Credit Derivatives Strategy
J.P. Morgan Securities
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Credit Option Pricing Model

- JPMorgan's implementation of the Black formula for pricing options
- Open, transparent approach to analysing volatility and price of both Index and Single name credit options
  - Observable calculations
  - No add-ins required
- Sensitivities and payoff at maturity are also calculated

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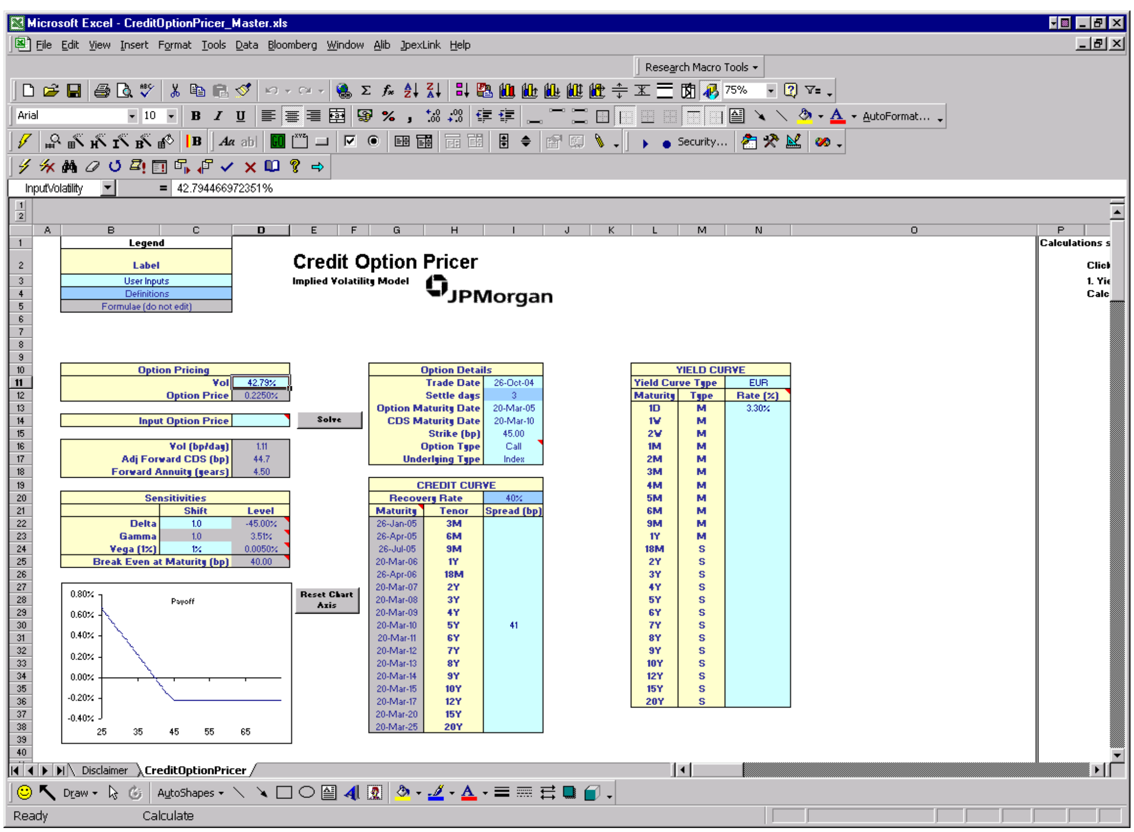

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Introduction

Over the past year the market for credit options has grown considerably. Up until now, market makers have been quoting volatilities for these options using internal models making the comparison between volatilities in the market difficult (for example two market makers may agree on the option price but quote very different volatilities). This document outlines a simple model that allows investors a standard way of analysing the volatility implied by the option price in the market and the tools to calculate how the volatility and price may change.

Click here to download the model directly from MorganMarkets. Please save locally before using it.

The model is provided as a description in the appendix and as a spreadsheet. Note that the spreadsheet and model description are provided as an educational and reference tool only. The model can be downloaded from MorganMarkets – www.morganmarkets.com.

Many thanks to Mehdi Chaabouni for his assistance in building and documenting this model.

