Université d’Evry
Séance 6

Strategies de courbe à base de swaptions
&
Straddles et strangles de swaptions

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Overview

- **An alternative to conventional bond/swap curve positions**
- **How to build the trade ?**
- **A duration weighted trade**
- **Importance of the volatility ratio**
- **A better risk/reward profile : case study**
- **Marked-to-market risk**
- **Appendix 1: Delta of an ATMF payer/receiver swaption**
- **Appendix 2: Link between the volatility ratio and the premium of a CCT**

An alternative to bond/swap curve positions

- **Conventional curve positions are built using bonds and swaps**

- **Steepeners**
  - **sell the longer term bond and buy the shorter one**
  - **pay the fixed rate in the longer term swap and receive it in the shorter one**

- **Flatteners**
  - **buy the longer term bond and sell the shorter one**
  - **receive the fixed rate in the longer term swap and pay it in the shorter one**

- **Conditional curve trades (CCT) through swaptions represent an interesting alternative to conventional positions**

How to build the trades ?

- **Bullish (bearish) curve trades are built with ATMF receiver (payer) swaptions**
- **Four kinds of trades**

	Steepeners	Flatteners
Bullish	Buy receiver on short tails Sell receiver on long tails	Sell receiver on short tails Buy receiver on long tails
Bearish	Sell payer on short tails Buy payer on long tails	Buy payer on short tails Sell payer on long tails

  - **Example: In a three month conditional 10s30s bull flattener, we sell the receiver swaption on the ten year tails and buy the receiver swaption on the 30 year tails**

How to build the trades ?

- **Bullish (bearish) curve trades are built with ATMF receiver (payer) swaptions**
- **Four kinds of trades**

	Barbells	Butterflies
Bullish	Sell receiver on short tails Buy receiver on body Sell receiver on long tails	Buy receiver on short tails Sell receiver on body Buy receiver on long tails
Bearish	buy payer on short tails sell payer on body buy payer on long tails	Sell payer on short tails Buy payer on body Sell payer on long tails

  - **Example: In a 3 month conditional 2s10s30s bullish barbell, we buy the receiver swaption on the ten year tails and sell the receiver swaptions on the two- and 30-year tails**

A duration weighted trade

- **The nominal on the longer tails is normalized to 100 million**
- **The nominal on the shorter tails is equal to**

$$100mn \times MD_l / MD_s$$

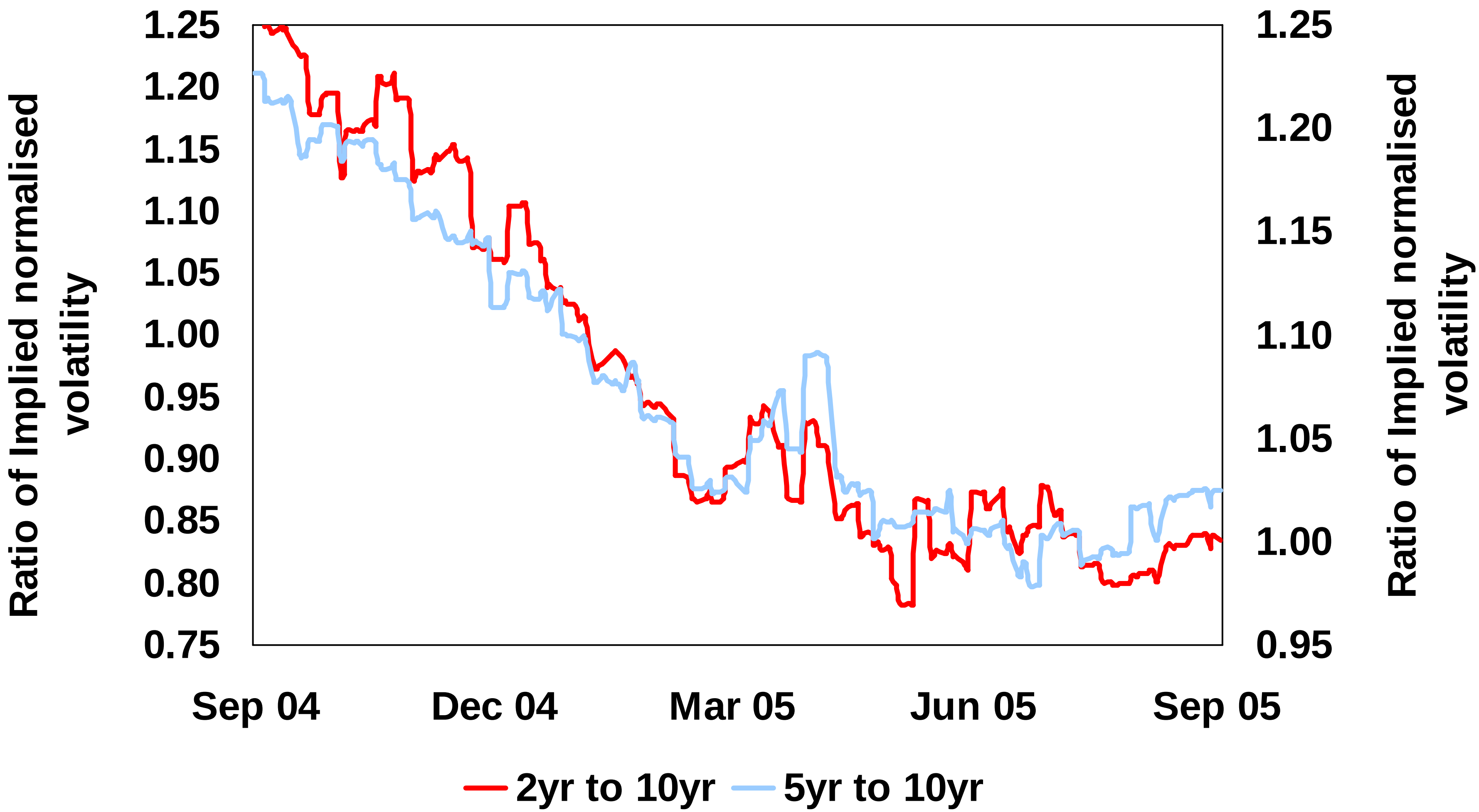
where $MD_l \text{ and } MD_s$ **are the modified durations of the long and short tails**

- **It is equivalent to a delta neutral trade ATMF because the delta of an ATMF payer (or receiver) swaption is very close to half the modified duration of the swap (see Appendix 2)**
- **Conditional curve trades can be compared with conventional as they give rise to the same economic exposure at expiration**

