

Global Asset Allocation J.P. Morgan Securities Inc. New York, 3 September 2008

# Hedging Default Risk in Portfolios of Credit Tranches

- As default concerns rise, credit tranche investors need to know if their books have name concentrations that expose them to large losses from multiple defaults.
- The non-linear response of tranches to defaults makes it very difficult to measure concentration. Tools that have served well to manage spread and correlation risks will not help here.
- Our solution is to develop an estimate of each name's average contribution to multiple default losses, based on a simulation of defaults.
- Using this measure, a "diversification swap" can be constructed, which hedges default exposure by purchasing protection on the highest concentration names, financed by selling protection on the lowest.
- The default risk framework we develop enables investors to tailor these trades to their views on future defaults and their tolerance for losses.
- The analysis of portfolio default risk described in this paper is available to J. P. Morgan clients. The authors or your J.P. Morgan contact can provide further information.

### **Global Asset Allocation**

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Hedging Default Risk in Portfolios of Credit Tranches September 3, 2008

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### 1. Introduction

This paper introduces simple tools for measuring the exposure to defaults of a "book" of tranches of credit default swap portfolios, and shows how to construct effective trades to hedge these risks. These tools and hedging strategies are needed as investors increasingly have taken credit risk not name by name, but bundled together in "bespoke" portfolios, to which they take exposure via tranches. Embedded in a book of these tranches, vulnerability to a name's default is hard to disentangle. It is a cocktail of the name's pattern of incidence across tranches, the tranches' nonlinear response to successive defaults, and its interaction with other names in the book.

It has been possible to get by without explicitly addressing default risk during the benign credit markets of recent years, when the use of bespoke tranches mushroomed. Until 2007, the scarcity of defaults and limited variation in spreads and correlation allowed bespoke tranche risks to be successfully managed by mapping them into index tranches. Concerns that books of tranches were diversified across the underlying names took a backseat. Now that credit spreads price in, and market participants anticipate, a rise in defaults, there are fears that credit investors have concentrations that excessively expose them to losses from defaults.

Concentration is simple to evaluate and adjust in a portfolio composed of corporate bonds or individual credit default swaps. But the credit portfolios we are concerned with are built directly from tranches, not single names. Unwinding existing tranche positions to limit default risk can be costly because of their illiquidity, and runs counter to the reason for accumulating them in the first place. Overlaying liquid index tranche positions -the preferred route for managing correlation risk- is a blunt instrument when it comes to defaults, because index and book names may not overlap enough. A tranche may well be the best hedge to mitigate concentration, but the underlying portfolio has to be constructed, bottom up, from the names whose cost of default is high. Identifying these culprits is problematic because tranches diffract exposure to individual names.

Tranche payoffs respond non-linearly to defaults. The effect of the first default sustained by a tranche is different from the second, third, etc. The result is that the "cost of default" of a name is ambiguous; it depends on which other names also default. For any name, there are many possible combinations of other names with which

it can default, and in each case, its marginal contribution to the losses of the book differs. To preserve the idea of the name's cost of default, we have to average in some way over all possibilities, but there are too many of them to make enumeration practical.

This paper introduces a cost of default measure that gets around these problems, and uses it to construct hedge strategies that are effective in limiting the downside risk from defaults. This amounts to identifying names that default when the book makes large losses. These names are not necessarily those with high credit spreads or default probability, which tend to default whether the book makes large losses or not. Rather, they are singled out by their membership in tranches, by the way these membership patterns interact with those of other names in the book, and by the non-linear default payoffs of the tranches.

Default risk thus inhabits a world very different from the spread and correlation sensitivities in most tranche investors' toolkits. Below, we develop the apparatus necessary for measuring the default risk of a book of tranches, and run through some elementary examples. After we introduce our cost-of-default measure, we apply it to a realistic book of tranches. From the ranking of names, we construct a hedge. This "diversification swap" is carry-neutral, buying protection on high cost-of-default names, financed by selling protection on low-cost ones. Adding this swap to the book lowers downside risk, with the greatest improvement when the overlay is structured in the form of tranches rather than CDS portfolios.

