Quantitative Strategies

TOPIC I: What drives breakeven inflation?[6]

- **Breakeven inflation levels represent not just the inflation expectations but also an inflation risk premium which depends upon the uncertainty over future inflation.**
- **We propose a simple model for breakeven inflation levels in the US. We find that the rise in the inflation risk premium explains a significant portion of the recent pick-up in breakeven inflation.**
- **A trading strategy based on our model results in attractive out-of-sample Sharpe ratios of 1.31 for 2028 TIPS and 1.33 for 2029 TIPS.**
- **Given the prevailing uncertainty over future growth and inflation, we believe that breakeven inflation levels, though much higher than consensus estimates, are likely to remain at current levels.**

Inflation risk premium

Nominal yields can be thought of as a sum of expected growth, expected inflation and a bond risk premium:

Nominal yield = Expected real growth rate + Expected inflation + Bond risk premium

Real bonds provide a protection against higher than expected inflation states. Thus an investor, in a world of only nominal and real bonds, would require a premium to compensate for the uncertainty in the expected inflation over the life of the bond. Assuming rational expectations, this premium can be expressed as the difference between the breakeven inflation rates and the expected inflation over the life of the bond. To put it more formally, inflation risk premium, as a first order approximation, can be expressed as

Inflation risk premium = Nominal rate – real rate – expected inflation.

In addition to inflation risk, the premium embedded in B/E rates should also include such not easily quantifiable risks as liquidity risk or other premia for structural shifts. We will assume that these are small or relatively stable through time.

Estimating the inflation risk premium

Using a consumption based asset pricing framework, with random endowments[7], we can show that the real rate can be expressed as

$$Y_r = \rho + \gamma g - \frac{1}{2}\sigma^2\gamma^2 \qquad (1)$$

where g is expected growth, σ is the variance of expected growth and ρ is the risk preference term.

Extending this framework to nominal bonds, we can show that nominal yields can be expressed as,

$$Y_N^T = \rho + \gamma g + i - \frac{1}{2}\sigma_N{}^2 \qquad (2)$$

where Y_N is the nominal yield and i is expected inflation. From equations (1) and (2), and using the relationship that real rate = nominal rate – breakeven inflation[8] we get

$$Y_N^T - Y_g^T = i + \frac{1}{2}(\sigma_i{}^2 - 2\operatorname{cov}(nom, i)) \qquad (3)$$

Thus the breakeven inflation, contrary to popular market belief, is not just the expected inflation over the remaining life of the bond. It includes a risk premium term which depends on the uncertainty of future growth and inflation and this should compensate investors exactly for the uncertainty of future economics.[9]

What drives breakeven levels?

Before we try to formally model breakeven rates, let us try to get some intuitive understanding of the inflation risk premium. As we showed above, the inflation risk premium depends on the uncertainty of expected inflation and expected growth over the remaining life of the bond. The higher the uncertainty – the higher the premium of inflation linked bonds over nominals – as, at least in theory, inflation linked bonds are a protection against unforeseen shocks to inflation.

A pertinent question arises - how do we quantify the uncertainty element? Disagreement among economists over the expected future inflation can be used a measure of this uncertainty. Diligent readers would recall that in our earlier publications, we had shown that the expected returns of inflation linked bonds versus nominals depend on the covariance between expected growth and inflation. We showed that such a model, based on consensus economist forecasts, performs well as a trading strategy. Unfortunately, forecasts data is available only monthly; thus we need to search for more frequent market variables that can be used in the model specification.

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[6] We would like to take this opportunity to thank Robin Lumsdaine who has given us valuable insight as well as contributing ideas for this piece.