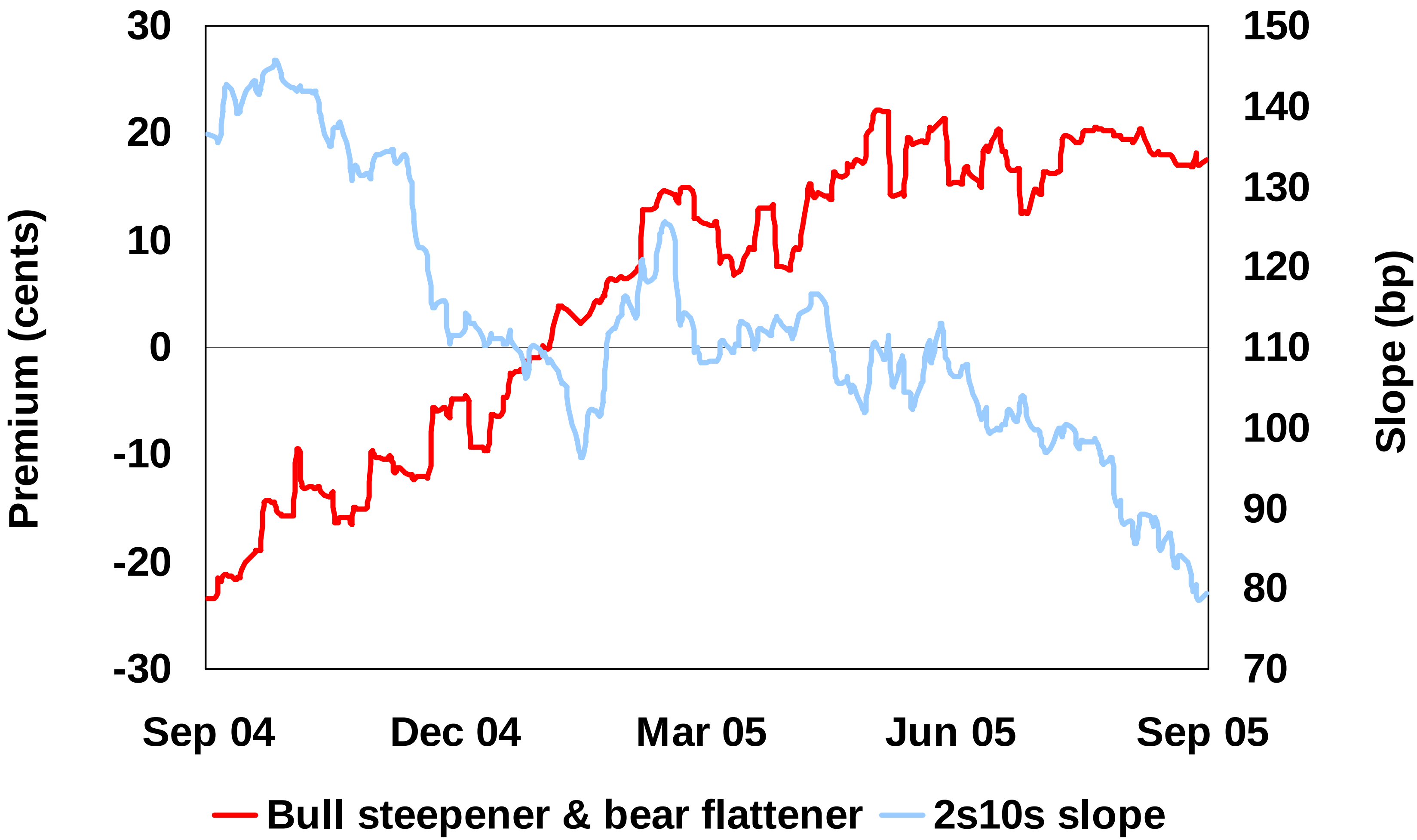
Importance of the normalized volatility ratio

- Premium of ATMF structure is mainly a function of the normalized volatility ratios between the two tails
- When the ratio shorter tails/ longer tails is superior to 1
  - bull flattener and bear steepener have a positive premium at inception
  - bear flattener and bull steepener cost money
- When the ratio shorter tails/ longer tails is inferior to 1
  - bull flattener and bear steepener cost money
  - bear flattener and bull steepener have a positive premium at inception
- See Appendix 3 for a mathematical proof
- By following these ratios, we can detect historical high or low premium

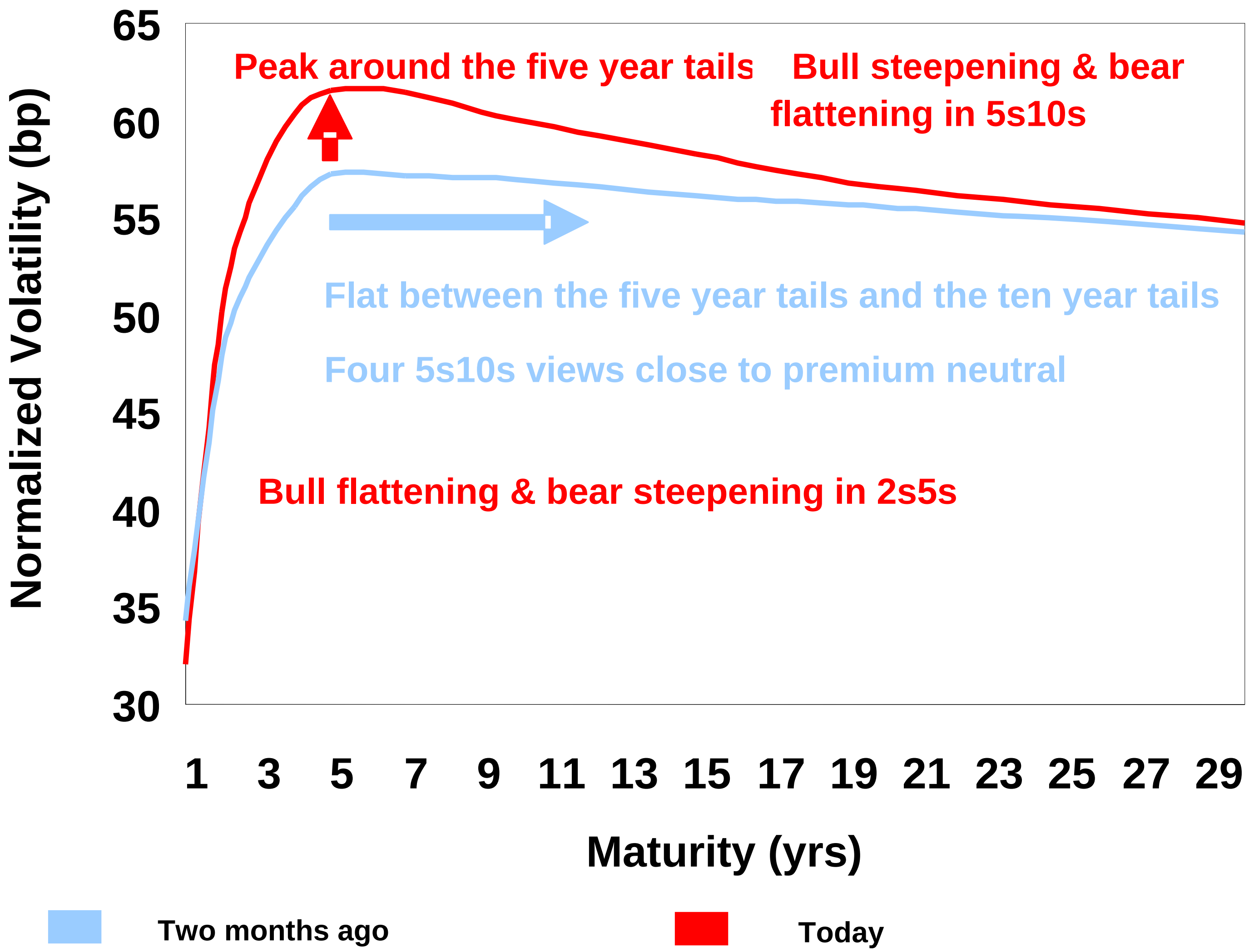
EUR 3m2y/3m10y volatility ratio



Historical premium of the EUR three month conditional 2s10s bear flattener



Term structure of short-dated volatility



EUR bullish curve trades through three month ATMF receiver swaptions (100mn on the longer maturity leg)

