Introduction to Hedge Ratio Estimation Methods

HUDSON AND THAMES

CONTENTS

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Research Interests:

- Distance Approach for Stats Arbitrage
- Hedge Ratio Estimation
- Dynamic Graph Embedding
- Graph Neural Network

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01. KEY CONCEPTS

Hedge Ratio Estimation Methods

1. The following presentation closely follows a paper by **Lopez de Prado, M.M. and Leinweber, D. (2012). Advances in Cointegration and Subset Correlation Hedging Methods**

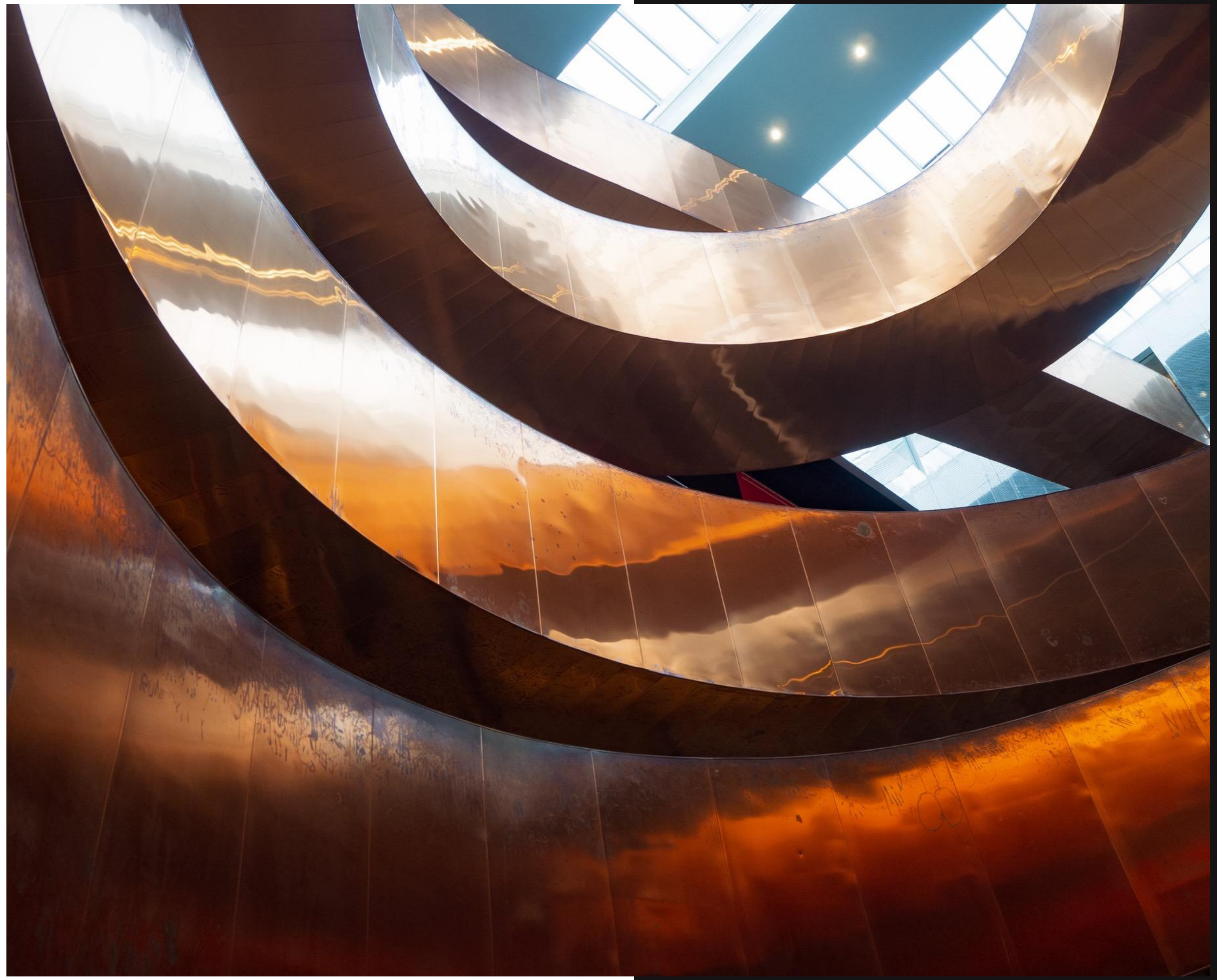
2. As well as a paper by **Lee et al. (2009). Alternative methods for estimating hedge ratio: Review, integration and empirical evidence**