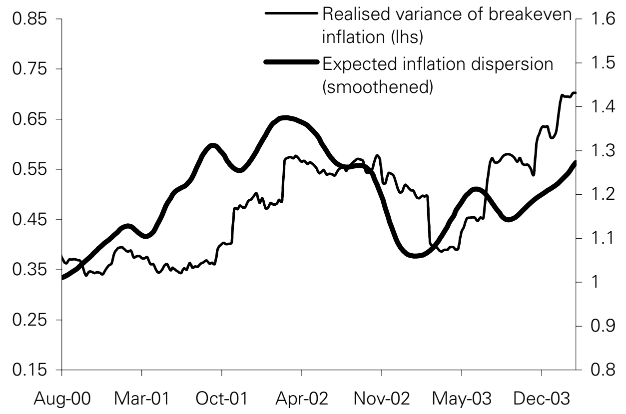
[7] We maximize our utility, subject to the constraint $c_t = e_t - e^{-r}\theta_t$ and $c_{t+1} = e_{t+1} + \theta_t$ with e_t being the endowment and θ_t being the savings (and r being the 1-period risk-free real rate). Solving for savings, we find $exp(-r_t)=E_t[e^{-\rho}(u'(c_{t+1})/u'(c_t))]$,
With lognormal consumption growth, $c_{t+1} = c_t\, exp(g(t) + \sigma_t^c \varepsilon)$ and power utility with risk-aversion γ, we get $r(t) = \rho + \gamma g_t - ½\gamma^2\sigma_t^2$

[8] We note that some definitions of the premium use ratios of real and nominal yields. We prefer additive premia for computational ease.

[9] i.e., in some sense equation (3) is the real “break-even”, although we refrain from using this terminology for fear of confusion.

As we have highlighted in previous Quantitative Strategies publications, implied volatility is the best predictor of realised volatility. The graph below shows the relationship between realised breakeven volatility and dispersion of consensus economists views (measured as a difference between the highest and the lowest estimate). As we can see, the higher the dispersion, typically, the higher the B/E variance in the future. A similar trend in the two series allows us to use realised breakeven volatility as a proxy for variance of expected inflation.

Exhibit 1: Realised volatility of breakeven inflation versus dispersion of expected inflation



Source: DB Global Markets Research

Empirically, we find that breakevens tend to be directional. This can be attributed to the lower volatility of real rates compared to nominals. Thus in a rally, we would expect breakeven inflation levels to decline, while they should widen in a sell-off. To capture the directionality component of the breakeven rates, we include the level of the 10Y nominal rates as one of our explanatory variables.

The model

For modelling the breakeven levels, we use realised volatility on breakeven inflation as a proxy for variance of future inflation. The covariance term is calculated as a rolling 1Y covariance between breakeven levels and nominal yields.

Exhibit 2 and 3 show the model specification and the model residuals for the 2028 TIPS. As expected, breakeven inflation levels increase with the increase in the uncertainty levels - increase in the variance of inflation and growth. The covariance term has a negative sign, in line with equation (3).

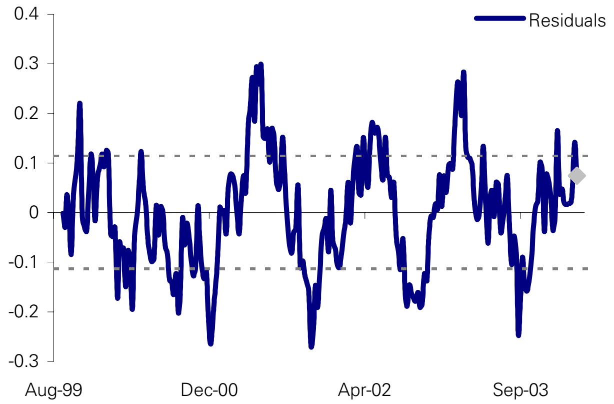
Exhibit 2: A simple model for breakeven inflation

Variable	*Beta*	*t-stat*	*Current value*
Expected inflation	0.12	2.1	1.57
Variance (inflation)	0.60	2.3	0.70
Variance (real rates)	1.64	8.3	0.49
Implied swaption volatility*	0.07	3.8	1.30
Cov (nominal, breakeven inflation)	-0.21	-1.7	0.64
10y nominal rate	0.12	3.6	3.79
C	0.89	4.6	
R-square	0.77		
Adj R-square	0.76		
Prob(F-stat)	0		

**We use 1Yx2Y volatility as a proxy for implied volatility*

Source: DB Global Markets Research

Exhibit 3: Model residuals: Current value still lies within the one standard deviation band



Source: DB Global Markets Research

Interestingly, we find that though the current breakeven levels are much higher than the long term consensus expectations, the present residual value (indicated by the grey dot in exhibit 3) still lie within the one standard deviation band, thus suggesting that breakeven levels are currently close to fair value.