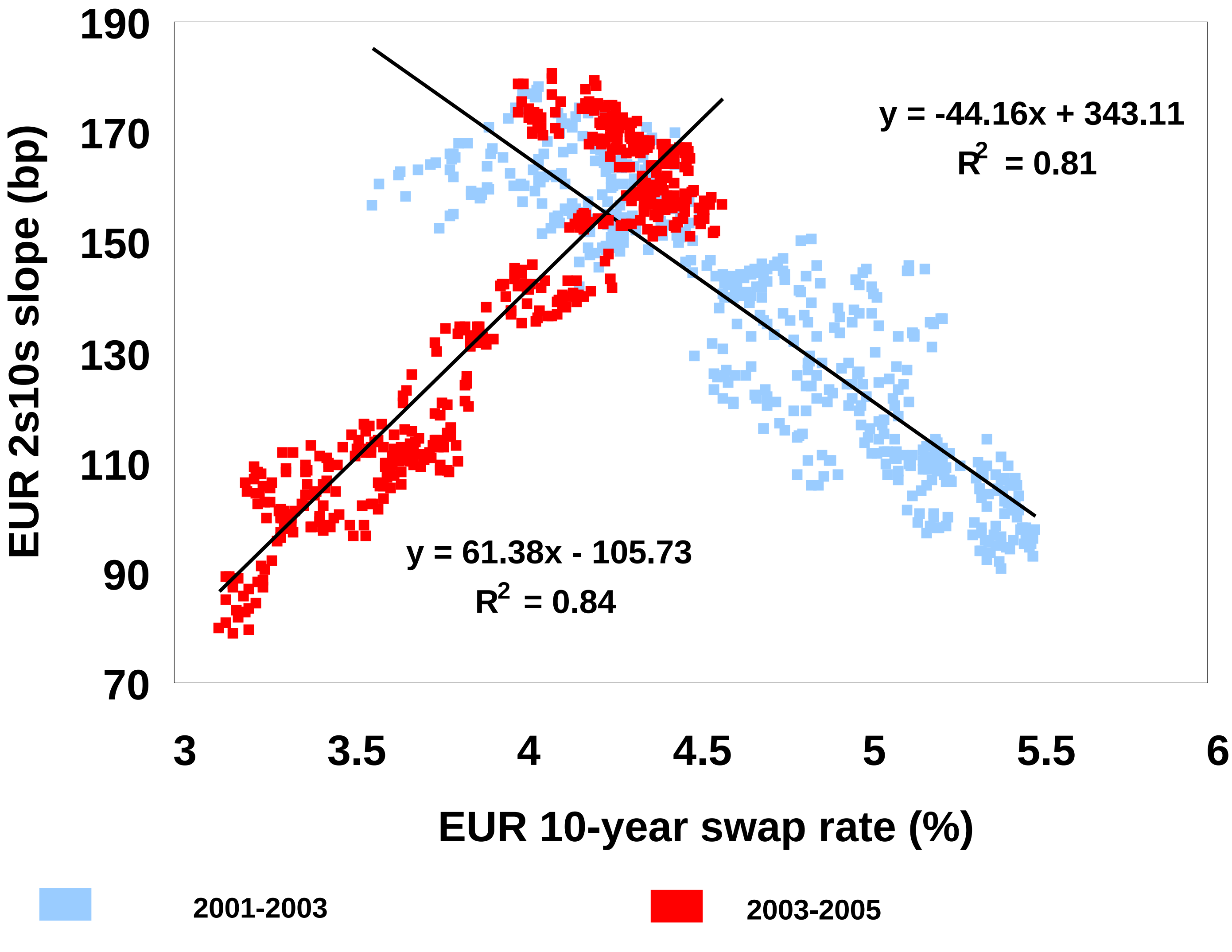
Sell

Buy	1yr	2yr	3yr	4yr	5yr	7yr	10yr	20yr	30yr
1yr		+64,897	+134,499	+202,721	+260,937	+354,248	+459,775	+682,170	+795,076
2yr	-74,463		+31,481	+67,459	+94,569	+129,208	+155,617	+176,498	+151,534
3yr	-149,330	-46,312		+16,389	+31,754	+44,241	+40,777	-14,426	-91,445
4yr	-223,061	-87,800	-36,729		-912	+54	-18,945	-113,717	-217,807
5yr	-286,968	-120,600	-57,785	-25,119		-16,318	-41,072	-150,504	-264,624
7yr	-392,042	-167,002	-82,034	-37,847	-21,475		-44,558	-156,298	-271,998
10yr	-515,470	-211,312	-96,472	-36,750	-14,623	-11,137		-128,517	-236,643
20yr	-787,641	-281,969	-91,044	+8,246	+45,033	+50,828	+23,047		-140,199
30yr	-932,200	-288,658	-45,678	+80,683	+127,500	+134,875	+99,519	+3,076	

--- Bull flatteners ---

------------------ Bull steepeners --

Directionality of steepeners trades w.r.t the general level of interest rates

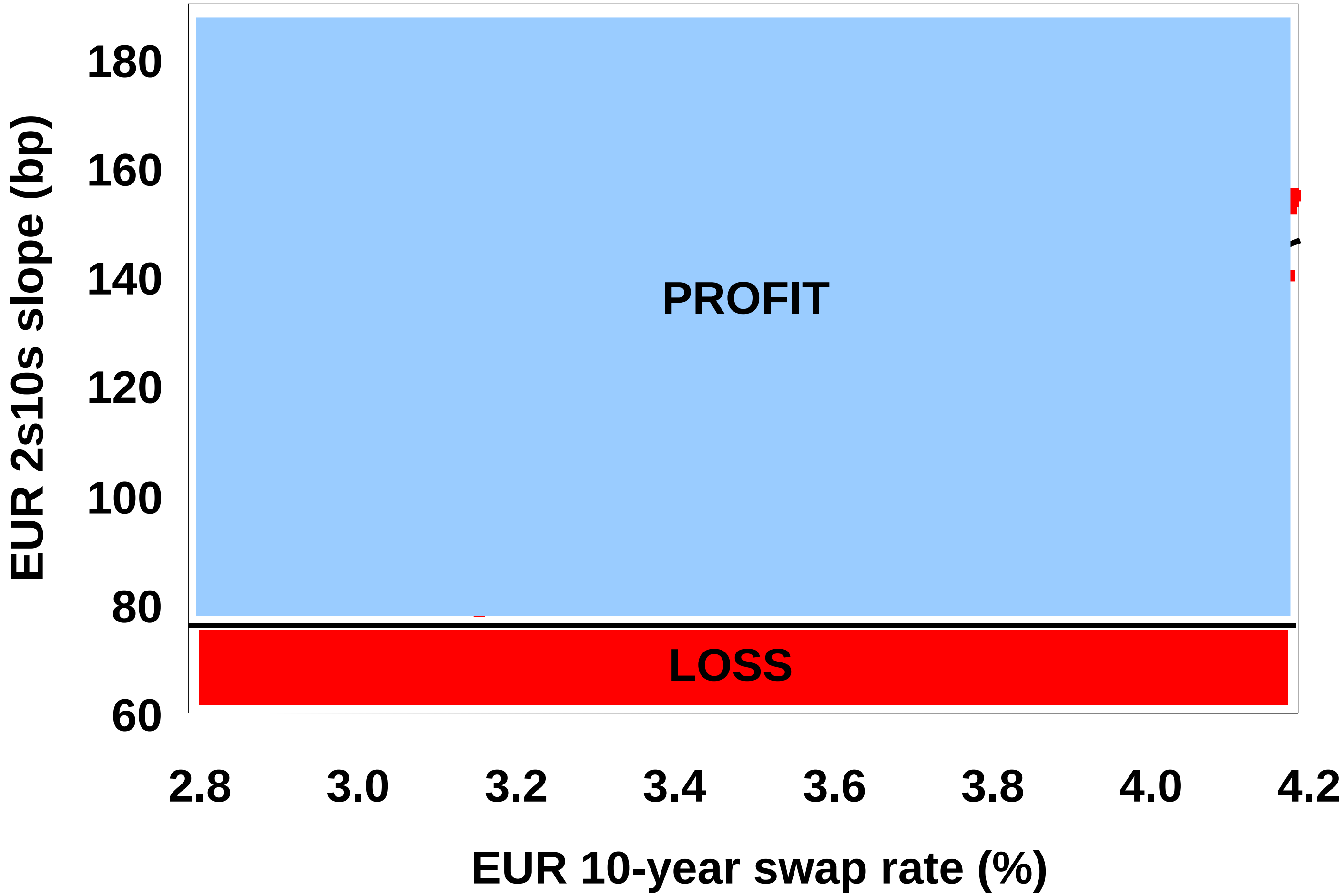


Trade Construction via swaps

- **Example : positioning a EUR 2s10s steepener three month forward**

	Spot	Fwd	Cost
Pay 100mn 3mth-into-10yr	3.25%	3.30%	0K
Receive 436mn 3mth-into-2yr	2.45%	2.53%	0k
	80bp	77bp	0k

