Optimal option delta-hedging

Uncovering the link between mean-reversion and options strategies across markets

- In this report we investigate the connection between mean-reversion and volatility trading for plain vanilla options. We find that mean-reversion on the underlying asset leads to a structural PnL cost for the volatility-sellers via the standard delta-hedging mechanism and that the effect tends to be stronger during high -volatility markets.
- We propose two alternative approaches for optimizing delta-hedging when mean-reversion is strong. The first one suggests lowering the frequency of delta-hedging, or alternatively introducing a smoothing mechanism on the delta calculated for hedging the positions. In this way the cost associated with delta-hedging is decreased and eventually delta-hedging can be even turned into an alpha generator.
- The second solution relies on a model which links daily and intra-day dynamics. Based on the model results, we find that daily-mean-reversion tends to reduce realized volatility at around mid-day in comparison to the standard choice of close-time. The latter result is confirmed by empirical data and suggests implementing delta-hedging throughout the day, rather than just at around closing time.
- Both solutions, either implemented individually or jointly, are supported by empirical results across different asset classes, including a number of actual options backtests carried out for the Equity, FX and Commodities markets.

Global Quantitative and Derivatives Strategy

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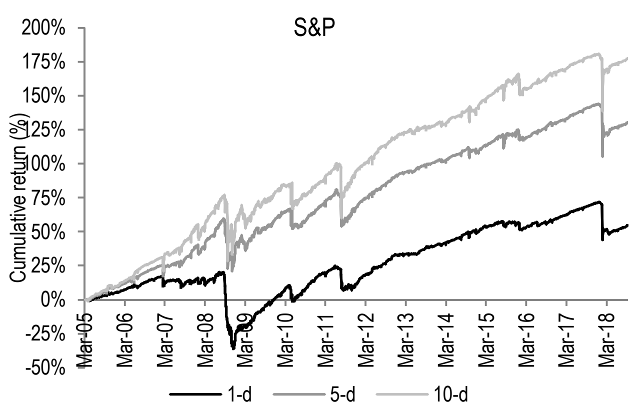
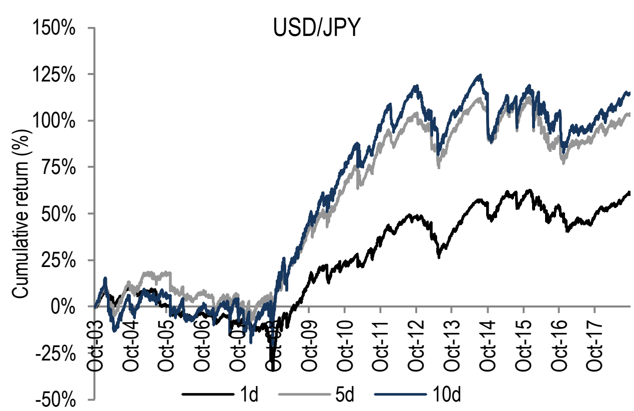
Short-volatility strategies performance can be improved if we smooth the delta of the positon (cumulative returns as a function of a smoothing parameter applied on the deltas)





Source: J.P. Morgan Quantitative and Derivatives Strategy

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Introduction

Over the past few years, interest from investors towards volatility as an asset class, both as a yield-enhancing strategy and as a replacement for long Equity positions, has grown considerably. However, the pitfalls of excessive vol selling strategies were exposed during the early February 2018 market sell-off, which called for careful risk-management of vol strategies and for the investigation of alternative solutions, other than high-leverage, for delivering consistent returns.

One natural mechanism for reducing drawdowns on a short vol portfolio is via cross-asset diversification, which allows mitigating the idiosyncratic risk factors that induce sharp losses for short-vol strategies on individual assets. In this paper we will focus our attention on the search for return-enhancing mechanisms which improve the risk profile of the vol strategies. While we will touch just marginally the topic of cross-asset diversification for short-vol strategies per se, which will be investigated more in detail in a series of future publications, the latter observations motivate our choice of naturally looking at the cross-asset space for investigating the properties of short-volatility portfolios.

This paper is organized as follows: in the Introduction, we quickly overview general properties of cross-asset short volatility strategies, with a focus on the choice of the instruments for trading the volatility premium. In the first section we investigate time series properties across asset classes and find a theoretical link between the mean reversion properties exhibited by an asset and the typical cost associated with delta-hedging: hedging at lower frequencies than daily can help to mitigate the impact of mean-reversion. In the second section, we consider the possibility of hedging intra-day: having found a link between the daily mean-reversion properties and the shape of the "intra-day vol curves", we find that realised vol tends to be lower at mid-day when mean-reversion is strong, and that hedging throughout the day rather than just at close should help reducing costs associated with delta-hedging. Additional formulas (most notably, one assessing the impact of trading costs as a function of the hedging frequency) are reported in the Appendix.

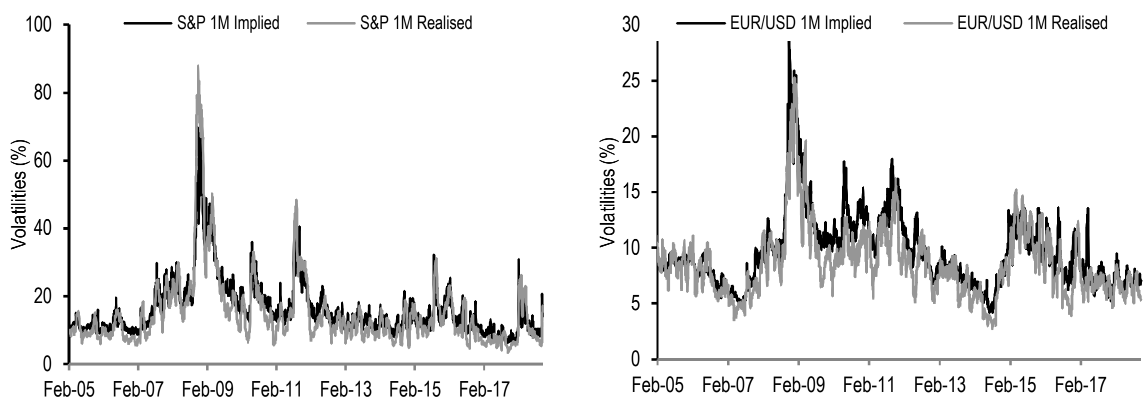
The volatility premium across asset classes

Risk premia are additional returns required by investors on top of risk-free rates as compensation from an extra source of risk. In the context of derivatives trading, it is customary to introduce a simple definition of volatility risk premium as (E standing for expected value):

$$\mathrm{Vol}_{RP} = E\left(\sigma_{imp} - \sigma_{real}\right)$$

We will see later that plain vanilla and exotic products both allow taking a position on the premium above (either directly or indirectly). A long-volatility position is akin to insurance, for which investors are more than willing to pay an extra premium, even in the (probable) scenario that the risk-events were not to materialize. Vol-sellers (banks dealers for instance), conversely, require an extra premium in making a market and possibly suffering sizeable losses (the maximum loss deriving from an unhedged short-volatility position is potentially unlimited). An investigation of the time series of 1M implied and realised volatilities for S&P and EUR/USD (Figure 1) shows that, while on average implied vols tend to be above realised vols. There are occasional episodes where both spike higher and where realised vols goes above implied vol: as it will become clearer later when we investigate more in detail the PnL generated by the trades, during such occurrences the vol-seller can suffer significant losses. These considerations motivate why in general $\mathrm{Vol}_{RP} > 0$.

Figure 1: Implied and Realised 1M Volatilities for S&P and EUR/USD



Source: J.P. Morgan Quantitative and Derivatives Strategy

Over the past few years, several papers have addressed the relationship between Sharpe ratio and skewness of a set of investable strategies, both traditional and alternative (see Lemperiere et al., 2017). With the exception of the trend-following style, all other "smart beta" strategies follow a linear relationship, with higher Sharpe ratios being associated with larger and more negative skew. For the same unit of volatility, large and negative skew calls for higher expected drawdown, materialising suddenly during spikes of volatility. Short volatility strategies perfectly fit within this high-Sharpe / negative skew classification: in other words, the possibility of large and unexpected drawdowns is what justifies the existence of the volatility risk premium.

While the considerations above which motivate the existence of the risk premium are general and common to different asset classes, a more granular investigation across different markets can reveal some marked differences as far as trading patterns are concerned (Table 1). Skew (a proxy of the difference in implied vols between calls and puts) in the Equity market is consistently negative, as a result of structural purchase of put options for hedging long positions in the underlying market. This feature is less prominent on other markets, most notably FX (especially for G7 than EM currencies), Commodities (supply/demand unbalances often lead to positive skew) and Volatilities (calls on the VIX, similarly as puts on the S&P, are commonly implemented as hedge trades). Systematic vol selling activity tends to suppress the vol premium in markets where the practice is commonly implemented, as for instance via ETFs/ETNs in Equities: lack of structural vol-selling in Credit, also due to higher trading costs, naturally leads to higher premia. For the FX market, symmetric and convex vol smiles typically call for higher vol premia for OTM than ATM options (see Ravagli, 2015).

Table 1: General features about the volatility premium across different asset classes

Asset class	Is there a Volatility Premium?	Structural considerations/flows	Other comments
Equity	Yes	Hedging (puts); auto-callable structures for some European and Asian indices	Higher premium due to higher volatility, negative skew
Commo	Yes	Hedging (mostly calls)	High volatility, typically positive skew (supply/demand dynamics)
FX	Yes	Hedging (puts/calls)	Very liquid market, low vols, balanced skews, high vol of vol, tighter premia
Rates	Yes	Hedging (puts/calls)	Balanced skews, vol suppressed by CBs QE programs and accommodative policies
Credit	Yes	Less active market for vol sellers	Higher trading costs, wider premia
Volatilities	Yes	Hedging (mostly calls)	Opposite skew than for Equities (higher premium for calls)

Source: J.P. Morgan Quantitative and Derivatives Strategy

As we have discussed earlier, the need of introducing cross-asset diversification for reducing the idiosyncratic risk embedded in short-vol systems will call for the choice of derivative instruments that can be consistently traded on different markets. Exploiting cross-asset diversification at full scale can help cutting the maximum drawdown typically incurred by short Equity vol systems by up to 50%. A detailed investigation of the properties of a cross-asset vol strategy, particularly regarding its risk management / portfolio construction, will be postponed to future publications, and so will be the topic of tackling systemic risk, whose impact cannot be reduced via diversification. Two effective solutions for reducing the directional exposure of a short-vol system are offered by the introduction of filters/timing indicators for the trades and the implementation of the strategy in a long/short format (either within the same market or across asset classes).

Instead, in this piece we will address a few tools for improving returns and Sharpe ratios of cross-asset vol strategies without compromising risk excessively. Moreover, we will see that the return enhancing features we will introduce, intervening on structural drag of performance for the strategies, in most cases also allow reducing the maximum drawdowns experienced when implementing vol premium strategy in practice.

The best instruments for monetising the volatility premium

There are several possibilities for taking a position on the volatility premium via financial derivatives. Perhaps the most natural implementation is offered via exotics products, like variance or volatility swaps. In the case of the variance swap, the PnL at expiry for each contract reads like:

$$\mathrm{PnL} = \mathrm{N}_{\mathrm{Var}}({\sigma_{real}}^2 - K_{Var})$$

The strike of the contract (in vol terms) is roughly given by the ATM volatility corrected by a convexity adjustment, strictly positive and higher for variance than for vol swaps. The main advantage offered by these first generation exotic products when trading the vol premium is offered by the ease in implementation: the strategy doesn't require delta-hedging and the payoff at expiry reproduces exactly the market parameter one would like to gain exposure on. This is a result of the fact that the Dollar Gamma of the variance swap is constant throughout the life of the contract, which reduces path sensitivity to a mark-to-market term whose contribution reduces to zero as expiry approaches.

Still, there are quite a few disadvantages for considering exotics as a benchmark implementation for monetising the vol premium. Trading costs are higher than for plain vanillas, and especially so during high-volatility markets. Internal risk-management and regulatory requirements can make the adoption of exotic products for trading purposes difficult in firms other than Hedge Funds. Poor liquidity in some asset classes render the adoption of diversified, cross-asset portfolios inviable. Also, despite the attempts over the years, var/vol swaps remain a fully OTC market, lacking the transparency offered by a listed market.

These arguments open the way for considering an implementation via plain vanillas. The well-known PnL formula for delta-hedged options, made possible thanks to the approximate proportionality between Theta and Gamma, reads like (assuming all second and higher-order Greeks other than Gamma to be negligible):

$$\Delta\mathrm{PnL} = \mathrm{Theta} * \Delta\mathrm{t} + \frac{1}{2}\mathrm{Gamma}\ \Delta\mathrm{S}^2 + \mathrm{Vega}\Delta\sigma_{imp} = \frac{1}{2}\mathrm{Gamma}\ \mathrm{S}^2\left({\sigma_{real}}^2 - {\sigma_{imp}}^2\right)\Delta\mathrm{t} + \mathrm{Vega}\Delta\sigma_{imp}$$

The compact formula above easily allows to isolating a breakeven term $\sigma_{BE} \equiv \sigma_{imp}$ for the realized vol required to produce a positive PnL for the long-option position: if $\Delta\sigma_{imp} = 0$, $\Delta\mathrm{PnL} > 0$ if ${\sigma_{real}}^2 > {\sigma_{BE}}^2$.

The need of dynamically delta-hedging positions for gaining an (approximate) exposure to the volatility premium via plain vanillas requires a continuous monitoring of the position and an additional execution effort compared to exotics: as this will be the main focus of this piece, we will come back to this aspect extensively in the following sections. The strong path sensitivity of the PnL generated by a delta-hedged option, due to the fact that the Gamma evolves with time and with changes in spot and volatilities, which at first appears as a drawback in the implementation, disguises in fact a natural stop-loss mechanism: a spike in volatility normally induces large moves in the spot, pushing it away from the strike and causing a drop in the Gamma, hence preventing further losses. Cheaper costs than for exotics, good liquidity across asset classes, with the possibility of implementation on listed markets, all represent obvious advantages for investors.

Table 2: Comparative advantages of plain vanillas vs exotics for trading the cross-asset vol premium

Product	Possibility of capturing the vol premium	Liquidity across all asset classes	Need delta-hedging	Suitable for all types of investors	Is there a listed market?	Natural switch off mechanism when short vol
Plain vanillas	YES	YES	YES	YES	YES	YES
Exotics	YES	NO	NO	NO	NO	NO

Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 2 summarizes the main merits/limits when using exotics and plain vanillas for capturing the volatility premium. Having considered pros and cons of both approaches, in the rest of the piece we will focus on the possibility of extracting a volatility premium via plain vanilla options. Two more aspects deserve attention, regarding the choice of strikes and maturities, and on the OTC vs listed implementation.

As a benchmark instrument for trading vol, we will consider strangles near the money (i.e., 95/105 for Equities vs 25delta for FX and precious Metals etc.). Compared to straddles, strangles offer a smoother Delta/Gamma profile against changes in market, reducing the impact of trading costs and the path-sensitivity of the PnL, while preserving the Gamma switch-off mechanism mentioned above. Regarding the choice of the maturity, an empirical observation usually points to short maturities for extracting the widest volatility premia. Given that very short expiries are typically associated with higher trading costs, we will focus on options of around 1M maturity for testing statistical properties across different asset classes and for carrying out actual backtests.

The final point we review here regards the choice of the venue (OTC vs listed) where to execute the trades. While OTC options might offer better liquidity in some markets, for instance FX, implementing trades in the listed market is typically the benchmark solution for systematic trading purposes. In the piece, we will adopt a hybrid strategy as far as the actual backtests are concerned, by relying on listed options for the Equity case, and on the OTC market for the FX and precious metals cases as overviewed.

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The link between volatility strategies and mean-reversion

We have seen in the Introduction that a Taylor expansion of the PnL of an option via its Greeks allows identifying a tradable risk premium, related to the interplay between the options' Theta and Gamma Greek letters, as the mismatch between implied and realised volatilities (additional risk premia could be highlighted for other smile parameters by taking into account other higher order Greeks in the expansion). In this section, we aim at isolating another possible source of extra returns by investigating the time-series properties of the linear (Delta) sensitivity of plain vanilla options.

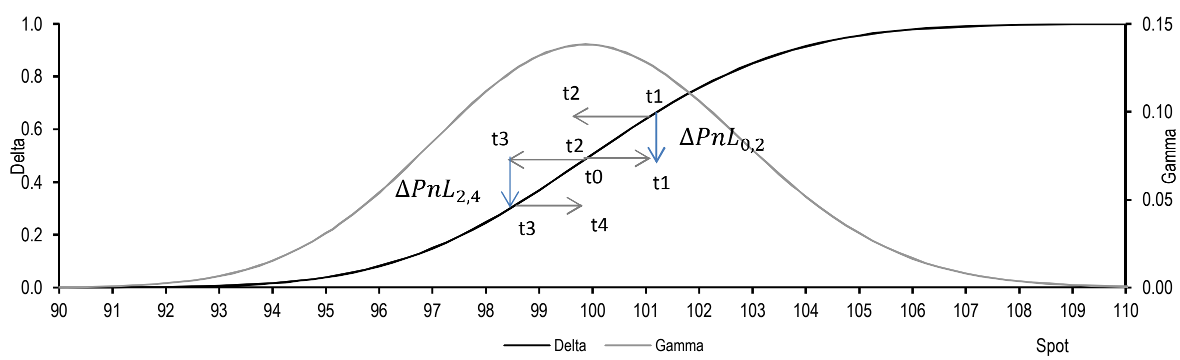
Delta-hedging and serial correlations

In the following, we will focus on the serial correlation properties embedded in the Delta sensitivity of an option, with the goal of identifying an optimal strategy for reducing the cost associated with delta-hedging. While most of this piece will be dedicated to short-volatility trades, we will start considering the impact of serial correlations on a long option position. As delta-hedging deals with cancelling out the linear sensitivity embedded in an option, its PnL over a short time interval will be the opposite (putting aside transaction costs) of that of the underlying delta position:

$$PnL_{DH,t} = -PnL_{Delta,t}$$

Keeping in mind the equation above, we will start by analysing the expected PnL of the underlying delta position in the framework of time series analysis, by working under physical (and not risk-neutral) probability measure (see Appendix for more details); delta-hedging PnL can be obtained accordingly. Figure 2 investigates a simple case study, of a 1M straddle with strike=100, for successive fluctuations of the spot around the strike at equally spaced times t_0, t_1, t_2, t_3, t_4: we assume that time step $\Delta t = t_i - t_{i-1}$ is small compared to the maturity of the option so that time decay of the Greek letters can be neglected, and that over each time step, the spot moves up or down by one unit. Between intervals t_0, t_1, t_2, the spot has moved from 100 to 101 and then back to 100, so flat over the period: still the return of delta-hedging is negative has the spot was sold when at the highest and sold at the lowest over the period. The vertical arrows in the chart refer to the change of Delta between such successive moves. A similar pattern takes place between t_2, t_3, t_4 when the spot first falls to 99 and then jumps back to 100; successive fluctuations in a tight range cost money when delta-hedging.

Figure 2: Impact of high-frequency fluctuations of the spot market near the strike on the delta-hedging strategy



Source: JP Morgan Quantitative and Derivatives Strategy

From the chart, we can see that the impact of the changes of the Delta over successive intervals is due to the Gamma (first derivative of the Delta), and that the cost of Delta-hedging due to mean-reversion will be highest when the spot moves in a range where the Gamma is highest.

We will try to make the impact of these serial correlations more evident by carrying out an expansion for a trading strategy whose dynamic position is set as the delta of an option. To start with, we consider daily trading intervals (i.e., Δt=1 day), and we will consider total trading periods corresponding to the maturity of the option at inception ($T\ \ \Delta t$ steps); in the

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following we will also consider different hedging intervals ($\Delta t \neq 1$ day). We will neglect fluctuations in volatility levels, by assuming the latter variable to remain constant.

By simplifying notations, the PnL generated by the Delta (linear) position over the scale Δt is:

$$PnL_t \equiv PnL_{Delta,t} = \Delta_{t-1}\Delta S_t$$

The intuition from the chart above suggests that strong mean reversion properties of the spot over short timeframes could imply a systematic bias as far as the expected value of the term above is concerned. For making the impact of serial correlations more evident, we carry out a further expansion, breaking down terms where serial correlation properties are expected to be strong and isolating them from others where they are not.