We fit our model for other maturity TIPS and find consistent explanatory power of the variables considered.

Recent market moves – a rationale

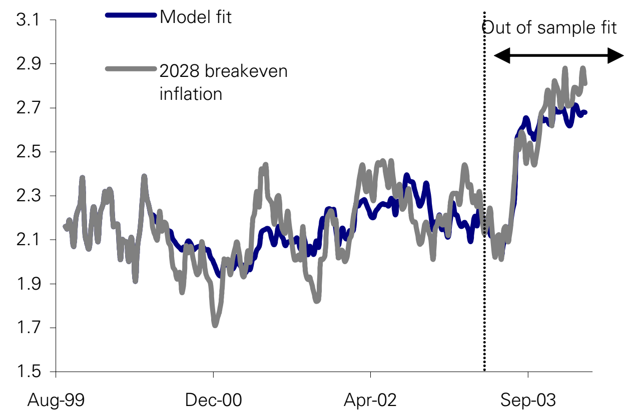
So what has driven the recent breakeven levels? Inflation expectations, as measured by consensus forecasts, have been grinding downwards in recent months while breakeven inflation levels across all maturities have risen sharply.

Breakeven inflation levels have risen sharply since the Jun-03 lows, as Fed fears of deflation fuelled uncertainty regarding timing and magnitude of possible reflationary attempts. Those fears have given way to uneasiness in the market concerning when we will see a balance of growth, employment, and inflation that (in the Fed's mind) warrants a departure from a policy of accommodation. In addition, in our view Fed rhetoric emphasising currently 'low, stable inflation' has injected uncertainty about future inflation. Thus in our view the past 9 months have been a period of increased uncertainty over future inflation and thus commanded a higher inflation risk premium.

While the rise in the uncertainty premium does explain a major portion of the breakeven rise, we note that some of this may be due not to inflation risk but to other structural factors – notably increasing demand from pension plans combined with global supply constraints. We therefore expect that breakeven levels should be well supported going forward.

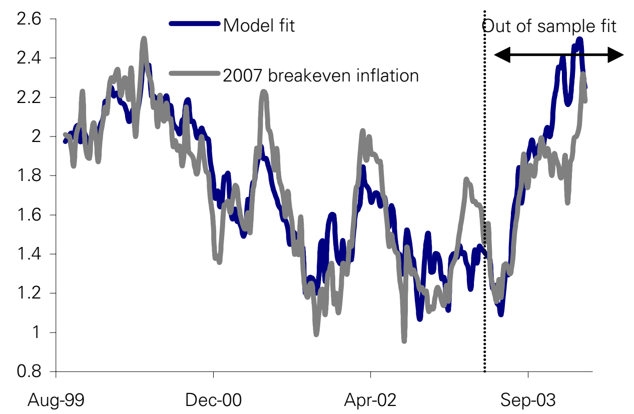
To estimate the predictive power of the model, we perform an out-of sample fit for the model. Exhibits 4 and 5 show the out-of-sample fit for the 2028 and 2007 TIPS. We find that a rise in uncertainty premium explains a major portion of the recent pick-up in breakeven inflation levels.

Exhibit 4: Pickup in inflation premium explains the pickup in inflation levels



Source: DB Global Markets Research

Exhibit 5: Model fit for the 2007 TIPS



Source: DB Global Markets Research

Trading the inflation risk premium

We back-test our model, using out of sample forecasts for the trading strategy. The table below shows the average return and Sharpe ratios for different maturity TIPS. The figures are calculated based on out-of-sample trading for the last two years[10]. We find attractive Sharpe ratios of 1.31 for the 2028 TIPS and 1.33 for 2029 TIPS.

