Measures of Asset Swap Spreads and their Corresponding Trades

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While the general purpose of asset swap spreads is well understood, the several distinct definitions of asset swap spreads are, to many, a source of some confusion. Furthermore, while trades based on asset swap spreads are quite common, the association of each definition with a particular trade is not fully appreciated.

This report shows that each commonly used definition corresponds to, and follows naturally from, a particular trade of bonds against swaps. The discussion not only makes the existence of the several definitions more comprehensible, but connects the weaknesses or drawbacks of each definition with the risks of its associated trade. Furthermore, this paper derives and intuitively explains the relationships across the various asset swap spreads as a function of the slope of the swap curve and of the coupon of the bond relative to swap rates. For a treatment of the credit risk in asset swap trades, see O'Kane (2000).

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1. DIFFERENCES ACROSS ASSET SWAP SPREADS: NUMERICAL EXAMPLES

Measures of asset swap spreads do differ from one another, particularly when the swap curve is far from flat and when the bond price is far from par. To illustrate these differences, five measures of spread are computed for low, medium, and high coupon bonds in upward-sloping, flat, and downward-sloping curve environments. Table 1 reports the parameters of these examples and Table 2 reports the numerical results and other associated quantities.

The choices described in Table 1 were made for several reasons. First, to focus on the differences across asset swap spreads, one particular spread, namely quarterly compounded LIBOR OAS, is fixed at 20 basis points. Second, to control for the level of rates across different curve scenarios, the rate on a par swap to the maturity date of the bond, called a **matched-date swap**, is held fixed at 4.50% (corresponding to the 4.475% quarterly compounded rate in Table 1). Third, the upward-sloping and downward-sloping curve environments are extreme so as to illustrate the impact of the curve on the various definitions of asset swap spreads. These examples are, at least in part, motivated by current market conditions: as of October, 2002, the 10-year swap rate was about 4.50% and the FNMA 6.125's of 3/15/12 traded at a LIBOR OAS of about 20 basis points.

Table 2 clearly demonstrates that measures of asset swap spreads vary with coupon and curve. Despite LIBOR OAS being fixed at 20 basis points, the matched-date and benchmark spreads range from 2.9 to 38.7 basis points while the par-par and market value spreads range from 17.4 to 22 basis points.

Table 1. **Parameters for the Numerical Examples in Table 2**

Today		9/15/02
Bonds		
Bond Maturity		3/15/12
Low Coupon		2.875%
Medium Coupon		4.500%
High Coupon		6.125%
LIBOR OAS (quarterly compounded basis points)		20
Swaps		
Par Swap Rate to 3/15/12		4.500%
	Quarterly Rates	
	Spot	10-Yrs Fwd
Upward-Sloping Swap Curve	2.000%	7.785%
Flat Swap Curve	4.475%	4.475%
Downward-Sloping Swap Curve	6.95%	1.202%

Table 2. **Numerical Illustration of the Relationships Among Asset Swap Spreads**

Curve Environment	**Downward-Sloping**			**Flat**			**Upward Sloping**		
Spread Measure\Coupon	Low	Medium	High	Low	Medium	High	Low	Medium	High
Par-Par Spread	19.0	20.5	22.0	18.2	19.6	21.1	17.4	18.8	20.3
Market Value Spread	22.0	20.8	19.9	21.1	20.0	19.0	20.3	19.1	18.2
LIBOR OAS	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
Matched-Date Spread	14.5	20.4	25.5	20.2	20.2	20.2	26.3	20.1	14.8
Banchmark Spread	27.6	33.6	38.7	20.2	20.2	20.2	14.5	8.3	2.9
Bond Prices	86.532	98.453	110.374	86.129	98.464	110.800	85.710	98.477	111.244
Bond Yields	4.645%	4.704%	4.755%	4.702%	4.702%	4.702%	4.763%	4.701%	4.648%
Bond DV01	0.06998	0.07520	0.08044	0.06961	0.07521	0.08082	0.06922	0.07523	0.08122
Bond PV01 at Swap Rates	0.07252	0.07809	0.08366	0.07173	0.07747	0.08322	0.07090	0.07683	0.08276
at Swap Rates + OAS	0.07120	0.07669	0.08219	0.07042	0.07609	0.08177	0.06961	0.07547	0.08133
SP01		0.07553			0.07817			0.08090	
10-Year Swap Rate		4.368%			4.500%			4.618%	
PV01 of 10-Year Swap		0.08039			0.07921			0.07802	

2. SUMMARY OF RESULTS

The first set of results in this paper associate each measure of asset swap spreads with a particular trade by linking the measure, the change in the measure, or both, to the P&L from that trade. Table 3 defines each asset swap spread, associates a trade with that definition, and describes the P&L characteristics of the trade assuming that the bond does not default. (In any asset swap trade, defaults easily generate substantial losses.) For clarity, Table 3 assumes that accrued interest is zero: a more general discussion is deferred to Section 5. Readers are strongly encouraged to read through Table 3 at this point.

Table 4 matches trader beliefs and objectives to particular asset swap trades. The table first divides traders into carry traders and convergence traders. Carry traders have a tolerance for interim mark-to-market volatility and believe that convergence to fair value will be slow. These traders are best served by spreads and trades based on the creation of synthetic floaters, described in Section 3. Convergence traders, on the other hand, have a tolerance for carry slippages and believe that convergence to fair value will be quick. These traders are best served by spreads and trades based on dynamic trading strategies, described in Section 4.

Table 4 then subdivides carry traders based on their tolerance for taking and posting collateral. Market value swaps do not require initial posting or taking of collateral while par-par asset swaps, by contrast, generally do: the par-par asset swapper of a discount bond takes collateral while the par-par asset swapper of a premium bond posts collateral.

Finally, Table 4 subdivides convergence traders based on their hedging and liquidity preferences. Traders using relatively complex hedging strategies are directed to LIBOR OAS trades whereas those content to hedge against parallel shifts are directed to benchmark or matched-date asset swaps. This latter group is then divided once more: traders valuing the extra liquidity of the benchmark swap do that trade while traders seeking to minimize maturity risk do the matched-date trade.

The next set of results in this paper describe how various measures of spread relate to one another as a function of the curve and bond coupon. Tables 5a and 5b give rules of thumb to summarize the relationship between semiannually compounded LIBOR OAS and the par-par asset swap spread based on the following approximation (see Section 4.2 and the Appendix):

$$\text{LIBOR OAS} \approx \text{Par-Par Asset Swap Spread} \times SP01 \div PV01 \tag{2.1}$$

For a 1-basis point parallel decline in swap rates, *PV01* is the change in the value of receiving scheduled bond coupons through the fixed side of a swap. *SP01* is the value of receiving another basis point in spread on the floating side of that swap. In the low-coupon, upward-sloping example of Table 2, $OAS \approx 17.4 \times .08090/.06961$ or 20.2 semiannually compounded basis points or 20 quarterly compounded basis points. Note that (2.1) is accurate so long as the usual *PV01* approximation is accurate, that is, so long as *OAS* is not too large. Under present market conditions this means that the approximation works well for agencies but less well for low-grade corporate bonds.

According to Table 5a, the case of positive OAS, OAS tends to exceed the par-par spread for low coupons while the par-par spread tends to exceed OAS for high coupons. The par-par spread tends to exceed OAS in downward-sloping curves while OAS tends to exceed the par-par spread for upward-sloping curves. According to Table 5b, these results are reversed for negative OAS. These results are discussed in Section 4.2.

Table 3. **Asset Swap Spreads: Definitions and Trade Correspondence**

Name	Definition	Underlying Trade	P&L (No Default)
Par-Par Asset Swap	Paying the bond coupon in a swap is fair against receiving LIBOR plus this spread on 100 plus the bond premium (or minus the discount) up-front.	Buy the bond, financing par with repo and the rest with an up-front flow from the swap desk; Pay the bond coupon vs. LIBOR plus the spread on 100 to bond maturity.	Locks in the spread plus LIBOR minus repo to bond maturity on 100; Subject to mark-to-market risk; Asset swapper posts the bond premium or takes the bond discount as collateral.
Market Value Asset Swap	Paying the bond coupon in a swap is fair against receiving LIBOR plus this spread on the bond price plus the bond premium (or minus the discount) at swap maturity.	Buy the bond financed with repo; Pay the bond coupon vs. LIBOR plus the spread on the bond price to bond maturity; Receive the bond premium from the swap desk (or pay the discount) at expiration.	Locks in the spread plus LIBOR minus repo to bond maturity on the bond price; Subject to mark-to-market risk; No collateral is posted at the time of the trade.
LIBOR OAS	The bond's price equals the value of its cash flows discounted at swap rates plus this OAS.	Buy the bond financed with repo; Dynamically hedge swap rate and curve risk with swaps according to some hedging model.	If the hedging model is correct, earns a total return of OAS plus LIBOR minus repo plus position risk times any change in OAS .
Benchmark Spread	The spread equals the yield of the bond minus the nearest benchmark par swap rate.	Buy the bond financed with repo; Dynamically hedge parallel shift risk with a benchmark swap.	Capital gain is the change in spread times the position's risk; Carry not as easily related to spread; Subject to maturity and curve risk.
Matched-Date Spread	The spread equals the yield of the bond minus the par swap rate to the maturity date of the bond.	Buy the bond financed with repo; Dynamically hedge parallel shift risk with a matched-date swap.	Capital gain is the change in spread times the position's risk; Carry not as easily related to spread; Subject to curve risk.

Table 4. **Choosing Among Asset Swap Definitions Based on Trading Beliefs and Objectives**

View Asset Swap as a Carry Trade	**Use Spreads/Trades Based on the Creation of Synthetic Floaters**	Do not Want to Post or Take Collateral at Initiation of Trade	**Market Value Asset Swap**
		Want to Take Collateral at Initiation of Trade	**Par-Par Asset Swap of a Discount Bond**
		Want to Post Collateral at Initiation of Trade	**Par-Par Asset Swap of a Premium Bond**
View Asset Swap as a Convergence Trade	**Use Spreads/Trades Based on Dynamic Trading Strategies**	Believe that Hedging Against Parallel Shifts is Sufficient; Want to Stay in a Benchmark Swap	**Benchmark Asset Swap Trade**
		Believe that Hedging Against Parallel Shifts is Sufficient; Want to Minimize Maturity Risk	**Matched-Date Asset Swap Trade**
		Rely on Hedging Strategies More Complex than PV01	**LIBOR OAS Trade**

Table 5a. **For OAS>0, the Relationship Between OAS and the Par-Par Asset Swap Spread as a Function of Curve and Coupon**

OAS > 0	**Downward-Sloping Swap Curve**	**Flat Swap Curve**	**Upward- Sloping Swap Curve**
Coupon < Matched-Date Swap Rate	OAS < Par-Par Spread, except for very low coupons	OAS > Par-Par Spread	OAS > Par-Par Spread
Coupon = Matched-Date Swap Rate	OAS < Par-Par Spread	OAS > Par-Par Spread	OAS > Par-Par Spread
Coupon > Matched-Date Swap Rate	OAS < Par-Par Spread	OAS < Par-Par Spread	OAS > Par-Par Spread, except for very high coupons

Table 5b. **For OAS<0, the Relationship Between OAS and the Par-Par Asset Swap Spread as a Function of Curve and Coupon**

OAS < 0	**Downward-Sloping Swap Curve**	**Flat Swap Curve**	**Upward- Sloping Curve**
Coupon < Matched-Date Swap Rate	OAS > Par-Par Spread, except for very low coupons	OAS < Par-Par Spread	OAS < Par-Par Spread
Coupon = Matched-Date Swap Rate	OAS > Par-Par Spread	OAS < Par-Par Spread	OAS < Par-Par Spread
Coupon > Matched-Date Swap Rate	OAS > Par-Par Spread	OAS > Par-Par Spread	OAS < Par-Par Spread, except for very high coupons

Table 6 gives rules of thumb to summarize the relationship between LIBOR OAS and the matched-date spread. When the swap curve is flat, OAS and the matched-date spread are equal. Relative to the center of the table, coupon levels and curve environments that tend to raise yields, i.e. a low coupon with an upward-sloping curve and a high coupon with a downward-sloping curve, have the matched-date spread exceeding OAS. By contrast, coupon levels and curve environments that tend to lower yields relative to the center of the table, i.e. a low coupon with a downward-sloping curve and a high coupon with an upward-sloping curve, have OAS exceeding the matched-date spread. For further discussion about this table see Section 4.4.

The final set of results allow for relatively quick calculations of par-par and market value asset swap spreads given a standard swap pricing tool. First, when accrued interest on the bond is zero, the par-par asset swap spread is given by the following equation:

$$\text{Par-Par Asset Swap Spread} = \frac{100 - P + NPV^{B}}{SP01} \tag{2.2}$$

where P is the price of the bond, NPV^B is the net present value of a swap with fixed payments matching those of the bond and *SP01* is the value of an additional basis point on the floating side. Section 5 generalizes this equation to incorporate accrued interest. In the low-coupon, upward-sloping example, P=85.710, NPV^B=-12.885, and *SP01*=.08090, so the par-par spread equals 17.4. Second, market value asset swap spreads are related to par-par asset swaps by the following simple equation:

$$\text{Market Value Asset Swap Spread} = \text{Par-Par Asset Swap Spread} \div P/100 \tag{2.3}$$

where P is the flat bond price. In the example, the market value spread is 17.4/.8571 or 20.3.

