Rates Alert | 19 August 2013

Introducing a relative-value tool for swaps

- **We have built a relative-value tool for swap curves, measuring the extent to which current spot and forward rates deviate from a constructed fair value**
- **We construct 'fair-value' estimations using PCA techniques**
- **We will use this methodology in two ways: to identify the best way in which to express an overall market view, and to identify market-neutral curve dislocation trade opportunities**

Summary

We have designed a relative-value (RV) model for swaps using principal component analysis (PCA). Using PCA techniques, we construct theoretical swap curves (including spot and forward curves). We then compare these constructed curves with current curves to assess the extent to which swap rates in both the spot and the forward space deviate from fair value. In this way, we are able to assess the relative cheapness or richness of each rate across the curves and, as such, can use this information as part of our overall trade-idea-generation process. An underlying assumption in our analysis is that rates will return to fair value – we model this 'return' to fair value as a mean-reverting process.

There are several methods to explain yield-curve dynamics. We have adopted a PCA approach, as it is a powerful statistical tool that works best on a highly collinear system, such as a term structure of interest rates. In rates, there are only a few important sources of information in the data that are common to all variables, and PCA allows one to extract these key sources of variation from the data, thus simplifying the representation of yield-curve dynamics.

This article explains and applies the RV tool framework using 2Y historical data sets for US dollar (USD) swaps and Korean won (KRW) swaps as examples curves.

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Figure 1: Current PCA trade lists

Currency	Trades	PCA weight (100k belly, short end)		Z-score	Entering level (PCA weight)	Target level	Investment horizon	Carry (1M)	Roll-down (1M)
Swap	19/08/13	Left (k)	Right (k)	Std dev.	bps	bps	Days	bps	bps
USD	Receive 2Y/7Y/30Y	147	72	2.11	-112	-125	35	-0.10	0.95
EUR	Receive 3Y/10Y/30Y	67	42	2.47	50	37	76	-0.10	-0.34
GBP	Pay 1Y/3Y/7Y	80	52	-2.80	-54	-43	115	-0.01	-1.70
KRW	3Y/10Y flattener	100	63	1.66	-78	-84	48	-0.39	-0.75
THB	6mf 2Y/5Y flattener	100	57	1.73	100	57	114	-0.59	-1.43
MYR	1Y/5Y flattener	100	49	1.72	-158	-169	31	-0.80	0.87

Source: Standard Chartered Research

The benefits of adopting a PCA approach

In order to increase and improve the flow of our trade recommendations in the swap space, we have built an RV tool using PCA. This methodology allows us to generate fair-value swap curves in both the spot and forward space based on just a few (in this instance, three) key explanatory risk factors – the principal components. Typically in rates, the most important risk factors are parallel shifts, changes in slope and changes in convexity of the curve. We find that the first three principal components capture over 99% of the variation in the curves, and as such justify the choice of only the first three principal components in the construction of fair-value curves. Given a fair-value curve, we can compare it with the current market curve to identify RV opportunities.

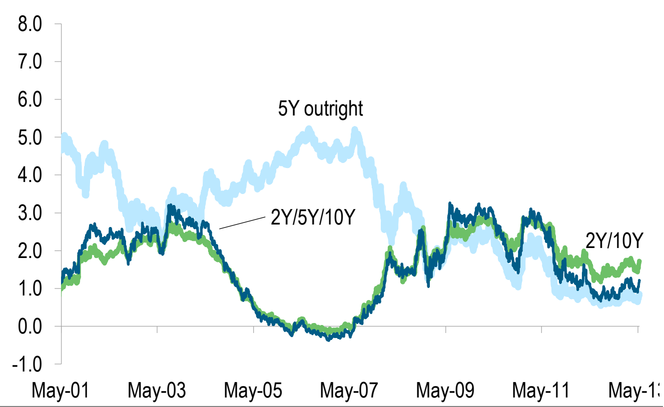
We have applied this methodology not only to G10 rates markets but also to the emerging-market (EM) rates markets that we cover. Using a two-stage approach, we first identify rich/cheap points on a given swap curve. As a second step, the RV tool uses the identified cheap/rich points to structure directionally neutral curve and curvature trades, such as butterflies. In this way the tool can also be used to assess whether a particular trade is indeed exposed to the curve in the way one would expect. For example, the tool can verify whether a given butterfly trade is actually exposed to changes in the convexity of the curve or mainly to slope and level changes.

For every trade our tool generates, it will provide a visualisation package of important analytics and graphics covering factors such as PCA weights, carry, roll-down, target profit and investment horizon. This visualisation package allows the characteristics of the trade to be captured in a more user-friendly way. The RV tool will form part of our framework for identifying and valuing trade opportunities in swap curves. In rates research, we will update our preferred trade ideas as and when identified by our RV tool. Eventually, we intend to roll out the complete package to our clients, and we will be happy to provide support and advice on the results generated whenever required.

We provide the mathematical details of the technical aspects of our PCA tool methodology in the Appendix.

Figure 2: US curve directionality increased after the financial crisis

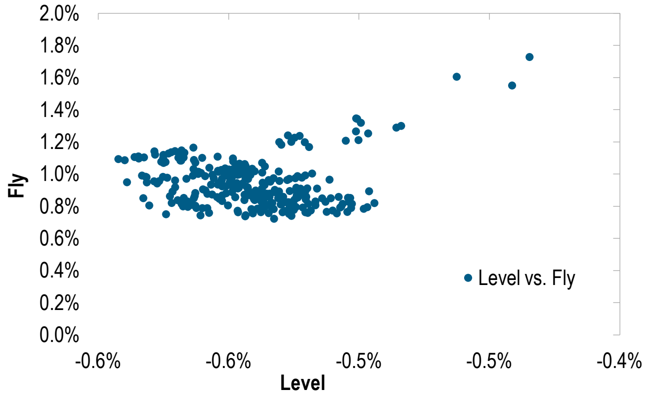
5Y swap rates, 2Y/10Y curve and 2Y/5Y/10Y butterfly



Source: Standard Chartered Research

Figure 3: No directionality in PCA trades

PCA weighted USD 2Y/5Y/10 butterfly vs. the 5Y level



Source: Standard Chartered Research

PCA framework – Building blocks for RV trades

Step 1 – Database

The PCA database is taken from trader-sourced closing levels

We use swap data taken from historical trader-sourced closing levels. Our trade identification is based two years of historical data, as it represents the curve's movement within a reasonable volatility range and the larger part of the investment horizon is within two years. However, the sample period used may be varied.

Forward curves for each market are derived from the spot curve using existing in-house quant functionality.