We can carry out an expansion of the delta of an option as a function of its Greeks, similarly as it is done for an option's price, in the limit of small time increments Δt. It's important to keep the relevant terms which are significant for a given time interval Δt ($\Delta t \ll T$). What matters here are the time decay and the convexity of the Delta, which can be expressed as the following higher order Greeks for the price of the option:

$$\Delta_t = \Delta_{t-1} + \Gamma_{t-1}\Delta S_t + \frac{\partial \Delta_{t-1}}{\partial t}\Delta t + \frac{1}{2} * \frac{\partial^2 \Delta_{t-1}}{\partial S^2} * \Delta {S_t}^2$$

$$Charm \equiv \frac{\partial \Delta}{\partial t} = \frac{\partial^2 C}{\partial S \partial t} \; ; \; Speed \equiv \frac{\partial^2 \Delta}{\partial S^2} = \frac{\partial^3 C}{\partial S^3}$$

If we consider fluctuations over a time scale Δt much shorter than the maturity of the options, the combined contribution of Charm and Speed is small, as the two Greeks tend to offset each other. So, we can assume that the dominant term in the expansion is just the delta's Delta, namely the option's Gamma:

$$\Delta_t = \Delta_{t-1} + \Gamma_{t-1}\Delta S_t$$

which leads to the following expansion for the daily PnL:

$$PnL_t = \Delta_{t-2}\Delta S_t + \Gamma_{t-2}\Delta S_{t-1}\,\Delta S_t$$

We compute the unconditional expectation of the PnL function $\mathrm{E}(PnL_t)$, by assuming to estimate the quantity at a future, far away time so that conditional effects can be neglected.

If we now assume that serial correlation properties are strong on the daily scale but negligible otherwise, the term $\Delta S_{t-1}\,\Delta S_t$ will be sensitive to these properties; similarly, we can imagine that Δ_{t-2} vs ΔS_t and Γ_{t-2} vs $\Delta S_{t-1}\,\Delta S_t$ will exhibit little serial correlations with each other. In other words, the expansion above allows breaking down the embedded time-series effects between the Greeks and the spot variable. Note that one could easily extend the reasoning by considering more (backward in time) steps in the expansion for taking into account higher relevant lags in the ACF structure. To summarize, by computing unconditional expectations and neglecting residual serial correlations as commented above, we get:

$$\mathrm{E}(PnL_t) = E(\Delta_{t-2}\Delta S_t) + E(\Gamma_{t-2}\Delta S_{t-1}\,\Delta S_t) \simeq \mathrm{E}(\Delta_{t-2}\,S_{t-1})\mathrm{E}(\frac{\Delta S_t}{S_{t-1}}) + \mathrm{E}(\Gamma_{t-2}S_{t-2}S_{t-1})\,\mathrm{E}(\frac{\Delta S_{t-1}}{S_{t-2}}\,\frac{\Delta S_t}{S_{t-1}})$$

And, by introducing the returns, $r_t = \log(1 + \frac{\Delta S_t}{S_{t-1}}) \approx \Delta S_t / S_{t-1}$:

$$\mathrm{E}(PnL_t) = \mathrm{E}(\Delta_{t-2}\,S_{t-1})\mathrm{E}(r_t) + \mathrm{E}(\Gamma_{t-2}S_{t-2}S_{t-1})\,\mathrm{E}(r_{t-1}\,r_t)$$

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We recall that the Greeks of an option (Delta and Gamma here) and the spot itself, if estimated at an earlier time, have to be treated as random variables, whose value depends on the future evolution of market parameters (spot, vol etc.). In principle, one could compute the expectation of the spot variable and of the Greeks as observed at a future time by relying on the forecasting properties under a suitable time series model, but that is not very practical (nor realistic, especially for far away horizons), given the strong path-dependency of such quantities. Therefore, in order to handle conveniently the expected value of the daily PnL, it is tempting to proceed as follows: we use the time series expectations for $\mathrm{E}(r_t), \mathrm{E}(r_{t-1}\, r_t)$, assuming $\mathrm{E}(\Delta_{t-2}\, S_{t-1}), \mathrm{E}(\Gamma_{t-2} S_{t-2} S_{t-1})$) given and equal to their path-dependent, sample realization.

By introducing the time series quantities for a smooth distribution (see Appendix for a summary on the conventions adopted):

$$\mathrm{E}(r_t) = \mu\,;\ \ \mathrm{E}(r_t\, r_{t-1}) = (\mu^2 + \sigma^2 ACF_1);\ \mathrm{E}(r_t{}^2) = (\mu^2 + \sigma^2)$$

We come to the following expression:

$$\mathrm{E}(PnL_t) = E(\Delta_{t-2}\, S_{t-1})\,\mu + E(\Gamma_{t-2} S_{t-2} S_{t-1})\,(\mu^2 + \sigma^2 ACF_1)$$

By introducing the spot-adjusted Greeks $Dollar\ Delta_t = Delta_t\, S_t$; $Dollar\ Gamma_t = Gamma_t\, S_t{}^2$ and by neglecting the difference between S_{t-1}, S_{t-2} for small Δt, we get:

$$E(PnL_t) = E(Dollar\ Delta\ _{t-2})\,\mu\ + E(Dollar\ Gamma\ _{t-2})\,(\sigma^2 ACF_1 + \mu^2)$$

For obtaining the expression above we have separated terms exhibiting little mutual correlation and have further introduced time-independent time series expectations for the first-two moments of the returns distribution, assuming that the latter (and in particular its properties σ^2, ACF_1, μ^2) stays unchanged during the life of the option, that there are no jumps, and by neglecting conditional effects when computing expectations. The statistical error introduced by replacing sample realisations with expectations will decrease when considering large statistical samples (for instance, the 1M maturity corresponds to roughly 21 daily samples).

From our compact expansion for the expected value of the PnL, we see that the quantity MR_{indic} :

$$MR_{indic} \equiv E(r_t r_{t-1}) = (\sigma^2 ACF_1 + \mu^2)$$

essentially expresses the expected gain (or loss, if negative) that the delta position is expected to return over the interval Δt (for a unit of Gamma). While the volatility expresses the size of the expected fluctuation over a given time horizon, the indicator above takes into account whether consecutive fluctuations are expected to occur in the same direction or not: as we have seen in Figure 1, cases where the indicator is negative and large are those where the impact on the delta-hedging strategy should be the most detrimental. We define mean-reversion as the regime where serial correlations are negative and average returns small. Conversely, a trend-following market regime will be characterized either by large sample average returns or by large and positive serial correlations. We refer to (Tzotchev 2018) for a more detailed review of the topic.

So far we have focused on the PnL generated over a given time step Δt, however, given the linearity of the procedure, the calculation for the cumulative PnL over the whole life of the option is straightforward:

$$Cumul(PnL)_T = \sum PnL_t = \sum (\Delta_{t-2}\Delta S_t + \Gamma_{t-2}\Delta S_{t-1}\,\Delta S_t)$$

By repeating the arguments above when introducing expectations, and swapping expectations with sample averages for the Greeks during the life of the option, as mentioned earlier, we get the following expression:

$$E(Cumul(PnL)_T) = \overline{Dollar\ Delta}\,\mu\,T + \overline{Dollar\ Gamma}\,(\sigma^2 ACF_1 + \mu^2)T$$

The formula above is quite appealing as it allows highlighting the impact of non-vanishing serial correlations of a time series on the PnL generated by a linear delta position. We recall that we have obtained the result above neglecting the impact of higher order Greeks and by assuming that only serial correlations of order one be significant. While this might appear as an oversimplification, it allows nonetheless an easy generalization of the BS framework for taking into account serial correlations. We will show later some numerical tests on the goodness of the aforementioned approximation. Also, as mentioned earlier, we have assumed that the distribution of the returns is constant over the life of the option and that there are no jumps, which allows separating time series expectations from the path-dependent Greek letters and estimating the latter as:

$$\overline{Dollar\ Delta} = \frac{\sum_{t=1}^{T} \Delta_{t-2}\, S_{t-1}}{T} \approx \frac{\sum_{t=1}^{T} \Delta_{t-1}\, S_{t-1}}{T};\ \overline{Dollar\ Gamma} = \frac{\sum_{t=1}^{T} \Gamma_{t-2}\, S_{t-2} S_{t-1}}{T} \approx \frac{\sum_{t=1}^{T} \Gamma_{t-1}\, {S_{t-1}}^{2}}{T}$$

In case of a large jump taking place at t_j, it would no longer be possible to disentangle time series and sample averages of the returns and the Greeks, what would rather matter would be the value at t_j of $\Delta_{\mathrm{t}_j-1}\Delta S_{\mathrm{t}_j}$. In that case, the jump would produce a positive or negative PnL depending on the sign of Δ_{t_j-1} relative to ΔS_{t_j}. From the perspective of reducing jump risk in a volatility strategy, frequent delta-hedging is the safest solution.

Considering the natural life of the option, from inception to expiry, is a natural interval for estimating the time-series properties introduced above. Over each such period, we estimate the quantities σ^2, ACF_1, μ^2 via:

$$\overline{\mu} = \sum_{t=1}^{T} r_t / T\ ;\ \overline{\sigma^2} = \sum_{t=1}^{T} (r_t - \overline{\mu})^2 \,/ (T-1);\ \overline{ACF_1} = \frac{\sum_{t=1}^{T} (r_t - \overline{\mu})(r_{t-1} - \overline{\mu})}{\overline{\sigma^2}}$$

For each sample, of length equal to the maturity of the option, the quantity $\overline{MR_{indic}} = \left(\overline{\sigma^2}\,\overline{ACF_1} + \overline{\mu}^2\right)$ indicates whether returns have exhibited a mean-reverting behavior, thus a cost for the delta-position, or not. As it will be clearer in the following sections, reviewing actual data, for each sample $|\overline{\sigma^2 ACF_1}| \gg \overline{\mu^2}$ generally holds, so that as a first approximation the trend contribution $\overline{\mu}$ is usually negligible in the Gamma. The main effect of the presence of a trend in the time series appears in the Delta term above. However, as $\overline{ACF_1}$ tends to oscillate in sign, depending on the sample, and as $\overline{\mu^2}$ does not, the latter can play a role when averaging over many samples. We will investigate how to assess this behavior over long periods of time (much longer than the maturity of the option) when dealing with actual data.

We now want to assess the impact of market conditions on delta-hedging when trading options in practice. Results above can be extended to the case of a book of options, in which case the averages would refer to the average value of the book's Greeks over the life of the options. Cumulating positions over time (i.e., selling vol every day and keeping trades on until expiry) produces a natural smoothing of the Delta and Gamma of the portfolio, by reducing path sensitivity. As a benchmark case, we will consider straddles or strangles, namely the most common implementation of short-vol strategies via plain vanillas: in this case, a few conclusions regarding the interplay between Gamma and Delta terms in the expansion above appear general.

The Gamma of plain vanillas, straddles/strangles is always positive (although that can approach zero for deep OTM strikes), so that for these structures, the sign of $\overline{MR_{indic}}$ drives the sign of the Gamma term of the PnL. Mean-reverting behavior over the daily time scale, as indicated by negative autocorrelations, is expected to impact negatively the long-vol position via its linear, delta sensitivity: when $\overline{MR_{indic}}$ is negative, an increase of volatility would make the indicator even more negative, thus making the expected contribution of the Gamma term even more negative. We recall the reader that the Gamma of a plain vanilla tends to spike for short maturities as spot values approach the strike: given the risk represented by a spiking Gamma when mean-reversion is present, reducing this "pin risk" would naturally smooth the PnL.

Delta at inception is zero for straddles and strangles, and, given that typically ATM/OTM (not ITM) options are traded for taking a position on the vol premium, the average delta of the portfolio will remain small unless a large sample trend will trigger its activation after inception. For these reasons, the Gamma term in the expansion above explains most of the PnL generated by the delta position during the life of the option, during "normal" market conditions (and especially so for range-trading markets). Conversely, when considering episodes where the trend has been large, it is easy to estimate the impact of $\overline{\mu}$, at least for straddles/strangles. If we assume that the delta at inception was zero, the effect of a large sample $\overline{\mu}$ will be to force the spot to move away from its initial value, which could produce a significant value for the Delta during the life of the option and at expiry (depending on the strikes). Given that, in such cases, a large, positive trend would trigger a positive Delta (and vice versa), the product of Delta and trend as appearing in the PnL expansion is always positive, regardless of the sign of the trend. Also, large $\overline{|\mu|}$ would naturally suppress mean-reversion by making $\overline{MR_{indic}}$ more positive (or less negative). In general, there is a μ_C such that for $\overline{|\mu|} > \mu_C$ the effect of trends will be dominant over volatilities/auto-regression. For ease of reading, in the following we will omit the overbar referring to the sample estimation of market parameters.

To summarize (Table 3), strong mean-reversion (with negative auto-correlation and high volatility) is detrimental to the long vol position due to the Gamma sensitivity. Large trends are supportive for the holder of a long vol position via the Delta term, and via the reduced mean-reversion impacting the Gamma term. In case of other linear combinations of plain vanillas, like risk reversals or butterflies, Gamma/Delta of the portfolio could change sign as the spot evolves, so the clean picture as highlighted above would break down. However, as these structures are normally not traded for taking a position on the volatility premium, it is safe to limit ourselves to the straddle/strangle case at this stage.

Table 3: Risk factors impacting the linear sensitivity of a long-volatility position

Factor impacting long vol position	ACF_1	Vol	μ	**Strike (pin risk)**	**Jumps**
Features impacting PnL	Negative autocorrelation induces mean-reversion and harms the long-vol position	When mean-reversion present, higher vol magnifies moves and impacts negatively PnL	Large trends (regardless of sign) support the holder of the long-vol position	Strong Gamma concentration at given strikes can induce large fluctuations in the PnL	Jumps increase PnL volatility and can induce significant losses

Source: J.P. Morgan Quantitative and Derivatives Strategy

We summarize the main risk factors when delta-hedging long and short vol positions (Table 4), by recalling that the PnL of the latter strategy is minus the PnL of the linear delta position investigated above. For the short-vol position, our main case of interest, the delta-hedging strategy benefits from large trends and is negatively impacted by strong mean-reversion: in other words, negative autocorrelations with high volatility constitutes a market regime where mean-reversion is magnified and daily delta-hedging is expected to suffer. Given that spikes in volatility levels tend to stimulate more negative auto-regressive coefficients (see empirical analysis later in the piece), this combined effect is particularly costly on the delta-hedging PnL during crisis-episodes that short-vol strategies are normally exposed to, therefore calling for alternative ways for reducing the linear sensitivity of the volatility position.

Table 4: Risks impacting the delta-hedging PnL of vol strategies (for both long and short volatility positions)

Expected impact on the delta-hedging PnL	**Long vol**	**Short vol**
Large trend and/or positive ACF with large vol	<0	>0
Negative ACF with large vol	>0	<0
Pin Risk (Gamma spiking at certain strikes)	>0	<0
Jumps	>0 or <0	>0 or <0

Source: J.P. Morgan Quantitative and Derivatives Strategy

Pin risk represents a large risk to the option seller: this risk can be reduced by limiting the concentration of volatility sold at a given strike, thus producing Gamma (and Delta) less exposed to the fluctuations in the spot. A smoother and closer to zero Delta reduces transaction costs when delta-hedging, and the exposure to jumps. These considerations favour the use of strangles over straddles for trading the volatility premium.

Vol trading vs serial correlation: two premia at play

We have seen that, for a short-dated, near the money, long option position (neglecting other second order Greeks, and assuming volatility to remain constant, so that Vega $\Delta\sigma_{imp} = 0$), the PnL expansion reads like:

$$\mathrm{PnL}_t = \mathrm{Theta}_{t-1}\Delta t + \frac{\mathrm{Gamma}_{t-1}}{2}\Delta \mathrm{S}_t{}^2 + \mathrm{Delta}_{t-1}\,\Delta \mathrm{S}_t = \frac{\mathrm{Gamma}_{t-1}}{2}\mathrm{S}_t{}^2\left(\sigma_{real}{}^2 - \sigma_{imp}{}^2\right)\Delta t + \mathrm{Delta}_{t-1}\,\Delta \mathrm{S}_t$$

We have seen that the expected value of the linear term due to serial correlations can be estimated as:

$$\mathrm{E}(\mathrm{Delta}_{t-1}\ \Delta \mathrm{S}_t) \cong \mathrm{E}(\mathrm{Delta}_{\,t-2}\,\mathrm{S}_{t-1})\,\mu + \mathrm{E}(\mathrm{Gamma}_{t-2}\mathrm{S}_{t-2}\mathrm{S}_{t-1})\,(\mu^2 + \sigma^2 * \mathrm{ACF}_1)$$

which leads to the following approximation for the expected PnL of the option:

$$\mathrm{E}(\mathrm{PnL}_t) \cong \frac{1}{2}\mathrm{E}(\mathrm{S}_t{}^2\ \mathrm{Gamma}_{t-1})\ \mathrm{E}\left(\sigma_{real}{}^2 - \sigma_{imp}{}^2\right)\Delta t + \mathrm{E}(\mathrm{Delta}_{\,t-2}\,\mathrm{S}_{t-1})\,\mu + \mathrm{E}(\mathrm{Gamma}_{t-2}\mathrm{S}_{t-2}\mathrm{S}_{t-1})(\mu^2 + \sigma^2 * \mathrm{ACF}_1)$$

By referring to the conventions introduced in the Appendix, in the equation above $\sigma^2 = \sigma_{real}{}^2\,\Delta t$:

Table 5: Interplay of vol premium and serial correlations contributing to the PnL of naked vol trades (non delta-hedged)

Impact on a naked vol trade	Vol premium	Mean-reversion	Trend-following
Long-volatility	<0	<0	>0
Short-volatility	>0	>0	<0

Source: J.P. Morgan Quantitative and Derivatives Strategy

Vol premium and serial correlations are both driving the PnL generated by a naked option position (Table 5), with the latter possibly contributing with an extra premium (assuming that delta-hedging is not performed). In particular, mean-reversion is seen as a possible source of positive PnL for the naked short-vol position. Delta-hedging is normally performed for reducing the risk of the position, however, in the next section we will investigate under which circumstances this could be turned into a generator of positive PnL.

Introducing autoregressive models for describing financial data

The results presented in the previous section depend mainly on the assumption that linear serial correlations are negligible beyond order one, an assumption we will check in practice later (and which could be relaxed by considering higher order lags the calculations); however, no particular time-series model was introduced yet. In this section, we will introduce autoregressive models which will come in handy for describing empirical data while being parsimonious parameters-wise, so that, from this point on, results presented will no longer be model independent. We introduce the $AR(1)$ time series model (more details in the Appendix):

$$r_t = a + \rho\ r_{t-1} + \varepsilon_t\,;\, E(r_t) = \frac{a}{1-\rho} \equiv \mu\,;\ Var(r_t) = \frac{\sigma_\epsilon{}^2}{1-\rho^2}\,;$$

$$ACF_1 = E((r_t-\mu)(r_{t-1}-\mu)) = \rho\,;\ ACF_p = E\big((r_t-\mu)(r_{t-p}-\mu)\big) = \rho^p$$

Negative value of ρ imply that negative returns tend to be followed by positive returns, i.e. a basic mean-reversion effect. By plotting the ACF as a function of the lag (Figure 3, for $\rho = -0.5, -0.1$), a negative ρ coefficient leads to a fluctuating ACF. However, we can see that if $|\rho|$ is sufficiently small (like -0.1 in the second chart), higher order lags are basically negligible. Therefore, the $AR(1)$ matches well the requirement that only the first lag in the ACF be significant. In the framework of the $AR(1)$ model, the mean reversion indicator is expressed as $MR_{indic} = \left(\frac{\sigma_\epsilon{}^2}{1-\rho^2}\rho + (\frac{a}{1-\rho})^2\right)$.

