Volatility Smile and Delta Hedging (Part 1) — Getting intimate with the vol surface.

Harel Jacobson · Follow

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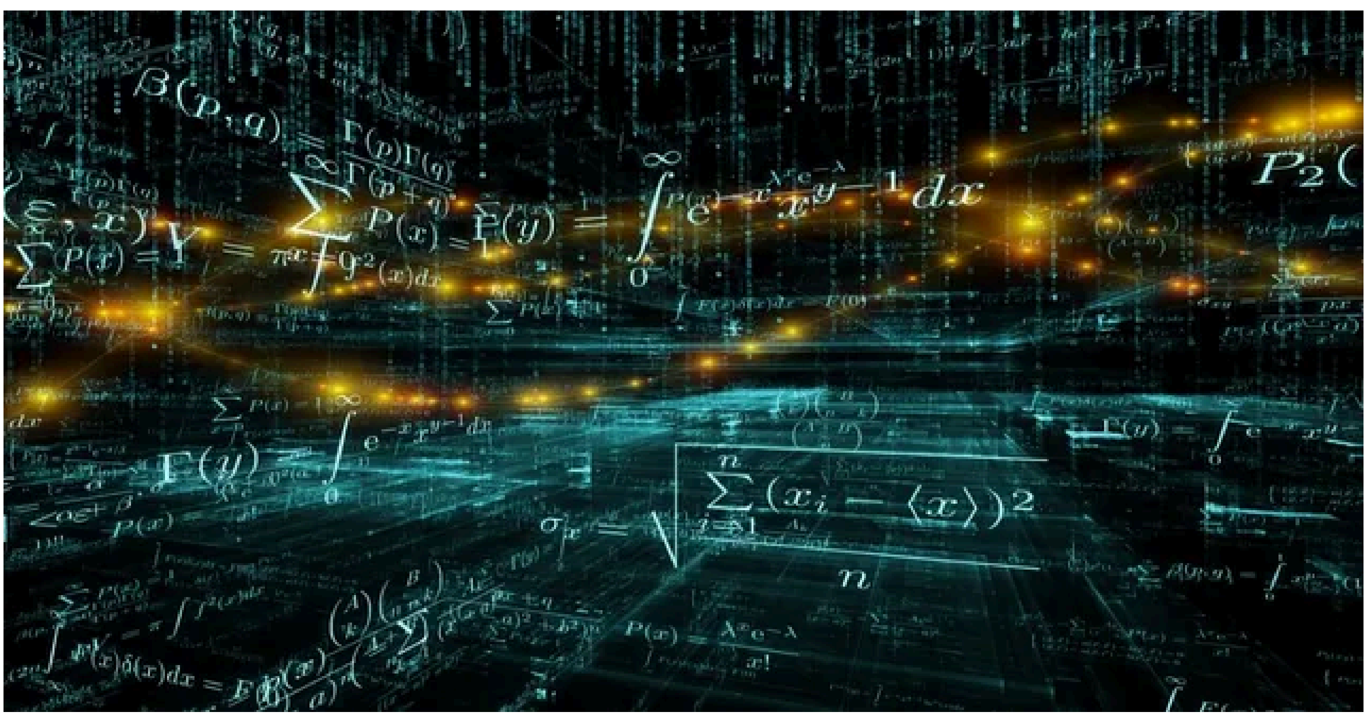
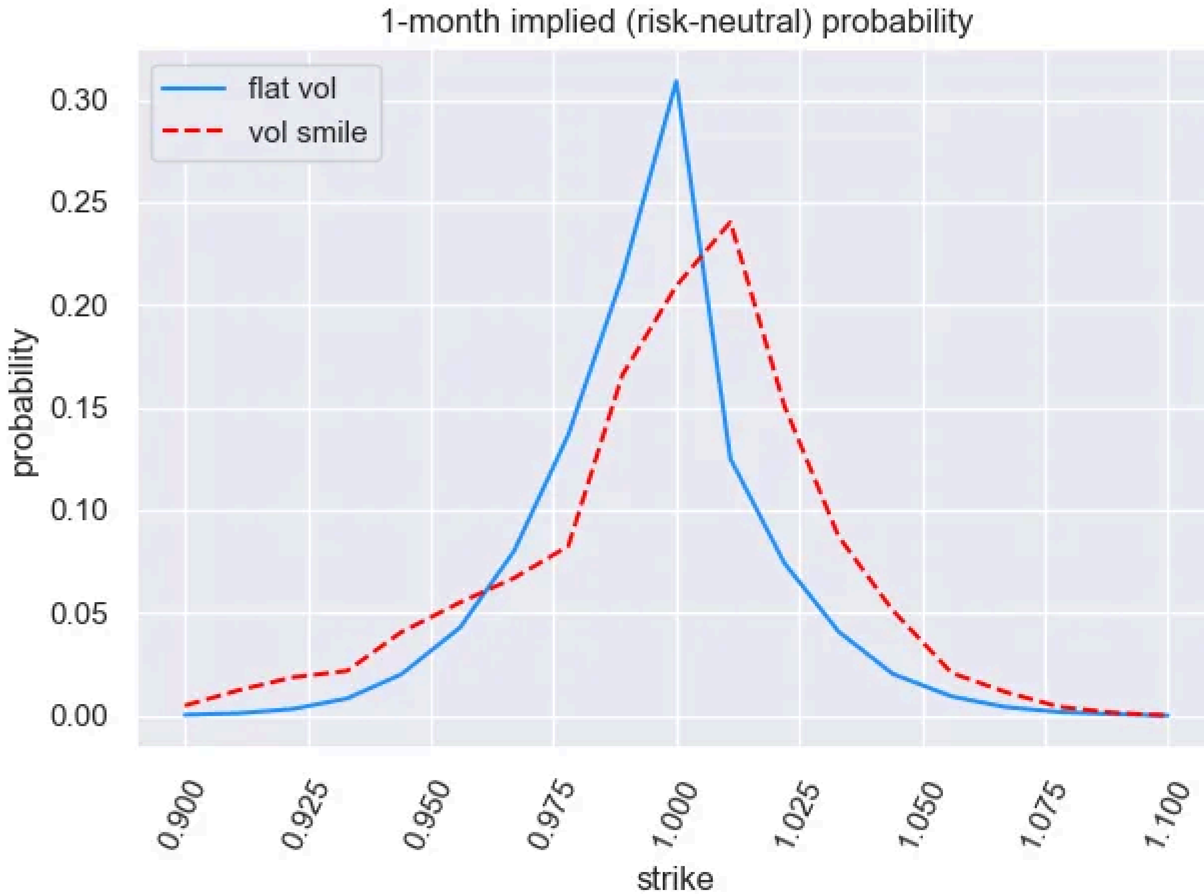