Step 2 – Three principal components

Three principal components capture the majority of the yield-curve dynamics

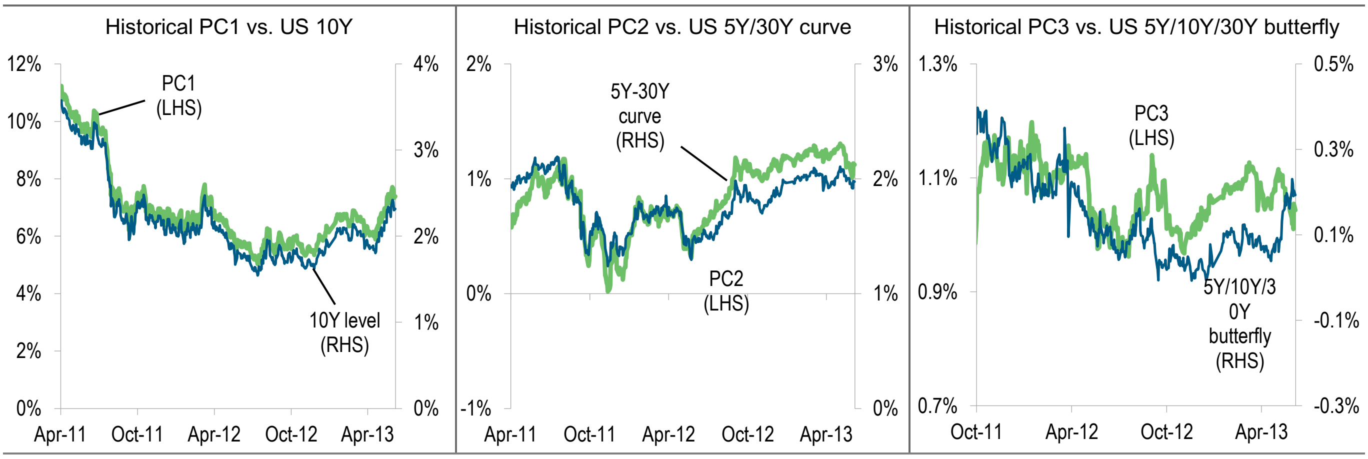
PCA allows us to reduce a covariance matrix of our set of interest rates into an importance-ordered subset of uncorrelated variables, i.e., the principal components (PCs). More simply, it allows us to express each rate as a linear combination of just a few (in this instance, three) variables – the PCs. The total number of PCs is equal to the number of observed variables. In our case, the observed variables are time series of interest rates of various maturities. The number of PCs used to express each rate depends on how much variance one wants to explain. Using all the components will explain all the variation in the observed variable; however, one may choose to use fewer PCs, as some of the minor variations may be viewed as noise.

The analysis explicitly reveals the importance of each PC (its factor), which is an expression of the contribution of that component to the variables in question, i.e., in this case to interest rates. Also associated with each PC is a set of 'factor loadings' that define how each rate will change (by how much and in what direction) to a shock to that component.

In interest rates, it is typical for the first three PCs to capture over 95% of the variation in the curve. As such, we choose to express each rate as a combination of the first three components.

We perform the PCA on the spot curve and forward curves separately. We then estimate the current fair value of each interest rate along a given curve using the first three PCs that are distinct to that curve. For example, the 1Y rate would be expressed as a linear combination of three PCs where those PCs are distinct to the spot curve. Similarly, the 1Y/1Y rate would be expressed as a linear combination of

Figure 4: Historical PCA



Source: Standard Chartered Research

the three PCs that are distinct to the 1Y forward curve. The result is that each rate is expressed as a linear combination of three PCs. As such, the time series of yield curve data is transformed into the time series of combinations of three PCs.

Interpretation of the principal components:

PC1 – Level

PC1 can be considered a measure of the outright level

In a perfectly correlated system of changes in interest rates or levels in interest rates, the factor loadings of the first principal component would be equal. More generally, the more correlated the system, the more similar the values of the loadings of the first principal component across the variables. As such, the first principal component captures a 'common trend' in the variables. So, if the first principal component changes and the other components remain fixed, then the variables in your system, in this case the interest rates, will all move by roughly the same amount. As such, in rates space, PC1 can be considered a measure of the outright 'level' of yields.

PC2 – Slope

PC2 is the proxy for the slope of the curve

If the system of variables has no natural order, then the second and higher-order principal components may have no intuitive interpretation. However, with interest rates, the system is ordered, i.e., there are set rates of various maturities. The loadings of the second principal component typically decrease (or increase) in magnitude with maturity. As such, if PC2 changes while the other components remain the same, then the rates at one end of the term structure will move up and the others will move down. For this reason, PC2 can be considered a proxy to the slope of the curve.

PC3 – Curvature

PC3 represents the curvature of the curve

Similarly for PC3, the loadings of PC3 typically decrease (or increase) in magnitude and then increase (or decrease). As such, if PC3 changes while the other components remain constant, then the rates at either end of the term structure will move up, and the rates in the middle will move down.

As mentioned above, each principal component has an associated factor that represents the importance of this principal component in capturing the variation in the

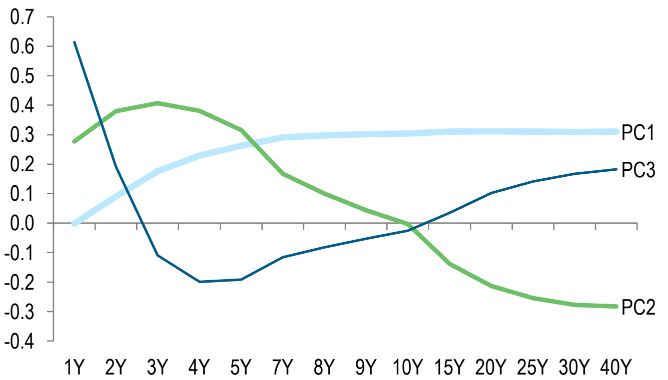
Figure 5: The three PCs and the loading vector

$$\begin{bmatrix} r1^{est} \\ r2^{est} \\ r3^{est} \\ \vdots \\ r(n)^{est} \end{bmatrix} = \begin{bmatrix} loading(1,1) & \cdots & loading(1,3) \\ \vdots & \ddots & \vdots \\ loading(n,1) & \cdots & loading(n,3) \end{bmatrix} \begin{bmatrix} PC1 \\ PC2 \\ PC3 \end{bmatrix} \qquad \begin{bmatrix} PC1 \\ PC2 \\ PC3 \\ \vdots \\ PC(n) \end{bmatrix} = \begin{bmatrix} loading(1,1) & \cdots & loading(1,n) \\ \vdots & \ddots & \vdots \\ loading(n,1) & \cdots & loading(n,n) \end{bmatrix} \begin{bmatrix} r1 \\ r2 \\ r3 \\ \vdots \\ r(n) \end{bmatrix}$$

Source: Standard Chartered Research

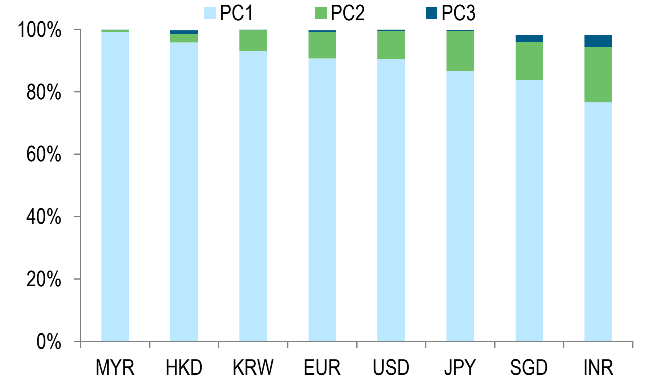
Figure 6: PC1, PC2 and PC3 explain 99% of the curve

