma_{BP}^2\tau)$ where $R(T|t)$ and $R(T|t+\tau)$ are the forward terminal date $t+T$ mortgage yield/rate level given the different information sets at time t and $t+\tau$. There are two unknowns: μ and σ_{BP} in our specification.

Using the distribution assumption above and assuming that Y is the sole random variable driving the value of $TBA(t+\tau)$, (3) and (4) can be written as:

[4] *For a generalised discussion of change of numeraire in option pricing, consult Geman, El Karoui & Rochet (1995)*

3 Normalised price vol and BP vol comparison for options on Fannie Mae 30Y 5.5 coupon April and May TBAs



Note: option maturity April 5, 2007 and May 7, 2007, pricing date March 12, 2007. The Black price vols are from a dealer survey. We take an average of the quotes from three dealers. The BP vols are generated using the price vols and a TBA price profile like that for figure 2. Both the Black vols and the BP vols are normalised by their corresponding base cases or at-the-money values. The two BP vol curves are much flatter than the Black vol cousins

$$C\left(TBA(t),\tau,T,K,.\right)= P(t,t+T)\frac{1}{\sqrt{2\pi\sigma^2\tau}}\int_{-\infty}^{Y_K}\left[TBA(t+\tau,Y)-K\right]e^{-\frac{(Y-\mu)^2}{2\sigma^2\tau}}dY \qquad (6)$$

where $TBA(Y_K) = K$ and we also assume that the TBA prices are monotonically decreasing against rate changes. And:

$$TBA(t)=\frac{1}{\sqrt{2\pi\sigma^2\tau}}\int_{-\infty}^{\infty}TBA(t+\tau,Y)e^{-\frac{(Y-\mu)^2}{2\sigma^2\tau}}dY \qquad (7)$$

In the search for the solution of the pair (μ, σ_{BP}), for a given functional relationship from Y to $TBA(t + \tau)$, which can be defined either analytically or numerically, equation (7) defines an implicit relationship between μ and σ_{BP}:

$$\mu=\mu(\sigma_{BP}) \qquad (8)$$

Substituting (8) into either equations (6) or the Gaussian version of (5), we arrive at the one-dimensional functional relationship from σ_{BP} to the option prices given the term and the strike price in which TBA prices are similar to those derived under the Black formula:

$$\begin{aligned}&C\left(TBA(t),\tau,T,K,\mu,\sigma_{BP}\right)=\\&C\left(TBA(t),\tau,T,K,\mu(\sigma_{BP}),\sigma_{BP}\right)=\\&C\left(TBA(t),\tau,T,K,\sigma_{BP}\right)\end{aligned} \qquad (9)$$

$$\begin{aligned}&P\left(TBA(t),\tau,T,K,\mu,\sigma_{BP}\right)=\\&P\left(TBA(t),\tau,T,K,\mu(\sigma_{BP}),\sigma_{BP}\right)=\\&P\left(TBA(t),\tau,T,K,\sigma_{BP}\right)\end{aligned} \qquad (10)$$

In our derivation, equation (8) is implemented as a one-dimensional numerical root finding scheme for a given BP volatility, σ_{BP}.[5] In solving the BP volatility given an option price, we again use a one-dimensional numerical root finding scheme for a function defined by numerical integration nested with equation (8). In solving an option price given σ_{BP}, we only need equation (8) and the numerical integration for the expectation defined in equations (3) or (5).

Implementation and some numerical examples of the BP volatility TBA option model

As noted before, the rate change to TBA price function is defined numerically in our implementation. We run finite number of parallel rate shocks to the benchmark yield curve, which is the Libor/swap curve in our case, and obtain the corresponding TBA prices.[6] We then use interpolation to arrive at the TBA prices for the rate levels that are not included in the rate shock scenarios. Using an analytical function form, as in Zhou & Subramanian (2004), would speed up the calculation even more. We opt for flexibility since the computation cost is small. The flexibility allows us to use the same routine for different rate shock scenarios and potentially different products that the analytical form might not fit well.

