Derivatives Strategy
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Research Note

Introduction to Bermudan Swaptions and A Framework for Analysis

- **Bermudan swaptions may be thought of as a basket of European swaptions but with a single exercise**
- **A Bermudan Swaption will "track" the richest European swaption in its basket (the "CTD"), with an additional premium for the right to choose other exercise possibilities**
- **We characterize Bermudan Swaptions by analyzing the "bases" of the European swaptions in the basket**
- **We discuss the replication of Bermudan Swaptions using European swaptions in its basket**
- **We develop a measure of relative value between Bermudan and European swaptions**
- **A new *Bermudan Swaptions Report* based on the approach outlined here may be found in the daily Interest Rate Derivatives Analytics package**

Introduction

Options markets today provide investors a diverse set of products to choose from – for example, CBOT Treasury options, Libor Caps & Floors and interest rate swaptions are all options with deep and liquid markets. These markets all offer investors transparent pricing and high liquidity as well as specific risk characteristics that can be exploited to hedge an investor's portfolio in an optimal manner and/or express a market view using options. Swaptions in particular have become arguably the most liquid options on rates that are available today. Participants in the swaptions market tend to focus their attention on the "European" swaptions – i.e., swaptions that can be exercised only on a single date. European swaptions are certainly very liquid products that are heavily used by investors. However, "Bermudan" swaptions (which can be exercised once on any of a predetermined set of dates) also present interesting characteristics and have in recent years become increasingly liquid. Bermudan Swaptions are another "tool in the box" for investors looking to hedge convexity/vega risk or express a market view.

Nevertheless, many investors do not focus on these, because of concerns regarding liquidity as well as their inherent complexity. We believe that liquidity is less of an issue today, and our goal in this research note is to develop a framework in which Bermudan swaptions can be analyzed vis-à-vis European swaptions.

Historically, the market for Bermudan swaptions developed in large part alongside the growth of the callable loan program. An investor who owns a bond that is callable on multiple dates (typically ex-coupon dates) is long a bond and is short a Bermudan call option on that bond. Frequently, when demand for callables creates opportunities, issuers seeking floating rate debt will issue callable fixed rate debt and strip the Bermudan option and sell it in the options market. The issuer can then enter into a swap in which they receive fixed. The net effect is to synthetically create floating rate debt, but by exploiting a mispricing in the embedded Bermudan option, the issuer can reduce borrowing costs.

The counterpart to callable bonds in the world of derivatives is a cancellable swap. A cancellable swap is just a regular interest rate swap that may be cancelled on any reset date. It is thus a swap plus an offsetting Bermudan swaption. For example, a cancellable swap in which an investor receives a fixed rate of 5% versus paying Libor is just a regular swap where the investor receives 5% versus Libor, plus a Bermudan payer swaption giving the investor the right to enter into another swap and

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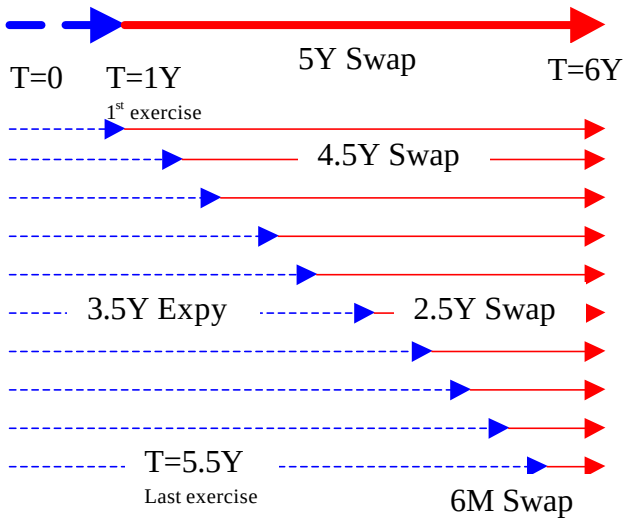
pay 5% versus receiving Libor on a swap with the same final maturity and notional amount. The net effect is just a swap that may be cancelled on any reset date. Cancellable swaps are a useful tool for corporate bond investors, and allows the unbundling of call risk from interest rate/credit risk. Consider an investor who seeks exposure to floating rate assets of a certain credit quality. He/she believes company ABC's bonds are attractively priced, but unfortunately they are callable fixed rate bonds. This investor can purchase the callable fixed rate debt, and enter into a cancellable swap with a counterparty where the investor pays fixed, thereby creating a synthetic floating rate asset as well as exploiting the pricing advantage of company ABC's bonds.

Applications such as these have in recent years contributed to the growth of the Bermudan Swaptions market. Today, many institutions participate in the Bermudan Swaptions market, ranging from the Agencies and Home Loan banks to Insurance companies, banks and hedge funds. This diverse group of participants has led to the market becoming much more efficient and liquid. While in the past Bermudan swaptions have been less liquid than their European counterparts, this gap continues to narrow. Improved pricing models and better technology has resulted in much more competitive pricing of Bermudan swaptions, and these products are nearly as liquid as European swaptions today. Bid-offer spreads are only marginally wider than those of European swaptions, and are of the order of 0.25-0.5 vega for most structures. Five years back, bid-offer spreads were more of the order of a full vega. Aggregate vega flows in the Bermudan swaptions market now rival flows in the European swaptions market, attesting to the depth of the market. Therefore, investors who fully understand the characteristics and risks of Bermudan swaptions (versus European swaptions) can potentially benefit by using trading strategies that utilize Bermudans as well as other options. Our goal in this research note is to provide an introduction to Bermudan swaptions and to provide a framework in which to analyze Bermudan swaptions versus their better understood European counterparts. We do this as follows:

- We begin by first describing Bermudan swaptions and their characteristics.
- We then develop a framework to compare Bermudans versus Europeans. We point out that a Bermudan may be thought of as a portfolio of Europeans (which are "spanned" by the Bermudan) with the constraint that at most one of them can be exercised, and we study Bermudans by analyzing the behavior of the "basis" between the two. For a given Bermudan Swaption, we define the "BE basis" (short for Bermuda/European) with respect to any European swaption spanned by it as the difference in premium between the Bermudan and the corresponding European swaption. For example, when discussing, say, a 1Yx5Y Bermudan swaption, the "2Y BE basis" is defined as the 1Yx5Y Bermudan premium minus the 2Yx4Y European swaption premium (with same underlying swap structure).
- In particular, we observe that a Bermudan swaption is always worth more than any European swaption in its basket, and we describe the notion of the "CTD" (terminology borrowed from the Treasury Futures world), and discuss the idea of trading BE bases in anticipation of CTD switches. Trading BE bases offer interesting ways to express views on the yield curve.
- We then look at replicating Bermudan swaptions using a small number of European swaptions. Replicating portfolios in some sense represent a decomposition of a Bermudan's characteristics, and allow us to think of a Bermudan as a combination of European swaptions. However, one should be somewhat cautious about this, because replicating portfolios will change over time and require rebalancing.
- We then develop a measure of relative value for Bermudan swaptions against European swaptions that is based on the implied volatility associated with the "deferral premiums" built into Bermudans.
- Bermudan swaptions, because of their multiple exercise possibilities, complicate the notion of "moneyness", and their value is affected by intrinsic value effects. For example, an investor seeking to purchase a 1Yx10Y Bermudan Payer swaption struck at the 1x10 at-the-money forward rate is actually purchasing an option that is in-the-money, for future exercise dates, since the curve today is so steep. We