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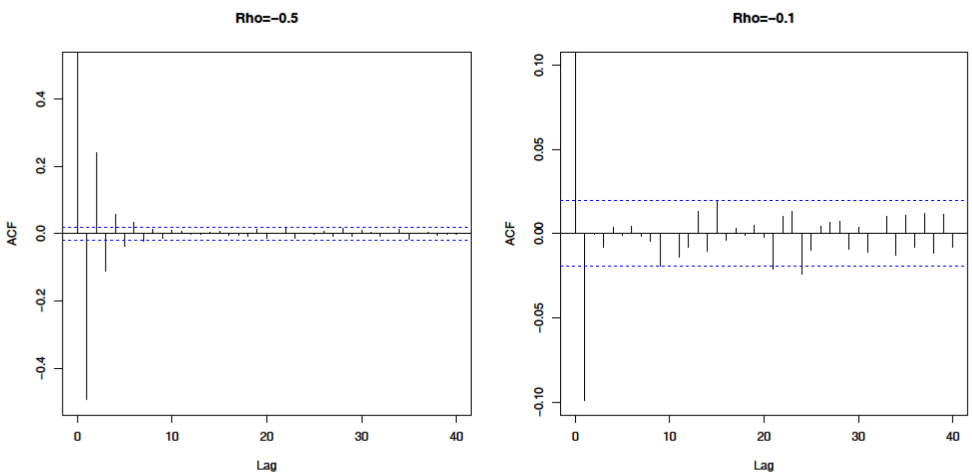
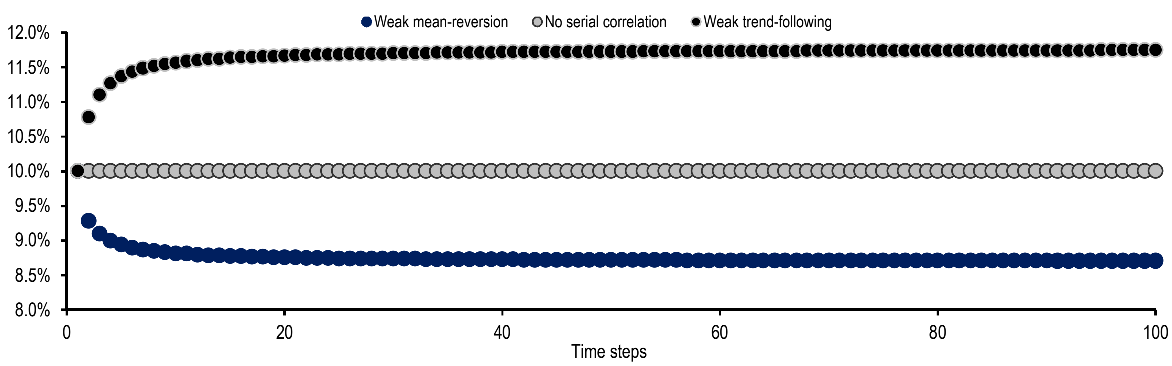
Figure 3: ACF functions for two simulated AR(1) models with ρ=-0.5,-0.1(lag 0 of ACF is 1 and is not displayed in the charts)



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 4: Signature plot of AR(1) model: mean-reversion supports downward sloping vol curves



Source: J.P. Morgan Quantitative and Derivatives Strategy

In the following sections, we will see that actual data support the choice of $AR(1)$ as a reasonable starting point for describing daily financial returns. Autoregressive models account naturally for the empirical observation that realised volatility depends on the frequency of the sampling. One can introduce the notion of statistical vol curve (or signature plot) and assess the shape of the curve as a function of the observation horizon. In the framework of the $AR(1)$ model, one gets the following expression for the signature plot σ_N[1]:

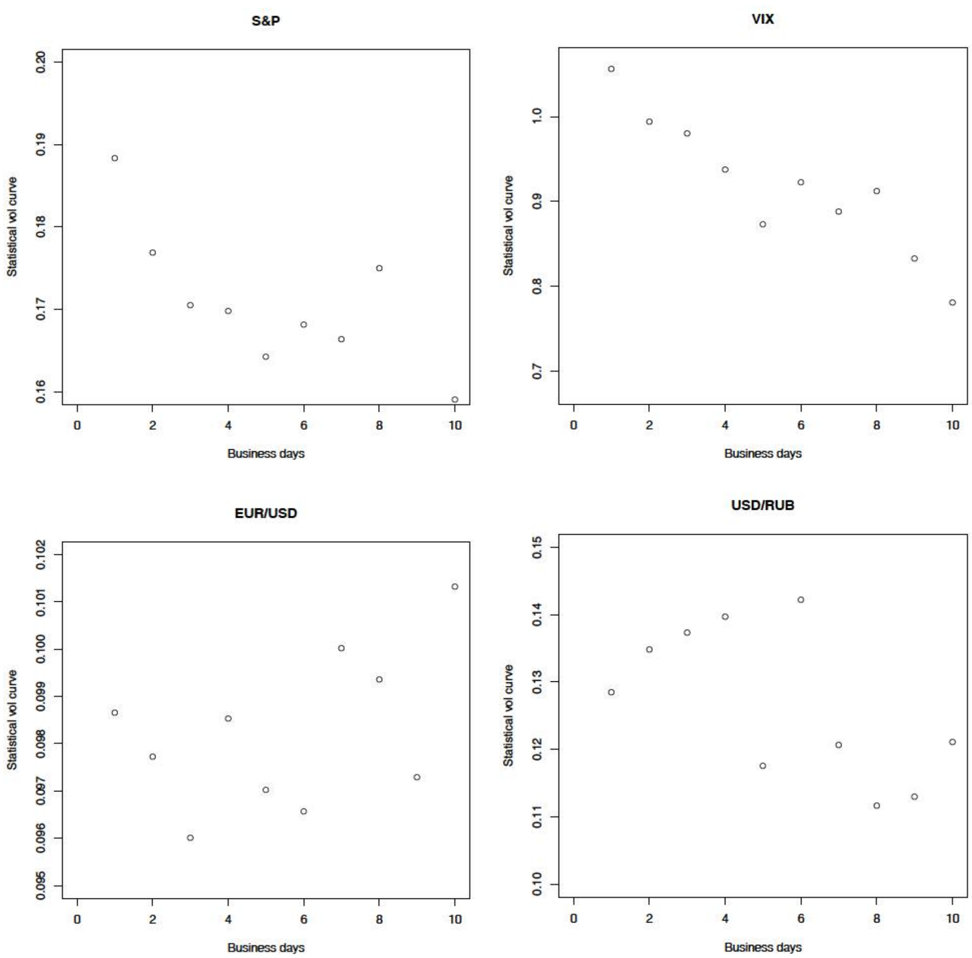
$$\sigma_N = \frac{\sigma_\epsilon}{1-\rho}\sqrt{1 + \frac{\rho^2}{N}\frac{1-\rho^{2N}}{1-\rho^2} - \frac{2\rho}{N}\frac{1-\rho^N}{1-\rho}}$$

where σ_ϵ is the daily standard deviation of the noise (linked to the daily unconditional volatility $\mathrm{Vol}(r_t) = \sqrt{\sigma_\epsilon{}^2/1-\rho^2}$) and N (in number of days) is the forecast horizon. Basically, there is a one to one correspondence between the shape of the vol curve and the AR(1) autoregressive coefficients, with positive (negative) coefficients supporting upward (downward) sloping curves (Figure 4, with $\sigma_\epsilon = 10\%,\ \rho = -0.5, 0, +0.5$); see for instance Bouchaud, Bonart, Donier, Gould 2018.

[1] The full calculation requires introducing conditional variances and is quite involved, we just report the final result here.

Figure 5 display actual statistical curves for four assets (S&P, VIX, EUR/USD and USD/RUB), with daily data since 1999. For the two Equities cases considered, we see the downward sloping shape is consistent with a mean-reversion scenario. For the two FX pairs, upward sloping curves favour more the presence of underlying trends (for USD/RUB, this is especially evident on the short-end of the curve).

Figure 5: Statistical vol curves on different assets



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 6 summarizes the results of the previous chart; by considering an average over all the assets considered in this piece (final row of the table), we see that the inverted vol curve is consistent with a mean-reverting pattern. Results above could also be obtained by considering $AR(p), p > 1$ models, where $r_t = a + \rho_1 \ r_{t-1} + \cdots + \rho_P \ r_{t-P} + \varepsilon_t$, although the maths would be more involved and will not be reported here. Different coefficients of the model $\rho_1, \cdots \rho_P$ might lead to competing effects in terms of mean-reverting vs trend-following behaviours over given time scales, so that the overall effect would be less straightforward. However, looking at the overall shape of the curve could provide with an operational definition of mean reversion, compared to what we have seen in the earlier section, based on $AR(1)$, in a more general case where higher lags in the $AR(p)$ expansion are not negligible.

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Table 6: Average statistical vol curves over different periods for a set of cross-asset financial variables

Asset	1d	1w	2w	1m
S&P	18.8%	16.4%	15.9%	15.8%
VIX	105.6%	87.3%	78.0%	67.6%
EUR/USD	9.9%	9.7%	10.1%	10.6%
USD/RUB	12.8%	11.8%	12.1%	12.6%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Delta-hedging and mean-reversion / trend-following investable strategies

So far, we have introduced mean-reversion and trend-following as general properties related to financial return time series. Beyond the intellectual interest of investigating such features, *it's important to stress that there exists investable instruments which allow trading these patterns in practice*. In this section we will establish a wider connection between delta-hedging, time-series properties and investable mean-reversion/trend-following strategies.

As a benchmark for an investable mean-reversion system, we consider a strategy which systematically buys daily variance and sells weekly variance (Daily-Weekly strategy, DW). We introduce:

$$DW_{var} = 5 * Daily_{var} - Weekly_{var} \; with \; Daily_{var} = \frac{\sum_{i=1}^{5} ({r^d}_i)^2}{5} \; ; \; Weekly_{var} = (\sum_{i=1}^{5} {r^d}_i)^2 = (r^w)^2$$

While sample means are typically removed when introducing sample variances, in this base case we simply define variances as sum of squares (the equation for the weekly variances derives from the fact that for log returns $r^w = \sum_{i=1}^{5} {r^d}_i$). There can be different implementations for the strategy, either via derivatives (variance swaps) or by replicating the payoff via delta-one positions. Of course, the payoff considered is just an example of a mean-reversion strategy, relevant for daily frequencies (there could be countless other implementations, ranging from intra-day to multi-months holding periods). As the topic itself is vast and would deserve a more in-depth analysis, we will dig into it in a future dedicated study.

What's relevant here is that, in the $AR(1)$ framework, the expected value of the payoff can be expressed as[2]:

$$E(DW_{var}) \approx T[\frac{-2\rho}{(1-\rho)}\sigma^2 - T\mu^2]$$

where $T = 5$ and ρ, μ, σ^2 are the autoregressive coefficient, unconditional mean and variances of the daily $AR(1)$ model respectively; the latter parameters will be estimated via statistical samples when dealing with actual data. In the limit where autoregression is weak compared to trends, $\sigma^2|\rho| \ll \mu^2 \cong MR_{indic}$:

$$E(DW_{var}) \approx -(T\mu)^2 = -T^2 \, MR_{indic} < 0$$

If we consider the opposite limit where trends are negligible ($\mu = 0$) and where autoregression is weak ($|\rho| \ll 1$), we get $MR_{indic} = \sigma^2\rho$ and :

$$E(DW_{var}) \approx -2T\rho\sigma^2 = -2T \, MR_{indic} \rightarrow \; E(DW_{var}) > 0 \leftrightarrow MR_{indic} < 0$$

[2] Similarly as for the signature plot formula, the full calculation requires introducing conditional expectations and a few algebraic expressions. For ease of reading, we just highlight the final formula here.

We will see later when dealing actual data that the latter condition usually holds for each monthly sample (trends account for about 15% of the values of MR_{indic} in each sample), so that the proportionality $E(DW_{var}) \approx -2N\ MR_{indic}$ is well respected in most practical cases. In that case, when delta-hedging an option over T days, a further analogy can be found:

$$E(Cumul(Delta - hedging)_N) \approx\ -\overline{Dollar\ Gamma}\ MR_{indic}T \approx \overline{Dollar\ Gamma}\ E(DW_{var})/2$$

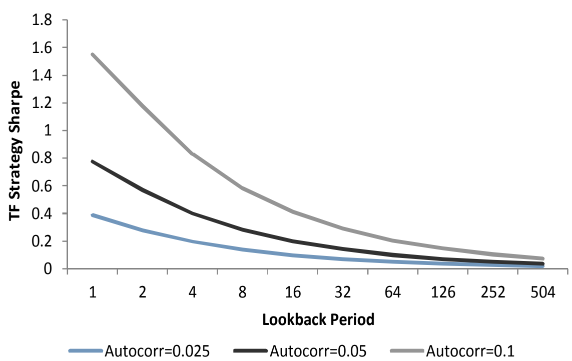
The result above is significant, and for several reasons. First, it establishes a one to one link between the DW payoff and the parameters of the $AR(1)$ model, thus motivating the extensive use of the latter (despite its simplicity) in this piece. Assets which exhibit mean-reversion (negative ρ) are those where the DW payoffs should deliver the best results (we will review this aspect in the following); this can be tested in the context of a linear regression, with a high corresponding R^2 (around 80%). Second, it allows interpreting the quantity MR_{indic} introduced earlier as the main driver of the performance of a mean-reversion strategy, thus confirming the intuition behind its definition: strong mean-reversion indicator is generated by negative autoregression and high volatility. Third, and in this context, most important conclusion, is the fact that we have found a direct correspondence between delta-hedging and a mean-reverting payoff.

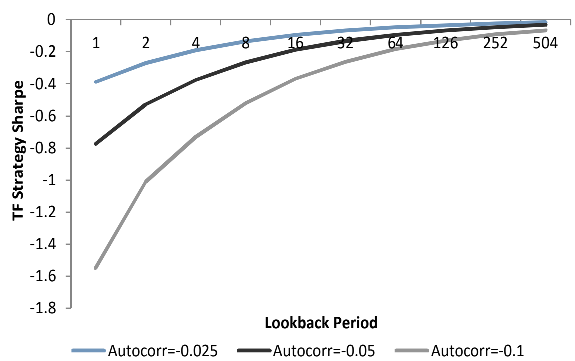
A second obvious comparison which comes to mind is the one with a trend-following strategy. In a former article by the QDS team (see Tzotchev et al. 2018) the authors had investigated in depth the relationship between the parameters of a $AR(1)$ model and the performance of a trend-following system, especially focusing on the noise impacting the estimation of the parameters. In the piece, a trend following signal to buy or sell the underlying asset was generated based on the current delta of a straddle T days before expiry and T days after inception. The strategy trades based on the following signal:

$$Signal = 2 * N(d1_t)\ ;\ d1_t = \frac{\ln\left(\frac{S_t}{S_{t-T}}\right) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

Where N is the Gaussian c.d.f., T the estimation period and σ an estimate of (realised) volatility.

Figure 6: Sharpe ratio of the trend-following strategy versus positive (left chart) and negative (right chart) autocorrelation for the underlying





Source: J.P. Morgan Quantitative and Derivatives Strategy

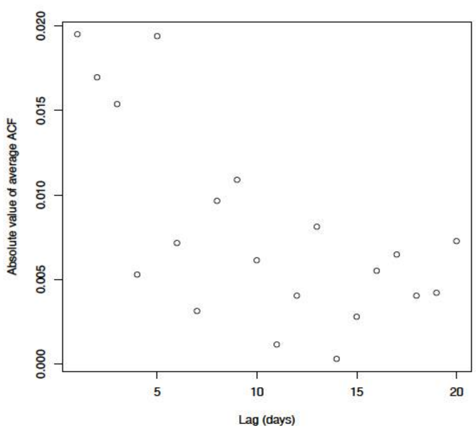
One may wonder how both mean-reversion and trend-following strategies can be profitable, by relying on the same property related to an asset (or set of assets), yet taking somehow opposite trading positions based on that shared property. One of the main results of the paper is that, when estimating trends over sufficiently long time horizons, the impact of statistical noise is reduced (Figure 6): in that limit, trends are the main drivers for supporting a positive expected PnL for the system, and positive auto-correlations concur in supporting the impact of the trends. Conversely, in the limit of short estimation periods, statistical fluctuations tend to dominate over trends and negative autoregressive coefficients would imply a negative expected value for a trend-following system. Basically, the article recommends estimating trends over windows ranging from a few weeks to several months for reducing the impact of statistical fluctuations. So, the conundrum of reconciling the positive performance of single asset mean-reversion vs trend-following systems can be resolved by understanding the different time scales/holding periods at play: mean-reversion typically is dominant over short time scales (days/weeks) whereas trends dominate over longer time scales (weeks/months).

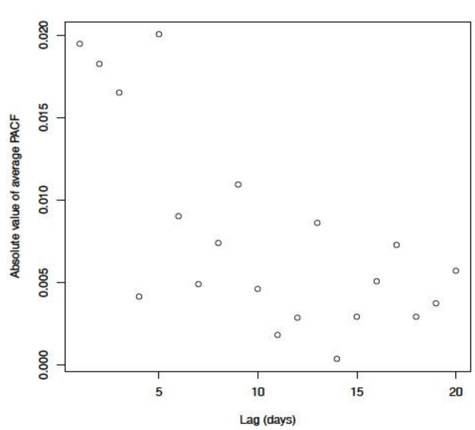
The application for the delta-hedging strategy is straightforward. When measuring PnL over the daily scale and using 1 month maturity options, as we have done when investigating the PnL of an option position, the corresponding estimation window would be $T = 11d$ (the average maturity of the option from opening the position till expiry). Based on the results of the piece, over such scale the negative daily auto-regression would dominate over trends and would be consistent with a cost of the daily delta-hedging strategy for the short volatility position. The topic will be discussed more extensively in the following section.

Delta-hedging through the lens of autoregression: empirical results

Having introduced some interesting theoretical results, it's time to check how the formalism developed so far applies in practice. We consider a universe of Equity, FX, Commodities, Vols, Rates, Credit assets, with data from Jan 1999 to June 2018. The first test we carry out regards the assumption that serial correlations are significant up to order one and that all higher lags are negligible. This assumption allowed a major simplification in the calculations around the PnL generated by the linear delta term for a vol position. We check this empirically by showing the average values of ACF and PACF across all assets (in absolute value for ease of reading) as a function of the lag (Figure 7). Linked to ACF, PACF is a statistical indicator whose behaviour is informative of the underlying auto-regressive structure (for a $AR(p)$ model, the PACF should converge to zero for lags>p). The empirical analysis reveals that, on average, ACF_1 is the largest, followed closely by ACF_5, ACF_2 and ACF_3. The other ones appear much smaller on average. The PACF analysis suggests that probably a more detailed description should take into account all lags up to order 5. So, our approximation is a reasonable starting point, although there could be room for refinements by taking into account a more structured dynamics.

Figure 7: Average ACF and PACF (in absolute values) across a set of cross-asset variables as a function of the lag (1 to 20)

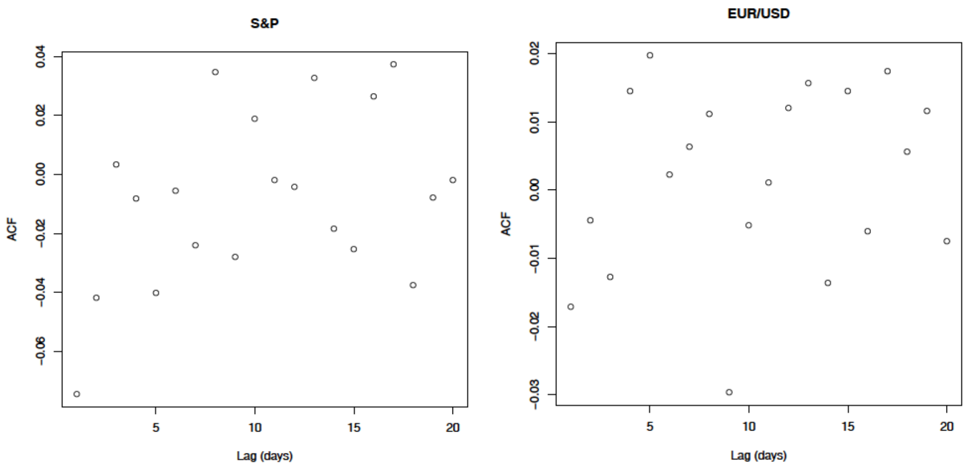




Source: JP Morgan Quantitative and Derivatives Strategy

By investigating the S&P and EUR/USD cases more in detail (Figure 8), we can see that while for the former the approximation holds well, for the latter ACF_9 is 50% bigger than ACF_1.

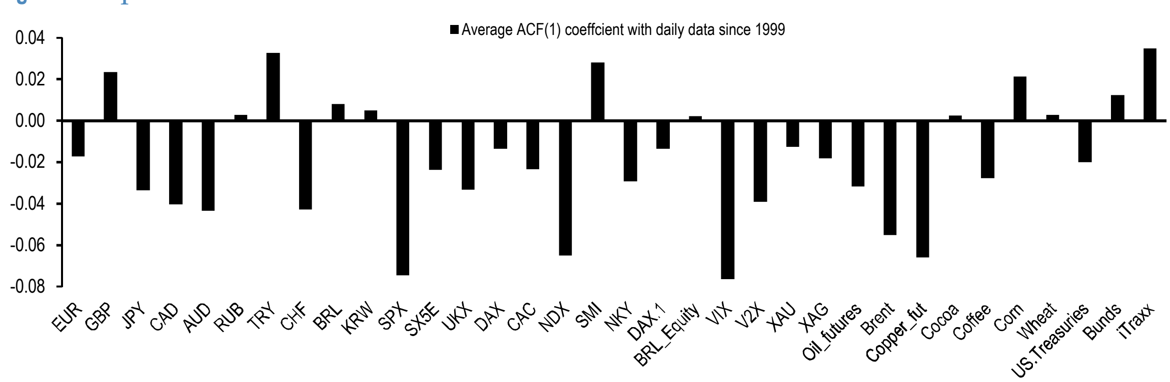
Figure 8: ACF as a function of the lag (from 1 to 20) for S&P and EUR/USD



Source: JP Morgan Quantitative and Derivatives Strategy

In Figure 9 we overview auto-regressive properties of a set of cross-asset variables (for most of these a listed option market is available). By examining the long-term properties of daily returns we can see that, amongst asset classes, FX is the one exhibiting the weakest mean-reverting properties, with positive ACF_1 in a few cases (GBP, TRY, BRL, KRW). This is not surprising, as currencies are often used by CTAs for trend-following trades. Other asset classes, in particular Equities and vols, exhibit stronger mean-reverting features on the daily scale.