P&L at expiration for a conventional steepener



Trade Construction via swaptions

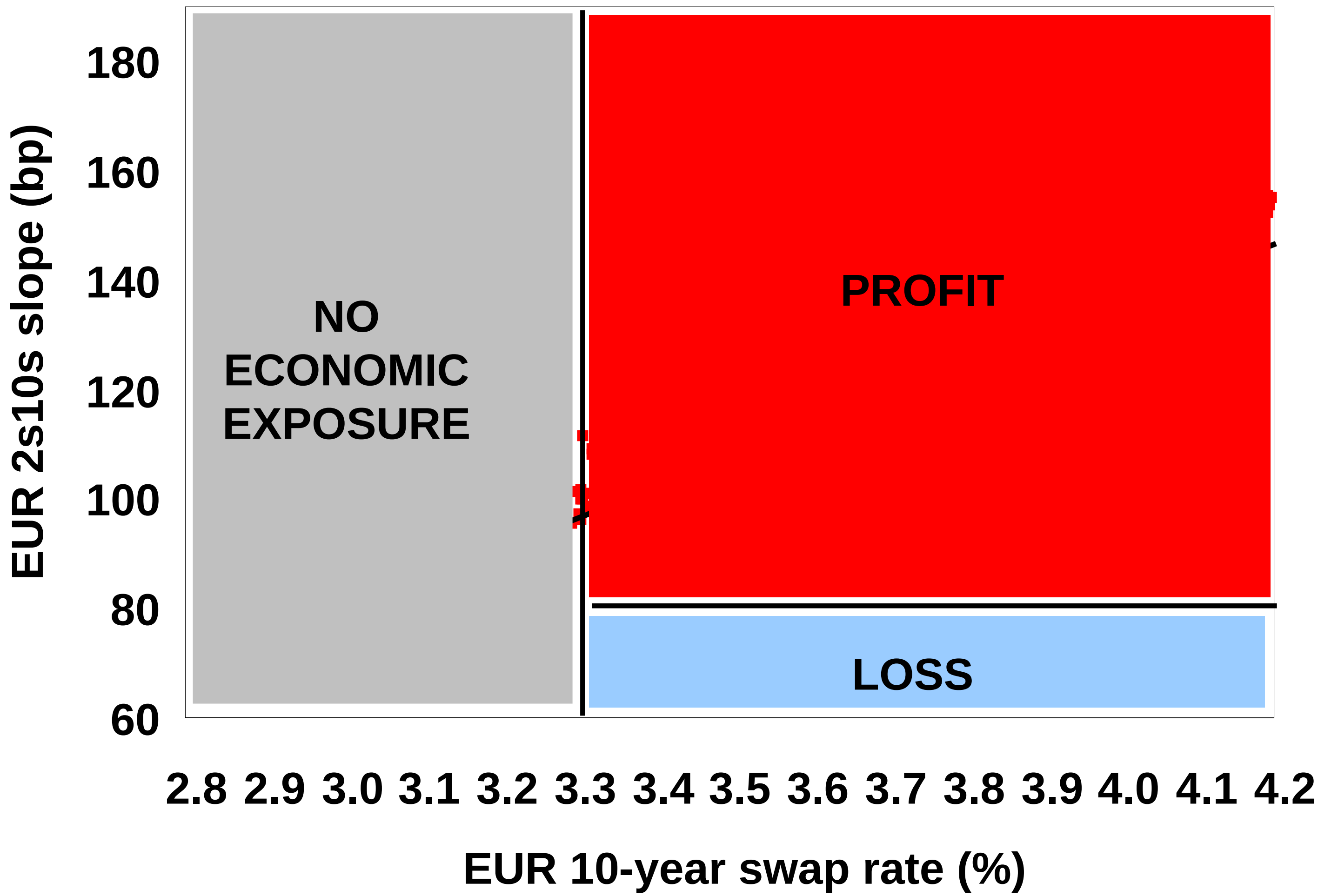
- Example : positioning a EUR three month 2s10s bear steepener

	Spot	Fwd	Strike	BPVol	Premium
• Buy100mn 3mth-into-10yr payer	3.25%	3.30%	3.30%	59.9	1,015K
• Sell 436mn 3mth-into-2yr payer	2.45%	2.53%	2.53%	50.7	840K
	80bp	77bp	77bp	1.18	175K

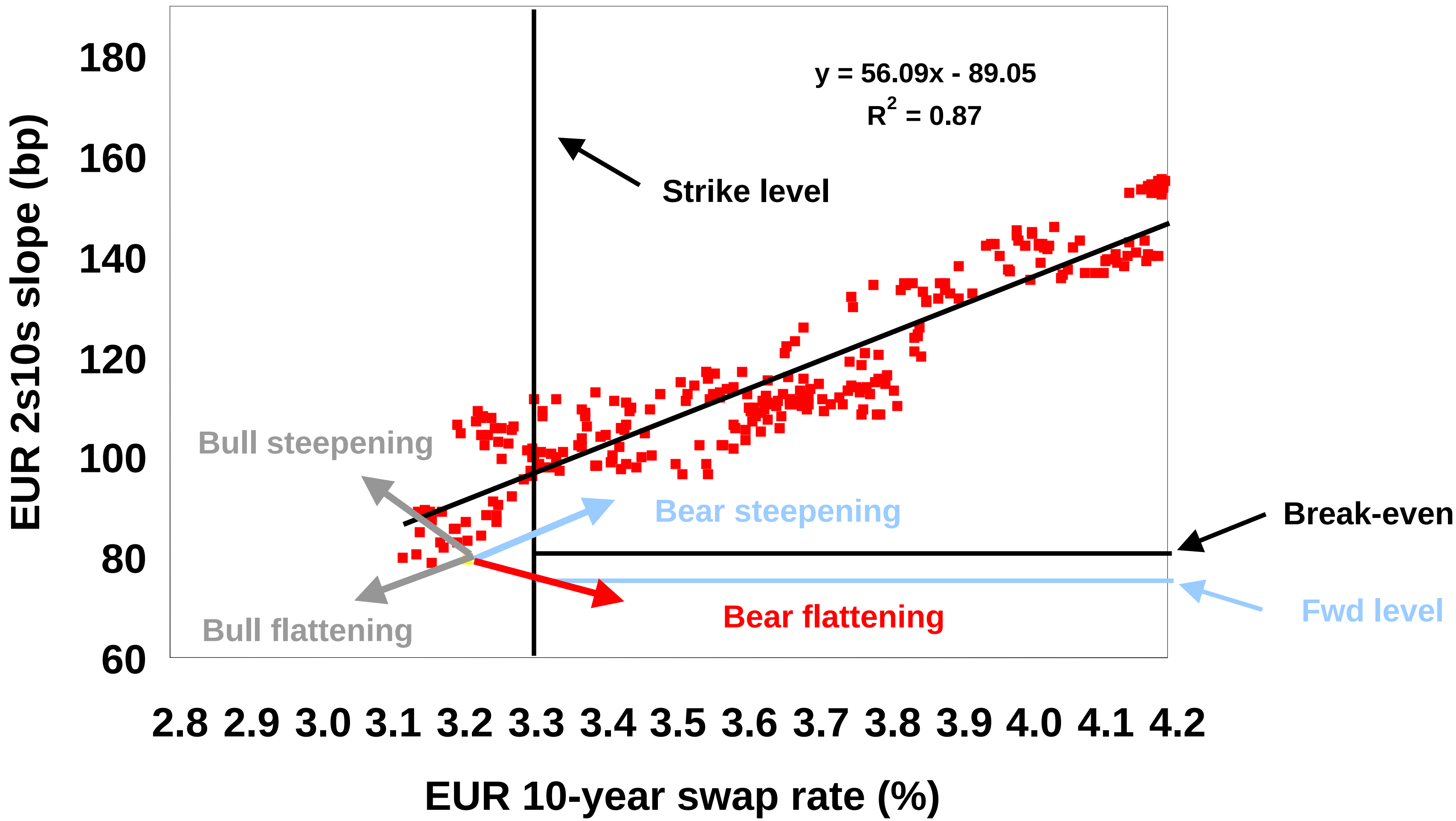
- Premium neutral structure is favoured

3.25%	3.30%	3.30%	59.8	1,015K
2.45%	2.53%	2.49%	50.7.	1,015K
80bp	77bp	81bp	1.18	0K