The topic of Implied Volatility Modeling has long been at the center of derivatives research. Ever since criticism regarding Black & Scholes's "flat volatility across strikes/maturities" assumption started gaining traction (somewhere around the late 70's/early '80s), volatility practitioners have tried to come up with ways to remedy that assumption that, empirically, didn't hold water.

Understanding the core problem with the "flat volatility" assumption becomes more straightforward when we move from the "volatility" space to the "probability" space. Suppose we assume that volatility is flat across all strikes. In that case, we essentially believe that the distribution of outcomes is a "Normal distribution" (i.e., "Bell-shaped" distribution with no "fat tails" or skewed). To demonstrate the effect of "volatility smile" on the risk-neutral (implied) probability, we can plot the distribution based on two assumptions :

1. Flat volatility across all strikes
2. Curved volatility smile corresponded with the same strikes (where we assume that as strikes decrease, volatility increases)



Now, we can obviously see that the two distributions vary significantly due to the change from flat volatility to curved volatility smile.

As volatility research evolved throughout the years (to read more about the history of volatility modeling, check out my blog post about this topic), practitioners have come to a conclusion that implied volatility follows a stochastic process, parallels to the process of the underlying asset, and is correlated to it (if not governed by the underlying asset dynamic), so why do people still use B&S model in real-world trading? Well, despite all its flaws, the B&S model has one great advantage, and that it is a robust mapper between implied volatility and price and greeks, and while it's debatable whether its assumptions are valid IRL trading, we can definitely use it to move from vol space to price and price derivatives.