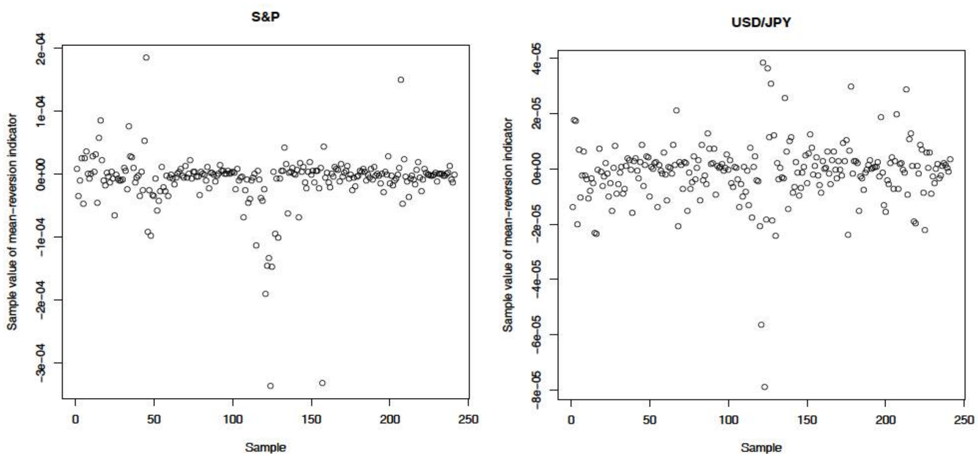
Figure 9: AR_1 coefficients estimated with data since 1999 across different markets



Source: JP Morgan Quantitative and Derivatives Strategy

We now want to examine the properties of the mean reverting indicator introduced earlier, for each financial asset. For carrying out the analysis, we break the whole sample into smaller intervals (around 250) of the same length, corresponding to the maturity of the options (1M); we estimate the time series parameters and the mean-reverting indicator for each sample. We then average over all samples to get a measure of average mean-reverting behaviour over the long run. This procedure should allow dealing better with non-linearities / changes of regimes, impacting the estimation of the auto-regressive parameters due to changing market conditions. Furthermore, such estimation will closely resemble the trading environment experienced when dealing with 1M options (our benchmark case for the short-vol trades) until expiry. The impact of statistical fluctuations will be assessed in the following section.

Figure 10: Sample values for the mean-reversion indicator for S&P and USD/JPY



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 10 displays the sample values of the mean-reverting indicators for S&P and USD/JPY over time as estimated over the monthly sub-periods. In both cases the indicator fluctuates over time, although a negative average value can be clearly discerned. We can also see that, post September-2008 high volatility market, the indicators have reached the most negative values, triggered by large, negative autoregressive and high sample volatilities. These conditions, implying elevated realised mean-reversion, should motivate elevated costs when delta-hedging short-volatility positions (this will be consistent with Equity options backtests presented later).

In Table 7, we consider an average over all assets/samples of the relevant quantities for our study. The average value of the autoregressive coefficient (-0.06) is consistent with a mild mean-reverting behaviour. There is roughly a factor 5 between the (average) effect of vol/autoregression compared to that of trends on the value of the mean-reversion indicator: this means that, as a first approximation, the impact of trends on the mean-reversion indicator is usually small, and the latter is determined by the auto-regressive properties.

Table 7: The effect of vol/autoregression tends to dominate over trends (average over all samples/assets)

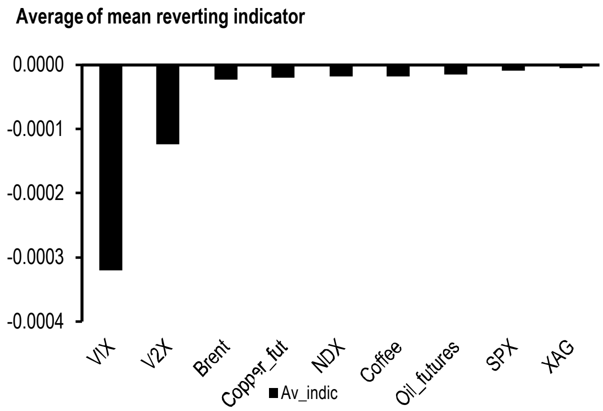
	AR(1)	Vol (yearly)	Mean (yearly)	Mean^2	|ACF|*vol^2	$\overline{\mathbf{MR}_{\mathbf{indic}}}$
Average over all assets	-0.06	23.3%	3.4%	1.6e-5	7.5e-5	-1.5e-5

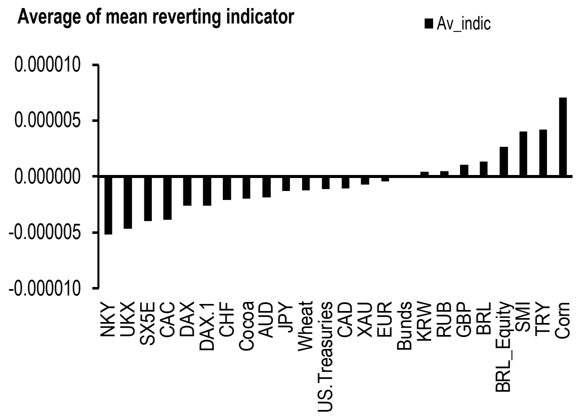
Source: J.P. Morgan Quantitative and Derivatives Strategy

Having introduced the mean-reversion indicator on each monthly sample, we also consider the average $\overline{MR_{indic}}$, over all assets and samples, which points to daily mean-reversion properties for the set of assets/markets overviewed.

By breaking results per asset (Figure 11), we see that volatility variables are those exhibiting the strongest mean reversion properties, due to the high volatility and the negative ACF_1 coefficients: the strong mean-reversion properties of the VIX have already been researched (see the JP Morgan piece “VIX Risk Premia and Volatility Trading signals” by Kolanovic, Kaplan and Mehra, 2014). Equities and Commodities follow in this respect. FX is the asset class exhibiting weakest mean-reversion properties on average. We stress that, for volatility variables, the mean-reversion effect is magnified by typical values of realized volatilities about ten times larger than for other markets, so that when vol-scaling positions in a diversified portfolio, the effect below would be diminished. Results below refer to the daily scale, but we will see in the following how to generalize them to other frequencies. We exclude from the chart our proxy of Credit variables: its value (+4e-5) would be roughly five times bigger than the second biggest in the chart. Credit appears therefore to exhibit positive linear correlation properties on the daily scale and its high volatility boosts the effect (as for VIX/VSTOXX).

Figure 11: Average value of the mean-reverting indicator for the different assets





Source: J.P. Morgan Quantitative and Derivatives Strategy

We can compute the expected value of the PnL of delta-hedging strategy per asset class for long and short vol positions (Table 8), by estimating an average behaviour for assets belonging to the same market. We stress that risk-management constraints might impose looser requirements when delta-hedging long (vs short) vol positions, and that rarely long options positions are implemented as pure vol plays (being able to obtain high leverage is normally the first requirement), so that in practice the relative impact might not be exactly symmetric. As we have seen Equities, Volatilities and Commodities are the asset classes where mean-reverting features are most persistent; given the results of the previous section, these are also the asset classes where the mean-reverting Daily-Weekly payoff should work best.

Table 8: Expected PnL contribution of daily delta hedging – split by asset class (average behaviors per market)

Expected contribution of DH	Long vol	Short Vol
Equities	>0	<0
FX	~0	~0
Commodities	>0	<0
Rates	~0	~0
Credit	<0	>0
Volatilities	>0	<0

Source: J.P. Morgan Quantitative and Derivatives Strategy

We recap results by summarizing (Table 9) in which cases can we expect daily delta-hedging to be the optimal choice: by that we mean that, if a hedging strategy not only reduces risk but also has a non-negative expected PnL, it surely should be implemented. Needless to say, the ones reported below refer to average patterns per asset class that individual assets might deviate from. In the majority of cases, reduction of risk comes at a cost, so that the suitability of a hedging strategy has to be assessed from this dual perspective (the Sharpe ratio of the resulting strategy is a more realistic indicator than just the PnL generated by delta-hedging). For FX and Credit daily delta-hedging appears a safe choice for short-vol positions, for the assets investigated (still, daily rebalancing might not be optimal for some currencies, see the following sections). For other asset classes, in particular Equities, Volatilities and Commodities, delta-hedging usually comes at a cost.

Table 9: For FX, Rates and Credit daily delta-hedging appear as a safe choice for short-vol trades

Is daily delta hedge is the optimal choice?	Long vol	Short Vol
Equities	YES	NO
FX	NO	YES
Commodities	YES	NO
Rates	NO	YES
Credit	NO	YES
Volatilities	YES	NO

Source: J.P. Morgan Quantitative and Derivatives Strategy

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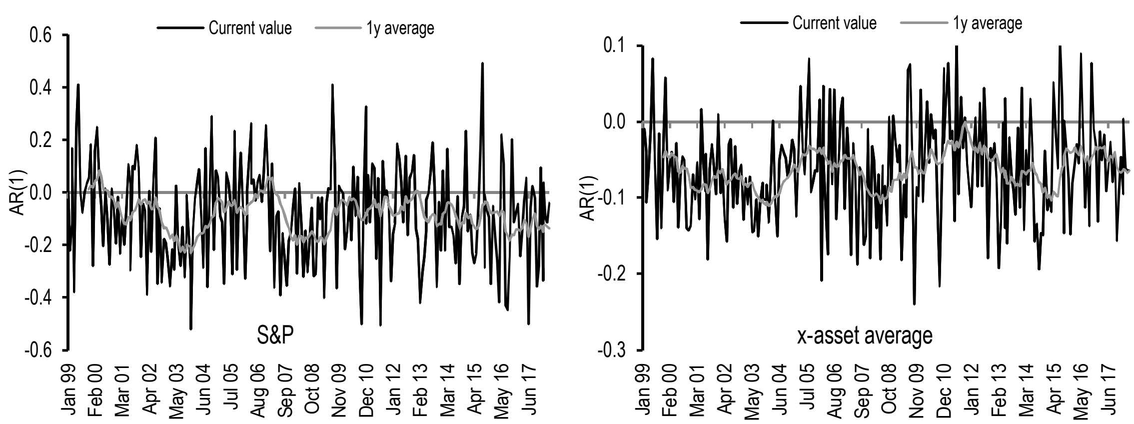
Even for the asset classes labelled as optimal choice cases in the table, there could be individual assets within that market for which daily delta-hedging represents a cost. Also, we will see in the following that reducing the hedging frequency could permit turning delta-hedging into a generator of positive returns: while its role as a hedging practice would then become questionable, such an investigation would be worth even for the cases where daily delta-hedging can be performed at no or little cost.

Assessing the persistence of mean-reversion over time

In the previous section we have seen that daily mean-reversion is a property that financial assets, and especially Equity markets, tend to exhibit in the long-run. Overreaction by market participants, herding, and intra-day trends (see Andersen, Bollerslev 1997, where the authors detect significant positive autocorrelation over the 5-min scale for the S&P) are commonly invoked for explaining mean-reverting patterns over the daily scale. On this topic, two aspects are worth further investigations. One deals with the stability of mean-reversion patterns against the impact of statistical fluctuations over short-periods of time. The other regards how these properties have evolved over time and whether they would be expected to hold in the future. We assess these aspects in this section.

We start by investigating more in detail the set of monthly samples introduced earlier. We focus on S&P and an average of all assets for illustrating the procedure at play (Figure 12). The whole period is split in roughly 250 monthly intervals. Sample average return, volatility, and autoregressive parameter $\overline{r_i}, \overline{\sigma_i}, \overline{\rho_i}$ are estimated over each sample. For the S&P, the value of the estimated $\overline{\rho_i}$ coefficient on each sample i, varies from -0.52 to 0.49, although in the majority of the cases it is comprised between -0.4 to 0.2; its 1y moving average is consistently in negative territory. For the cross-asset case, the average over all assets $\overline{\rho_i}$ coefficient varies in a tighter range (between -0.24 and 0.12), and its 1y average is strictly always negative.

Figure 12: Time-varying estimates of AR(1) coefficient over monthly periods – S&P and average on all assets



Source: JP Morgan Quantitative and Derivatives Strategy

Taking into account the role of sample volatilities and averages, one can recover the corresponding time series for the mean-reversion indicators (Figure 13). For both S&P and average x-asset case, conclusions remain similar to the behaviour of the $AR(1)$ coefficient, in that mean-reversion is a rather stable feature over time. For S&P, the indicator ranges from -3.4e-4 to 1.9e-4 (average -9.1e-6), for the cross-asset case from -2.5e-4 to 3.7e-4 (average -1.5e-5). We have seen earlier that large sample averages (either up or down) tend to support trend-following patterns, by reducing the mean-reversion strength accordingly. We can also see that high-volatility regimes (September 2008, Summer 2011, August 2015, February-March 2018) are those when mean-reversion is overall strongest, which can be understood by recalling that the mean-reversion indicator $MR_{indic} = (\sigma^2 ACF_1 + \mu^2)$ is in fact linear in autocorrelation and quadratic in volatility. For the vol-seller, delta-hedging daily, this constitutes an additional bleed of PnL on top of the obvious impact due to the negative Gamma and Vega of the position.

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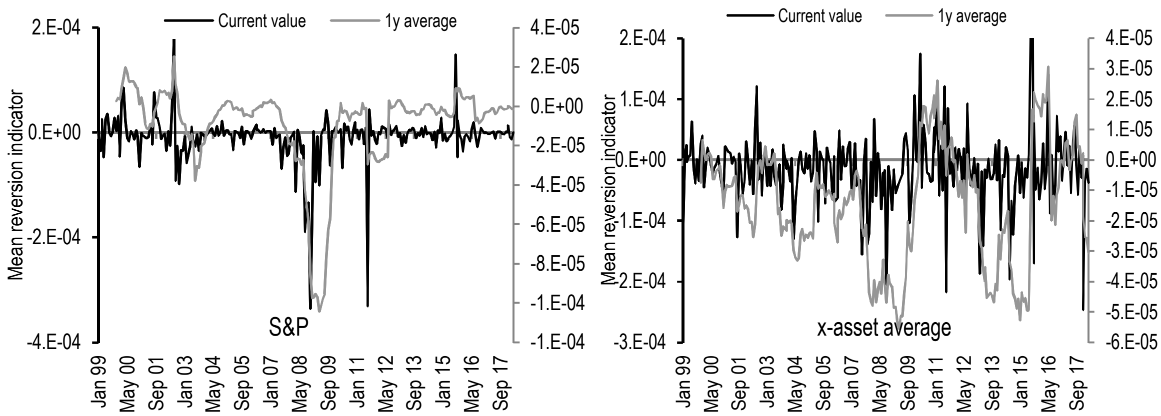
Figure 13: Time-varying mean reversion indicator – S&P and average on all assets



Source: J.P. Morgan Quantitative and Derivatives Strategy

As estimated parameters are subject to large fluctuations over different samples, due to statistical noise, one can account for statistical fluctuations by scaling the average value $\overline{MR_{indic}}$ for each asset by its sample standard deviation over the whole time interval (Table 10): NDX, Copper, VIX, Brent and S&P are the assets exhibiting the most persistent mean reverting properties after accounting for statistical fluctuations.

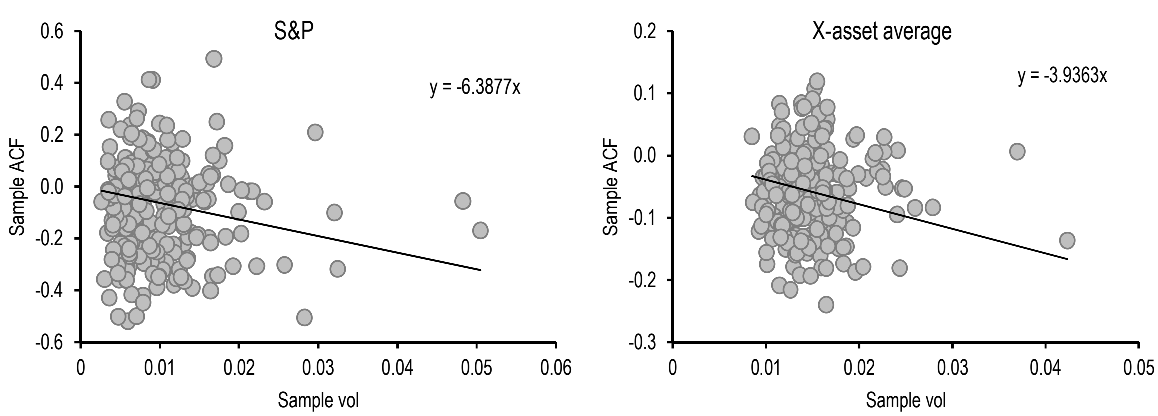
Table 10: Top 5 most persistent mean-reverting assets after accounting for sample fluctuations

Asset	NDX	Copper_fut	VIX	Brent	S&P
Rank	I	II	III	IV	V
Normalised MR indicator	-0.232	-0.229	-0.213	-0.203	-0.199

Source: J.P. Morgan Quantitative and Derivatives Strategy

Confirming the link between sample autocorrelation and vols would increase the interpretability of our conclusions and reduce the impact of statistical noise. In Figure 14 we compare sample vols (x-axis) and autocorrelations (y-axis), for S&P and by averaging over all assets. Via a linear regression, we find the effect to be significant in both cases, despite the noise (with low R^2, less than 10%): it is confirmed that higher volatility triggers more negative autocorrelation. A gradual reduction of central bank balance sheets, whose unprecedented activity over the past decade has acted as a catalyst in dragging volatilities lower, should support the re-pricing of market volatility to higher levels and stimulate mean-reversion going forward.

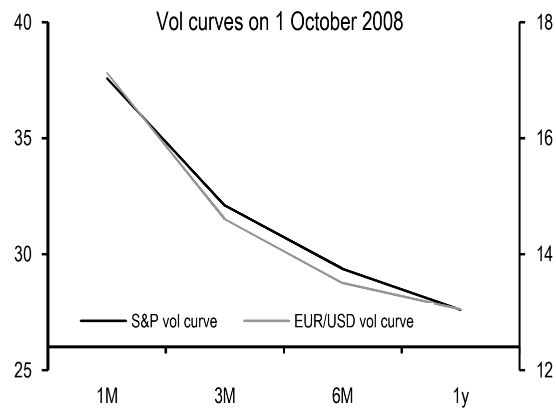
Figure 14: Higher volatility tends to stimulate more negative autoregression (S&P and average across assets)

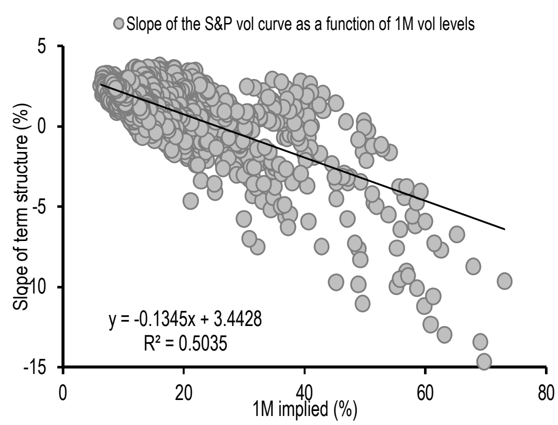


Source: J.P. Morgan Quantitative and Derivatives Strategy

We have already seen earlier in the piece that inverted volatility curves can be informative of a mean-reversion dynamics on the underlying returns. In Figure 15, we consider the S&P and EUR/USD implied vol curves as measured on 1 October 2008: in both cases, extreme levels of volatilities corresponded to a maximal inversion of the curve, and therefore to elevated "implied" mean-reversion property via negative autocorrelation coefficients. The relationship between inversion of the curve and volatility levels is confirmed empirically (rightmost chart): if we take S&P (but similar results hold for other Equity Indices and asset classes), with data since 2008, the slope of the vol curve is negatively correlated with the vol level (with a significant $R^2 = 50\%$). Heston-like stochastic volatility models, introducing an explicit mean-reversion *à la* Vasicek of short-dated vols towards long-term average values, automatically account for stronger implied mean-reversion during high-volatility markets.

Figure 15: High-volatility levels typically call for inverted vol curves at the front end





Source: J.P. Morgan Quantitative and Derivatives Strategy

We now investigate the stability of mean-reversion over time and across asset classes, by breaking the whole set of data into two intervals (1999-2008 and 2009-2018) of roughly the same length (Table 11). By looking at all asset classes, the strength of mean-reversion has been weaker in the recent interval (average of -1.2e-5 vs -1.9e-5 for the first decade). For the different asset classes, mean-reversion in Equities dropped sharply in the second interval (for S&P, it would move from -1.3e-5 to -5.2e-6), remained constant on vols and increased in FX. One should not be overly surprised by these results, by recalling that the first interval contains the episode of September 2008 when cross-asset volatilities reached all-time highs, and that market volatilities were suppressed in the second period also due to unprecedented balance sheet expansion of world-wide Central Bank, as a way for spurring economic activity following the Global Financial Crisis of 2008.