### 2. Profit & Loss Risk Profiles

We now outline the steps involved in calculating a book's default risk profile. The first type of information we need is a description of each individual trade: notional amount, attachment, detachment point, coupon, maturity, and spreads of CDS in the underlying portfolio. To measure default exposure, we look at the P&L of the book over a three-month horizon. P&L for each tranche will vary according to the number of defaults that occur over the three-month period, as shown in Figure 1<sup>1,2</sup>. We

<sup>&</sup>lt;sup>1</sup> A three-month horizon is chosen for convenience only, and in specific instances an alternative may be more appropriate. For example, examining defaults until maturity may be more relevant for an investor who is not affected by mark to market or short-term reporting needs. There, we would be looking at the probability distribution of the present value of cashflows in each scenario. In contrast, the specific time horizon approach looks at the probability distribution of cashflows

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define P&L as the sum of carry, change in mark-to-market value, and principal loss during the three-month interval. To calculate these figures, we need to value the tranche today and at the end of the horizon. This requires an assumption about spreads and correlation three months from now, and in our examples we assume they remain at their current values. While it is straightforward to conduct our analysis using any other spread and correlation level, or even a probability distribution over a range of values, our assumption conforms to the practice of isolating a particular source of risk –here, defaults – by holding everything else constant.

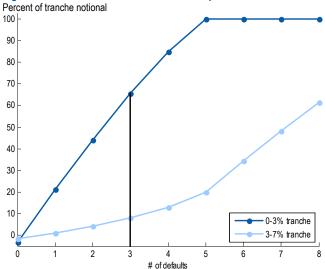
A single default scenario simply designates a list of names that default. To calculate the effect of these defaults on the book, we need to know how names are distributed across tranches. This name interaction is captured by the "Name Allocation Table", or NAT (Figure 2) that relates tranches to the CDSs in the underlying portfolios. Summing the defaulting names in each column of the NAT gives us the number of defaults sustained by each tranche. For example, in the default scenario illustrated by Figure 2 (where defaulting names are indicated in blue), Tranche 2 experiences one default from Name 3, while Tranche 8 has three defaults caused by Names 3,7, and 14. We then refer the defaults for each tranche to its P&L curve, and sum the resulting P&Ls, weighted by tranche notionals. So, to continue with our example, if Tranche 8 is an equity tranche, the loss from its three defaults in this scenario is 66% of the tranche notional (see Figure 1).

To go from a collection of default scenarios to risk measurements that can form the basis for decisions, we need to assign a probability to each scenario. These probabilities will be the consequence of our assumptions on individual names' default probabilities, and on correlation. It is useful to start with a very simple case, where each name has the same default probability. In this case, we can go a remarkably long way without further assumptions. Say five defaults have occurred. The probability that a name is among them is the same for each name, independent of whether the common probability of default is 1% or 10%, and irrespective of

over the horizon, plus the price of the tranche at the end of the horizon, which is an average over the future "stubs" of lifetime default scenarios. The difference between the two approaches will thus be greater, the more the riskiness of names differs between the first part of the period and the second.

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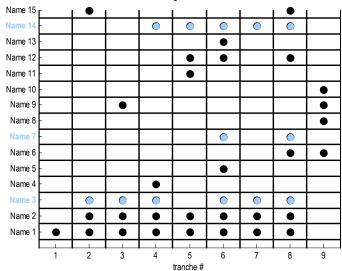
Figure 1: Loss from successive defaults for protection seller



Source: JPMorgan

Figure 2: Name allocation table (NAT)

Read by column, the NAT identifies all the names belonging to a tranche. Read by row, it identifies all tranches to which a name belongs



Source: JPMorgan

the correlation among them. This motivates Figure 3, which shows the distributions of P&L, conditional on total defaults.