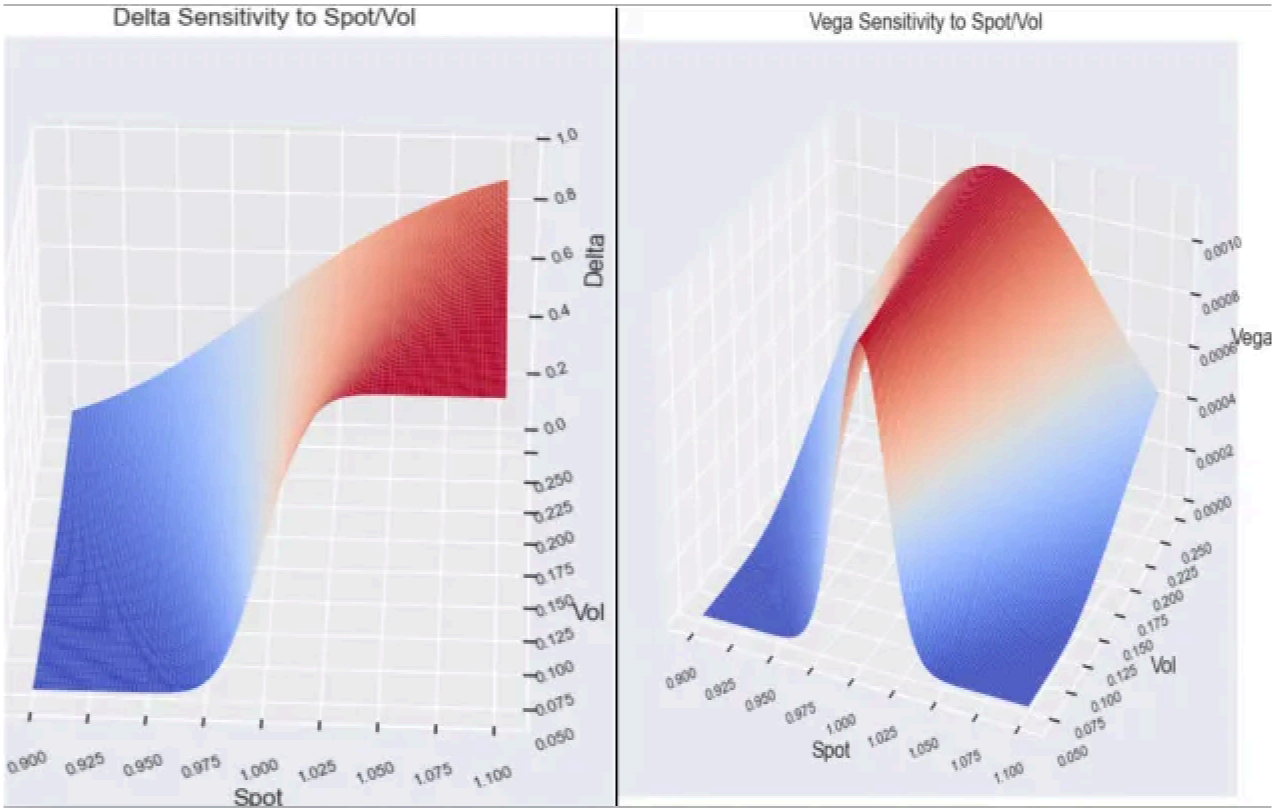
Breaking Options

If we think about it, options can be broken down into two parts:

1. Intrinsic Value — How far ITM the option is
2. Time Value — The portion of the option's value attributed to the amount of time remaining until the expiry of the option. The time value is driven by two factors: variance and time (or in other words, σ·sqrt(t), where t=time, σ =volatility)

We can also represent these two parts in derivatives terms: "intrinsic value" can be viewed as the "Delta" exposure, while the "time value" can be viewed as the "Vega" exposure of our options book.

To understand the sensitivity of these derivatives to spot/vol, it makes sense to run a sensitivity analysis, in which we plot spot/vol on the axes to generate a 3D surface.



Now, we can already see that both Delta and Vega are sensitive to the change in the underlying spot/vol (especially around-the-money), which means that our options book is likely to incur profits/losses as they move. That leads us to a very important (and interesting question IMO) — "what vol should we use to price (and risk manage) our options book? should we even bother moving it?"

We know that as the time-to-expiry decreases, our "time-value" decreases by the square-root of time and approaches 0 as we near expiry (meaning that the options book has almost no sensitivity to the change in volatility), so it only makes sense to ask ourselves a simple two questions:

1. Assuming that we hold our option to maturity, should we even bother to account for the change in volatility?
2. If we do change options volatility, what type of volatility should we use? should we use implied volatility, historical volatility, or just our "future"

realized volatility (i.e., the volatility we forecast the underlying asset will realize).

With regard to the 2nd question, I strongly recommend reading Riaz Ahamed's paper about it, as it really addresses this question in an elegant way.