Figure 1: A Bermudan Swaption
Symbolic representation of a 1Yx5Y Bermudan Swaption



discuss a way to structure the underlying swap (the so-called "***stepped at-the-money*** swap") to eliminate these effects. The basic idea here is to use a schedule of coupons on the fixed leg of the underlying swap where each semiannual accrual coupon equals the forward rate for that accrual period.

Hedgers can use the basis analysis to understand the performance of Bermudans versus Europeans and decide which style of swaption is most appropriate to hedge a specific risk. Alternatively, hedgers can replicate a Bermudan using a small number of European swaptions for a given horizon. Speculators can trade the BE basis as an efficient way to express a particular rate or curve view.

Bermudan Swaptions Basics

A Bermudan or Multi-European swaption is an option to enter into an interest rate swap (either receive or pay fixed versus paying or receiving floating), which can be exercised on a set of predetermined dates. Typically, the final maturity date of the swap is the same regardless when the option is exercised (and the swap is entered into). In a fairly typical structure, the exercise dates are set 6 months apart after the first exercise date. For example, a 1Yx5Y Bermudan receiver swaption allows the long to exercise into a 5Y swap and receive fixed (vs. paying Libor) a year from the purchase date, or exercise into a 4.5 year swap 1.5 years from purchase date, and so on. Clearly, the last exercise opportunity would be 5.5 years from purchase date, at which time the long could enter into a 6 month swap. Figure 1 demonstrates these possibilities.

Bermudan swaptions, like American options and other options allowing "early exercise", are priced in a tree framework, based on the volatilities of all of these European swaptions that are spanned by it, as well as market expectations of mean reversion.

Thus, a Bermudan swaption is really a basket of European swaptions as depicted in Figure 1 – however, once it is exercised (into the swap corresponding to the exercise date), any "subsequent" expiration European swaptions cease to exist. Thus, a Bermudan swaption may be thought of as a basket of European swaptions of which at most one can be exercised. This is, for example, in contrast to caps or floors, where each "caplet" or "floorlet" in the portfolio is an option that is individually exercised or not exercised. (We will frequently refer to the collection of Europeans

Chart 1: Bermudan Swaptions spread their curve and vega risk across the yield curve
% of curve and vega risk in each bucket for a 1Yx10Y Bermudan swaption

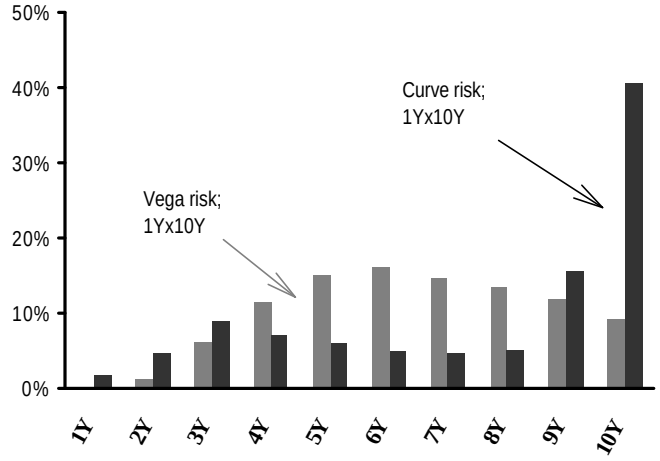


Chart 2: Bermudan swaptions behave like short dated Europeans on the up-side…
Premium as a % of original premium under parallel shifts of the yield curve for Bermudan and European swaptions;

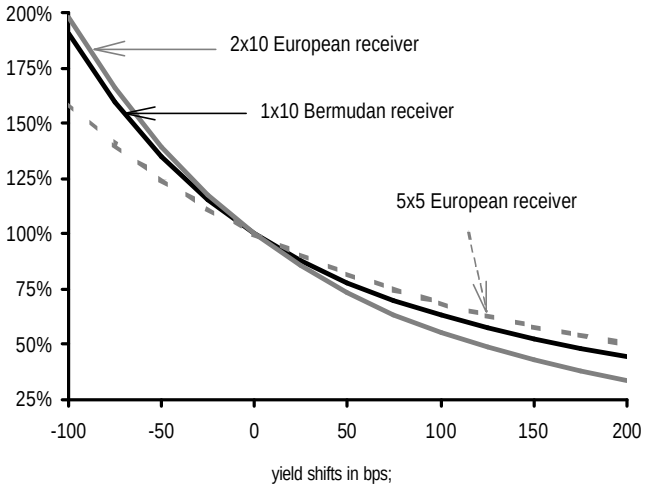
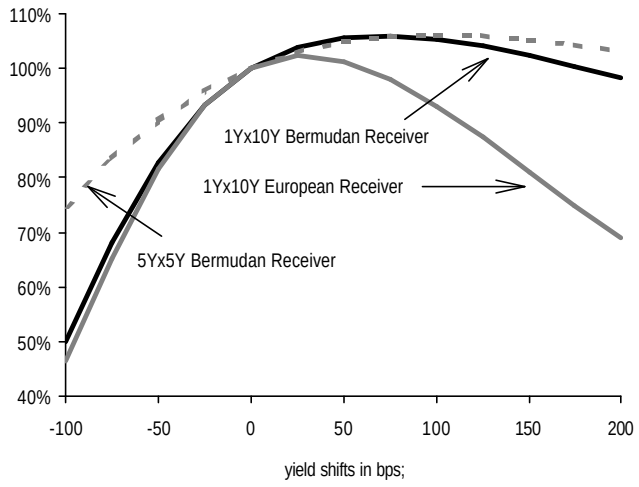


Chart 3: … but hold their vega in the down-side
Vega as a % of original vega under parallel shifts of the yield curve for Bermudan and European swaptions;



"spanned" by a Bermudan as its basket.) Bermudans thus offer a way to buy significant optionality (i.e., the basket) at lower cost than owning each option in the basket. The downside of course is that it can be exercised only once.