Three PC loadings for the US swap curve



Source: Standard Chartered Research

Figure 7: PC – Explanation level for yield-curve variance

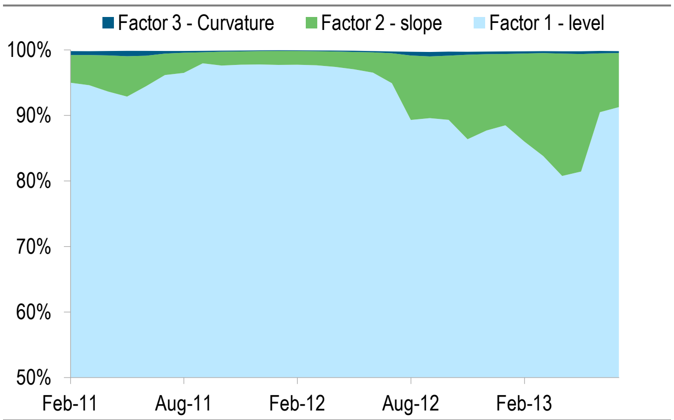


Source: Standard Chartered Research

system of variables (the interest rates). While we find in rates that PC1, PC2 and PC3 explain more than 95% of the variation in the curve, the relative importance of these components can change – i.e., their factors can change. This is to say that at any given time, PC1 may capture 91% of the variation in the curve, PC2 may capture 7% and PC3 may capture 1%. One year later, PC1 may capture 85%, PC2 may capture 13% and PC3 may capture 1%. Parallel curve shifts result in little change to the factor composition; however, if the curve inverts or the curvature changes, then the factor composition changes – see Figures 8-11. Tracking the changes in the factor composition allows one to identify changes in the structural behaviour of a given curve, and as such suggests the time period over which to run the PCA analysis, which is important in order to obtain an undistorted fair value, especially for Asian curves.

Figure 8: USD factor contribution shows stability

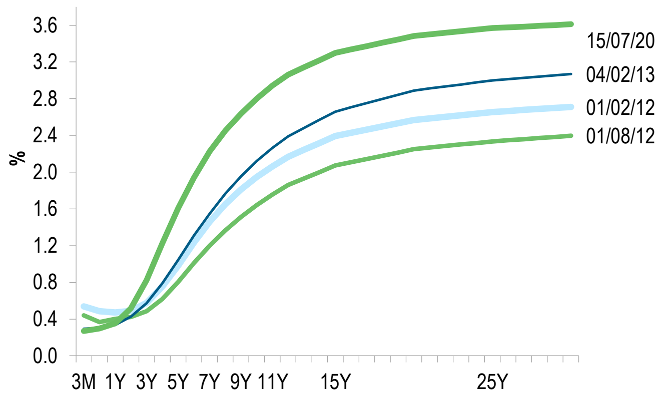
1M rolling contribution changes – USD



Source: Standard Chartered Research

Figure 9: USD swap curves shifted in a normal pattern

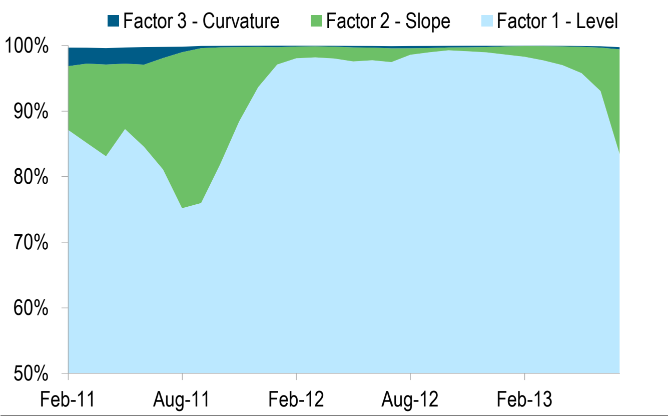
Term structure changes – USD



Source: Standard Chartered Research

Figure 10: KRW factor contribution shows stability

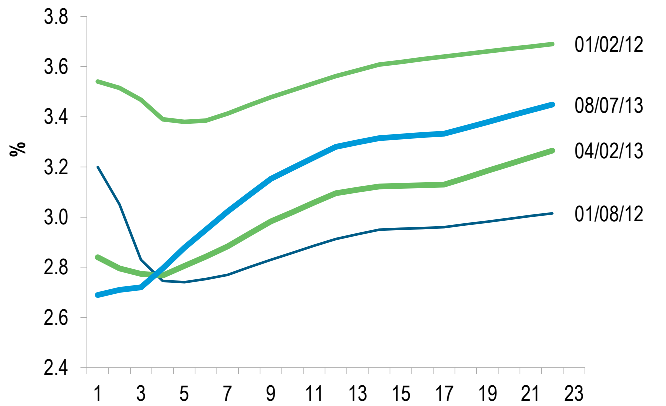
1M rolling contribution changes – KRW



Source: Standard Chartered Research

Figure 11: KRW swap curves inverted for some tenors

Term structure changes – KRW



Source: Standard Chartered Research

Step 3 – Constructing a rich/cheap representation of swap curves: first PCA on the entire curve

We conduct a PCA to calculate the relative richness/cheapness of all tenors on the curve

We perform the PCA on the spot and forward curves individually. For the spot curve, we use historical swap data from 6M-30Y; for the 1Y forward curve, we use historical forward swap data from 1YF6M-1YF30Y; similarly, for all forward curves out to 10Y. We take the three most important explanatory factors (PC1, PC2 and PC3) and use them to construct the fair value of each rate on the curve. We subtract these estimated rates from current rates to assess whether the current market rate is rich or cheap with respect to the estimated PCA rate. We calculate the historical Z-score of these differences (see the Appendix for details on Z-scores) to adjust rates that appear to be consistently rich or cheap versus the curve. Users can vary the time horizon for the Z-score – 6M to 3Y. A Z-score table for the forward curves is then obtained (see Figure 12), and we can easily observe which rates across each curve are comparatively rich or cheap.