Exhibit 6: Sharpe ratios of the trading strategy

Bond	*No trigger*	*σ/2*	*σ*
2007	0.25	0.74	0.89
	(100%)	(57%)	(33%)
2028	0.32	0.75	1.31
	(100%)	(71%)	(35%)
2029	0.33	0.49	1.33
	(100%)	(63%)	(36%)

Note: Figures in parenthesis indicate trading frequency

Source: DB Global Markets Research

Conclusions

The above analysis shows that breakeven inflation levels are not just a reflection of inflation expectations but also include a risk premium term which depends on the uncertainty of future inflation and growth. The recent pickup in breakeven levels is largely explained by an increase in the rise in the uncertainty premium. We find that these uncertainty levels are likely to persist for some time till we get a clearer clue from the Fed. This, combined with a large pent up demand from pension plans, makes us constructive on breakevens in the near to medium term.

Trade Recommendation

- Hold B/E spreads across the TIPS curve

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TOPIC II: How tame is the Condor?

- **We analyse the stability of condor weights derived from PCA or cointegration analysis by using the Bootstrapping method.**
- **We find that most of the instability of condor weights depends on the sample length and period.**

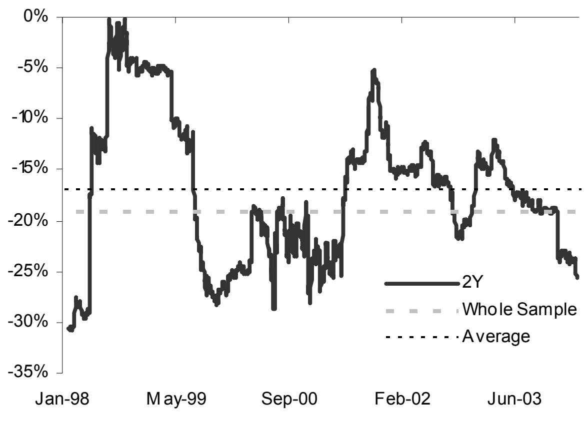
[10] We estimate the model using data from Jan-99 to the day of the trading. If the residuals are positive and higher than the threshold, implying that breakevens are too wide, we go short the TIPS and long an equivalent maturity nominal. If the residuals are negative, a reverse trade is put on.

- **Weights mean-revert to a long term average and variance in PCA estimates is due to smaller sample size and not changing market conditions. Taking too small a sample may lead to deriving sample-biased weights.**
- **We recommend a 3-7-15-20 condor in Euroland, one of the few remaining condors still offering value.**

We formally introduced the PCA weighted condor on March 5th (*Quantitative Strategies*, *FIW*, March 5th 2004). A main advantage of the PCA approach is that we can easily find a set of weights that are explicitly level, slope and concavity-neutral. Indeed, one of the features of PCA analysis is that each principal component (PC) is independent of all the others. Hence, if we extract all four PCs of our four rates, the fourth will be independent of the first three (which represent none other than the level, slope and curvature). So the fourth component gives a set of neutral weights for a trade which is level, slope and curvature-neutral. Also, as we expect there to be three (non-stationary) PCs of the yield curve and we should only care about their stability up to rotation and consequently look at the fourth PC (i.e., we don't care if a little slope gets mixed into our first PCA estimate and a little level gets mixed into our second PCA estimate, so long as only the first three get mixed leaving the fourth unchanged).

It has been observed that PCA weights seem to very sample sensitive. For example, Exhibit 1 shows the 1Y rolling PCA weight of the 2Y leg of the 2-5-10-30 butterfly.

Exhibit 1: 1Y rolling PCA weight of 2Y leg



Source: DB Global Markets Research

The 2Y weight is by far the most volatile of the four weights as shown in table 1. However, we notice that the rolling weights drift close to their mean. But more importantly we see that the average rolling weight is very close to the whole sample PCA weight. This is even clearer when looking at the other weights. This is suggestive of the possibility that the variance in PCA estimates is not necessarily due to changing market conditions but can instead be just a higher standard error in estimates due to smaller sample sizes.