Bermudan swaption sensitivities reflect their "basket" nature – Bermudan at-the-money swaptions typically offer much higher vega and less gamma relative to European swaptions with the same tenor. Furthermore, while European swaptions concentrate their curve risk and vega risk at a single point on the curve, Bermudan risk parameters are more spread out across the whole curve. For example, Chart 1 shows the curve and vega risk profiles for a 1Yx10Y Bermudan swaption. This distributed risk profile has advantages for clients such as mortgage market participants, who face exposure to the whole curve rather than a particular point on the curve. This means that Bermudan swaptions can offer more robust hedge performance, not just on an instantaneous basis, but also more "durable" as we slide forward in time.

A Bermudan swaption combines the vega profile of a long dated European swaption on the down side, and the convexity profile of a shorter dated European swaption in the upside. For example, as shown in Charts 2 & 3, a 1Yx10Y Bermudan Receiver swaption mimics a 5Yx5Y European Receiver in a selloff, but more closely tracks a short-dated (2Yx10Y) European receiver in a rally.

The Notion of a "CTD"

In this section, we draw parallels between Bermudan swaptions (which we will think of as a basket of Europeans) and Treasury Futures, which encompass a basket of Treasury bonds (or notes). The Futures price always "tracks" one bond in the basket of bonds/notes that are deliverable into the contract, which is called the "Cheapest to Deliver" or CTD bond. Differences between the futures price and the CTD bond price are driven by the cost of carry (the futures contract earns no coupon), as well as the delivery option cost. The delivery option cost represents a premium paid by the short to the long in exchange for the right to deliver one of the other bonds in the basket. Drawing parallels, we can make a few observations about Bermudan swaptions:

1. A Bermudan swaption is at least as valuable as any European swaption spanned by it. If this were not true, one could buy the Bermudan, sell the European that was richer, and ignore all other exercise possibilities except the one corresponding to the particular European swaption, and make a riskless profit.
2. Consequently, the Bermudan is worth at least as much as the richest European spanned by it. Borrowing terminology from the Treasury Futures world, we call the richest swaption in the basket as the "CTD". The price of the Bermudan swaption is equal to the CTD price, plus an additional premium for the right to switch to one of the other European swaptions and enter into one of the other swaps at a later date. (Even though the "CTD" swaption is actually the richest in the basket, and not the cheapest as the term might imply, we borrow the abbreviated term because we believe that it highlights the similarities between the concepts.)

We now describe some characteristics of Bermudans vis-à-vis the Europeans in the basket. Chart 4 depicts the premiums of a 1Yx5Y Bermudan payer swaption as well as selected European swaptions spanned by it under parallel shifts of the yield curve. For each European swaption spanned by a Bermudan, as defined earlier, its BE basis (or simply basis) is just the premium of the Bermudan minus the European's premium. It follows that the CTD swaption should have the smallest basis in the basket. The BE basis of any European swaption in the basket may be thought of as the additional premium being paid for the privilege of having all the other exercise possibilities available. Thus, the basis is affected by all factors affecting this "chooser option" value - yield levels, steepness of the curve as well as implied volatility and the steepness of the vol surface, in addition to being driven by the choice of strike. Under large yield shifts or curve twists, this CTD can change – in this case the CTD's basis could potentially widen. Trading the basis opens interesting ways to express particular rate or curve views.

Chart 4: Bermudan Swaptions are at least as valuable as the richest European in the basket…

Premiums of 1Yx5Y Bermudan Payer and selected European swaptions in its basket plotted against parallel shifts of the yield curve;
bps

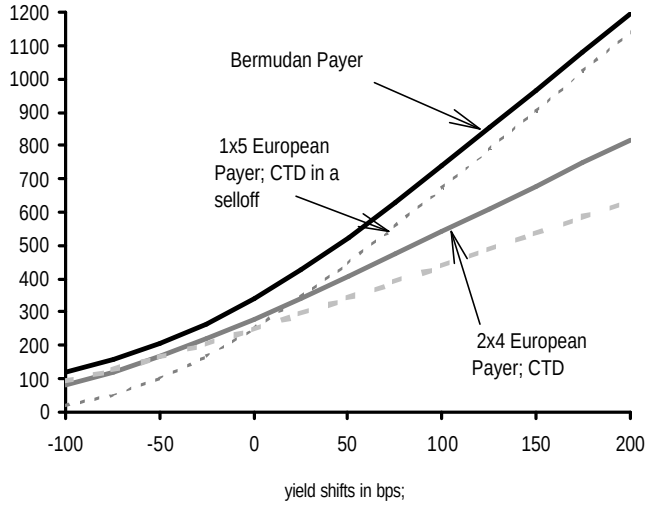
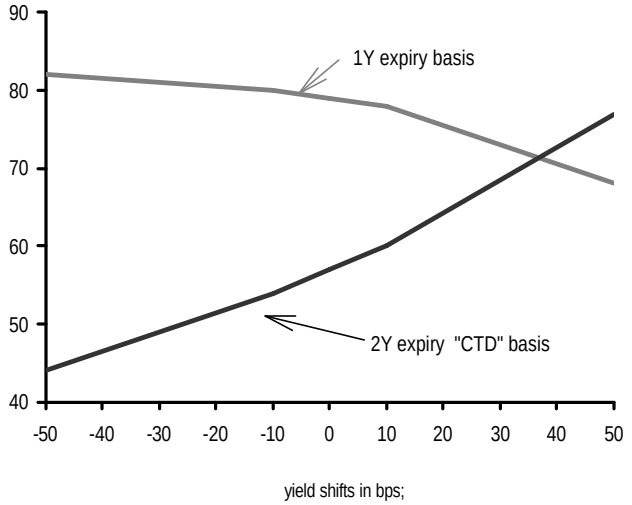


Chart 5: BE Bases can represent options on the yield curve

1Y and 2Y BE bases (with respect to 1Yx5Y stepped at-the-money payer Bermudan) under parallel shifts of the yield curve;
bps



For example, Chart 5 shows the 1Y and 2Y expiry BE bases with respect to a 1Yx5Y (stepped at-the-money) Bermudan Payer swaption. An investor wishing to position for a large selloff could buy the 2Y BE basis (which is long gamma) and profit in a selloff. In this example, an even better way to position for a large selloff is to sell the 1Y BE basis. Since the 1Y basis is short gamma in this case, the short basis position is long gamma, and provides a very asymmetric payoff. This asymmetry is because in a selloff (see Chart 4), the 1Y expiry becomes the CTD causing the basis to shrink. If one has a particular market view, then structuring a basis trade that would result in a CTD switch can offer this type of interesting asymmetric risk/reward characteristics. We highlight these opportunities in a scenario analysis that may be found in the new ***Bermudan Swaptions Report*** in JPMorgan's *Interest Rate Derivatives* package.