Figure 12: Some tenors (green) are rich while others (blue) are cheap

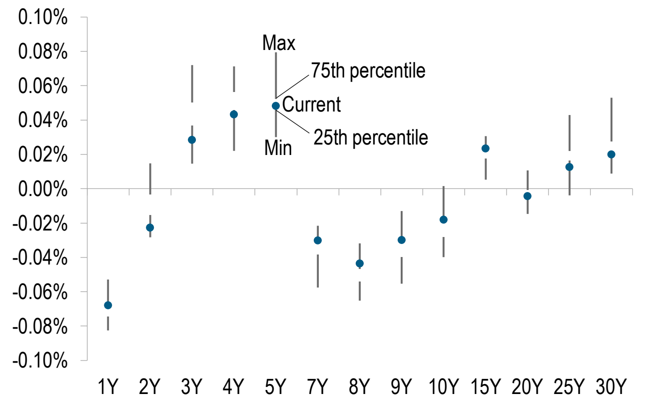
The richness or cheapness on the spot and fwd surface

		Forward							
Tenor	**Spot**	**1M**	**3M**	**6M**	**1Y**	**2Y**	**5Y**	**7Y**	**10Y**
1Y	(1.84)	**(2.21)**	**(2.68)**	**(3.12)**	**(2.59)**	(0.14)	(0.75)	0.10	0.45
2Y	**(3.31)**	**(3.37)**	**(3.44)**	**(3.57)**	1.75	**3.05**	**(2.17)**	(2.11)	0.50
3Y	**(3.70)**	**(3.69)**	**(3.62)**	**(3.16)**	**2.94**	2.30	(2.15)	(0.57)	(1.49)
4Y	**(2.47)**	(1.92)	(0.47)	**2.78**	**2.78**	0.11	(1.44)	(0.42)	(1.56)
5Y	**2.98**	**3.08**	**3.29**	**3.37**	1.36	**(3.43)**	0.53	**2.67**	(0.32)
7Y	**2.87**	**2.87**	**2.78**	**2.61**	**(3.28)**	**(2.91)**	**3.66**	1.68	0.94
8Y	**3.02**	**2.90**	**2.78**	**2.57**	**(3.14)**	(1.47)	**2.88**	1.73	1.75
9Y	**2.88**	**2.81**	**2.70**	**2.55**	(1.89)	(0.49)	**2.97**	1.57	2.35
10Y	**3.00**	**2.94**	**2.79**	**2.50**	(1.04)	0.79	**2.54**	1.58	2.38
15Y	1.48	1.29	0.93	0.20	0.70	2.24	1.85	0.41	(0.75)
20Y	(2.00)	(1.98)	(1.96)	(1.99)	1.74	2.17	(1.25)	**(2.19)**	(1.83)
25Y	**(3.40)**	**(3.36)**	**(3.29)**	**(3.20)**	(1.27)	(0.36)	**(3.36)**		
30Y	**(3.73)**								

Source: Standard Chartered Research

Figure 13: Historical relative position

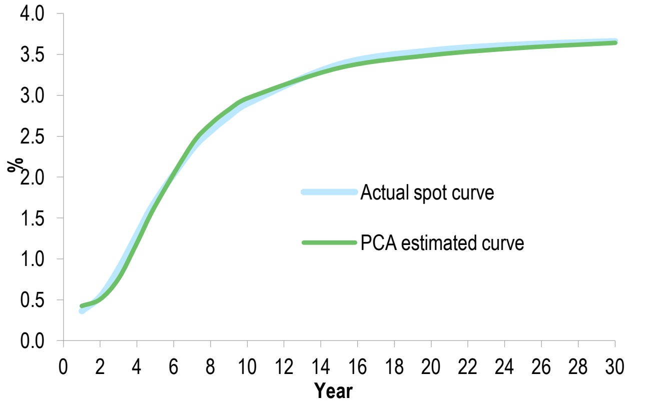
Actual-estimated level historical deviation



Source: Standard Chartered Research

Figure 14: The deviation of the two term structures

Actual spot curve vs. PCA term structure – USD



Source: Standard Chartered Research

Step 4 – Trade identification: second PCA on three selected points of the curve

We use the Z-score tables to identify trades that we weight with respect to the PCA weightings

Given the Z-score table of the spot and forward curves, our RV tool identifies trades. For butterfly trades, it automatically detects two cheap points and one rich point or vice versa, and displays a list of the selected trades. For a curve trade, it chooses one rich and one cheap point (or vice versa).

Once the trades are identified, we calculate the optimal weights for the trade by running the PCA again on these selected points. Rather than applying DV01-neutral weights – 1:2:1 (left wing: belly: right wing) in the case of a fly – we calculate the level and curve-neutral weights, i.e., we weight the trade such that it is neutral to PC1 and PC2 and exposed only to PC3. Similarly, we can retrieve level-neutral and curvature-neutral weights and leave the trade exposed to only slope risk (see the Appendix for more details).

Step 5 – Trade analytics

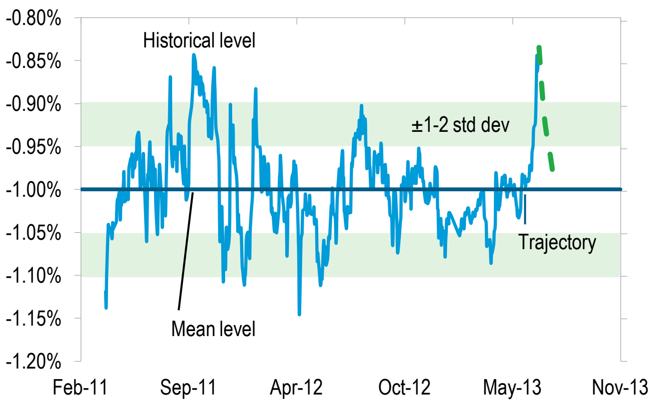
The trades from Step 4 are not ranked. The selection criterion is only the Z-score of the difference between the actual and estimated rates. To prioritise the listed trades, we conduct a further analysis. Our RV tool provides analytic details such as investment horizon, lifetime Z-score, target level, carry/roll-down and historical behaviour. We elaborate on each component of this analysis below.

Mean reversion: investment horizon

The overriding assumption in our work is that the trades based on PCA analysis will revert to their estimated mean level. As such, we model our trades as a mean-reverting process to calculate the mean-reversion speed of our PCA-weighted trades, including using statistical filters to exclude trades that do not exhibit statistically significant mean-reversion. We then use this to estimate an anticipated holding period. A PCA-weighted historical trade level is taken as the data set into a mean-reverting regression. We run the regression on this historical level to calculate a half-life – which is the amount of time estimated to move halfway between the current level and the estimated mean. We assume the time to reach the target is 87.5% (50% + 25% + 12.5%) of the lifetime. The shorter the investment horizon and the higher the target, the greater the possibility of realising profit (see the Appendix for more details).

Figure 15: Investment trajectory converges to mean

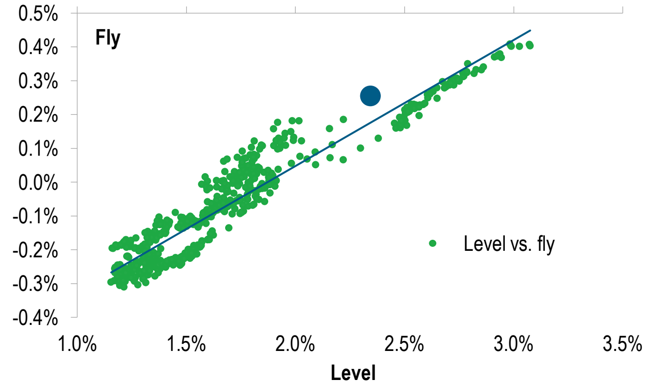
Mean reverting trajectory of PCA swap 1MF 2Y7Y30Y



Source: Standard Chartered Research

Figure 16: DV01-weighted fly is cheap

Fly vs. level of DV01-weighted swap 1MF 2Y7Y30Y



Source: Standard Chartered Research

It is also useful to look at the DV01-weighted fly and compare it with the PCA-weighted fly (Figure 16). For the US receiving 1MF 2Y7Y30Y trade, both weighting approaches support the receiving trade. The blue circle point is the current position, and the solid line is the regression between the historical rate level and the butterfly spread, which in this case shows the current direction of the fly. The current DV01-weighted position lies above the regression line and suggests that the belly is historically cheap within the fly.

