Derivatives 101

03 July 2008

Contents

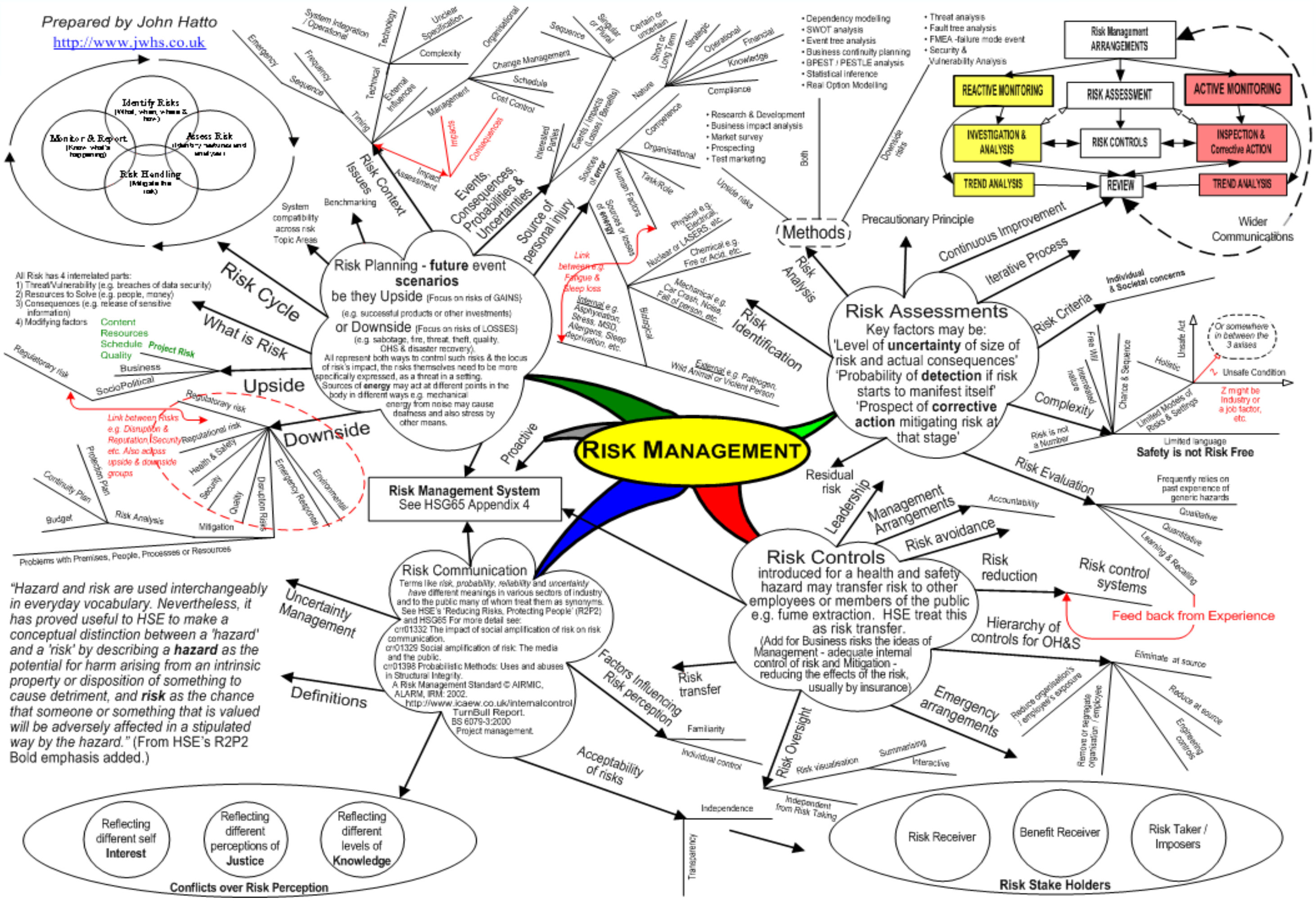
SECTION 1

Applications of derivatives :
the liability management context

"The policy of being too cautious is the greatest risk of all."

Jawaharlal Nehru - Indian politician (1889 - 1964)

Risk management ?



Passive versus active liability management

PASSIVE MANAGEMENT (UNTIL THE 90'S)

One takes the rates as they are – Given a 3% or 6% 10 years rate, the corporate used to support this cost for a very long period

Fixed or floating rate – It was of course possible to manage the ratio between fixed and floating rate debt taking more or less short term loan (Euribor + spread) in the portfolio. Nevertheless it was often impossible to get a floating rate loan on 10 years before the interest swaps developed.

Debt profile – If the capital markets insisted to lend money on the long term (10 years), the corporate had no choice but submit to it and accept on one side to pay a high coupon (in case of a period of high economic activity) and on the other side to increase its refinancing risk if the cashflow schedule did not allow a good smoothing between years.

Debt characteristics determine interest rate characteristics.

Passive versus active liability management

ACTIVE MANAGEMENT (SINCE THE 90'S)

New financial tools – The emergence of new financial tools such as interest rate swaps has allowed the development of a more active management of liabilities.

The separation of decisions – While before the fixing of the rates was made at the same time than the issue of the obligatory loan, those two decisions can now be taken separately. The right time for the fixing of the rates is rarely the right one for the capital markets.

The separation of the risks – The interest rate risk can now be managed separately from its underlying (public loan, private investment). A fixed rate loan can be turn into a floating rate one (and conversely) and this in a confidential way (OTC Deal).

Reduction of the average cost/ Rating – The active management allows very often to reduce the average cost of the debt. Moreover it is very appreciated by the rating agencies.

The new financial products allow to widen the range of tools available for debt manager.
An active management is now possible.

Mapping of active liability management

Debt Characteristics ⟷ Derivatives ⟷ **Interest Rate Characteristics**

Liquidity requirements
Debt Profile : short-term versus long-term cashflows

Market access and investor's preferences
Benchmarks, Credit, Fixed versus Floating

Nature of assets
Financial Ratios

Business environment
Overall Budget and flexibility of budget

Accounting environment
IAS, Hedge Accounting

Corporate Environment
Management, Analysts

Cost of debt and its sensitivity
Cost-At-Risk

Sensitivity to change in market value of debt
Value-At-Risk

Nature of revenues
Income-At-Risk

Business Opportunities and targets

Market Opportunities and views

Active Liability Management allows to differentiate traditional debt characteristics and interest rate characteristics : this flexibility brings more market opportunities

Interest rate risk

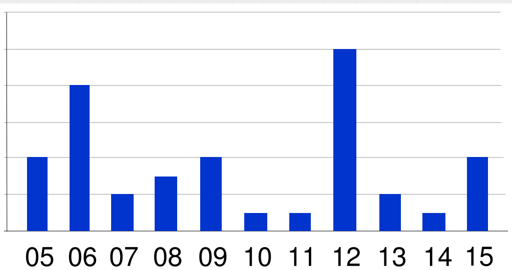
DEFINITION

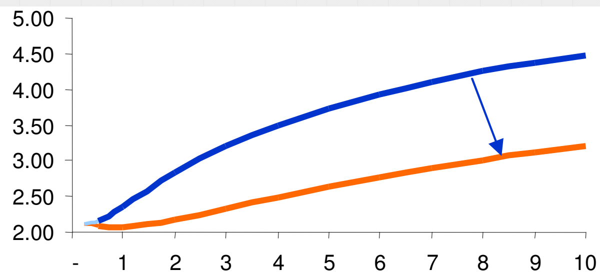
The risk can be defined as the uncertainty concerning the **direction and the extent of the future moves of the interest rates.**

The interest rate risk can be defined as the risk related to the fluctuations of both the value and the cost of the debt following the fluctuations of the interest rates in the capital markets.

- If the Euribor rates rise from 2.00% to 3.00%, the cost of a floating debt increases by 50%
- If the yield curve decreases by 1.00%, the present value of a 10y debt increases by EUR 80,000,000 (EUR 1,000,000,000 debt x 8 duration x 1%)

Debt Profile → **change in interest rate curve** → **impact on debt**





Cost of Debt:
unchanged because 100% fixed rate

Market Value of Debt:
+10% or EUR 80,000,000 in one year

2 aspects of the interest rate risk: impact on the cost of the debt and on its market value

Present value

CONCEPT

If one asks somebody if he would prefer to have a amount of EUR 1,000,000 today or in one year, the answer will be straightforward.

The money you have today has a higher value than the money you will have tomorrow, because this money can be deposited between today and tomorrow and thus brings in interests.

The today's value of money is also called present value.

The **present value is synonymous of market value** when the rates used for discounting are the rates observed on the capital markets for similar maturities.

By this way, the rates of the capital markets can be used to compute the present value of any series of cashflows maturing in the future: Nevertheless, it is important to respect the congruency of the length of the interest rates, to use a market rate, and to respect the level of risk.

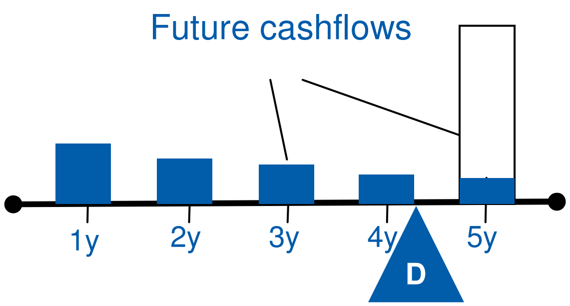
Today $>$ **Worth more than**

Tomorrow

The present value allows to compare cashflows which are separated in time by bringing them back on a common basis.

Duration

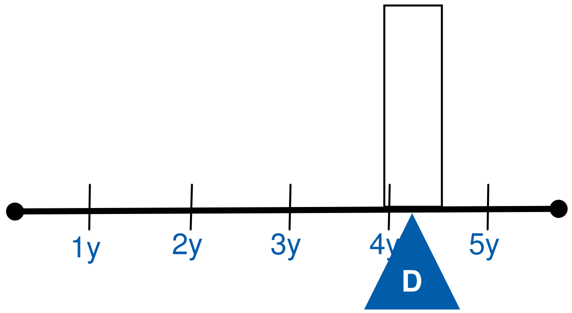
FIRST MEANING : AVERAGE MATURITY OF CASHFLOWS



The duration as a "balance"

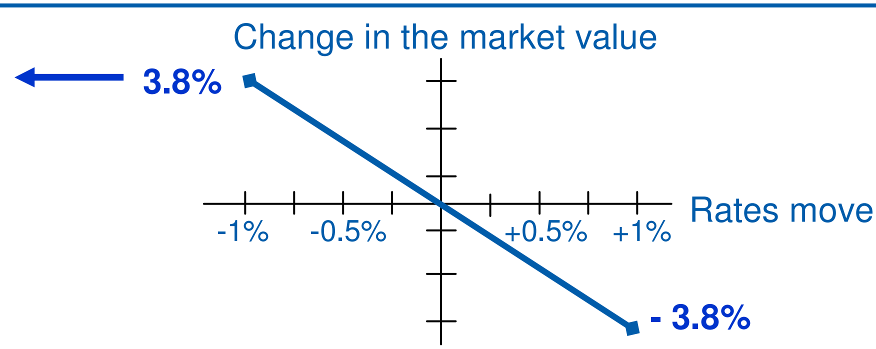
- Redemption (Notional)
- Payments of interests (=present value, discounted)

D Duration



SECOND MEANING : MEASURE OF SENSITIVITY TO RATES

Duration in % and no longer in number of years



Duration = sensitivity to the parallel moves of the interest rates (in %)

Market value of debt and value at risk

CONCEPT : WHICH INCREASE IN THE MARKET VALUE OF DEBT CAN I BEAR ?

The value at risk is one of the principal values in risk management.

It is the **maximum value that the debt can take within a specified time frame and confidence Interval.**

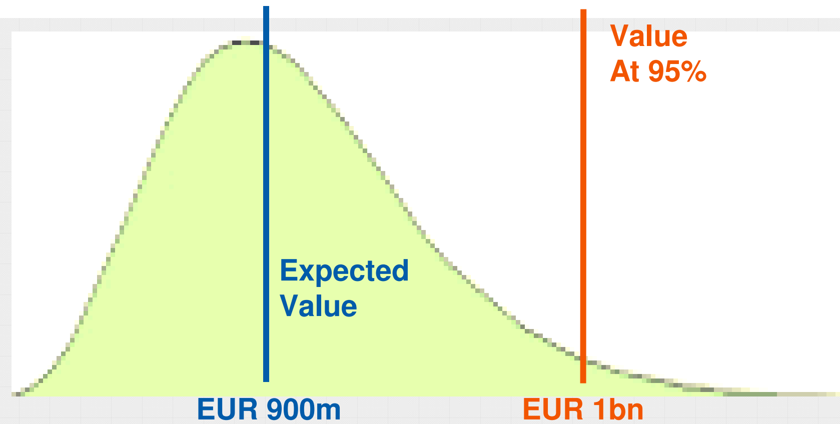
The **value at risk of a fixed rate loan is high** because its market value depends on the level of rates

The **value at risk of a floating loan is low** because its market value does not depend on the level of rates

Example:

In one year and in 95% of the cases, the market value of a loan will not exceed EUR 1bn.

This means that if a company plans to buy back this loan in one year time, there is less than 5% probability that EUR 1bn will not be sufficient.



The more volatile the interest rates are and the higher the proportion of fixed rate debt is, the higher the value at risk is.

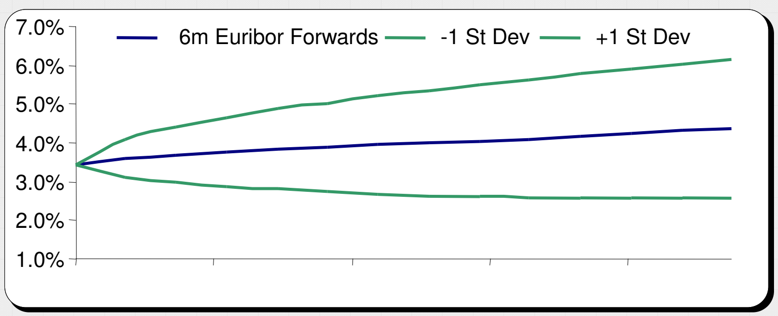
Cost of debt and cost at risk

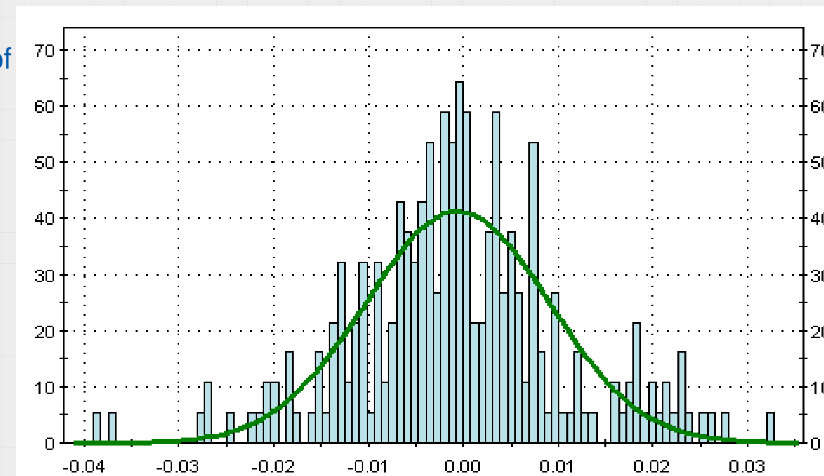
CONCEPT : WHICH INCREASE IN THE COST OF DEBT CAN I BEAR ?

By analogy with the value at risk, it is possible to estimate **future variations of interest costs** - based on implied or historical volatility of rates.

Example:

In 68% of the cases the 6 month Euribor will be between 3.00% and 4.40% in Nov 07 (forward 3.70%) according to a normal distribution of rates.





The more volatile the interest rates are and the higher the proportion of floating rate debt is, the higher the cost at risk is.

Value at risk and cost at risk

THE RIGHT BALANCE ?

The distribution of the debt in fixed rate and floating rate has an important impact on the two measures of risk the VaR and the CaR.

One of the objective of risk management is to find the **right balance between the two**, given the “environment” of the debt

		VAR	CAR
Fixed Debt proportion	100%	Very high	Low
	75%	Quite high	Quite Low
	50%	Low	High
	25%	Very low	Very High

The level of risk of an interest rate linked strategy has two dimensions

Proposed pillars of liability management

1 - MINIMISE THE “BETS” ON THE MARKETS

Information flow is difficult to handle : too many markets and too many parameters change too quickly

Ability to react and change positions in very fast moving environments is often limited

Technology is key but expensive

Prop-trading, even for investment banks, is a high risk business

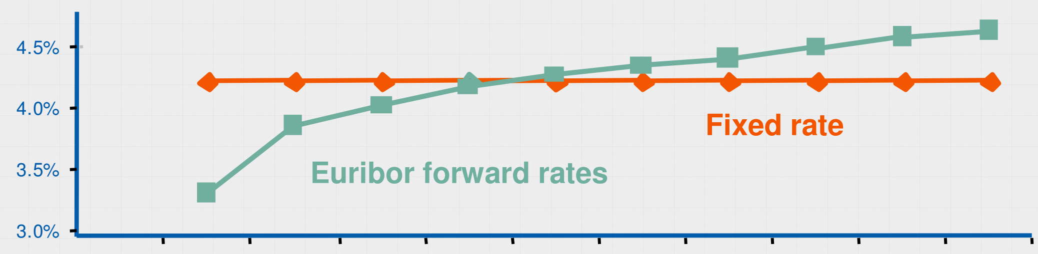
Too much tactics destroy strategies

Minimise the “bets” on the markets

Proposed pillars of liability management

2 - AVOID NEGATIVE CARRY

Negotiating down by a few basis points the rate paid on a loan requires lots of efforts : is it really worth adding on top up to several hundred basis points for insurance against future rates rise ?



Alternatives to high fixed rate by two families of strategy : one based on floating rates, and one based on a fixed rate subsidised by a structured derivatives.

Negative carry is both a certain cost and an opportunity cost !

Avoid negative carry

Proposed pillars of liability management

3 - EXPLOIT MARKET "INEFFICIENCIES"

While past performance is not a guarantee for future performance, there are certain themes that offer strong opportunities.

Theme 1 : The upward interest rate curves imply that rates will rise !

Theme 2 : The slope of interest rate curves imply that curve will flatten !

Theme 3 : Differences between interest rate curves lack of substance …

Theme 4 : Combining previous themes offer even more opportunities !

Even if there is no "free lunch", some meals look really good for their price …

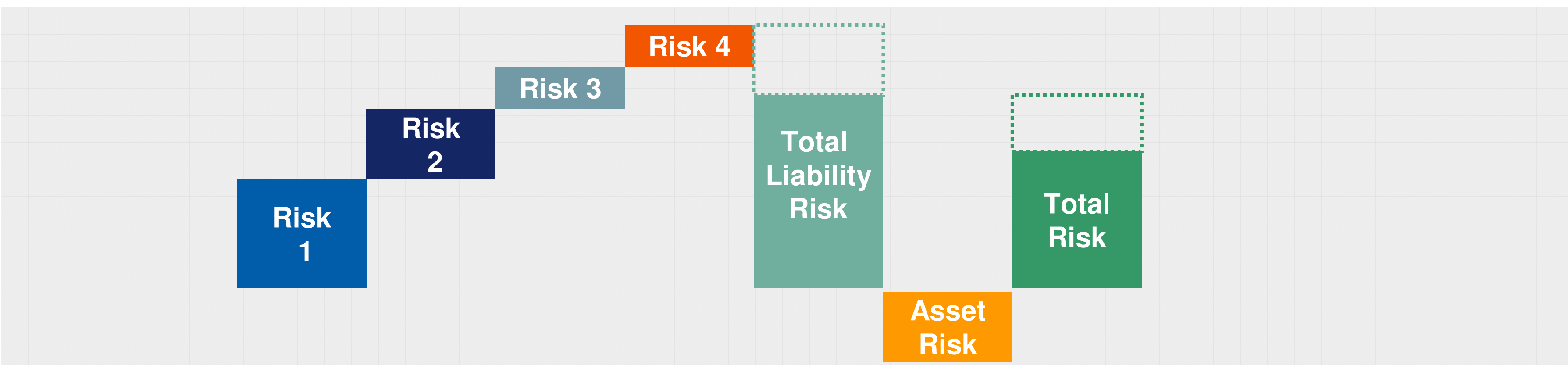
Exploit market "inefficiencies"

Proposed pillars of liability management

4 - DIVERSIFY RISKS

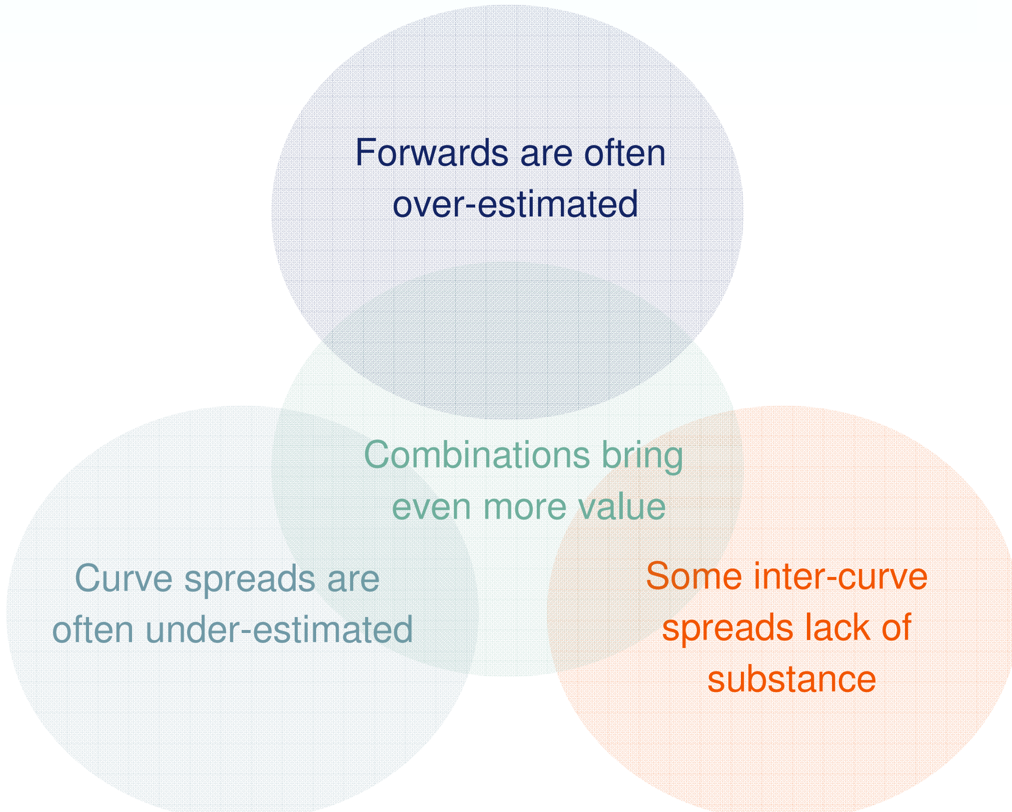
Diversifying risks, i.e. exploiting different themes at the same time, allows for a global lower risk (cost-at-risk or value-at-risk) while still offering substantial benefits.