Next, in order to measure risk, or judge strategies to mitigate it, one needs to assign probabilities to each total number of book defaults. Implicitly, these probabilities involve an assumption about correlation. They could, for example, follow the Gaussian copula beloved of tranche pricing models, but there is no reason why they should take this form, or indeed conform to any value of

<sup>&</sup>lt;sup>2</sup> Throughout the paper, we assume a 40% recovery rate for all names, with no uncertainty.

deptember 5, 2000

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correlation that can be backed out of market pricing. The goal here is not to price so as to avoid arbitrage, but rather to capture the actual likelihood of defaults occurring, and their tendency to be simultaneous. In other words, a "view" is what is required, and (for example) historical estimates of default frequencies and correlation may be as good a place to start as the values implied by the market. Alternatively, the view may be that there is a liquidity premium in CDS spreads so default probabilities are lower than risk-neutral probabilities implied by spreads.

It is not even necessary to lay out an explicit probability distribution, in order to evaluate hedging strategies. It may be sufficient to designate a particular range of defaults as negligibly probable, and just look for strategies that provide unambiguous improvement in downside risk in the range of defaults considered likely.

Figure 3 will be used extensively below as the vehicle for evaluating and comparing hedge trades. It is the natural way to extend Figure 1 to the entire book. We prefer it to the more standard unconditional probability distribution of P&Ls that results from the same simulation exercise, an example of which is shown in Figure 4. One disadvantage of Figure 4 is that it requires the view on future defaults to take the form of a completely specified probability distribution. None of the partial or qualitative expressions of default views described above will work. In contrast, Figure 3 measures P&L risk against the natural focus of a view: the number of defaults that are likely to occur across the book, and so it is easy to impose partially specified default views. For related reasons, Figure 4 is also much less useful for comparisons among hedge strategies and the original book. It is capable of showing the effect on downside risk of a hedge overlay, but not whether these effects come from default scenarios that there is a high premium on avoiding. Again, to use Figure 4, the view needs to be fully baked before any progress can be made.

When default probabilities differ among names (the usual case), the risk profiles for each level of book defaults in Figure 3 are no longer independent of CDS names' default probabilities and correlation. In this case it is necessary to take a view on these two, which again need not correspond to risk-neutral levels. As a matter of convenience here, our examples use risk-neutral probabilities. We still separate this view, which gives rise to the distribution conditional on each total number of book defaults, and the view on the likelihood of each

Figure 3: P&L distributions conditional on total number of book defaults
The lines trace out the percentiles of the conditional distributions. The highlighted
distributions for 2 and 5 defaults correspond to those shown in Figure 4.

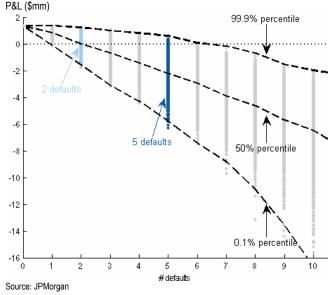
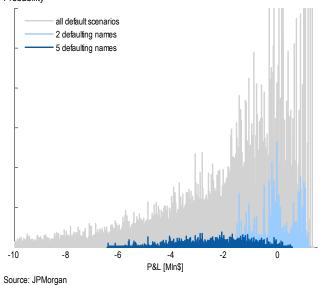


Figure 4: Profit and loss from simulated defaults Probability



total number of book defaults, which, as discussed above, need not necessarily be explicit.

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# 3. Default Exposure: Some Simple Examples

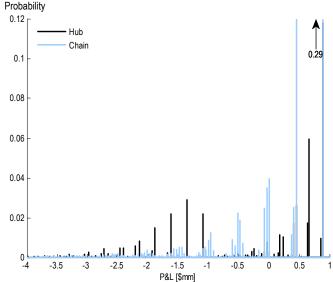
The default risk profile is the result of the interaction of the shape of the tranches in the book, and the way names are distributed across them. Here we provide some examples. The first highlights the contribution of the name interaction as depicted by the Name Allocation Table. The book of tranches comprised 11 copies of a 3-7% 5-year mezzanine tranche. The underlying portfolios of the tranches each contain 100 names, with equal CDS spreads. Together, the 11 tranches span 550 names.