Table 1: 1Y rolling PCA weights

	2Y	5Y	10Y	30Y
Whole Sample	-19.1%	54.8%	-73.1%	35.8%
Average	-17.0%	51.1%	-73.5%	37.5%
Median	-17.5%	55.3%	-73.7%	35.4%
Std Dev	7.2%	11.7%	3.5%	9.8%
Std Dev/Average	-42.0%	22.9%	-4.7%	26.1%
Min	-30.9%	13.6%	-79.5%	23.6%
Max	-0.1%	66.4%	-63.9%	61.6%

Source: DB Global Markets Research

In other words, it looks like the variation in the PCA weights is due to the sample chosen, and is not a consequence of modelling error. We will show this formally by implementing the bootstrapping method.

Bootstrapping

PCA gives us a method for extracting the co-movements of the four rates (we limit our PCA analysis to these four rates alone rather than the entire curve). Common practice is to assume that there are three principal components (or common trends) driving the yield curve. If we do our PCA in levels rather than in differences, we will be extracting exactly the first three modes of the four bonds, the three non-stationary components of their movement. The fourth principal component is stationary by design (e.g., mean-reverting). In essence, by doing PCA in levels we are doing a *cointegration* model. The fourth principal component is similar to the cointegrating vector[11] and will consequently be mean-reverting. As such we will look at the stability of the condor weights as given by these two methods.

We will use bootstrapping to look at the stability of the weights. The bootstrapping method is based on resampling. For each resampling we can re-estimate the model, and see how stable the results are.

PCA Bootstrap

A PCA enables us to represent the data as linear combinations of independent factors, F. The linear combinations are given by the loadings or rotation matrix, L. So we can represent the data as:

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[11] In particular, the common trends of a set of time series (e.g., the level, slopes and curvature) are non-stationary and thus have the highest variance and naturally, a PCA will tend to search them out as the first few factors. The stationary component (e.g., the condor) will generally have less variance and be among the last few principal components. This can be made more formal for large-samples and has been suggested as an alternative to the level-regressions method of Engle-Granger as a means of finding cointegration relations. Irrespective, the Johansen method which involves solving a generalized eigenvalue problem is the most proper for small samples.

$$\text{Data}_t = LF_t$$[12]

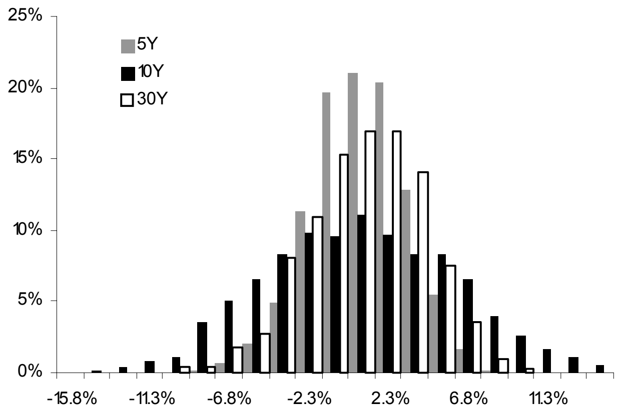
The second step of the bootstrap method is to resample from F_t. In other word we randomly draw a new set of factors, $\{F_t^*\}$, from the set $\{F_t\}$ of factors (where we sample with replacement to get a new sample of the same length). We can then create a new set of data, Data* using the above relationship:

$$\text{Data}_t^* = L\,F_t^*$$

By doing a new PCA on Data* we will obtain a new loadings matrix L*. We can then save L* and repeat the procedure to obtain distributions of the bootstrapped loadings. These distributions will then give us an idea of the stability of the PCA methodology or allow us to derive a bootstrap standard error. For the purposes of this article we will resample 1,000 times.