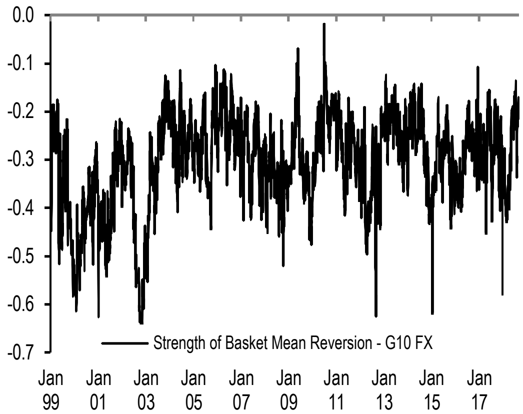
Table 11: Persistence of mean-reversion (S&P and average over assets) over time

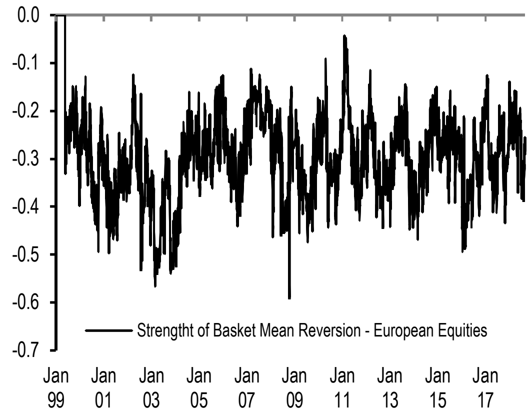
Averages of mean-reversion indicator	Equity	FX	Commodities	Rates	Credit	Other (Vols)	Average of all assets
1999-2008	-8.5e-6	1.7e-6	-1.2e-5	-7.2e-8	1.2e-5	-2.3e-4	-1.9e-5
2009-2018	3.3e-9	-1.7e-6	-6.4e-6	-9.7e-7	7.4e-5	-2.1e-4	-1.2e-5

Source: JP Morgan Quantitative and Derivatives Strategy

So far, we have assessed mean-reversion as a property regarding each individual asset. While we will not investigate the topic in detail here, another possibility would be to consider basket mean-reversion, by generalizing the $AR(1)$ formalism to a multi-asset $VAR(1)$ case. Just for the purpose of investigating general properties related to mean-reversion, and by skipping all technical details, we report the time series of a relevant parameter for describing the "strength" of basket mean-reversion for G10 FX and Equity Indices (Figure 16), namely the most negative eigenvalue of the corresponding $VAR(1)$ matrices (the more negative the value, the stronger the mean reversion for the corresponding basket).

Figure 16: An indicator measuring the strength of *basket mean-reversion* over time (FX and Equity Indices)





Source: J.P. Morgan Quantitative and Derivatives Strategy

For both cases, we don't detect a considerable attenuation of this property over the past twenty years: if ever, we observe occasional spikes in the indicators on occasions which often correspond to bursts in market volatility levels (Q2 2000, H2 2002, H2 2008, Q1 2016). Compared to the single-asset case (where mean-reversion in Equities tends to be more pronounced), we don't notice significant differences in the property between the FX and Equity cases considered here.

To summarize, we have seen that mean-reversion is a property commonly shared by financial assets which is relatively stable over time. As mean-reversion tends to be stronger when volatility levels are high, it tends to reduce the effectiveness of the delta-hedging procedure that vol sellers implement for reducing their risks; therefore, it makes sense to investigate alternative delta-hedging procedures which might prove beneficial when vol levels are high. Two possible solutions are presented in the following sections.

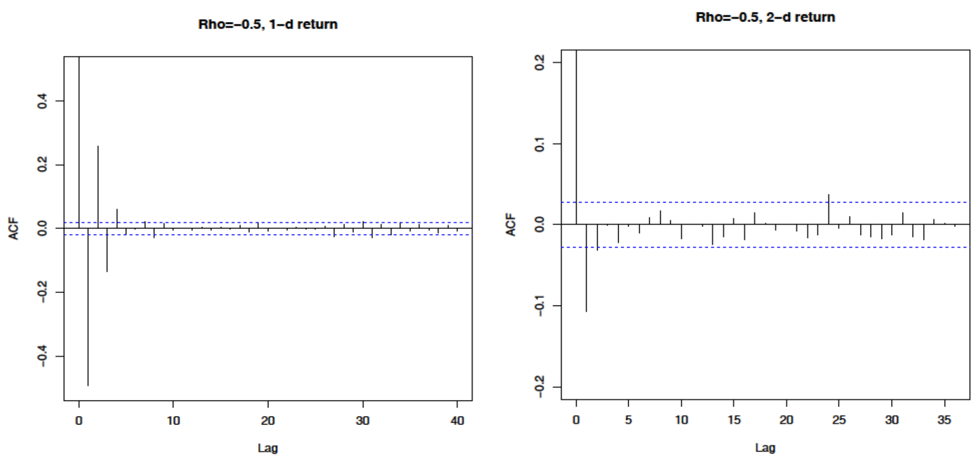
Finding an optimal frequency for delta-hedging

We have seen in the previous sections that mean-reversion on the daily scale can make delta-hedging of short-vol positions costly. Despite the tight bid/ask spreads in delta one instruments, rebalancing delta positions frequently will also entail higher costs in the long-run (see more details and formulas in the Appendix). It is therefore legitimate to investigate whether delta-hedging at a frequency lower than (1/) daily might prove beneficial in PnL-terms, which we do in the following.

What we have found is that, assuming delta-hedging at frequency ν, a negative ACF_1 for data sampled at that frequency implies an expected cost for delta-hedging (neglecting the impact of trends). The first question we ask ourselves is: can we show that ACF_1 diminishes with the sampling frequency υ? As a starting point, we can carry out a Monte-Carlo simulations by assuming a AR(1) dynamics on the daily scale to start with (by recalling that log returns allow decomposing a weekly return as the sum of daily returns ${r_t}^w = \sum_i {r_t}^{i,D}$).

For a pure $AR(1)$ process with $\rho = -0.5$ (Figure 17), it is confirmed that sampling at a lower frequency (in the chart, daily vs bi-daily) reduces markedly autocorrelations, which should make delta-hedging less costly. Furthermore, we have seen in the previous section that daily mean-reversion would support a downward sloping vol curve, whose effect would also cut hedging costs. In practice, we have seen that higher order lags in the AR dynamics are present, so we need to tackle the problem from a more general perspective. One possibility would be using more evolved mathematical derivations, along the lines of (Tzotchev 2018), defining an optimal frequency which maximizes the PnL from the delta-hedging component (taking into account costs).

Figure 17: ACF series for a AR(1) model sampled at daily bi-daily frequencies (lag 0 = 1 is not displayed in the charts)



Source: JP Morgan Quantitative and Derivatives Strategy

In the following we will adopt a more hands-on approach entirely based on data themselves. We repeat the calculations as above, and find that, if hedging over a different frequency/time interval, the formula above for the PnL over the life of an option becomes:

$$PnL_{Delta}{}^{M} = \overline{Dollar\ Delta}_M\ \mu_M\ T/M + \overline{Dollar\ Gamma}_M\ (\sigma_M{}^2 ACF_1{}^M + \mu_M{}^2)T/M$$

Here, T is the option maturity at inception (in days), M the new holding period for the delta position; $\overline{Dollar\ Delta}_M, \overline{Dollar\ Gamma}_M, \mu_M, \sigma_M, ACF_1{}^M$ are now estimated over the M days scale (rather than daily). Internal consistency for implementing hedging at a viable frequency requires $M \leq T$ for a fixed T.

We now want to assess the sensitivity of $PnL_{Delta}{}^{M}$ on M. We expect little dependence of the average Greeks as a function of M. Similarly, given the linearity of the averaging procedure, $\mu_M \frac{T}{M} \sim M\mu_1 \frac{T}{M} = \mu_1\ T$; this means that when a large trend takes place, it will impact the PnL regardless of the hedging frequency. What remains to assess is then the scaling of the following quantity as a function of M:

$$MR_{indic}^{M} = (\sigma_M{}^2 ACF_1{}^M + \mu_M{}^2)/M$$

A non-trivial scaling of MR_{indic}^{M} as a function of M could help defining empirically an optimal holding period for the delta: if MR_{indic}^{1} is significant and negative on the daily scale, and less negative or even positive for $M > 1$, this should reduce the cost embedded in the delta-hedging procedure. For instance, a Brownian motion (Wiener process) with drift exhibits a non-trivial scaling for the squared return $E(r_{t,M}{}^2) = (\sigma_M{}^2 + \mu_M{}^2) = (M\sigma_1{}^2 + M^2\mu_1{}^2) = M^2(\mu_1{}^2 + \sigma_1{}^2/M)$. For large holding periods, the drift the dominates over the volatility term, which is conversely dominant over short time scales. In general, serial correlations will impact the scaling of both $\sigma_M, ACF_1{}^M$ with M (depending on the higher lags of a $AR(p)$ expansion) therefore we assess the question numerically based on actual data.

We investigate the sensitivity of MR_{indic}^{M} on M by studying empirical data, sampled at different frequencies. If we consider the weekly frequency as an example, in principle one should consider five different analyses corresponding to the five days of the week: intra-week "seasonal" patterns and statistical fluctuations could lead to different results for the different days of the week in terms of optimal hedging patterns. Here, for simplicity, we will display the results corresponding always to the same starting day, chosen arbitrarily, which can introduce a bias in the analysis: we will discuss later how to reduce this bias when hedging in practice.

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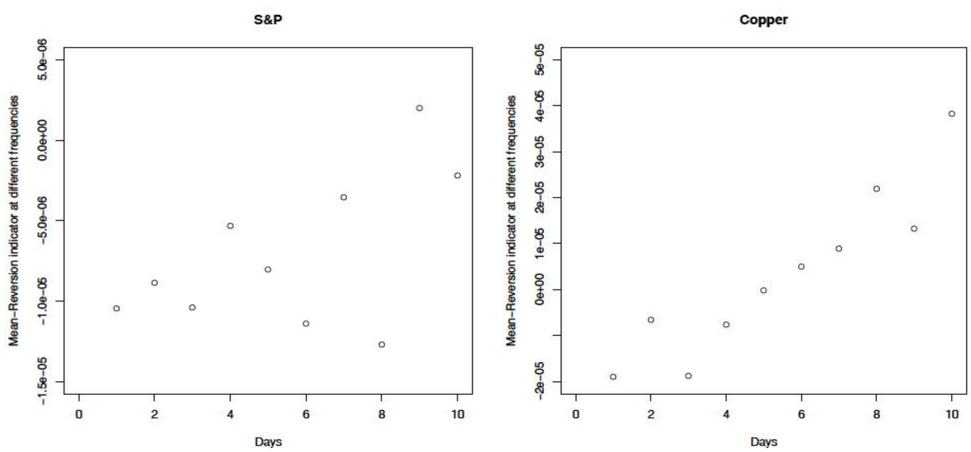
We consider four case studies more in detail by plotting the values of MR_{indic}^{M} for $M = 1 \cdots 10$. On S&P and Copper (Figure 18), we see that there appears to be value in reducing the hedging frequency. For S&P, there is value in considering optimal holding periods of four or five (working) days. Much longer holding periods around two weeks would also be supported; similarly, not delta-hedging at all (i.e., rebalancing over the monthly scale, not displayed in the chart) should also beat daily delta-hedging from a PnL-perspective. However, given that hedging at relatively high-frequencies is normally preferred from a risk-management perspective, we find that the weekly scale offers a good trade-off between protecting the portfolio and reducing the impact of hedging costs. Very similar results are obtained for Copper (and other Commodities, like for instance Brent). For both cases, the curve appears relatively smooth as a function of the hedging frequency, as a sign that the bias due to the arbitrary choice of the starting day could be modest.

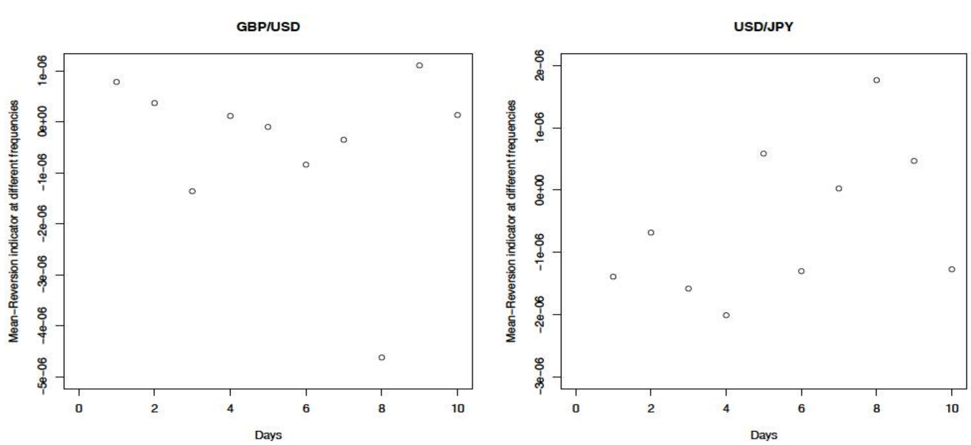
Figure 18: Average mean-reversion indicator as a function of the hedging frequency for S&P and Copper



Source: JP Morgan Quantitative and Derivatives Strategy

We then consider two FX pairs, GBP/USD and USD/JPY (Figure 19). For GBP/USD, it's difficult to justify hedging at lower frequencies than daily, given the limited benefit on the MR_{indic}^{M}; this result is in line with our earlier finding that for FX daily delta-hedging can be generally assumed to be the benchmark choice. The picture is different for USD/JPY, in this case supporting delta-hedging around the weekly scale. In both cases, we see that the curves are less smooth than for Equities and Commodities.

Figure 19: Average mean-reversion indicator as a function of the hedging frequency for GBP/USD and USD/JPY



Source: JP Morgan Quantitative and Derivatives Strategy

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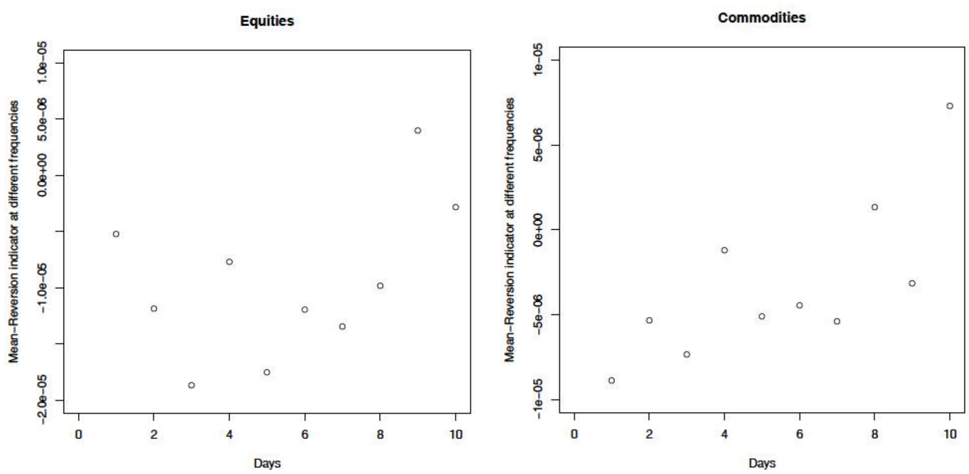
The advantage of hedging at lower frequencies is not homogeneous on all assets within a market. If we consider an average value of the MR_{indic}^{M} on Equities and Commodities (Figure 20), we see that the added value of reducing the frequency of the hedging is more pronounced for the latter case. If both cases, hedging on the monthly scale would be seen as a viable opportunity (tested empirically but not displayed in the chart).

Figure 20: Average value of the mean-reversion indicator o as a function of the holding period – Equities/Commodities



Source: JP Morgan Quantitative and Derivatives Strategy

We can apply consistently the criterion on individual assets per market and find cases where hedging at a lower frequency is supported (Table 12). For each asset, we preferably look for optimal hedging periods up to one week. The procedure can be systematically applied to any assets of interest. These results will be compared with actual backtests in the following section. Despite the difficulty of forecasting whether an asset will exhibit trend-following or mean-reverting features in the future (statistical fluctuations can be dominant over short time intervals), results in the table, referring to average behaviour over long-term periods, can be indicative of structural features of different markets/asset classes, and therefore prove useful for reducing the impact of mean-reversion, by considering hedging at lower frequencies than daily.

Table 12: When daily delta-hedging of short-vol positions is not optimal across different asset classes

Asset class	Equities	FX	Commodities	Rates	Credit	Vols
Best asset	S&P	USD/JPY	Copper	US Treasuries	iTraxx	VIX
Optimal holding period (days)	4	5	5	4	1	21
Daily value of indicator	-1e-5	-1.4e-6	-1.9e-5	-9.9e-7	3.5e-5	-3.3e-4
Value of indicator at lower frequency	-5e-6	5.8e-7	-2.2e-7	7.4e-7	3.5e-5	3.0e-4
Average value – 1d	-5.2e-6	-1.1e-6	-8.9e-6	-4.2e-7	3.5e-5	-2.4e-4
Average value – 1w	-1.76e-5	-1.3e-6	-5.1e-6	3.5e-7	-5.5e-5	-5.7e-4
Average value – 2w	-2.8e-6	1.9e-6	7.3e-6	7.4e-7	5.1e-5	-3.5e-4
Average value – 1m	3.2e-5	8.2e-6	1.3e-5	-2.5e-6	5.3e-5	1.6e-4

Source: J.P. Morgan Quantitative and Derivatives Strategy

In the bottom part of the table, we consider the average results per asset class at different frequencies. For the volatility variables, the impact of higher-order lags in the ACF series is not negligible, so that the scaling of MR_{indic}^{M} is not trivial. For both VIX and VSTOXX, we see value in hedging over 21 days, the lowest frequency allowed for monthly options, which essentially corresponds to *not hedging at all* the vol position. For FX, while we have already noted that daily delta-hedging is less costly than for other asset classes (and therefore, already appealing from a risk-management perspective), we see from the table that for holding periods of 2-weeks or longer, delta-hedging should deliver a positive PnL. Similar conclusions are found for Commodities and Rates (in the latter case, 1-w to 2-w holding periods appear optimal). Results reported in the table don't take into account trading costs. In the Appendix, we show that the expected contribution of trading costs decreases when delta-hedging at lower frequencies, scaling as $1/\sqrt{M}$ when hedging every M days; when

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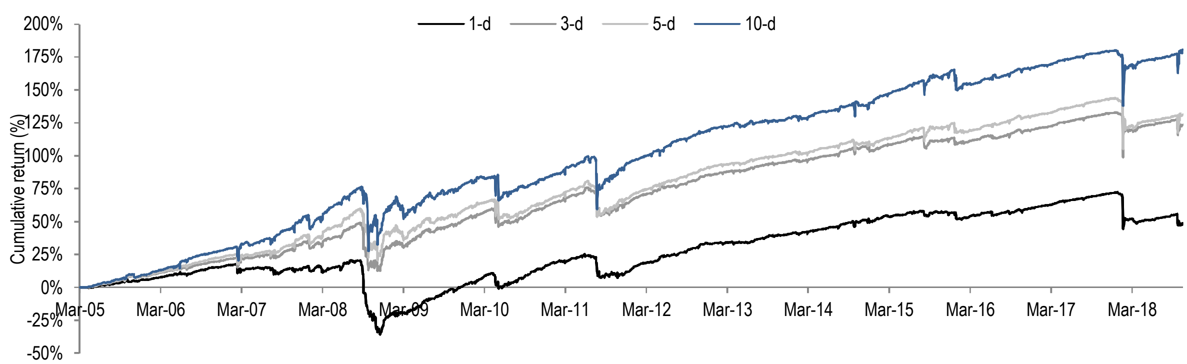
overviewing actual option backtests in the next section, the latter effect will motivate further the possibility of hedging at lower frequencies than daily, especially for the cases like Rates or Credit where daily mean reversion is weaker than for Equities.

So far, we haven't addressed how to implement in practice delta-hedging at other frequencies than daily. The most obvious and natural way for doing so would be simply to hedge positions every N-days, with $N > 1$ chosen as to reduce the impact of mean reversion. However, doing so would expose the hedger to an undesirable bias related to the choice of the day when to implement the hedging, as discussed earlier. Also, this implementation would be overly sensitive to sudden price movements: if a large move starts unfolding on Monday during business hours, hedging at Monday EOB would reduce risk compared to hedging on Friday EOB. A second, more robust option, could involve still rebalancing positions daily, but by computing the delta over a $N - day$ moving average of the spot (or forward), as obtained via the Black-Scholes formula. We will pursue the latter implementation in the following, and will generally refer to it as lower frequency smoothing for the options' delta, with N as the smoothing parameter.

From theory to practice – investigation of actual options backtests

In this section we consider actual options backtests on Equity, FX and Commodities (Precious Metals) markets, with the goal of finding an empirical, practical counterpart to the more "theoretical" results as introduced in the earlier sections. For the Equity case, we will rely on listed options (95/105 strangles): delta-hedging is performed on the underlying futures market. Costs are taken into account for the vol and delta trades. On a daily basis, we cumulate positions (held until expiry) by trading the first or second monthly contract depending on the one which is nearer to the target monthly maturity. The notionals of the new options entering the portfolio are chosen so that their Vega at inception is constant over time.