Lifetime Z- score

The lifetime Z-score represents the trade distance from the mean

The lifetime Z-score for the listed trade is the historical Z-score of the difference between the actual and estimated butterfly trades. A high positive Z-score for the US 2Y/7Y/30Y butterfly trade indicates the actual butterfly trade is too cheap and suggests receiving the 7Y. A negative Z-score favours a pay 7Y position. By looking at the size of the Z-score, we can see how far from the mean the butterfly is trading.

Target and stop-loss level

We set the target level as the historical mean of PCA-weighted trades

We assume the target is the mean of the historical fly level and that it would be reached in 87.5% of the derived lifetime as the trajectory approaches the target asymptotically. The stop-loss level is set to the point that is half the target distance from the mean in the opposite direction. The target is based on the statistical mean regardless of the investor's risk appetite; i.e., if the investor is aggressive about risk, the target can be set to ±1 or ±2 standard deviation away from the mean.

We apply a filter to rule out unprofitable trades

In order to incorporate transaction costs, we applied a filter that rules out trade ideas for which the estimated profit potential – the difference between the target and entry levels – is less than twice the in-and-out cost of implementing and unwinding the position (as implied by the relevant bid-ask spreads). Given the level of bid-ask spreads in EM rates markets covered, this filter ensures that the trades we recommend are viable.

Carry/roll-down: ex-ante and ex-post

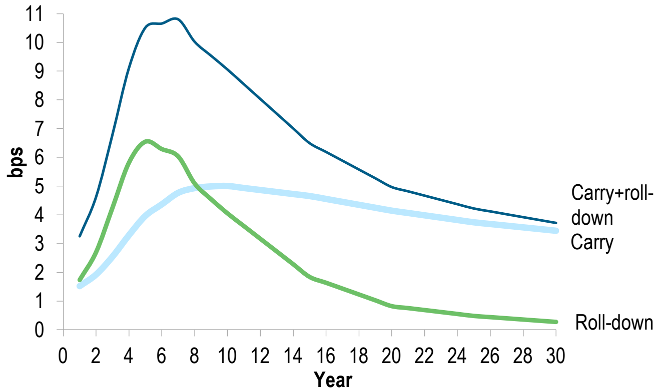
The carry/roll-down of each tenor on the curve represents carry attractiveness

We look closely at the carry/roll-down of each PCA trade. It is especially important in an environment of low volatility, as carry/roll-down represent the expected return in an unchanged rate scenario.

We incorporate carry into our analysis by including carry for each curve node. For our default trade selection, we calculate the one-month carry for each node and use this

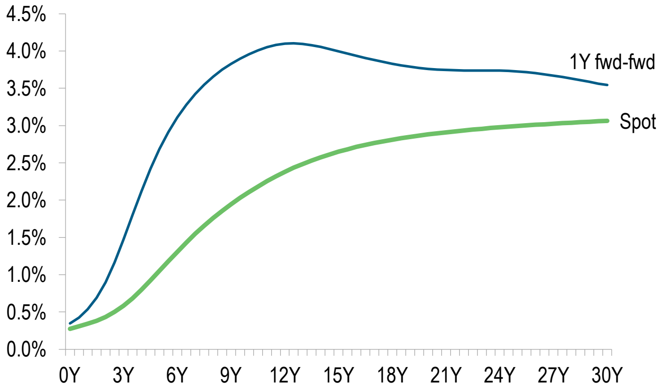
Figure 17: Carry/roll-down term structure – US swaps

Carry, roll-down and carry + roll-down



Source: Standard Chartered Research

Figure 18: 1Y fwd-fwd vs. spot curve – US swaps



Source: Standard Chartered Research

as the default. This is because often PCA-weighted butterfly trades that appear attractive before carry do not appear attractive once carry is incorporated (ex-post). Our analysis then shows the ex-ante carry and roll-down for the whole PCA-weighted butterfly strategy (e.g., a result of 2bps for a receiving butterfly represents the total carry/roll-down over the anticipated holding period). In our trading tools, we also intend to include options to calculate carry/roll-down both ex-ante and ex-post, but our default analysis includes carry on an ex-ante basis.

Step 6 – Trade prioritisation

PCA trades are prioritised using the analytical results in Step 5

We use US swap butterfly trades and Korea swap curve trades as examples of prioritisation, using the analysis in Step 5.

The Z-score table (Figure 12) suggests that the two most attractive US butterfly trades are receiving 3Y/10Y/30Y and receiving 1MF (1-month forward starting) 2Y/7Y/30Y. Figure 21 provides model-based estimates for the anticipated trade performance. Most of the analytical figures suggest that receiving the 1MF 2Y/7Y/30Y fly is a more favourable trade than 3Y/10Y/30Y; the lifetime Z-score is far away from the mean for both trades, but the investment horizon is shorter for the 1MF 2Y/7Y/30Y trade. Carry/roll-down is positive for the forward-start but it is negative for the receiving 3Y/10Y/30Y trade. The profit/cost ratio is also higher in the receiving 1MF 2Y/7Y/30Y trade.

The US example shows a clear preference for one trade that can be easily prioritised; however, the analytics for the Korea curve trades show much more mixed results. The lifetime Z-scores for both trades indicate a strong bias towards the mean, and suggest these trades should be realised within their investment horizons if the mean reversion holds. While the 7Y/20Y steepener has more favourable features in terms of the Z-score and carry/roll-down, the 2Y/7Y flattener has a shorter investment horizon. We suggest that investors who seek profit in a relatively shorter time horizon select the 2Y/7Y flattener, while the 7Y/20Y steepener is preferable for the investor who cares about carry.

Our strategists assess trades to identify possible structural anomalies

As a final step, each trade that we identify and consider sufficiently attractive and viable is assessed by our strategist for the market in question. This is in order to reduce the risk that the ‘anomaly’ identified is due to some fundamental or structural break in the development of the curve which could result in the mean reversion assumption being compromised.