Replication with European Swaptions

The analogy of comparing Bermudans against a basket of Europeans opens up a natural question – can we replicate the performance of a Bermudan swaption using a weighted combination of European swaptions from its basket? Obviously, we cannot replicate with exactness in all scenarios since European swaptions can each be individually exercised. However, as we will see, it is possible to find a portfolio of European swaptions that matches the characteristics of a Bermudan under a predefined range of curve and vol scenarios to a horizon date that is prior to the first exercise date of the Bermudan. An investor can then decide between a Bermudan swaption and its replicating portfolio based on relative value considerations. The tradeoff is that the replicating portfolio changes with horizon date because of the greater time decay of the shorter dated European swaptions in the basket, and will require rebalancing.

As we saw in the previous section, analyzing the bases is a very good framework to characterize Bermudan swaptions versus European swaptions. For instance, we may argue that the Bermudan is most like the European in its basket whose basis is relatively stable across a reasonable range of scenarios. Extending this logic further, we may find a 'replicating portfolio' (i.e., a weighted combination of European swaptions from the basket) by minimizing the variance of the portfolio's BE basis under the range of scenarios. Indeed, this is our approach to finding portfolios of European swaptions that replicate a Bermudan, with respect to a predetermined set of scenarios. Specifically, we look for a portfolio whose weighted BE basis is as stable as possible over rate shifts, curve movements and volatility shifts at a chosen horizon date. We find the portfolio by choosing notional weights for the European swaptions in the basket that minimize the sum of absolute deviations in the weighted BE basis. The replicating portfolio may be thought of as a succint way of summarizing the characteristics of a Bermudan swaption. When viewed in this framework, Bermudans become simpler to characterize and analyze as linear combinations of the much better understood European swaptions.

Table 1: Replication of a 3x5 Bermudan Payer with European swaptions in its basket

Premiums of Bermudan and European swaptions under different scenarios; all scenarios except BASE are at 3 month forward horizon date; premiums in bps of notional; as of 6/9/2003; tweaks applied to spot curve; steepening/flattening scenarios anchor short end and move 30Yr rates 25bps, and other rate moves are linearly interpolated

3Yx5Y BERMUDAN PAYER, STRIKE = 4.32						
	Bermudan	3.5Yx4.5Y	4Yx4Y	7Yx1Y	Portfolio	Portfolio Basis
WEIGHTS	NA	0.88	0.12	0.31	-	
BASE	414	347	338	109	380	34
+100 SHIFT	643	563	535	161	610	33
-100 SHIFT	191	137	146	62	157	34
CURVE STEEPENS	440	369	363	123	406	34
CURVE FLATTENS	347	282	281	98	312	35
VOLS RISE	416	344	339	114	379	37
VOLS FALL	368	304	302	104	336	32

Thus, in our approach we define a robust definition of replication, rather than one based on instantaneous greeks. The example in Table 1 shows that a $100MM notional 3Yx5Y Bermudan Payer swaption may be compared to a portfolio comprised of $88MM 3.5 year X 4.5 year,

$12MM of the 4Yx4Y and $31MM of the 7Yx1Y European payer swaptions, all struck at 4.32%. Better replication may be possible by choosing different strikes, but we don't explore that here. The base scenario is simply the spot environment on the trade date. All other scenarios are as described on a 3M forward projection. In this case, the replicating portfolio costs $340K less than the Bermudan, and behaves almost like the Bermudan under these predefined curve scenarios. The replicating portfolio has less vega than the Bermudan, since the basis widens when vols increase. The difference between the cost of the Bermudan and the replicating portfolio should be compared with the anticipated cost of dynamically rebalancing the replicating portfolio in order to mimic the Bermudan's characteristics – this cost could be larger or smaller than $340K.

Details about the replicating portfolio of European swaptions for benchmark Bermudans may be found in JPMorgan's *Bermudan Swaptions Report.*

A Relative Value Measure

As we saw earlier, it is possible in a limited way to replicate Bermudans using European swaptions. The replication, unfortunately, is not static and may require rebalancing as the market moves. The natural question for a convexity hedger is then whether or not the rebalancing costs outweigh any benefits of rebalancing a portfolio of European swaptions relative to hedging with Bermudans. Phrased differently, an investor needs to have a view on (or a measure of) the value of Bermudans relative to European swaptions. In this section we develop such a measure.

Consider a 1Yx10Y Canary option, which is a Bermudan swaption with only two exercise dates – 1Y from today and 1.5 years from today. (These options are so named because the Canary Islands are halfway between Bermuda and Europe!) One year from today, either the swaption will be exercised and the investor will hold a 10Y swap, or the exercise will be *deferred* and the investor will be long a 6M X 9.5 year European swaption. The investor pays for this deferral option – the premium paid for this is the difference between a 1Yx10Y Canary option and a 1Yx10Y European swaption. We call this premium the *deferral premium* at the 1Y point. Note that the deferral option itself is a European compound option that expires in 1Y. Its payoff is given by:
Max(0, 6Mx9.5Y Swaption Value-Max(0,Value of 10Y Swap))
at expiration. In other words, the deferral option pays off when the 1Yx10Y European swaption expires worthless (in which case the payoff is the value of the surviving 6M x 9.5Y European swaption), or when the 1Yx10Y European swaption has some exercise value, but the surviving 6Mx9.5Y European swaption is worth more than the exercise value (in which case the payoff is the difference between the two).

Of course, the value of the 0.5 year X 9.5 year European swaption 1Y from today is going to depend on the *implied volatility of the 0.5 year X 9.5 year swaption 1Y-forward*. Thus, the deferral option premium implies a 6M volatility of the 9.5 year rate, 1Y from today. We estimate the deferral premiums and the implied forward volatility for each benchmark Bermudan for the first 10 exercise dates, which, taken together, measure the relative value between Bermudan and European swaptions. Chart 6 shows the term structure deferral premiums and implied forward volatilities associated with the 1Yx10Y stepped Bermudan swaption. Note that the 1Y point refers to the implied forward volatility between time 1Y and 1.5Y of the 9.5Y swap rate, the 2Y point refers to the implied forward volatility between 2Y and 2.5Y of the 8.5Y rate, and so on. This data may be found in the *Bermudan Swaptions Report* or in JPMorgan's *Dataquery* tool. We recommend using the average implied forward volatility of a Bermudan swaption as a measure of relative value. By comparing it to a historical time series (say, 1 year of data) an investor can decide if Bermudan swaptions trade rich or cheap to European swaptions at a given time.