The results demonstrate that the PCA weights are very stable. For ease of comparison between methods and with the cointegration results, we will set the 2Y weight equal to 100%. Exhibit 2 shows the density plots for the percentage deviation of bootstrap weights from their mean for the three sets of weight, where we have normalised the 2Y weight to be 1. We see that in most cases the bootstrapped weight is within 10% of its actual PCA weight.

Exhibit 2: Bootstrapped weights: density around the mean



Source: DB Global Markets Research

Table 2 further highlights the stability of the PCA methodology.

Table 2: Bootstrapped Weights

	2Y	5Y	10Y	30Y
PCA Weight	100.0%	-309.9%	329.6%	-120.7%
Average	100.0%	-310.1%	330.0%	-120.9%
Median	100.0%	-310.1%	329.9%	-120.7%
Std Dev	0.0%	2.6%	5.4%	3.3%
Std Dev/Average	0.0%	-0.8%	1.6%	-2.7%
Min	100.0%	-319.3%	315.6%	-131.9%
Max	100.0%	-303.3%	349.1%	-112.2%

Source: DB Global Markets Research

In addition, we look at alternative bootstrap methods. The results in the above table are based on drawing the set of four factors at the same point in time. But we could have drawn each of the factors independently, so that a set of four resampled factor may come from different point in time, (loosening the assumptions and probably increasing the variance). Also we may want to resample directly from the rates (but are not allowed to sample each rate independently since the resulting sample covariance will be completely non-sensible). Table 3 gives the results of these other bootstrap methods. The results confirm our previous ones.

Table 3: Alternative bootstrap methods

Resampled from factors at different times

	2Y	5Y	10Y	30Y
PCA Weight	100.0%	-309.9%	329.6%	-120.7%
Average	100.0%	-309.9%	329.5%	-120.6%
Median	100.0%	-309.9%	329.5%	-120.5%
Std Dev	0.0%	2.7%	5.7%	3.4%
Std Dev/Average	0.0%	-0.9%	1.7%	-2.8%
Min	100.0%	-321.2%	312.3%	-134.2%
Max	100.0%	-302.0%	352.5%	-110.0%

Resampled from rates

	2Y	5Y	10Y	30Y
PCA Weight	100.0%	-309.9%	329.6%	-120.7%
Average	100.0%	-309.9%	329.6%	-120.8%
Median	100.0%	-309.8%	329.6%	-120.8%
Std Dev	0.0%	3.1%	6.2%	3.5%
Std Dev/Average	0.0%	-1.0%	1.9%	-2.9%
Min	100.0%	-321.2%	310.8%	-133.9%
Max	100.0%	-301.0%	352.2%	-110.7%

Finally, we look at PCA on rate changes. Even though a PCA on levels is our preferred method, many market participants calculate PCA on changes so we show the results for completeness. We note that, although not altogether bad, the standard error for the PCA in differences is much higher than that in levels.

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[12] We note that there is no error term as we use four components to fully describe the motion of the four bonds.

Table 4: Bootstrap on PCA of rate changes

Resampled from factors at same time

	2Y	5Y	10Y	30Y
PCA Weights	100.0%	-287.4%	383.2%	-187.8%
Average	100.0%	-290.4%	391.9%	-193.7%
Median	100.0%	-287.6%	384.9%	-189.9%
Std Dev	0.0%	21.1%	52.9%	34.7%
Std Dev/Average	0.0%	-7.3%	13.5%	-17.9%
Min	100.0%	-391.0%	282.6%	-353.7%
Max	100.0%	-241.2%	645.2%	-118.2%

Resampled from factors at different times

	2Y	5Y	10Y	30Y
PCA Weights	100.0%	-287.4%	383.2%	-187.8%
Average	100.0%	-287.8%	384.1%	-188.5%
Median	100.0%	-286.9%	383.1%	-187.1%
Std Dev	0.0%	13.8%	32.0%	20.2%
Std Dev/Average	0.0%	-4.8%	8.3%	-10.7%
Min	100.0%	-347.5%	299.9%	-274.2%
Max	100.0%	-253.3%	520.9%	-135.9%