Risk diversification should also be seen in the broader Asset and Liability environment where some risks may already be over-weighted or under-weighted in the assets.



Diversify risks

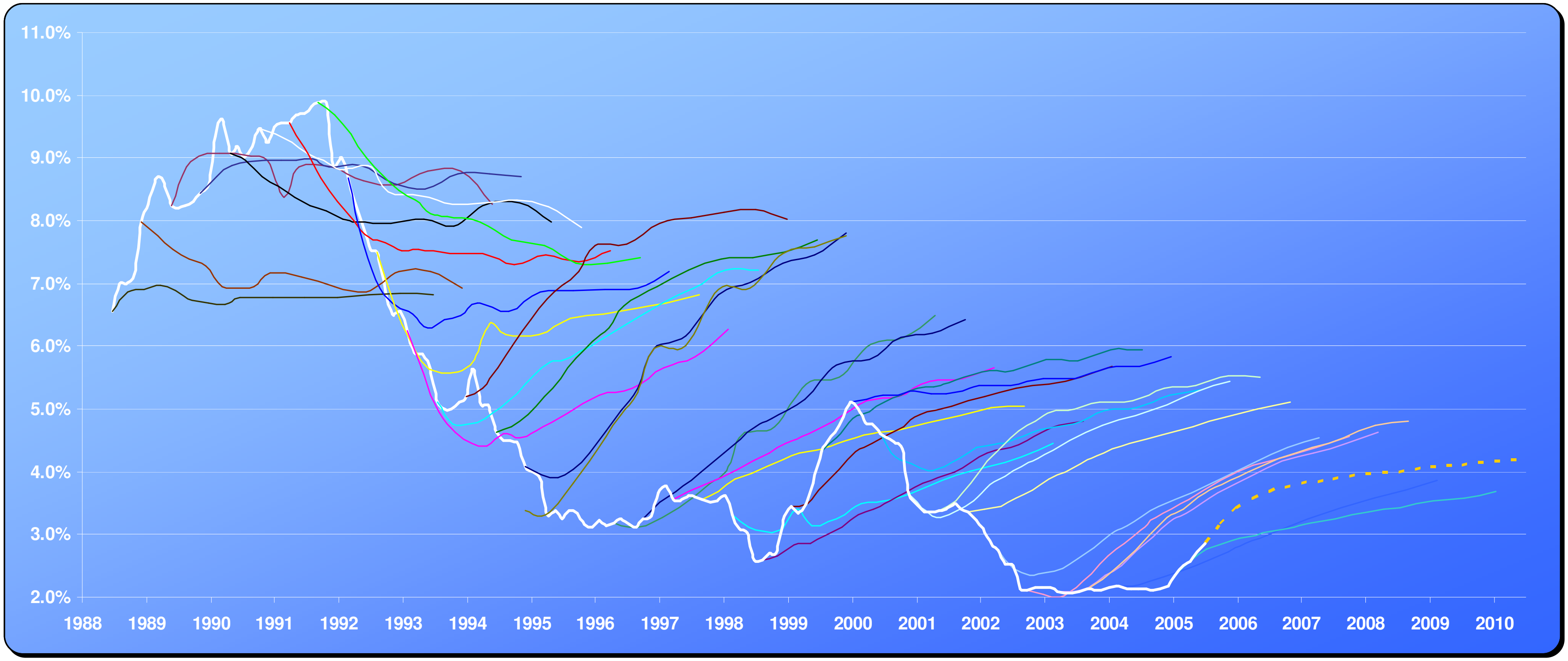
The 4 themes



4 Themes observed during the past

3m euribor forwards

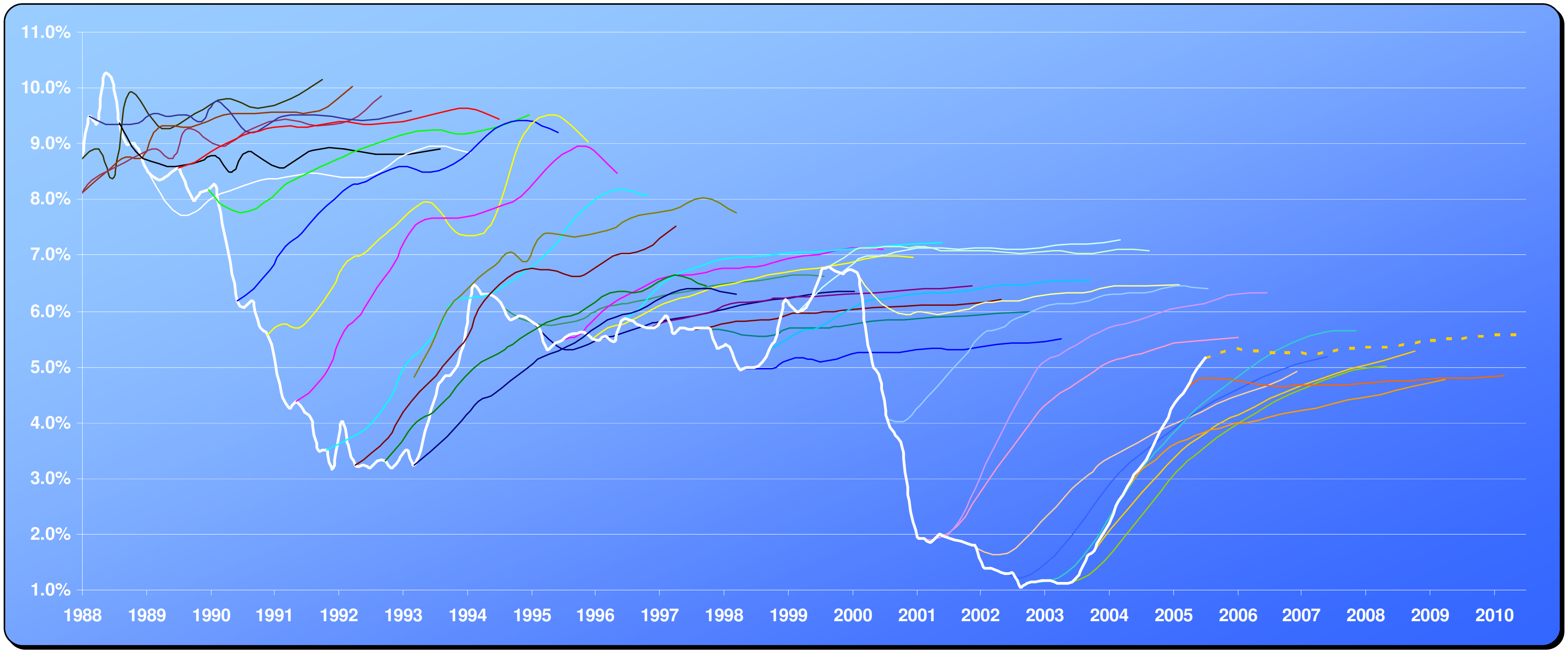
Theme 1
Forwards



Historically, the EUR market has over-estimated the forwards...
but "history" refers to a downward trend

Theme 1
Forwards

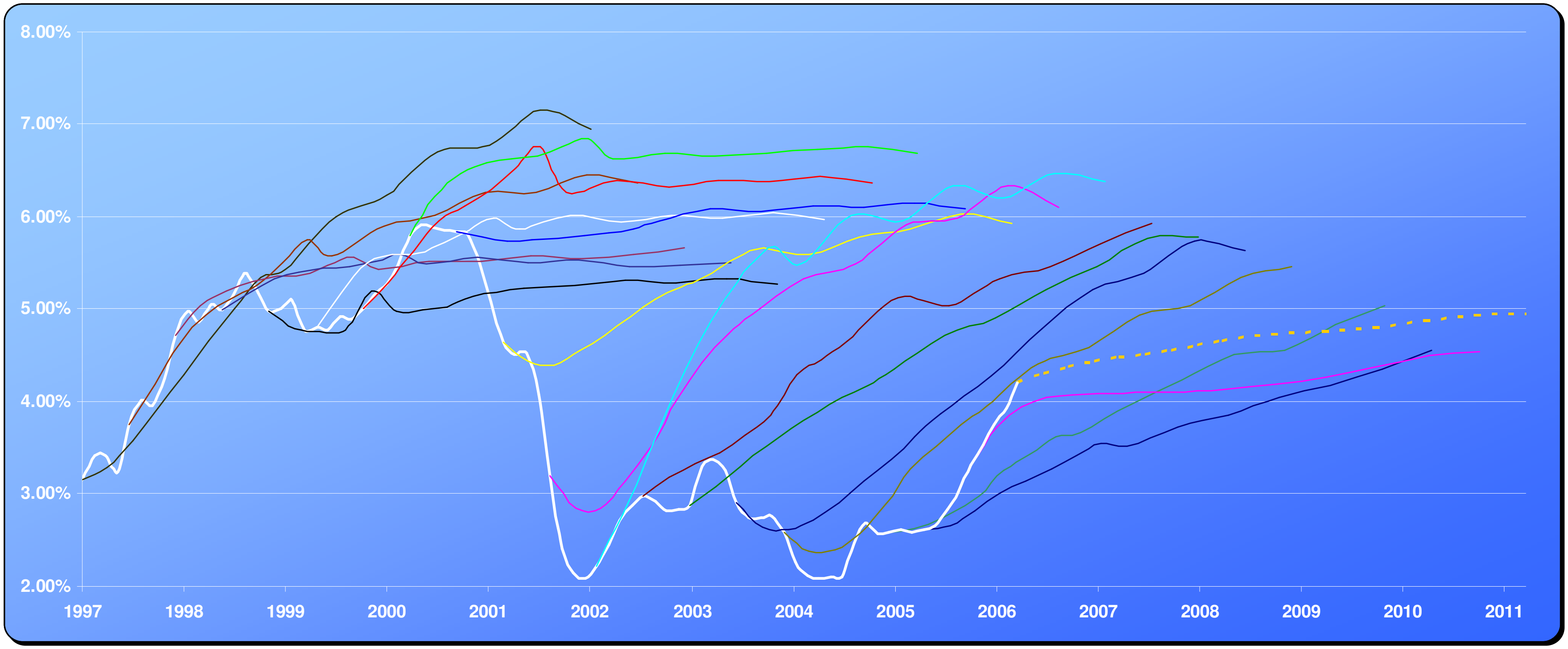
3m usd libor forwards



USD similar to EUR but with a slightly better estimation in upward trends

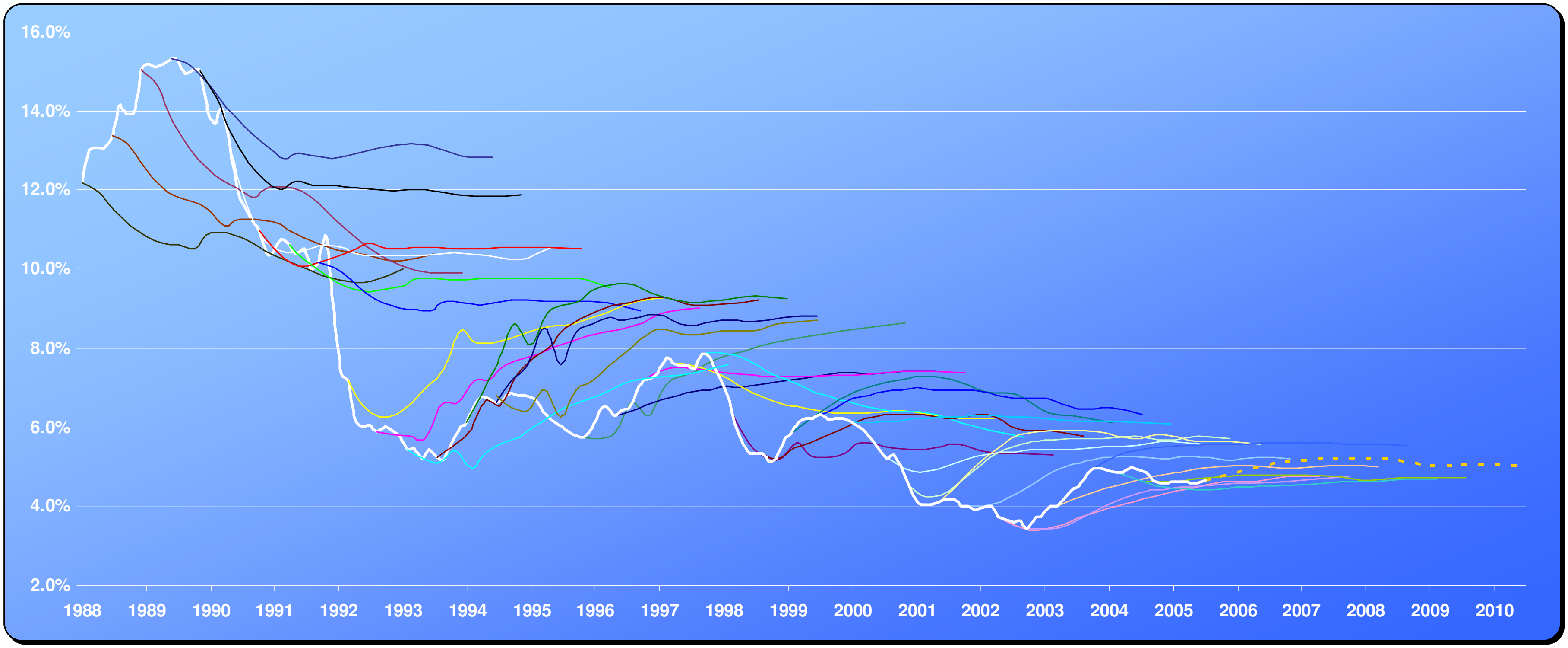
3m cad libor forwards

Theme 1
Forwards



When trend is upward, CAD forwards tend to be more accurate than during downward trends…

3m gbp libor forwards



In GBP, the flatter structure of forwards means that the over-estimation of forwards is of a lower extent but that in a upwards trend the market is less accurate

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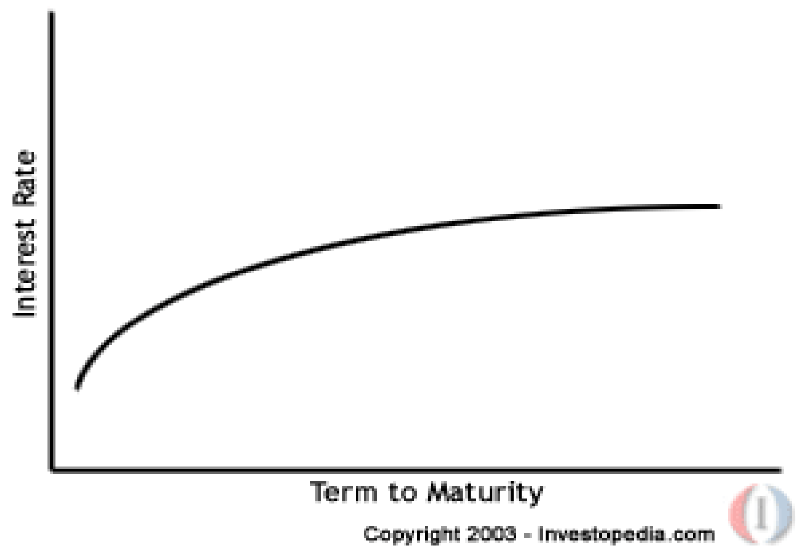
Theme 1
Forwards

Curves assume rates will rise !



Given the observed nature of interest rate curves, future short-term rates are assumed to be higher

Range accruals

Theme 1
Forwards

Range Accrual Example

Liability Manager receives Euribor x (n/N)

Liability Manager pays 75% x Euribor

n is the number of days in each period when the Euribor is between a lower and an upper barrier

N is the total number of days in each period

Power Range Accrual Example

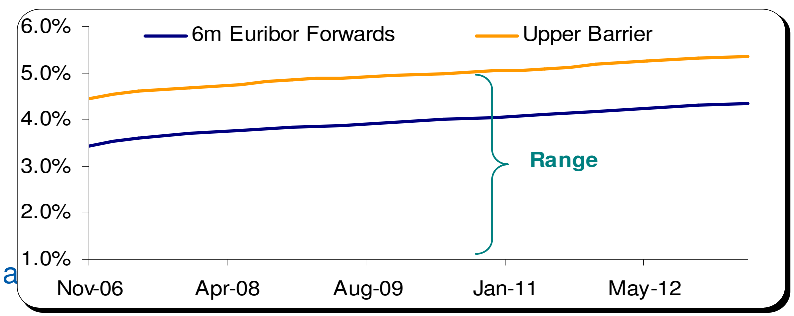
Liability Manager receives G x Euribor

Liability Manager pays 65% x Euribor

G is n/N initially and then is equal to previous G x (n/N)

n is the number of days in each period when the Euribor is between a lower a

N is the total number of days in each period



The upper barrier can be set at the forward rates plus one percent

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Theme 1
Forwards

Backtesting of 5y structures in eur (91-95)

Date	SWAP	COLLAR	CALLABLE	RA	PRA	Euribor Trend	
01-Jan-91	-12.26%	-10.74%	-10.25%	13.98%	19.77%	▼	-2.66%
01-Jul-91	-12.94%	-11.12%	-11.31%	11.00%	11.42%	▼	-3.10%
01-Jan-92	-15.79%	-13.03%	-14.15%	12.88%	17.80%	▼	-4.02%
01-Jul-92	-18.84%	-15.85%	-17.27%	12.62%	17.11%	▼	-4.86%
01-Jan-93	-13.18%	-10.82%	-11.69%	10.37%	14.31%	▼	-3.81%
01-Jul-93	-11.50%	-9.56%	-10.54%	8.26%	11.75%	▼	-2.86%
03-Jan-94	-6.15%	-5.50%	-14.25%	6.18%	4.77%	▼	-1.77%
01-Jul-94	-15.71%	-15.27%	-12.99%	6.43%	9.42%	▼	-1.33%
02-Jan-95	-20.40%	-19.61%	-17.41%	6.59%	9.47%	▼	-2.00%
03-Jul-95	-15.20%	-14.73%	-12.27%	4.54%	7.19%	▼	-1.11%

In declining periods, vanilla structures under-perform

BNP PARIBAS

Backtesting of 5y structures in eur (96-01)

Date	SWAP	COLLAR	CALLABLE	RA	PRA	Euribor Trend	
01-Jan-96	-8.21%	-7.97%	-5.71%	3.49%	6.09%	—	-0.05%
01-Jul-96	-10.46%	-10.32%	-7.40%	3.69%	6.37%	—	0.27%
01-Jan-97	-6.95%	-7.03%	-4.28%	4.40%	7.16%	▲	0.58%
01-Jul-97	-5.16%	-5.23%	-2.87%	4.61%	7.42%	▲	0.60%
01-Jan-98	-6.96%	-5.92%	-4.92%	4.83%	7.60%	—	-0.10%
01-Jul-98	-5.56%	-4.84%	-3.85%	6.37%	9.23%	—	-0.16%
01-Jan-99	-1.09%	-1.47%	-5.43%	2.25%	-4.51%	—	0.22%
01-Jul-99	-4.31%	-5.12%	-7.69%	5.17%	7.79%	▲	0.58%
03-Jan-00	-9.79%	-9.58%	-7.68%	4.49%	6.96%	—	-0.32%
03-Jul-00	-13.29%	-11.77%	-11.53%	6.52%	9.06%	▼	-1.84%
01-Jan-01	-11.52%	-9.90%	-10.04%	5.33%	7.60%	▼	-2.08%

Even in rising periods vanilla structures under-perform !

Theme 1
Forwards

Backtesting of 5y structures in usd (96 -01)

Date	SWAP	COLLAR	CALLABLE	RA	PRA	Euribor Trend	
01-Jan-96	0.96%	-0.31%	-0.30%	14.33%	6.09%	—	0.39%
01-Jul-96	-5.38%	-4.16%	-3.23%	17.83%	6.37%	—	-0.03%
01-Jan-97	-5.86%	-4.86%	-3.84%	18.15%	7.16%	—	-0.30%
01-Jul-97	-8.38%	-7.12%	-6.41%	18.77%	7.42%	▼	-0.96%
01-Jan-98	-8.01%	-7.26%	-6.35%	16.72%	7.60%	▼	-1.34%
01-Jul-98	-9.40%	-8.80%	-9.60%	17.12%	9.23%	▼	-1.73%
01-Jan-99	-8.31%	-8.37%	-14.09%	-10.22%	-4.51%	▼	-1.47%
01-Jul-99	-15.83%	-15.27%	-17.01%	13.75%	7.79%	▼	-2.33%
03-Jan-00	-21.31%	-20.02%	-19.20%	11.42%	6.96%	▼	-3.28%
03-Jul-00	-23.17%	-21.22%	-21.29%	11.32%	9.06%	▼	-4.35%
01-Jan-01	-17.72%	-15.61%	-16.13%	8.65%	7.60%	▼	-3.72%

Same observations than in EUR

Theme 2
Curve

10y – 2y eur swap spread forwards

Historically, the EUR market has under-estimated the forwards...