P&L at expiration for a bear steepener 2s10s



Scenario analysis



Scenario analysis

- **We consider either a 25bp rally or sell-off resulting in a 15bp flattening or steepening (beta of 0.6)**

- **Bull steepening scenario : probability of 10%**

	Swaps	Swaptions
Slope at expiration	95bp	95bp
P&L at expiration	+18bp	0bp

- **Bull flattening scenario : probability of 40%**

	Swaps	Swaptions
Slope at expiration	65bp	65bp
P&L at expiration	-12bp	0bp

Scenario analysis

- Bear steepening scenario : probability of 40%

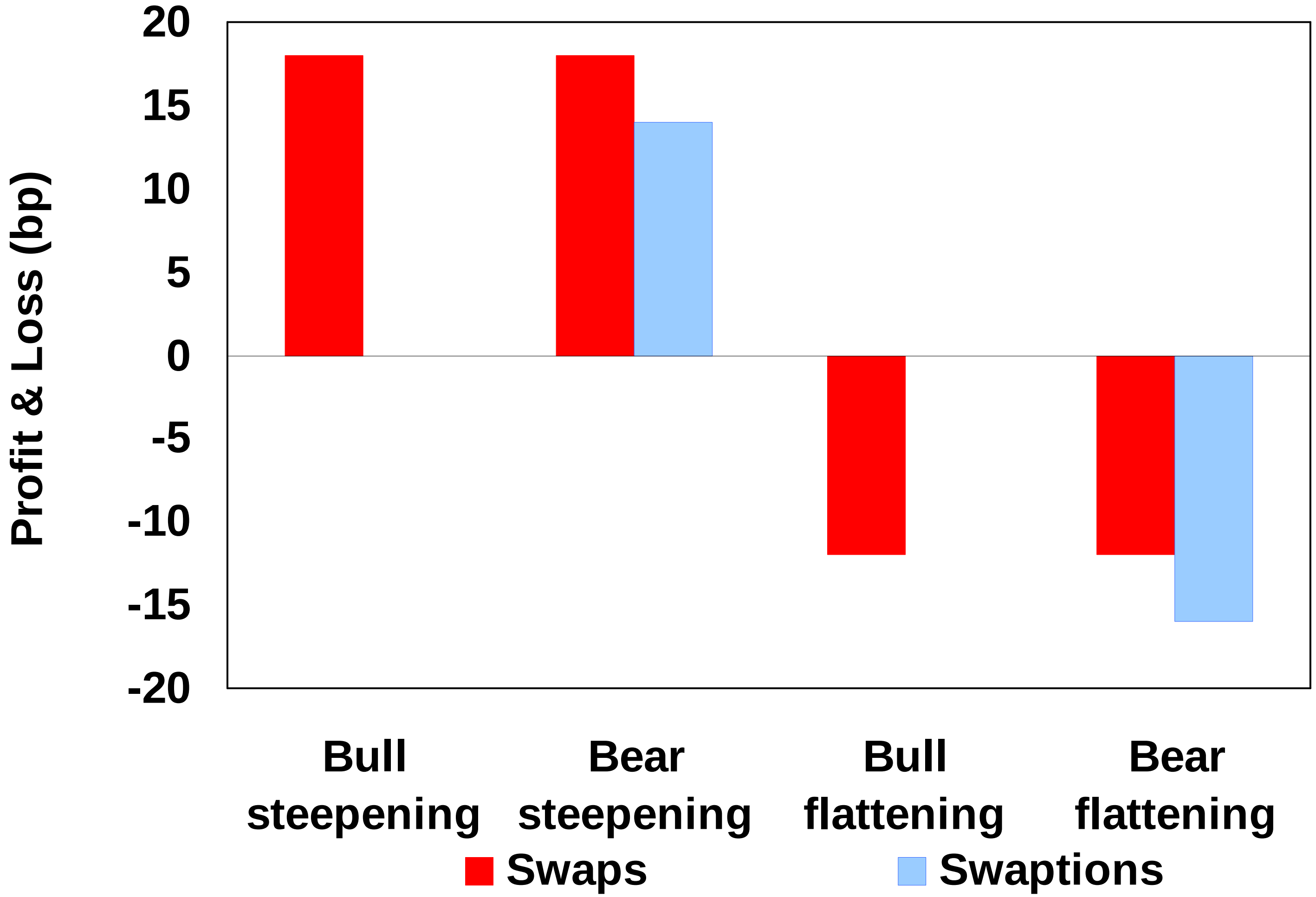
	Swaps	Swaptions
Slope at expiration	95bp	95bp
P&L at expiration	+18bp	+14bp

- Bear flattening scenario : probability of 10%

	Swaps	Swaptions
Slope at expiration	65bp	65bp
P&L at expiration	-12bp	-16bp

- Additional protection costs money

Summary of the scenario analysis at expiration



Risk Return analysis

- **We calculate the expected return of each trade**

$$E(R)=\sum R_i P(R=R_i)$$

- **Expected return of the conventional position**

40% * 18 + 40%* -12 + 10% * 18 + 10%*-12 = 3bp

- **Expected return of the conditional trade**

40% * 14 + 40%* 0 + 10% * 0 + 10%*-16 = 4bp

- **Volatility of the return of the conventional trade = 15bp**

- **Volatility of the return of the conditional trade = 9.7bp**

- **So Risk adjusted return of 0.4 for the conditional trade versus 0.2 for the unconditional trade**

A more flexible risk/reward profile

- Allow investor to take exposure conditional on a view
- Less risky and a more flexible way to position a curve trade compared with an outright position in swap/bond position as :
  - Contrarians views offers better conditional P&L than outright positions due to the premium take-in at trade inception
  - Market's views offer less favourable conditional P&L than outright positions due to the initial outlay of the positions (cost of additional protection)
- Variations of the trade
  - setting one of the strikes OTMF to make the trade premium-neutral
  - setting the strikes OTMF to make the trade will not create any economic exposure at expiration in an adverse scenario unless the market moves significantly (below\above strikes) but skew effects
  - Position contrarian trades with the view to keep premium take-in
- but the investors still remains exposed to a mark-to-market risk

Marked-to-Market risk

- **Before expiration, the trade is also exposed to a mark-to-market risk due to**
  - **an adverse movement in the slope**
  - **an adverse movement in the implied volatilities spread**

- **The marked-to-market risk can be approximated in the following way :**

$$
\begin{aligned}
P/L \approx & \left(\frac{Delta\ at\ inception + Delta\ in\ 1\ mth}{2}\right) \times yield\ change \\
& + 0.5 \times \left(\frac{Gamma\ at\ inception + Gamma\ in\ 1\ mth}{2}\right) \times (yield\ change)^2 \\
& + \left(\frac{Vega\ at\ inception + Vega\ in\ 1\ mth}{2}\right) \times volatility\ change \\
& + Theta\ effect
\end{aligned}
$$