Stepped "At-The-Money" Swaps

Thus far, we have focused on Bermudan swaptions without focusing much on the structure of the underlying swap. Typical practice is to have the fixed leg of the swap be determined by a constant fixed rate regardless of when the swap is exercised into. For example, a 1Yx5Y Bermudan payer swaption with strike 5% gives the holder the right to

Chart 6: Implied Forward volatility
Term structure of deferral premiums and implied forward volatility associated with a 1Yx10Y Stepped At-The-Money Bermudan swaption; premiums in bps of notional; implied forward volatility in bp/day

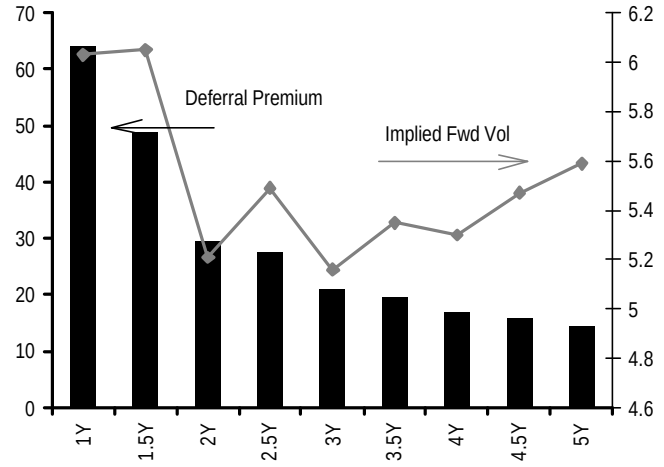
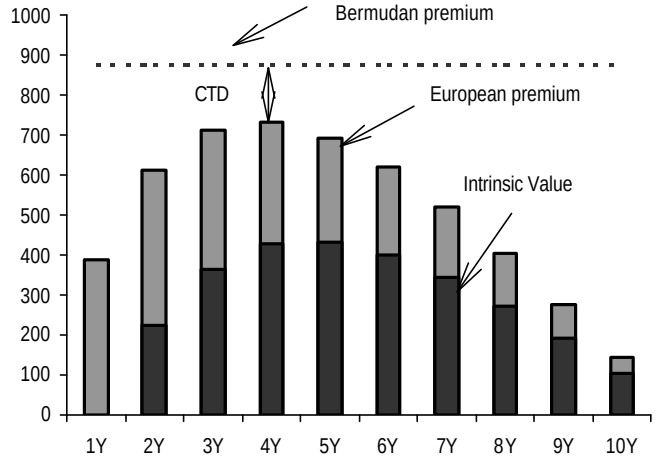


Chart 7: Flat Strike Bermudans are never strictly "at-the-money"

Premiums and intrinsic values of Europeans in the 1x10 Bermudan 4% Payer swaption's basket; bps; data as of 6/9/2003



pay 5% in a swap beginning on any exercise date with the final maturity being 6Y from the value date. If the holder exercises at the 1Y point, he/she pays 5% in a 5Y swap. If he/she exercises at the 4.5 year point, then he/she pays 5% in a 1.5 year swap.

Clearly, such a swaption is never strictly at-the-money, except if the yield curve is perfectly flat. Otherwise, for instance, 5% may be the at-the-money forward rate for a 5Y swap 1Y from today, but may be in-the-money for a 4Y swap beginning 2Y from today. Market convention defines "at-the-money" Bermudan swaptions similar to European swaptions – a 1Yx5Y Bermudan swaption with fixed strike equal to the 5Y rate 1Y forward is said to be "at-the-money".

Because constant strike structures are never strictly at-the-money, Bermudan swaptions typically span options that are either deep in-the-money or deep out-of-the-money. This is especially true in steep yield curve environments. For example, an investor buying a 1Yx5Y "at-the-money" Bermudan payer swaption struck at the 1x5 at-the-money forward rate is buying potential exercise possibilities that are deep-in-the-money today. This is not particularly useful – deep in-the-money options will trade with a delta of nearly 1, and will create characteristics somewhat like that of the underlying, to the extent that those in-the-money options are CTD. Chart 7 illustrates this, using a 1Yx10Y Bermudan Payer swaption as an example. While the 4Yx7Y European swaption is the CTD for this case, it is clear that this is largely because of the intrinsic value of the underlying swap. This creates a situation where the Bermudan swaption might behave more like a swap, if the intrinsic value of the swaptions in the basket is large enough, at least for small rate moves. The question then is, can we structure a Bermudan swaption that somehow eliminates these intrinsic value effects?

One way to mitigate these effects is to focus on structures further out the curve such as 3Yx7Y swaptions or 5Yx5Y swaptions. This is because forward rates beyond the 3Y point are generally somewhat flat, and these Bermudans do not suffer as much from moneyness effects. However, this restricts the ability to capitalize on relative opportunities at the front end of the curve. Also, the very steepness of the curve that creates these moneyness effects can (and does) present opportunities to structure curve trades using Bermudan swaptions. Thus, it is useful to be able to structure Bermudan swaptions with shorter lockouts (i.e., closer first exercise date) but without these moneyness effects.

There is a way to eliminate intrinsic value effects when structuring Bermudan swaptions. Here we define the notion of a "Stepped At-The-Money" swap, where the fixed leg coupons are structured such that the underlying swap is at-the-money (as of the value date) regardless of which exercise date it corresponds to. The basic idea is to set the accrual rate for each semiannual fixed leg payment period to equal the corresponding forward rate for that semiannual period.

For simplicity, we assume that swap fixed leg payments are made semiannually, and coincide with exericise dates. Specifically, let $t_1, t_2, \ldots, t_n$ be the allowable exercise dates, and let T denote final maturity. Then, fixed leg are payable on dates $t_2, \ldots, t_n$ and T, subject to prior exercise. Let F(i,j) denote the forward rate (as of the value date) from t_i to t_j. Now consider a swap where the first coupon rate equals F(1,2), the second coupon rate equals F(2,3), and so on. ***Because the accrual rates for each semiannual period is the same as the corresponding forward rate, this swap is at-the-money (as of the value date) regardless of which exercise date it is entered into.*** We call this structure a "stepped-at-the-money" swap, and we refer to a Bermudan option on this swap as a stepped swaption. It is insightful to note that, for valuation purposes, a European option on such a stepped at-the-money structure is indistinguishable from a "standard" at-the-money European swaption struck at the at-the-money forward term rate. It is also possible to shift all coupons by a single offset – say, 50 basis points - leading to a "stepped ATM+50" structure.