This description is consistent with a wide range of exposure to defaults. Figure 5 displays two distributions of the P&L of the book over the course of three months, each corresponding to a different NAT. In one case there is a 19.2% chance of a loss in excess of \$1mm. In the other, a loss bigger than \$1mm has a 13.5% chance. The crucial difference between the two cases is revealed by their respective NATs, Figures 6 and 7. Figure 5's blue curve is a "chain" where each name is in only two tranches. The black curve results from a "hub", in which 55 names are in all tranches, while the remaining 495 are each in only one tranche<sup>3</sup>. Hub P&L has a wider range, because any given list of defaulting names can be all in or all out of the hub. Variation in chain P&L is driven by whether the names are in the same tranche or not.

The next step for the hub book may be to find trades to limit its large losses. In this case, it is pretty easy to eyeball the NAT, and point to the concentration in the hub as the source of this downside risk, suggesting that hub names should be hedged. In general, the NAT will not be so clean and simple, and it would be desirable to have a statistic we can calculate that would measure each name's contribution to default risk. The standard jump-to-default measure simply calculates the effect of a name defaulting in isolation. This statistic (which we abbreviate as "JTD1", for reasons that will become clear below) fits the bill here: it is higher for hub names than for non-hub names<sup>4</sup>.

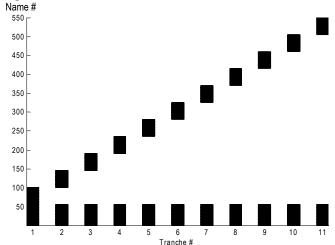


Figure 5: Profit and loss distributions for hub and chain NATs



Source: JPMorgan

Figure 6: Hub name allocation table



Source: JPMorgan
Figure 7: Chain name allocation table

Name #

550

500

450

400

350

250

200

150

100

50

Tranche #

Source: JPMorgan

 $<sup>^{\</sup>rm 3}$  To give a real life example, books with substantial CDX index hedges show hub formations.

<sup>&</sup>lt;sup>4</sup> One inconsequential difference between the JTD measures we shall use in this paper and standard JTD measures is that ours incorporate changes in value over a (3-month) period, whereas the standard statistic is instantaneous. Our approach is based on our need to consider the probabilities of different JTD scenarios, so that the scenarios can be aggregated.

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Unfortunately, it is not difficult to concoct examples where JTD1s provide a misleading guide for hedge strategies. Consider the NAT in Figure 8, where the first five tranches are 0-3% 5 year equity on 100 names each, and the last five are the same as the mezzanine tranches of the last example. Damage from successive defaults increases for a mezzanine tranche and decreases for equity, as the lower panel of Figure 9 shows. The different shapes of these two profiles provide ample opportunity for mischief. JTD1s will be highest for equity names, as E1 (P&L of equity at 1 default) exceeds five times M1. However, once a hub name has defaulted, the subsequent default of another hub name will be more damaging than the default of an equity name (five times point M2 exceeds E1). In other words, the default risk of the book is not the sum of its parts.

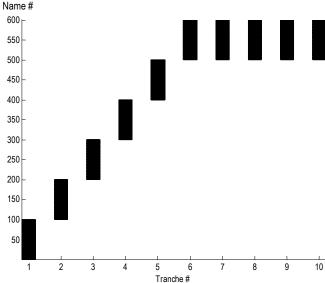
This last example highlights that it is not just the shape of the NAT that presents a challenge, but also its interaction with the tranches in the book. On top of this, our examples simplify considerably by including tranches that all sell (or all buy) protection. A long-short portfolio on a hub can actually be much less risky than one on a chain. Thus, in general, it is not possible to separate the impacts of the NAT and the tranche configuration. In addition, we are rarely confronted with such simple patterns of interaction among names. Even a book containing a small number of tranches can generate configurations that defy simple classification, as we will show in the example of Section 5. These complications, and the absence of tools to deal with them, make it likely that investors' tranche books have not been built up with a view to minimizing default exposure.