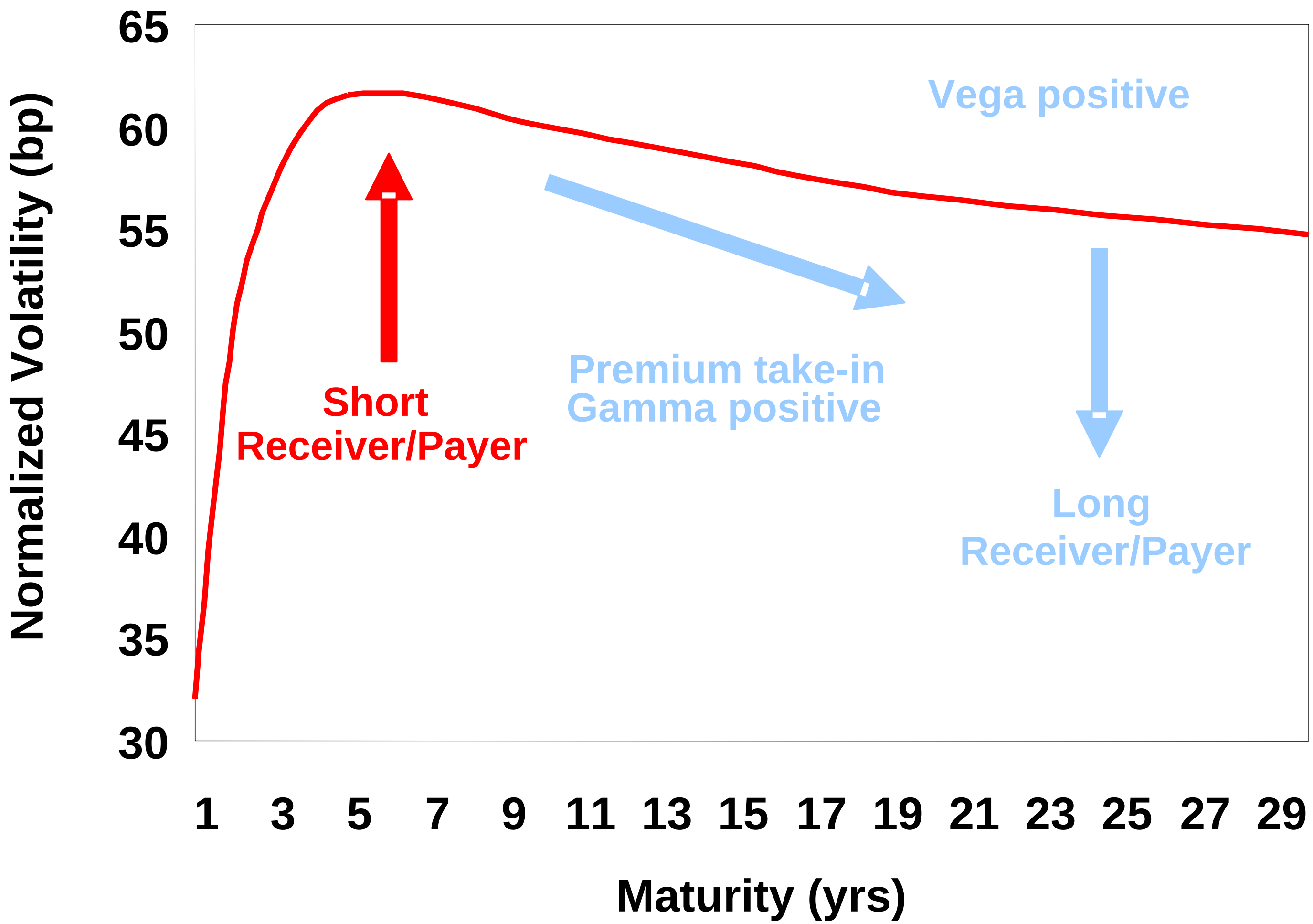
Marked-to-market risk

- **Conditional curve trades outperform conventional bond/swap positions in terms of risk/reward profile**
- **Example**
  - **the 5yr and 30yr swap rates are equal to 3.07% and 4.04%. The spot 5s30s slope is 97bp**
  - **As we expect the curve to bull flatten, we put on a 3-month conditional 5s30s bull flattener**
  - **ATMF strikes are equal to 3.15% and 4.07%. Forwards imply a 5bp flattening from spot levels over three month**
  - **the strikes are set 30bp OTMF rather than ATMF. The trade construction implies buying a receiver swaption on 30 year tails struck at 3.77% and selling a receiver swaption on 5 year tails struck at 2.85%**
  - **the trade provides an initial premium take-in of +0.6bp**
  - **at the option maturity, we compare our trade with a conventional 5s30s three month forward swap position in different yield scenarios**

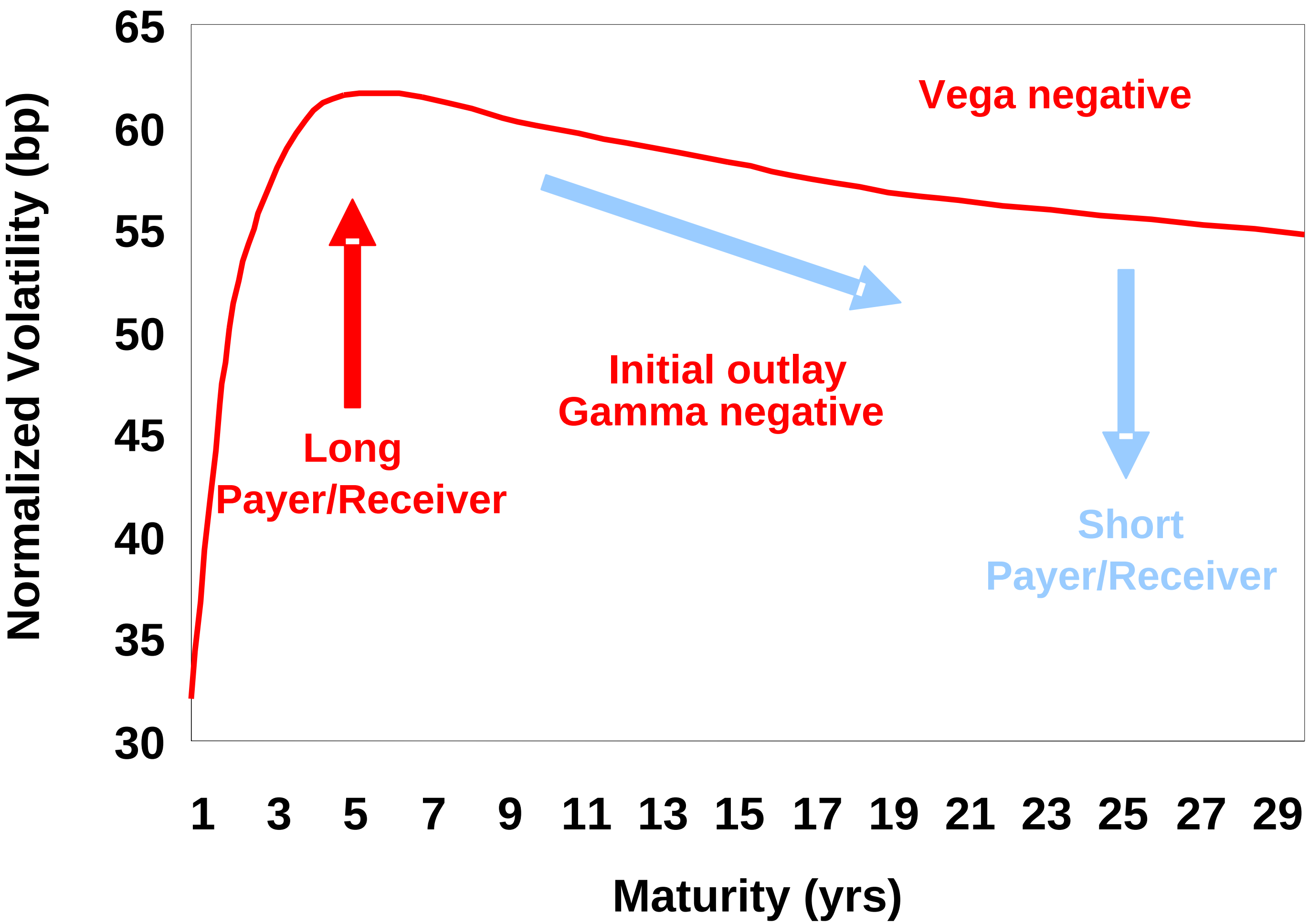
One month later

- We reprice the 3mth conditional 5s30s bull flattener
- The curve has experienced a bear steepening move
- P/L = EUR +50,000 (equivalent to 0.3bp) which can be broken down into:
  - the P/L due to the yield change is negligible (delta + gamma effects)
  - EUR 30,000 due to the volatility change (vega effect)
  - EUR 20,000 due to the theta effect
- A conventional swap position would have lost –EUR 2,400,000 equivalent to -14bp