Stepped Bermudan swaptions are particularly interesting because they completely eliminate moneyness effects, and allow trading of just the option characteristics. This structure also eliminates smile effects. Stepped coupon

options trade often, although not the special "stepped at-the-money" structure we outline here. Interestingly, such stepped at-the-money swaps do trade, and are referred to as Curve Efficient Swaps. They allow corporate borrowers to fix funding costs for a given term while exploiting the steepness of the curve to lower interest expense in the initial years. Stepped structures are slightly less liquid as compared to constant strike structures, but offer more interesting characteristics. For instance, because of the elimination of intrinsic value effects, CTD switches are more likely with Stepped at-the-money Bermudans, creating interesting ways to express curve views. We discuss such a trade in the next section.

Trading Strategies

Here we highlight a few applications of Bermudan swaptions.

- **Creating forward volatility exposure**: For simplicity, consider an investor who is long a Canary straddle than can be exercised 3Y from today into a 7Y swap, or 5Y from today into a 5Y swap. At the first exercise date, the investor is either long a 2Yx5Y straddle, or a 2Yx5Y receiver and a 7Y payer swap that is in-the-money, or a 2Yx5Y payer and a 7Y receiver swap that is in-the-money. In all circumstances, the investor has exposure to 2Yx5Y implied volatility, three years forward. This could be of use to mortgage market participants, who are exposed to the MBS current coupon-Swaps basis risk. This basis is well correlated with rates as well as implied volatility in the intermediate part of the curve, such as 2Yx5Y swaption volatility. Thus, forward volatility exposure is needed to hedge risk that depends on the mortgage current coupon index rate. While this example is illustrative, it suggests that Bermudan swaptions may provide stable hedging alternatives to mortgage servicers and banks.
- **Carry Trading**: BE switches can sometimes look attractive as carry trades. Consider an investor who is uncertain about yield levels, curve as well as implied volatility over a three month horizon. All that this investor believes is that large rate moves (+/- 100 bps) are unlikely over this period, and is looking to earn carry. This investor could buy a 3Yx10Y Bermudan Payer swaption struck at the 3x10 forward rate, sell a 3Yx10Y European payer swaption for a net cost of 234 bps of notional. He/she faces a 3-month horizon P&L profile shown in Table 2, suggesting that this is an attractive carry trade if rates are not expected to move sharply in either direction.
- **Conditional Trades through CTD switches:** Currently, we believe that the yield curve will flatten, especially in a rally. One could position for this by selling calls on the short end of the curve versus buying calls at the long end. A problem with this trade is that it carries negatively. It would be attractive to have a bullish flattener that carries positively. One way to structure such a trade is by using a 1x10 Bermudan receiver swaption, which will benefit from a rally as well as a flattening. However, in this current steep yield curve environment, the 1x10 at-the-money forward rate is out of the money for several exercise possibilities, this option will not appreciate very rapidly unless there is a significant rally and/or

Table 2: Some BE switches can be good carry trades
BE Bases under various scenarios; steepening / flattening defined by anchoring short end rates, moving 30Yr by 25bps and other rates moved by a linearly interpolated amount

3Yx10Y Payer B/E Switch, 4.85 strike	
Switch Premium, spot (bps)	**234**
3M Horizon P&L under scenarios	
Rates rise 25bp	17
Rates fall 25bp	19
Rates rise 100bp	(7)
Rates fall 100bp	(9)
Curve steepens 25bp	25
Curve flattens 25bp	12
Imp. Vol increases 0.5 bp/day	37
Imp. Vol decreases 0.5 bp/day	4

Data as of 06/09/2003 Close

Table 3: Positioning for bull flattening by buying the 1Yx10Y Bermudan receiver's CTD Basis
3Y expiry BE Bases (with respect tp 1Yx10Y Bermudan receiver) under various scenarios; steepening / flattening defined by anchoring short end rates, moving 30Yr by 25bps and other rates moved by a linearly interpolated amount

1Yx10Y Stepped ATM Receiver CTD (3Y expiry) Basis	
3Y Expiry Basis premium, spot	**120**
3M Horizon P&L under scenarios	
Rates selloff 25bp	(2)
Rates rally 25bp	13
Rates selloff 100bp	(18)
Rates rally 100bp	73
Curve steepens 25bp	(2)
Curve flattens 25bp	10
Imp. Vol increases 0.5 bp/day	12
Imp. Vol decreases 0.5 bp/day	(4)

Data as of 06/09/2003 Close

Table 4: Specification of fixed leg on a 1Yx10Y stepped at-the-money swaption

Fixed Leg accrual coupons; data as of 6/9/2003

1Yx10Y Stepped ATM Swap *Fixed Leg Accrual Coupons*					
Start	**End**	**Rate**	**Start**	**End**	**Rate**
11-Jun-04	10-Dec-04	**1.39**	11-Jun-09	10-Dec-09	**4.67**
11-Dec-04	10-Jun-05	**1.85**	11-Dec-09	10-Jun-10	**4.97**
11-Jun-05	10-Dec-05	**2.35**	11-Jun-10	10-Dec-10	**4.87**
11-Dec-05	10-Jun-06	**2.75**	11-Dec-10	10-Jun-11	**5.12**
11-Jun-06	10-Dec-06	**3.18**	11-Jun-11	10-Dec-11	**5.11**
11-Dec-06	10-Jun-07	**3.60**	11-Dec-11	10-Jun-12	**5.33**
11-Jun-07	10-Dec-07	**3.88**	11-Jun-12	10-Dec-12	**5.32**
11-Dec-07	10-Jun-08	**4.28**	11-Dec-12	10-Jun-13	**5.53**
11-Jun-08	10-Dec-08	**4.32**	11-Jun-13	10-Dec-13	**5.43**
11-Dec-08	10-Jun-09	**4.66**	11-Dec-13	10-Jun-14	**5.61**

flattening. A better way to position for this is to buy a Bermudan swaption on a stepped at-the-money swap, so that all exercise possibilities are at-the-money. As Table 3 shows, buying the 1Yx10Y "stepped" at-the-money receiver CTD basis (i.e., buying the 1Yx10Y Bermudan stepped receiver, selling the CTD 3Yx8Y European stepped or plain vanilla at-the-money receiver) benefits from a rally as well as a flattening, and offers attractive risk reward. The fixed leg accrual coupons associated with the underlying stepped at-the-money swap are shown in Table 4. This trade benefits from CTD switches (which causes the 3Y basis to widen) in a rally or in a flattening.