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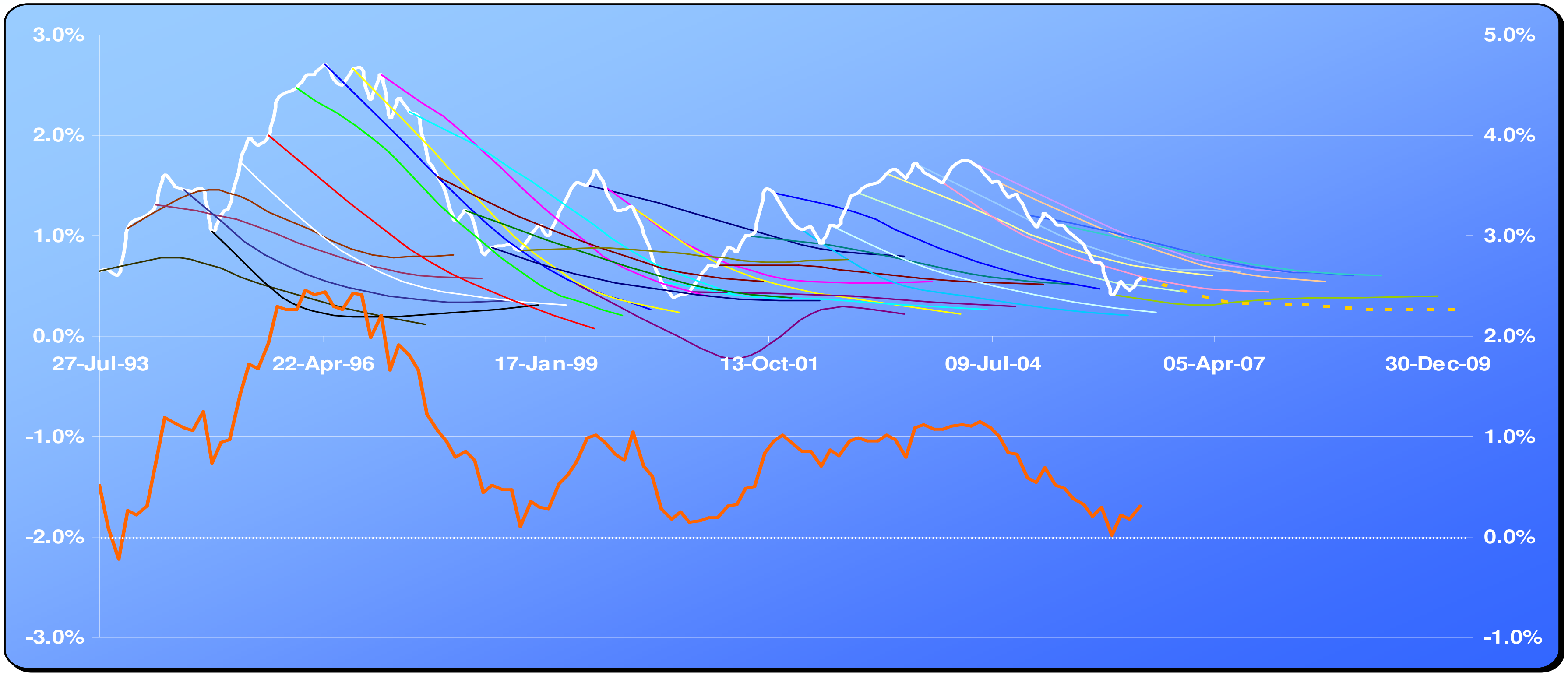
Theme 2
Curve

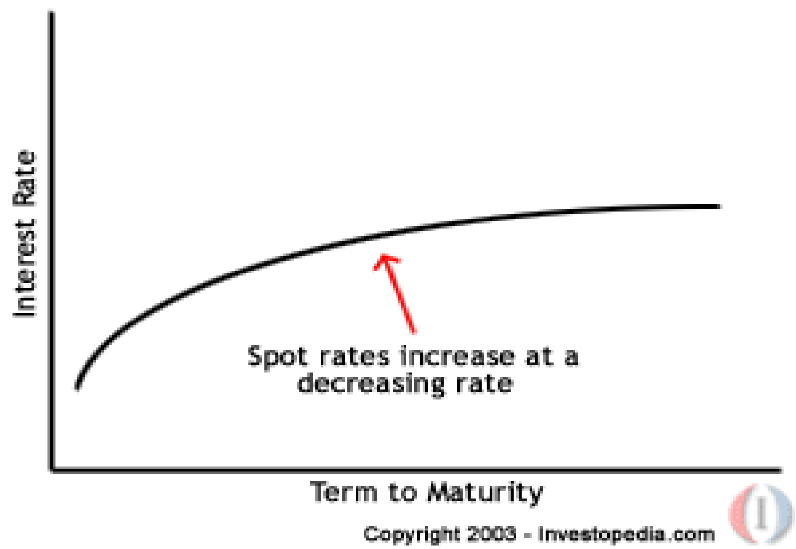
10y – 2y eur swap spread forwards



Forward spread curve is mean-reverting to a level that has never been reached historically !

Theme 2
Curve

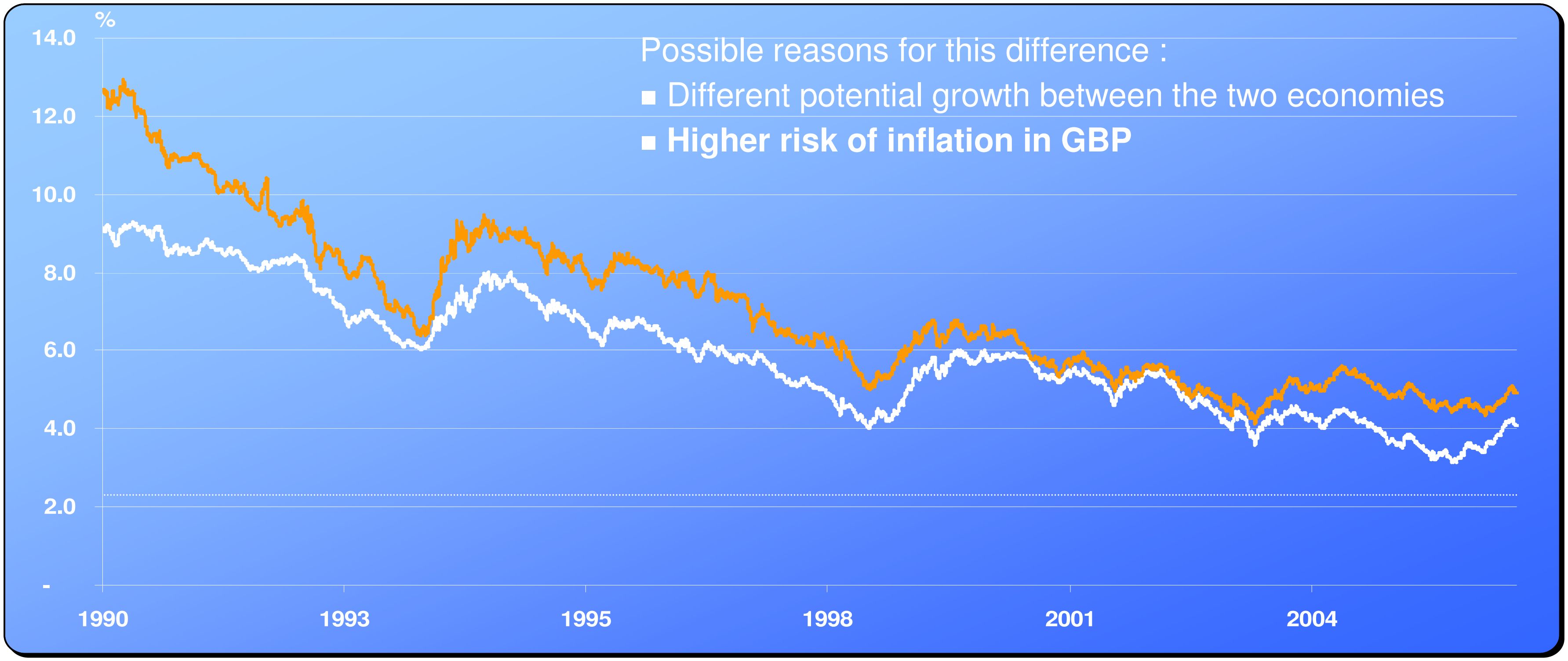
Curves assume spreads will decrease !



Given the observed nature of interest rate curves,
future curves are assumed to be flatter

Historical eur & gbp 10y swap rates

Theme 3
Inter-curves



Historically, the GBP rates have always been above EUR rates

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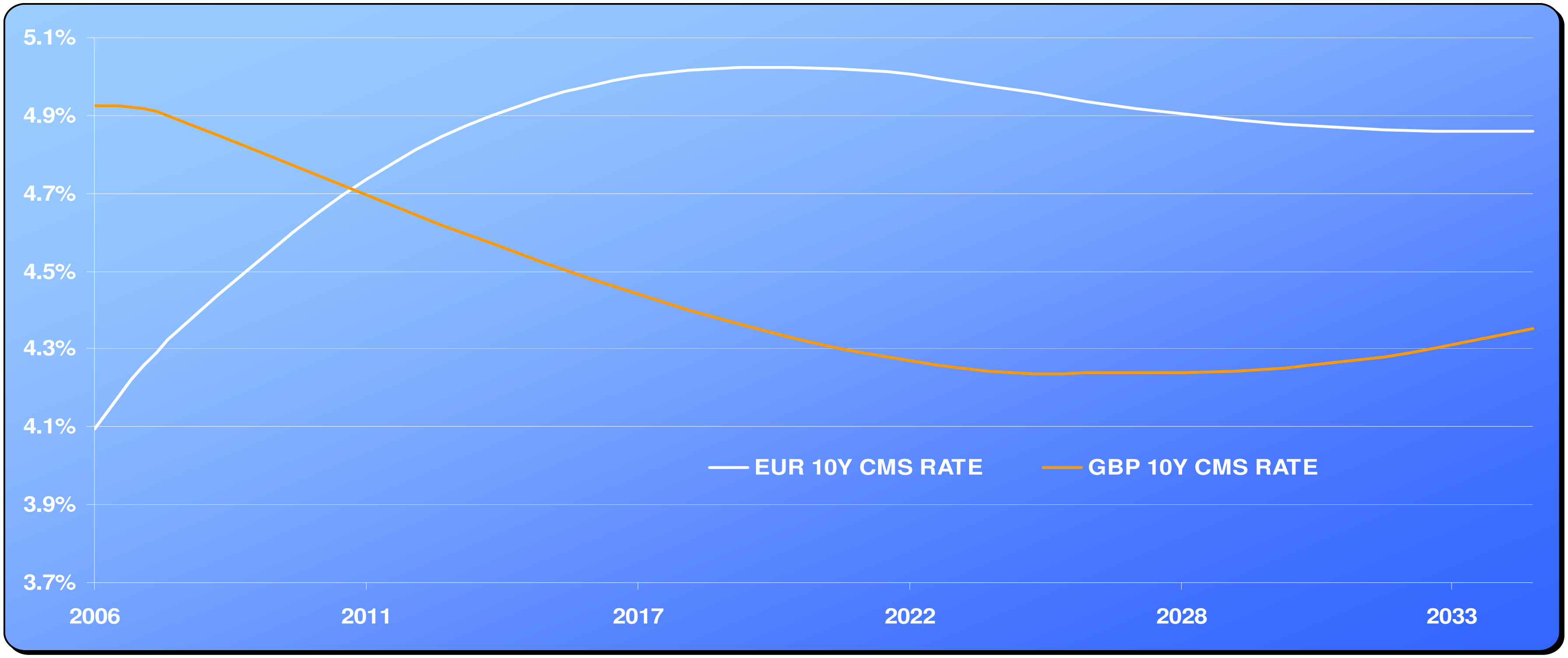
Theme 3
Inter-curves

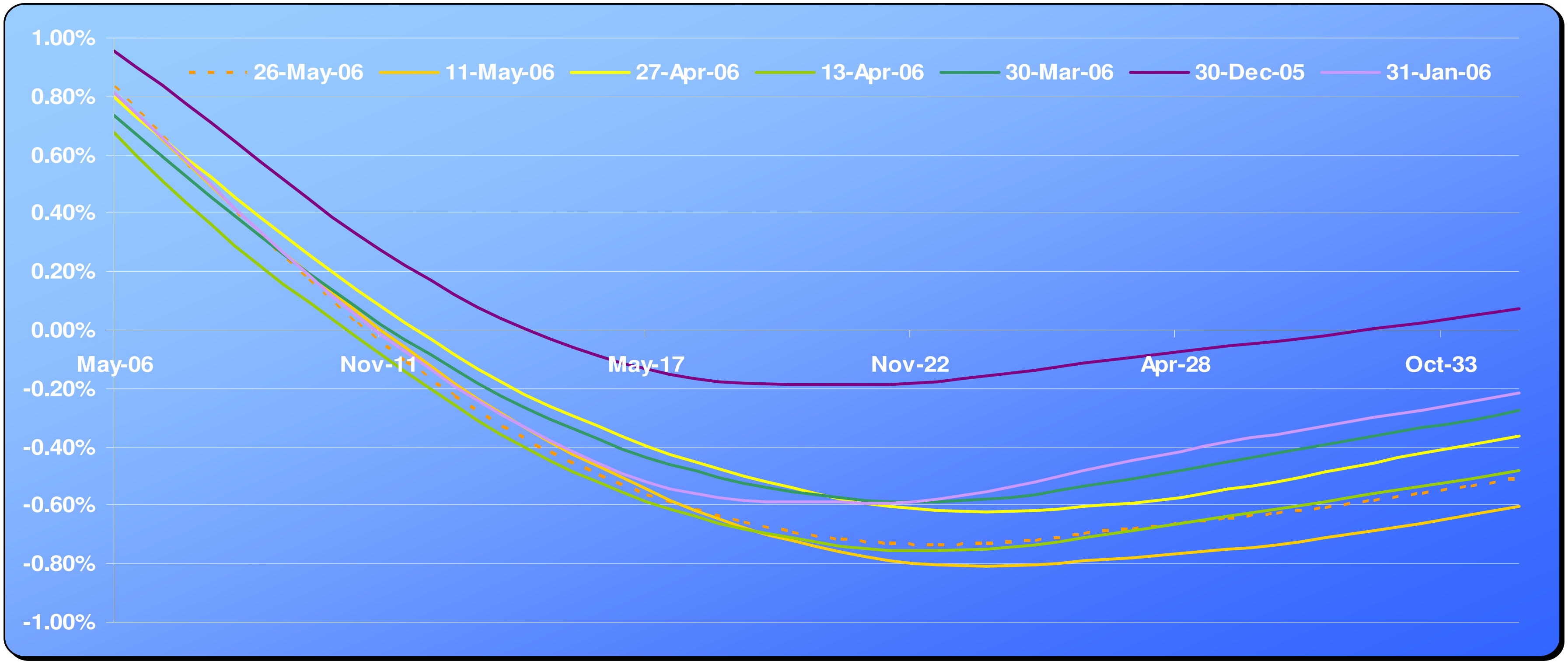
Forward eur & gbp 10y swap rates



According to forwards, GBP rates are lower than EUR after 2012

Theme 3
Inter-curves

Forward spread in last months



A good window for trading ?

Theme 3
Inter-curves

Possibility of UK joining the union

Feb. 8 (Bloomberg) -- **Britain's currency is increasingly being treated by investors as a member of the European Monetary Union,** negating bets that the pound will weaken versus the euro, according to Morgan Stanley. Volatility in the pound's exchange rate against Europe's single currency last year fell to the lowest since 2002, Bloomberg data shows. The U.K. has ruled out adopting the euro, which was introduced in 1999 across 10 nations in the European Union, until the ``right'' economic conditions are in place……..etc…

``**The U.K. is a small boat** with a big hole in it, **but it's tied to a big ship**, that being Europe,'' said Jen. ``Even though it seems like it should sink, it just won't go down.''

In September 2003, Prime Minister Tony Blair said it would be ``madness'' to rule out adopting the currency forever. At the beginning of that year, he said joining the dozen EU nations using the currency was Britain's ``destiny.'' The euro region accounts for about 53 percent of U.K. trade. Volatility on the one-month euro-sterling options contract declined to 5.18 percent, its lowest since Dec. 20……etc…

Author : Rodrigo Davies

The FX market increasingly views the Pound parallel to EUR

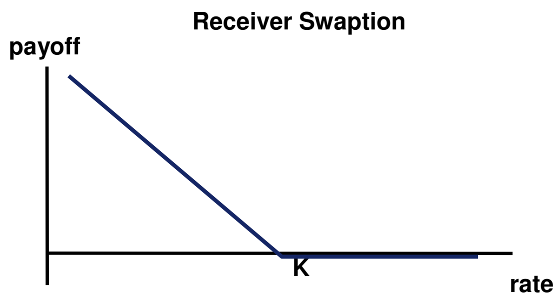
SECTION 2

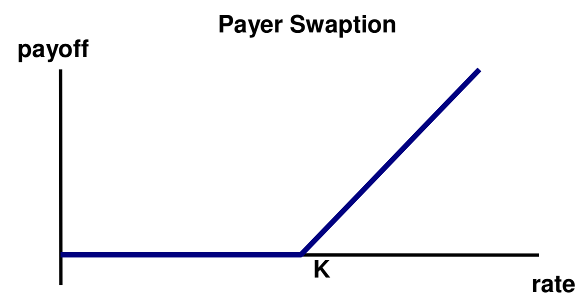
Vanilla options - basics

Option Basics

Swptions

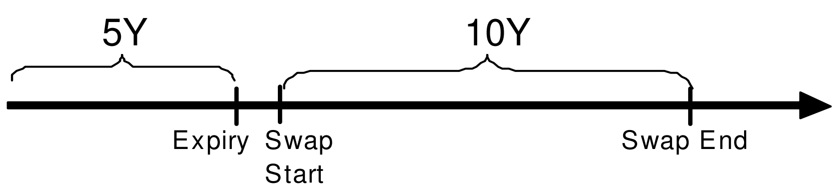
- **A swaption is an option to enter into an interest swap:**
  - **receiver** swaption gives the right to receive a fixed rate.
  - **payer** swaption gives the right to pay a fixed rate.

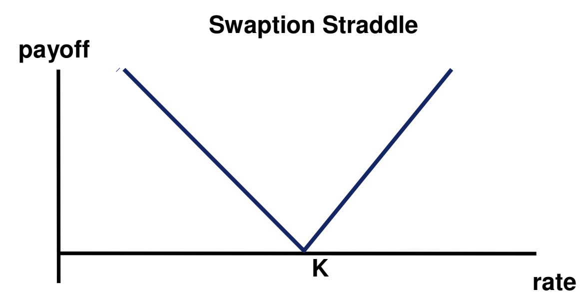




- **A swaption is defined by expiration, underlying, strike.**

Example: 5Y10Y ATM straddle





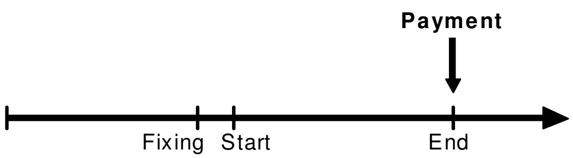
Option Basics

Caps & floors

- **Payoff of a caplet:**

$$\max\left(L(t)-K,0\right)\cdot\frac{d}{360}$$

  - Caplet = call on Libor
  - Defined by Strike K, Start Date, End Date



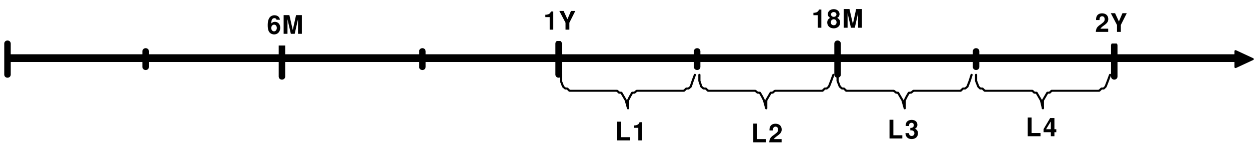
- **Payoff of a floorlet:**

$$\max\left(K-L(t),0\right)\cdot\frac{d}{360}$$

  - Floorlet = put on Libor

- **Cap = series of caplets**
- **Floor = series of floorlets**

Example: 1x2 cap on 3m Libor



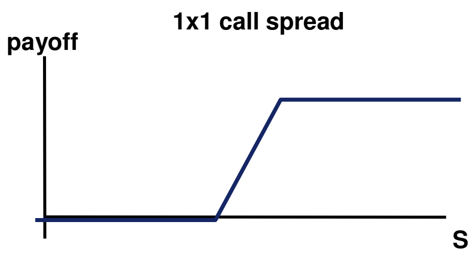
Option Basics

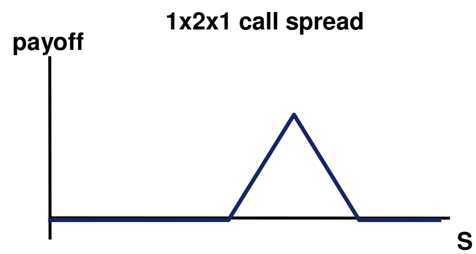
EuroDollar Options

- **Options on EuroDollar Futures (CME)**
  - Calls & Puts on first 8 Eurodollar contracts, strikes every 25bp.

Example: EDH7 95.25 Call

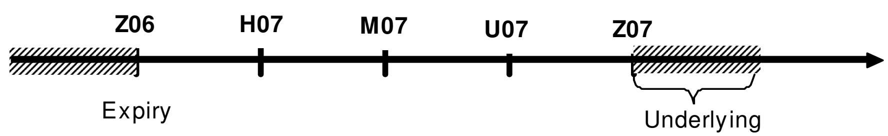
Basic payoffs: 1x1, 1x2, 1x2x1 call spreads





- **Mid-Curve options.**
  - Serials: EDV6 expiry on EDZ6, EDX6 on EDZ6
  - 1Yr mid curve: the underlying contract starts one year after the option expiry.