Hedge Ratio Estimation Methods

1. Hedging problem is posed as the following equation

$$S_t = P_{1,t} + \sum_{n=2}^{N} \omega_n P_{n,t}$$

   a. where P_1 represents the market value at observation t of a portfolio we wish to hedge
   b. and P_n represents a set of variables(instruments or portfolios) available for building a hedge
2. The hedging problem is in computing the vector w_n (holdings of each variable)



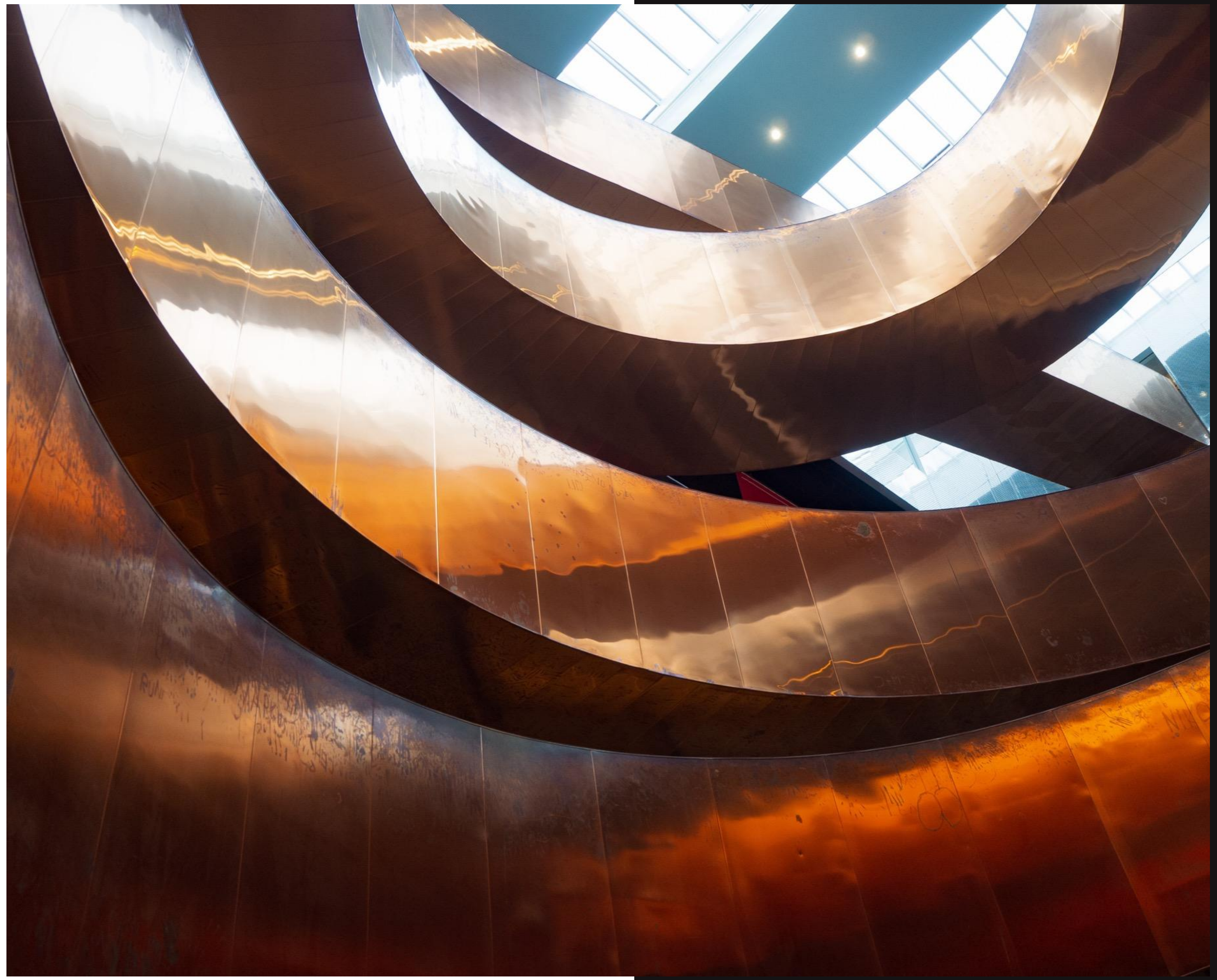
Hedge Ratio Estimation Methods

3. The hedging error is as follow

$$e(h) = S_{T+h} - S_T$$

   a. which is the error after h observations