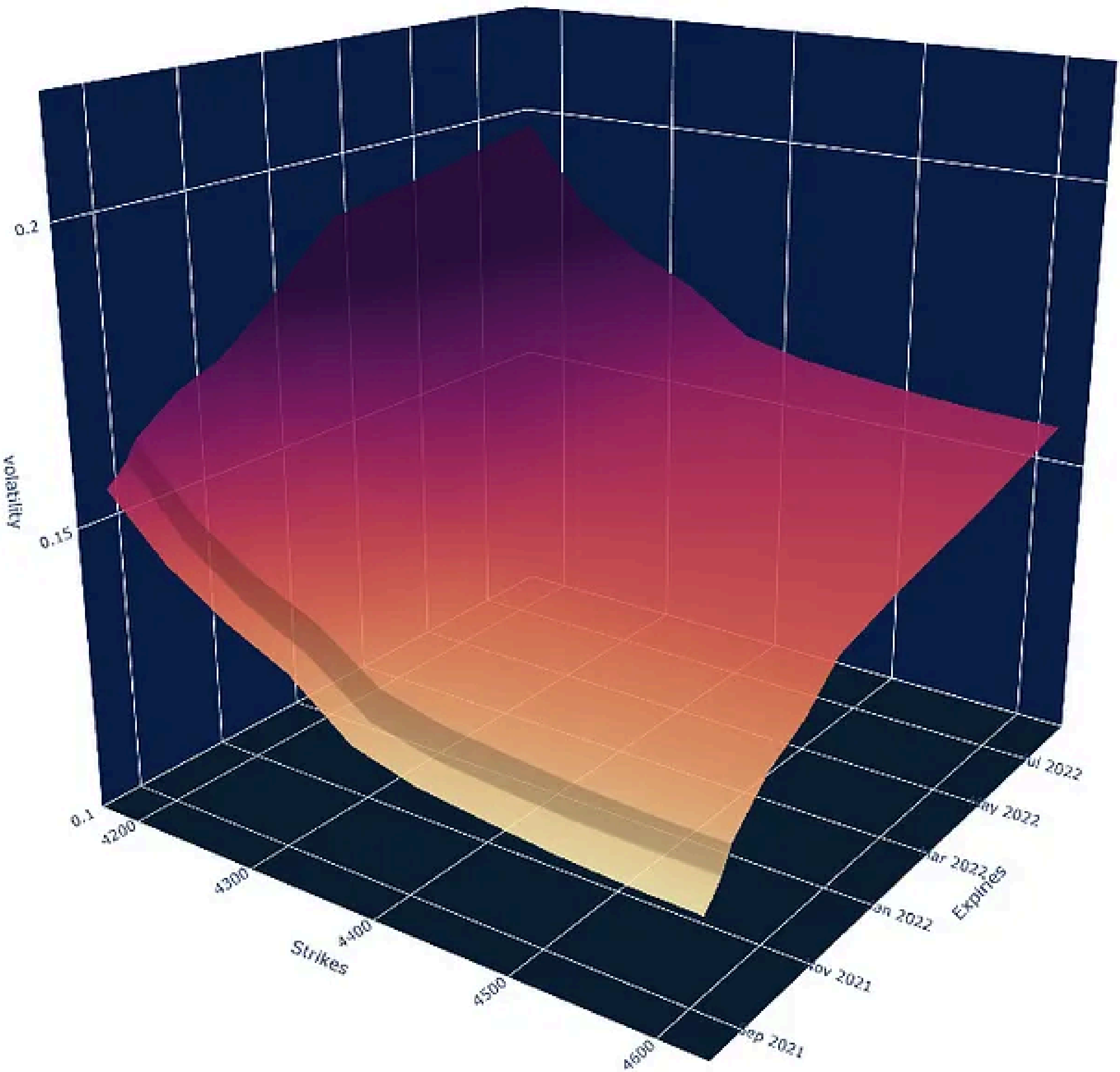
With regard to the first question, we need to note three important factors:

1. Our options book is marked-to-market — if we incur mark-to-market loss due to an increase in volatility (i.e., vega loss), our broker/dealer/bank will ask us to post margin to avoid liquidation of our positions
2. Moves in implied and realized volatility tend to be correlated to some degree (although that degree of correlation tends to change as market regimes change). Hence, a move in implied volatility is likely to come in conjunction with a move in realized volatility.
3. If we aim to exploit implied volatility dislocation, it only makes sense to mark our positions according to the traded volatility (and manage our greeks based on that volatility)

Putting a Smile on Volatility

Now that we established why we should use volatility “somewhat” close to the market/implied volatility let’s try to understand what actually the volatility surface tries to tell us.

As we already know, the volatility smile gives us a snapshot of the (market) expected risk-neutral probability distribution. If we want to plot the expected distribution across different maturities, we get the well-known "volatility surface."