# 4. Generalized Jump-to-Default Risk (JTDn)

The last example showed that there is more to the P&L distribution that results from defaults than can be explained by JTD1. The impact of a marginal default by company XYZ depends on the other defaults to which it is marginal. Of course, we do not know which names experience these other defaults; their identity is random from one scenario to the next. What we can do, however, is calculate the *expected value* of a marginal default by XYZ, which we measure as follows. In each scenario where *n* names, including XYZ, have defaulted, we calculate the P&L as usual, and we also calculate the P&L if XYZ had not defaulted (but all the other *n*-1 names defaulting in the scenario continue to do so). Subtract the second P&L from the first, and we have the effect of XYZ defaulting in the scenario. Average over



Figure 8: Equity/mezzanine name allocation table



Source: JPMorgan

Figure 9: Total loss (upper panel) and marginal loss (lower panel) from successive defaults for protection seller

See text for explanation of labels
Percentage of tranche notional

100
40
20
0 1 2 3 4 5 6
# of defaults

0-3% tranche
2.5-4.5% tranche
5x 2.5-4.5% tranches

Source: JPMorgan

all such scenarios (weighted by their probabilities), and we have the expected P&L impact of a default by XYZ.

3

# of defaults

By analogy with JTD1, this is the JTD loss associated with XYZ when exactly *n* companies default, hence the "n". Obviously, we can calculate JTDns conditional on any range of total defaults, or subset of scenarios, or unconditionally. This measure is a natural extension of the standard jump-to-default statistic, JTD1. Company XYZ's JTD1 is simply the P&L when it is the only

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default, minus the P&L when it does not default (and so there are no defaults). It is not necessary to think about probabilities in this case, because there is only one possible scenario where XYZ is the only default.

JTDn is independent of whether the name has a high likelihood of default, because it calculates losses attributable to the name, given that it defaults.

To illustrate, we return to the examples used in the last Section. In each case, we show each name's JTDn, conditional on specific numbers of defaults. In these simple examples, it is possible to trace the shape of the JTDn curves back to the P&L curves of the book's tranches. For example (Figure 10), in the chain case, the effect on P&L is low for small numbers of book defaults (n), then it rises as the marginal impact of a default on tranche P&L rises. It falls slowly to zero as the chance that the tranche has been wiped out goes to 1. In the hub example, there are two distinct types of name, and part of the dispersion comes from the fact that a name can default alongside differing numbers of the two types of name. In the third example (Figure 10), the differences emanate not only from the two types of name, but also from the differing P&L profiles of the equity and mezzanine tranches.

We propose to use JTDn to design overlay trades in exactly the same way as one would use JTD1: buy protection against the high-JTDn names, and sell protection against low-JTDn names. We structure these trades so that they are carry-neutral.

### 5. Hedging Default Risk: An Example

Here, we attempt to reshape default risk of a particular book. It contains 12 mezzanine 3-7% tranches of the type described above, with a notional of \$10mm each. The book spans 454 names, and the NAT is shown in Figure 11. This pattern of name allocation borrows from some investor portfolios we have examined. It clearly does not closely fit the generic examples of Section 3, but exhibits aspects of both hub (darker bars) and chain formations (lighter bars). Thus, while there is no one name in all tranches, there are 15 in ten or more tranches. Similarly, there are 67 names that are only present in a single tranche. In contrast to our earlier examples, spreads also vary widely across individual names. Figure 12 shows the outcome of simulating defaults over the next 3



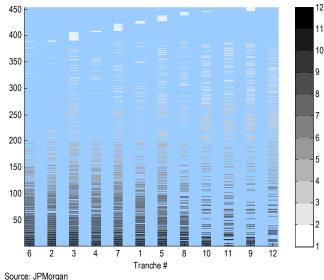
Figure 10: Generalized Jump-to-Default (JTDn)

\$mm Hub model Chain model name in hub Eqty-Mezz model name in mezz 3 name in equity 2 name out of h 20 30 50 60 70 # defaults

Figure 11: Sample name allocation table

Source: JPMorgan

The darker the bars the more tranches the corresponding name appears in Name number



months, using the risk-neutral default probabilities, and a correlation figure of 10%<sup>5</sup>.

To evaluate hedging strategies, we require an operational definition of what it means to improve risk. We shall assume the goal is to improve outcomes in which there is more than a \$5mm loss over the 3-month horizon. This is represented by the horizontal line in Figure 12. We also

<sup>&</sup>lt;sup>5</sup> However, tranches are priced at 50% correlation to reflect current market conditions.