3. SPREADS BASED ON SYNTHETIC FLOATERS

Par-par and market value asset swap spreads are based on the creation of synthetic floaters. To illustrate, consider the FNMA 6.125's of 3/15/12 at a price of 110.800 on 9/15/02. If an investor buys the bond and pays fixed on a swap at a rate of 6.125% to 3/15/12, the coupon and the fixed rate swap payments cancel. (Issues arising from accrued interest are deferred to Section 5.) The remaining cash flows from the position are repo interest on money borrowed to finance the purchase of the bond and interest of LIBOR plus or minus a spread from the floating side of the swap. In this sense, the bond and swap positions are synthetic floaters with offsetting principal flows, leaving an interest stream on some notional amount.

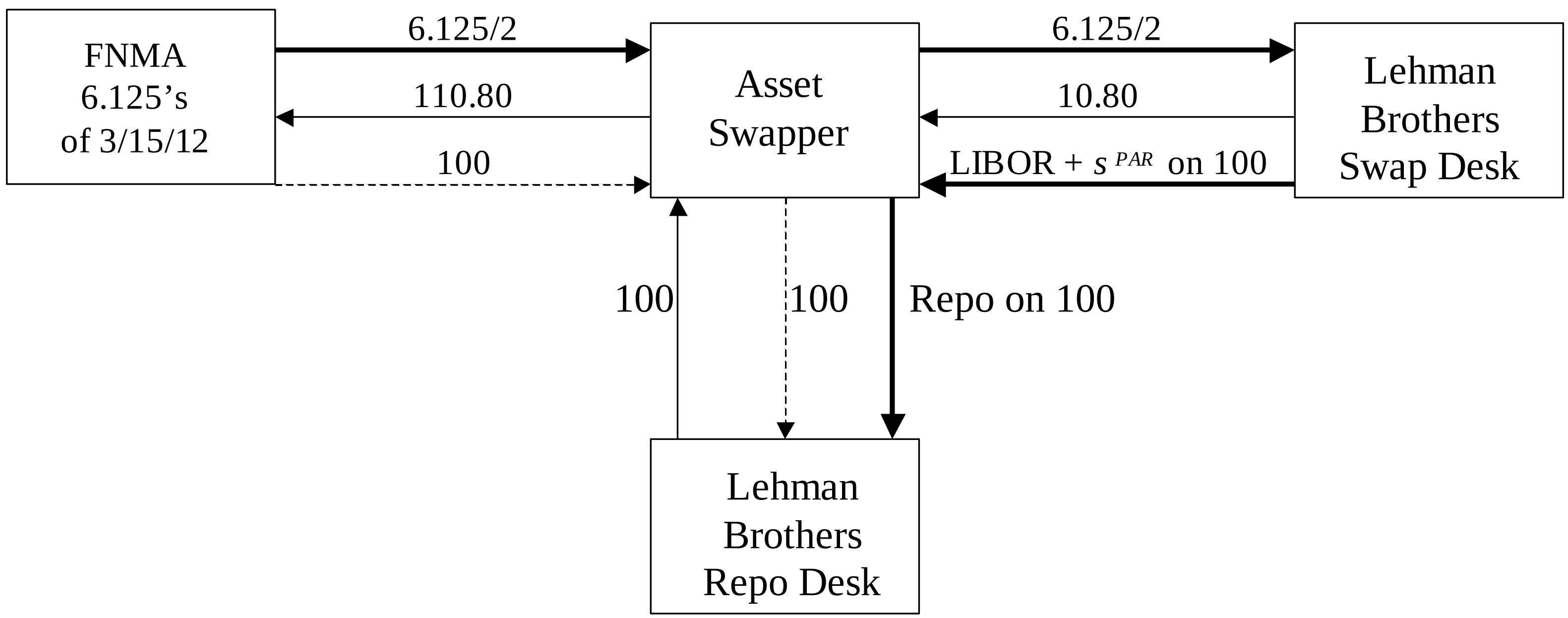
3.1 Par-Par Asset Swap

In a par-par asset swap, the par amount of the bond is financed through repo and the notional amount of the floating side of the swap is set to par. The difference between the purchase price of the bond and par is obtained from the swap desk. With the par-par asset swap spread denoted by s^{Par}, Figure 1 illustrates the transaction for 100 face amount of the bond. The plain arrows represent initial cash flows, the bold arrows represent periodic cash flows, and the dashed arrows represent terminal cash flows. Note that, for the asset swapper, the initial, final and fixed periodic cash flows cancel. Assuming for the moment that the repo agreements are quarterly so as to match the frequency of the floating cash flows, the net quarterly flow to the asset swapper is $\text{LIBOR} + s^{Par} - \text{repo}$ on 100 face amount.

Table 6**. The Relationship Between OAS and the Matched-Date Asset Swap Spread as a Function of Curve and Coupon**

OAS > 0	**Downward-Sloping Swap Curve**	**Flat Swap Curve**	**Upward- Sloping Curve**
Coupon < Matched-Date Swap Rate	OAS > Matched-Date Spread	OAS = Matched-Date Spread	OAS < Matched-Date Spread
Coupon > Matched Date Swap Rate	OAS < Matched-Date Spread	OAS = Matched-Date Spread	OAS > Matched-Date Spread

Figure 1. **A Par-Par Asset Swap**



The LIBOR-repo spread over the life of the trade is the market's compensation for bearing the credit and liquidity risk embedded in the swap market relative to the repo market. The spread s^{Par} is the market's compensation for bearing the credit and liquidity risk of the agency relative to the swap market. Hence, the total spread represents the market's compensation for bearing agency risk relative to repo risk.

Letting *c* be the bond coupon per 100 face value and *r* the fixed payment on 100 notional of a par swap to the maturity of the bond, Figure 2 depicts the par asset swap trade in general terms. Note that in the case of a discount bond, the arrow indicating a payment of *P*-100 should be interpreted as a receipt of 100-*P*.

Equation (2.2), a special case of Proposition 3 of the Appendix, is reproduced here for easy reference:

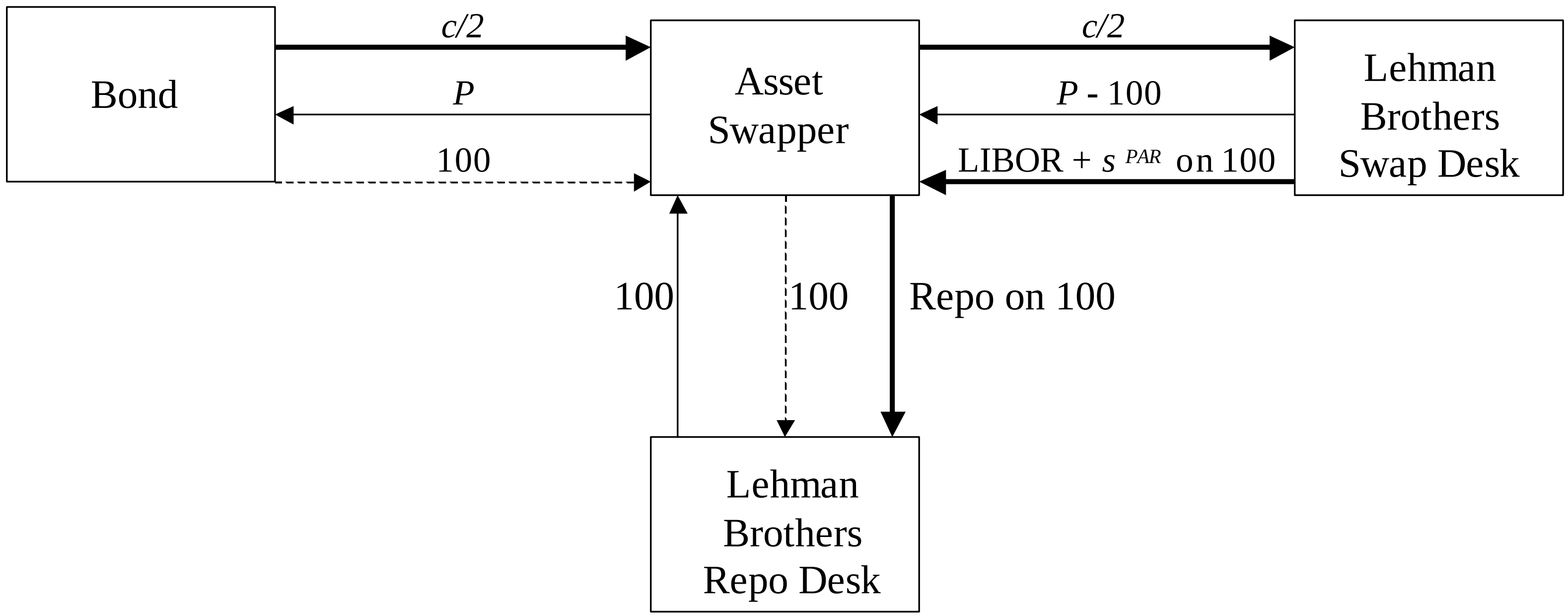
$$\text{Par-Par Asset Swap Spread} = \frac{100 - P + NPV^{B}}{SP01} \tag{3.1}$$

The intuition behind this equation is best understood from the perspective of the swaps desk. The spread times *SP01* gives the value of the floating payments in excess of LIBOR paid by the swap desk. NPV^B plus 100-*P* gives the value of the fixed payments in excess of the par swap rate received by the swap desk. For the swap as a whole to be fair, these two values must be equal.

While the synthetic floating structure locks in a spread over the life of the bond (assuming no default), mark-to-market risk remains. First, if the asset swap spread widens then long positions record a loss while if the spread narrows long positions record a gain. This implies, by the way, that after the narrowing of a spread a trade might be unwound at a profit. In other words, the periodic spread over the remaining life of the bond may be exchanged for an immediate capital gain. A second source of mark-to-market risk is that the position is not hedged against changes in the level or shape of the swap curve: a trader can be right about the asset swap spread but suffer mark-to-market losses because of changes in the interest rate environment. To take a simple example, because the bond's cash flows are essentially discounted at swap rates plus an *OAS* while the swap's fixed cash flows are discounted at swap rates, the *PV01* of the long bond position will differ from the *PV01* of paying fixed on the swap. Hence, the position is not hedged against parallel shifts to the swap curve. In the case of \$100 million 6.125% bonds and an upward-sloping curve, this particular mismatch is worth $\$100\text{mm} \times (.08276 - .08133)/100$ or \$1,430 per basis point.

A final aspect of the par-par asset swap to be considered is its implications for taking and posting collateral. After the up-front payment of *P*-100 to the asset swapper, the *NPV* of the swap to that party is $100 - P$. Hence the asset swapper of a premium bond must usually post collateral equal to the bond premium while the asset swapper of a discount bond may usually take collateral equal to the bond discount. To the extent that posting collateral is costly (e.g. earning a rate on cash below the opportunity cost of funds or holding a non-optimal portfolio of securities), the net floating rate falls short of the par-par spread plus the LIBOR-repo spread. Similarly, to the extent that taking collateral is profitable, the net floating rate exceeds that level. Note that, on average, the *NPV* tends to fall over time as the above- or below-market coupon is due over shorter and shorter time periods. But, unless the trade is held over a relatively long horizon, this average decline in *NPV* is unlikely to affect trading decisions.

Figure 2. **A General Par-Par Asset Swap**



3.2 Market Value Asset Swap

In a market value asset swap the purchase price of the bond is financed through repo. At expiration, then, today's purchase price must be repaid to the repo desk even though the bond returns only 100. This shortfall is made up by a terminal payment from the swap desk. Denoting the market value asset swap spread s^{Mkt}, Figure 3 illustrates the transaction for the FNMA 6.125's of 3/15/12 and Figure 4 illustrates the transaction in general. Note that all fixed and one-time payments cancel, leaving the asset swapper $\text{LIBOR} + s^{Mkt} - \text{repo}$ on the price of the bond.

As recorded in equation (2.3) and proved in Proposition 5 of the Appendix, the market value swap spread equals the par-par swap spread divided by the flat bond price per dollar face value. The intuition behind this result is explained by Table 7 with reference to the examples in Figures 1 and 3. As is often useful in analyzing swaps, assume that a notional of 100 is exchanged at the termination of the swaps so that each swap may be viewed as a fictional fixed note and a fictional floating rate note. (Abstracting from counter-party risk, this doesn't change the value of the swaps.) Then Table 7 demonstrates that, aside from the spread component, the sum of the parts for each swap has the same value. The fixed flows are exactly the same so they certainly have the same value. Since LIBOR on 100 plus a terminal payment of 100 is worth 100 today and LIBOR on 110.8 plus 110.8 at termination (100 of fictional principal and a 10.8 terminal swap payment) is worth 110.8 today, the values of the up-front amounts and fictional floaters sum to 110.8 for each swap. But if these components sum to the same value for each swap and both swaps are fair, it must be the case that s^{Par} on 100 is equal in value to the market value spread on 110.8. In other words, as stated, the market value spread equals the par-par spread divided by the bond price per dollar face value.