S&P500 Volatility Surface

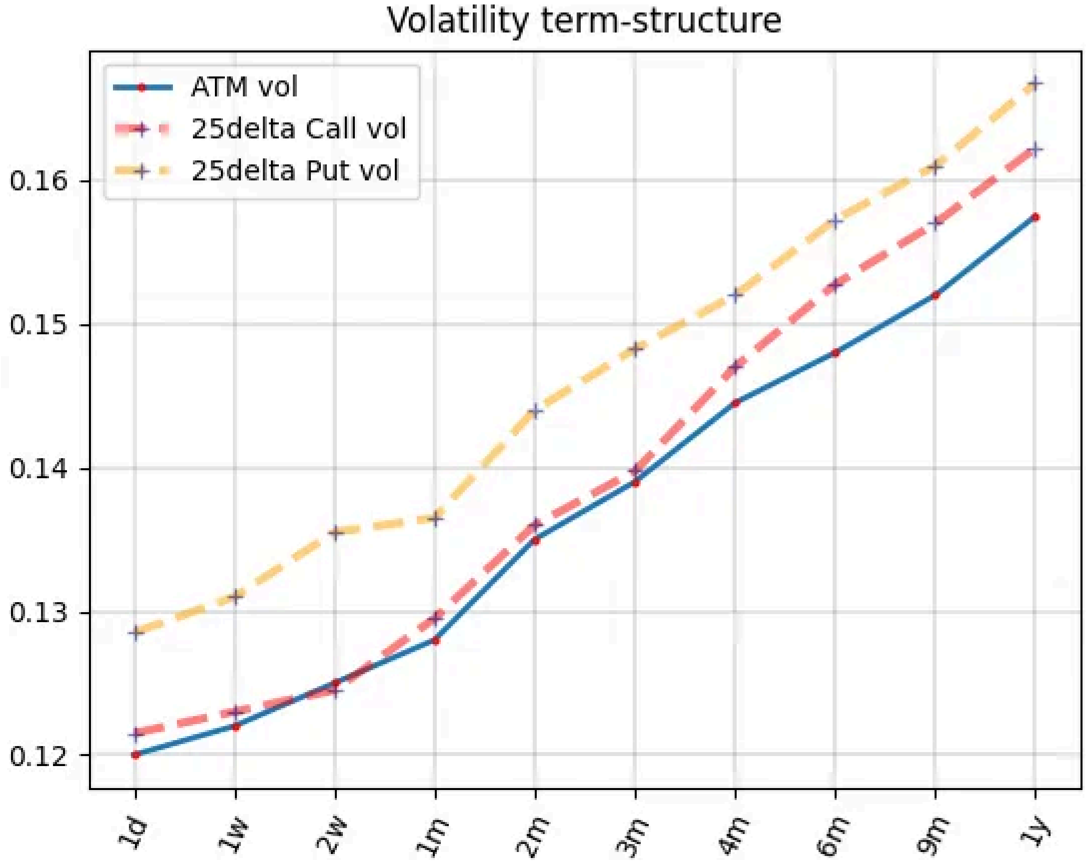
Except for being a very cool plot on our screen, the volatility surface holds some valuable information for us as traders (and practitioners in general). What information do we exactly need to manage our options book risk? well, we would be better off knowing what will happen to implied volatility as our parameters (spot, vol, and time) change. Understanding the "expected" evolution of these parameters can help us risk manage our positions.

As we can observe in the above volatility surface, as strikes move lower, implied volatility increases. This implied volatility behavior is a typical dynamic in the equity market, where phenomena like "The Leverage Effect" and down-side protection buying by large asset-managers (like pension funds) keep the volatility of the OTM puts marginally higher than OTM calls (we will call it going forward — volatility skew).

So what can the volatility skew tell us?

We already see that there seems to be a negative relation between the change in strike price and the change in volatility (as we move lower on the strike axis, the volatility increases), so we can comfortably assume that COV(Spot,σ) is negative. As noted before, volatility follows a stochastic process and is driven by the move of the underlying asset itself. That means that as the underlying moves, implied volatility will move as well. We also see that the increase in volatility is not linear (i.e., the volatility skew is not a linear function of strike difference) but more of a polynomial function (some sort of cubic/quadratic spline), which means that the rate of change of the volatility increases (we will call it "vol convexity") as we move lower on the strike axis (or in other words, the volatility-of-volatility is larger than zero).

Another property of the volatility surface that is worth noting is the volatility term structure (often called "volatility curve"). The volatility term structure gives us information w.r.t the volatility level of a certain strike (mostly ATM, specific delta, or fixed strike) across different expiries.



If you feel a bit overwhelmed with the last paragraph, take a small pause before we dive deeper into the effect of volatility on our options book risk management.