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take the view that the probability of more than 10 defaults over the 3-month horizon is negligible. Consequently, the goal of an overlay trade becomes lowering the number and/or severity of scenarios that fall in the shaded region. Of course, this could entail worse performance at higher levels of defaults, but our view assigns negligible likelihood to those outcomes.

The JTDn for each name is accordingly calculated using only the scenarios that fall in the shaded region. Figure 13 graphs JTDns against the simple count of the number of tranches in which each name occurs, while Figure 14 graphs them against credit spreads. Evidently, JTDns are tracking incidence in this case, rather than spread. Since all tranches are long, we would expect the largest JTDs from hub names, as occurs.

We now use this information to assemble a carry-neutral trade, buying protection on the high-JTDn names, and selling protection on the low-JTDn names to finance it (all trades being priced at mid-market). We consider the case where we use the 100 names with the highest JTDn, in equal amounts. The question now is: do we implement the strategy: via CDS or via tranches? The answer will determine the amount of protection sold to finance the trade, and therefore the risk profile. Figure 15 shows two possible combinations. In one case, we buy a total of \$440m of CDS protection in the 100 highest-JTDn names, financing it by selling CDS protection on \$920mm of the 100 lowest-JTDn names. In the other case, we buy and sell tranches on the respective legs of this trade. We buy protection on the 0-3% tranche, for a notional of \$13.2mm, and sell protection on the 3-7% mezzanine tranche, for a tranche notional of \$50mm. The tranche version improves downside risk more inside the shaded region. Of course, it has to make up for this by underperforming somewhere, which appears to be in the mid-range of defaults. The effect on the JTDn of the portfolio hedged with the equity and mezzanine tranche is shown in Figure 16, where the tradeoff is apparent. The hedge has thus flattened JTDns across names.

The particular view involved –that more than 10 defaults occurring over the time horizon is highly unlikely, makes hedging with tranches more efficient than hedging with CDS. The equity tranche in our example provides protection on up to five defaults (under a 40% recovery rate assumption), some large proportion of which materialize within the 10-default range our view deems likely. Much less benefit is to be gained in this region from the \$440m of straight CDS protection on 100 names. This will cover us outside the 10-default region,



Figure 12: Sample book P&L distributions conditional on total number of defaults

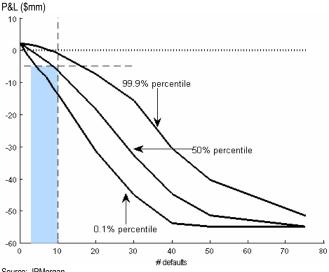
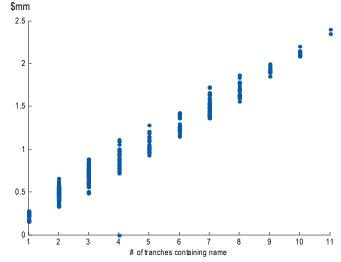
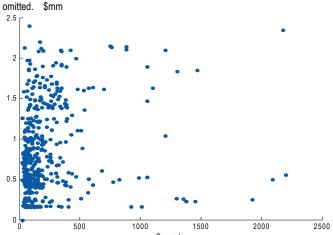


Figure 13:JTDn vs. incidence of names in tranches



Source: JPMorgan

Figure 14: JTDn vs. 5 year CDS spreads (bp)
Two names with spread of 7000 and 3500bp abd JTDns of 1.6 and 0.2 have been



Source: JPMorgan

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of course, but that does not fit with our view. It also entails selling a large volume of protection. Roughly, the extra volatility (in dollar terms) created by the volume of the CDS hedge swamps any benefit it may have in terms of tilting JTDn exposure. Of course, a different view could change the ranking between these strategies