Table 7. **Intuition Behind the Relationship Between s^{Par} and s^{Mkt}**

	Par-Par Swap				Market Value Swap		
	Now	Flow	Terminal	Value	Flow	Terminal	Value
Fictional Fixed Note		-6.125/2	-100		-6.125/2	-100	
Up-Front Amount	10.8			10.8			
Fictional Floater		LIBOR on 100	100	100	LIBOR on 110.8	110.8	110.8
Spread		s^{Par} on 100			s^{Mkt} on 110.8		

Like the par-par asset swap, the market value asset swap locks in a spread over the life of the bond but is subject to mark-to-market risk. The collateral implications of the two asset swap structures, however, are quite different. Since there is no up-front payment in the market value structure, the *NPV* of the swap is zero and there is no collateral requirement at initiation of the trade. As time passes, however, the terminal payment draws near and, on average, collateral will be required. But, unless the trade is held over a relatively long horizon, this average change in the collateral requirement is unlikely to affect trading decisions. Therefore, when parties want to avoid taking and posting collateral, market value asset swaps have an advantage over par-par asset swaps.

Figure 3. **A Market Value Asset Swap**

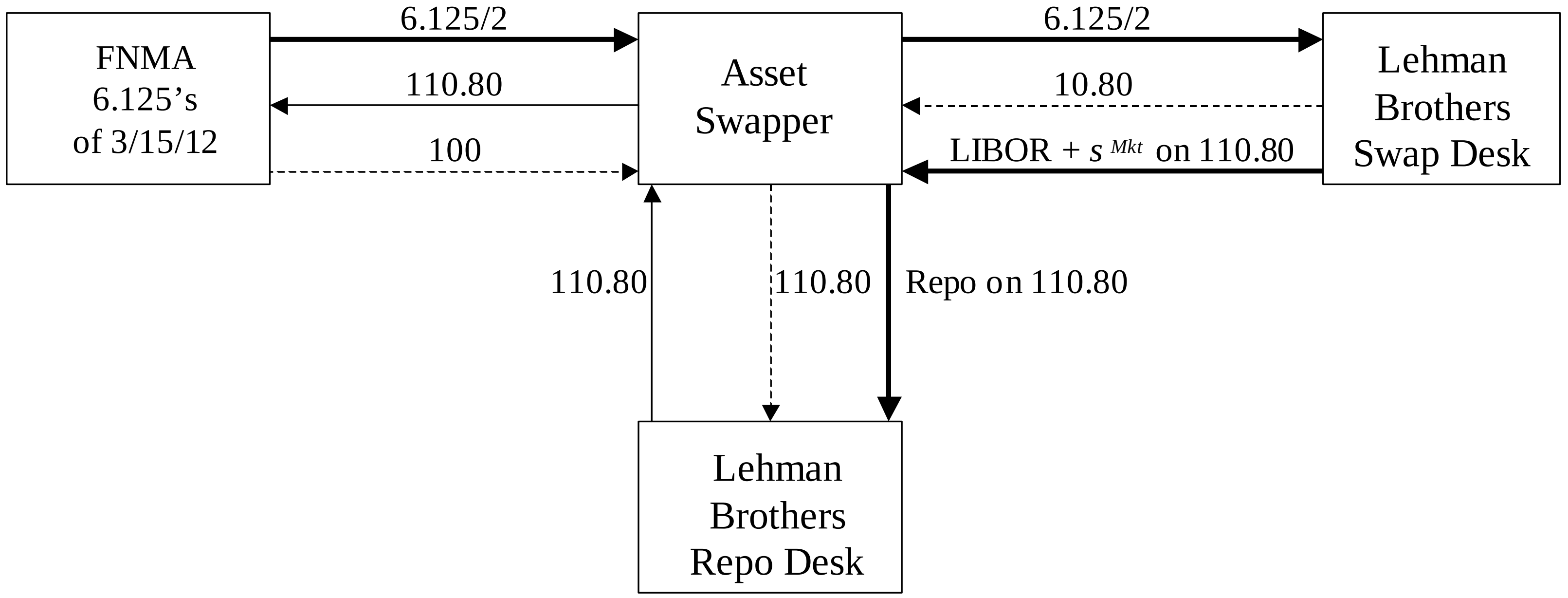
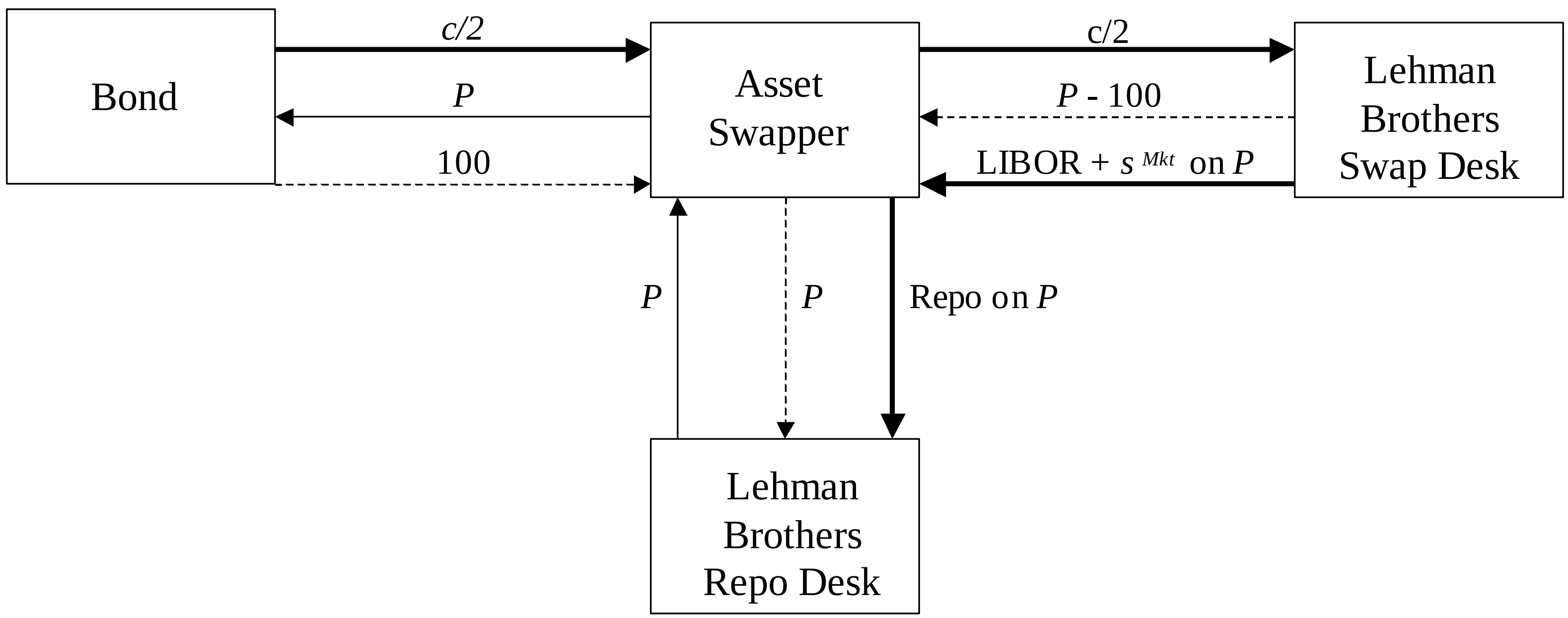


Figure 4. **A General Market Value Asset Swap**



4. SPREADS BASED ON DYNAMIC TRADING STRATEGIES

The three remaining definitions of asset swap spreads are motivated by trading strategies that establish an initial position and then periodically adjust hedges with changing market conditions. These are called *dynamic trading strategies* to distinguish them from buy-and-hold or *static* trading strategies like the par-par and market value asset swaps.

4.1 LIBOR OAS

The *LIBOR OAS* of a bond is defined such that the present value of the bond's cash flows, when discounted at swap rates plus that OAS, equals the bond's market price. In the examples, the OAS was computed as a spread to quarterly forwards but other compounding conventions are also common.

Appreciating the appeal of LIBOR OAS requires some understanding of dynamic trading strategies.[2] To take a simple case, assume that OAS is constant, that the swap curve moves up and down in parallel, and that *PV01* gives the capital gain of a bond for a 1-basis point decline in swap rates. Then it can be shown that the total P&L, carry as well as capital gains, from a financed position in that bond is given by

$$d\overline{P} = \left(L + OAS - R\right)\overline{P}dt - PV01\left\{dr - E\left[dr\right]\right\} + \lambda PV01dt \tag{4.1}$$

where $\overline{P}$ and $d\overline{P}$ are the full bond price and change in the full bond price, L is a short-term LIBOR rate,[3] R is a short-term repo rate, dt is the change in calendar time, dr is the change and $E\left[dr\right]$ the expected change in swap rates, and λ is a risk premium. While (4.1) holds at any given time, various quantities in the relationship, including the *PV01*, change over time.

The intuition behind this relationship is that every bond priced according to some rule or model earns the short-term rate of interest plus a risk premium proportional to the risk borne. If a bond is priced not according to that rule or model, but according to that rule or model with an *OAS* added to the discount rates, then the bond earns the *OAS* above that return. Finally, if the bond is financed, then the return is reduced by the repo rate. These effects describe all the terms multiplying dt. The remaining term, the *PV01* times $dr - E\left[dr\right]$, gives the capital gain or loss depending on the realization of future rates.

By the same logic, the change in the value of the fixed side of a swap, dP^S, with sensitivity $PV01^S$ and with financing at the short-term LIBOR rate is given by

$$dP^S = -PV01^S\left\{dr - E\left[dr\right]\right\} + \lambda PV01^S dt \tag{4.2}$$

Now consider hedging the rate risk of a bond by paying fixed on $PV01/PV01^S$ notional amount of swaps. Appropriately weighting (4.1) and (4.2), the net return from this position is

$$d\overline{P} - \left(PV01/PV01^S\right)dP^S = \left(L + OAS - R\right)\overline{P}dt \tag{4.3}$$

The rate of return of the net position, obtained by dividing the dollar return by $\overline{P}$, is

$$\left(L + OAS - R\right)dt \tag{4.4}$$

In words, if a pricing and hedging model is correct, then a hedged and financed position in a bond will earn a rate of return equal to the bond's *OAS* plus the difference between the short-term LIBOR and repo rates. With an added hedge to lock in term investment and financing rates, the return may be expressed in terms of 3-month LIBOR and term repo.

While most investors and traders are familiar with the hedging dimensions of the previous discussion, many may be new to the relationship between OAS and carry. To illustrate with a very simple example, consider buying the FNMA 6.125's of 3/15/12 on 9/15/02 in the flat curve environment of Table 1, hedging with a par swap, and holding the position for six months. If swap rates and the bond's OAS remain unchanged, then, after six months, the price of the bond may be computed to be 110.3426. Table 8 shows that the return on the bond is 2.35%, representing a half year LIBOR return of 4.50%/2 or 2.25% plus a half year OAS return of 20/2 or 10 basis points. The hedging swap costs 2.25% in fixed payments, but, since the swap curve is flat, earns that exact amount back on the floating side. Hence, the return on the trade, omitting only the repo component, is LIBOR+OAS for a six-month period. Subtracting the repo return gives a total return consistent with (4.4).

Table 8. **P&L and Return from Holding the FNMA 6.125's of 3/15/12 Over 6 Months with an Unchanged OAS and an Unchanged, Flat Swap Curve at 4.50%.**

Bond Price Change	110.3426-110.800=-.4574
Bond Coupon	3.0625
P&L	2.6051
Return	2.6051/110.800=2.35%

If the OAS of the bond changes then the LIBOR OAS trade will experience a capital gain or loss. The subsequent return from the trade, the new OAS plus the LIBOR-repo spread, will fall after a capital gain and rise after a capital loss. Like the par-par and market value asset swaps, P&L may be experienced as carry, capital gain, or both.

The discussion here assumed parallel shifts, but the results do not require so strong an assumption. Any hedging model that turns out to be correct will generate a total return of OAS plus the LIBOR-repo spread over the subsequent period. However, if the hedging model is not correct, this return will not be realized. More precisely, the position will experience capital gains or losses even if OAS does not change.

4.2 The Relationship Between the Par-Par Asset Swap Spreads and LIBOR OAS

Using *PV01* to approximate the difference between the value of a bond's cash flows when discounted at swap rates and when discounted at swap rates plus an OAS gives an approximate relationship between par-par asset swap spreads and LIBOR OAS. As reported earlier as equation (2.1) and as proved in Proposition 6 of the Appendix,

$$\text{LIBOR OAS} \approx \text{Par-Par Asset Swap Spread} \times SP01 \div PV01 \qquad (4.5)$$

Intuitively, this result says that the value in the bond relative to the swap curve is approximately the same under either measure. Since the OAS measures value through discounting, multiplying by *PV01* gives the relative value. Since the par-par spread measures value as a spread on the floating side, multiplying by *SP01* gives the relative value. Hence, (4.5) says that the two measures give approximately the same relative value. This in no way negates the argument of previous sections that realizing a particular spread requires a particular trade.