From Vega exposure to options book risk management

By now, we are pretty much experts when it comes to why (and how) volatility surface dynamic works. We know that implied volatility is driven by the underlying spot dynamic, and we know that its change is not constant (or, in other words, it has a convex nature).

This is where things get even more interesting. Because we understand that our options book vega exposure has a significant effect on our book's P&L, we want to measure how this exposure changes (or how sensitive it is) w.r.t changes in spot and vol. These two sensitivities also have pretty cool names — "Vanna" ($\partial \text{Vega} \partial \text{Spot}$) and "Volga" ($\partial \text{Vega} \partial \text{Vol}$, aka "vol-gamma" or "vomma"). These two 2nd-order derivatives even got their own names on a volatility surface model (mostly used in FX options market) — the Vanna-Volga Model (if you want to read more look it up in my shared google-drive)

Now, knowing that our Vega exposure has 2nd order sensitivities we can rewrite our options book P&L as a function of two parts :

1. Delta exposure+2nd order derivatives
2. Vega exposure+ 2nd order derivatives

$$\mathbf{\Pi}(\boldsymbol{S}, \boldsymbol{\sigma}\sqrt{\boldsymbol{t}}) = \sum_{i=1}^{n}(\boldsymbol{Delta} + \boldsymbol{Gamma}) + \sum_{i=1}^{n}(\boldsymbol{Vega} + \boldsymbol{Vanna} + \boldsymbol{Volga})$$

- *I intentionally left out theta (options sensitivity to time passage) and rho (options sensitivity to interest rate change).*

Now, as we can see above, our delta exposure has only one 2nd order derivative, while the vega has two, and the reason for that is that Vanna is actually a 2nd order derivative to the delta (w.r.t to volatility), so to avoid double counting we will treat the Vanna as a vega related exposure.

To emphasize the importance of measuring (and risk managing) vega related exposures, let’s consider the following scenario:

We are seasoned traders, and we know that S&P500 low-delta Put options are priced at a premium relative to low-delta Call options, which is why we run a systematic “Short 3-months Risk-Reversal” strategy (meaning that we sell OTM puts and buy OTM calls expiring in 3-months). Because we have no opinion about the level of volatility (i.e., we have no conviction whether it’s going to go up or down), and we don’t have a directional bias (we don’t know whether SPX is going to rally or sell-off) we initiate a delta-neutral/vega-neutral position (so we have no vega, nor delta exposure at time=0).

We call our broker for a 3-month 4130/4610 (the equivalents of 25delta Put/Call Risk Reversal (RR) in 100k vega (because we are also old skool traders, and we don’t like trading on the screen), and get quoted 20.5/12.5 vol. This means that we sell the put at 20.5 vol and buy the call at 12.5, which we can convert to premium term and see that we net receive ~440K USD off spot 4407 for ES Dec21 future, and ATM vol for the same expiry around 15.

A day goes by, and the following headlines flash on our Bloomberg terminal:

***FED POWELL: WE WERE WRONG, INFLATION IS NOT TRANSITORY**

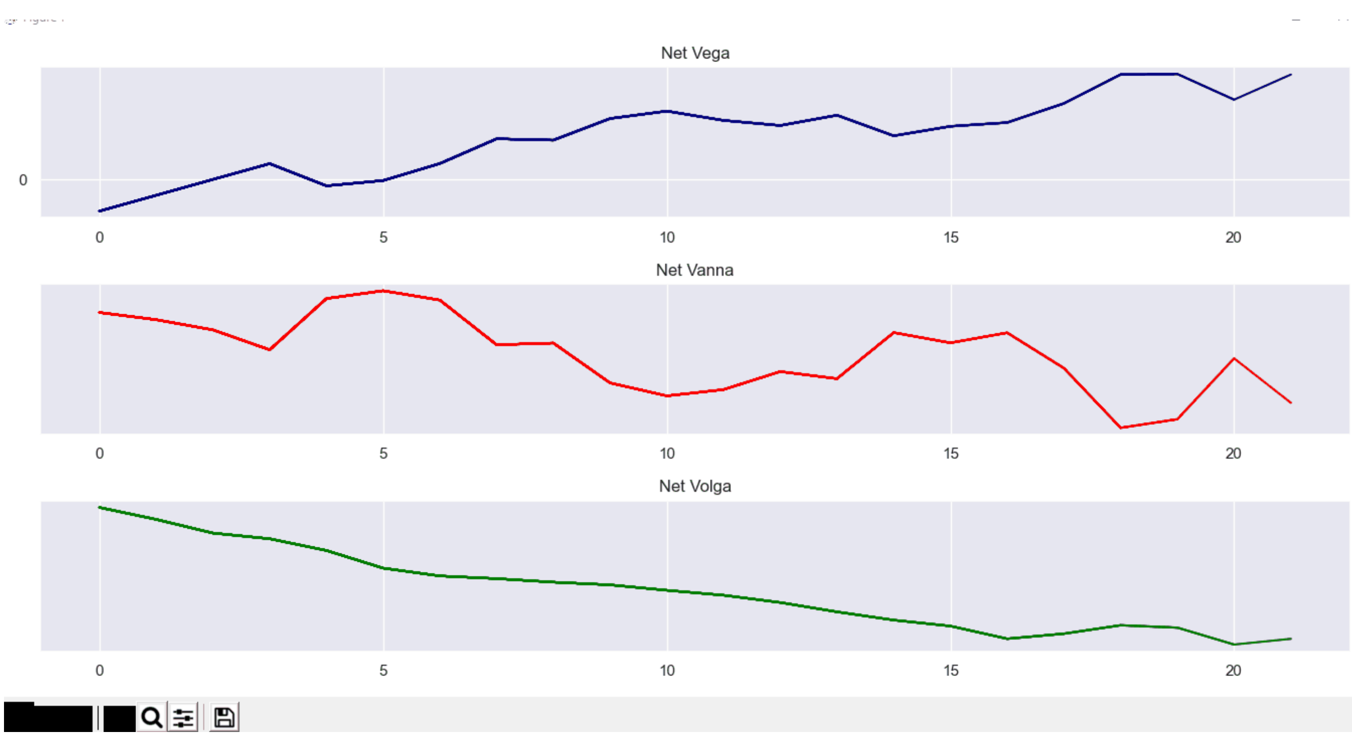
***FED POWELL: RATE HIKE IS LIKELY TO BE DISCUSSED IN THE UPCOMING FOMC MEETING**

Suddenly, the S&P500 future drop 5% and implied volatility gaps 10 volatility points higher. After our first reaction (which usually is: “WHAT DA FCK JUST HAPPENED???”), we quickly open our ES tab on our position management system and see that we are now -77K vega. We see on the screen that the ATM vol for the 3-month tenor is around 25vol. We also know that our 4130 strikes turned from a 25-delta Put to a 41-delta Put (slightly OTM now) and now trading at 26.5vol at the broker.

We quickly calculate our P&L (ignoring our Delta/Gamma P&L for now), and we see that our vega P&L is about -480K $ ((26.5–20.5)*(-80K)). To sum things up, our risk manager ain’t gonna be happy tomorrow morning.

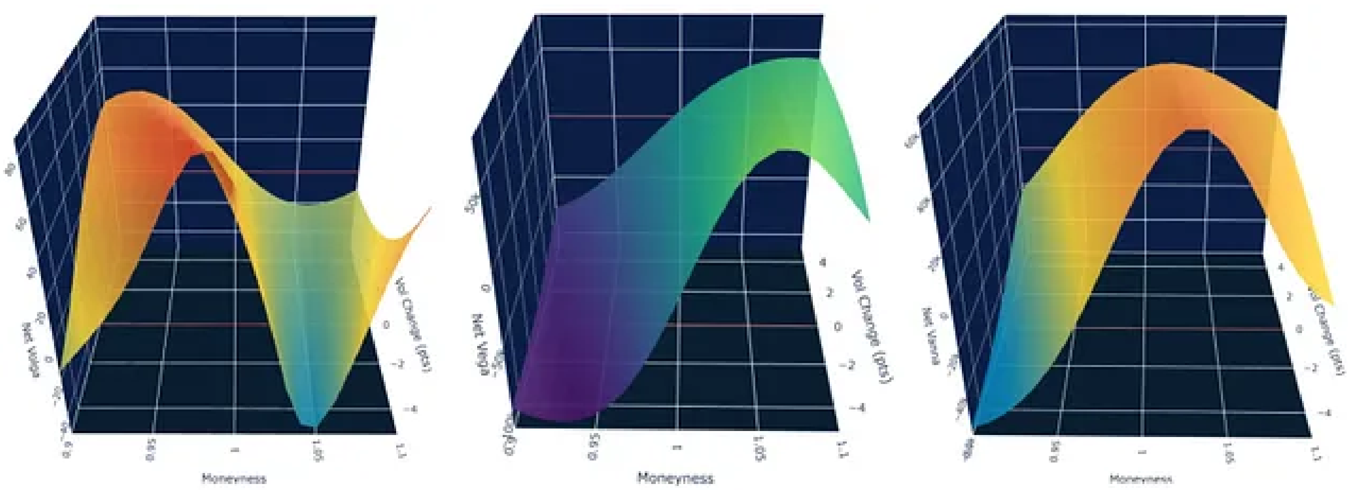
SPX/ 3-month atm vol price action (5-min intervals)

When we examine the evolution of our Vega/Vanna/Volga exposures over the same time frame, we can definitely spot that the sudden drop in spot (in conjunction with a spike in vol) massively changed our vega profile.