Example: EDZ6 expiry on EDZ7 underlying



Option Basics

Other products

- **Options on CBOT**
  - Calls & Puts on FV, TY, US contracts

- **Bond Options**
  - Calls & Puts on US Treasuries
  - Repo and carry are additional parameters

- **Bermudean**
  - Callables : the payer of the fixed rate has the right to cancel the swap.
  - Putables : the receiver of the fixed rate has the right to cancel the swap.

Option Basics

Option valuation: straddle premium

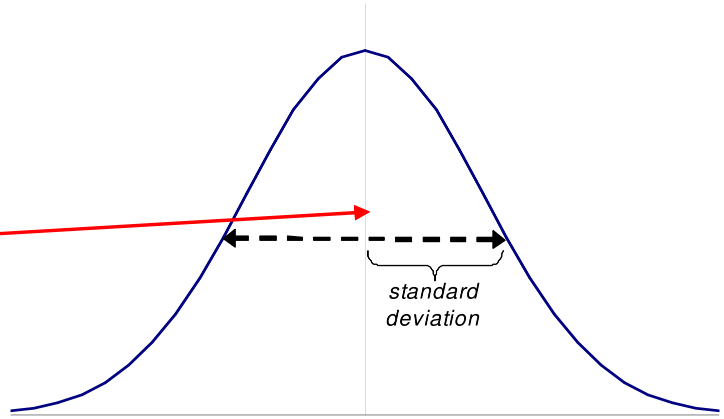
- **Let us consider a straddle with maturity T. (strike at the money)**

  Assume the underlying S follows a normal distribution with standard deviation $\sigma = Vol \cdot \sqrt{T}$

We get the premium of the straddle by: $\Pi = E(|S|) \cdot PV01$

$$E(|S|) = \int_{-\infty}^{\infty} \frac{|s|}{\sigma\sqrt{2\pi}} e^{-\frac{s^2}{2\sigma^2}} ds = \frac{2\sigma}{\sqrt{2\pi}}$$

$$\boxed{\Pi = \frac{2}{\sqrt{2\pi}} \cdot Vol \cdot \sqrt{T} \cdot PV01}$$



Example: 1Y 10Y ATM straddle: $\sigma = 78bp, SwapLevel = 7.35$

$$\Pi = 78bp \times 7.35 \times \sqrt{\frac{2}{\pi}} = 458bp$$

Note: the straddle price is a linear function of the volatility.

Option Basics

Black-Scholes Model

- If S has a lognormal dynamic, the volatility of S being a deterministic function : $$\frac{dS_t}{S_t} = \sigma\, dW_t$$

We can value European options written on S, struck at strike K at maturity T with the risk free rate r :

$$\Pi_0 = S_0 N(d_1) - K e^{-rT} N(d_2)$$

With
$$d_{1,2} = \frac{1}{\sigma\sqrt{T}} \ln \frac{S_0 e^{rT}}{K} \pm \frac{1}{2}\sigma\sqrt{T}$$

- Black formula for swaptions:

$$\Pi_0 = Level_{Swap}\left(F \cdot N(d_1) - K \cdot N(d_2)\right)$$

$$d_{1,2} = \frac{1}{\sigma\sqrt{T}} \ln \frac{F}{K} \pm \frac{1}{2}\sigma\sqrt{T}$$

Option Basics

Risk management: Black Scholes Greeks

- **Delta : sensitivity to the forward rate:** $\Delta = \frac{\partial \Pi}{\partial S}$
  - We look at the PV change for 1bp move

Example: 1M10Y f-25 rec $\Delta = 8.6\%$, $\mathrm{SwapLevel} = 7.72$

N=100,000,000 $\Delta = 8.6\% \cdot 7.72\mathrm{bp} \cdot 100{,}000{,}000 = 6{,}640$

- **Gamma : sensitivity of the delta to the underlying forward:** $\Gamma = \frac{\partial^2 \Pi}{\partial S^2}$
  - We look at the Delta change for 10bp move

Example: 100M 1M10Y ATM straddle: Gamma=33,000

If we rally 10bp, we get longer in Delta by 33,000

- **Vega : sensitivity to the volatility:** $V = \frac{\partial \Pi}{\partial \sigma}$
  - We look at PV change for 10% change of normal vol.

Example: 100M 1Y10Y ATM straddle:

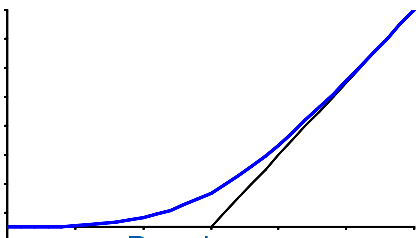
$$\sigma = 80\% \text{ , } \Pi = 470\,bp \text{ , } Vega = 46\,bp$$
$$\sigma = 88\% \quad \Pi = 516\,bp$$

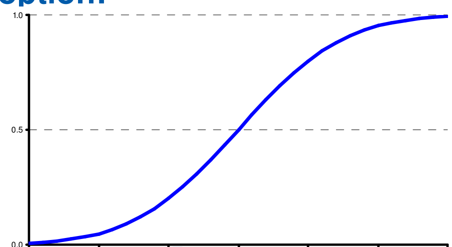
- **Theta: cost of carry:** $\Theta = \frac{\partial \Pi}{\partial t}$

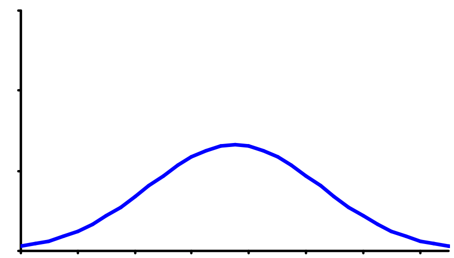
Option Basics

Risk management: Black Scholes Greeks

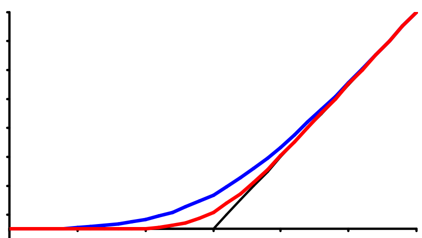
- **Let us have a look at the greeks of a call option:**

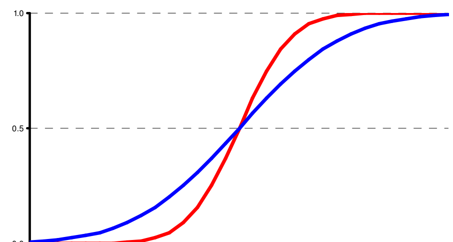

Premium

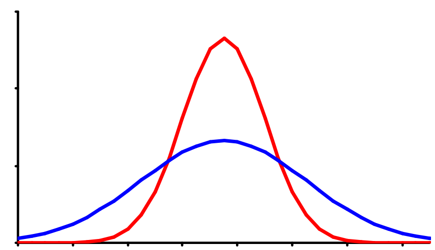

Delta


Gamma

- **As time to maturity approaches:**


Premium


Delta

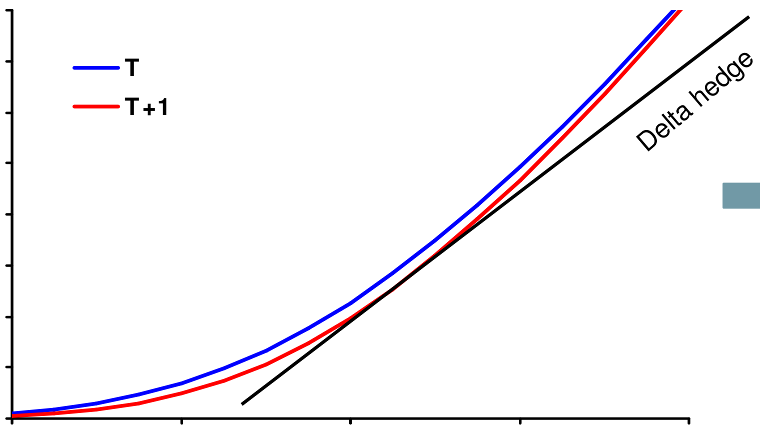

Gamma

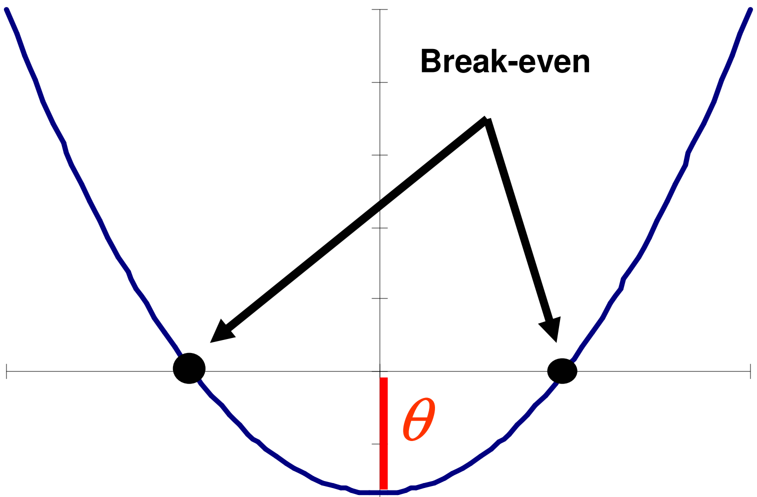
- **Gamma** risk increases as we get closer to maturity.
- **Theta** gets larger as we approach to maturity
- **Vega** risk decreases as we get closer to maturity.

Option Basics

PL decomposition

- PL of a delta hedged option:





PL of a delta hedged option

- Loss on Theta (Cost of carry) : $-\theta$
- Gain on Gamma: $\frac{1}{2}\Gamma(\Delta S)^2$

$$TotalPL = -\theta + \frac{1}{2}\Gamma(\Delta S)^2$$

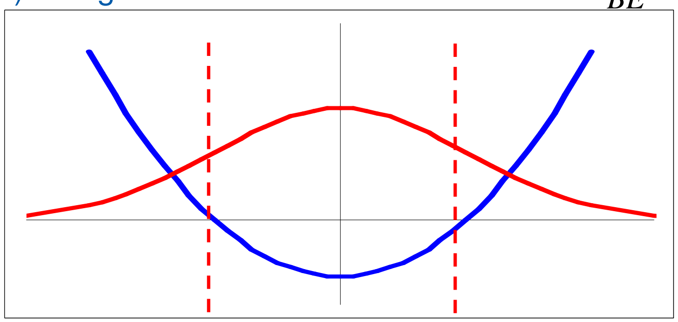
Black-Scholes :

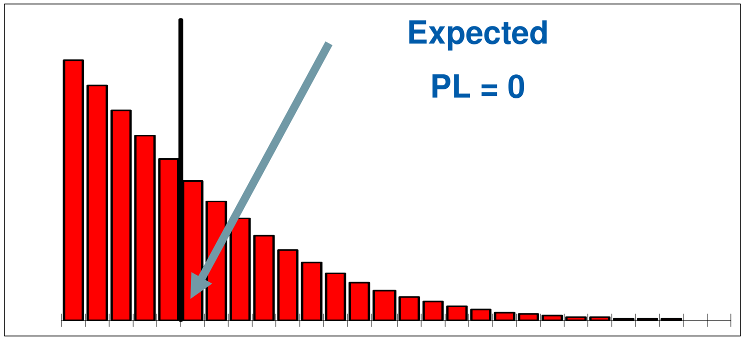
$$\Theta = \Gamma$$

Option Basics

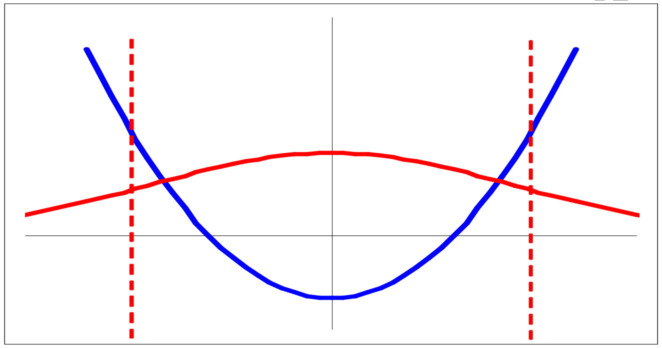
Realized Volatility vs Implied Volatility

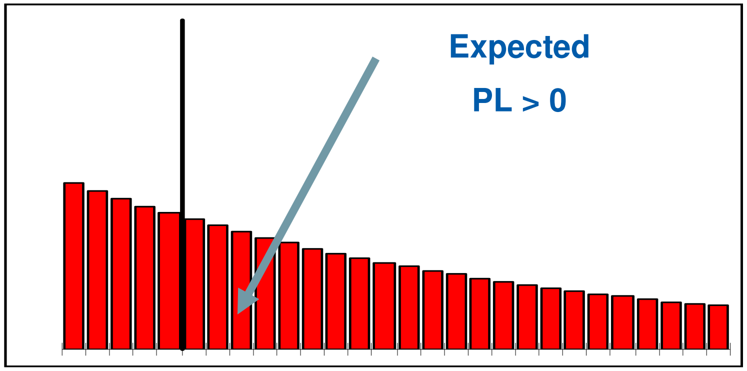
- 1) Using the break-even vol: $\sigma = \sigma_{BE}$





- 2) If the delivered vol is higher: $\sigma > \sigma_{BE}$





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SECTION 3

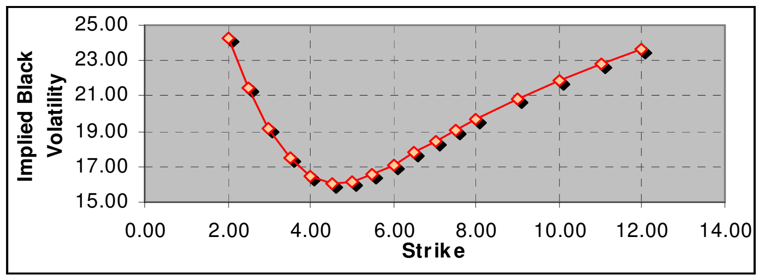
SABR Volatility Model

Sigma Alpha Beta Rho model

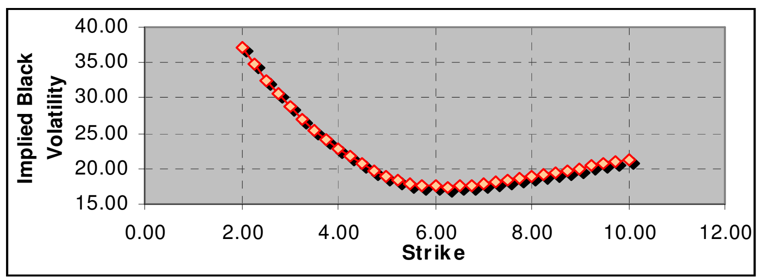
SABR

- **Significant Market skew and kurtosis (Here 1Y cap smile)**

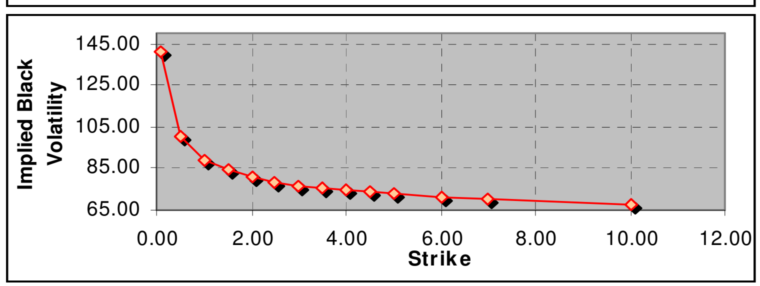
EUR



USD



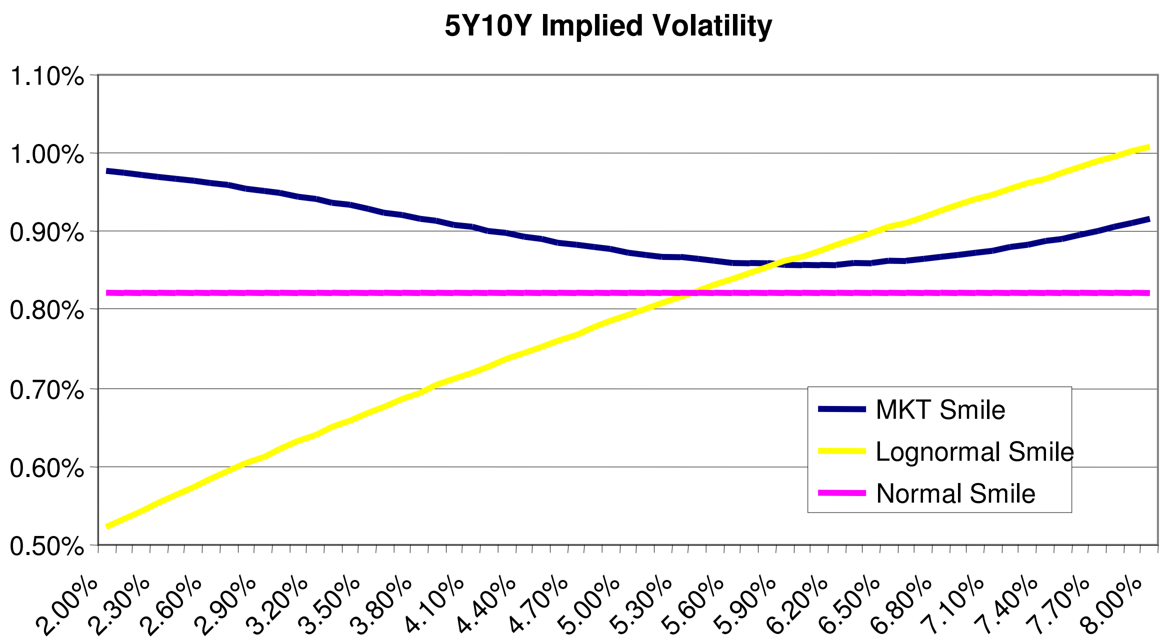
JPY



SABR

Market Smile

- The option markets display a smile i.e. a different Black-Scholes implied volatility for different strikes



SABR

- **Example of models:**
  - Market Models: dynamics of all forward Libor rates (BGM) or some forward swap rates (Jamshidian)
  - Short Rate Models (Vasicek, BDT): dynamics of the short rate

- **Example of structured products:**
  - Cancelable Swaps
  - Callable Inverse Floaters
  - Callable Yield Curve Swaps
  - Knock-out Power Dual Notes

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SABR

Black-Scholes risk management drawbacks

- Incapacity to aggregate risks coming from options of different strikes on the same underlying
  - Greeks are computed for each option with a fixed implied volatility

- Inability to perform cheap-rich analysis
  - No modeling of the volatility

- Wrong Greeks calculated since the assumption that the volatility remains constant when the forward moves does not fit the market smile
  - Fake arbitrage opportunity by creating both theta and gamma positive portfolios

SABR

Smile models

- To overcome this risk management issues, we need a consistent smile model that matches the market prices of European options of all strikes

- 3 types of models achieve that goal :
  - **Local Volatility models :** the local volatility σ of the underlying S depends on the underlying value. Ex :
    Ex : $\sigma = f(S)$

  - **Jump models :** they assume the smile to move with market jumps
    Ex : $dS(t)= \sigma\, dN(t)$; N being a Poisson process

  - **Stochastic volatility models :** the smiles come from the intrinsic moves of the volatility
    Ex : $dS(t)= \sigma(t)\, dW(t)$ with $\sigma(t)$ random variable

SABR

- **Strategy 1: Estimate volatility**
  - Problem: possible loss if realized volatility doesn't correspond to estimation

- **Strategy 2: Calibrate volatility**
  - Fix the value of volatility so as to hit the market prices of European instruments: caps and swaptions
  - Mark to market of volatility
  - Allows to hedge the volatility risk (vega) of complex instruments with more liquid European ones

SABR

SABR dynamic

- SABR is a combination of forward and stochastic volatility models :
  - A forward volatility equation :

$$dF = \sigma_\beta F^\beta dW$$

  - A stochastic volatility equation :

$$d\sigma_\beta = \alpha.\sigma_\beta dZ$$

Where W and Z are brownian motions with correlation ρ

SABR

SABR formula

- The expectations are computed through closed-form approximations

$$\sigma(K,f)=\frac{\sigma_B}{(fK)^{\frac{1-\beta}{2}}*\left\{1+\frac{(1-\beta)^2}{24}\log^2\frac{f}{K}+\frac{(1-\beta)^4}{1920}\log^4\frac{f}{K}+\ldots\right\}}.\left(\frac{z}{x(z)}\right).$$

$$\left\{1+\left[\frac{(1-\beta)^2}{24}.\frac{\sigma_B{}^2}{(fK)^{1-\beta}}+\frac{1}{4}\frac{\rho\beta\alpha\sigma_B}{(fK)^{\frac{1-\beta}{2}}}+\frac{2-3\rho^2}{24}\alpha^2\right]t_{ex}+\ldots.\right\}$$