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Bermudan Swaptions Report

Benchmark Bermudan Receiver Swaptions Summary

	1Y5Y	1Y10YS	3Y5YS	3Y10YS	5Y5YS	1Y5Y	1Y10Y	3Y5Y	3Y10Y	5Y5Y
Strike	STP	STP	STP	STP	STP	3.17	4.06	4.32	4.85	4.96
Premium	293	609	382	676	402	244	458	361	610	392
Delta	-1.99	-3.72	-2.08	-3.97	-2.03	-2.21	-3.92	-2.10	-3.96	-2.03
Gamma	10.30	11.05	7.94	10.45	5.64	9.72	9.58	7.98	10.31	5.45
Vega	35.53	76.45	46.99	89.09	51.63	32.45	65.81	45.68	83.91	50.97
Expected Yrs to Exercise	2.24	3.55	3.82	5.02	5.63	1.47	2.25	3.50	4.25	5.42
Std. Dev of Yrs to Exercise	1.33	2.28	1.30	2.71	1.13	1.01	2.04	1.08	2.37	0.95
Premiums Under Scenarios										
Parallel Shift of +25 bps (S1)	275	551	355	609	369	219	390	334	544	358
Parallel Shift of -25 bps (S2)	375	737	459	808	470	330	586	439	742	459
Parallel Shift of +100 bps (S3)	179	361	243	399	256	121	216	222	339	246
Parallel Shift of -100 bps (S4)	607	1149	668	1214	670	586	1051	651	1152	659
Steepen 25 bps around 5Y (S5)	292	563	367	618	373	238	406	346	554	362
Flatten 25 bps around 5Y (S6)	353	718	444	795	464	304	562	424	728	454
Vols increase 0.5 bp/day (S7)	338	674	427	746	442	284	510	405	677	431
Vols decrease 0.5 bp/day (S8)	303	598	380	657	390	254	446	360	593	380
B/E Switch										
Switch Price	69	228	36C	114C	28C	24C	91C	19C	61C	20C
Best scenario	S3	S7	S3	S7	S7	S3	S3	S7	S7	S7
Switch Price Chg	25	36	4	12	4	13	24	2	10	3
Worst Scenario	S4	S4	S4	S4	S4	S4	S4	S4	S4	S4
Switch Price Chg	-51	-137	-13	-36	-4	-18	-62	-7	-20	-3
CTD										
CTD Expiry	2.0Y	3.0Y	3.0Y	3.0Y	5.0Y	1.0Y	1.0Y	3.0Y	3.0Y	5.0Y
CTD Basis	45C	120C	36C	114C	28C	24C	91C	19C	61C	20C
Best scenario	S4	S4	S3	S7	S7	S3	S3	S7	S7	S7
Chg in CTD basis	51	73	4	12	4	13	24	2	10	3
Worst scenario	S3	S3	S4	S4	S4	S4	S4	S4	S4	S4
Chg in CTD basis	-3	-18	-13	-36	-4	-18	-62	-7	-20	-3
Replication										
1st European (expy, wt)	1.0/0.66	1.0/0.30	3.5/0.96	3.5/0.48	5.5/0.14	1.0/0.24	1.0/0.44	3.0/0.12	3.0/0.65	5.5/0.64
2nd European (expy, wt)	5.5/2.18	2.0/0.15	4.5/0.08	4.0/0.17	6.5/1.22	1.5/0.92	2.0/0.51	3.5/0.99	6.0/0.28	6.0/0.33
3rd European (expy, wt)	5.5/2.18	10.0/3.73	7.5/0.48	12.0/1.71	9.5/0.11	1.5/0.92	10.0/0.11	7.5/0.29	12.0/0.47	6.5/0.28
Replicating Portfolio basis	45	63	22	40	6	15	62	19	56	18
Deferral Premium/Implied Fwd Vols										
1st exercise date	28.22/5.59	64.01/6.03	11.58/4.98	25.29/5.07	11.08/5.42	58.74/4.41	117.78/5.49	20.48/4.16	42.92/3.80	14.70/5.14
2nd exercise date	20.51/5.77	48.65/6.05	10.39/5.23	24.05/5.26	9.70/5.49	40.98/4.60	90.71/5.14	16.00/4.93	36.63/4.51	11.50/5.52
3rd exercise date	11.17/5.02	29.44/5.21	8.82/5.28	19.83/5.02	8.08/5.47	22.67/4.25	58.36/3.88	12.63/5.14	29.19/4.39	9.96/5.49
4th exercise date	9.68/5.40	27.59/5.49	7.40/5.40	18.55/5.12	6.67/5.52	16.52/5.05	47.91/4.32	9.01/5.54	24.17/4.93	7.32/5.61
5th exercise date	5.31/4.65	20.91/5.16	6.42/5.66	17.70/5.26	3.97/4.67	8.83/4.72	34.09/4.30	8.03/5.70	23.15/5.02	5.01/4.72
6th exercise date	4.25/4.91	19.50/5.35	4.92/5.75	16.35/5.30	3.09/4.72	5.57/5.23	26.97/4.93	5.26/5.94	19.29/5.30	3.44/4.81
7th exercise date	3.76/5.56	17.05/5.30	2.78/5.09	12.57/4.91	1.23/3.50	4.20/6.03	21.90/5.07	3.28/5.38	15.58/4.79	1.65/3.78
8th exercise date	2.39/5.66	15.84/5.47	1.68/5.19	11.37/4.91	0.78/3.31	2.03/6.50	17.56/5.59	1.57/5.56	12.75/5.02	0.85/3.88
9th exercise date	1.10/5.75	14.54/5.59	0.77/5.00	11.00/5.07	0.36/3.12	1.12/6.50	16.25/5.70	0.94/5.38	12.98/5.16	0.50/4.25
10th exercise date	1.10/5.75	13.02/5.68	0.77/5.00	9.85/5.12	0.36/3.12	1.12/6.50	12.92/5.98	0.94/5.38	10.72/5.38	0.50/4.25

1.All prices are indicative only; execution premiums may be differ.
2.E=European, R=Receiver, P=Payer, C=the 'CTD' - richest European swaption in the basket.
3.A "Stepper" swaption is structured so that the value of the underlying swap is par (as of the value date) regardless of the exercise date.
4.Base scenario column includes bases - for other scenarios, changes in the basis relative to the base scenario are presented.