The evolution of the option book vega/vanna/volga over time

Clearly, our options book has some significant exposure to both spot and vol, and a sharp move in either one can create massive P&L swings. This is why it's worth taking a look at the sensitivity of vega related exposures to a move in spot/vol



Volga/Vega/Vanna sensitivity matrix (x=spot move,y=vol move)

As we can see in the above matrices our options book vega exposure turns quite negative as we drift away from our long side (i.e. the 4610 Call strike), which is exactly when vol moves higher, which is mostly a function of our positive Vanna (i.e., when the spot moves lower we become more short vega). Furthermore, as volatility moves higher, the vega increases, which might have helped us if we were not short vega…

To sum our position and risk right now — ignoring delta/gamma P&L, a large portion of our P&L is now a function of our very negative vega x (change in vol, which we can only guestimate, not working in our favor at the moment…)

This entire introduction was merely scratching the surface of volatility surface and its properties for options traders. In Part II, we will take a walk down the rabbit hole of understanding the practice of smile hedging and the dynamic across different markets when it comes to volatility pricing and delta hedging.

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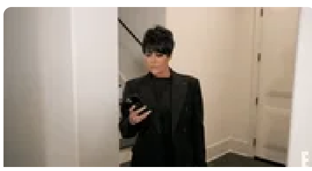

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Calvin Lai
almost 3 years ago

good article. But as you said, the RR was vega neutral (with delta exchange so also delta neutral), you dont have vega pnl from the mkt move. You do have vanna pnl, which can be approximated as 77k*(25-15)/, so roughly 350k. [As vega moved by -77k......

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Devank Sriram
about 3 years ago

Loved it, ty!

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Alain Chenier
over 3 years ago

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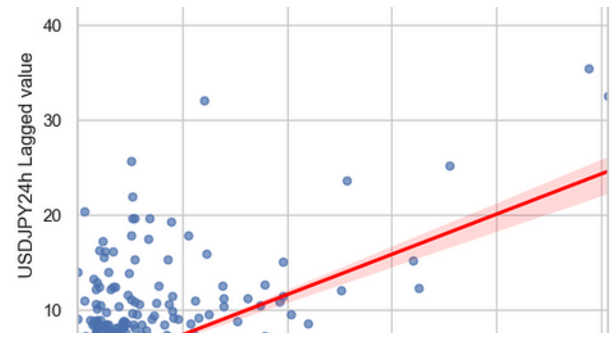

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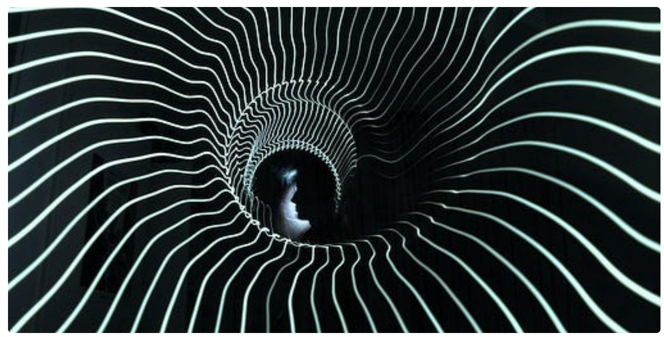

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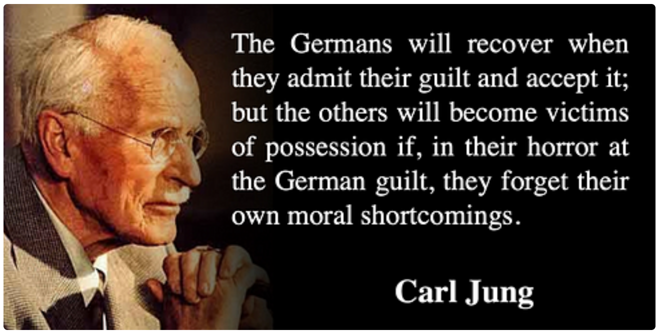

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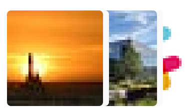

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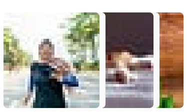

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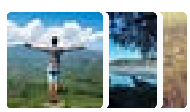

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