IR Swaps - Curve sensitivity at maturity node

Asked 8 years, 3 months ago Modified 8 years, 3 months ago Viewed 6k times

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I was recently trying to price some IR swaps in BBG. I noticed that when I shock the yield curve up by 1bps at a single specific node, the DV01 is close to zero except at the node nearest the maturity. Nearly 100% of the DV01 for a parallel shift comes from the shock to the node near maturity.

I don't really understand this, since I would expect every node to have similar risk, perhaps slightly increasing the further away you are.

I see this trend with every IR Swap that I look at.

Clearly I am missing some understanding of the exposure of IR Swaps, could anyone here help me?

Thanks!

Note: I'm looking at the combined legs in this case.

risk bloomberg interest-rate-swap sensitivities

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asked Aug 24, 2016 at 2:42

user3365528

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4 Answers

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The value of a T year payer swap on a coupon payment date at time t, or a new swap that is about to be traded today time t, is given by

$$V(t) = (S(t,T) - C)\sum_{i=1}^{N} Z(t_i)\Delta_i$$

where C is the swap coupon, $S(t,T)$ is the current market swap rate to the swap maturity time T, $Z(t_i)$ is a LIBOR discount factor to time t_i and Δ_i is the year fraction over $[t_{i-1}, t_i]$. We assume annual coupons so set $\Delta_i = 1$ for simplicity so that

$$V(t) = (S(t,T) - C)\sum_{i=1}^{N} Z(t_i)$$

Bumping the swap curve can change the T-maturity market swap rate $S(t,T)$ and the discount factors $Z(t_i)$. Suppose we bump the $S(t,T^*)$ rate where T^* is one of the input swap rates used to construct the curve. This is done in a way that keeps ALL other input swap rates CONSTANT. To first order we have

$$\partial V(t)/\partial S(t,T^*) = \frac{\partial S(t,T)}{\partial S(t,T^*)}\sum_{i=1}^{N} Z(t_i) + (S(t,T) - C) \times \sum_{i=1}^{N} \frac{\partial Z(t_i)}{\partial S(t,T^*)}$$

where the DV01 equals $\partial V(t)/\partial S(t,T^*) \times 1$ basis point. The second term is multiplied by $(S(t,T) - C)$ and so if the market swap rate is close to the swap coupon, i.e. $S(t,T) \simeq C$, this term will be small. For a new swap the second term is exactly zero as $C = S(t,T)$.

Now consider an example.

We build our curve from 1Y, 2Y, 3Y, 4Y, 5Y swap rates and we are valuing a $T = 5$ year swap. The value of T^* can be $1, 2, 3, 4$ or 5. Consider then two scenarios:

I) $T^* = T$.

The term of the bumped swap rate and swap are same. Suppose $T = 5$ and $T^* = 5$ then $\partial S(t,T)/\partial S(t,T^*) = 1$. Also, bumping $S(t,T^* = 5)$ will lower discount factors between 4Y and 5Y. This means that

$$\partial V(t)/\partial S(t,T^*) = \sum_{i=1}^{N} Z(t_i) + (S(t,T) - C) \times \sum_{i=1}^{N} \frac{\partial Z(t_i)}{\partial S(t,T^*)}$$

The second term will usually be small, especially for a new swap, so we can write $\partial V(t)/\partial S \simeq \sum_{i=1}^{N} Z(t_i)$. This term is usually known as the swap PV01.

II) $T^* <> T$. The term of the bumped swap rate and swap are different. Consider $T = 5$ and $T^* = 4$ then $\partial S(t,T)/\partial S(t,T^*) = 0$. Also, bumping $S(t,T^* = 4)$ will only lower discount factors between 3Y and 4Y but those between 4Y and 5Y will need to rise to compensate so that the 5Y rate is still matched. The effect at 5 years on the sum of the discount factors will be almost cancelling. This means that

$$\partial V(t)/\partial S(t,T^*) = 0 + (S(t,T) - C) \times \sum_{i=1}^{N} \frac{\partial Z(t_i)}{\partial S(t,T^*)}$$

.

The second term will be very small due to partially offsetting changes in the discount factors, and especially if the swap rate is close to its initial coupon.

So in summary,

For an existing swap

- If $T^* = T$, the DV01 is approximately equal to the swap PV01
- If $T^* <> T$, the swap DV01 will be close to zero.

For a new swap

- If $T^* = T$ the DV01 exactly equals the swap PV01
- If $T^* <> T$ the swap DV01 is exactly zero.

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edited Aug 24, 2016 at 15:26

answered Aug 24, 2016 at 7:57

Dom
2,187 15 23

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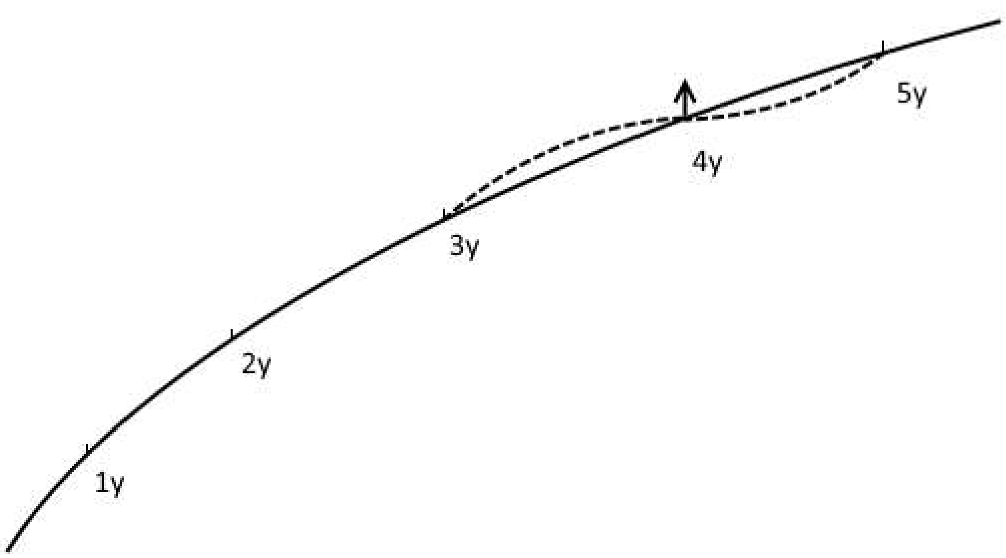
This is a common confusion, and it comes down to the difference between forward rates and swap rates. Swap rates are essentially the integral of forward rates (just like zero coupon rates).