Figure 21: Empirical backtests for S&P show value in implementing the lower frequency smoothing on the delta



Source: JP Morgan Quantitative and Derivatives Strategy

We start by focusing on S&P, with data covering the period March 2005 to 2 November 2018, before focusing on other Equity Indices. Figure 21 displays the performances of short-vol strategies on S&P by varying the smoothing parameter. It is reassuring to see that the strategy smoothing the delta easily allows to outperforming the benchmark case (daily delta-hedging with no smoothing). It is also striking that, during the latest market sell-off as occurred in October 2018, the strategies smoothing the delta were the quickest to recover (for smoothing parameters of 5 days or longer, short-vol strategies are in positive territory since early October).

Increasing the smoothing parameter, being analogous to reducing the delta-hedging frequency, obviously increases the exposure to market fluctuations and, therefore, the volatility of the resulting strategy. Despite that, the reduced sensitivity to market mean-reversion as occurring in high-volatility markets allows for a reduction of the maximum drawdown of the strategy, especially if expressed in proportion of their volatility. This effect was particularly evident in the late 2008 – early 2009 period, when high market volatilities combined with large and negative autocorrelations induced additional losses on the standard daily delta-hedging strategy: choosing a smoothing parameter between 3 to 5 days would have helped reducing the maximum drawdown by roughly 30%.

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More detailed results of the backtests are reported in table 13. We see that increasing the smoothing parameter up to 10 days allows improving the yearly returns, compared to the benchmark case also thanks to the lower impact of costs (see Appendix); the rise of the volatility of the strategy with the smoothing parameter is confirmed. A choice of 3 business days for the smoothing parameter delivers the best performance in terms of Sharpe ratio and maximum drawdown. A sudden spike in volatility levels, especially for the US Equity market, caused sharp losses on short-volatility strategies in the first ten days of February. From the table we see that, while the benchmark strategy is the one which suffered the least maximum drawdown (24.5%) in the occurrence, is also the one for which the episode has had by far the most enduring consequences (year-to-date): this once again was due to the extra mean-reversion cost cumulating on top of the natural losses due to the short-Gamma and –Vega sensitivities of the strategy, acting as a drag on it even when volatilities started to fade shortly thereafter.

Table 13: Smoothing the daily fluctuations of the options delta can help improving performance measure: focus on S&P

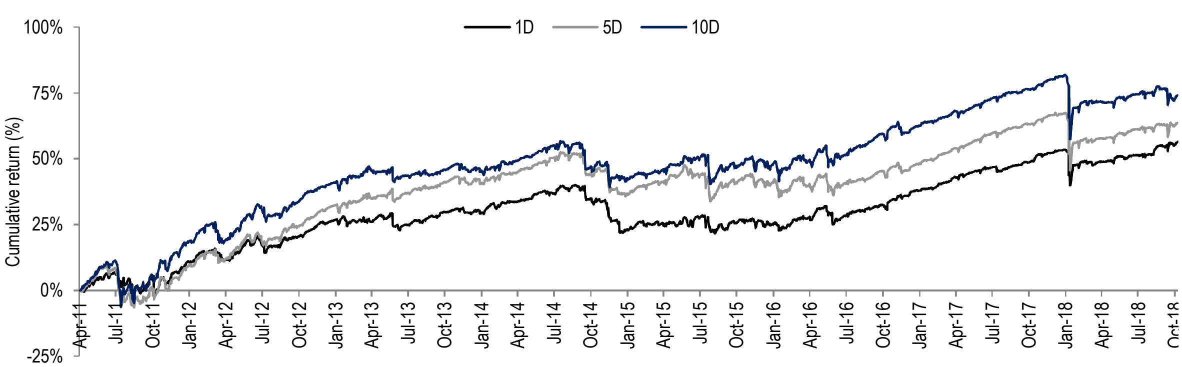
Smoothing parameter (business days)	Return	Vol	Sharpe	MDD	MDD/vol	MDD - 2018	Loss since Feb 18
1	3.5%	12.0%	0.29	43.3%	3.61	24.5%	19.5%
2	6.3%	14.2%	0.45	39.6%	2.80	27.8%	15.8%
3	8.8%	15.2%	0.58	30.8%	2.02	29.2%	7.5%
4	9.0%	16.5%	0.55	33.0%	2.01	30.5%	10.0%
5	9.3%	17.8%	0.52	32.5%	1.82	32.2%	9.5%
6	10.4%	19.2%	0.54	33.6%	1.75	33.6%	6.7%
7	11.0%	20.5%	0.53	34.0%	1.66	34.0%	5.7%
8	11.3%	21.8%	0.52	35.1%	1.61	34.4%	4.8%
9	12.0%	23.0%	0.52	36.4%	1.58	34.6%	0.3%
10	12.8%	24.1%	0.53	38.8%	1.61	34.6%	-4.2%
21	8.1%	30.5%	0.26	47.7%	1.56	33.4%	-23.8%

Source: JP Morgan Quantitative and Derivatives Strategy

These results are generally in good agreement with the earlier investigation of time series properties for the S&P, especially when taking into account the impact of trading costs: a rough estimate of the reduction of trading costs on the delta when considering weekly and bi-weekly smoothing is around 55% and 68%, respectively (see formulas in the Appendix for the case of hedging at lower frequencies than daily). Furthermore, as we had already highlighted, trading positions daily, while still allowing to smoothing the delta, permits to reducing the sensitivity to the rather arbitrary choice of start/end days when delta-hedging at lower frequencies. Therefore results here appear more stable as a function of the smoothing parameter than it was for the earlier section investigating data sampled at lower frequencies than daily. More results on the S&P will be presented at the end of the following section, when considering the possibility of hedging intraday.

We then consider a set of other Equity Indices: Nasdaq (since March 2005), Euro Stoxx 50 (since April 2011, Figure 22) and Nikkei (since January 2006), by applying the same trading rules as for the S&P earlier (with a common end date of 2 November 2018). For simplicity, we just consider 1-d (i.e., no smoothing), 5-d and 10-d as choices for the smoothing parameter as applied onto the delta. The case study of Euro Stoxx 50 is displayed in Figure 22.

Figure 22: Smoothing the deltas allows improving yearly returns on the Euro Stoxx 50



Source: JP Morgan Quantitative and Derivatives Strategy

As for the S&P, introducing a smoothing parameter allows improving substantially the yearly returns, although at the cost of an increased volatility (Table 14): for the Nikkei, also thanks to reduction in trading costs, the yearly return can turn from negative to positive. Choosing a weekly smoothing parameter also tends to reduce the maximum drawdown experienced by the strategies, especially if measured against their volatility. Compared to the case of S&P, introducing a smoothing on the delta does not help improving performance in 2018, and particularly across the February sell-off episode; during the October 2018 sell-off, the smoothing procedure outperformed the benchmark for Nasdaq, was roughly on par for Eurostoxx and underperformed for Nikkei. Overall, the picture across the Equity indices investigated appears consistent with each other and with the earlier analysis on the time series properties of the underlying assets.

Table 14: Using smoother deltas can boost short-vol strategies on other Equity Indices too

Asset	Smoothing parameter	Return	Vol	Sharpe	MDD	MDD/vol	MDD-Feb 18	Loss since 1Feb 18
NDX	1	1.5%	10.0%	0.15	39.0%	3.89	27.5%	25.79%
NDX	5	2.3%	12.2%	0.19	35.4%	2.89	28.8%	26.75%
NDX	10	2.4%	13.8%	0.18	44.1%	3.20	30.1%	27.07%
SX5E	1	7.3%	9.6%	0.76	16.9%	1.76	12.7%	-4.12%
SX5E	5	8.2%	12.6%	0.65	19.7%	1.56	19.7%	2.92%
SX5E	10	9.5%	13.8%	0.69	21.7%	1.57	21.7%	5.83%
NKY	1	-0.7%	11.4%	-0.06	32.3%	2.84	10.4%	9.88%
NKY	5	0.4%	15.5%	0.03	43.7%	2.81	20.5%	19.48%
NKY	10	1.5%	20.3%	0.07	53.9%	2.66	23.4%	20.85%

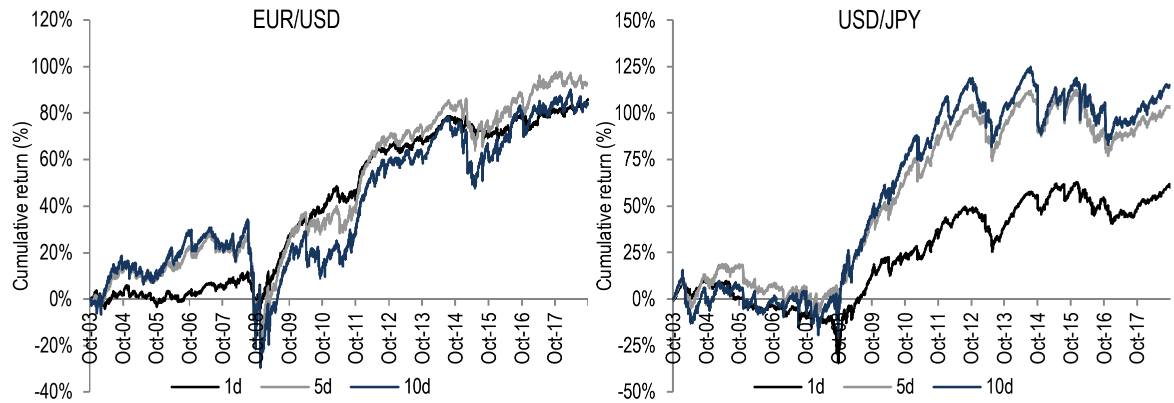
Source: JP Morgan Quantitative and Derivatives Strategy

We then move on to the FX (since November 2003) and Precious Metals (since January 2005) cases, for which we implement short-vol trades via selling each day 1M, 25delta strangles (held to expiry), by setting a constant Vega an inception for each; we take into account trading costs on both vol and deltas, having seen that the latter variable plays an important role when determining an optimal hedging frequency[3]. For simplicity, we will consider three values for the smoothing parameter for all the simulated performances considered: 1 (i.e., no smoothing), 5 and 10 business days. 30 October is a common end date in the backtests.

In Figure 23, we start by reviewing the case of EUR/USD and USD/JPY. For EUR/USD, smoothing the spot value entering the Black-Scholes formula for the delta does not seem to add much value. Returns for the 1-d (i.e., no smoothing) and 5-d smoothing parameters are comparable, but in the latter case the PnL is more volatile. For USD/JPY, the methodology does add considerable value, with the cumulative return almost doubling over a 15-yrs period when considering weekly or by-weekly smoothing. The maximum drawdown is also reduced. The added value for USD/JPY is consistent with the results of Table 12 (page 26), where it was shown that USD/JPY tends to exhibit mean-reversion behaviour in the long run. From the chart, we can also see that most of the extra performance introduced from the smoothing strategy was contributed to in the 2008-2010 high-volatility regime.

[3] We consider here hedging each option via the corresponding fixed-expiry forward, rather than via a common spot value, which might introduce an overestimation of trading costs.

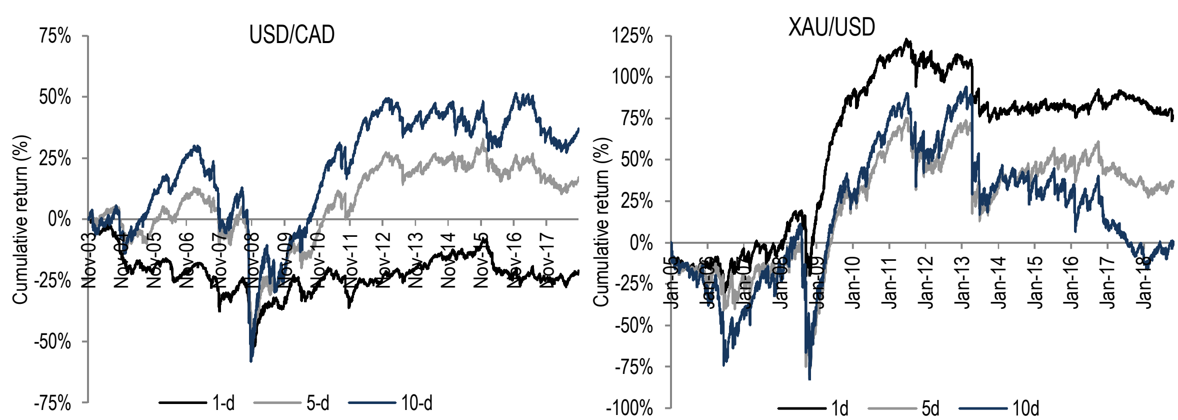
Figure 23: Introducing a smoothing parameter on the delta improves substantially the performance of USD/JPY short-vol, not so much so for EUR/USD



Source: JP Morgan Quantitative and Derivatives Strategy

We then consider backtests for USD/CAD and Gold (Figure 24). For USD/CAD, the methodology does add considerable value in the long run: the period where the extra return generated by the delta-smoothing strategy was highest has been between 2009 to 2013, when range-trading price action in USD/CAD, and the corresponding strong realised mean-reversion, supported reducing the fluctuations of the delta. For Gold, the methodology does not appear to bring value in the long run.

Figure 24: Smoothing the delta improves substantially the performance on USD/CAD. It does not add value on XAU/USD



Source: JP Morgan Quantitative and Derivatives Strategy

Results for the FX and Precious Metals assets here considered are reported in Table 15. We first highlight that short vol strategies in these markets have performed well over 2018, despite the two Equity sell-offs of February and October[4]: maximum drawdowns have remained very contained (often less than 10%) and in several cases, the strategies are in positive territory year to date (negative numbers in the rightmost column refer to actual gains throughout 2018). This observation confirms that implementing cross-asset diversification can help tacking better drawdown episodes than just on the Equity case, where still average returns of short-vol strategies tend to be higher. The trading costs when entering new option positions are higher for precious metals (and especially so for Silver) than for FX, which justifies to some extent the corresponding lower Sharpe ratios.

[4] Short-volatility strategies for the three currencies reviewed here actually made money in October 2018.

Table 15: Summary of the performance of FX and Precious Metals short-vol strategies for different smoothing parameters

Asset	Smoothing parameter	Return	Vol	Sharpe	MDD	MDD/vol	MDD-Feb 18	Loss since 1 Jan 18
EUR/USD	1	5.6%	8.2%	0.68	20.1%	2.46	3.2%	-5.31%
EUR/USD	5	6.1%	12.2%	0.50	45.9%	3.77	6.5%	3.14%
EUR/USD	10	5.5%	15.8%	0.35	63.7%	4.04	10.1%	0.49%
USD/JPY	1	3.9%	10.9%	0.36	46.1%	4.25	2.7%	-7.07%
USD/JPY	5	6.7%	15.2%	0.44	35.7%	2.35	6.2%	-6.26%
USD/JPY	10	7.4%	18.4%	0.40	42.3%	2.30	7.2%	-10.45%
GBP/USD	1	-1.7%	9.4%	-0.18	53.7%	5.69	34.8%	-2.66%
GBP/USD	5	0.5%	12.4%	0.04	59.2%	4.78	9.6%	4.37%
GBP/USD	10	0.1%	15.5%	0.01	77.5%	4.98	15.3%	9.34%
USD/CHF	1	-5.4%	22.0%	-0.24	98.2%	4.47	4.0%	-3.98%
USD/CHF	5	-5.2%	26.3%	-0.20	122.9%	4.67	7.9%	3.29%
USD/CHF	10	-5.1%	27.5%	-0.19	123.9%	4.51	10.3%	8.39%
AUD/USD	1	-4.4%	13.0%	-0.34	105.9%	8.17	3.7%	-5.55%
AUD/USD	5	-4.5%	19.5%	-0.23	134.2%	6.89	10.6%	7.12%
AUD/USD	10	-6.2%	27.4%	-0.23	172.1%	6.29	12.2%	5.11%
NZD/USD	1	-5.4%	12.8%	-0.42	97.2%	7.61	3.4%	-2.23%
NZD/USD	5	-6.0%	19.2%	-0.31	128.3%	6.69	15.3%	12.17%
NZD/USD	10	-4.9%	25.1%	-0.20	127.2%	5.06	19.1%	14.23%
USD/CAD	1	-1.4%	7.9%	-0.17	53.9%	6.83	4.8%	-4.19%
USD/CAD	5	1.1%	11.6%	0.10	63.8%	5.49	6.7%	-1.22%
USD/CAD	10	2.4%	15.1%	0.16	88.2%	5.85	8.8%	-2.73%
USD/NOK	1	-2.6%	11.4%	-0.23	62.3%	5.48	2.8%	-7.81%
USD/NOK	5	0.3%	15.6%	0.02	87.8%	5.65	11.0%	6.87%
USD/NOK	10	3.1%	19.9%	0.15	98.3%	4.93	17.0%	5.87%
USD/SEK	1	-1.6%	10.0%	-0.16	45.4%	4.52	9.0%	3.01%
USD/SEK	5	2.8%	14.4%	0.20	50.3%	3.49	22.5%	20.93%
USD/SEK	10	4.0%	18.9%	0.21	68.7%	3.63	22.9%	18.93%
XAU/USD	1	5.4%	18.9%	0.28	50.3%	2.66	11.4%	7.95%
XAU/USD	5	2.6%	28.4%	0.09	79.9%	2.81	6.8%	-4.17%
XAU/USD	10	0.0%	34.1%	0.00	110.5%	3.24	10.6%	-11.00%
XAG/USD	1	1.2%	39.3%	0.03	150.2%	3.82	16.3%	1.78%
XAG/USD	5	-5.3%	56.4%	-0.09	245.8%	4.35	18.5%	-20.03%
XAG/USD	10	-9.9%	67.1%	-0.15	283.6%	4.22	23.0%	-33.37%

Source: JP Morgan Quantitative and Derivatives Strategy

When applying the smoothing methodology to G10 currencies, we find that in several cases it helps increasing considerably average returns and Sharpe ratios, also thanks to the reduction of trading costs. The assets where the methodology brings the most evident improvement are USD/JPY, GBP/USD, USD/CAD, USD/NOK and USD/SEK. Compared to Equities, maximum drawdown, typically experienced across the late 2008 episodes, tend to increase vs the benchmark case (i.e., no smoothing for the delta), and the overall added value introduced by the methodology is less consistent across all assets. This result is consistent with the earlier empirical analysis on the underlying asset classes, which was finding that generally FX tends to exhibit more of a trend-following behaviour. Still, it has to be stressed that, for the assets like USD/JPY where mean-reversion appears to be stable, the smoothing methodology can be useful, and could be even more so if vols were to increase from the current levels (current levels for most FX volatilities are still below long-term averages). Therefore, it can make sense to assess the applicability of the methodology on a case by case basis, by overviewing a wider set of assets and by potentially investigating more granularly the sensitivity to the smoothing parameters (i.e., including other values than 1, 5, 10 days). A more comprehensive review of options backtests across other asset classes (Rates, Credit, Vols and other Commodities) and the risk-management / portfolio construction of optimal cross-asset volatility portfolios, will be postponed to future studies.

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Allowing the possibility of hedging intra-day

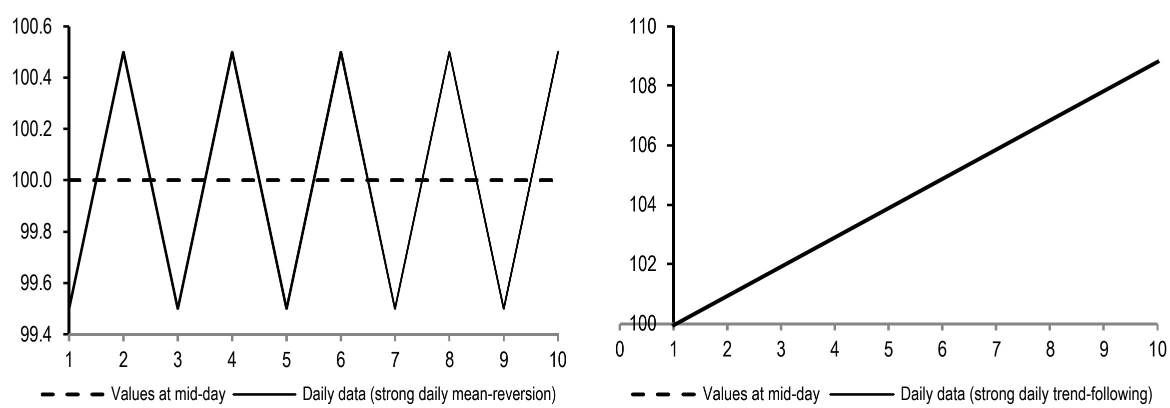
Having investigated earlier in the piece alternative delta hedging strategies, by considering reducing the frequency of the hedging in order to mitigate the impact of mean-reversion on the daily scale, in this section we focus our attention to the intra-day space. We introduce a possible link between the daily mean-reversion properties of an asset and its "intra-day" vol shape, and a pragmatic solution for turning intra-day vol patterns into viable return-enhancing solutions.