Model Theory Description

The model used is a standard Black model. However there are a number of differences to using this model in the credit market compared to interest rate markets.

This model is designed for use with options on credit derivatives. In the credit market, a call is defined as an option to buy risk at maturity, i.e. sell protection at a set strike at maturity. A put is the reverse; the option to sell risk/buy protection at a set strike at maturity. The standard contract terms for credit options is that options on single names have an embedded knock out option where the option knocks out if the name defaults (or the spread goes above a certain level). Index options, however, do not knockout even if one of the underlying names defaults. Please refer to the Appendix as to how these are incorporated into the calculations. In addition all options are European style options (i.e. can only be exercised at maturity)

Using the Model

On opening the model, you will be presented with a disclaimer. To proceed to the main valuation page, click on the “Start Analysis” button.

The model allows users to type in a volatility and from that calculate an option price. It can also be used in reverse to backout a volatility from an input option price.

The spreadsheet looks like Figure 1 and is split into five main sections. At the upper left hand side the user inputs the current volatility or option price from the market. To the right of this the option and underlying details need to be input (eg. Option strike and maturity, type of option and underlying maturity). Below the option details the credit curve for the underlying credit is input as well as the current yield curve. At the bottom left the sensitivities, or Greeks, are calculated along with a payoff diagram.

Finally, on the right hand side of the screen are the calculations. These are broken into a number of sections and are fully detailed in the appendix. These can be revealed by clicking on the relevant cross in the top border. These are not needed for day-to-day use, but are provided in as transparent way as possible to ensure the model is open and simple to understand. Almost all of the calculations are provided

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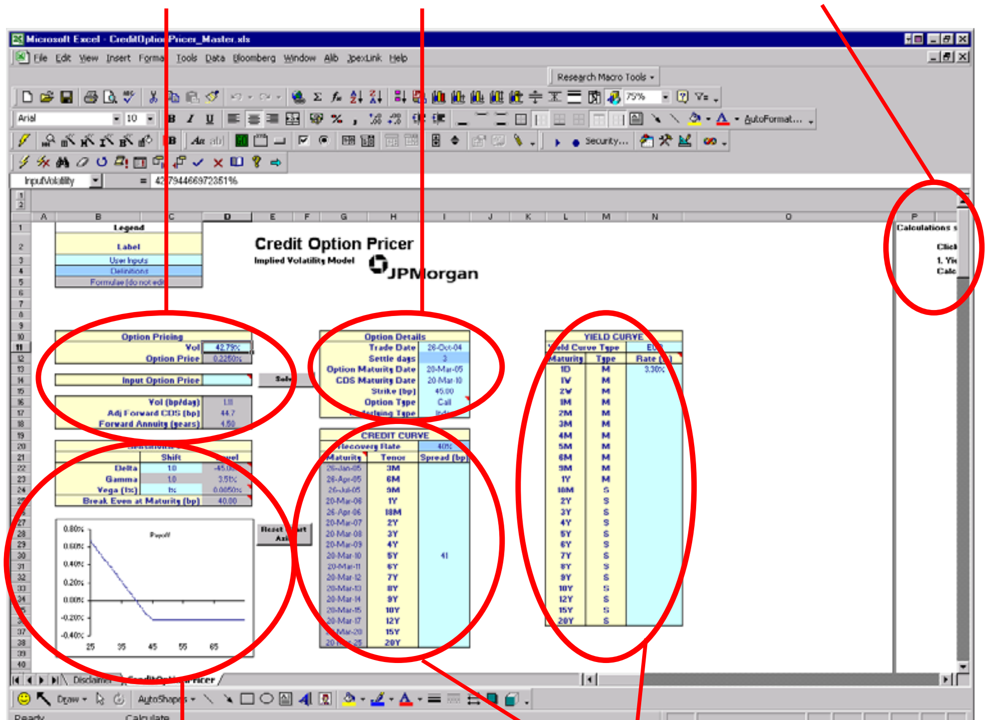
as simple worksheet functions – there are macros in the spreadsheet, but they deal only with solving for the price and formatting the payoff chart.

There is a consistent colour scheme on the model to help users find their way around. Cells with a yellow background are labels and grey cells have formulae in and should not be changed. Users should only enter values in cells with a blue background. The darker blue cells are used for relevant conventions. For instance we define the recovery rate to be 40% and that there are three days from the trade date to the settlement date.