4. Whether e(h) is stationary or non-stationary in variance is a crucial problem to hedge ratio estimations (Lopez de Pedro, 2012)



Hedge Ratio Estimation Methods

1

Single Period (Static Method)

- Assume that the returns are IID

2

Multi Period (Dynamic Method)

- Does not assume IID random perturbations

02. SINGLE PERIOD METHODS

Single Period Methods

- OLS in Differences (OLSD)
- Minimum Variance Portfolio (MVP)
- Principal Components Analysis (PCA)

Single Period Methods

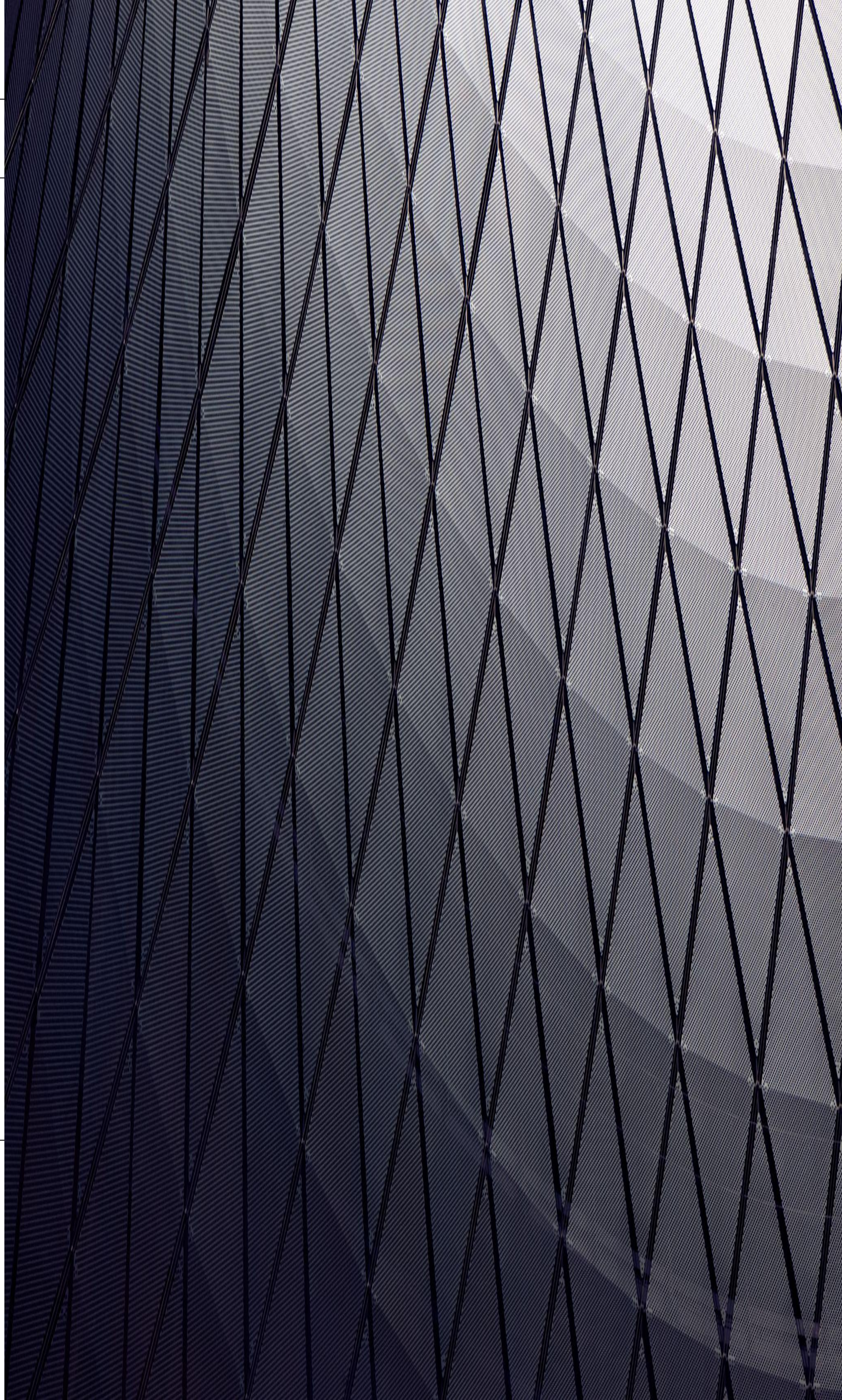
- OLS in Differences (OLSD)
- Minimum Variance Portfolio (MVP)
- Principal Components Analysis (PCA)