Marked-to-market risk



Marked-to-market risk



Appendix 1

Delta of a payer/receiver swaption

- **The delta of a payer swaption ATMF with EUR 1 nominal amount is given by**

$$MD_{swap} \times \Phi(d)$$

where $d = 0.5\sigma\sqrt{Option\ maturity}$ with σ the Black volatility of the forward swap and MD the modified duration

- **As d is close to zero, $\Phi(d)$ is equal to 0.5 in first approximation and a duration weighted trade is equivalent to a delta neutral trade**

- **In the same way, the delta of a receiver swaption ATMF is given by**

$$MD_{swap} \times \Phi(-d) \approx 0.5 MD_{swap}$$

Appendix 2

Link between the volatility ratio and the premium of a conditional curve trade

- **The premium of a duration weighted ATMF conditional bear flattener with 100mn nominal amount on the longer tails is given by**

$$100mn \times MD_l \times Fwd_l \times [2\Phi(d_l) - 1] - 100mn \times \frac{MD_l}{MD_s} \times MD_s \times Fwd_s \times [2\Phi(d_s) - 1]$$

where $d_l = 0.5\sigma_l \sqrt{Option\ maturity}$ and $MD_l, Fwd_l\ and\ \sigma_l$ the modified duration, forward swap rate and Black volatility of the longer tails

- **Using a one order Taylor expansion we obtain**

$$2\Phi(d_l) - 1 \approx 0.392 \times \sigma_l \times \sqrt{Option\ maturity}$$

- **The premium then equates to**

$$100mn \times MD_l \times 0.392 \times \sqrt{Option\ maturity} \times [Fwd_l \times \sigma_l - Fwd_s \times \sigma_s]$$

- **It shows that the premium is proportional to the spread between the normalised volatility of the longer tails and of the shorter tails, and negative if the shorter tails/ longer tails ratio is superior to 1**

Swaption straddles & strangles

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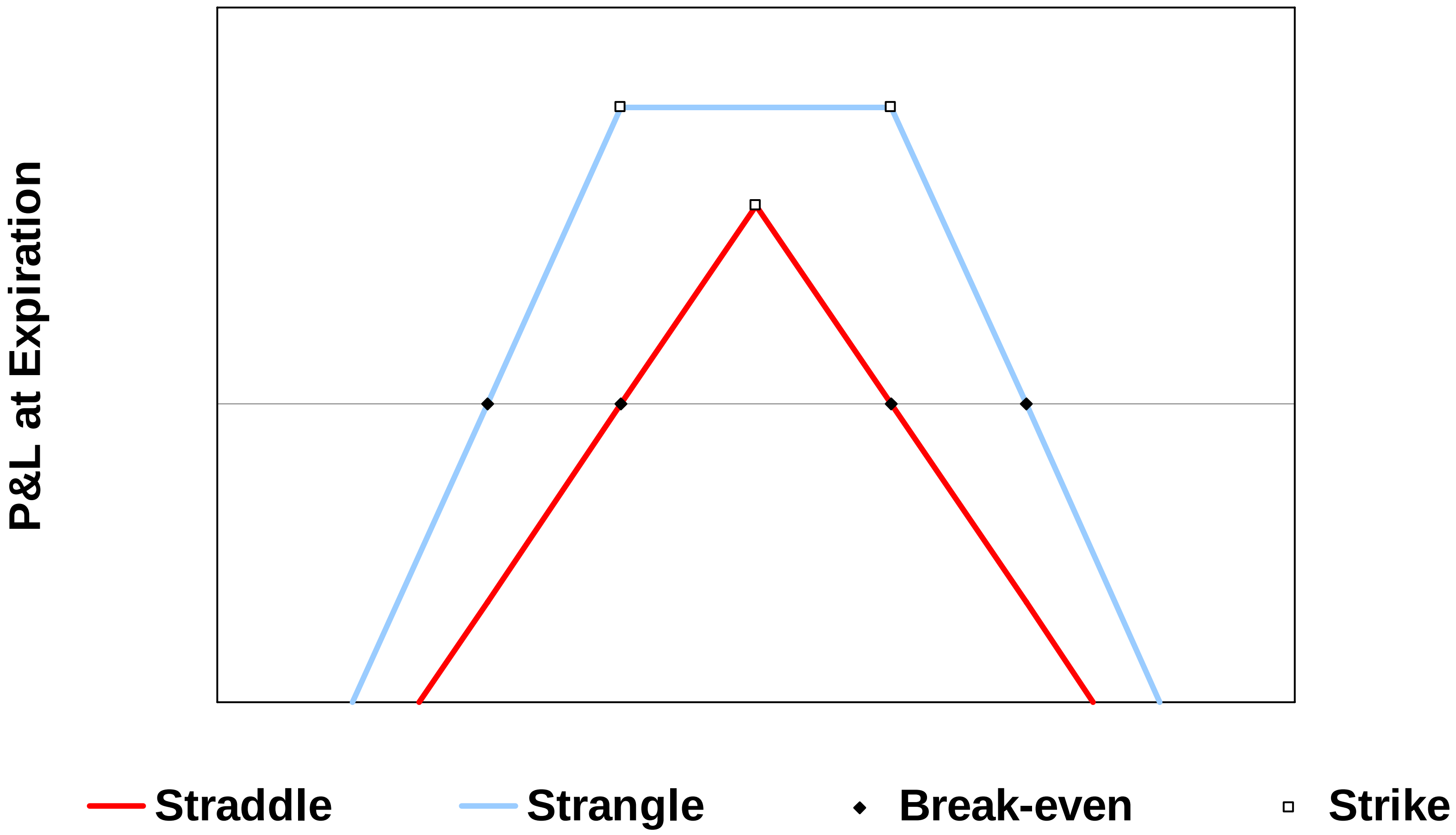
Overview

- Trade construction
- P&L at expiration
- When to position it ?
- Implied vs realized volatility
- Roll-down effect
- Mark-to-market risk
- Combinations: swaption butterfly, condor, chinese hat and batman

Trade construction

- Straddle
  - bottom straddle: buy the payer and receiver swaptions with the same maturity, ATMF and based on the same forward swap
  - top straddle: the opposite position
  - in a bottom (top) straddle, the investor pays (gets paid) to take the position
- Strangle
  - buy the payer and receiver swaptions with the same maturity, OTMF and based on the same forward swap
  - or the opposite position by selling the two options
  - the strikes of the two options can be positioned at different levels from the money forward
  - example: +10bp OTMF for the payer swaption and -20bp for the receiver one
- By selling a straddle or a strangle, the investor plays the stability of the underlying swap rate in a range

Bottom straddle and strangle P&L at expiration



Properties

- **A straddle is quasi delta-neutral**
- **It is also the case of a strangle when the strikes of the two options are positioned at the same level in absolute value from the money forward**
- **Price of a receiver swaption ATMF = Price of a payer swaption ATMF = P**
  - **where P is expressed in bp of the underlying swap rate**
- **The two break-evens at expiration of a straddle are obtained according to**

BE1 = ATMF Strike - 2P **BE2 = ATMF Strike + 2P**

- **The two break-evens at expiration of a +/-xbp strangle are obtained according to**

BE1 = ATMF Strike -xbp - (P1 + P2)

BE2 = ATMF Strike +xbp + (P1 + P2)

  - **where P1 and P2 are the prices of the receiver and payer swaptions, both expressed in bp of the underlying swap rate**

When to position it ?