Once the market has been defined in terms of constants, option details and underlying spread levels, a price or volatility should be input into the relevant cell. If you input a volatility all that is required is a refresh of the spreadsheet. If a price is input hit the "Solve" button, which is next to the input price cell.

Figure 1 Screenshot of Credit Option Pricer Model in Excel



Source: JPMorgan

Model Applications

This model can be used to back out the volatility or the price from levels in the market. In addition the price or volatility of an option can be calculated as various factors, such as underlying spreads, time, option volatility or price, change. The payoff diagram here is at maturity.

Standard method for quoting volatility

One of the purpose of this model is to provide a simple, transparent and replicable method of quoting volatility implied in the credit market. For options on indices the price/volatility of the option has little sensitivity to the shape of the credit curve or the yield curve – only to the absolute levels with the same maturity of the underlying. So as to provide transparent and replicable prices, JPMorgan's published prices and volatility will be quoted using this model with the only pricing inputs being the current index level and the swap rate which has the nearest maturity to the underlying. This will ensure users can have a standard framework for analysing option prices. All option prices and index levels will be available from the JPCI page on Bloomberg. The swap rate will be the nearest swap rate in maturity to the index maturity that can be obtained from the relevant country page, from the Bloomberg page (IRSB)

The index is insensitive to the shape of the curve due to the fact that there is no knock out on default (please see the Appendix for further information). However single name contracts are sensitive to the curve shapes. As such these prices will be quoted using full credit and yield curves.

Appendix: Model Details

Introduction

In this appendix we give a detailed background to the calculations performed within the Credit Option Pricer Model. The emphasis is on making the model as transparent as possible. We will concentrate more on the actual option pricing and assumptions and give only a brief overview of the yield curve calculations.

Definitions

This pricer is concerned with options on single name credit default swaps (CDS) and CDS indices (such as DJ iTraxx Europe and DJ CDX.NA). In the credit market a call is defined as an option to buy risk at maturity, i.e. sell protection at a set strike at maturity. A put is the reverse, the option to sell risk/buy protection at a set strike at maturity. The spreadsheet provides the choice between pricing a single name option or an index option. Both of these are European style options so can only be exercised at maturity.

We assume that the underlying CDS follows the usual conventions of the market. It has a short front fee payment followed by a quarterly fee. In case of default, the accrued fee is paid. For single names, the option knocks out on default. However, for a credit index, if one (or more) names in the index defaults, the option does not knock out and can still be exercised at its maturity. See the Forward calculation to see how this difference is taken into account.

We consider an option maturing at time t, with a strike of K, on a CDS maturing at time T. We define S_t as the CDS spread at t and A_t as its annuity at t. The following table gives the payoff of each option at its maturity.

		No Default	Default
Single Name	Put	Max ((S_t – K) × A_t , 0)	0
	Call	Max ((K - S_t) × A_t , 0)	0
Index	Put	Max ((S_t – K) × A_t , 0)	
	Call	Max ((K - S_t) × A_t , 0)	

Standard Black Model

The calculations for the option price can be revealed in the model by clicking on the cross in the top border above column CO. Our approach is similar to the standard market methodology for pricing swaptions. The model used is a Black model where the Black price produced is a running basis point payment. This is then multiplied by the forward annuity of the underlying CDS in order to convert it from a basis point payment to an upfront payment.

The main input to the Black model is the forward level of the underlying. This forward is the forward on the underlying CDS that starts at the option expiry and with the same maturity as the CDS. For single name options, as the convention is that the option knocks out on default, we can apply this forward level to the Black formula. However, for an index, the forward spread is increased by a certain level equal to the expected loss in the index.

See the "Forward Calculation" section for more details.