Figure 19: US swap butterfly trades

US swap butterfly trade

USD	Rec. 3/10/30Y	Rec. 1MF 2/7/30Y
PCA weight	88k:100k:65k	147k:100k:49k
Lifetime Z-score	3.78 std	3.25 std
Entry level	-22.7bps	-83.3bps
Target level	-33.3bps	-99.8bps
Stop-loss level	-17.4bps	-75.1bps
Investment horizon	36 days	29 days
Profit/cost ratio	4.25	4.81
Carry/roll-down (1M)	-2.52bps	0.76bps

Source: Standard Chartered Research

Figure 20: Korea swap curve trades

Korea swap curve trade

KRW	2Y/7Y flattener	7Y/20Y steepener
PCA weight	100k:100k	100k:102k
Lifetime Z-score	2.69 std	-3.03 std
Entry level	32.4bps	51.3bps
Target level	10.5bps	59.9bps
Stop-loss level	43.3bps	47.0bps
Investment horizon	79 days	91 days
Profit/cost ratio	1.56	1.82
Carry/roll-down (1M)	-0.94bps	0.38bps

Source: Standard Chartered Research

Visualisation

We use a visual representation of the trades and analytics to enhance understanding

We highlight various details and facilitate understanding of the trade analysis via visual representations. These allow us to capture the characteristics of the trade in a more user-friendly way.

All the graphs represented here are available for users to generate automatically using the Standard Chartered RV tool.

In addition to individual graphs, we provide a PDF that contains all the analytical details and corresponding graphs. We present an example in Figures 21-24.

Figure 21: Analysis of the two most attractive trades – butterfly and curve

Trade summary with Z-score table

USD	Butterfly – top 2 trades		Curve – top 2 trades	
	Rec 0MF 3Y/7Y/30Y	Rec 1MF 2Y/7Y/30Y	10YF 4Y/10Y flattener	1YF 3Y/8Y flattener
PCA weights (left wings: belly: right wings)	130k : 100k : 47k	146k : 100k : 68k	100k : 102k	100k : 70k
Lifetime Z-score (standard deviation)	1.35	1.19	1.38	-1.99
Initiated (belly × PC weight - wing × PC weight - wing × PC weight)	-49.18bps	-93.68bps	-10.40bps	50.14bps
Target (mean level)	-58.54bps	-110.10bps	-16.81bps	41.56bps
Stop-loss	-44.50bps	-85.47bps	-7.20bps	54.44bps
Investment horizon (days to target)	32	34	7	46
Profit when target achieved (when investing 100k)	14	25	11	17
Carry (bps)	-0.45bps	-0.56bps	0.10bps	-1.55bps
Roll-down (bps)	-2.22bps	-1.37bps	0.01bps	6.99bps
Carry + roll-down (bps)	-2.67bps	-1.93bps	0.11bps	5.44bps

Forward curve Z-score table

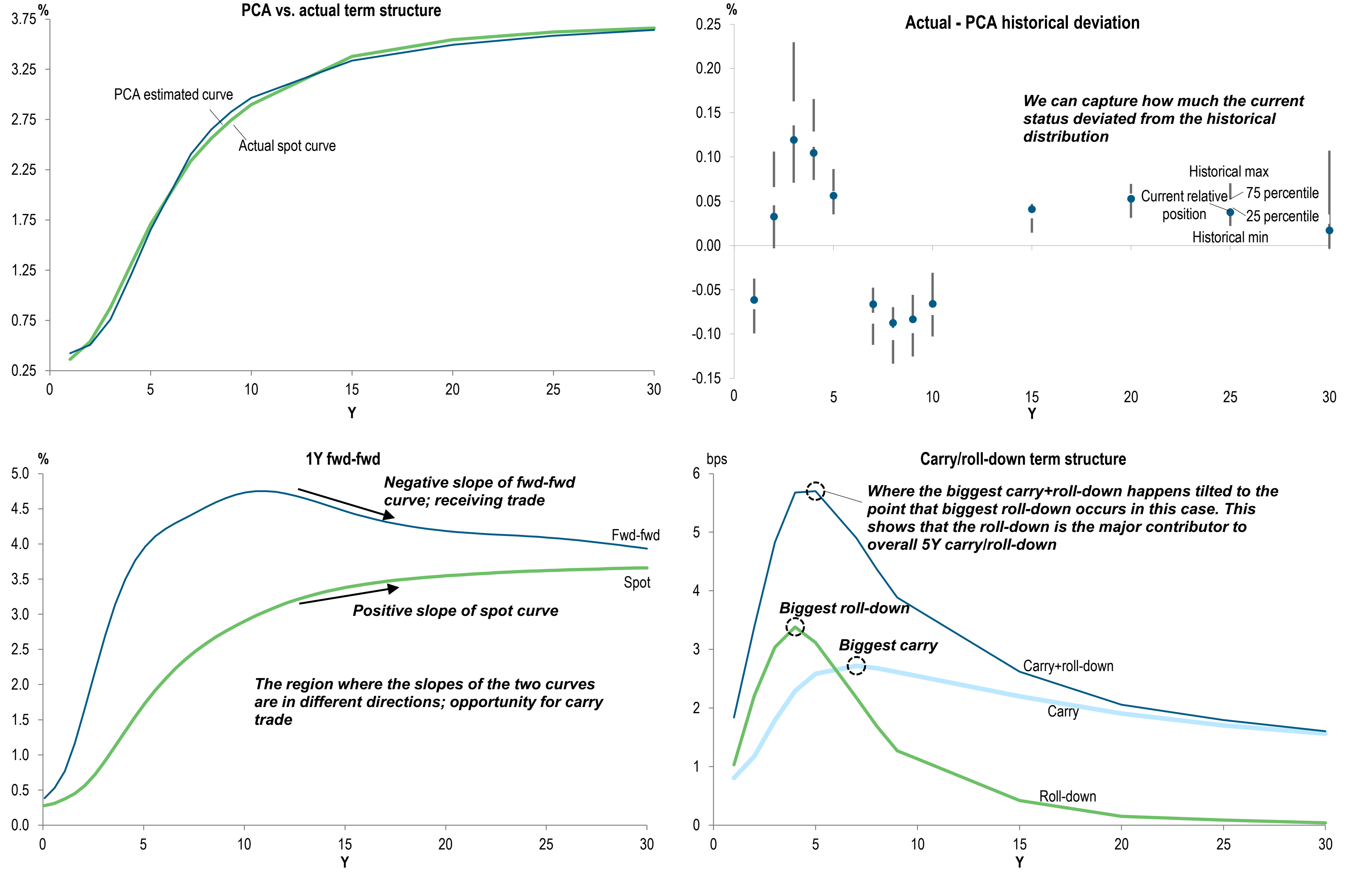
Blue highlighted:
Actual value > PCA estimated value
Rates should go down