OLS in Differences (OLSD)

1. Because of its simplicity, this is one of the most widely used methods (Moulton and Seydoux, 1998)

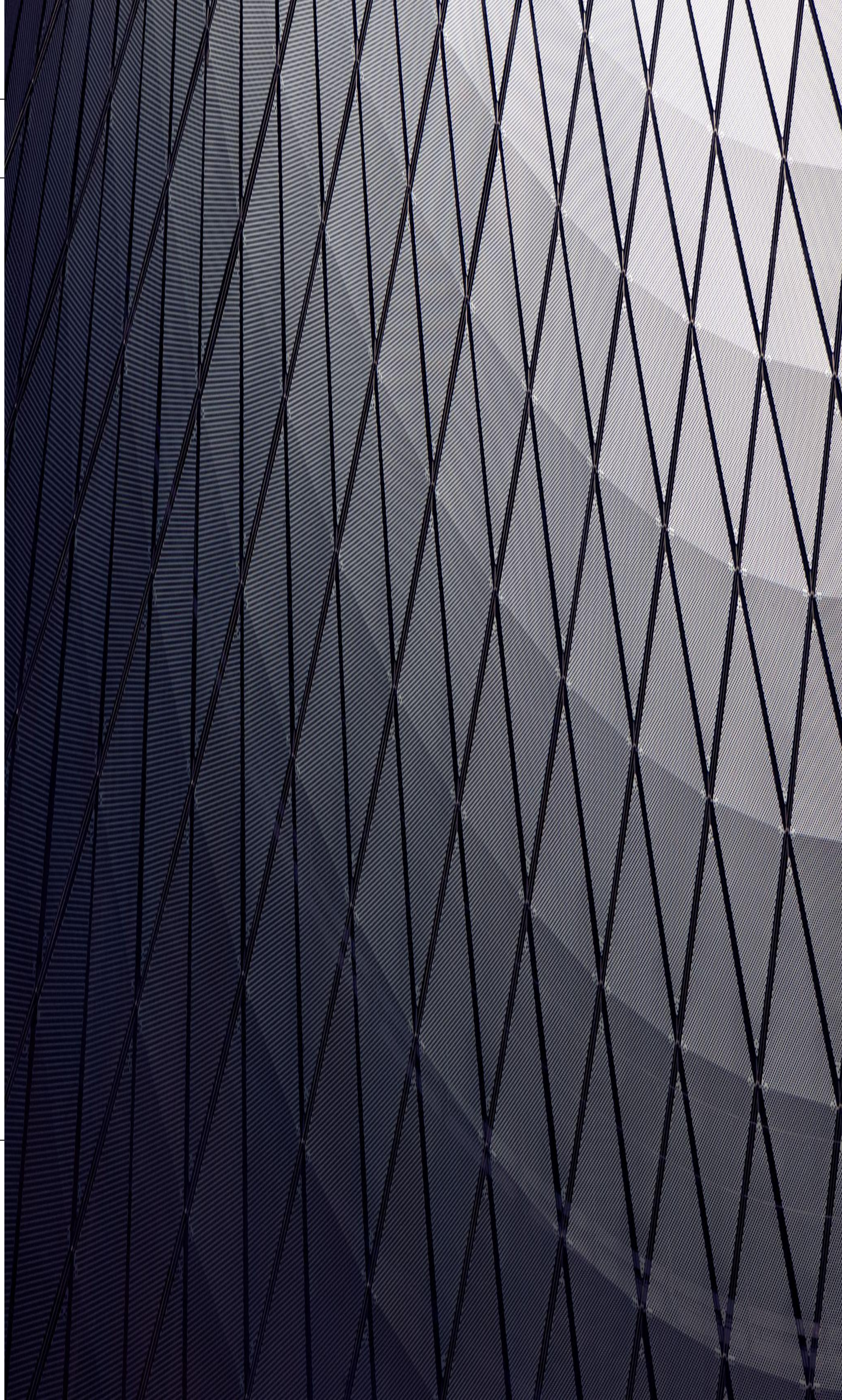
$$\Delta P_{1,t} = \alpha + \sum_{n=2}^{N} \beta_n \Delta P_{n,t} + \varepsilon_t$$

   a. where ΔP represents the change in market value between observations
2. Necessary condition
   a. alpha(α) is statistically insignificant
3. Solution : $\omega_n = -\beta_n$



OLS in Differences (OLSD)

- **Limitations**
  - alpha has to be zero
  - epsilon(ε) is IID Normal
  - assuming any change in the target portfolio(P_1) must be offset by the hedging portfolio(weighted sum of 2 to n portfolios)



- OLS in Differences (OLSD)
- Minimum Variance Portfolio (MVP)
- Principal Components Analysis (PCA)

Single Period Methods

Minimum Variance Portfolio (MVP)

1. Introduced by Markowitz (1952)
2. Settings:
   a. ΔP observations are IID Normal
   b. V is the covariance matrix of ΔP, where its first column represents the covariances against the portfolio we wish to hedge(P_1)

$$\begin{aligned} \mathrm{Min}_\beta \quad & \beta' V \beta \\ \text{s.t.} \quad & \beta' a = 1 \end{aligned}$$

Minimum Variance Portfolio (MVP)

3. Solution:
    a. Can be solved using lagrangian

$$L(\beta, \lambda) = \frac{1}{2}\beta' V \beta - \lambda\left(\beta' a - 1\right)$$

    b. Optimal beta and weights are:

$$\beta = \frac{V^{-1}a}{a'V^{-1}a} \qquad \omega_j = \frac{\beta_j}{\beta_1}$$

4. Results:
    a. MVP results in the minimum risk solution under the assumption of normality

Single Period Methods

- OLS in Differences (OLSD)
- Minimum Variance Portfolio (MVP)
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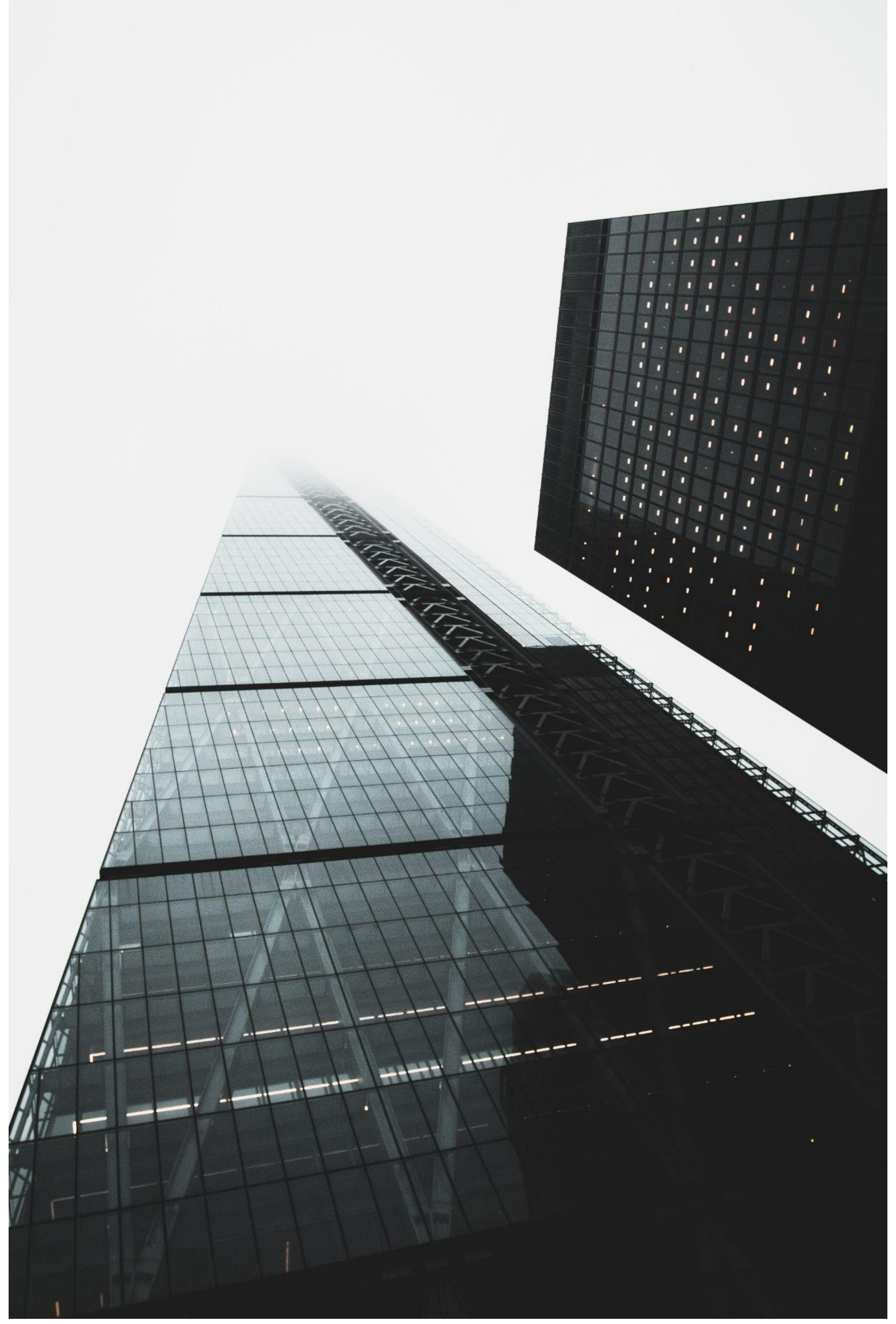
Principal Components Analysis (PCA)