Note that the quality of the approximation (4.5) depends on the magnitude of OAS: the larger the absolute value of OAS, the worse the approximation. Note also that in (4.5) the *PV01* of the bond may be computed at the swap curve or at the swap curve plus OAS. The former is readily available from swap calculators by setting up a swap with its fixed side equal to the bond's cash flows.

Propositions 7 and 8 of the Appendix show that if the swap curve is flat and if the coupon equals the matched-date swap rate, then $PV01 < SP01$. Together with (4.5) this means that, under these conditions, OAS exceeds the par-par asset swap spread for positive OAS, as indicated in the central case of Table 5a. As it turns out, *SP01* exceeds *PV01* in this case mostly due to the difference between the actual/360 floating rate convention governing *SP01* and the 30/360 fixed rate convention governing *PV01*. Conceptually, however, it is useful to think of the OAS and par-par spreads as being roughly equal in this case and vary coupon, curve, or both, to understand the other inequalities of Table 5a. Discussion focuses on Table 5a since, according to (4.5), any inequality derived for positive OAS may be reversed to obtain the corresponding result for negative OAS in Table 5b.

To explain intuitively the remaining relationships in Table 5a, Proposition 9 of the Appendix proves that, for a single cash flow *C* at time *t*, the ratio of the value of a basis-point decrease in the discount rate to the value of a basis-point increase in cash flow is[4]

$$\text{Value of a Discount Rate bp} \big/ \text{Value of a Cash Flow bp} = tC \big/ \left(1 + r_t/2\right) \qquad (4.6)$$

Using this fact, maintain a flat swap curve and raise the coupon above the matched-date swap rate. Raising the coupon raises the ratio for every cash flow. Hence, *PV01*, the sum of the value of discount rate basis points across cash flows, will now exceed *SP01*, the sum of the value of cash flow basis points across cash flows. Thus, by (4.5), s^{PAR} will exceed OAS. Conversely, for a coupon rate below the matched-date swap rate, OAS will exceed s^{PAR}.

Next consider moving away from the flat curve scenario of Table 5a. Changing from a flat to an upward-sloping curve, r_t rises for high *t* and falls for low *t*. This curve effect lowers the ratio of (4.6) for high *t* and raises the ratio for low *t*. However, since the ratio increases directly with *t*, the effect of the later cash flows and lower ratios dominates. Furthermore, the ratio for the last cash flow, i.e. the principal payment, is relatively large. Therefore, the net effect of the curve change will be to lower *PV01* relative to *SP01*. Hence, leaving coupon at the matched-date swap rate, OAS will exceed s^{PAR} if the curve slopes upward. Conversely, OAS will be less than s^{PAR} if the curve slopes downward.

The preceding paragraphs explain Table 5a except for its four corners. The case of a high coupon bond and a downward-sloping curve is straightforward. It has already been shown that raising the coupon rate increases the par-par spread relative to OAS and that a downward-sloping curve has the same effect. Hence, the combination of a high coupon and a downward-sloping curve must also be characterized by a high par-par spread relative to OAS. Conversely, for a low coupon bond and an upward-sloping curve, the coupon and curve effects both depress the par-par spread relative to OAS.

The remaining two corners of Table 5a are not as straightforward since the coupon and curve effects work in opposite directions. Hence, for a coupon below the swap rate in a downward-sloping curve, the par-par spread will exceed the OAS unless the coupon is particularly low. Similarly, for a coupon above the swap rate in an upward-sloping curve, the OAS will exceed the par-par spread unless the coupon is particularly high.

4.3 Benchmark Spread

The Benchmark Spread is the difference between the yield of the bond and the rate of its benchmark swap. With a flat curve the yield of the FNMA 6.125's of 3/15/12 is 4.702% and its benchmark, the 10-year swap, has a rate of 4.50%. The difference between the two gives a benchmark spread of 20.2 basis points. In this special case this semiannually compounded benchmark spread is equivalent to the quarterly compounded LIBOR OAS of 20 basis points, but this equivalence does not carry over to other curve environments.

In an upward-sloping curve, the yield of the FNMA 6.125's of 3/15/12 is 4.648%, the 10-year swap rate is 4.618% and, therefore, the benchmark spread is 3 basis points—substantially below the measures of spread considered so far. Intuitively, upward-sloping curves lower the yield of a premium security[5] and raise the 10-year swap rate relative to the 9.5-year bond yield. The opposite distortion occurs for the bond in a downward-sloping environment: the yield is 4.755% and the 10-year swap rate is 4.368% giving a benchmark spread of 38.7 basis points. Downward-sloping curves raise the yield of a premium security and lower 10-year rates relative to 9.5-year rates for a net increase in the benchmark spread. In short, this measure of asset swap spreads is affected by curve shapes and bond coupons even when LIBOR OAS is fixed. In other words, the benchmark spread is affected by factors that cannot be connected to the liquidity and credit characteristics of a bond relative to the swap curve.

The trade underlying this spread is to buy 100 face of the bond and pay fixed on a weighted notional amount of the benchmark swap, weighting the notional amount so as to immunize the trade against parallel shifts in rates. It is easy to show that the capital gain from this trade for small rate changes is minus the *DV01* of the bond times the change in the benchmark spread. In other words, for a 1-basis point tightening of the spread, the capital gain equals the *DV01* of the bond. Mathematically, if y^B is the bond yield, r^{Bmk} the benchmark swap rate, $s^{Bmk} \equiv y^B - r^{Bmk}$ the benchmark spread, and if Δ denotes a change, then

$$\text{Capital Gain} = -DV01\left(\Delta y^B - \Delta r^{Bmk}\right) = -DV01\Delta s^{Bmk} \tag{4.7}$$

For example, in the upward-sloping curve environment, the *PV01* of the 10-year swap is 0.07802 and the *DV01* of FNMA 6.125's of 3/15/12 is .08122. Therefore, hedging the purchase of 100 face amount of the bonds requires paying fixed on a notional amount of $100\times 8.122/7.802$ or 104.102. If the benchmark yield spread were then to narrow from 3 to 0 basis points, the trade would enjoy a capital gain of $(3-0)\times 8.122\%$ or 24.4 cents per $100 face amount of the bonds.

One problem with the benchmark trade is that while the capital gain is straightforward and proportional to the benchmark spread, the carry from the trade is not at all straightforward nor so associated. To see this, use the fact that the total return from a bond with an unchanged yield over a short time period is its yield.[6] Then, the carry from the benchmark trade, as a fraction of bond price, is

$$y^B - \left[\frac{DV01/PV01^{Bmk}}{\overline{P}}\right] r^{Bmk} - \text{Repo} + \left[\frac{DV01/PV01^{Bmk}}{\overline{P}}\right]\text{Libor} \tag{4.8}$$

A second problem with the benchmark trade is curve risk. In the example of a premium 9.75-year bond versus the 10-year swap, for example, a flattening of the curve would hurt the premium bond more than the par swap and would hurt the 9.75-year bond more than the 10-year swap. Consequently, the "spread" position will perform poorly without any deterioration of the credit or liquidity characteristics of the bond relative to the swap curve.

In practice, this criticism of the benchmark trade is somewhat unfair. The LIBOR OAS trade is, in theory, protected from curve risk by an appropriate hedging model. But if a parallel shift model is used, then the LIBOR OAS trade may be subject to substantial curve risk as well.

Perhaps the biggest advantage of the benchmark spread is that it is particularly easy to observe as both the bond yield and the benchmark swap rate are readily available. Similarly, the biggest advantages of the associated trade are simplicity and relative liquidity. The benchmark trade is simpler than a LIBOR OAS trade that uses a more complex hedging model. And, by using only the benchmark swap, the benchmark trade is more liquid than a par-par swap asset swap, a market value asset swap, or the matched-date trade described in the next section.

4.4 Matched-Date Spread

The Matched-Date Spread is the difference between the yield of the bond and the rate on the matched-date swap. In the flat curve example, the yield of the FNMA 6.125's of 3/15/12 is 4.702% and the par swap rate to 3/15/12 is 4.50%. The difference gives a matched-date spread of 20.2 basis points, equivalent to the quarterly LIBOR OAS of 20 basis points.

In an upward-sloping curve, however, the yield of the FNMA 6.125's of 3/15/12 is 4.648% while the matched-date rate is, by construction, still 4.50%. Therefore, the matched-date spread is 14.8 basis points—below the LIBOR OAS, par-par and market value measures. The intuition is that upward-sloping curves lower the yield of a high-coupon security relative to the par-coupon matched-date swap rate. (Also, as shown in Table 6, since upward-sloping curves raise the yield of a low-coupon security relative to the matched-date par swap rate, the matched-date spread exceeds LIBOR OAS in this case.[7]) While similar to the intuition for the benchmark yield spread, the use of a matched-date swap instead of a benchmark swap removes any maturity distortion from the matched-date spread, i.e. any difference between the swap rates to the bond maturity and to the benchmark maturity.

In a downward-sloping environment the yield of the FNMA 6.125's of 3/15/12 is 4.755% and the matched-date swap rate is 4.50% so the matched-date spread is 25.5 basis points. This is higher than the LIBOR OAS, the par-par, and the market value spreads because downward-sloping curves raise the yield of a high-coupon security relative to the par-coupon matched-date swap rate. (Also, as shown in Table 6, since downward-sloping curves lower the yield of a low-coupon security relative to the par-coupon matched-date swap rate, LIBOR OAS exceeds the matched-date spread in this case.) In short, this measure of asset swap spreads is affected by coupon and curve in ways that cannot be explained by the liquidity and credit characteristics of the bond relative to the swap curve.

The trade associated with the matched-date spread is to buy the bond and pay fixed on a weighted notional amount of the matched-date swap, weighting the notional amount to immunize the trade against parallel shifts in rates. It is easy to show that the capital gain from this trade for small rate changes is minus the *DV01* of the bond times the change in the matched-date spread. Mathematically, with r the matched-date swap rate and $s^{MD} \equiv y^B - r$ the matched-date spread,

$$\text{Capital Gain} = -DV01\left(\Delta y^B - \Delta r\right) = -DV01\Delta s^{MD} \tag{4.9}$$

For example, in the upward-sloping curve the *PV01* of the swap to 3/15/12 is 0.07547 and the *DV01* of the FNMA 6.125's of 3/15/12 is .08122. Hedging 100 face of the bond requires paying fixed on a notional amount of $100 \times 8.122/7.547$ or 107.619. Were the spread to narrow from 14.8 to 11.8, the trade would enjoy a capital gain of $(14.8-11.8)\times 8.122\%$ or 24.4 cents per $100 face amount of the bonds.

Compared with the benchmark trade, the matched-date trade reduces maturity risk. Equivalently, the matched-date spread eliminates the maturity effect in the benchmark spread. Curve risk remains, however, created by the difference between the coupon of the bond and the matched-date swap rate.

5. TRADING BETWEEN COUPON DATES

For expositional simplicity, previous sections have, for the most part, assumed that par-par and market value assets swaps are executed on coupon dates, or, equivalently, that accrued interest is zero. This section generalizes the results to trades initiated at any time.

Accrued interest complicates par-par and market value asset swaps in two ways. First, the full price rather than the flat price must be financed. Second, despite the presence of accrued interest, the trades are still constructed so that the par-par asset spread is earned on 100 while the market value spread is earned on the flat price of the bond at the initiation of the swap.

The marketplace has two conventions for dealing with the first fixed cash flow of the swap in an asset swap trade. In the full-coupon convention, the first cash flow is set to the bond coupon payment. For easy reference, a swap set up this way will be called a **matched-coupon swap**. In the partial-coupon convention, the first fixed cash flow is some fraction of the coupon payment, determined using the bond's coupon rate and swap payment conventions. A swap set up this way will be called a **matched-rate swap**. The next few sections show how par-par and market value asset swaps may be constructed using either the full- or partial-coupon conventions while imposing the constraints that the full price of the bond is financed and that the floating notional amount is indeed either 100 or the flat price. Propositions 1 and 2 of the Appendix prove that these are the only constructions that obey these constraints.

5.1 The Full-Coupon Par-Par Asset Swap

For the FNMA 6.125's of 3/15/12 settling on 12/16/02, a full-coupon par-par asset swap trade, illustrated in Figure 5, is constructed as follows: 1) Buy 100 face of the bond for its flat price of 110.556 plus accrued interest of 1.548 for a full price of 112.104. 2) Finance 100 from the repo desk. 3) Pay fixed on a matched-coupon swap, receiving the full bond premium, 12.104, up-front and receiving LIBOR plus a spread on 100. The initial proceeds of the trade are zero: 100 from the repo desk and the 12.104 up-front payment equal the full bond price. Also, all fixed cash flows due on the swap are exactly offset by coupon payments from the bond. Letting *AI* denote the accrued interest on the bond, Figure 6 depicts the general case of the full-coupon par-par asset swap.