Green highlighted:
Actual value < PCA estimated value
Rates should go up

	Forward								
Tenor	Spot	1M	3M	6M	1Y	2Y	5Y	7Y	10Y
1Y	0.20	0.24	0.38	0.03	**(1.13)**	(0.48)	(0.75)	**(0.95)**	0.14
2Y	**(1.15)**	**(1.15)**	**(1.02)**	(0.72)	**1.58**	**1.54**	(0.54)	(0.88)	0.66
3Y	**(1.29)**	**(1.22)**	**(1.13)**	(0.61)	**1.88**	**1.41**	**(1.63)**	(0.81)	(0.80)
4Y	**(1.00)**	(0.93)	(0.71)	(0.03)	**1.66**	0.65	**(1.50)**	(0.27)	**(1.12)**
5Y	0.05	0.37	0.55	**0.94**	0.77	**(1.08)**	(0.92)	**2.08**	(0.41)
7Y	**1.42**	**1.14**	**1.09**	0.43	**(1.84)**	**(2.15)**	**2.46**	**1.53**	0.29
8Y	**1.13**	**0.97**	0.84	0.27	**(2.12)**	**(1.79)**	**1.57**	**1.69**	0.77
9Y	0.83	0.71	0.51	(0.05)	**(2.01)**	(0.31)	**1.84**	**1.47**	**1.15**
10Y	0.60	0.52	0.47	0.21	(0.69)	0.46	**1.93**	**1.14**	**1.22**
15Y	**1.20**	**1.17**	**1.13**	**1.17**	0.93	**1.38**	0.91	(0.09)	(0.34)
20Y	(0.37)	(0.30)	(0.22)	0.21	**1.52**	**1.15**	(0.80)	**(1.50)**	(0.43)
25Y	**(1.16)**	**(1.08)**	**(1.00)**	(0.61)	0.28	0.08	**(1.82)**		
30Y	**(1.12)**								

Source: Standard Chartered Research

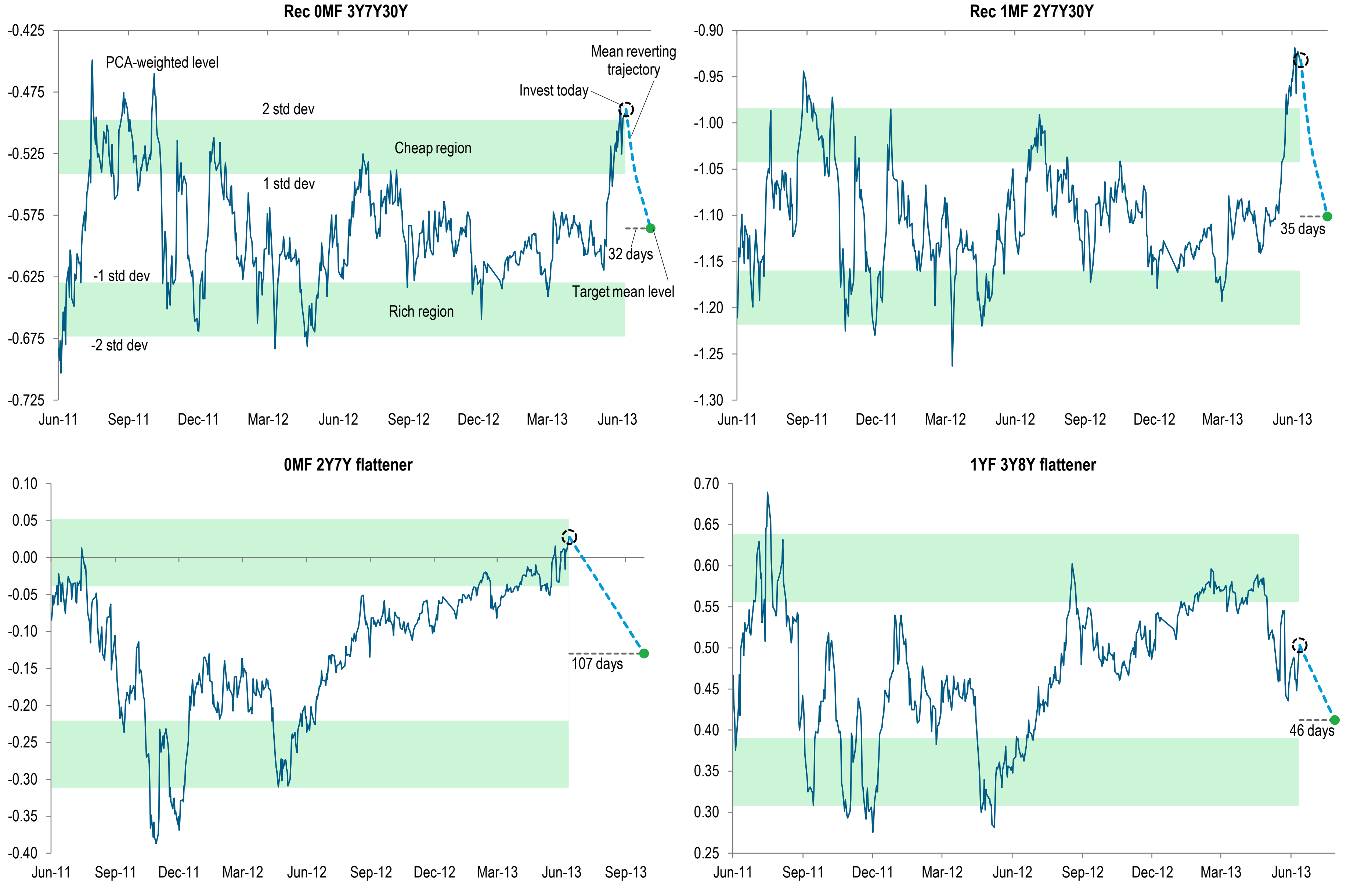
Figure 22: Analysis of the two most attractive trades – butterfly and curve – with Z-score table



Source: Standard Chartered Research

Figure 23: Trade trajectory, %

These charts represent how the PCA trades would perform



Source: Standard Chartered Research

Appendix: Principal component analysis – PCA

1. What is PCA?

   $Y = b_1X_1 + b_2 X_2 + \ldots b_n X_n$

   We typically have a data matrix of *n* observations on *p* correlated variables $x_1, x_2, \ldots x_p$. PCA looks for a transformation of the x_i into *p* new variables y_i that are uncorrelated.

   Principal components are linear combinations of the observed variables. The coefficients of these principal components are chosen to meet three criteria:

   1) There are exactly (n) principal components (PCs), each being a linear combination of the observed variables;
   2) The PCs are mutually orthogonal (i.e., perpendicular and uncorrelated);
   3) The components are extracted in order of decreasing variance.

   PCA is useful for finding new, more informative, uncorrelated features; it reduces dimensionality by rejecting low variance features.