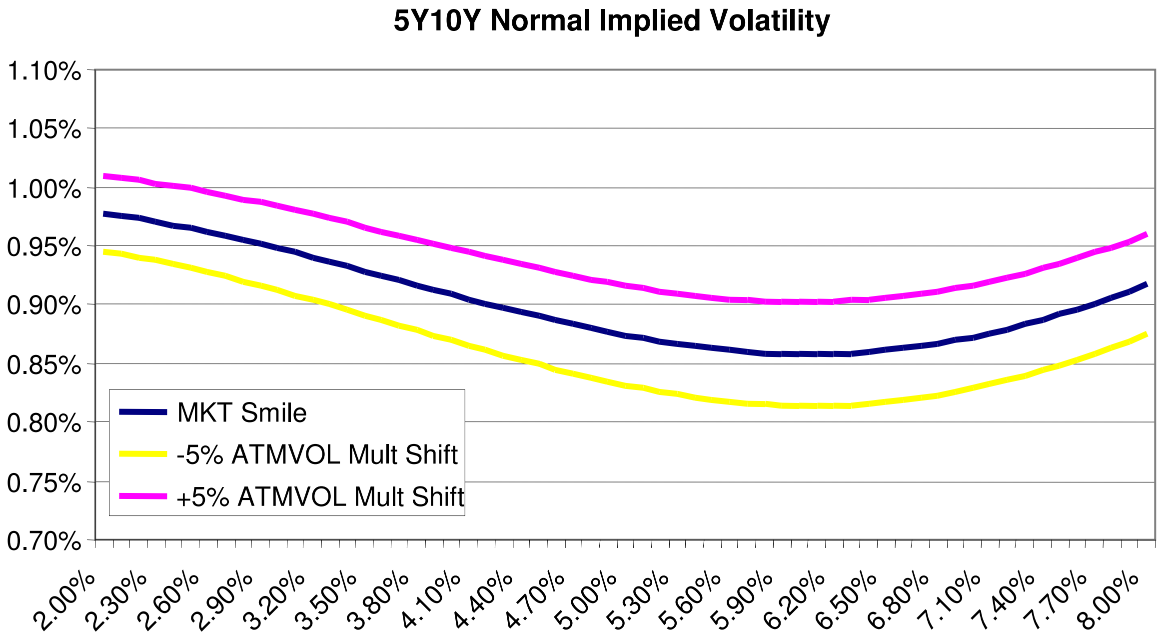
with $z=\frac{\alpha}{\sigma_B}(fK)^{\frac{1-\beta}{2}}\log\frac{f}{K}$

and $x(z)$ is defined by $x(z)=\log\left\{\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right\}$

SABR

SABR parameters

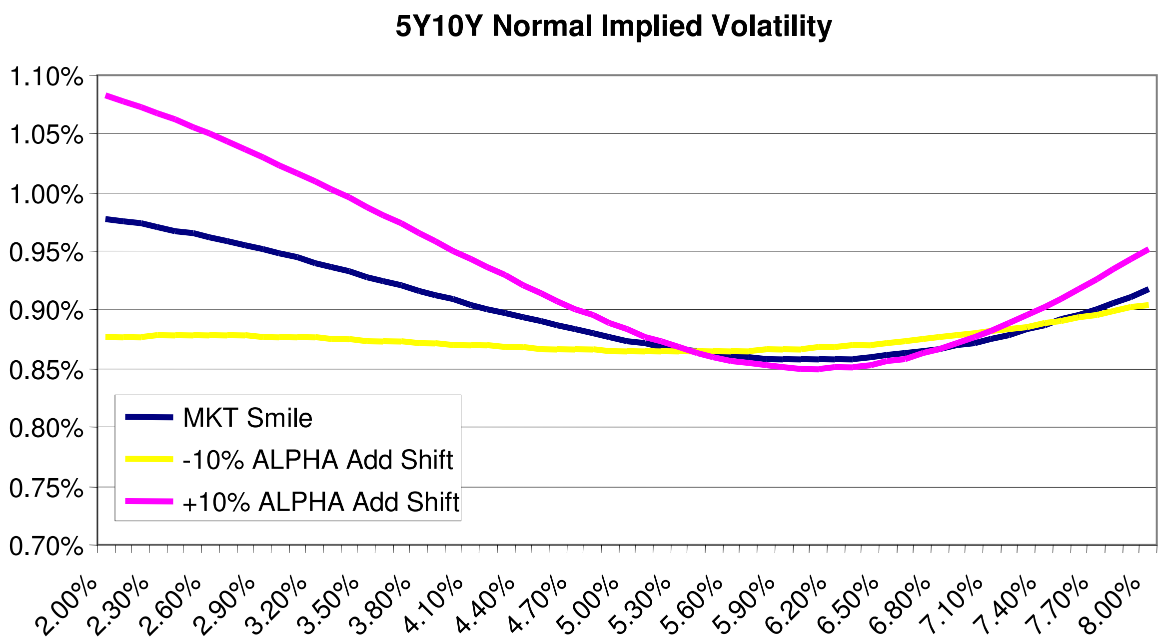
- Sigma : it’s the beta-ATM volatility and is always given by the market



SABR

SABR parameters

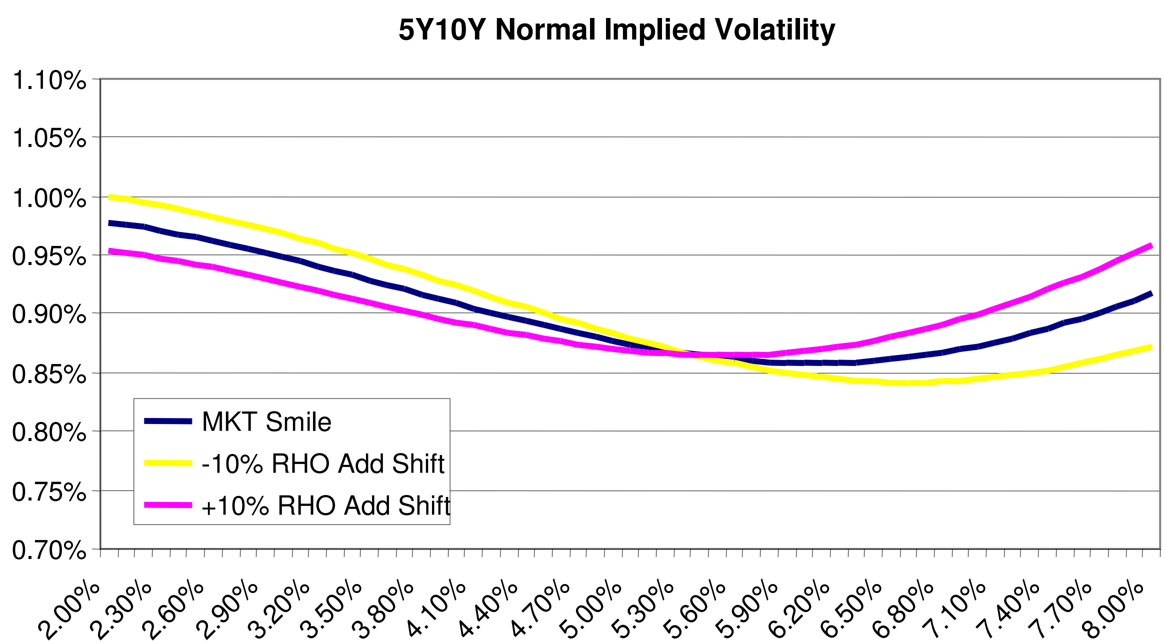
- Alpha : it's the volatility of the volatility and controls the convexity of the smile. It can be calibrated to the market smile or come from historical analysis



SABR

SABR parameters

- Rho : it's the correlation between the forward and the volatility and also controls the skew



SABR

SABR parameters

- Beta : it's the functional form that links the level of the volatility to the underlying forward value. It controls the skew. Note that the only role of β is to determine the σ_β volatility

BETA = 0.5									
RHO	1M	3M	6M	1Y	5Y	10Y	20Y	25Y	30Y
6M	-38.01%	-38.00%	-38.00%	-33.97%	-26.91%	-22.96%	-22.96%	-22.95%	-22.96%
12M	-37.00%	-37.00%	-37.01%	-38.26%	-32.93%	-28.96%	-29.45%	-29.69%	-29.93%
5Y	-33.00%	-33.00%	-33.00%	-33.67%	-37.99%	-39.00%	-38.01%	-37.51%	-37.01%
10Y	-31.00%	-31.00%	-31.00%	-32.33%	-39.00%	-43.00%	-42.00%	-41.50%	-41.00%
15Y	-31.00%	-31.00%	-31.00%	-32.33%	-39.00%	-43.00%	-42.00%	-41.50%	-41.00%
20Y	-31.00%	-31.00%	-31.00%	-29.67%	-37.00%	-41.00%	-40.00%	-39.50%	-39.00%
30Y	-30.00%	-30.00%	-30.00%	-29.33%	-36.00%	-40.00%	-39.00%	-38.50%	-38.00%
40Y	-28.99%	-29.01%	-28.99%	-28.99%	-36.00%	-40.00%	-39.00%	-38.50%	-38.00%

BETA = 0									
RHO	1M	3M	6M	1Y	5Y	10Y	20Y	25Y	30Y
6M	-30.12%	-30.33%	-29.95%	-22.08%	-12.07%	-8.83%	-8.44%	-8.40%	-8.32%
12M	-24.11%	-24.39%	-24.11%	-21.65%	-12.53%	-8.64%	-8.53%	-8.60%	-8.61%
5Y	-0.47%	-0.87%	-1.44%	-0.94%	-7.38%	-11.16%	-11.46%	-10.88%	-10.29%
10Y	10.84%	10.23%	9.67%	9.85%	-0.32%	-9.14%	-11.21%	-10.76%	-10.43%
15Y	13.29%	14.09%	12.14%	14.10%	4.76%	-3.90%	-7.09%	-6.69%	-6.04%
20Y	17.35%	18.31%	16.01%	21.18%	12.69%	4.40%	-0.91%	-0.58%	0.73%
30Y	26.97%	28.24%	25.04%	30.57%	20.10%	10.96%	3.82%	3.93%	5.21%
40Y	28.87%	30.25%	26.97%	31.96%	20.89%	11.42%	3.46%	3.42%	4.55%

BETA = 1									
RHO	1M	3M	6M	1Y	5Y	10Y	20Y	25Y	30Y
6M	-44.86%	-44.69%	-44.98%	-43.85%	-39.29%	-35.14%	-35.42%	-35.44%	-35.50%
12M	-47.40%	-47.22%	-47.41%	-50.65%	-47.94%	-44.48%	-45.21%	-45.51%	-45.84%
5Y	-53.33%	-53.17%	-52.98%	-53.86%	-56.42%	-56.28%	-54.99%	-54.61%	-54.25%
10Y	-54.68%	-54.50%	-54.36%	-55.65%	-59.37%	-61.10%	-59.52%	-59.12%	-58.69%
15Y	-55.28%	-55.44%	-55.00%	-56.61%	-60.42%	-62.24%	-60.60%	-60.21%	-59.88%
20Y	-56.16%	-56.32%	-55.87%	-55.82%	-60.16%	-62.03%	-60.15%	-59.77%	-59.59%
30Y	-57.02%	-57.18%	-56.72%	-57.08%	-60.47%	-62.24%	-60.26%	-59.85%	-59.66%
40Y	-56.54%	-56.72%	-56.24%	-57.00%	-60.59%	-62.33%	-60.23%	-59.78%	-59.56%

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SABR

USD SABR Parameters

Contributor	NYK: ChoE
Contrib Da	25-Sep-06
Contrib Tim	19:48:00

TMNORMA	1M	3M	6M	1Y	5Y	10Y	15Y	20Y	25Y	30Y
6M	0.6222	0.6075	0.6339	0.7660	0.7809	0.7407	0.7268	0.7129	0.7076	0.7023
12M	0.8319	0.8200	0.8342	0.8625	0.8364	0.7975	0.7747	0.7596	0.7539	0.7482
5Y	0.9575	0.9575	0.9545	0.9486	0.9192	0.8775	0.8285	0.7963	0.7854	0.7744
10Y	0.8950	0.8950	0.8922	0.8866	0.8510	0.7990	0.7439	0.7084	0.6966	0.6836
15Y	0.7847	0.7999	0.7822	0.7924	0.7606	0.7192	0.6649	0.6331	0.6226	0.6153
20Y	0.7135	0.7273	0.7113	0.7206	0.6998	0.6695	0.6118	0.5826	0.5729	0.5729
30Y	0.6263	0.6384	0.6244	0.6325	0.6000	0.5740	0.5245	0.4995	0.4911	0.4911
40Y	0.5427	0.5529	0.5407	0.5477	0.5196	0.4971	0.4542	0.4325	0.4253	0.4253

ALPHA	1M	3M	6M	1Y	5Y	10Y	15Y	20Y	25Y	30Y
6M	0.5000	0.5000	0.5000	0.5225	0.5000	0.4900	0.4600	0.4450	0.4375	0.4300
12M	0.4700	0.4700	0.4700	0.4475	0.3900	0.3700	0.3450	0.3325	0.3263	0.3200
5Y	0.3400	0.3400	0.3400	0.3175	0.3100	0.3100	0.2950	0.2875	0.2837	0.2800
10Y	0.2634	0.2634	0.2634	0.2460	0.2406	0.2408	0.2322	0.2278	0.2256	0.2234
15Y	0.2205	0.2205	0.2205	0.2059	0.2014	0.2015	0.1944	0.1907	0.1888	0.1870
20Y	0.1957	0.1957	0.1957	0.1828	0.1788	0.1789	0.1725	0.1693	0.1676	0.1660
30Y	0.1678	0.1678	0.1677	0.1567	0.1533	0.1534	0.1479	0.1451	0.1437	0.1423
40Y	0.1526	0.1526	0.1526	0.1425	0.1394	0.1395	0.1345	0.1320	0.1307	0.1294

BETA	1M	3M	6M	1Y	5Y	10Y	15Y	20Y	25Y	30Y
6M	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
12M	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
5Y	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
10Y	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
15Y	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
20Y	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
30Y	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
40Y	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

RHO	1M	3M	6M	1Y	5Y	10Y	15Y	20Y	25Y	30Y
6M	(0.3200)	(0.3200)	(0.3200)	(0.3000)	(0.2300)	(0.1900)	(0.1900)	(0.1900)	(0.1900)	(0.1900)
12M	(0.3300)	(0.3300)	(0.3300)	(0.3501)	(0.2900)	(0.2500)	(0.2525)	(0.2550)	(0.2575)	(0.2600)
5Y	(0.3100)	(0.3100)	(0.3100)	(0.3167)	(0.3600)	(0.3700)	(0.3650)	(0.3600)	(0.3550)	(0.3500)
10Y	(0.2900)	(0.2900)	(0.2900)	(0.3033)	(0.3700)	(0.4100)	(0.4050)	(0.4000)	(0.3950)	(0.3900)
15Y	(0.2900)	(0.2900)	(0.2900)	(0.3033)	(0.3700)	(0.4100)	(0.4050)	(0.4000)	(0.3950)	(0.3900)
20Y	(0.2900)	(0.2900)	(0.2900)	(0.2767)	(0.3500)	(0.3900)	(0.3850)	(0.3800)	(0.3750)	(0.3700)
30Y	(0.2800)	(0.2800)	(0.2800)	(0.2733)	(0.3400)	(0.3800)	(0.3750)	(0.3700)	(0.3650)	(0.3600)
40Y	(0.2700)	(0.2700)	(0.2700)	(0.2700)	(0.3400)	(0.3800)	(0.3750)	(0.3700)	(0.3650)	(0.3600)

BNP PARIBAS

SABR

SABR Greeks

- **Delta :** it ‘s computed by shifting the forward while leaving unchanged the sigma-beta vol
- **Gamma** : it’s the sensitivity of the delta to the underlying forward value. Again it’s computed with a frozen value of the sigma-beta volatility
- **Vega :** it’s the sensitivity to a move in the ATM volatility, with α , β , ρ being unchanged.
- **Volga :** it’s the convexity in the volatility
- **Vanna :** it’s the cross convexity in the forward and its volatility

SABR

SABR P&L decomposition (1)

- Black-Scholes : $\theta = rV - \frac{1}{2}\Gamma\sigma^2 F^2$

Black-Scholes only charges for the convexity in the forward underlying because it does not expect other parameters to move.

- SABR : $\theta = rV - \frac{1}{2}\Gamma\sigma_B{}^2 F^{2\beta} - \frac{1}{2}\mathrm{volga}\,.\alpha^2.\sigma_B{}^2 - \mathrm{vanna}\,.\rho.\alpha.\sigma_B{}^2.F^{\beta}$

The theta charged by SABR takes into account volatility moves and cross moves

SABR

SABR P&L decomposition (2)

- The time-value θ is split into 4 components :
  - The carry : rV
  - The cost of gamma : $-\frac{1}{2}\Gamma\sigma_B^{2}F^{2\beta}$; it works exactly as in BS with an extra coefficient
  - The cost of volga : $-\frac{1}{2}\text{volga}.\alpha^{2}.\sigma_B^{2}$; conversely to BS, SABR expects the volatility to move and charge the convexity in volatility
  - The cost of vanna : $-\text{vanna}.\rho.\alpha.\sigma_B^{2}.F^{\beta}$; if the model expects the forward and the volatility to move together (ρ > 0), it will charge a positive convexity , negative otherwise.

SABR

SABR drawbacks

- SABR is pretty inefficient for short term options and gamma trading because it does not involve jumps
- SABR can not value options that depend on more than one underlying, therefore it can not be used for exotic options
- SABR does not provide tools to aggregate the risks on several underlyings

SABR

Questions

- How can we estimate α , β , ρ ?
- Does β impact the delta ? And ρ ?
- What's the link between the sigma-beta ATM volatility and the lognormal ATM volatility ?

SABR

Answers !!

- How can we estimate α , β , ρ?
  - α : historically or calibrated to the market smile
  - ρ : calibrated to fit the smile
  - β : trader choice

- What's the link between the sigma-beta ATM volatility and the lognormal ATM volatility ?

$$\sigma_{LOG}^{ATM} = \sigma^{ATM}{}_{\beta} * F^{\beta-1}$$

- Does β impact the delta ? And ρ ?

$$\sigma_{LOG}^{ATM}(F + \delta F) = \sigma_{LOG}^{ATM}(F) * (1 - (1 - \beta)\frac{\delta F}{F})$$

  - ATM, β impacts the delta and ρ does not.

SABR

Impact of SABR parameters

			OTM Receivers	ATM	OTM Payers
Sigma	σ	↑	↑	↑	↑
		↓	↓	↓	↓
Alpha	α	↑	↑	-	↑
		↓	↓	-	↓
Rho	ρ	↑	↓	-	↑
		↓	↑	-	↓

SECTION 4

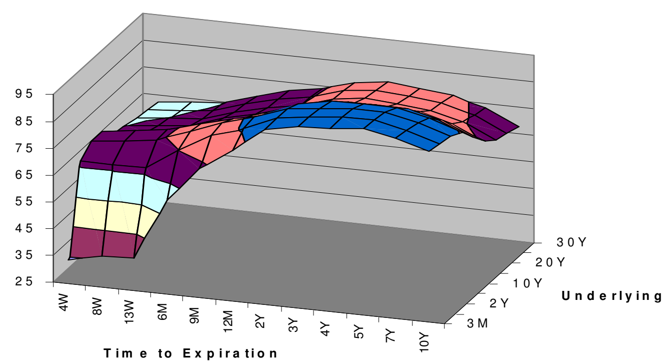
Generating the volatility surface

BNP PARIBAS

Generating the volatility surface

- **How do we generate the volatility surface?**
  - The volatility surface in Ramp spans 40 years of optionality on underlyings as short as 1m to as long as 30yrs.
  - Understanding the inputs to the volatility cube can provide intuition to this large set of data

BP vol	3M	1Y	2Y	5Y	10Y	15Y	20Y	25Y	30Y
4W	36.8	67.1	70.7	70.7	**66.7**	66.0	65.2	64.7	64.3
8W	36.8	67.1	72.1	72.1	**68.1**	67.1	66.1	65.6	65.1
13W	37.8	67.2	73.6	73.6	**69.5**	68.3	67.1	66.6	66.0
6M	**60.8**	**75.1**	**78.3**	**77.3**	**73.1**	**71.8**	**70.4**	**69.9**	**69.3**
9M	73.5	81.4	82.4	80.1	**76.0**	74.6	73.1	72.6	72.0
12M	**81.6**	**85.3**	**85.9**	**82.8**	**78.8**	**76.5**	**75.0**	**74.4**	**73.9**
2Y	90.3	90.9	90.6	88.6	**84.4**	80.9	79.1	78.3	77.5
3Y	92.8	92.7	92.2	91.1	**86.8**	82.8	80.5	79.7	78.9
4Y	93.6	93.9	93.2	91.5	**87.1**	82.6	79.7	78.7	77.7
5Y	**95.0**	**94.2**	**93.1**	**91.2**	**86.9**	**82.0**	**78.8**	**77.8**	**76.7**
7Y	92.6	91.8	90.7	88.7	**84.1**	79.0	75.7	74.4	73.3
10Y	88.8	88.0	87.0	84.5	**79.1**	73.6	70.1	69.0	67.7



Generating the volatility surface

- **We have 3 different sources of volatility information**
  - **(1) CME Eurodollar Options**

Code	Strike	Price	MTM	Implied	% adj	Tenors	Dates	Interp	Used
Quarterlies									
EDZ06	94.625	11.0	15.4	26.9	0.0%	1W	4-Oct	26.9	26.9
EDH07	94.875	31.5	32.3	58.4	0.0%	2W	11-Oct	26.9	26.9
EDM07	95.000	47.5	48.1	72.1	0.0%	4W	25-Oct	26.9	26.9
EDU07	95.250	60.5	62.1	79.4	0.0%	8W	22-Nov	26.9	26.9
EDZ07	95.250	70.5	70.9	83.9	0.0%	13W	27-Dec	30.0	30.0
EDH08	95.250	79.0	79.7	86.3	0.3%	6M	27-Mar	59.6	59.6
EDM08	95.250	86.5	87.0	88.2	0.5%	9M	27-Jun	72.8	72.8
EDU08	95.250	94.0	93.9	90.3	0.5%	12M	27-Sep	79.9	79.9
						18M	27-Mar	86.8	86.8
						2Y	29-Sep	91.0	91.0

    - Using straddle prices from the CME, we construct 2 years of caplet volatility
    - We now have the upper left hand corner of the surface