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Multi-European Swaptions Report
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Bermudan Swaptions Report

1Yx10Y 4.06 RECEIVER

Strike	4.06
Premium (bps)	458.00
Premium (bps) (MR UP/DN)	460/457
Delta (bps notional per bp)	-3.92
Gamma (chg in delta per bp)	9.58
Vega (chg in premium per 1 bp/day)	65.81
Intrinsic Value(bps notional)	-1.08
Probability of Exercise	59%
Expected Years to Exercise	2.25
Std. Dev. of Yrs to Exercise	2.04

Premiums under scenarios

Baseline (BASE)	458
Parallel Shift of +25 bps (S1)	390
Parallel Shift of -25 bps (S2)	586
Parallel Shift of +100 bps (S3)	216
Parallel Shift of -100 bps (S4)	1051
Steepen 25 bps around 5Y (S5)	406
Flatten 25 bps around 5Y (S6)	562
Vols increase 0.5 bp/day (S7)	510
Vols decrease 0.5 bp/day (S8)	446

Bermudan/European bases

	BASE	S1	S2	S3	S4	S5	S6	S7	S8
1.0Yx10.0Y,ERC WT=0.44	91C	22	-4C	24	-62C	17	2C	23	0C
1.5Yx 9.5Y,ER	103	-3C	9	-18	49	-3C	8	10C	-5
2.0Yx 9.0Y,ER WT=0.51	122	-11	29	-45	137	-8	22	12	0
2.5Yx 8.5Y,ER	151	-24	37	-74C	197	-19	27	8	-3
3.0Yx 8.0Y,ER	175	-29	50	-93	250	-21	37	11	0
3.5Yx 7.5Y,ER	200	-34	58	-111	293	-25	43	12	0
4.0Yx 7.0Y,ER	221	-37	68	-124	330	-26	50	16	1
4.5Yx 6.5Y,ER	242	-41	74	-136	360	-28	55	18	1
5.0Yx 6.0Y,ER	259	-42	81	-146	387	-29	61	21	2
5.5Yx 5.5Y,ER	277	-46	86	-157	411	-32	64	23	1
6.0Yx 5.0Y,ER	293	-47	92	-165	433	-32	69	27	2
6.5Yx 4.5Y,ER	312	-51	95	-176	451	-36	71	27	0
7.0Yx 4.0Y,ER	329	-53	100	-184	471	-37	76	30	0
7.5Yx 3.5Y,ER	347	-56	103	-193	488	-40	79	32	-3
8.0Yx 3.0Y,ER	364	-57	108	-200	506	-41	83	35	-3
8.5Yx 2.5Y,ER	382	-60	111	-209	521	-44	86	37	-5
9.0Yx 2.0Y,ER	398	-61	115	-215	537	-45	90	41	-5
9.5Yx 1.5Y,ER	416	-64	117	-224	550	-48	92	42	-8
10.0Yx 1.0Y,ER WT=0.11	431	-66	120	-230	564	-50	96	45	-10
10.5Yx 0.5Y,ER WT=4.67	446	-68	123	-237	577	-52	99	47	-11
Replicating Portfolio	62	1	-2	0	0	1	-3	0	0

1Yx10Y 4.06 Deferral Premium Analysis

	1.0Y	1.5Y	2.0Y	2.5Y	3.0Y	3.5Y	4.0Y	4.5Y	5.0Y	5.5Y
Deferral Option Premium	117.78	90.71	58.36	47.91	34.09	26.97	21.90	17.56	16.25	12.92
Imp. Fwd. Vol (bp/day)	5.49	5.14	3.88	4.32	4.30	4.93	5.07	5.59	5.70	5.98

1Yx10Y 4.06 PAYER

Strike	4.06
Premium (bps)	832.00
Premium (bps) (MR UP/DN)	841/823
Delta (bps notional per bp)	4.06
Gamma (chg in delta per bp)	-8.76
Vega (chg in premium per 1 bp/day)	76.63
Intrinsic Value(bps notional)	1.08
Probability of Exercise	75%
Expected Years to Exercise	4.99
Std. Dev. of Yrs to Exercise	2.32

Premiums under scenarios

Baseline (BASE)	832
Parallel Shift of +25 bps (S1)	907
Parallel Shift of -25 bps (S2)	703
Parallel Shift of +100 bps (S3)	1234
Parallel Shift of -100 bps (S4)	439
Steepen 25 bps around 5Y (S5)	898
Flatten 25 bps around 5Y (S6)	711
Vols increase 0.5 bp/day (S7)	840
Vols decrease 0.5 bp/day (S8)	765

Bermudan/European bases

	BASE	S1	S2	S3	S4	S5	S6	S7	S8
1.0Yx10.0Y,EP	464	52	51	-31	-57	69	40	77	41
1.5Yx 9.5Y,EP	346	31	45	-33	-3	45	33	56	28
2.0Yx 9.0Y,EP	257	22	39	-23	20	34	29	45	21
2.5Yx 8.5Y,EP WT=0.74	203	9	22	-15	16	17	14	28	5
3.0Yx 8.0Y,EP	164	7	12	3C	8	11	7	22	-1
3.5Yx 7.5Y,EPC	148C	4	-1	23	-10	4	-3	12	-11
4.0Yx 7.0Y,EP	148	3C	-14C	46	-33	0C	-13C	6C	-19C
4.5Yx 6.5Y,EP	160	5	-26	73	-58	0	-22	2	-25
5.0Yx 6.0Y,EP	187	6	-39	98	-88C	-2	-33	-3	-33
5.5Yx 5.5Y,EP	216	14	-46	130	-111	3	-38	-1	-34
6.0Yx 5.0Y,EP	257	16	-58	155	-142	4	-49	-5	-41
6.5Yx 4.5Y,EP	301	23	-66	185	-168	10	-55	-3	-43
7.0Yx 4.0Y,EP	354	25	-77	209	-198	11	-66	-6	-49
7.5Yx 3.5Y,EP	405	35	-81	240	-221	21	-69	-1	-48
8.0Yx 3.0Y,EP	462	39	-91	263	-250	24	-79	-3	-53
8.5Yx 2.5Y,EP	519	46	-97	290	-274	32	-85	0	-54
9.0Yx 2.0Y,EP	581	50	-105	312	-300	38	-94	0	-58
9.5Yx 1.5Y,EP	643	57	-111	336	-324	45	-101	1	-60
10.0Yx 1.0Y,EP WT=1.93	709	60	-119	357	-350	50	-111	1	-64
10.5Yx 0.5Y,EP WT=0.35	769	69	-122	382	-369	60	-114	6	-63
Replicating Portfolio	108	0	2	0	0	0	1	10	-8

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2.E=European, R=Receiver, P=Payer, C=the 'CTD' - richest European swaption in the basket.
3.A "Stepper" swaption is structured so that the value of the underlying swap is par (as of the value date) regardless of the exercise date.
4.Base scenario column includes bases - for other scenarios, changes in the basis relative to the base scenario are presented.

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