We first give an example using the price profile in figure 2, in which we list the prices from –100 to +100 basis-point rate shocks in 25bp increments. As of March 12, 2007, the premium for an at-the-money call option on a Fannie Mae 30-year 5.5 coupon May-settled TBA option (expiry on May 7, 2007) that corresponds to a price volatility of 2.52% is 0–123, or 12⅜ ticks. We use an actual/365-day count in calculating the option term and time to settlement for the TBA option. To solve for the BP volatility: a) take a cubic spline function that goes through the points in the price profile as given; b) start with an initial guess of BP volatility and solve for the corresponding drift using the TBA price function – equation (7); c) substitute the BP volatility and drift into the option price equation (6) or its counterpart for a put option, and obtain an option price; d) if the option price from c) is higher than market price, shrink the BP volatility and go through b) and c) again, otherwise do the opposite until the price from c) is equal to the market price. For our example, the BP volatility model gives a corresponding BP volatility of 70.14 a year. The vega for a 1bp volatility increase is 0.53bp in option premium.

In figure 3, we provide some BP volatility results and compare them with the corresponding Black price volatilities. We select the Fannie Mae 30-year 5.5 coupon TBA options for April and May settlement with a pricing date of March 12, 2007. On that day, the 5.5 coupon TBA option is closest to par in price, where the potential pricing effect due to difference in prepayment models used by different shops is least dramatic. We normalise the BP (Black) volatilities across the various strike prices by at-the-money option BP (Black) volatility. Once volatilities are expressed in BP volatilities, the skew and smile effects are significantly smaller than those measured in Black price volatilities. For April (May) TBA options, the normalised BP volatility is 1.05 (1.04) for those one-point out-of-the-money puts and decreases to 1.00 (0.99) for those one-point out-of-the-money calls. The difference is only 5% (5%) of the at-the-money BP volatility. The corresponding normalised price volatility numbers are 1.13 (1.12) for one-point out-of-the-money puts and 0.90 (0.88) for one-point out-of-the-money calls. The difference is 23% (24%) of the at-the-money price volatility number. However, we are aware that the BP volatility model is imperfect. If

[5] *Obviously μ would be zero if the TBA price as a function of yield is linear. When the function is non-linear, μ is like a convexity adjustment. The search of (8) is localised to around zero to be meaningful and efficient given that the TBAs have short lives to settlement dates*

[6] *We assume that mortgage yield moves one to one with the general rate level given other things are fixed*

the world matches what the BP volatility model describes, we should have a same BP volatility number for all strike prices with respect to the same underlying TBA option.

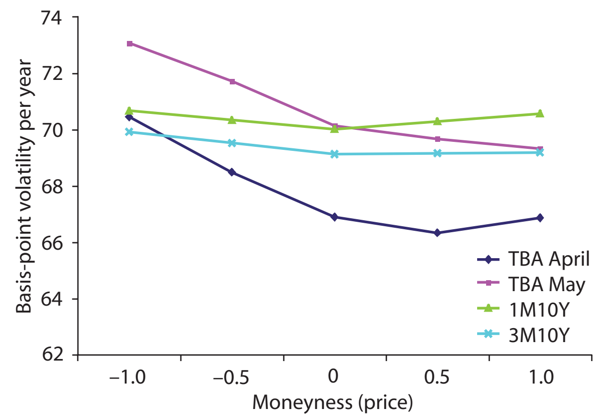
Comparison of BP vols between swaptions and TBA options