Generating the volatility surface

- (2) CBOT Options

Expiry date	Underlying	Strike	Type	Price straddle	Price call	Price put	Price 64ths	Vega multi	Vega/ 1000	Vol daily	Vol bp
X06	US	112-000	STR	1-380	0-590	0-430	**1-380**	0-113	178,343	3.98	63.7
Z06	US	112-000	STR	2-143	1-151	0-632	**2-143**	0-155	243,407	3.96	63.4
F07	US	112-000	STR	2-503	1-307	1-19+	**2-503**	0-20+	319,767	4.11	65.8
H07	US	112-000	STR	3-436	1-59+	1-482	**3-436**	0-265	415,788	4.15	66.4
X06	TY	108-000	STR	0-605	0-323	0-283	**0-605**	0-063	99,952	4.36	69.8
Z06	TY	108-000	STR	1-212	0-445	0-405	**1-212**	0-087	138,573	4.36	69.7
F07	TY	108-000	STR	1-436	0-573	0-50+	**1-436**	0-11+	179,779	4.50	72.0
H07	TY	108-000	STR	2-143	1-105	1-036	**2-143**	0-151	236,110	4.54	72.6
X06	FV	105-160	STR	0-406	0-207	0-197	**0-406**	0-041	65,134	4.56	72.9
Z06	FV	105-160	STR	0-571	0-290	0-280	**0-571**	0-056	90,478	4.54	72.7
F07	FV	105-160	STR	1-080	0-36+	0-35+	**1-080**	0-073	115,958	4.70	75.2
H07	FV	105-160	STR	1-307	0-477	0-467	**1-307**	0-096	152,695	4.72	75.6

  - CBOT straddle prices provide us with gamma volatility.
  - Assuming a ratio between between CBOT options and swaptions, we can create the short-dated sector of the volatility surface

Generating the volatility surface

- **(3) OTC swaption prices**

Expiry date	Swap Start	Swap End	Type	Price	Vol bp
1y		10y	str	464.34	78.9
2y		10y	str	668.78	84.4
5y		10y	str	929.94	86.9
10y		10y	str	917.38	79.1

  - Through the inter-dealer market, these 4 relatively liquid swaption points build the rest of the volatility surface
  - We define the rest of the swaption matrix as a percentage to the 10yr underlyings
  - For example, we assume 5y5y swaption vol is 105% of 5y10y swaption vol, as seen on the next page…

Generating the volatility surface

- We combine these small baskets of CME, CBOT, and OTC options, and add a sensible layer of interpolation between these points to get…

Input

NORMAL	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	25Y	30Y
1W	3.5%	26.9	-5.0%	100.0%	100.0%	1.3%	0.5%	100.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	97.8%	0.0%	97.8%
2W	3.5%	26.9	-5.0%	100.0%	100.0%	1.3%	0.5%	100.0%	0.0%	0.0%	0.0%	0.0%	100.0%	0.0%	97.3%	0.0%	97.8%
4W	3.5%	26.9	-5.0%	100.0%	98.0%	1.3%	0.5%	98.0%	0.0%	0.0%	0.0%	0.0%	97.5%	0.0%	97.3%	0.0%	97.8%
8W	3.5%	26.9	-5.0%	99.8%	98.0%	1.3%	0.5%	97.8%	0.0%	0.0%	0.0%	0.0%	97.8%	0.0%	96.5%	0.0%	97.8%
13W	3.5%	30.0	-5.0%	89.5%	94.0%	1.3%	0.5%	95.3%	0.0%	0.0%	0.0%	0.0%	94.5%	0.0%	96.0%	0.0%	97.8%
6M	2.5%	59.6	-4.0%	91.8%	94.8%	0.8%	0.5%	96.3%	0.0%	0.0%	0.0%	0.0%	96.0%	0.0%	96.0%	0.0%	97.8%
9M	2.0%	72.8	-1.0%	95.5%	96.0%	0.5%	0.3%	96.8%	0.0%	0.0%	0.0%	0.0%	96.5%	0.0%	96.0%	0.0%	98.3%
12M	1.5%	79.9	0.0%	**85.6**	**86.1**	0.5%	0.3%	**105.3%**	0.0%	0.0%	0.0%	0.0%	**78.9**	-0.5%	95.3%	0.0%	98.5%
18M	0.5%	86.8	0.0%	87.5%	78.5%	0.0%	0.0%	105.1%	0.0%	0.0%	0.0%	0.0%	0.0%	-0.8%	94.9%	0.0%	98.5%
2Y	0.0%	91.0	0.0%	88.5%	80.5%	0.0%	0.0%	105.1%	0.0%	0.0%	0.0%	0.0%	**84.4**	-1.0%	93.8%	0.0%	98.0%
3Y	0.0%	**92.9**	0.0%	88.9%	81.8%	0.0%	0.0%	105.0%	0.0%	0.0%	0.0%	0.0%	1.9%	-1.0%	92.8%	0.0%	98.0%
4Y	0.0%	**93.6**	0.0%	89.1%	83.4%	0.0%	0.0%	105.0%	0.0%	0.0%	0.0%	0.0%	1.3%	-1.0%	91.5%	0.0%	97.5%
5Y	0.0%	**95.0**	0.0%	88.1%	85.3%	0.0%	0.0%	105.0%	0.0%	0.0%	0.0%	0.0%	**86.9**	-1.0%	90.8%	0.0%	97.3%
6Y	7.8	**7.75**	7.75	7.75	7.75	7.50	7.50	**7.75**	7.40	7.30	7.20	7.10	**7.50**	7.06	6.88	6.69	**6.75**
7Y	7.0	**7.00**	7.00	7.00	7.00	6.75	6.75	**6.75**	6.70	6.65	6.60	6.55	**6.50**	6.38	6.25	6.13	**6.00**
8Y	5.5	**5.50**	5.50	5.50	5.50	5.50	5.50	**5.50**	5.45	5.40	5.35	5.30	**5.25**	4.97	4.69	4.69	**4.50**
9Y	5.0	**5.00**	5.00	5.00	5.00	4.83	4.67	**4.50**	4.40	4.30	4.20	4.10	**4.00**	3.72	3.44	3.44	**3.25**
10Y	3.5	**3.50**	3.50	3.50	3.50	3.42	3.33	**3.25**	3.10	2.95	2.80	2.65	**2.75**	2.41	2.31	2.31	**2.25**
12Y	1.8	**2.00**	1.75	2.00	2.00	2.00	2.00	**2.00**	2.00	2.00	2.00	2.00	**2.00**	2.00	2.00	2.00	**2.00**
15Y	1.3	**1.75**	1.25	1.75	1.75	1.75	1.75	**1.75**	1.75	1.75	1.75	1.75	**2.00**	1.75	1.75	1.75	**2.00**
20Y	1.0	**1.00**	1.00	1.00	1.00	1.08	1.17	**1.25**	1.25	1.25	1.25	1.25	**1.50**	1.25	1.25	1.25	**1.50**
30Y	0.8	**0.75**	0.75	0.75	0.75	0.67	0.58	**0.50**	0.50	0.50	0.50	0.50	**0.50**	0.50	0.50	0.50	**0.50**
40Y	0.0	**0.00**	0.00	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	0.00	**0.00**	0.00	0.00	0.00	**0.00**

Output

Normal	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	15Y	20Y	25Y	30Y
1W	34.93	33.75	42.59	66.98	70.72	71.54	70.95	70.54	69.59	68.65	67.70	66.76	65.81	65.07	64.33	63.61	62.88
2W	34.93	33.75	42.59	66.98	70.72	71.54	70.95	70.54	69.59	68.65	67.70	66.76	65.81	64.90	64.00	63.28	62.56
4W	34.93	33.75	42.59	66.98	70.72	71.54	70.95	70.54	69.59	68.65	67.70	66.76	65.81	64.90	64.00	63.28	62.56
8W	37.65	36.38	44.25	66.98	72.17	73.00	72.40	71.98	71.08	70.19	69.29	68.39	67.50	66.32	65.13	64.40	63.67
13W	38.68	37.38	44.93	67.15	73.64	74.56	74.00	73.63	72.72	71.80	70.88	69.97	69.05	67.67	66.29	65.54	64.80
6M	61.88	60.38	62.65	75.03	78.34	78.58	78.04	77.31	76.46	75.61	74.76	73.92	73.07	71.61	70.15	69.36	68.57
9M	74.59	73.13	75.25	81.77	82.68	82.30	81.31	80.32	79.48	78.64	77.80	76.96	76.11	74.59	73.07	72.43	71.79
12M	82.47	81.25	82.71	85.63	86.13	85.51	84.26	83.02	82.19	81.36	80.53	79.70	78.88	76.62	75.13	74.56	74.00
18M	87.69	87.25	87.73	88.70	88.92	87.88	86.85	85.81	84.97	84.13	83.30	82.46	81.63	78.94	77.44	76.86	76.28
2Y	90.63	90.63	90.78	91.10	90.66	90.00	89.35	88.70	87.83	86.97	86.10	85.24	84.38	80.92	79.10	78.31	77.52
3Y	92.88	92.88	92.85	92.79	92.19	91.84	91.49	91.15	90.28	89.41	88.54	87.67	86.81	82.82	80.51	79.71	78.90
4Y	93.63	93.63	93.72	93.92	93.31	92.70	92.09	91.47	90.60	89.73	88.86	87.99	87.12	82.58	79.71	78.72	77.72
5Y	95.00	95.00	94.73	94.18	93.13	92.49	91.85	91.22	90.35	89.48	88.61	87.74	86.88	82.03	78.84	77.76	76.67
6Y	93.44	93.44	93.17	92.63	91.60	90.76	90.14	89.72	88.58	87.65	86.72	85.79	85.25	80.17	76.92	75.73	74.72
7Y	92.57	92.57	92.30	91.77	90.74	89.70	89.09	88.68	87.50	86.54	85.58	84.62	84.06	78.95	75.66	74.40	73.32
8Y	91.35	91.35	91.09	90.56	89.55	88.52	87.92	87.51	86.31	85.32	84.34	83.35	82.76	77.52	74.09	72.86	71.67
9Y	90.43	90.43	90.17	89.65	88.65	87.50	86.76	86.22	84.96	83.90	82.85	81.81	81.15	75.81	72.26	71.06	69.77
10Y	88.80	88.80	88.54	88.03	87.04	85.84	85.05	84.45	83.10	81.95	80.80	79.67	79.10	73.65	70.13	68.97	67.68
12Y	83.90	84.30	83.65	83.57	82.64	81.50	80.74	80.18	78.89	77.80	76.71	75.64	75.10	69.92	66.58	65.48	64.25
15Y	77.85	79.36	77.63	78.67	77.79	76.72	76.01	75.48	74.27	73.24	72.22	71.20	71.20	65.82	62.68	61.64	60.92
20Y	70.79	72.16	70.59	71.54	70.74	70.04	69.67	69.45	68.34	67.39	66.45	65.52	66.28	60.57	57.68	56.72	56.71
30Y	62.14	63.34	61.96	62.79	62.09	61.00	60.20	59.54	58.59	57.78	56.97	56.17	56.83	51.93	49.45	48.62	48.62
40Y	53.81	54.85	53.66	54.38	53.77	52.83	52.14	51.57	50.74	50.03	49.34	48.64	49.21	44.97	42.82	42.11	42.11

Generating the volatility surface

- On top of the volatility matrix, we add an event calendar, where we can make certain dates more or less valuable
  - In the 1990s, many option pricing systems used a simple calendar weighting, so that an option decayed 3 days from Friday to Monday, which is incorrect
  - Several years ago, market players demanded 1 day options that reduced their risk on certain events, like NFP or CPI or FOMC.
  - Subtracting the weekends and adding additional days for major events alleviated pricing and risk management issues

Event Weight	Weight	Manual	Weekends (1.00)	NYK hol (0.75)	LON hol (0.30)	NFP 2.00	Fed 1.75	CPI 1.75	Fed mins 1.00	CBOT 0.00	Blank 0.00
25-Sep-06	0.00	0.00								0	
26-Sep-06	0.00	0.00								0	
27-Sep-06	0.00	0.00								0	
28-Sep-06	0.00	0.00								0	
29-Sep-06	0.00	0.00								0	
30-Sep-06	(1.00)	0.00	1							0	
1-Oct-06	(1.00)	0.00	1							0	
2-Oct-06	0.00	0.00								0	
3-Oct-06	0.00	0.00								0	
4-Oct-06	0.00	0.00								0	
5-Oct-06	0.00	0.00								0	
6-Oct-06	2.00	0.00				1				0	
7-Oct-06	(1.00)	0.00	1							0	
8-Oct-06	(1.00)	0.00	1							0	
9-Oct-06	(0.75)	0.00		1						0	
10-Oct-06	0.00	0.00								0	
11-Oct-06	0.00	0.00								0	
12-Oct-06	0.00	0.00								0	
13-Oct-06	0.00	0.00								0	
14-Oct-06	(1.00)	0.00	1							0	
15-Oct-06	(1.00)	0.00	1							0	
16-Oct-06	0.00	0.00								0	
17-Oct-06	0.00	0.00								0	
18-Oct-06	1.75	0.00						1		0	
19-Oct-06	0.00	0.00								0	
20-Oct-06	0.00	0.00								0	

BNP PARIBAS

SECTION 5

Trading Options 101

Trading Options 101

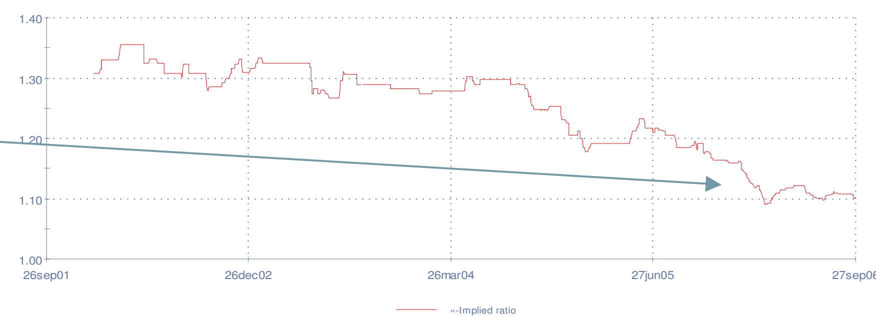
- **Options trade on a price basis.**
  - The implied vol helps us obtain the mid market price but bid-offer is dictated by upfront premium.
  - The model mid-mkt may not necessarily be where dealers are willing to execute
    - For example 5y30y 10% payers swaption prices in the system at 12.7 bps. Does that mean we are willing to sell it at 18.7 bps because the vega is 6 bps?
      - No. The cost of risk-managing an option for the life of the trade can cost way more than 6bps. The negative vanna and volga of this position is very high. A more sensible bid-offer on this particular structure could be 15 bps bid – 35 bps offered

- **The choice of date for option expiration is extremely important**
  - The calendar of events is critical for option pricing.
    - Owning a gamma option that includes Bernanke’s congressional testimony is worth much more than a regular day between Christmas and New Year’s

Trading Options 101

- **Large mortgage and hedge fund flows can disturb certain volatility sectors**
  - Many mortgage players are currently concerned with a further 50 bp rally on the yield curve, and have hedged themselves accordingly.
    - As a result, 2y2y -50 recvrs have richened vs 2y2y +50 payers

- **Large exotic flows can also heavily influence the volatility surface**
  - The vol differential between 3y10y and 3y30y collapsed from exotic yield curve option business



- In short, we must always be mindful of flows in pricing options

Trading Options 101

- **There are many ways to make money, or lose money, using options**
  - Naked option position
    - Hoping that a long option position expires in-the-money, more than the premium
  - Delta-hedging an option until maturity
    - If you are long an option, and the underlying moves more than the implied volatility for the life of the trade, you can generate gamma profits, even if the option expires out-of-the-money
  - Spread play between 2 different underlyings that expire on the same day
    - ex. buying 6m2y -25 otm recvrs vs selling 6m10y -25 recvrs
  - Calendar play between 2 different expirations on the same underlying
    - ex. sell 3m2y and buy 6m2y, hoping that gamma will outperform in 3 months time
  - Strike play. Same option expiry and underlying, but different strikes
    - ex. long 1 unit of edh07 95.00 call and short 1 unit of edh07 95.25 call. Synthetic long delta position

- **In order to use some of these strategies, we need a set of relative value tools gauge richness/cheapness…**

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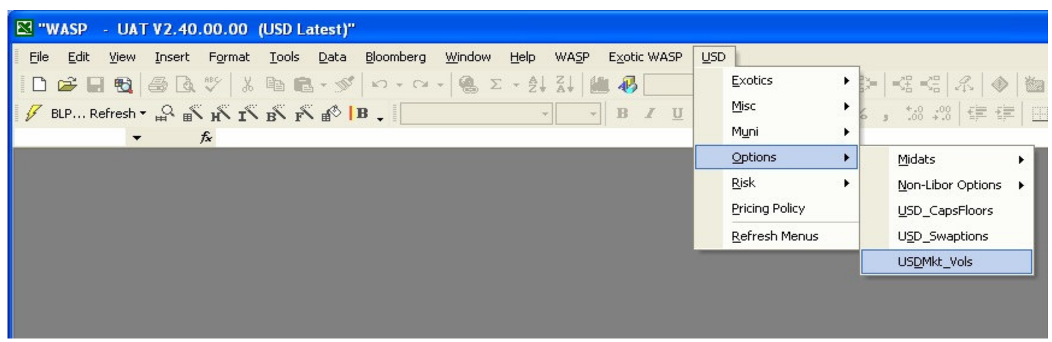
SECTION 6

Relative value on the volatility surface

Relative value on the volatility surface

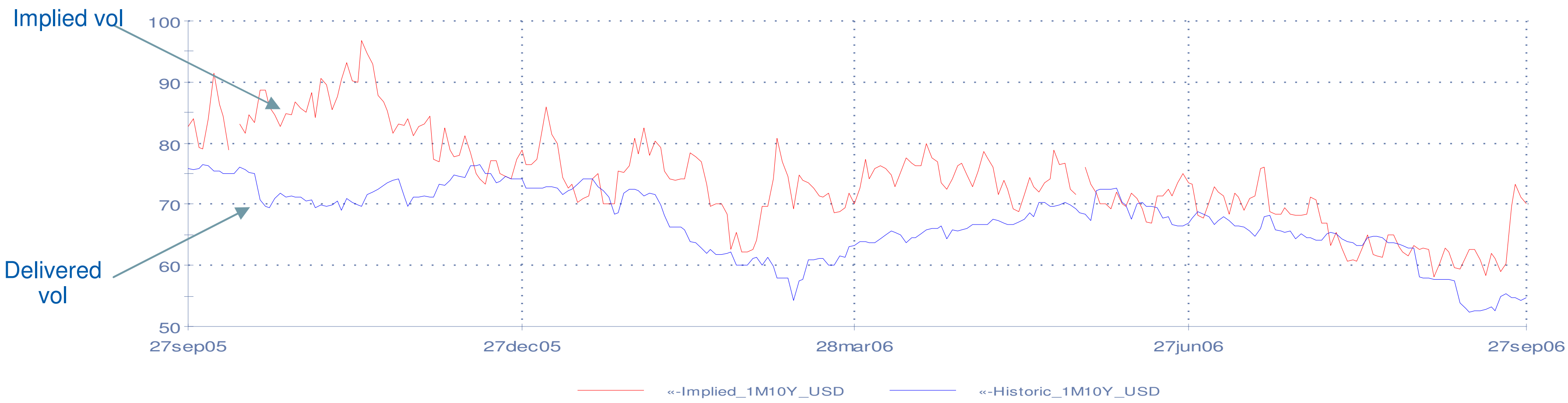
- Many option strategies involve holding an option position for days, months, or even years in order to realize a positive payout.
- Therefore, we need to look at the carry of a volatility position differently than just the option sensitivity to the five black-scholes greeks.
- Our first tool is historical volatility, as a measure of whether implieds is outperforming or underperforming daily breakevens
- MAG is a critical tool to obtain historical data. Using the USD_vols_pricer spreadsheet, we can easily access MAG.

	Single	Spread	Fly	
	1	2	3	
	USD	USD	USD	
	1M	1Y	1M	
	10Y	10Y	15Y	
	64.6	78.2	63.6	
		82.5%	122.1%	



Relative value on the volatility surface

- Let's look at 1m10y implieds versus delivered volatility of the 10yr swap rate



- Notice that implieds have been considerably greater than delivered volatility. A short gamma strategy, where you re-hedge your gamma every day, would have generated profits most of this year.

Relative value on the volatility surface

- Our second tool is "sliding volatility," which helps us measure the value of rolling down the volatility surface. This concept is similar to rolling down a swap curve.
  - For example, 9m2y swaption vol = 81.375 and 1y2y swaption vol = 85.375. If the vol surface retains this same shape as we move forward 3months, how much volatility must the 1y2y swaption deliver to breakeven?

$$\text{total variance} = \sigma * \sigma * \mathrm{T}$$

$$\text{total variance 9m2y} = 81.625 * 81.625 * 0.75 = 4966.42$$

$$\text{total variance 1y2y} = 85.25 * 85.25 * 1 = 7267.56$$

$$\sigma_{3m} = \sqrt{\frac{\text{total variance 1y2y - total variance 9m2y}}{\mathrm{T}_{3m}}}$$

$$\sigma_{3m} = \sqrt{\frac{7267.56 - 4966.42}{0.25}} = 95.30$$

- *** note that this is only a rough approximation ***
- *** this is not forward volatility ***

Relative value on the volatility surface

- Using the sliding volatility measure, we have another perspective on whether a certain vol sector is cheap or rich

BP vol	3M	1Y	2Y	5Y	10Y	15Y	20Y	25Y	30Y
4W	30.75	64.02	67.84	68.37	**63.62**	62.74	61.87	61.17	60.48
8W	30.75	64.02	69.40	70.12	**65.42**	64.27	63.13	62.42	61.71
13W	30.75	64.67	71.55	72.29	**67.61**	66.26	64.91	64.18	63.45
6M	58.88	**72.86**	**76.53**	**76.30**	**71.74**	**70.30**	**68.87**	**68.10**	**67.32**
9M	72.25	80.51	81.63	79.48	**75.12**	73.62	72.12	71.48	70.85
12M	80.38	**84.75**	**85.25**	**82.36**	**78.25**	**76.01**	**74.53**	**73.97**	**73.42**
2Y	90.13	90.80	90.34	88.17	**83.88**	80.44	78.63	77.85	77.06
3Y	92.25	92.35	91.85	90.66	**86.34**	82.38	80.08	79.28	78.48
4Y	93.13	93.55	92.97	91.03	**86.70**	82.18	79.33	78.33	77.34
5Y	94.63	**93.84**	**92.81**	**90.83**	**86.50**	**81.67**	**78.50**	**77.42**	**76.34**
7Y	92.20	91.43	90.43	88.29	**83.70**	78.61	75.34	74.08	73.01
10Y	88.45	87.71	86.74	84.09	**78.76**	73.33	69.83	68.67	67.39

Sliding	3M	1Y	2Y	5Y	10Y	15Y	20Y	25Y	30Y
4W	30.45	63.39	67.18	67.70	**62.99**	62.13	61.26	60.57	59.88
8W	30.45	63.39	71.94	73.04	**68.47**	66.79	65.10	64.37	63.64
13W	30.45	65.96	77.12	77.91	**73.35**	71.42	69.49	68.71	67.93
6M	110.80	**87.77**	**85.40**	**83.34**	**79.04**	**77.46**	**75.88**	**75.03**	**74.18**
9M	106.47	100.60	95.05	87.71	**83.93**	82.25	80.58	80.34	80.10
12M	108.12	**99.54**	**97.88**	**92.36**	**89.15**	**84.27**	**82.90**	**82.60**	**82.30**
2Y	103.16	98.81	96.12	98.39	**93.59**	87.40	84.42	82.86	81.30
3Y	97.54	95.99	95.38	96.98	**92.65**	87.21	83.53	82.70	81.86
4Y	95.63	97.29	96.45	91.61	**87.25**	80.59	75.52	73.75	71.98
5Y	100.91	**94.34**	**91.06**	**88.91**	**84.68**	**78.33**	**73.54**	**72.01**	**70.48**
7Y	85.09	84.38	83.46	79.94	**74.28**	69.06	65.48	63.70	62.09
10Y	69.79	69.21	68.45	63.78	**54.59**	47.23	43.99	43.26	41.81

- Note that the 1y2y swaption sector is 85.25, roughly midway between 9m2y @ 81.63 and 2y2y @ 90.34
- Looking at sliding vol, 1y2y is 97.88, actually more expensive than 9m2y @ 95.05 and 2y2y @ 96.12
- Why is this?
- Intuitively speaking, the volatility drop of -3.62 over a 3 month period has a more adverse effect on 1y2y than the volatility drop of -5.09 over a 1 year period on the 2y2y.
- Notice the 10y10y sliding vol is 54.59, 24.2 cheaper than the bpvol. Is this a screaming buy?