Let α^B and α^S be the fractions of a semiannual period from settle to the first coupon date according to bond and swap day-count conventions, respectively, and let d_1 be the swap discount factor to that date. Proposition 3 of the Appendix shows that the full-coupon par-par asset swap spread is given by the following expression:

$$s_F^{Par} = \frac{100 - P + NPV^B - \left\{\left(1-\alpha^B\right) - \left(1-\alpha^S\right)d_1\right\} c/2}{SP01} \qquad (4.10)$$

Figure 5. **A Full-Coupon Par-Par Asset Swap**

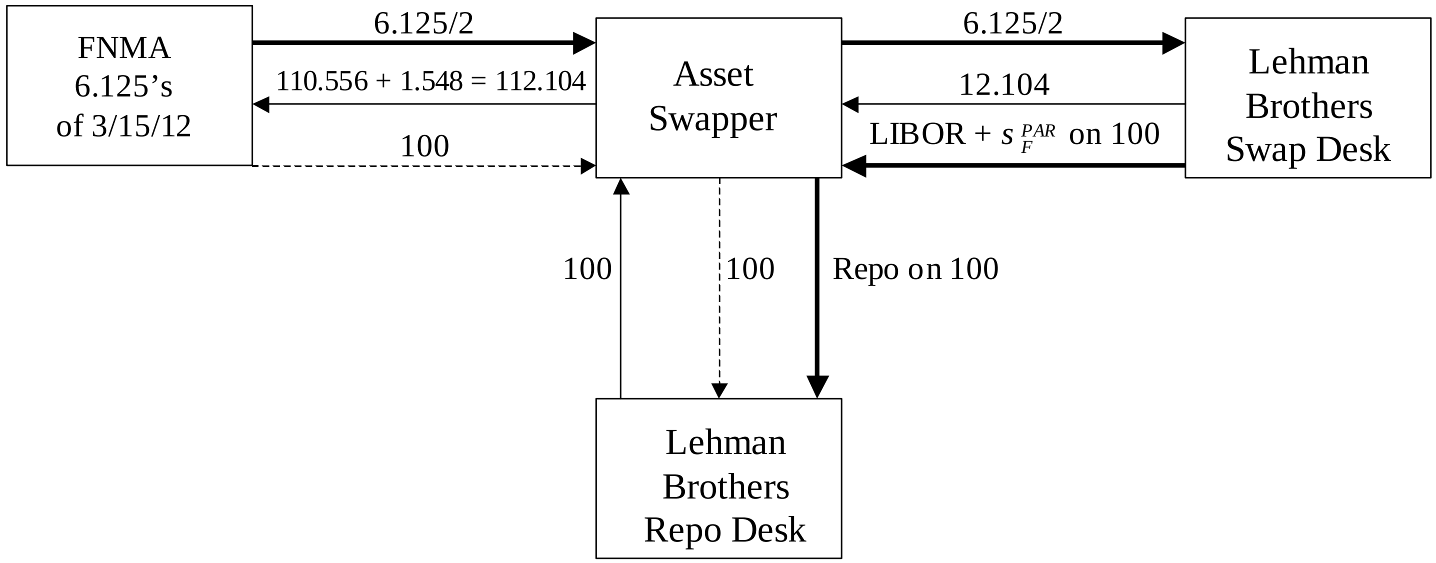
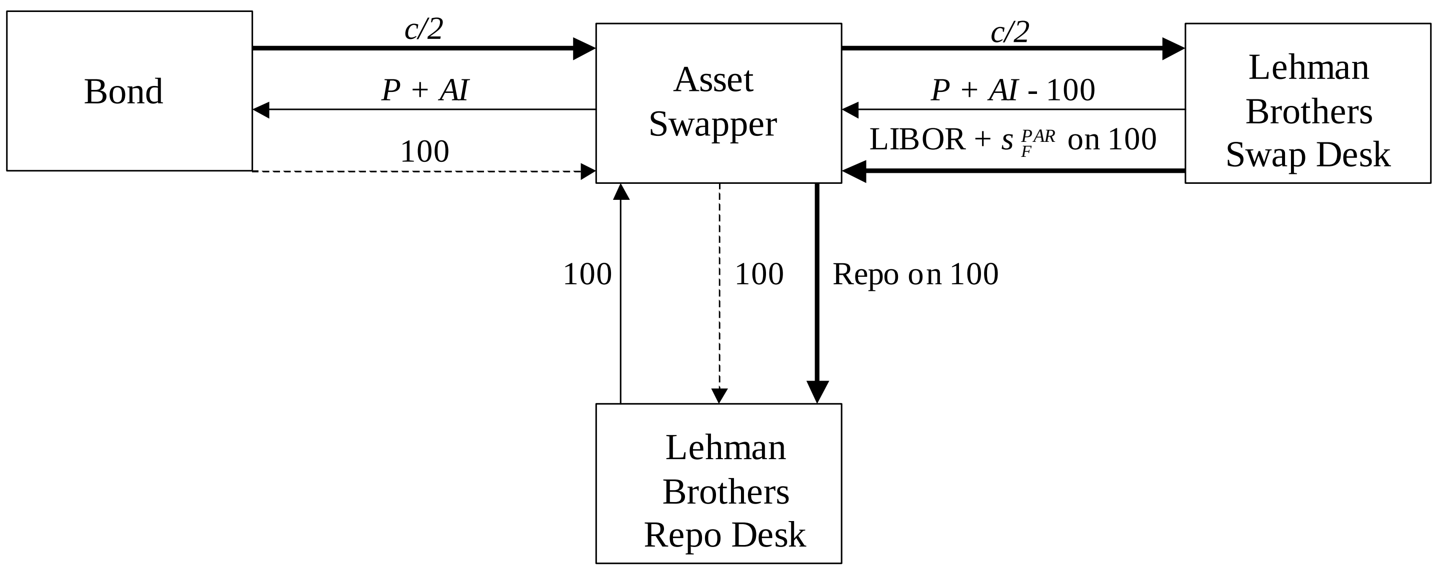


Figure 6. **A General Full-Coupon Par-Par Asset Swap**



P is the flat bond price and NPV^B is the net present value of the fixed side of a matched-rate swap. Equation (4.10) differs from the case of accrued interest equal to zero, (2.2) and (3.1), by the last term in the numerator. This extra term has two parts best understood by, as in Section 3, taking the perspective of the swap desk. First, $\left(1-\alpha^B\right)c/2$, that is, accrued interest, must be subtracted from the numerator to reflect the value of paying this accrued interest as part of the up-front payment. Second, $\left(1-\alpha^S\right)d_1\,c/2$ must be added to NPV^B to reflect the receipt of the full first coupon in the full-coupon par-par structure. If accrued interest is zero, $\alpha^B=\alpha^S=1$ and (4.10) reduces to the special case of (2.2) and (3.1).

5.2 The Partial-Coupon Par-Par Asset Swap

In the partial-coupon version of the par-par asset swap, the asset swapper pays fixed on a matched-rate swap, making a partial coupon payment on the first coupon date. The amount remaining from receiving the full coupon from the bond is used to repay borrowing from the repo desk. Figure 7 illustrates using the example of the FNMA 6.125's of 3/15/12 and a repo rate of 1.20%. The dotted arrows represent cash flows on the first coupon payment date. Figure 8 illustrates the general case; the *PV* function gives present values using repo rates.

With 89 30/360 days from 12/16/02 to the next coupon date of 3/15/03, the first fixed swap payment will be $6.125\times 89/360=1.514$. The difference between the bond coupon payment of 6.125/2 or 3.063 and this fixed payment is 3.063-1.514 or 1.549, with a present value, using the repo rate for discounting, of $1.549/\left(1+89\times 1.20\%/360\right)=1.544$. With these results, the partial-coupon par-par asset swap may be described as follows: 1) Buy 100 face of the bond for a full price of 112.104. 2) Finance 100 plus 1.544 or 101.544 from the repo desk. 3) Pay fixed on a matched-rate swap, receive an up-front payment of 112.104-101.544 or 10.560, and receive LIBOR plus a spread on 100.

Note that the initial net proceeds of the trade are zero: 101.544 from the repo desk plus a 10.560 payment equals 112.104, the full bond price. On the first coupon date, the asset swapper collects a coupon of 6.125/2 or 3.063, pays 1.514 as the first fixed swap payment, and uses the remainder, 1.549, to reduce the repo loan balance from 101.544 to 100. All subsequent fixed cash flows clearly cancel. Finally, note that the notional of the floating side of the trade, even over the first period, is 100: the initial excess repo borrowing of 1.544 is paid off with interest using the 1.549 excess of the first coupon over the first fixed payment.

Proposition 4 of the Appendix shows that the expression for the partial-coupon par-par asset swap is

$$s_F^{Par}=\frac{100-P+NPV^B-\left\{\left(1-\alpha^B\right)-\left(1-\alpha^S\right)d_1^R\right\}c/2}{SP01} \qquad (4.11)$$

where d_1^R is the discount factor for the first coupon date using repo rates for discounting. The only difference between (4.10) and (4.11) is that d_1, the discount factor using swap rates, appears in (4.10) while d_1^R appears in (4.11). Intuitively, in the full-coupon version the term $\left(1-\alpha^S\right)d_1\,c/2$ is part of the value of the coupon flow received by the swap desk. In the partial-coupon version, $\left(1-\alpha^S\right)d_1^R\,c/2$ is subtracted from the up-front payment the swap desk must make as the asset swapper is financing that amount through repo.

Once again, if the next coupon payment is one full coupon period away, (4.11) reduces to the expression for the par-par asset swap presented earlier in the paper.

Figure 7. **A Partial Coupon Par-Par Asset Swap**

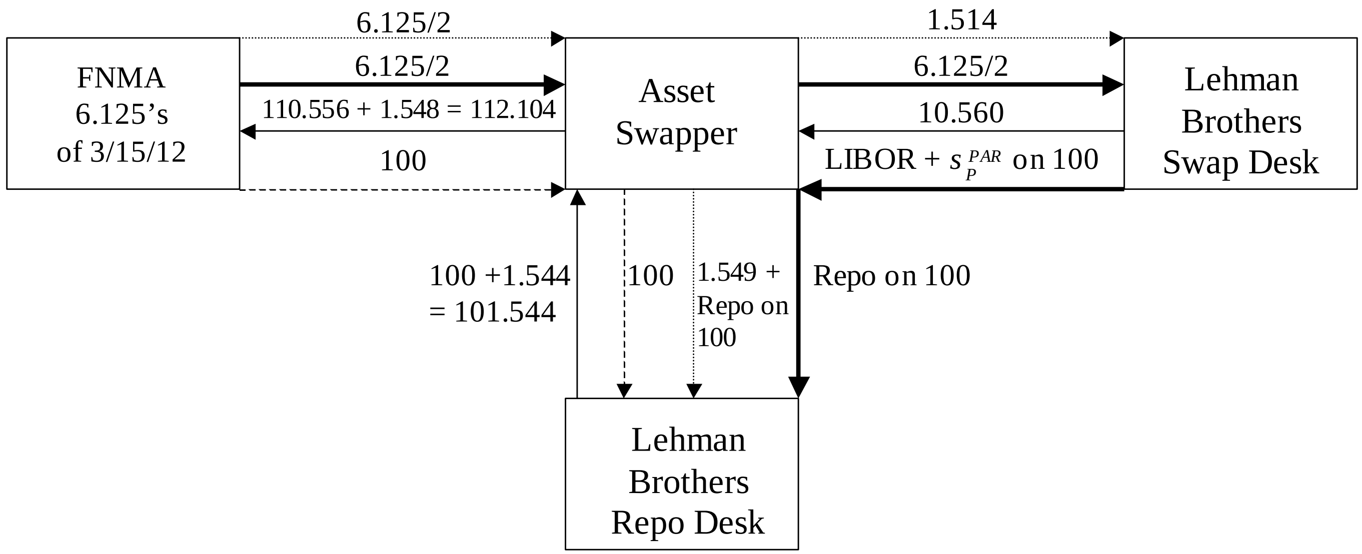
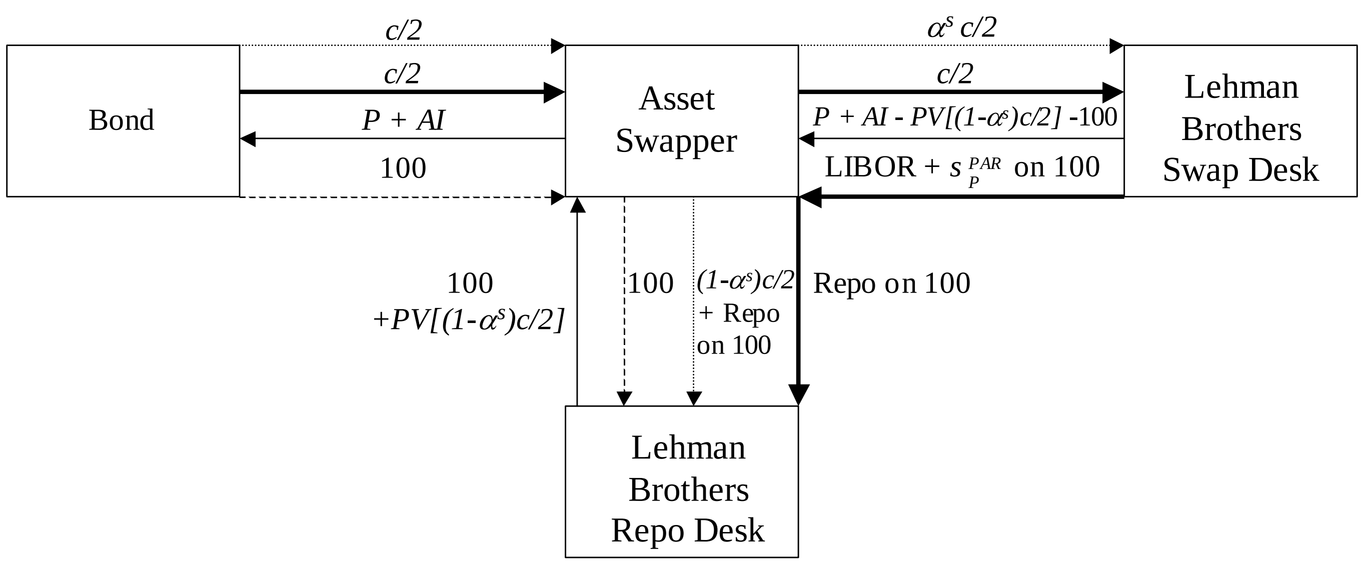