The price of a call is:

$$C = \left[-Forward \times N(-d_1) + Strike * N(-d_2) \right] \times Annuity$$

The price of a put is:

$$P = \left[Forward \times N(d_1) - Strike \times N(d_1) \right] \times Annuity$$

Where

$$d_1 = \frac{Ln(forward / Strike) + (Vol^2) \times OptionMaturity / 2)}{Volatility \times \sqrt{OptionMaturity}}$$

$$d_2 = \frac{Ln(forward / Strike) - (Vol^2) \times OptionMaturity / 2)}{Volatility \times \sqrt{OptionMaturity}}$$

$$d_2 = d_1 - Volatility \times \sqrt{OptionMaturity}$$

This formula takes in a volatility and calculates a price. To backout the volatility from the price a solving algorithm is used.

Yield curve, spread curve and annuities calculation

To compute the yield curve and the spread curve, we decided to opt for simple methodologies.

The user can input a whole yield curve (in column N) or only one money market rate if assuming a flat yield curve. We use a simple bootstrapping methodology in order to derive the approximate riskless discount factors.

For the spread curve, we use a flat forward approach to interpolate between the input market spreads. The curve is assumed to be flat before the first spread and after the last spread. This method is a good trade off between accuracy and simplicity. It allows us to get the spot spread curve and the survival probability curve.

Using the discount factors and the survival probabilities, we are able to calculate any spot or forward annuity. The risky annuity is defined by the present value of one basis point paid each year during the life of the CDS as long as no default occurs.

Forward calculation

To compute the forward spread of the CDS beginning in 1 year and maturing in 5 years from today (i.e. a 4y CDS), then we have the following relationship between spreads and annuities:

$$Spread_{0,5} \times Annuity_{0,5} = Spread_{0,1} \times Annuity_{0,1} + Spread_{1,5} \times Annuity_{1,5}$$

Where $Spread_{m,n}$ is the spread of the CDS starting in M years from today and maturing in N years from today. $Spread_{1,5}$ is what we want to compute. $Spread_{0,5}$ and $Spread_{0,1}$ are spot spreads that can be derived from the spread curve. The annuities are calculated as detailed in the “Yield curve, spread curve and annuities calculation” section.

For single names, the option knocks out at default so we do not have to take a possible default occurring before the option maturity into account. However, for the index, if a name in the index defaults the option will not knock out. Hence, we need adjust the forward to take into account for any losses due to default before the option maturity. This adjustment may be understood as the expected loss (induced by defaults) in the basket of names constituting the index. The expected loss is given by the size of loss (1-Recovery) multiplied by its probability (1-P(survival)). As the option is only exercised at its maturity, we discount this expected loss by the discount factor at the maturity of the option. Moreover, we divide it by the forward annuity in order to express it in basis points. This adjustment is always positive leading to a higher forward.

$$FwdAdjustment = \frac{(1 - \text{Recovery}) \times FwdDiscountFactor \times (1 - P(Survival))}{ForwardAnnuity}$$

Here, P(Survival) is the probability of survival until the maturity of the option and FwdDiscountFactor is the riskless discount factor at the maturity of the option. The adjusted forward index spread is then the sum of this adjustment with our computed forward.

Sensitivities / Greeks

The sensitivities calculated in the spreadsheet are: delta, the change in the option price for a shift in the underlying credit curve; gamma, the rate of change of the delta with the underlying spread changing; vega, the change in the option price for a change in the volatility of the option.

These calculations can be revealed by clicking on the cross in the top border above column DA. The delta and gamma calculations use the TABLE function in excel which provides an efficient way of recalculating the various outputs for a range of inputs.

The purpose of delta and gamma is to know the sensitivity of the option price to a change in the spread levels which can give us a way to hedge the option. Theoretically, the best instrument to use as a hedge is the forward CDS starting at the option maturity. Due to the illiquidity at the short end of the CDS market, practitioners typically hedge with spot starting CDS. We have implemented this second, more practical, approach. In order to compute delta, we move the market spreads (for example shifting them up in parallel by 1bp) and we observe the impact on the option price and on the MtM of the spot starting CDS. The Delta is then calculated as the ratio of the change in the option price to the change in the MtM of our hedging instrument. This defines the level of hedge that you should get today to be insensitive to a slight change in the spread curve.

Gamma is simply the change in delta if we shift again the market spreads. The closer gamma is to zero, the more stable the delta hedge will be.

Vega is a measure of the option price sensitivity to a change in the volatility. We shift (in percentage terms) the volatility up by 1% and the vega is the amount the option price will change.

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