2. Philosophy of PCA

   Introduced by Pearson (1901) and Hotelling (1933) to describe the variation in a set of multivariate data in terms of a set of uncorrelated variables

3. Data matrix – We perform a PCA on each of the historical forward strips on whole forward surface,

	Fwd								
Tenor (y)	SPOT	1m	3m	6m	1y	2y	3y	4y	5y
1y	0.24	0.19	0.42	0.45	0.63	0.96	1.63	2.14	3.28
2y	0.44	0.33	0.42	0.46	0.66	1.11	1.67	2.18	3.30
3y	0.72	0.37	0.43	0.48	0.70	1.19	1.72	2.24	3.34
4y	1.05	0.41	0.45	0.52	0.78	1.30	1.83	2.36	3.42
5y	0.48	0.50	0.55	0.66	1.05	1.58	2.11	2.56	3.44
7y	0.59	0.63	0.71	0.88	1.31	1.84	2.32	2.70	3.40
8y	0.97	1.06	1.16	1.36	1.80	2.24	2.61	2.90	3.42
9y	1.40	1.49	1.58	1.77	2.14	2.50	2.81	3.04	3.36
10y	1.88	1.96	2.04	2.18	2.48	2.74	2.96	3.14	3.34
15y	2.65	2.68	2.71	2.77	2.88	2.98	3.06	3.12	3.15

Date/Tenor	6M	1Y	2Y	--	15Y
01/01/10	0.02	0.03	0.04	--	0.07
02/01/10	--	--	--	--	--
--	--	--	--	--	--
14/08/12	0.02	0.04	0.07	--	0.08
15/08/12	0.03	0.04	0.05	--	0.08

4. Mathematical form of PCA data matrix

   We are looking for a transformation of the data matrix X such that

   $Y = \delta^T X = \delta_1 X_1 + \delta_2 X_2 + .. + \delta_p X_p$

   Where $\delta = (\delta_1, \delta_2, .., \delta_p)^T$ is a column vector of weights with $\delta_1^2 + \delta_2^2 + .. + \delta_p^2 = 1$

5. Maximise the variance

   Maximise the variance of the projection of the observations on the Y (swap rates) variables

   Find δ so that

   Var(δTX) = δT Var(X) δ is maximal

   The matrix C = Var(X) is the covariance matrix of the Xi variables

$$
\begin{bmatrix}
v(x_1) & c(x_1, x_2) & \cdots & c(x_1, x_p) \\
c(x_1, x_2) & v(x_2) & \cdots & c(x_2, x_p) \\
\vdots & \vdots & \ddots & \vdots \\
c(x_1, x_p) & c(x_2, x_p) & \cdots & v(x_p)
\end{bmatrix}
$$

The direction of δ is given by the eigenvector γ1 corresponding to the largest eigenvalue of matrix C

The second vector that is orthogonal (uncorrelated) to the first is the one that has the second-highest variance that comes to be the eigenvector corresponding to the second eigenvalue

New variables Yi that are linear combinations of the original variables (xi):

$Y_i = a_{i1}x_1 + a_{i2}x_2 + \ldots a_{ip}x_p$; $i = 1..p$

The new variables Yi are PCs and are derived in decreasing order of importance

6. Eigenvalues and eigenvectors
   a) Eigenvalues
      For a square matrix *A* of order n, the number λ is an eigenvalue, if and only if there exists a non-zero vector *C* such that $AC = \lambda C$
      Using the matrix multiplication properties, we obtain $(A - \lambda I_n)C = 0$
      This is a linear system for which the matrix coefficient is $A - \lambda I_n$
      Since the zero-vector is a solution and *C* is not the zero vector, then we must have det(C-λI)=0
   b) Eigenvector
      An eigenvector is a direction for a matrix
      Every square matrix has at least one eigenvector
      An *n x n* matrix should have *n* linearly independent eigenvectors

7. Calculating eigenvalues and eigenvectors
   a) The eigenvalues λi are found by solving the equation det(C-λI)=0
   b) Eigenvectors are columns of the matrix A such that A x = λx, C=ADA^T

$$where\ D = \begin{bmatrix} \lambda 1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots & \lambda_p \end{bmatrix}$$

A is a covariance matrix of the order of tenor and a vector x is a vector specified by historical rates and tenor n
The new variables (PCs) have a variance equal to their corresponding eigenvalue *Var*(Y_i)= λ_i for all *i=1…p*
Small λ_i ⇔ small variance ⇔ data change little in the direction of component Y_i

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{n-1} \\ r_n \end{bmatrix} = \begin{bmatrix} eig(1,1) & \cdots & eig(1,n) \\ \vdots & \ddots & \vdots \\ eig(n,1) & \cdots & eig(n,n) \end{bmatrix} \begin{bmatrix} PC1 \\ PC2 \\ \vdots \\ PC(n-1) \\ PC(n) \end{bmatrix}$$

8. Calculating the PC and fair value
   Take only the three PCs (cumulative variance explained by the PCs is > 99%) – disregard the rest of the PCs, as we regard the rest of the factors a surprise.

$$\begin{bmatrix} PC1 \\ PC2 \\ \vdots \\ PC(n-1) \\ PC(n) \end{bmatrix} = Inverse \begin{bmatrix} eig(1,1) & \cdots & eig(1,n) \\ \vdots & \ddots & \vdots \\ eig(n,1) & \cdots & eig(n,n) \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{n-1} \\ r_n \end{bmatrix} \qquad \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{n-1} \\ r_n \end{bmatrix} = \begin{bmatrix} eig(1,1) & \cdots & eig(1,3) \\ \vdots & \ddots & \vdots \\ eig(n,1) & \cdots & eig(n,3) \end{bmatrix} \begin{bmatrix} PC1 \\ PC2 \\ PC3 \end{bmatrix}$$

9. Calculating the PCA weights
   Calculate the market-neutral weights – no exposure to the first (level) and the second factors (curve) by performing the second PCA on three mispriced points from the first PCA.

$$\begin{bmatrix} r_2 \\ r_5 \\ r_{10} \end{bmatrix} = \begin{bmatrix} eig(1,1) & \cdots & eig(1,3) \\ \vdots & \ddots & \vdots \\ eig(3,1) & \cdots & eig(3,3) \end{bmatrix} \begin{bmatrix} PC1 \\ PC2 \\ PC3 \end{bmatrix}$$

$$r_2 = eig(1,1) \times PC1 + eig(1,2) \times PC2 + eig(1,3) \times PC3$$
$$r_5 = eig(2,1) \times PC1 + eig(2,2) \times PC2 + eig(2,3) \times PC3$$
$$r_{10} = eig(3,1) \times PC1 + eig(3,2) \times PC2 + eig(3,3) \times PC3$$

$$weight(r_2) = \frac{eig(1,3)}{eig(2,3)} \quad weight(r_{10}) = \frac{eig(3,3)}{eig(2,3)}$$

No exposure to PC1: $eig(1,1) \times weight(r_2) + \ eig(2,1) + \ eig(3,1) \times weight(r_{10}) = 0$
No exposure to PC2: $eig(1,2) \times weight(r_2) + \ eig(2,2) + \ eig(3,2) \times weight(r_{10}) = 0$

10. Investment horizon (half the lifetime)

    Half life time $H_{half} = \frac{\ln(2)}{\mu}$ where μ = reversion speed

    75% life time = $H_{75\%} = \frac{\ln(4)}{\mu}$

    87.5% life time = $H_{87.5\%} = \frac{\ln(8)}{\mu}$

11. Dickey-Fuller test: Unit Root test for the stationary

    If the time series appears to be fluctuating around a sample average, use the following test equation:

    $Y_t - Y_{t-1} = b_0 + b_1 Y_{t-1} - Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim iid(0,\sigma^2)$

    $\Delta Y_t = b_0 + \beta Y_{t-1} + \varepsilon_{tgg\ ak}$

Disclosures Appendix

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