Figure 8. **A General Partial-Coupon Par-Par Asset Swap**



5.3 The Full-Coupon Market Value Asset Swap

Whereas the notional floating balance of the full-coupon par-par asset swap is par, the notional floating balance of the full-coupon market value asset swap is the bond's flat price. Returning to the example of the FNMA 6.125's of 3/15/12, Figure 9 illustrates: 1) Buy the bond for a flat price of 110.556 plus accrued interest of 1.548. 2) Borrow 110.556 from the repo desk. 3) Pay fixed on a matched-coupon swap, receive an up-front fee of the accrued interest, i.e. 1.548, a terminal payment of the flat premium, or 10.556, and LIBOR plus a spread on 110.556. Note that unlike the market value asset swap when accrued interest equals zero, this trade entails an up-front as well as a terminal payment from the swap. Figure 10 illustrates the general case.

The initial proceeds of the market value asset swap are zero since borrowing the flat price from the repo desk and receiving an up-front swap payment of the accrued interest cover the full price. All fixed cash payments due on the matched-coupon swap are offset by coupon payments from the bond.

Proposition 5 of the Appendix shows that the full-coupon market value asset swap spread equals the full-coupon par-par asset swap spread divided by the flat price of the bond.

5.4 The Partial-Coupon Market Value Asset Swap

The remaining permutation of the static asset swap trades is the partial-coupon market value asset swap. Figure 11 depicts the example of the FNMA 6.125's of 3/15/12 with a repo rate of 1.20%. As in the case of the partial-coupon par-par asset swap, the first fixed payment on the matched-rate swap is $6.125 \times 89/360 = 1.514$ and the difference between the bond coupon payment and this fixed payment is 1.549 with a present value under repo rates of 1.544. The trade is as follows: 1) Buy 100 face of the bond for 110.556 plus accrued interest of 1.548 for a total of 112.104. 2) Finance 110.556 plus 1.544 or 112.100 from the repo desk. 3) Pay fixed on the matched-rate swap, receive an up-front payment of 112.104-112.100 or .004, and receive LIBOR plus a spread on 110.556. The up-front payment in this structure, equal to the difference between accrued interest and its present value over a short-time period, will always be relatively small.

Note that the net proceeds at the initiation of the trade are zero: 112.100 from the repo desk and an up-front payment of .004 equal 112.104, the full price of the bond. On the first coupon payment date, the asset swapper collects a coupon payment of 6.125/2 or 3.063, pays 1.514 as the first fixed swap payment, and uses the remainder, 1.549, to reduce the repo loan balance from 112.1 to 112.1-1.544 or 110.556, the flat price. All subsequent fixed cash flows clearly cancel. Note that the flat price of 110.556 is the notional amount earning the spread from initiation to the first coupon date: the extra repo borrowing of 1.544 at initiation is paid off with interest using the 1.549 difference between the bond's first coupon payment and the first fixed swap cash flow.

Proposition 5 of the Appendix shows that the partial-coupon market value asset swap spread equals the partial-coupon par-par asset swap spread divided by the flat price of the bond.

Figure 9. **A Full-Coupon Market Value Asset Swap**

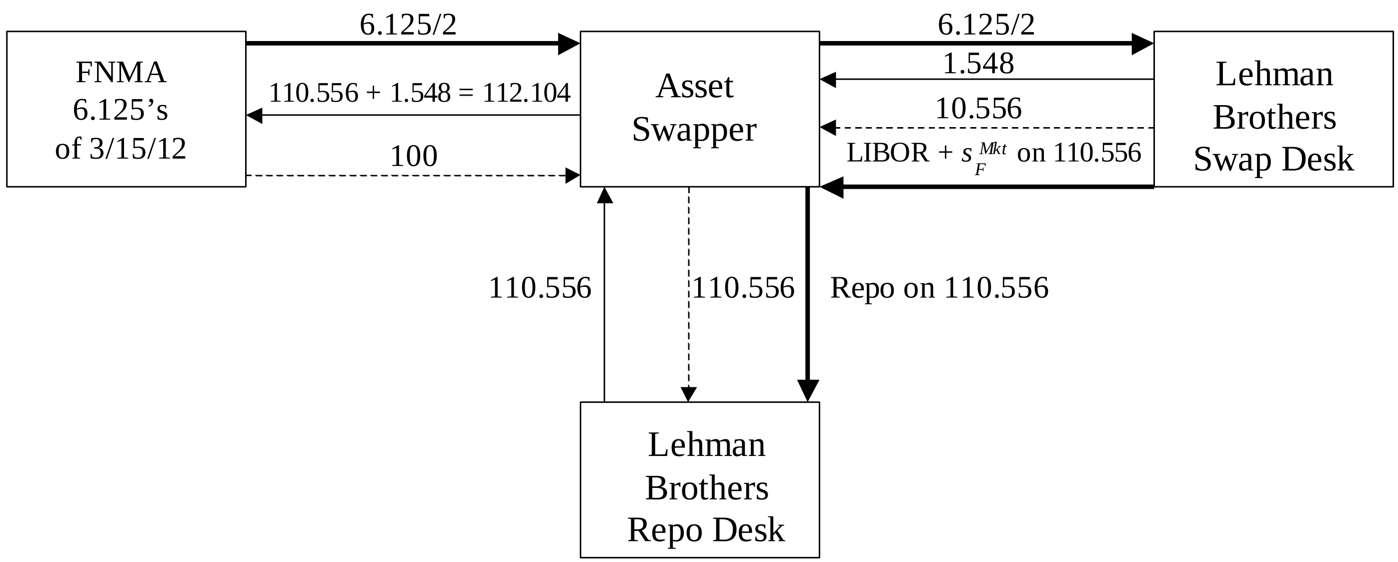


Figure 10. **A General Full –Coupon Market Value Asset Swap**

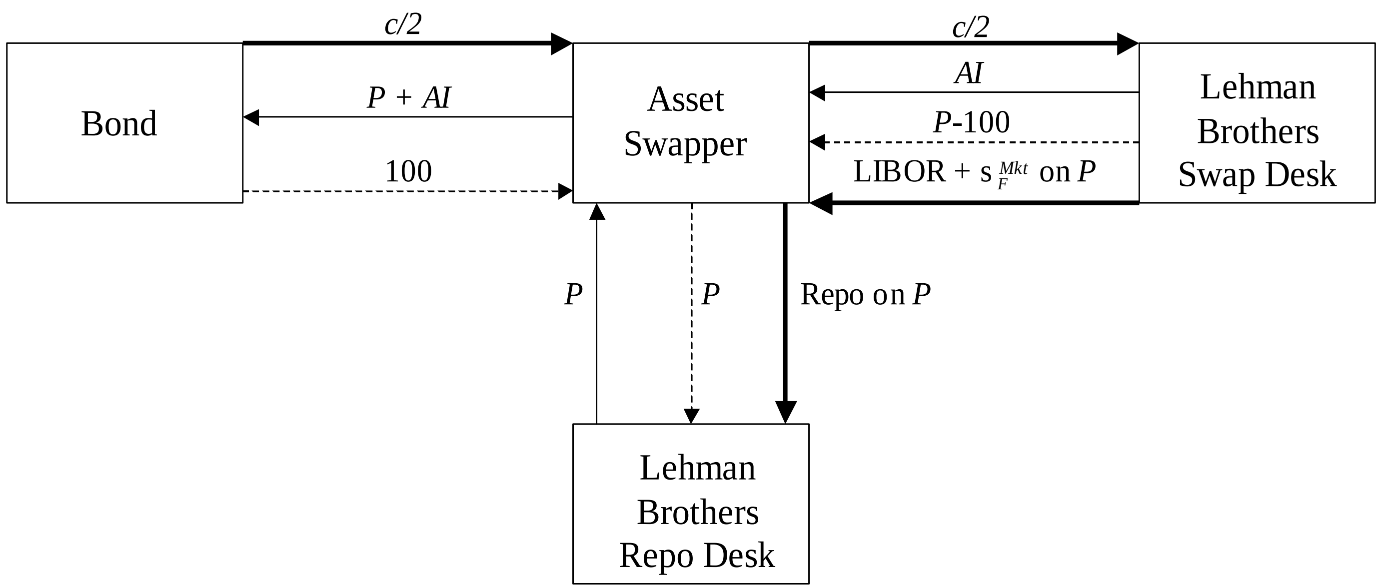


Figure 11. **A Partial-Coupon Market Value Asset Swap**

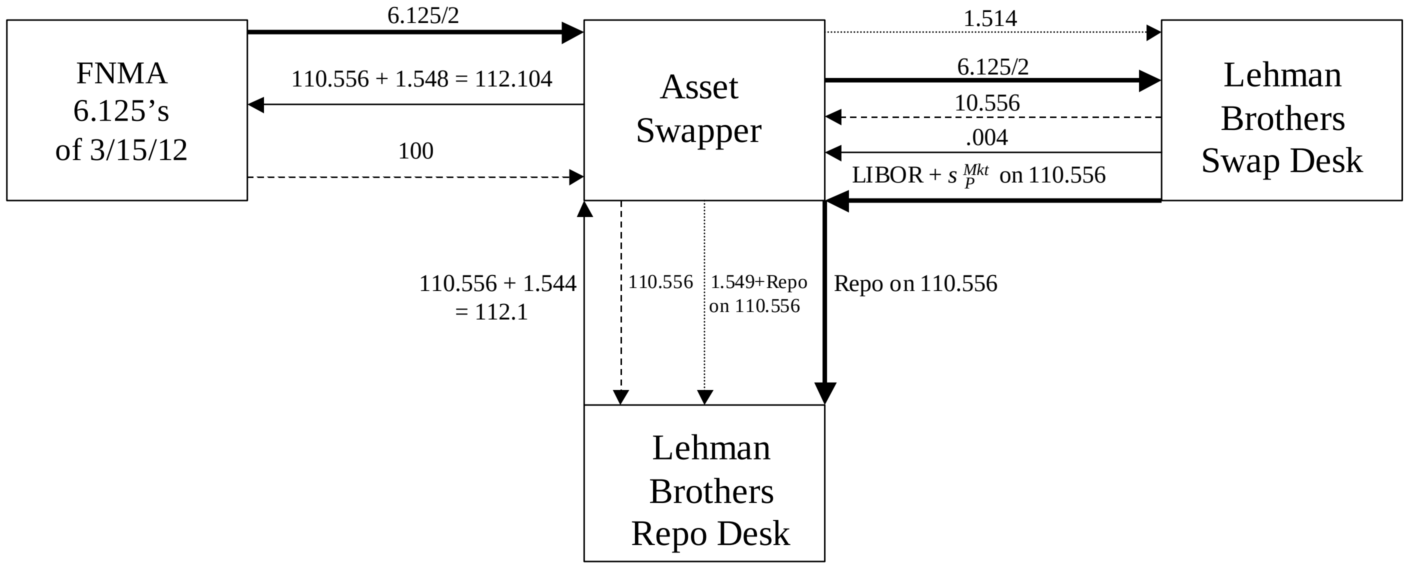
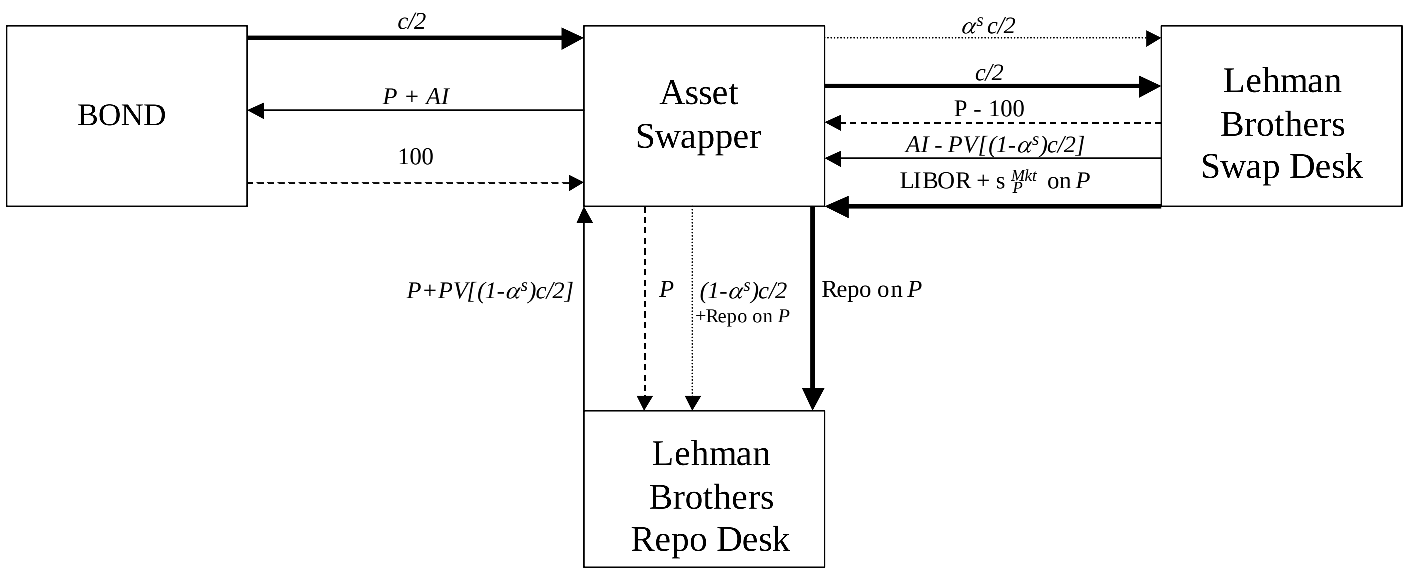