1. The target is to compute the vector of weightings β such that is hedged against moves of the m largest principal components (typically, m=N-1), leaving the combined position solely exposed to moves of the N-m components with lowest variances (eigenvalues)
2. Our goal:
   a. Find β which follows $W^{*\prime}\beta = \mathbf{0}_m$
   b. where (W*)` is the transposed eigenvector matrix after having removed the columns associated with the unhedged eigenvectors
3. This approach presents the advantage of searching for a solution which hedges against the principal sources of risk

03. MULTI PERIOD METHODS

- OLS in Levels (OLSL)
- Error Correction Model (ECM)

Multi Period Methods

Multi Period Methods

- OLS in Levels (OLSL)
- Error Correction Model (ECM)

OLS in Levels (OLSL)

1. The goal looks similar to OLSD method, but the condition that the hedge is effective when S is stationary in mean and variance gives a different approach to OLSD

$$P_{1,t} = \sum_{n=2}^{N} \beta_n P_{n,t} + S_t$$

2. However, as the error correction component is not separated from the observed levels in the equation, the calculated weight, beta, may not be the optimal

Multi Period Methods

- OLS in Levels (OLSL)
- Error Correction Model (ECM)

Error Correction Model(ECM)

1. If an error correction representation is verified, the series of it are cointegrated (Engle and Granger, 1987)
2. Although this model only shows a hedge ratio between two portfolios, the extension of this method will be more discussed in the advanced methods

$$\Delta p_{1,t} = \beta_0 + \beta_1 \Delta p_{2,t} + \gamma(p_{2,t-1} - p_{1,t-1}) + \varepsilon_t$$

   a. where p is the natural log of market value P
   b. $\boldsymbol{\gamma}$ has to be tested positive(>0) in order to be effectively hedged

3. The optimal holdings will be $(\omega_1, \omega_2) = (1, -K)$

04. ADVANCED METHODS

- BOX-TIAO CANONICAL DECOMPOSITION (BTCD)
- DICKEY-FULLER OPTIMAL (DFO)

Advanced Methods

Advanced Methods

- BOX-TIAO CANONICAL DECOMPOSITION (BTCD)
- DICKEY-FULLER OPTIMAL (DFO)

BOX-TIAO CANONICAL DECOMPOSITION (BTCD)

1. Box and Tiao (1977) introduced a canonical transformation of a N-dimensional stationary autoregressive process
   a. The components of the transformed process can then be ordered from least to most predictable
2. AR(1) → VAR(1) → VAR(L)

$$\boxed{P_t = \beta P_{t-1} + \varepsilon_t}$$

AR(1)

$$\boxed{P_{t,n} = \sum_{i=1}^{N} \beta_{i,n} P_{t-1,i} + \varepsilon_{t,n}}$$

VAR(1)

$$\boxed{\sum_{l=1}^{L} \sum_{i=1}^{N} \beta_{i,l,n} P_{t-l,i} + \beta_{n,0} X_{t-1,n} + \varepsilon_{t,n}}$$

VAR(L)