Resampled from rates

	2Y	5Y	10Y	30Y
PCA Weights	100.0%	-287.4%	383.2%	-187.8%
Average	100.0%	-289.8%	389.8%	-192.4%
Median	100.0%	-287.3%	382.3%	-187.7%
Std Dev	0.0%	20.6%	52.0%	34.4%
Std Dev/Average	0.0%	-7.1%	13.4%	-17.9%
Min	100.0%	-377.0%	265.4%	-314.8%
Max	100.0%	-239.3%	592.4%	-114.5%

Cointegration Bootstrap

We finally look at the cointegration weights. First, let's review the VEC methodology. Given a set of multivariate data, econometricians will often forecast using a series of OLS for each variable in terms of lags of all of the given variables (a reduced form model, usually called a VAR or vector-autoregression). When our data is not *mean-reverting*[13] (see for example the discussion in *Trading Butterflies: Traditional Approaches can be Misleading*, Quantitative Strategies, FIW, 21-Nov 2003 and *Butterflies: just another way of taking a view on slopes?*, Quantitative Strategies, FIW, 12-Dec 2003), it is necessary to decide if a linear combination of the variables is mean-reverting (*cointegration).* Cointegrated series are more accurately modelled through a vector error correction (VEC, a restricted form of a VAR) than through the general form of a VAR.

A VEC formulation essentially forecasts a vector of variables X_t, we model changes through:

$$\Delta X_t = \alpha(\beta \cdot X_{t-1}) + \varepsilon_t$$

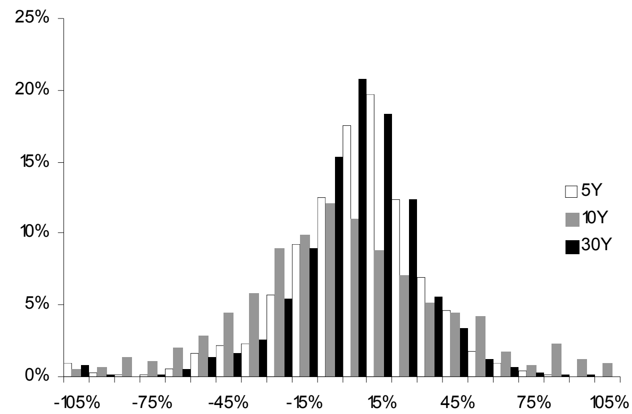
For a VEC formulation the (parametric) bootstrap method consists in resampling from the estimated model errors. So we draw a set $\{\varepsilon_t^*\}$ from the set $\{\varepsilon_t\}$ of model errors. By fixing $X_0^*=X_0$ as a starting point, we can reconstruct a set $\{X_t^*\}$ of observations:

$$X_t^* = X_{t-1}^* + \alpha(\beta'. X_{t-1}^*) + \varepsilon_{t-1}^*, \text{ for } t=1,\ldots,T$$

We can then refit the VEC model on $\{X_t^*\}$ to obtain a new set of parameters α^* and β'^*.

Just like for the PCA bootstrap, we resample 1,000 times. We obtain the densities shown in exhibit 3. We see that the VEC method is not as stable as the PCA. Table 5 further highlights that point.

Exhibit 3: Bootstrapped weights: density around the mean



Source: DB Global Markets Research

While the VEC methodology is, in our estimation, superior in terms of the parametric assumptions for the DGP, the Johansen estimation method introduces a good deal more error than does standard PCA.[14]

However, a VEC obviously implies estimating a model. In addition there are several equations underlying a VEC estimation. On the other hand, PCA is a purely mathematical result and has no distributional assumption with it. As such it should come as no surprise that the VEC is less stable. Nevertheless, the VEC weights exhibit a reasonable degree of stability.

[13] We establish mean-reversion through a unit-root test, e.g., an Augmented Dickey-Fuller Test or a Phillips-Perron test, under the null of a unit root (no mean-reversion). Failure to reject the null results in claiming a series has a unit root. In particular, most yield series are known (using say, anything from 5-10 years of data) to have unit roots. In reality, they may be stationary or mean-reverting with regime shifts, but the speeds of mean-reversion cannot be determined over so short a time-frame and several business cycles are needed.