A toy model for describing intra-day realised volatilities

So far, we have implicitly assumed all delta transactions to take place at close time on each trading day: this is in fact a standard approach, especially in the real-money community. In fact, nothing prevents from delta hedging at a different time during the day, if that proves convenient from a PnL standpoint. Numerous pieces from academia and practitioners (see Andersen and Bollerslev, 1997) support the finding that intra-day realised vol curves tend to be U-shaped, with mid-day vol being usually lower than end-of-day vol: here, by intra-day vol, we refer to the volatility around successive high-frequency transactions at around a given time during a day. Intra-day properties of financial time series and market micro-structure of high-frequency trades need to be introduced for explaining these effects; trading activity/volumes/liquidity vary during the day and higher volumes at open/close tend to support higher realised vols around these times. Empirical evidences also support significant autocorrelation of returns up to the 5 min scale (see Bouchaud, Potters, 2000).

From the perspective of delta-hedging a short-volatility position, rather than the high-frequency intra-day vol as occurring during the day, what matters the most is the realised vol based on daily returns measured at different times during the day. Finding a time during the day when daily realised vol is lowest would naturally offer the opportunity of monetising a higher vol premium via short-vol trades. In the following, rather than addressing directly the market forces stimulating price action at the high-frequency scale, we will investigate empirical properties of the daily vol sampled at different hours during the day (for simplicity, we will refer to it as intra-day vol). We will show that for most assets delta-hedging at close is not optimal, as lower realised vols can be observed earlier in the day; by introducing an intuitive toy model linking daily and intra-day dynamics, we will be able to explain most of this peculiar intra-day pattern via the *daily* autocorrelation properties of an asset.

Figure 25: Daily trading patterns should naturally impact the daily vol measured at different times during the day



Source: JP Morgan Quantitative and Derivatives Strategy

We start by considering a simple case study (Figure 25), where daily returns follow a strict mean reverting or trend-following pattern over a given timeframe (ten trading days for simplicity). By simply invoking a continuity argument for the observations sampled at different times during the day (without paying attention to the actual intra-day dynamics), we see that daily mean-reversion should be a powerful catalyst for drastically reducing the daily volatility sampled at mid-day (in this limit case, mid-day realised vol would be strictly zero). In case of a persistent trend-following regime, we see a priori no reason why the intra-day vol should be altered (from the right-hand side chart, we can see that in this limit case end-of day and mid-day curves would overlap, leading to the same daily returns/vols). Basically, within this framework, the impact of daily mean-reversion/trend-following properties on the shape of the intra-day curve *is not symmetric*.

In practice, a realistic description of market data will involve some interplay between intra-day and lower frequencies dynamics. We introduce a toy-model description which can allow to assessing quantitatively the effect which seems evident by just looking at the chart. The starting point is a AR(1) model for the daily returns, as supported by the empirical evidences described in the previous section. For the intra-day dynamics, we consider another AR(1) model, with N_{ID} intra-day steps, with parameters chosen as to guarantee the consistency of the end-of day mean values and variances with the ones set by the daily model.

$$r_t^{\,D} = \mu^D + \rho^D \; r_{t-1}^{\,D} + \varepsilon_t^{\,D} \,; \sigma^2(r_t^{\,D}) = \frac{(\sigma^D)^2}{1-(\rho^D)^2};\; E(r_t^{\,D}) = \frac{\mu^D}{1-\rho^D}$$

$$r_i^{\,ID} = \mu^{ID} + \rho^{ID} \; r_{i-1}^{\,ID} + \varepsilon_i^{\,ID};\; \sigma^2(r_t^{\,ID}) = \frac{(\sigma^{ID})^2}{1-(\rho^{ID})^2};\; E(r_t^{\,ID}) = \frac{\mu^{ID}}{1-\rho^{ID}}$$

$$E(r_t^{\,D} \,|\, r_{t-1}^{\,D}) = \mu^D + \rho^D \; r_{t-1}^{\,D} = N_{ID}\frac{\mu^{ID}}{1-\rho^{ID}};\; \sigma^2(r_t^{\,D} \,|\, r_{t-1}^{\,D}) = (\sigma^D)^2 = \frac{N_{ID}}{(1-\rho^{ID})^2}(\sigma^{ID})^2$$

The two latter equations allow setting the values of mean and vol intra-day parameters (for a given value of autocorrelation coefficient for the intraday process). The intra-day mean parameter is adjusted on a daily basis as to match that day's end-of-day conditional expected return. We will investigate later the impact of the intra-day correlation parameters on the intra-day vol and illustrate ways for setting this parameter based on empirical evidence, so for the moment we will treat it as a free parameter.

Amongst the strong assumptions of the toy-model is the fact that there are no overnight effects, i.e. close (at t-1) = open (at t). While certainly this represents an oversimplification of actual financial data, if we take the example of S&P, we can see that this can represent a decent first step approximation. The correlation (since 2008) of close to close vs open to close returns is just +30%, and the number of occurrences when open to close and close to open returns have the same sign is just above 50% (56%), both indicating that the price formation mechanism develops mostly during trading hours rather than overnight. Another assumption behind the toy model regards the importance attributed within the model to the daily frequency on the intra-day dynamics. By assuming complete market efficiency, there should be no reason why the new inflow of information should be conditioned on some sort of lower frequency averaging of the high-frequency trades/prices. In practice, not all market players monitor price activity at the high-frequency scale, either for hedging or for trading, whereas the daily scale remains the benchmark for a multitude of institutional investors. Furthermore, the presence of small if significant serial correlations on the daily scale is well acknowledged, so assessing their impact on the determination of intra-day vol, even without digging into the precise dynamics driving formation of prices at the high-frequency scale, is interesting on its own.

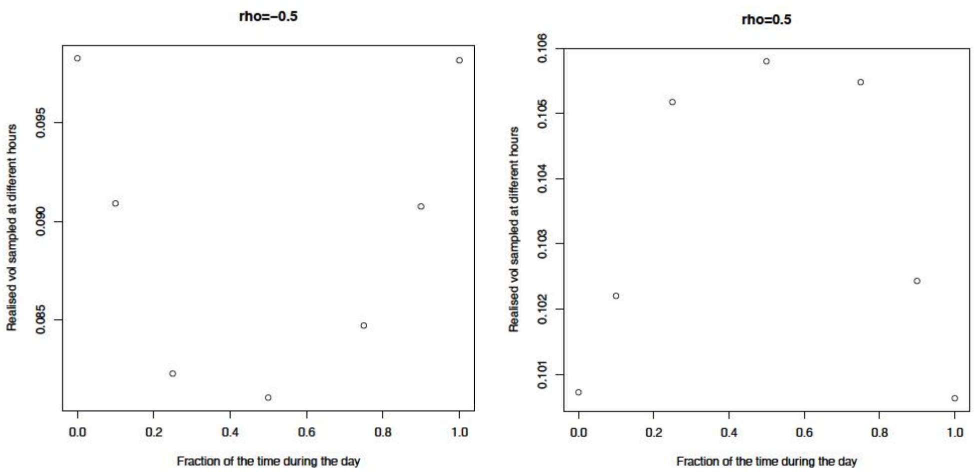
In the two charts below (Figure 26) we display a typical behaviour of intra-day vol patterns by simulating the model above with N_{ID} equally spaced intra-day steps (1000) and number of days (1000): being a MC simulation, it is important that the number of iterations is large enough to ensure that statistical fluctuations don't play a significant role. Y-values in the charts correspond to the average daily vols as observed at a given time during the day, expressed as a percentage, or quantile, or the closing time (i.e., 0% stands for open, 50% for mid-day and 100% for close time). We set daily parameters to be consistent with a 10% daily vol target and 5% average return (both in yearly terms). We consider two case studies of $\rho^D = -0.5$ (daily mean reversion) and $\rho^D = 0.5$ (daily trend-following) and start by considering $\rho^{ID} = 0$.

The comparison of the two charts below explains very clearly how the daily serial correlation properties are expected to impact the intra-day patterns of realised volatility. Mean reversion produces a peculiar bell-shaped curve with a minimum located at around mid-day. For a parameter of $\rho^D = -0.5$ the reduction of mid-day vol vs close-day vol is around 20% compared to the end-of-day vol, which is a large effect. A choice of $\rho^D = 0.5$ (daily trend-following properties) triggers the opposite effect, now with an inverted bell shape: however, here the effect is much smaller in absolute term (about 5%

increase of mid-day vs end-of-day volatilities). So, the main takeaway from this analysis is that daily mean reversion naturally justifies a reduction of the volatility sampled at mid-day compared to end-of day.

Figure 26: Daily mean reversion creates a bell-shaped curve for the vol sampled at different hours in the day

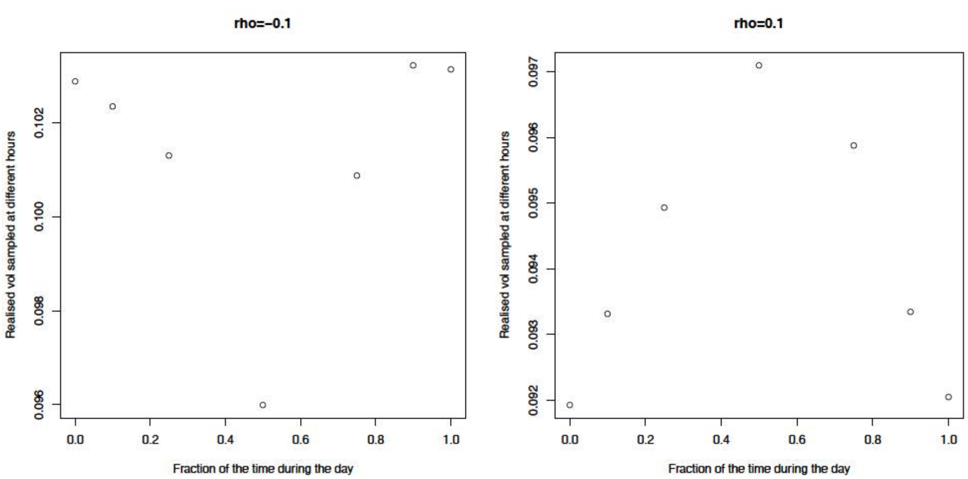
For two different values of the daily AR(1) parameter rho=-0.5, +0.5



Source: JP Morgan Quantitative and Derivatives Strategy

In practice, serial correlation effects tend to be weaker than considered above. We repeat the calculation by considering values of $\rho^D = -0.1$ and $\rho^D = 0.1$ (Figure 27). Now effect is much smaller in absolute value (the change of the mid-day vol is now 7% and 5% respectively), but the effect on the shape of the curves is confirmed. Again, the shape of the vol curve is mostly impacted in presence of daily mean-reversion.

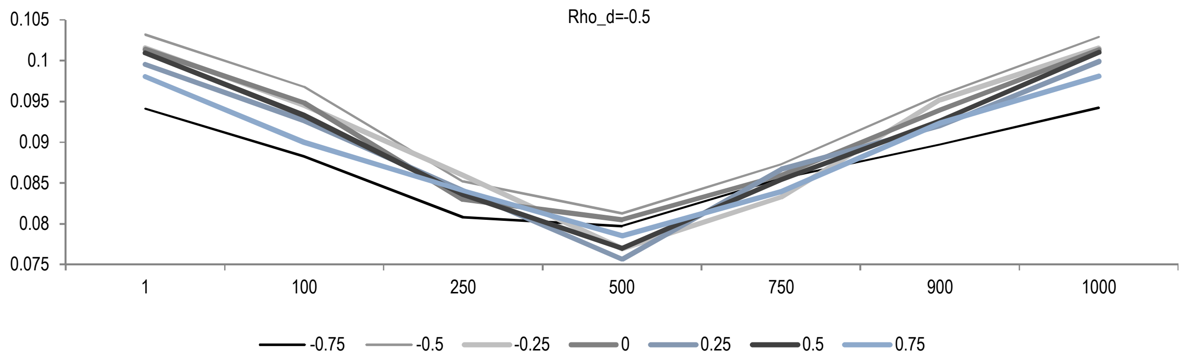
Figure 27: Daily mean reversion creates a bell-shaped curve for the vol sampled at different hours in the day



Source: JP Morgan Quantitative and Derivatives Strategy

The calculations above were carried out in the limit where intra-day correlations were absent. Now, we want to address what the impact of the intra-day correlation parameter might be. In the chart below (Figure 28), we consider a value of $\rho^D = -0.5$, by varying the intra-day correlation parameter from -0.75 to 0.75 (staying away from the sensitive +-1 limit). As we can see, the impact on the intra-day parameter is modest and the shape of the intra-day vol is not qualitatively altered.

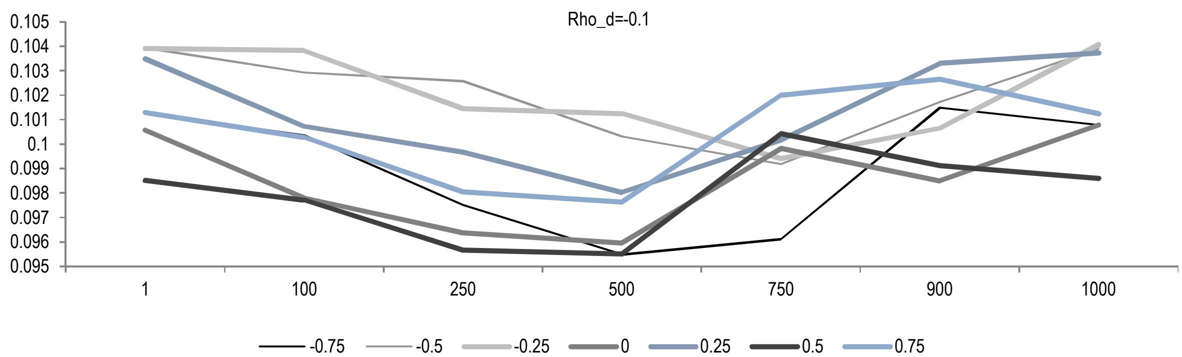
Figure 28: Intra-day serial correlation seem to play a lesser role in driving the shape of the vol curve intra-day



Source: JP Morgan Quantitative and Derivatives Strategy

In the more realistic case of $\rho^D = -0.1$, the picture is more noisy given the weaker daily-mean reversion effect (Figure 29): for getting a smoother picture, a larger sample would be needed for the simulation. But the main point here is that the general picture is unchanged in that daily mean reversion, albeit weak, still leads to a bell-shaped curve for the intra-day vol. We also see that the intra-day correlation parameter does not seem to alter the symmetricity of the shape of the curve.

Figure 29: The effect of intra-day correlations is more relevant when daily mean-reversion is weak



Source: JP Morgan Quantitative and Derivatives Strategy

One can intuitively expect that other measurable quantities could be more sensitive to this parameter, therefore allowing for an estimation based on empirical data. In the following chart (Figure 30), we display the average spread between min and max values of the asset measured during the day (in % vs the opening value) as a function of the intra-day correlation parameter. Two effects emerge clear from the study: 1) the average spread rises with the strength of intra-day mean-reversion (i.e., more negative ρ^{ID}); 2) the average spread is highest for $\rho^D = 0$ and tends to drop as daily serial correlation effects become more relevant (i.e., rise of $|\rho^D|$). The Garman-Klass estimator of volatility (Garman, Klass, 1980), relying on (OPEN, CLOSE, MAX and MIN) for each trading day should therefore allow an estimation of the intra-day correlation parameter based on market data.

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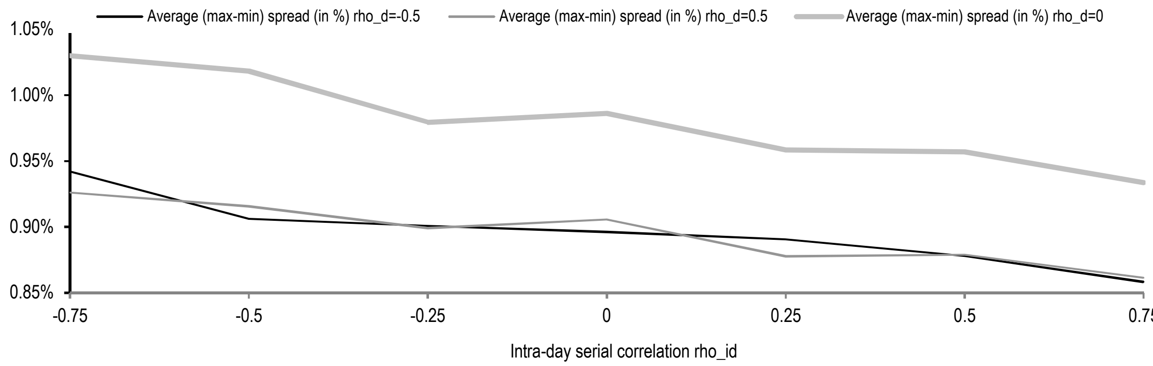
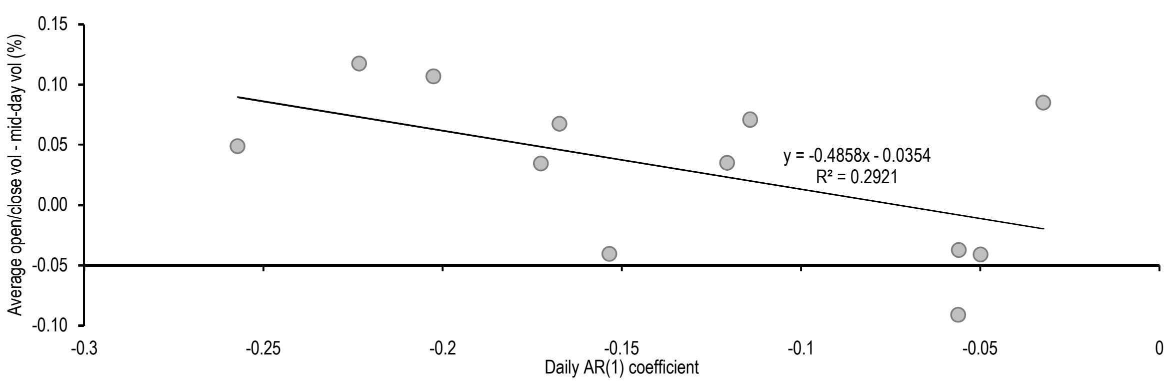
Figure 30: The spread between maximum and minimum values of an asset observed intra-day are sensitive on the intra-day correlation



Source: JP Morgan Quantitative and Derivatives Strategy

The prescriptions above allow an estimation of the two parameters ρ^D, ρ^{ID} via market data. However, the main result of our basic toy-model above is that the daily serial correlation parameter seems to have the bigger impact for driving the shape of the intra-day vol pattern. Figure 9 (see also Table 16 on page 37) recalls that the average value of the ρ^D parameter across asset classes tends to be negative, and more so for Equities if compared to other asset classes. In Figure 31, we consider a set of 12 Equity Indices, with data from February to August 2018, and compare the daily $AR(1)$ coefficient of the time series with the average spread between average of open/close to mid-day realised vols (in percentage terms). Empirical analysis supports that stronger mean-reversion tends to support lower realised vol at mid-day, with a decent R^2 at around 30%, and a slope coefficient in the regression fairly close to -50%. For VIX, VSTOXX (leftmost point) and Nasdaq, assets typically exhibiting strong mean-reversion, the relationship is nicely confirmed: the relationship has held up well for other European Equity Indices (SMI, UKX, FTSE-MIB, DAX) too during this period.

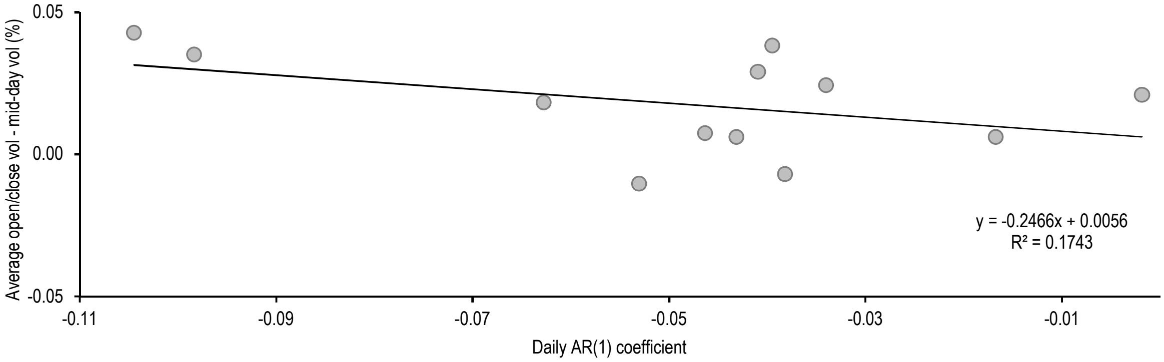
Figure 31: There is a reasonable agreement between daily autoregression and reduction of realized vol at mid-day for Equities (12 Equity Indices, sampled in the period February-August 2018)



Source: JP Morgan Quantitative and Derivatives Strategy

We replicate the exercise for 12 FX/precious metals assets (Figure 32) and find similar results (this time, we use a longer sample of data starting in 2008). We group assets by region and sample intra-day values from 8am to 5pm (London time for Precious metals and EMEA, Tokyo for Asian currencies, New York for CAD and Latam). The regression this time has a lower R^2 of 17%, and the slope of the regression is now ~-25%: basically, the impact is weaker in absolute terms, and the reduction of mid-day vol compared to open/close is less significant than for Equities, still, results appear consistent with our toy model. Of course, we have to keep in mind that the FX market is for the most part OTC, and that currencies can be traded 24-hrs during the week, so that market microstructure is considerably different from Equities: for establishing a comparison, we have focused on the trading hours where liquidity can be assumed to be highest on each asset.