The behaviour you're seeing is easy to understand if you think about the effect on the *forward* rates of bumping a *swap* rate. Here's a sketch of some implied forward rates for a swap curve built from 1y, 2y, 3y, 4y, and 5y swaps:



Suppose I bump the 4y swap rate only. The dotted line shows the new forward rates.

- Forward rates up to 3y won't move at all, since they must continue to reprice the existing swap rates up to 3y.

- Thus, to increase the implied 4y rate, the forward rates must increase between the 3y and 4y swap maturities.
- We didn't move the 5y swap rate, so the forward rates between 4y and 5y need to *decrease* to compensate for the heightened rates between 3y and 4y.

Now consider the effect of this perturbation on the value of a swap (where we receive the floating leg). It all depends on how much of the forward rate perturbation is sampled by the swap in question:

- If the swap maturity is 3y or less it will show no effect at all, since none of the forward rates it depends on fall in the perturbed region.
- If the maturity is between 3y and 4y the value of the swap will increase, and more so the closer the maturity is to the 4y point, as this samples more of the positively shifted region.
- If the maturity is between 4y and 5y the value of the swap will *also* increase, but the change will tend to zero as the maturity tends to 5y because we increasingly sample the negatively shifted region (offsetting the positive contribution from 3y-4y).
- If the maturity is greater than 5y we see no effect, as we fully sample both the positively and negatively shifted regions, which cancel each other out.

Thus, only swaps with maturity between 3y and 5y will show sensitivity to the 4y point.

Conversely, a swap with maturity between 3y and 4y will show sensitivity to the 3y and 4y points, and nothing elsewhere.

Note that the exact results depend on the particular interpolation scheme used - smoother interpolators will tend to result in more dispersed forward rate perturbations.

Share Improve this answer Follow edited Aug 24, 2016 at 15:17 answered Aug 24, 2016 at 15:10

atkins
264 1 7

0

For simplicity, let's consider the government par yield curve (ignoring nuances such as market conventions, OIS discounting, etc., this is conceptually equivalent to the par swap curve). The par curve, by definition, represents the yields & coupon rates of bonds trading at par ($100). Now let's shock the 10-year par rate by 100bp, while holding all other points on the par curve unchanged. So after this shock, how much should the value of the 30-year par bond change by? Zero. This is because after shocking the par curve, you continue to **ASSUME** that the new curve is still a par curve. So by definition, the 30-year par rate still represents the yield/coupon of a bond trading at 100. There's virtually no sensitivity elsewhere – "by definition."

Share Improve this answer Follow edited Aug 24, 2016 at 5:42 answered Aug 24, 2016 at 5:14

Helin
11.9k 1 25 44

This is not exactly true as it depends on how the curve was constructed. – Dom Aug 24, 2016 at 8:18

0

First of all, what are your values of swap rate ?

In the mono curve case where r stands for the curve, and $B^r(T)$ is the zero-coupon bond of maturity T associated to the curve r.

$$\mathrm{PV}_{\mathrm{swap}}(r) = 1 - B^r(T) - k_0 \times \sum_{i=1}^{n} \delta_i B^r(t_i) = 0$$

the $= 0$ is because k_0 is designed to have the pv at zero.

So now, you want to compare

$$\mathrm{DV01(swap)} = \mathrm{PV}_{\mathrm{swap}}(r + \epsilon \text{ parallel shift}) - \mathrm{PV}_{\mathrm{swap}}(r)$$

and

$$\mathrm{Bump}(\mathrm{swap}, t) = \mathrm{PV}_{\mathrm{swap}}(r + \epsilon \text{ bump on } t \text{ node of the yield curve}) - \mathrm{PV}_{\mathrm{swap}}(r)$$

I assume that what you call yield curve is such that:

$$r : t \to r(t) \text{ st } B(t) = \frac{1}{(1 + r(t))^t}$$

since to compute above quantities, you need to evaluate them on B let us focus on:

$$\begin{aligned}\mathrm{Bump}(B(T), t) = \mathrm{PV}_{B(T)}(&\text{bumped } r : s \to r(s) \text{ if } s \neq t \text{ and } r(s) + \epsilon \text{ if } s = t) \\ &- \mathrm{PV}_{B(T)}(r)\end{aligned}$$

by definition of B and r, you get:

$$\mathrm{Bump}(B(T), t) = 0 \text{ if } t \neq T \text{ and } -\epsilon \frac{T}{1 + r(T)} B(T) \text{ if } t = T$$

from that you easily see that:

$$\mathrm{DV01}(B(T)) = \mathrm{Bump}(B(T), T)$$

and thus you get:

$$\mathrm{DV01(swap)} = -\mathrm{Bump}(B(T), T) - k_0 \times \sum_{i=1}^{n} \delta_i \mathrm{Bump}(B(t_i), t_i)$$

if $k_0 T$ is small relatively to 1 then you can prove that the k_0 part is small relatively to $\mathrm{Bump}(B(T), T)$