- **These products are positioned depending on the**
  - **expectation of stable (or unstable) rates in the market**
  - **expectation of higher (or lower) volatility**
  - **the roll-down effect**
  - **the degree of mark-to-market risk**
- **The stability (or unstability) of rates depends on many aspects, and in particular the central bank policy, the state of the supply and demand of securities in the different maturities of the curve...**
- **The Implied vs realized volatility ratio, which can show some mean reversion properties, is a standard criterium used to judge if the volatility will go up or down**
- **The roll-down effect measures the P&L of the product at a given horizon by assuming that the yield and volatility curve will not move overt time**
- **Before expiration, the trade is also exposed to a mark-to-market risk**

Implied vs realized volatility

- **Even if the implied volatility is a forward measure while the realized volatility is a past measure, the implied vs realized volatility ratio usually shows a strong mean reversion around a certain level which can be different from one**
- **When the ratio is abnormally high (low), it tends to mean-revert (see following slides)**
- **But we cannot conclude that the implied volatility will go lower (higher) because the decrease (increase) of the ratio can be caused by a higher (lower) realized volatility**
- **It is common in the market to use three different realized volatilities based on different historical periods: one month, three and six months**
- **Based on different periods, it is possible to check if the value of the volatility ratio is robust over time**

Roll-down effect

- **Positions with positive roll-down are preferred to negative ones**
- **It is decomposed into three different effects and enables to judge what is the more important component**
  - **the roll-down due to the yield change**
  - **the roll-down due to the volatility change**
  - **the roll-down due to the time passing**
- **The roll-down due to the yield change at the one-month horizon is approximated by**

$$RD_y \approx \left(\frac{Delta\ at\ inception + Delta\ in\ 1\ mth}{2} \right) \times yield\ change$$
$$+ 0.5 \times \left(\frac{Gamma\ at\ inception + Gamma\ in\ 1\ mth}{2} \right) \times (yield\ change)^2$$

Roll-down effect (2)

- **The roll-down due to the volatility change is approximated by**

$$RD_V \approx \left(\frac{Vega\ at\ inception + Vega\ in\ 1\ mth}{2} \right) \times volatility\ change$$

- **The roll-down due to the time passing is approximated by**

$$RD_\theta \approx \left(\frac{Theta\ at\ inception + Theta\ in\ 1\ mth}{2} \right) \times (nb\ of\ days)$$

Mark-to-market risk

- **Straddle short position at trade inception:**
  - **quasi delta neutral and negative gamma**
  - **negative vega**
  - **positive theta**
- **Strangle short position at trade inception:**
  - **quasi delta neutral for a +/-xbp OTMF strangle, and undetermined for other kind of strangles**
  - **negative gamma**
  - **negative vega**
  - **positive theta**
- **Before expiration, a short position is then exposed to**
  - **a large movement in yield**
  - **an increase of the implied volatility**

Mark-to-market risk (2)

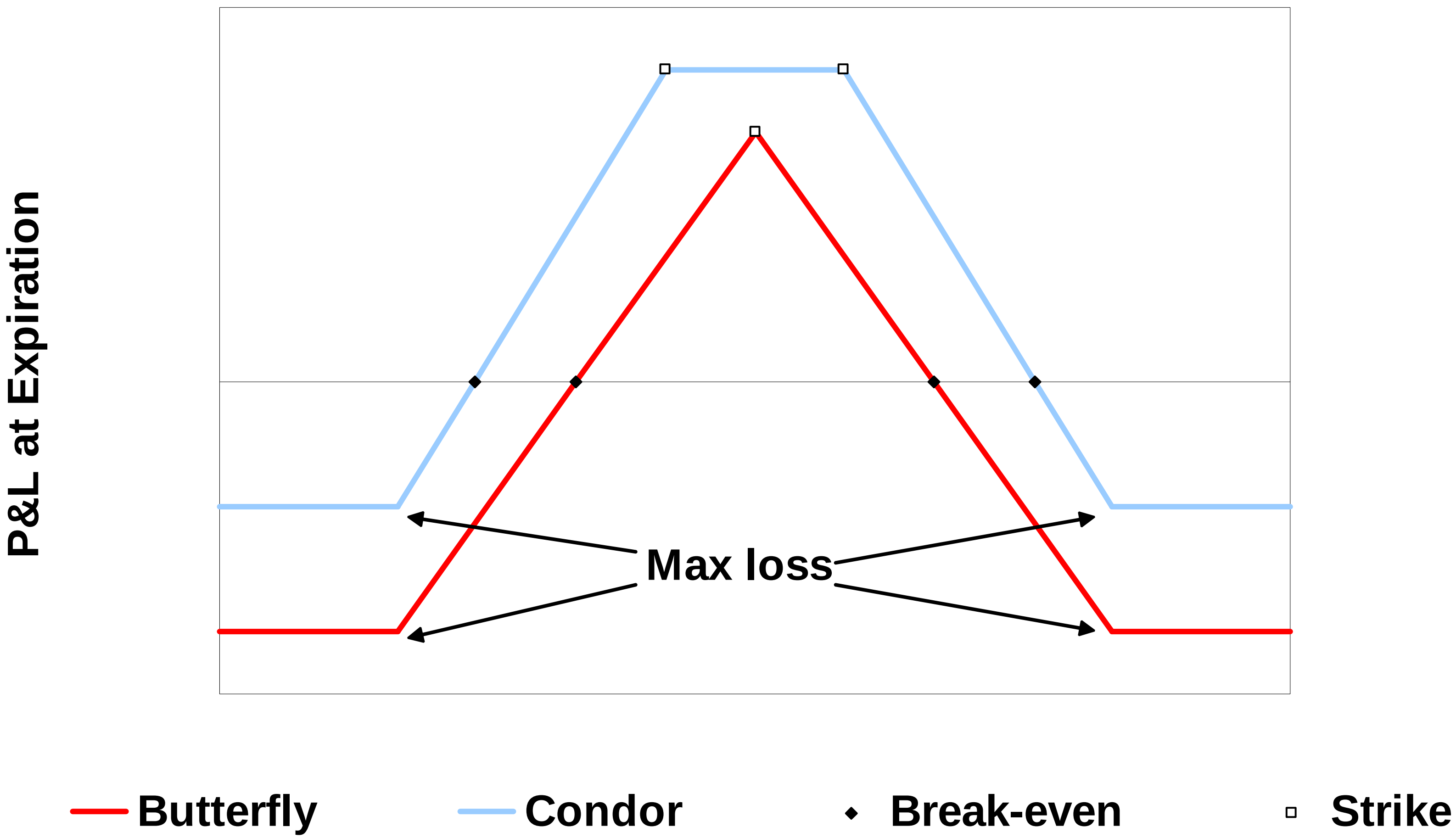
- **The mark-to-market P&L one month after trade inception can be approximated in the following way**

$$
\begin{aligned}
P/L \approx\ & \left(\frac{Delta\ at\ inception + Delta\ in\ 1\ mth}{2}\right) \times yield\ change \\
& + 0.5 \times \left(\frac{Gamma\ at\ inception + Gamma\ in\ 1\ mth}{2}\right) \times (yield\ change)^2 \\
& + \left(\frac{Vega\ at\ inception + Vega\ in\ 1\ mth}{2}\right) \times volatility\ change \\
& + Theta\ effect
\end{aligned}
$$

Combinations

- **swaption butterfly**
  - the investor simultaneously sells a straddle and buys a OTMF strangle, or the opposite position
- **swaption condor**
  - the investor simultaneously sells a +/-xbp OTMF strangle and buys a +/-ybp OTMF strangle with x < y, or the opposite position
- **chinese hat**
  - the investor simultaneously buys a straddle for a 100 mn nominal amount and sells a +/-xbp OTMF strangle for 200mn nominal amount
- **swaption batman**
  - the investor simultaneously buys a +/-xbp OTMF strangle for a 100 mn nominal amount and sells a +/-ybp OTMF strangle for 200mn nominal amount with x < y

Swaption butterfly and condor P&L at expiration



Properties

- **Swaption butterfly**
  - The two break-evens at expiration of a +/-xbp butterfly are obtained according to

BE1 = ATMF Strike - butterfly price **BE2 = ATMF Strike + butterfly price**

  - The maximum loss ML of the position is given by

ML = -x + butterfly price

- **Swaption condor**
  - The two break-evens at expiration of a condor (sell the +/-xbp OTMF strangle and buy the +/-ybp one with x < y) are obtained according to

BE1 = ATMF Strike - x - condor price **BE2 = ATMF Strike + condor price**

  - The maximum loss ML of the position is given by

ML = -(y-x) + condor price