Figure 12. **A General Partial-Coupon Market Value Asset Swap**



6. SOME DETAILS ON FINANCING AND COLLATERAL

For ease of exposition, in the context of par-par and market value asset swaps, this paper has simplified a few issues regarding the financing of bonds in the repo market and the collateral implications of embedded net present values in the swap market. This section comments accordingly.

Previous sections implicitly assumed that the bond is financed over the same period as the next floating cash flow of the swap. In this way the total spread earned on the notional amount is 3-month LIBOR plus the asset swap spread minus term repo. In practice, bonds are often financed using overnight repo. But there is interest rate risk in borrowing overnight through the repo market and investing term through the floating leg of the swap. Therefore, if term financing cannot be obtained, eurodollar futures, fed fund futures, or some other short-term securities should be used to hedge the exposure to short-term rates.

The paper has assumed that the asset swapper could freely choose the cash raised from a repo of 100 face amount of bonds. In practice, the proceeds are almost always the full price of the bond. So, for example, the discussion in the paper would call for financing 100 in a par-par asset swap of a discount bond with a full price of 90 even though no repo desk would allow that. To effect the par-par asset swap in this case, the asset swapper must finance the extra 10 in the repo market using other securities. Similarly, when the discussion in the paper calls for financing 100 in a par-par asset swap of a premium bond with a full price of 110, the practical application would be to finance the full 110 and then invest 10 in the repo market.

The last comment of this section concerns the effect of collateral requirements on the notional amount earning LIBOR plus the asset swap spread minus repo. The claim in the paper that, at initiation of the trades, the notional amount is 100 for par-par swaps and bond price for market value swaps is true by construction. However, as market prices move, collateral requirements can alter these notional amounts. Say, for example, that by the first floating cash flow date interest rates have fallen so that the bond price has risen and the *NPV* of the swap to the asset swapper has become negative. In rolling over a term repo agreement, the higher bond value will be financed. Also, the asset swapper will have to post collateral earning, at least in theory, LIBOR. Hence, collateral requirements can change the notional amount earning the LIBOR-repo spread.

7. CONCLUSION

Traders and money managers use measures of asset swap spreads to make decisions about trades of bonds relative to swaps. This paper relates the various measures of asset swap spreads to one another and associates each spread with a trade. Understanding this association is important for ensuring that the decision making process behind a trade and the resulting P&L are as closely related as they should be.

8. NOTATION

This section lists the notation used in the paper and in the Appendix.

α^B : Days from today to the first coupon date as a fraction of a semiannual coupon period, counting days with the bond's day-count conventions.
α^S : Days from today to the first coupon date as a fraction of a semiannual coupon period, counting days with the day-count conventions for the fixed side of swaps.
β^S : Days from today to the first floating payment date as a fraction of a quarterly payment period, counting days with the day-count conventions of the fixed side of swaps.
A_Q : Quarterly annuity factor using swap rates for discounting and fixed-side day-count conventions for cash flows.
A_{SA} : Semiannual annuity factor using swap rates for discounting and fixed-side day-count conventions for cash flows.
c: Bond coupon per 100 face value.
d: Discount factor for the maturity date of the bond using swap rates for discounting.
d_1 : Discount factor for the first coupon date using swap rates for discounting.
$d_{1/2}$: Discount factor for the first floating payment date when it occurs before the first fixed payment date.
d_1^R : Discount factor for the first coupon date using repo rates for discounting.
f: Factor to convert a 30/360 cash flow (e.g. from the fixed side of swaps) to an actual/360 cash flow (e.g. from the floating side of swaps). This factor is, on average, *(365.25/4)/360* or *.253646*.
λ: Risk premium in basis points per unit of *DV01* or *PV01* risk.
NPV^B: The net present value of the fixed side of a matched-rate swap. Mathematically, $NPV^B = \frac{1}{2}(c-r)A_{SA}$.
P: Flat bond price per 100 face value.
$\bar{P}$: Full bond price per 100 face value.
P^S: Value of a swap per 100 notional amount.
PV01: Change in the value of the cash flows from 100 face of a bond when discounted using swap rates or swap rates plus an OAS for a 1 basis point decline in swap rates.
$PV01^S$: *PV01* of a swap.
r: The fixed payment per 100 face value on a matched-date swap that is fair against LIBOR flat.
s_P^{Mkt} : Partial-coupon market value asset swap spread.
s_F^{Mkt} : Full-coupon market value asset swap spread.
s_P^{Par} : Partial-coupon par-par asset swap spread.
s_F^{Par} : Full-coupon par-par asset swap spread.
SP01: Value of an additional basis point on the floating side of a 100 notional of a matched-date swap, identically equal to fA_Q .

9. APPENDIX

Proposition 1: Par-par asset swaps are constructed as follows. In the full coupon version,

1) Buy the bond.
2) Borrow 100 from the repo desk.
3) Pay fixed in a matched-coupon swap against LIBOR plus a spread and collect an up-front payment of the full price of the bond minus 100.

In the partial coupon version,

1) Buy the bond.
2) Borrow $100+\left(1-\alpha^{S}\right)d_1^R\, c/2$ from the repo desk.
3) Pay fixed in a matched-rate swap against LIBOR plus a spread and collect an up-front payment of

$$P+\left\{\left(1-\alpha^{B}\right)-\left(1-\alpha^{S}\right)d_1^R\right\}c/2-100 \qquad (7.1)$$

Proof: In general, the legs of a par-par asset swap are the following.

1) Buy the bond for $P+\left(1-\alpha^{B}\right)c/2$.
2) Borrow 100+*u* from the repo desk.
3) Pay fixed based on the coupon *c* against LIBOR+*s* on 100 and collect an up-front payment of *P*+*v*-100.

In any asset swap the proceeds of the asset swap at initiation must net to 0 and the fixed cash flows on any date must also net to 0. In a par-par asset swap, the floating cash flows must be earned on a principal of 100.

To satisfy the zero proceeds condition,

$$100+u+P+v-100-\left(P+\left(1-\alpha^{B}\right)c/2\right)=0 \qquad (7.2)$$

Or,

$$u+v-\left(1-\alpha^{B}\right)c/2=0 \qquad (7.3)$$

In a full coupon asset swap, the first swap coupon is $c/2$. Furthermore, in a par-par asset swap, the amount *u* borrowed from the repo desk must be paid off on the first coupon date with interest so that the floating interest is earned on 100. Therefore, the netting of the first fixed cash flow requires that

$$c/2-c/2-u/d_1^R=0 \qquad (7.4)$$

This implies that $u=0$, and, by (7.3),

$$v=\left(1-\alpha^{B}\right)c/2 \qquad (7.5)$$

Note, of course, that all subsequent fixed cash flows cancel by construction.

In words, in the full coupon asset swap the asset swapper borrows *100* from the repo desk and finances the premium or discount plus the accrued interest through the swap.

In a partial coupon asset swap, the first coupon is $\alpha^S\, c/2$. In this case, then, the netting of the first cash flow requires that

$$c/2-\alpha^S\, c/2-u/d_1^R=0 \tag{7.6}$$

Or,

$$u=\left(1-\alpha^S\right)d_1^R\, c/2 \tag{7.7}$$

And, from (7.3),

$$v=\left\{\left(1-\alpha^B\right)-\left(1-\alpha^S\right)d_1^R\right\}c/2 \tag{7.8}$$

Note again that all subsequent fixed cash flows cancel by construction. ***Q.E.D.***

Proposition 2: Market value asset swaps are constructed as follows. In the full coupon version,

4) Buy the bond.
5) Borrow the flat price from the repo desk.
6) Pay fixed in a matched-coupon swap against LIBOR plus a spread, collect an up-front payment of the accrued interest of the bond, and collect a payment at the termination of the swap of the flat price minus 100.

In the partial coupon version,

4) Buy the bond.
5) Borrow $P+\left(1-\alpha^S\right)d_1^R\, c/2$ from the repo desk.
6) Pay fixed in a matched-rate swap against LIBOR plus a spread, collect an up-front payment of

$$\left\{\left(1-\alpha^B\right)-\left(1-\alpha^S\right)d_1^R\right\}c/2 \tag{7.9}$$

and collect a payment at the termination of the swap of the flat price minus 100.

Proof: In general, the legs of a market value asset swap are the following.

4) Buy the bond for $P+\left(1-\alpha^B\right)c/2$.
5) Borrow $P+u$ from the repo desk.
6) Pay fixed based on a coupon c against LIBOR+s on 100, collect an up-front payment of v, and collect a payment at the termination of the swap of P-100.

In any asset swap the proceeds of the asset swap at initiation must net to 0 and the fixed cash flows on any date must also net to 0. In a market value asset swap, the floating cash flows must be earned on a principal of *P*.

To satisfy the zero proceeds condition,

$$P + u + v - \left(P + \left(1-\alpha^{B}\right)c/2\right) = 0 \tag{7.10}$$

Or,

$$u + v - \left(1-\alpha^{B}\right)c/2 = 0 \tag{7.11}$$

In a full-coupon asset swap, the first swap coupon is $c/2$. Furthermore, in a market value asset swap, the amount *u* borrowed from the repo desk must be paid off on the first coupon date with interest so that the floating interest is earned on *P*. Therefore, the netting of the first fixed cash flow requires that

$$c/2 - c/2 - u/d_1^{R} = 0 \tag{7.12}$$

This implies that $u = 0$, and, by (7.11),

$$v = \left(1-\alpha^{B}\right)c/2 \tag{7.13}$$

Note, of course, that all subsequent fixed cash flows cancel by construction.

In words, in the full coupon asset swap the asset swapper borrows the flat price of the bond from the repo desk and finances the accrued interest through the swap.