Relative value on the volatility surface

- Let us examine two ratios to develop a sense of richness and cheapness:
  - (1) the ratio of implied volatility / delivered volatility
  - (2) the ratio of sliding volatility / delivered volatility
    - Sectors less than 100% can be considered cheap, sectors greater than 100% can be considered expensive.
    - Notice 10y10y swaption vol does not look cheap @ 175% of delivered vol, even though it looked cheap on a sliding vol basis.
    - Notice 9m3m caplet is currently delivering well, but looks rich on the sliding vol ratio

History	3M	1Y	2Y	5Y	10Y	15Y	20Y	25Y	30Y
4W	213%	128%	106%	110%	113%	118%	122%	123%	123%
8W	138%	114%	104%	111%	115%	120%	123%	125%	125%
13W	93%	100%	102%	112%	119%	124%	127%	129%	129%
6M	99%	96%	103%	116%	125%	131%	134%	136%	136%
9M	95%	99%	108%	122%	132%	138%	141%	143%	144%
12M	96%	106%	114%	128%	139%	143%	147%	149%	150%
2Y	127%	127%	135%	148%	156%	158%	160%	161%	161%
3Y	142%	147%	153%	164%	169%	168%	168%	169%	168%
4Y	162%	163%	166%	174%	176%	173%	170%	170%	168%
5Y	175%	171%	173%	181%	180%	176%	172%	171%	168%
7Y	175%	184%	186%	186%	181%	174%	168%	166%	163%
10Y	194%	187%	185%	181%	175%	165%	157%	155%	152%

Sliding	3M	1Y	2Y	5Y	10Y	15Y	20Y	25Y	30Y
4W	213%	128%	106%	110%	113%	118%	122%	123%	123%
8W	138%	114%	109%	116%	122%	126%	128%	130%	130%
13W	93%	103%	111%	122%	131%	136%	138%	140%	141%
6M	189%	117%	116%	128%	140%	146%	150%	152%	152%
9M	142%	125%	126%	136%	149%	155%	159%	163%	165%
12M	131%	126%	132%	145%	160%	160%	165%	168%	169%
2Y	147%	140%	145%	167%	177%	174%	174%	174%	172%
3Y	152%	154%	160%	177%	183%	180%	177%	178%	177%
4Y	168%	171%	174%	177%	178%	171%	164%	162%	158%
5Y	188%	174%	171%	179%	178%	170%	162%	160%	157%
7Y	163%	171%	173%	170%	163%	155%	148%	144%	140%
10Y	154%	149%	147%	139%	123%	107%	100%	99%	95%

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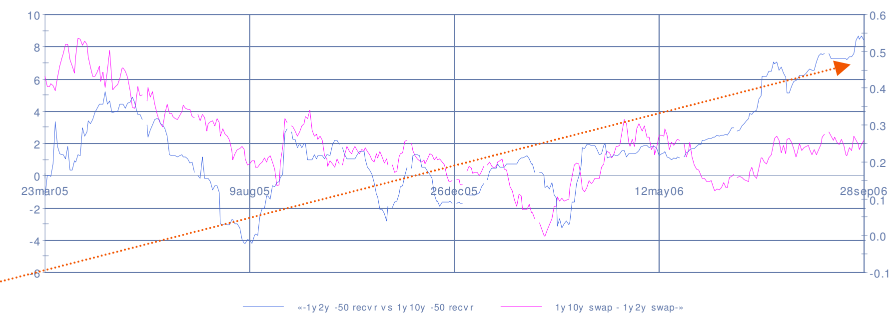
Relative value on the volatility surface

- **Trade Idea: Buy 1y2y -50 otm recvrs vs selling 1y10y -50 otm recvrs, to express the view that the curve will steepen if/when Fed eases**

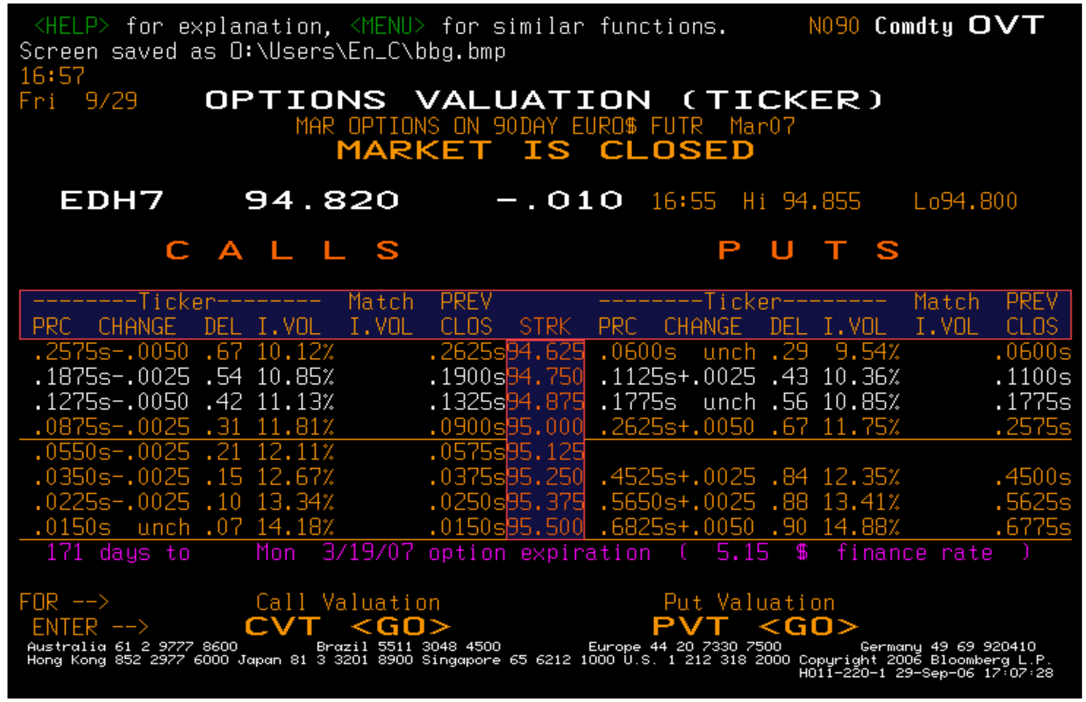
  - We saw hedge funds/mortgage accounts executing this trade in late spring
  - Notice how the normal vol spread jumped from +2 bpvol in May to currently +8 bpvol, even though the swap curve has been relatively static
  - Now it makes sense why 1y2y is one of the highest vol points on the sliding vol chart
  - Finding good trade ideas will involve extensive work in MAG, and an understanding of flows in that sector



Sliding	3M	1Y	2Y	5Y	10Y	15Y	20Y	25Y	30Y
4W	30.75	63.40	67.21	67.72	62.52	61.66	60.80	60.12	59.43
8W	30.75	63.40	71.86	72.95	67.84	66.18	64.53	63.80	63.08
13W	30.75	65.97	77.15	77.23	73.47	71.54	69.61	68.83	68.04
6M	111.90	88.37	86.03	84.32	80.31	78.44	76.58	75.71	74.85
9M	107.53	101.70	96.15	88.76	84.92	83.12	81.31	81.07	80.82
12M	109.40	101.25	99.58	93.96	90.69	86.09	85.03	84.71	84.40
2Y	104.38	99.72	97.72	99.76	94.90	88.61	85.57	83.99	82.40
3Y	98.49	97.22	96.64	97.92	93.54	88.06	84.35	83.51	82.66
4Y	96.58	97.91	97.10	92.52	88.11	81.39	76.27	74.48	72.69
5Y	101.91	95.24	92.03	89.79	85.52	79.11	74.27	72.73	71.18
7Y	85.94	85.19	84.34	80.73	75.02	69.74	66.14	64.34	62.71
10Y	70.49	69.88	69.18	64.42	55.13	47.70	44.43	43.69	42.22

Relative value on the volatility surface

- **Trade Idea: EDH7 Fed Ease play**
  - Buy 1 Unit EDH7 95.125, Sell 1 Unit EDH7 95.375 call
    - Strategy costs 5.5 – 2.25 = 3.25 mid-mkt
    - If you believe in the possibility of 2 Fed eases, paying 3.25 ticks for a maximum payout of 25 appears attractive
    - A quick sharp rally can also provide an opportunity to make a few ticks profit, because this trade is a synthetic long delta position
    - The sharp call skew works against us, as do the low forward rates. Theta is our enemy.

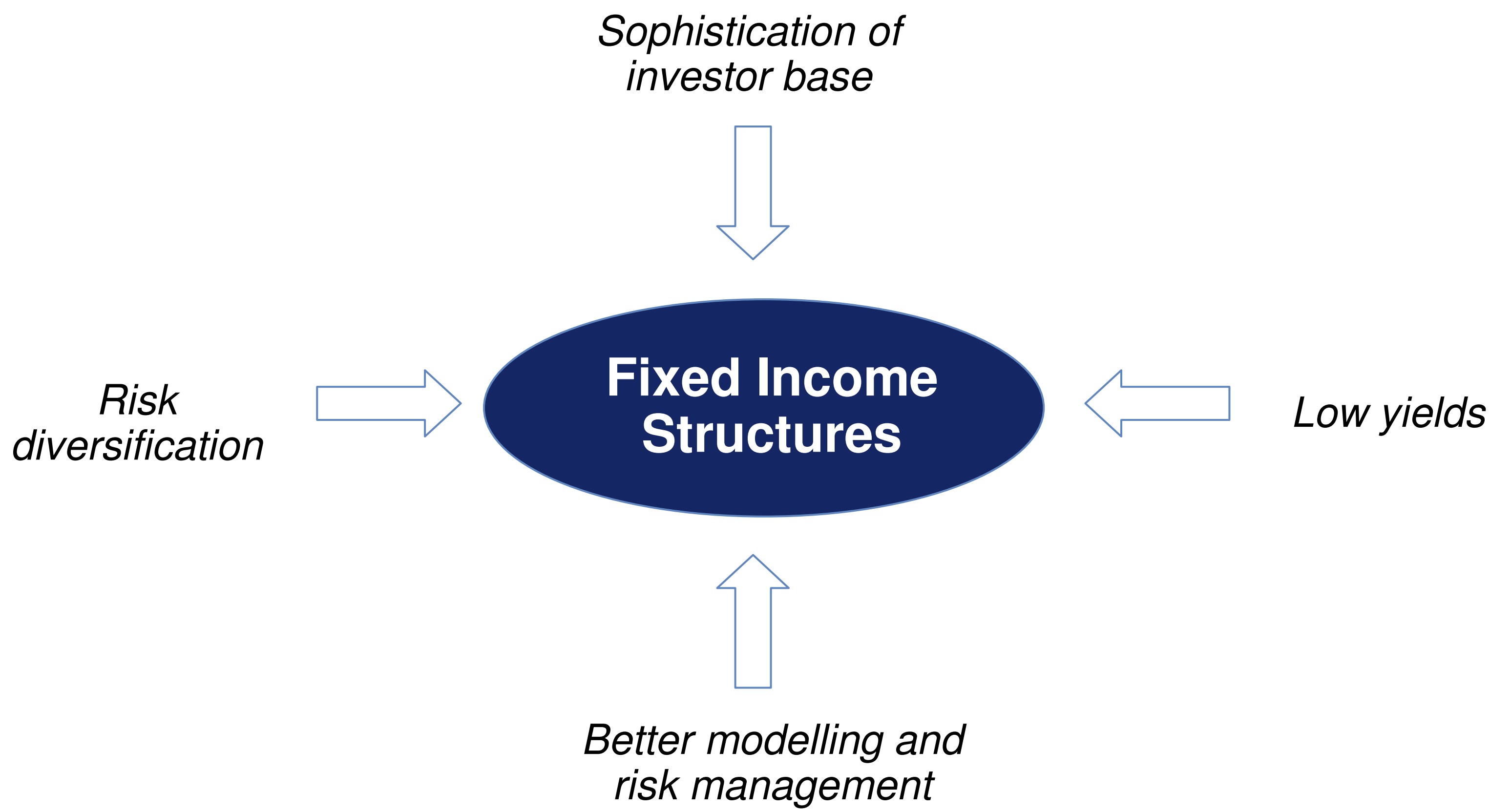


Expiry	Underlying	Strike	Type	Size	Price	Vega multi	Vega /1000	Vol bp	Skew bp
H07	H07	95.125	CALL	2,000	5.32	1.21	30.2	59.36	3.09
H07	H07	95.375	CALL	(2,000)	1.97	0.74	18.4	62.57	6.30

SECTION 7

Recent innovations and trends in structured products

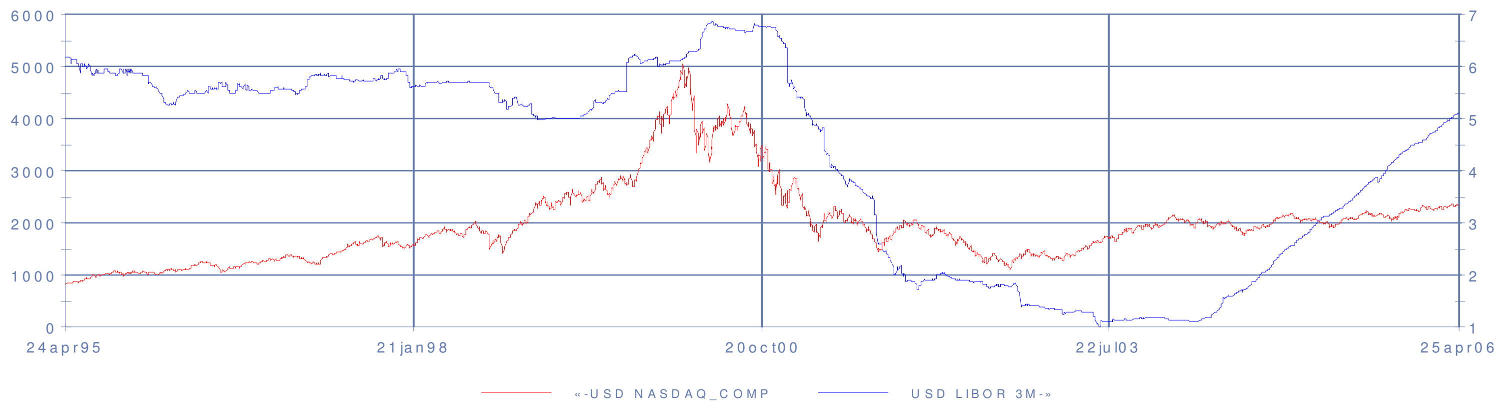
2006 a fine vintage for structured products



Where have we come from?

1990's: "Exotic" structures dominated callable bonds and Libor-linked inverse floaters

Equity bull run: *"But how do we know when irrational exuberance has unduly escalated asset values?"* (Alan Greenspan, 5-Dec-1996)



2001: Return of the structured rate product

In Japan, **Power-Reverse Dual Currency notes**

- Risk management questions => correlation between rates and FX

In Europe, **Callable Range Accrual notes**

- Modelling questions: How to calibrate on digitals and integrate callability?

What has helped?

Analytical approaches adopted in finance – stochastic calculus

Raw computing power: BNP Paribas' worldwide processor "farms" rank in Global Top 10

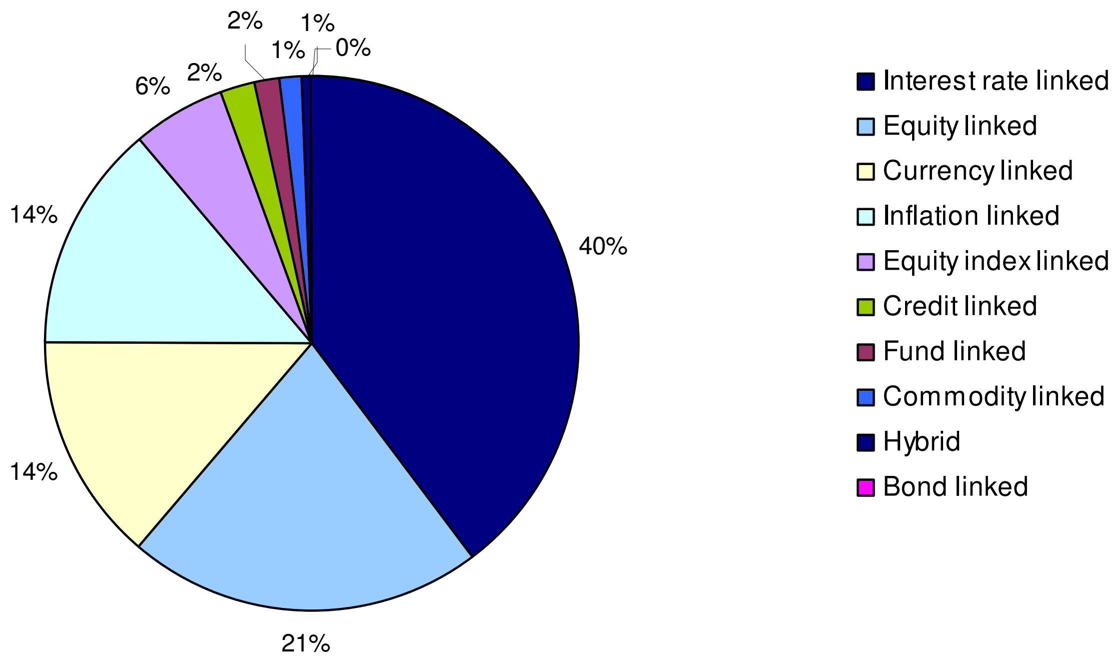
Deeper, **liquid European markets**

- European Monetary Union in January 1999 => swap curve the benchmark
- Active Euro option markets and two-way flows => ability to trade in size and to recycle risk
- Warehousing of large risks by well-capitalised financial institutions

BNP PARIBAS — Page 1 of 2

6 Month Euribor EUR IRS	Size	BNPP Pays	BNPP Receives	Size	Time
1Y ABB vs 6m					8:49
18M ABB vs 6m					8:49
2Y ABB vs 6m	300	3.600	3.620	300	9:19
3Y ABB vs 6m	300	3.745	3.765	300	9:20
4Y ABB vs 6m	300	3.847	3.867	300	9:24
5Y ABB vs 6m	300	3.924	3.944	300	9:24
6Y ABB vs 6m	300	3.990	4.010	300	9:24
7Y ABB vs 6m	300	4.052	4.062	300	9:24
8Y ABB vs 6m	300	4.105	4.115	300	9:24
9Y ABB vs 6m	300	4.154	4.164	300	9:24
10Y ABB vs 6m	300	4.199	4.209	300	9:23

Medium-term Notes: What have investors bought in 2007 so far?



Source: MTN-I

Underlyings of notes issued between 1 Jan and 9 March 2007, based on the value of issues.