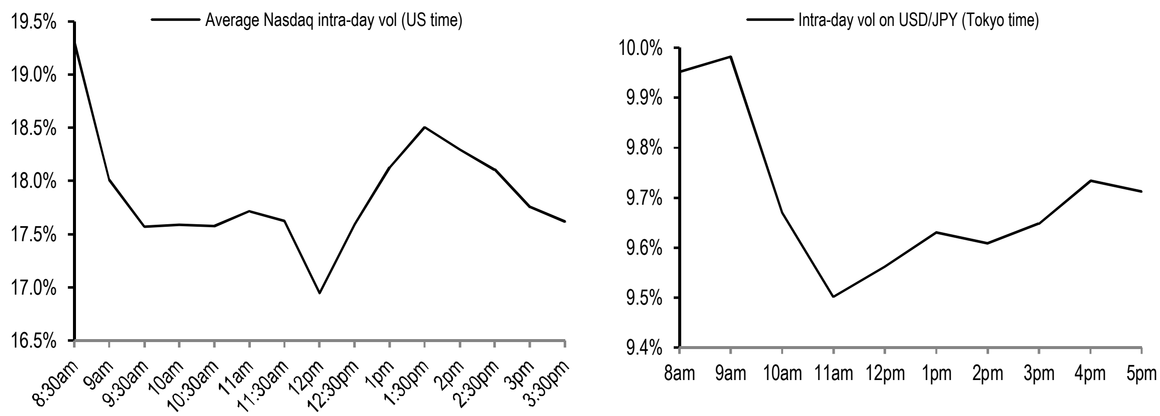
Figure 32: The relationship holds well in FX/Precious metals too, although the effect is weaker in absolute terms



Source: JP Morgan Quantitative and Derivatives Strategy

We report a couple of case studies (Figure 33). For assets, Nasdaq and USD/JPY, both displaying mean-reversion features (see earlier sections): in both cases, the reduction of vol at mid-day is significant.

Figure 33: Average intra-day vols for Nasdaq (Feb-Aug 18) and USD/JPY (Jan 2008-Aug 2018)



Source: JP Morgan Quantitative and Derivatives Strategy. Bloomberg

The effect of daily mean-reversion on the shape of the intra-day vol curves for different asset classes (average behaviour) is reported in Table 16. Being weak mean-reversion a feature which tends to apply on average for most assets, we can expect the effect above to hold on a wide number of cases. We have already seen that mean-reversion properties tend to oscillate over different samples, so that the one reported below is an average behaviour after averaging over many samples.

Table 16: Expected impact due to daily mean reversion on the shape of the intra-day vol curves

Asset class	Average MR indicator	Average AR(1) indicator	Typical MR behaviour	Expected shape of intra-day vol curve
Equities	-4.4e-6	-0.079	Significant	Lower at mid-day
FX	7.3e-8	-0.066	Weak	Slightly lower at mid-day
Commodities	-9.5e-6	-0.068	Significant	Lower at mid-day
Rates	-5.0e-7	-0.057	Weak	Slightly lower at mid-day
Credit	4.2e-5	-0.002	Negligible	Flat – no impact due to mean reversion
Vols	-2.2e-4	-0.065	Significant	Lower at mid-day

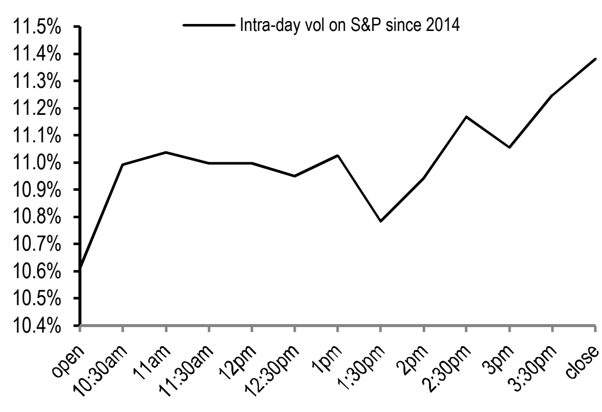
Source: JP Morgan Quantitative and Derivatives Strategy

By recapping our results (Table 16), we have established a theoretical link between the shape of the intra-day vol curve and the daily mean-reversion properties of an asset; daily mean-reversion supports a lower realised vol at mid-day than for open/close. We have seen that the impact of the intra-day serial correlation parameter, if assumed constant during the day, on the intra-day vol is modest. The main assumption behind the model is that information flows “continuously” during open trading hours and that overnight effects are small (close-to-open return is small vs close-to-close return). For the assets trading 24-hours a day (FX, Commodities, some futures etc.), the main assumption is that price action would mostly take place during a certain high-liquidity window during the day, and that little moves would occur outside that window. The relation highlighted by the toy-model is well supported empirically, despite the fact that the latter completely disregards market factors stimulating price activity throughout the day.

A more realistic description would need to account for time-varying intra-day parameters. In a well-known JP Morgan report by Marko Kolanovic, the author finds that an increase of Gamma-hedging activity by market-makers towards the end of the day would push volatility higher (see the report, "Market Impact of Derivatives Hedging", Kolanovic et al., 2008); basically, the author finds that the required hedging activity by market makers over each day's last trading hour is expected to amplify the moves occurred up until that point, stimulating an intraday trend-following pattern and causing a significant increase of vol. As we have seen, our toy-model is consistent with lower vol at mid-day in case of mean reversion, but finds rather symmetric effects when comparing open and close times, as it does not accounts for a variation of time-series parameters during the day. Combining the two effects (market-makers hedging activity and daily mean reversion) is a natural extension of the formalism presented in this piece. Another limitation of our approach is that intra-day microstructure and daily models might in fact share deeper connections, beyond agreeing on just daily average returns and vols (i.e., daily and intra-day autoregressive parameters might not be independent). As pointed out earlier, the neglected overnight effect might also play a role as far as the symmetricity of the intra-day vol curve as a function of the time during the day is concerned. So, we should see our toy model as useful in highlighting an interesting effect but not as the ultimate tool for investigating the matter more in depth.

These results are intellectually appealing but contain practical input for trading decisions too: if we assume to have full flexibility for implementing delta-hedging over the course of the day, doing so when realised vol is lower should systematically prove advantageous to the vol seller. In Figure 34, we focus on S&P, by using intra-day data since August 2014 (a longer sample than for the other Equity indices shown earlier), sampled every 30 minutes. From the chart, we notice a sharp rise of daily volatility as measured towards the close, and a drop of vol at around mid-day is confirmed, in this case particularly at 1:30pm. For this specific example, however, the lowest level of vol is reached at open rather than at mid-day.

Figure 34: S&P intra-day data (since 2012) show a clear increase of vol towards the close



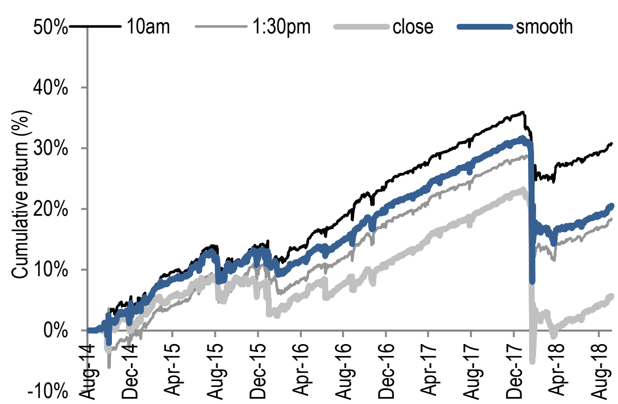
Average values	10am	1:30pm	close
Realized vol	10.6%	10.8%	11.4%
Vol premium	1.4%	1.2%	0.6%

Source: JP Morgan Quantitative and Derivatives Strategy/Bloomberg

The right-hand table in Figure 34 reports the corresponding vol premium, simply defined as difference between implied and realised vols (averaged since 2014). Compared to the value at close (0.6%), higher vol premia could be monetised at open (1.4%) and mid-day (1.2%). If we consider, as an illustrative example, a strategy rolling 12 monthly vol swaps per year, with a Vega notional of $1M each, for an initial capital of $100M, the possibility of locking-in the vol premium at mid-day rather than at close would increase the yearly return from 7.2% to 14.4% (very rough estimate, neglecting trading costs and volatility fluctuations).

In the following, we carry out an empirical analysis on the S&P, with data since August 2014 until end of September 2018 (by using the same trading instruments/settings as in the previous section). A more comprehensive examination of other Equity Indices and asset classes will be postponed to future studies. The findings above suggest at least two ways for exploiting the intra-day pattern in favour of the vol seller. One deals with executing the whole daily delta budget at an earlier time than at close. Another option involves spreading the delta-budget over the day (from open to close), by executing 1/6 of the daily delta budget at each hour from 10am to 3pm, and thus reducing the exposure to end-of-day high realised variance but also the sensitivity to statistical fluctuations. Both cases are reported in Figure 35.

Figure 35: Executing delta-hedging at times earlier than close brings significant value to the vol sellers



Hedging time	Return	Vol	Sharpe	MDD	MDD/Vol
10am	7.3%	14.6%	0.50	-23.6%	-1.62
10.30am	5.1%	14.4%	0.35	-24.2%	-1.68
11am	5.4%	14.1%	0.38	-22.7%	-1.61
11.30am	3.9%	13.6%	0.29	-22.0%	-1.62
12pm	4.2%	13.6%	0.31	-21.8%	-1.60
12.30pm	4.9%	13.6%	0.36	-20.3%	-1.49
13pm	4.9%	13.6%	0.36	-20.7%	-1.52
13.30pm	4.3%	12.9%	0.34	-18.8%	-1.46
14pm	2.8%	13.1%	0.21	-19.8%	-1.51
14.30pm	3.9%	11.8%	0.33	-18.7%	-1.59
15pm	3.6%	11.4%	0.31	-18.5%	-1.62
Close	1.3%	14.6%	0.09	-24.8%	-1.70
10am to 3pm	4.8%	13.2%	0.37	-21.1%	-1.60

Source: JP Morgan Quantitative and Derivatives Strategy

With data from 2014, and taking into account trading costs, delta-hedging at open would deliver by far the best performances, in terms of Sharpe ratio (0.50) and yearly return (7.3%). Trading at 1:30pm would deliver the best maximum drawdown (at -18.8%) and the lowest volatility (12.9%). In both cases, the benchmark (delta-hedging at close) scenario is largely outperformed. The implementation where the delta-budget is smoothed throughout the day also delivers convincing results. The empirical results of this analysis confirm the great added value of allowing delta-hedging during the day and are consistent with the earlier empirical findings concerning intra-day volatilities as outlined in Figure 34.

Combining intra-day and lower frequency hedging

In the previous two sections we have investigated the serial correlations properties of financial assets over a spectrum of frequencies (from intra-day to multi-weeks), assessing in particular the impact on the optimal delta-hedging strategy from the perspective of a vol-seller. We have seen that daily mean reversion should generally support implementing delta-hedging at a lower frequency than daily (or alternatively, reducing the daily fluctuations by averaging the value of the forward as appearing in the BS delta), and that empirical patterns in realised volatilities as measured during the day should support hedging at mid-day rather than at close. We summarize some results across asset classes in Table 17, by referring to the earlier sections investigating either directly options backtests or the statistical properties of the underlying assets.

Table 17: Combining intra-day and lower frequency hedging for short vol strategies

Delta hedging strategy	Equities	FX	Commodities	Rates	Credit	Vols
Low-frequency: value in trading at lower frequencies than daily?	High	Medium	Medium	Medium	Low	Medium
Intra-day: value in trading at other times than at close?	High	Medium	High	Medium	Low	High

Source: JP Morgan Quantitative and Derivatives Strategy

The question of how to combine the two hedging approaches emerges naturally, given that as we have seen the two phenomena are inter-related and, based on our analysis, both could be linked to the daily mean-reversion properties of the underlying assets. In order to define an “optimal” hedging strategy, one could choose to rely on just lower frequency

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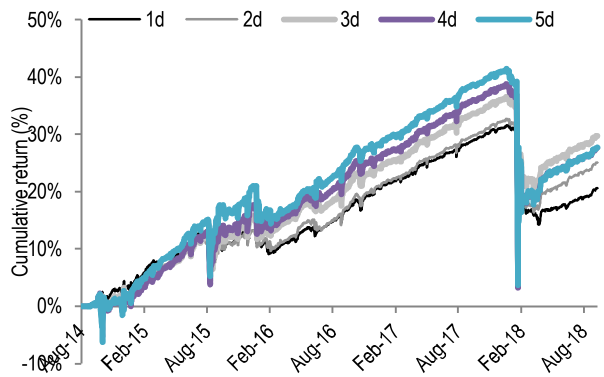
smoothing or intra-day hedging depending on the solution delivering the best performance measures historically (highest average return or Sharpe, biggest reduction of vol or max drawdown). Otherwise, one can combine the two procedures together, although finding the optimal combination of parameters might not be straightforward as we have seen the two effects are linked. In the following, we will test on S&P listed options (with data from August 2014) the possibility of combining the two approaches. As a benchmark for the intra-day hedging procedure we consider the case where delta is executed between 10am to 3pm over the day, as discussed before. We then vary the "smoothing" parameter as applied on the forward entering the BS formula for the delta. Results are collected in Figure 36 below, for delta smoothing parameter up to five days.

Figure 36: Combining intra-day hedging with delta averaging at lower frequencies than daily allows boosting returns further



Smoothing parameter (days)	Return	Vol	Sharpe	MDD	MDD/vol
1	4.8%	13.2%	0.37	-21.1%	-1.60
2	5.9%	16.6%	0.35	-25.2%	-1.51
3	7.0%	19.2%	0.36	-28.1%	-1.46
4	6.5%	21.0%	0.31	-29.9%	-1.42
5	6.5%	22.7%	0.29	-31.4%	-1.38

Source: JP Morgan Quantitative and Derivatives Strategy

As we have seen before, smoothing the actual delta used for hedging (by inserting a moving average of the forward as entering the delta expression) allows improving the daily returns, while also increasing the volatility of the strategy ($3-day$ averaging is the combination with highest average return, close to highest Sharpe). The maximum drawdown rises as well, but declines when scaled by the volatility of the strategy. More generally, having defined a theoretical framework for understanding the interplay of the two effects, one can feel more comfortable in ultimately relying on an optimization procedure for selecting an optimal set of parameters for an optimal hedging strategy, asset by asset.

Appendix

Assessing the impact of serial correlations when trading derivatives

A strong assumption of the Black-Scholes pricing framework is that linear serial correlations are not taken into account, i.e., the latest price is assumed to embed all the relevant information available on the market (the process is Markovian). This description is perfectly consistent with the strictest definition of markets efficiency (see Taylor, Sarno 2003). Stochastic volatility (SV) models, which are normally introduced as natural extensions of Black-Scholes for dealing with volatility smiles and heteroscedasticity (volatility varying with time), introduce time series dynamics for the vol (or variances), which are nonetheless quadratic, and not linear, functions of the returns themselves. Still, the issue of introducing a dependence of future on past linear returns is not assessed in the SV models framework (processes remain Markovian). At the same time, there is an active statistical-arbitrage community that aims at extracting positive returns by investigating time series patterns, and extensive academic material has been written on the topic. Therefore, the question of analysing the impact that serial correlations might have on the delta position of an option emerges naturally.

Without entering into technicalities, a key concept in derivatives pricing deals with risk-neutrality, which entails that the expected return of an asset does not impact the price of derivatives on that asset. An intuitive explanation is that, by working in continuous times, an option can be perfectly replicated via a dynamic strategy involving the underlying asset, regardless of the actual moves of the latter (i.e., one can build a self-financing portfolio). If we assume no arbitrage opportunities, the resulting long derivative / short asset portfolio is risk-free and therefore yields the risk-free rate, thus eliminating the sensitivity on the actual asset's return in the pricing formulas (see Hull, 1988). The other common (but math-heavier) argument involves a change of probability distribution from the real-world to the so-called risk-neutral world (see Baxter, Rennie, 1996), where the expected return of the asset is the risk-free rate and its discounted asset price is a Martingale (in simple terms, a driftless stochastic process): derivatives can then be priced by discounting expected payoffs under this risk-neutral measure. In either case, while the expected return of the underlying asset would have a role in determining its real-world distribution, it does not impact prices of that asset's derivatives.

However, the motivation of our study was very pragmatic, namely to assess whether delta-hedging is a costly strategy from an investor perspective and whether this cost can be reduced (or even turned into a profit) by adjusting the frequency of hedging; for these reasons, we have investigated the impact of both expected returns and serial correlations on the PnL generated by a derivatives strategy by working in the physical (not risk-neutral) world. We leave it to the immense literature on the topic to assess how to properly price derivatives in presence of linear correlations (see Lo, Wang 1995, Wang, Wu, Tzang 2012).

Conventions adopted – Continuous vs discrete times

Throughout this paper, we have worked with models defined on both continuous (e.g., Black-Scholes) and discrete times. In order to avoid any confusion, we recap here the main differences, especially as far as conventions on annualization factors are concerned. In the continuous-time Black-Scholes framework:

$$\frac{dS}{S} = (\mu_{CT}\, dt + \sigma_{CT} dW);\; r_{t,t+dt} = \left(\mu_{CT} - \frac{{\sigma_{CT}}^2}{2}\right) dt + \sigma_{CT} dW;$$

$$E(r_{t,t+dt}) = \left(\mu_{CT} - \frac{{\sigma_{CT}}^2}{2}\right) dt;\; Var(r_{t,t+dt}) = {\sigma_{CT}}^2 dt$$

Parameters μ_{CT}, σ_{CT} refer to the instantaneous, continuous-time scale; quantities like average returns and/or standard deviations depend on the time scale dt considered. In a discrete-time framework, for instance an AR(1) time series model:

$$r_t = a_{DT} + \rho\ r_{t-\Delta t} + \varepsilon_t\,; E(r_t) \equiv \mu_{DT} = \frac{a_{DT}}{1-\rho} = \left(\mu_{CT} - \frac{{\sigma_{CT}}^2}{2}\right)\Delta t;\ Var(r_t) = \frac{{\sigma_\epsilon}^2}{1-\rho^2} = {\sigma_{CT}}^2 \Delta t$$

where Δt is a natural time involved ($\Delta t = 1\ day$ for the daily data investigated throughout this piece). The consistency between the two approaches is reconciled by matching the values for the relevant quantities (e.g., means, variances) as calculated in the two formalisms over that given scale Δt. Given the motivation of investigating actual delta-hedging properties over different frequencies, for most of the paper we have used discrete-time (time-series) conventions: in any case, for facilitating the reading, we have skipped the subscript referring to continuous/discrete times and have left it to the context to assess whether we are referring to one or the other.

Assessing the impact of trading costs on delta-hedging

When delta-hedging at time t, one has to pay trading costs on the difference between the delta at different times. If we introduce a trading cost charge TC (1/2(bid-ask), in basis points), the total cost paid at t is:

$$\Delta Cost_t = TC\ S_t |Delta_t - Delta_{t-1}| \simeq TC\ S_t \Gamma_{t-1}\ |\Delta S_t| \simeq TC\ Dollar\ Gamma_{t-1} |r_t|$$

By introducing the estimate[5] $E|r_t| \simeq \sqrt{E({r_t}^2)} = \sqrt{(\mu^2 + \sigma^2)}$, we get:

$$\mathrm{E}(\Delta Cost_t) = TC\ Dollar\ Gamma_{t-1} \sqrt{(\mu^2 + \sigma^2)}$$

It is interesting to investigate the expected impact of cumulative trading costs for different hedging frequencies. By using the same conventions introduced earlier, when hedging over M trading days we get:

$$E(Cost) = TC\ \overline{Dollar\ Gamma_M}\ \sqrt{({\mu^2}_M + {\sigma^2}_M)}\ T/M$$

If now, for the sake of simplicity, we neglect the impact of trends and of serial correlations, and just approximate $({\mu^2}_M + {\sigma^2}_M) \simeq {\sigma^2}_M \simeq M\,\sigma^2$ ($\sigma_1 \equiv \sigma$), we get for the expected cumulative cost:

$$E(Cost) = TC\ \overline{Dollar\ Gamma_M}\ \sigma\, T/\sqrt{M}$$

This simple analysis shows that the expected cumulative impact of trading costs decreases when hedging at lower frequencies (higher M). By incorporating costs, the expected ${PnL_{DH}}^M$ generated when delta-hedging a short-volatility position over the $M -$ day scale becomes:

$$E({PnL_{DH}}^M) = \overline{Dollar\ Delta_M}\ \mu_M\, T/M + \overline{Dollar\ Gamma_M}\ ({\sigma_M}^2 {ACF_1}^M + {\mu_M}^2 - TC\sqrt{{\mu^2}_M + {\sigma^2}_M})T/M$$

[5] More precisely, $E|r_t| \leq \sqrt{E({r_t}^2)}$ holds due to Jensen's inequality. For simplicity, we neglect such convexity adjustments.

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