In a partial coupon asset swap, the first coupon is $\alpha^{S}\, c/2$. In this case, then, the netting of the first cash flow requires that

$$c/2 - \alpha^{S}\, c/2 - u/d_1^{R} = 0 \tag{7.14}$$

Or,

$$u = \left(1-\alpha^{S}\right)d_1^{R}\, c/2 \tag{7.15}$$

And, from (7.11),

$$v = \left\{\left(1-\alpha^{B}\right) - \left(1-\alpha^{S}\right)d_1^{R}\right\}c/2 \tag{7.16}$$

Note again that all subsequent fixed cash flows cancel by construction. ***Q.E.D.***

Proposition 3: The full coupon par–par asset swap spread satisfies the following expression.

$$s_F^{Par} = \frac{100 - P + \tfrac{1}{2}(c-r)A_{SA} - \left\{\left(1-\alpha^B\right) - \left(1-\alpha^S\right)d_1\right\}c/2}{SP01} \tag{7.17}$$

Proof: From the point of view of the swap desk, the value of the swap has the following components. First, the up-front payment is worth

$$100 - \left(P + \left(1-\alpha^B\right)c/2\right) \tag{7.18}$$

Second, the value of receiving the coupon as the fixed side of the swap is

$$c/2A_{SA} + c/2\left(1-\alpha^S\right)d_1 \tag{7.19}$$

where the second term accounts for the fact that the full coupon is being paid through the swap while the annuity factor, as defined, assumes a partial coupon payment. Third, the value of paying LIBOR plus the spread is

$$-r/2A_{SA} - fs_F^{Par}A_Q \tag{7.20}$$

The first term invokes the fact that, by definition of the matched-date swap payment *r*, paying $r/2$ semiannually is fair against LIBOR. The second term is the value of paying the spread, where *f* converts the annuity value, defined using the fixed side day-count convention, to the floating side convention. For the swap to be fair, the sum of (7.18), (7.19), and (7.20) must equal 0:

$$100 - \left(P + \left(1-\alpha^B\right)c/2\right) + c/2A_{SA} + c/2\left(1-\alpha^S\right)d_1 - r/2A_{SA} - fs_F^{Par}A_Q = 0 \tag{7.21}$$

Solve for the spread and recall that $SP01 \equiv fA_Q$. ***Q.E.D.***

Proposition 4: The partial coupon par–par asset swap spread satisfies the following expression.

$$s_P^{Par} = \frac{100 - P + \tfrac{1}{2}(c-r)A_{SA} - \left\{\left(1-\alpha^B\right) - \left(1-\alpha^S\right)d_1^R\right\}c/2}{fA_Q} \tag{7.22}$$

Proof: From the point of view of the swap desk, the up-front payment is worth

$$100 - P - \left\{\left(1-\alpha^B\right) - \left(1-\alpha^S\right)d_1^R\right\}c/2 \tag{7.23}$$

Receiving the coupon as the fixed side of the swap, where the first cash flow is a partial coupon, is worth

$$\tfrac{1}{2} c A_{SA} \tag{7.24}$$

And, lastly, by the definition of r, paying LIBOR plus the spread is worth

$$-\tfrac{1}{2} r A_{SA} - s_P^{Par} f A_Q \tag{7.25}$$

Add (7.23), (7.24), and (7.25), set the sum to zero, and solve for the spread. ***Q.E.D.***

Proposition 5: The market value asset swap spread equals its corresponding par-par asset swap spread divided by the flat price of the bond.

Proof: Following the logic of the earlier propositions, in the case of the full coupon market value swap, the spread must satisfy the following equation:

$$(100-P)d - \left\{\left(1-\alpha^B\right)-\left(1-\alpha^S\right)d_1\right\} c/2 + \tfrac{1}{2} c A_{SA} - \tfrac{1}{2} r\left(P/100\right) A_{SA} - s_F^{Mkt} P f A_Q = 0 \tag{7.26}$$

Note that this expression resembles (7.21), the equation for the full coupon par-par asset swap, with three differences. First, the term 100-P is multiplied by a discount factor in (7.26) because, in a market value swap, this payment is made at termination. Second, paying LIBOR is worth $\tfrac{1}{2} r\left(P/100\right) A_{SA}$ instead of $\tfrac{1}{2} r A_{SA}$, because, in a market value swap, the notional of the floating side is P not 100. Third, the spread is multiplied by P because, again, the notional of the floating side is P.

Solving for the spread,

$$s_F^{Mkt} = \frac{(100-P)d - \left\{\left(1-\alpha^B\right)-\left(1-\alpha^S\right)d_1\right\} c/2 + \tfrac{1}{2}\left(c - rP\right) A_{SA}}{P f A_Q} \tag{7.27}$$

Next, noting that, by the definition of r,

$$\tfrac{1}{2} r A_{SA} + 100 d = 100 \tag{7.28}$$

it follows from (7.27) that

$$\begin{aligned} s_F^{Mkt} &= \frac{100 - P - \left\{\left(1-\alpha^B\right)-\left(1-\alpha^S\right)d_1\right\} c/2 + \tfrac{1}{2}\left(c-r\right) A_{SA}}{f P A_Q} \\ &= s_F^{Par} / P \end{aligned} \tag{7.29}$$

In the case of the partial coupon market value swap, the equation to be satisfied is

$$(100-P)d - \left\{\left(1-\alpha^B\right)-\left(1-\alpha^S\right)d_1^R\right\} c/2 + \tfrac{1}{2}\left\{c - r\left(P/100\right)\right\} A_{SA} - s_P^{Mkt} P f A_Q = 0 \tag{7.30}$$

This resembles the terms (7.23), (7.24), and (7.25) determining the partial coupon par-par asset swap spread except for the same three differences mentioned above: the term 100-*P* must be discounted and the terms coming from the floating side must reflect a notional amount of *P*.

Solving for the spread,

$$s_P^{Mkt} = \frac{(100-P)d - \left\{(1-\alpha^B)-(1-\alpha^S)d_1^R\right\}c/2 + \tfrac{1}{2}(c-rP)A_{SA}}{PfA_Q} \tag{7.31}$$

Then using the definition of *r* in (7.28),

it follows from (7.31) that

$$\begin{aligned} s_P^{Mkt} &= \frac{100-P-\left\{(1-\alpha^B)-(1-\alpha^S)d_1^R\right\}c/2 + \tfrac{1}{2}(c-r)A_{SA}}{fPA_Q} \\ &= s_P^{Par}/P \end{aligned} \tag{7.32}$$

Q.E.D.

Proposition 6: $OAS \approx fs_F^{Par}A_Q / PV01$.

Proof: By the definition of *PV01*, the difference between the value of the bond's cash flows evaluated at the swap curve and the value of the bond's cash flows evaluated at the swap curve plus an *OAS* approximately equals the *PV01* times the *OAS*. Mathematically,

$$PV01 \times OAS \approx \tfrac{c}{2}A_{SA} + \tfrac{c}{2}(1-\alpha^S)d_1 + d - \left(P + (1-\alpha^B)\tfrac{c}{2}\right) \tag{7.33}$$

(Note that the second term on the right hand side is added to include the entire first coupon in the present value of the cash flows as the entire first coupon is included in the full price of the bond.) Using the definition of the matched maturity swap rate in (7.28),

$$PV01 \times OAS \approx \tfrac{c}{2}A_{SA} + \tfrac{c}{2}(1-\alpha^S)d_1 + 100 - \tfrac{r}{2}A_{SA} - \left(P + (1-\alpha^B)\tfrac{c}{2}\right) \tag{7.34}$$

Rearranging terms,

$$PV01 \times OAS = 100 - P + \tfrac{1}{2}(c-r)A_{SA} - \left\{(1-\alpha^B)-(1-\alpha^S)d_1\right\}c/2 \tag{7.35}$$

But, by inspection of (7.17), the right hand side of (7.35) equals $fs_F^{Par}A_Q$. ***Q.E.D.***

Proposition 7: For a flat swap curve, $\tfrac{1}{2} A_{SA} = PV01\big|_{c=r}$.

Proof: The present value of a bond's payments using a discount rate of r equals

$$\frac{c/2}{(1+r/2)^{\tau}}+\frac{c/2}{(1+r/2)^{1+\tau}}+\cdots+\frac{1+c/2}{(1+r/2)^{N-1+\tau}} \tag{7.36}$$

where $\tau \equiv \alpha^S$ and N is the number of coupon payments. Writing this using summation notation,

$$\frac{1}{(1+r/2)^{\tau}}\left[c/2\sum_{t=0}^{N-1}\frac{1}{(1+r/2)^{t}}+\frac{1}{(1+r/2)^{N-1}}\right] \tag{7.37}$$

Performing the summation and simplifying,

$$(1+r/2)^{1-\tau}\left[c/r\left\{1-(1+r/2)^{-N}\right\}+(1+r/2)^{-N}\right] \tag{7.38}$$

Differentiating with respect to r, multiplying by -1, and evaluating at $c=r$ shows that

$$PV01\big|_{c=r}=\frac{(1+r/2)^{1-\tau}}{r}\left(1-(1+r/2)^{-N}\right)-\tfrac{1}{2}(1-\tau)(1+r/2)^{-\tau} \tag{7.39}$$

The semiannual annuity factor with a swap rate of r is, by definition,

$$A_{SA}=\frac{\alpha^S}{(1+r/2)^{\tau}}+\frac{1}{(1+r/2)^{1+\tau}}+\cdots\frac{1}{(1+r/2)^{N-1+\tau}} \tag{7.40}$$

Performing the summation,

$$A_{SA}=\frac{2(1+r/2)^{1-\tau}}{r}\left(1-(1+r/2)^{-N}\right)-\frac{1-\alpha^S}{(1+r/2)^{\tau}} \tag{7.41}$$

Comparing (7.39) and (7.41), recalling that $\tau \equiv \alpha^S$, proves the proposition. ***Q.E.D.***

Proposition 8: For a flat swap curve, $\frac{1}{2} A_{SA} < SP01$.

Proof: Let y be the quarterly compounded swap rate and write $\frac{1}{2} A_{SA}$ in terms of this rate:

$$\tfrac{1}{2} A_{SA} = \tfrac{1}{2}\alpha^S d_1 + \left(1+y/4\right)^{-2\tau} \tfrac{1}{2}\sum_{t=1}^{N-1}\left(1+y/4\right)^{-2t} \qquad (7.42)$$

Performing the summation,

$$\tfrac{1}{2} A_{SA} = \tfrac{1}{2}\alpha^S d_1 + \tfrac{1}{2}\left(1+y/4\right)^{-2\tau}\left[1-\left(1+y/4\right)^{-2(N-1)}\right] \Big/ \left[y/2 + y^2/16\right] \qquad (7.43)$$

There are two cases two consider when writing an expression for *SP01*. In the first case, the next floating cash flow occurs on the same date as the next fixed cash flow. In this case, since α^S is the fraction of a semiannual period to the next cash flow, $2\alpha^S$ is the fraction of a quarterly period. Hence,

$$SP01 = 2\alpha^S f d_1 + \left(1+y/4\right)^{-2\tau} f \sum_{t=1}^{2(N-1)}\left(1+y/4\right)^{-t} \qquad (7.44)$$

Performing the summation and rearranging terms slightly,

$$SP01 = 2\alpha^S f d_1 + \tfrac{1}{2}\left(1+y/4\right)^{-2\tau}\left[1-\left(1+y/4\right)^{-2(N-1)}\right] \Big/ \left[y/8f\right] \qquad (7.45)$$

Noting that $f > .25$ (see Section 8), each of the two terms in (7.45) exceeds its counterpart in (7.43). This proves the result for this case.

In the second case, the next floating cash flow occurs before the next fixed cash flow and the second floating cash flow occurs at the same time as the next fixed cash flow. In this case,

$$SP01 = f\left[\beta^S d_{1/2} + d_1\right] + f\left(1+y/4\right)^{-2\tau} \sum_{t=1}^{2(N-1)}\left(1+y/4\right)^{-t} \qquad (7.46)$$

This expression differs from (7.44) only in the first term. Hence, to prove the result in this case, it need only be shown that the first term of (7.46) exceeds the first term of (7.43):

$$\tfrac{1}{2}\alpha^S d_1 < f\left[\beta^S d_{1/2} + d_1\right] \qquad (7.47)$$

Noting that $d_{1/2} > d_1$ and that $1+\beta^S = 2\alpha^S$, since both sides of this equality denote the fraction of a quarterly period to the next fixed cash flow, (7.47) may be proved as follows:

$$
\begin{aligned}
f\left[\beta^S d_{1/2} + d_1\right] &> f\left(\beta^S + 1\right) d_1 \\
&= f \times 2\alpha^S d_1 \\
&> \tfrac{1}{2}\alpha^S d_1
\end{aligned}
\tag{7.48}
$$

The last inequality follows from the fact that $f > .25$. Hence the proposition is proved for this case as well. ***Q.E.D.***

Proposition 9: If V is the present value of a single cash flow C payable at time t and the appropriate semiannual discount rate is r_t, then

$$
-\frac{\partial V}{dr_t} \Big/ \frac{\partial V}{dC} = tC \big/ \left(1 + r_t/2\right) \tag{7.49}
$$

Proof: Set $V = C\left(1 + r_t/2\right)^{-2t}$, differentiate with respect to each variable, and divide. ***Q.E.D.***

10. REFERENCES

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[1] The author would like to thank David Chan, Robert Durie, James Iorio, Doug Johnston, Vinay Pande, Evan Sheinberg, Michael Sung, and Fei Zhou for helpful comments.
[2] For a more detailed treatment see, for example, Tuckman (2002), chapter 14.
[3] This rate is a pedagogical device. Strictly speaking, it is a non-existent rate that integrates up to a 3-month LIBOR rate.
[4] This result is for intuition rather than derivation since, for simplicity, it assumes semiannual rather than quarterly floating payments.
[5] For a more detailed treatment see, for example, Tuckman (2002), chapter 3.
[6] See Tuckman (2002), chapter 3.
[7] The matched-date spread and OAS are not easily compared when the coupon rate is very close to the swap rate. In these situations the interaction of the curve and coupon level, shown in Table 6 and discussed in the text, can be overwhelmed by the interaction of the curve and the fact that bond cash flows are discounted at swap rates plus an OAS while swap cash flows are discounted at swap rates.

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