Here, we perform cross-market comparison in BP volatilities between swaptions and mortgage TBA options. The comparison is reasonable because the price profiles for the TBA options were constructed using rate shocks to the Libor/swap curve. For TBA options, we still use the options on the two immediate settlement month TBAs that are closest to par. The swaptions we choose are the one-month and three-month receiver options into a 10-year swap. We choose those very short-dated swaptions that have more or less the same option terms as the TBA options. We choose the 10-year swap rate since, typically, the 30-year mortgage has the highest key rate exposure at the five- to 10-year swap rate range. One could do a BP volatility comparison using weighted average swap rates with the weights equal to the key rate weights of the TBA options on the key swap rate points.[7] Relative value signals can be extracted using this 'blended swap rate' approach, but we will stick with the 10-year swap for the illustration here. We also match the moneyness of the options. As explained previously, typically there are only five strike prices quoted for options of a TBA contract for a given expiry date. The strikes are quoted in term of percentage price for TBA options. However, the strikes are quoted in rates for swaptions. To be consistent, we scale the moneyness for the swaptions by the (notional amount-based) duration of the underlying swap and express the moneyness in terms of (notional amount-based) percentage prices.[8] We then linearly interpolate the BP volatilities for the swaptions based on the moneyness to find the matches for the TBA option moneyness. The BP volatility comparison is given in figure 4.

As we can see from figure 4, the BP volatilities for the TBA options are very close to those of the comparable swaptions. Especially for the options that expired in May, the BP volatilities for that contract are basically the same as the one-month and three-month swaptions. The slope of the BP volatility curves across the various strikes for TBA options is more negative, that is, there is more skew than for the swaption BP volatility curves. But, as we show in figure 3, the skews are only 4–5% of the at-the-money BP volatilities for the TBA options. In contrast with the swaptions, the BP volatilities for the longer-term TBA options are higher than those of the shorter-term options.

Given that the TBA option markets and swaption markets are closely related and market participants move between those two markets with ease, it is not surprising to see that the BP volatilities for those two markets are consistent with each other when we make the comparison a level field, that is, using the BP volatility for rates as measurement.

4 TBA option and swaption BP volatility comparison: Mar 12, 2007



Conclusion

We provide pseudo closed-form BP volatility formulas for mortgage TBA options and compare some empirical results with those from the closed-form BP volatility formulas for swaptions. BP volatility models make direct comparison of volatilities across interest rate derivatives markets possible. The empirical results confirm the intuition that mortgage TBA options are trading very closely with the swaption market. We also confirm that BP volatility models remove much of the skewness that is typically observed for volatilities quoted using the Black model. The smile effect seems to persist in the BP volatility model to some extent, especially for the short-dated options. The BP volatility methodology can potentially be extended to other interest rate derivatives such as Treasury futures options. It will then be very easy to make comparisons of BP volatilities across different interest rate derivative markets and proceed to further convergence of the volatility markets. ■

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[7] *In that comparison, the key rate weights, the ATM BP vols, the BP vol skews at different (rate) moneyness levels and the correlation structure among the different swap rates all play their parts. However, the result is not too different from the single point proxy case (that is, 10-year swap) that we present here, especially in term of the shape of the BP vol against the moneyness plot. The reasons are as follows: 1) suppose we pick one, two, three, five, 10, 20 and 30 years as the key swap points, from our calculation the weight on one to three year is about 25%, the weight for five to 10 year is about 55% (about even splits between five and 10 year), 20 year accounts for about 15% and 30 year accounts for the last 5%. 10 year is right at the central peak of the weight distribution; 2) for ATM BP vols, it turns out that 10 year is also in the middle section of the downward sloping curve (for very short option expiries we see a hump going from one year to two year swap tails for the data we have) in the BP vol against swap tenor plot; 3) in our data, the BP vol skews across swap tenors lie in a very tight band (1–2 basis points for short-dated options) for a given moneyness in rate level. At least this is true for five-year to 30-year tails and –100 to +100 BP (in rate) moneyness levels that contains the moneyness range of –1 to 1 point in price since the durations of most of the underlying instruments that we consider are greater than one. The BP vol skews for the shortest tenors could be quite different, but as discussed above, they are weighted much less; 4) given 1)-3), the correlation structure and the weights only change the general BP vol level to some extent, they do not change the shape of the curve in the BP vol against the moneyness plot significantly*

[8] *Alternatively, one can convert the moneyness in price for the TBA options to moneyness in rate level and do the BP vol comparison in rate space rather than price space. This is recommended for the 'blended swap rate' approach*