Example of structured notes

USD CMS Steepener Note, quantoed in EURO

Issue Price:	90%	
Tenor: 10y, non-call 12m		
Coupon:	Y 1:	6.00%
	Y 2-10:	10 x($10y - 2y swap spread) + 2%
	Coupon capped at 9%, floored at 0%	

"ICE" Note *Protect against inflation*

Tenor: 10y

Coupon: Min (6m USD LIBOR + 0.45%, 245%*Inflation)

Example of structured notes (2)

Basket Options –multiple underlyings *Bullish particular sector / region*

Typically combinations of emerging market currencies, commodities, and equities

Tenor: 1y

Redemption: 100% + 157% * max(0, Basket performance)

Basket: average of USDIDR, USDINR, USDJPY, USDMYR, USDTWD returns

Hybrid – Bull/ Bear notes *Benefit from correlation*

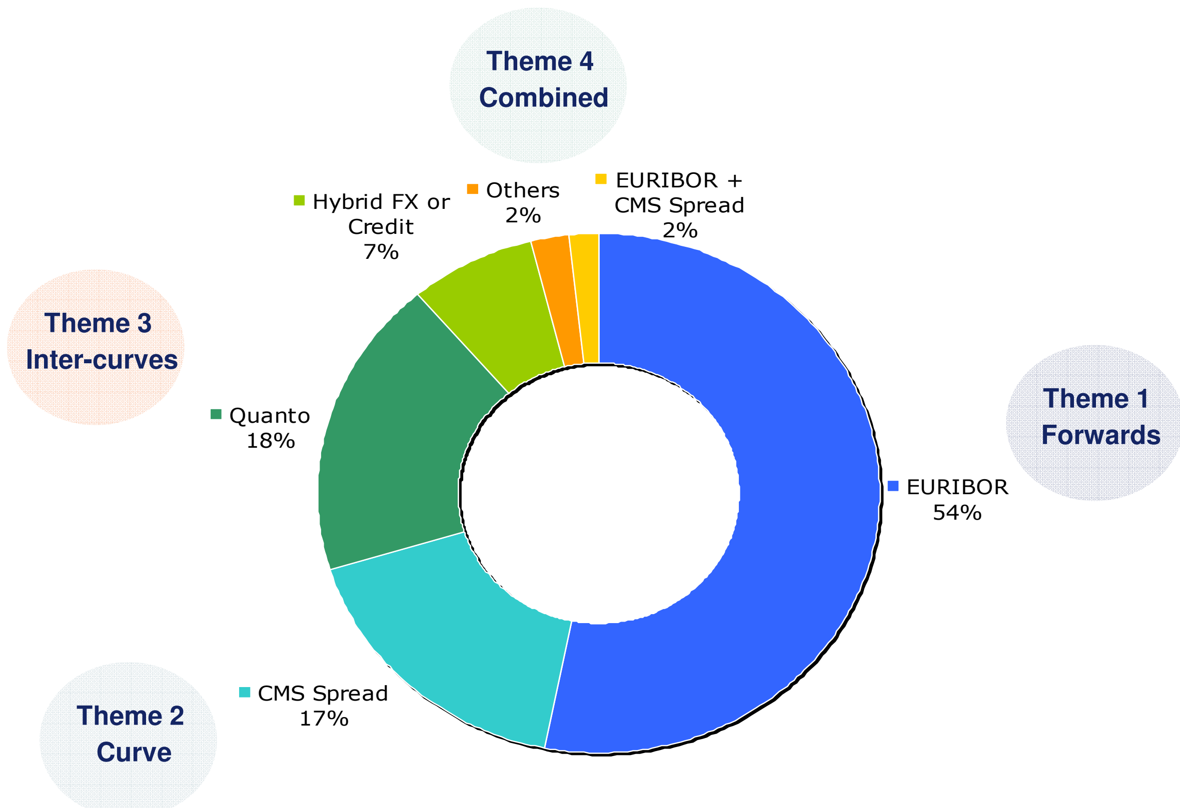
Investor benefits when all baskets/ or underlyings move together up or down from a reference price

Tenors: 5y

Baskets: FX (CNYUSD, SGDUSD, JPYUSD), Equity (SDY, Hang Seng, Nikkei), Commodity (Aluminium, Copper, Zinc)

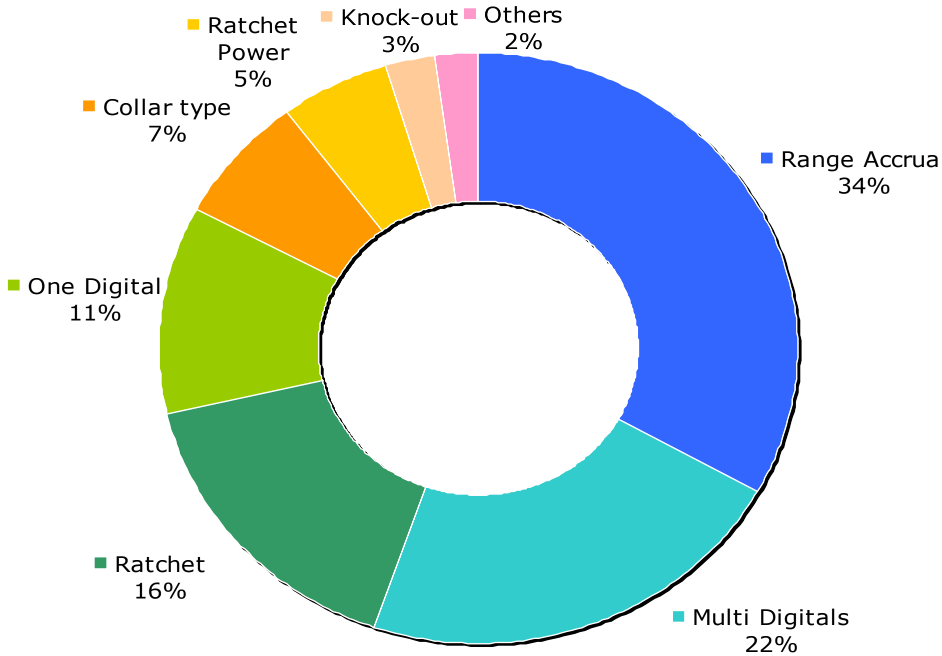
Coupon: Y 1: 7.00% Y2-5: If all baskets return <0 or >0, than 8.50% Otherwise: 0.00%

Popular views by liability managers



From 2005/2006 BNP Paribas trade-blotters with European Liability Managers

Popular features by liability managers



From 2005/2006 BNP Paribas trade-blotters with European Liability Managers

Range Accruals, Ratchets and Digital Combinations formed 75% of structures

Proposed structures: over-estimation of forwards

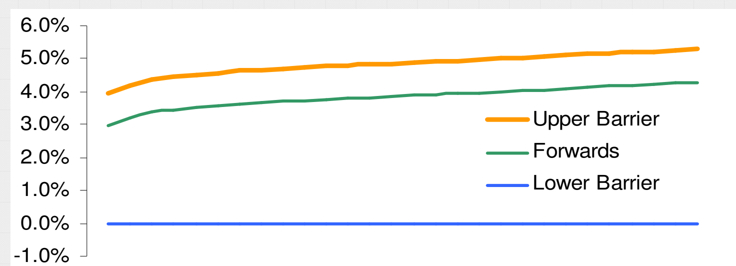
Theme 1
Forwards

POWER LIABILITY SWAP ON 3M EURIBOR

- Provides a high discount when Euribor stays within a range. **(RANGE ACCRUAL FEATURE)**
- Rate paid in a certain period also depends on previous behaviour of Euribor. **(RATCHET FEATURE)**
- Once discount decreases, it can not increase later. **(POWER FEATURE)**
- **Expected positive carry : 1.00%**

Fixed Rate Version

7y swap (Quarterly payments, Act/360)



Liability Manager Receives 3.80%

Liability Manager Pays 6.80% - Discount

Discount = 4.00% for first period then Discount = Previous Discount x (n/N)

n is number of days when 3m Euribor is between 0% and current forwards + 1.00%

BNP PARIBAS

Proposed structures: over-estimation of forwards (2)

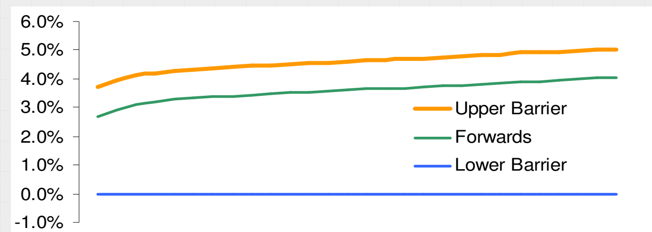
Theme 1
Forwards

RESETTABLE POWER LIABILITY SWAP ON 3M EURIBOR

- Provides a high discount when Euribor stays within a range. **(RANGE ACCRUAL FEATURE)**
- Rate paid in a certain period also depends on previous behaviour of Euribor. **(RATCHET FEATURE)**
- Discount is reset after 3 years to the initial discount. **(RESETTABLE POWER FEATURE)**
- **Expected positive carry : 1.25%**

Floating Rate Version

7y swap (Quarterly payments, Act/360)



Liability Manager Receives 3.80%

Liability Manager Pays 130% x 3m Euribor - Discount

Discount = 1.25% for first period then Discount = Previous Discount x (n/N)

n is number of days when 3m Euribor is between 0% and current forwards + 0.75%

At the end of year 3 Discount is reset to 1.25% and then is Previous Discount x (n/N)

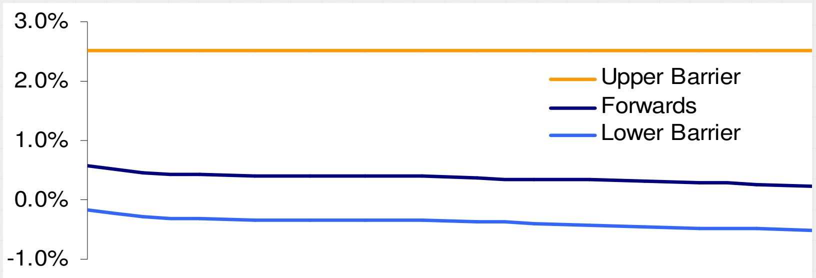
Proposed structures: under-estimation of spreads

Theme 2
Curve

POWER STEEPENER SWAP ON 10Y – 2Y EUR SWAP SPREAD

- Provides a high discount when the spread stays within a range. **(RANGE ACCRUAL FEATURE)**
- Rate paid in a certain period also depends on previous behaviour of Euribor. **(RATCHET FEATURE)**
- Once discount decreases, it can not increase later. **(POWER FEATURE)**
- **Expected positive carry : 1.20%**

7y swap (Quarterly payments, Act/360)



Liability Manager Receives 3.80%

Liability Manager Pays 6.80% - Discount

Discount = 4.20% for first period then Discount = Previous Discount x (n/N)

n is nb of days when 10y-2y spread is between current forwards - 0.75% and 2.50%

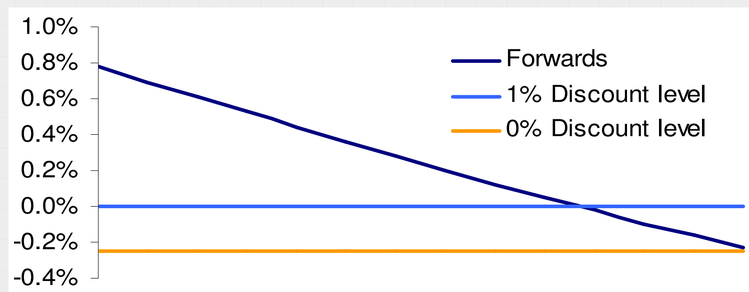
Proposed structures: inter-curve lack of substance

Theme 3
Inter-curves

QUANTO 10Y GBP – 10Y EUR SPREAD

- Provides a good discount in the first years.
- Thereafter discount stays positive as long as the spread is above -25bp. **(QUANTO FEATURE)**
- Discount can increase or decrease in any period.
- **Expected positive carry : 1.00%**

30y swap (Annual payments, Act/360)



Liability Manager Receives 12m Euribor
Liability Manager Pays 12m Euribor - 0.85% for 5y
Thereafter Pays 12m Euribor - 1.00% - 4* (10Y GBP - 10Y EUR)

Theme 4
Combined

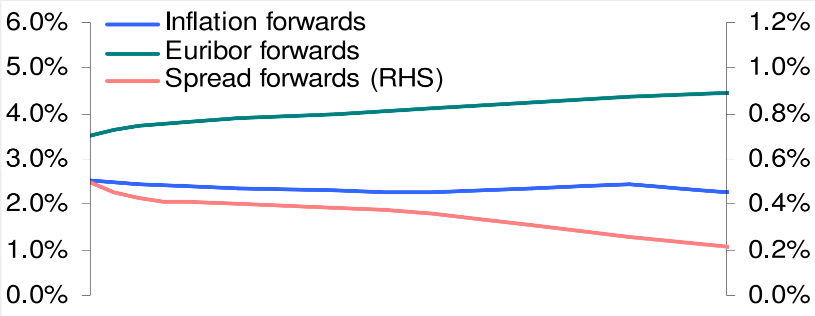
Proposed structures: hybrids

MAGIC LIABILITY ON 10Y – 2Y EUR SWAP SPREAD

- Provides a good discount as long as the spread is positive.
- Total rate paid is floored at the Euro zone inflation.
- Inflation floor provides **extra value** compared to a fixed floor (high volatility).

<u>EUR Version</u>

10y swap (Annual payments, Act/360)



Liability Manager Receives 12m Euribor

Liability Manager Pays 12m Euribor - 5* (10Y EUR - 2Y EUR - 0.28%), floored at YoY European HICP (and floored at 0%)

Proposed structures: hybrids (2)

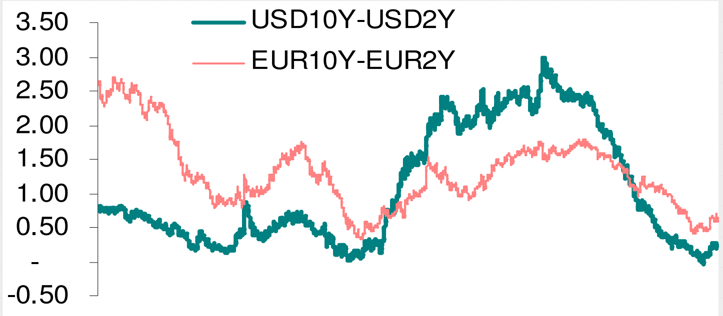
Theme 4
Combined

MAGIC LIABILITY ON 10Y – 2Y USD SWAP SPREAD

- Provides a good discount as long as the usd spread is positive.
- Total rate paid is floored at the Euro zone inflation.
- Inflation floor provides **extra value** compared to a fixed floor (high volatility).

<u>Quanto Version</u>

20y swap (Annual payments, Act/360)



Liability Manager Receives 12m Euribor

Liability Manager Pays 12m Euribor - 5* (10Y USD - 2Y USD - 0.17%), floored at YoY European HICP (and floored at 0%)

Recap : popular structures in 2006

Forwards are often over-estimated
POWER LIABILITY SWAPS

Combinations bring even more value
MAGIC LIABILITY SWAPS

Curve spreads are often under-estimated
POWER STEEPENER SWAPS

Some inter-curve spreads lack of substance
QUANTO CMS SPREAD SWAPS

SECTION 8

Case study: the EUR 10y-2y Steepener

CASE STUDY: the EUR 10y-2y Steepener

- Birth of the product
  - Forwards spread were very low compared to historical data
  - Different structures capture the same client view but have opposite correlation sensitivities. For example, digital on the spread versus call on the spread

- Risk management and pricing
  - Specific platform using the in-house copula pricing consistent with the in-house yield curve models
  - Risk and reserves defined in a unified framework allowing netting between correlation positions
  - Proper cross-risk management is

Typical structure

- BNPP Receives EURIBOR3M quarterly
- BNPP Pays 4 x (SWAP10Y - SWAP2Y-K) floored at 0%
- The vast majority of EUR structures are European
- In the callable case BNPP has the right to call the structure after 1Y and annually thereafter
- The forward price is E 4 x max (SWAP10Y - SWAP2Y-K,0)
- The above expectation refers to the forward measure to the option maturity
- Consequently we need to specify the joint distribution of CMS10Y and CMS2Y

Dynamic model

- From the market we get information about the distributions of CMS10Y and CMS2Y (under the forward measure to the option maturity)
- Assumptions must be made about the joint distribution of CMS10Y and CMS2Y
- One way to proceed is to specify a joint dynamic model for the two rates
- One could adopt a SABR like model for both. This would mean a 4 dimensional diffusion process with 4 correlated Brownian motions
- A Heston like model would also do a good job
- The cross-gamma – rate & rate, rate & vol and vol & vol are linked with the relevant correlations

Correlation: What is the correct value?

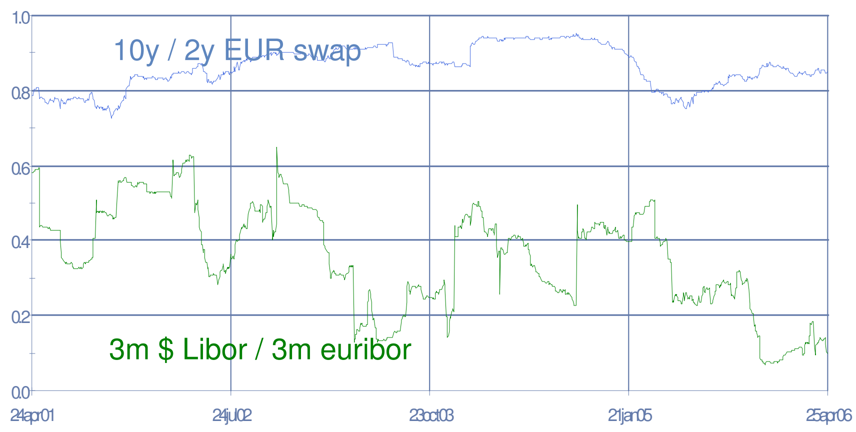
Historical correlation is easily calculated, but is it stable?

- Bad events tend to be highly correlated

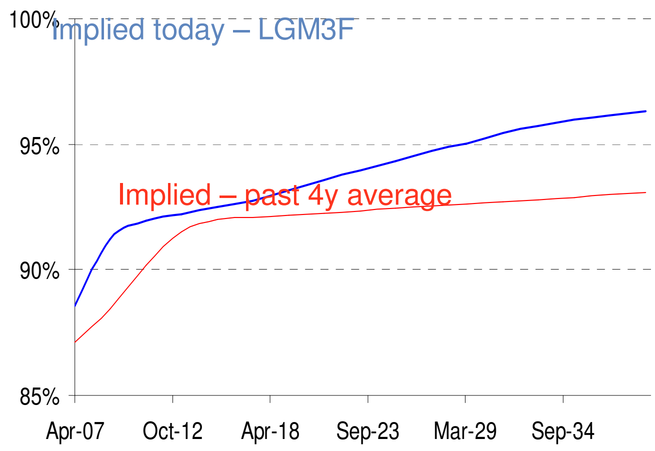
Implied correlation is perhaps a better indicator

- Represents the experience of many traders in capturing cross-gamma
- Unfortunately there is no liquid market currently

Realised correlation – 126 days



Term structure of EUR 10y / 2y correlation



Copula

- As previously we take the marginal distributions of CMS10Y and CMS2Y from their respective smiles for a given maturity. Note that we do not need to specify the dynamics but only the distribution at maturity

- Moreover, the CMS distributions can be implied using a replication with cash swaptions

- We need to specify the joint distribution of CMS10Y and CMS2Y at maturity. In fact the distribution of CMS10Y – CMS2Y is enough

- To this end we choose a copula function. In principle we can take any, but each choice will generate different risk management and price

- Gaussian copula is a market standard but other copula functions are more suitable given the spread option sensitivity

Copula choice

- For example, we can try to choose a copula which is consistent with and close to the copula generated by our favourite dynamic model
- We gain a lot in terms of speed, precision and representation of risk
- A Delta and Vega hedged spread option book will show important cross Delta/Gamma Vega/Vanna risks which can be linked to the copula parameters (forward correlations, volatilities, correlations,...)
- The Copula approach is hence natural
- However, we may generate inconsistencies across products and cannot price callable structures

Impact of a call option – Analysis of risk

- For example, the structure can specify that BNPP has the right to call the structure after 1Y and annually thereafter
- The right to call (bermudan) can be broken down into the right to call the trade at a given date (OTC) and a switch option
- The OTC is an option on the difference between a string of spread options and a funding leg
- The Switch Option is mainly dependent on the forward volatility of the underlying

Impact of a call option – Pricing and risk management

- The structure is priced in a framework consistent with the one used for other structures which allows possible netting of exotic risks

- A term structure model is used only to price the right to call

- The identification of the relevant hedging instruments (the ones that are linked to the underlying of the option) lead to the specification of the best calibration set

Impact of a call option – Term structure models

- LGM3FSV model
  - 3 factors on the curve
  - 1 factor on the volatility
  - Product specific calibration
  - Product specific pricing and risk management

- LIBOR MARKET MODEL
  - Models directly forward LIBOR dynamics and hence multifactor on the curve
  - 1 factor on the volatility
  - Generic calibration
  - Benchmark pricing and risk management

Can we generate two-way correlation flows?

Parallel with Credit

- Developing CDO tranche market on Itraxx (Europe) and CDX (USA) indices
- 0-3% equity tranche: long correlation Senior tranches: short correlation

In FX and rates markets

- Very limited inter-bank and broker flows as dealers rebalance their positions

But some structures with **opposite correlations can be built**

CMS Spread Floater

Tenor: 9y, non-call 1y

BNPP rec: Euribor 3m

BNPP pays: **7 x (CMS 10 – CMS2) + 1.00%**
Floored at 0%, capped at 7%

BNPP LONG CORRELATION

"SCAN" - Spread Callable Accrual Note

Tenor: 10y, non-call 1y

BNPP rec: Euribor 3m

BNPP pays: **6.10% * (n/N)**
n : # of days when CMS10-CMS2 > 0

BNPP SHORT CORRELATION

The 10y-2y steepener: a unique opportunity today

CMS Spread Floater - NOW	
Tenor:	9y, non-call 6m and quarterly thereafter
BNPP rec:	Euribor 3m
BNPP pays:	**20 x (CMS 10 – CMS2) + 4.00%** Floored at 0%, capped at 20%

- The CMS Spread Floater on the previous page was in fact priced in 2005.
- The same structure priced under current market conditions gives a much higher gearing, plus a comfortable margin and a high cap.
- The morale is the following: buy cheap, not expensive!

Afterword

NUMBER CRUNCHING

15’000

FOR NUMBER OF STRUCTURED TRADES BOOKED AT BNP PARIBAS

2’848’148’805

FOR NUMBER OF WEEKLY RISK CALCULATIONS IN BNP PARIBAS STRUCTURED TRADES SYSTEM

5^{th}

FOR BNP PARIBAS RISK COMPUTER RANKING WORLDWIDE

1^{st}

FOR BNP PARIBAS IN INTEREST RATE DERIVATIVES

INTEREST RATE DERIVATIVES
HOUSE OF THE YEAR 2006

“BNP Paribas’ scaling up and integration of its fixed-income marketing, combined with its new-found confidence in sharing its exotic trading ideas, has made the French dealer one of the premier crus in 2005.”

“Risk”, January 2006

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