[14] We note that PCA can be made more formally correct by writing our series into the stochastic common trends (a state-space model): $Y_t = L F_t + \varepsilon^1_t$, $F_t = F_{t-1} + \varepsilon^2_t$. Estimates of L are usually made from PCA loadings and found to be asymptotically equivalent to likelihood estimators.

Table 5: Bootstrapped Weights

	2Y	5Y	10Y	30Y
Coint. Weight	100.0%	-270.1%	234.5%	-64.4%
Average	100.0%	-278.0%	253.1%	-80.4%
Median	100.0%	-274.6%	248.0%	-78.6%
Std Dev	0.0%	29.4%	48.3%	26.0%
Std Dev/Average	0.0%	-10.6%	19.1%	-32.3%
Min	100.0%	-481.5%	106.0%	-230.0%
Max	100.0%	-201.9%	523.6%	5.7%

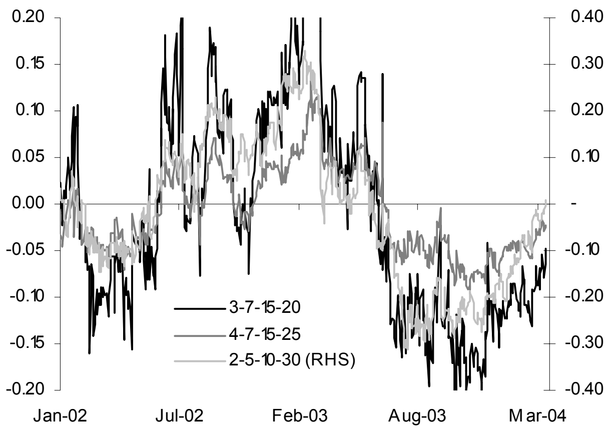
Source: DB Global Markets Research

Altogether we can conclude that PCA in levels is much more stable than PCA in differences and both more stable than Johansen's method for estimating VEC's. While there is always some possibility of a structural shift or parameters that drift which requires that only recent data should enter into an estimation, results suggest that longer samples increase stability without sacrificing accuracy.

One final point that we have not addressed is the time-constancy of our cointegration or PCA parameters. While our results suggest that variation in parameter estimates is due to increased standard errors from lower sample sizes, we have not formally established the parameter constancy. This is a (hyper)-technical discussion which we leave to later studies, but we suffice it to say that our study should leave us well-assured that the parameters are stable.

With that in mind we take a look back at our recent recommendations. As pointed out in our article on March 5th, the 2-5-10-30 condor had run most of its course. It is now at fair value, with today's dislocation being only 1bp. We therefore recommended implementing a 4-7-15-25 condor. This condor has performed well, going from -6bp to -2bp. It is therefore now close to fair value and we look at alternative condors for value. While most condors have recently gone back to fair levels, we still find value in the 3-7-15-20 condor, with a current dislocation of 5bp, as shown in exhibit 4.

Exhibit 4: PCA-weighted condor dislocations



Source: DB Global Markets Research

The following table summarises our recommended DV01 neutral weights. We recommend to repo the bonds wherever possible. Real money can initiate this trade through underweights and overweights but it is not cash neutral and has a cash take-out of €26mm.

Table 6: 3-7-15-20 DV01-neutral weights

	Y3	Y7	Y15	Y20
Weight (€mm)	-100.00	151.72	-194.67	116.88
Net Cash Position (€mm)	-26.07			
1M Carry (bp)	-1.41			

Source: DB Global Markets Research

Trade Recommendation

- **Initiate a 3Y-7Y-15Y-20Y condor.**
- **Hold on to 4Y-7Y-15Y-25Y condor.**
- **Exit 2Y-5Y-10Y-30Y condor.**

Data Set for Stability Analysis

- **Sample: Jan-97 to date**
- **Source:**
  - **Jan-97 to Dec-98: German swap rates.**
  - **EUR swap rates thereafter**
- **Frequency: daily**

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