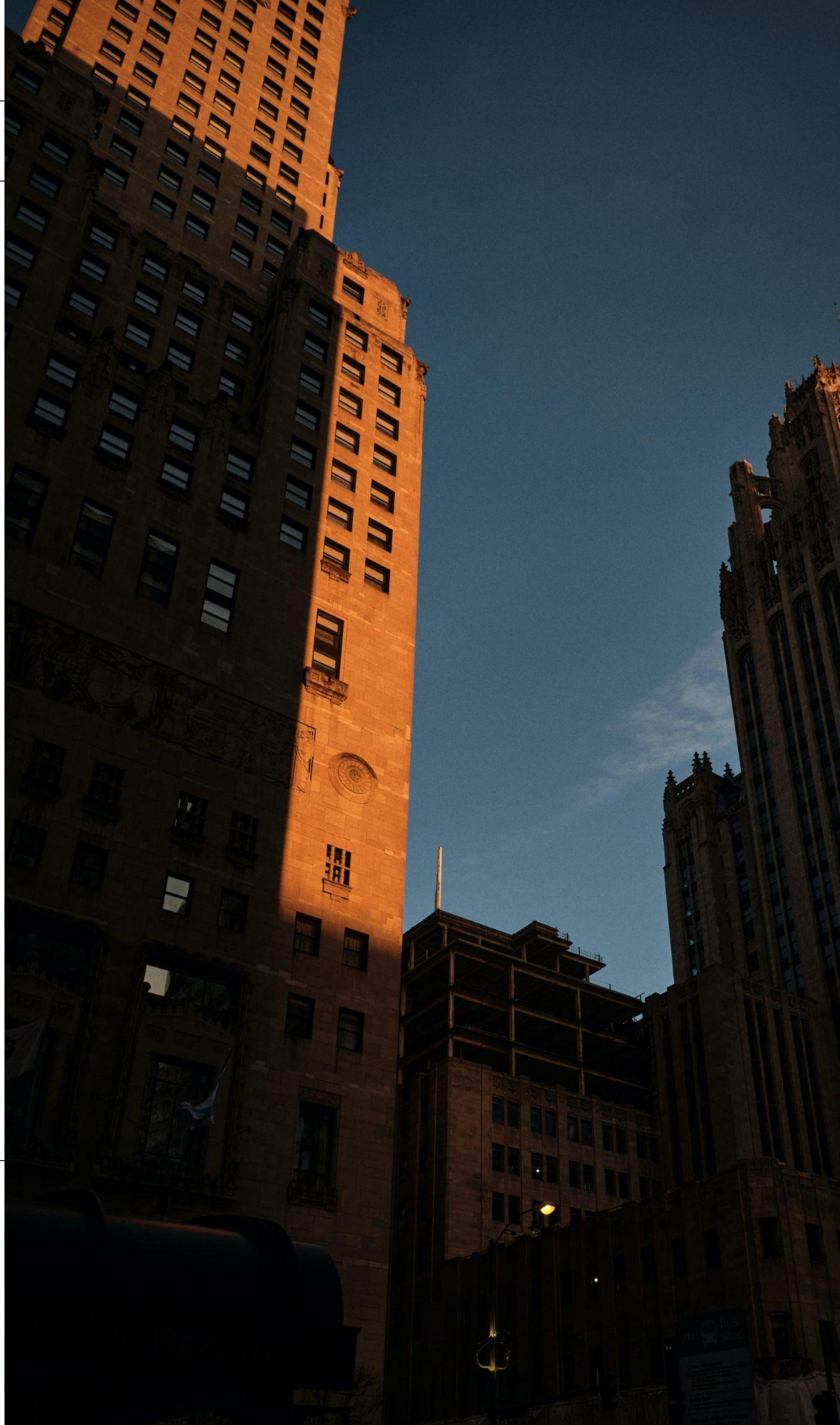
- BOX-TIAO CANONICAL DECOMPOSITION (BTCD)
- DICKEY-FULLER OPTIMAL (DFO)

Advanced Methods

DICKEY-FULLER OPTIMAL (DFO)

1. In the previous chapter, ECM is a dynamic model limited to two dimensions
2. This limitation could be solved through a canonical transformation of a multivariate, multi-equation specification as we did for BTCD
   a. This approach will give stronger structure through a system of equations, each imposing an individual autoregressive equilibrium condition
3. The target is to find an optimal w where the probability of having a unit root in the spread(S) is minimized

$$S_t = P_{1,t} + \sum_{n=2}^{N} \omega_n P_{n,t}$$



References

- **Lopez de Prado, M.M. and Leinweber, D. (2012).** Advances in Cointegration and Subset Correlation Hedging Methods. SSRN Electronic Journal. [Available here]
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THANK YOU!

Does anyone have any questions?

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