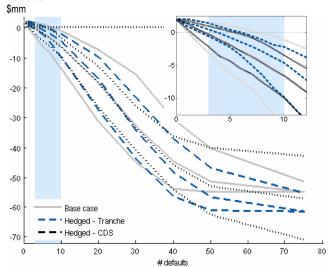
Our JTDn measure is an estimate of something that is impractical to calculate, as it would require enumeration of all possible default scenarios. Instead, we base our estimate on the subset of default scenarios generated by a Monte Carlo simulation. Nevertheless, the hedge strategy derived from ranking JTDns, and the rankings of JTDns themselves, are quite robust across different simulations. Figure 17 compares the performance in two simulations: the "in-sample" measures the performance using the same scenarios that we used to find the hedge, whereas the "out-of-sample" uses a completely independent simulation run to measure performance. As shown in the pictures and highlighted in the inset, the difference between them remains very small across a large range of defaults. Figure 18 compares the rankings of JTDns for two different sets of simulations. It displays the name overlap of the first n names across the two. A perfect match would be represented by a horizontal line at 100%, i.e. all first 5,10, etc. names in one set of simulations correspond to all the first 5, 10 etc. names in the second set. Figure 18 indicates that our algorithm provides rankings that match consistently to a degree of at least 90%.

### 6. Hedge mechanics and scaling the trade

Using the framework we have developed, we can elucidate the functioning of trades like the one discussed in the last section. Ideally, we would like to reduce the variance of the distribution, in particular the downside risk. Loosely speaking, the hedge has caused a folding in of the wings of the P&L distribution, as depicted in Figure 19. We illustrate with a slice of the P&L distribution of the original portfolio. Figure 20 shows the P&L distribution conditional on five defaults occurring. Our objective in creating the hedge trade is to limit the downside, that is, to raise P&L in as many as possible of the scenarios that fall below the -\$5mm line. For this to happen, the hedge trade must make a profit in some of these scenarios. Of course, this does not come without a cost, because we have sold protection to finance the insurance we have bought against large-JTDn names.

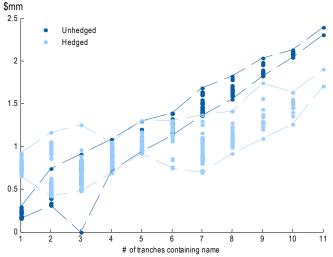


Figure 15: Sample book P&L distributions conditional on total number of



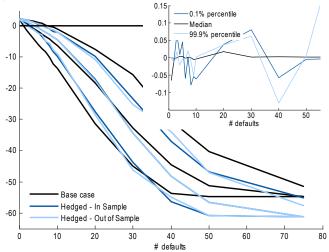
Source: JPMorgan

Figure 16: Improvement and tradeoff of JTDn



Source: JPMorgan

Figure 17: Robustness of hedging strategy



Source: JPMorgan

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This cost could appear in the right tail of Figure 20<sup>6</sup>. In those scenarios, one would expect low JTDn names to predominate among the five defaults, and so selling protection tends to push P&L down. A more precise rendition is given by the probability distributions shown in Figure 20. The success of the hedge is evident here in the fact that the points have moved above the -\$5mm threshold. Also note that the hedging strategy benefits from some continuity and significantly improves the downside risk even for scenarios in the range up to -\$2mm.

In general, a hedge overlay that buys protection on high and sells protection on low JTDn names will move scenarios in the direction of the arrows in Figure 20. The size of the arrows is determined by the scale of the trade: the larger the notional, the more protection is transacted on both sides of the trade, and the larger the arrows. If the notional of the trade is large enough, it may well push previously upside scenarios below the -\$5mm threshold, which would be counterproductive. Sizing the trade is therefore a matter of balancing its upside and downside effects.

### 7. Conclusion

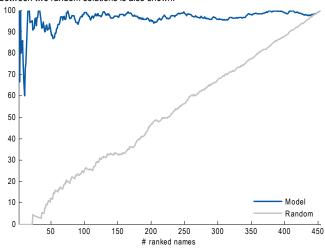
This paper has addressed how to measure default risk in a book of credit tranches. While defaults are the ultimate source of risk in credit instruments, they have not been the direct focus of hedging and risk management activity in the synthetic credit market, which has been devoted to immunizing P&L against movements in market spreads and correlation.

Default risk presents formidable complexity. While it is simple to measure the impact of the first default in a book (JTD1), the non-linearity of tranche payoffs means that multiple default P&Ls are not the sum of their constituent JTD1s. To use an overworked term, default "gamma" is not zero, and it is also very badly behaved. Beyond the first default, the marginal effect of a default by each name depends on which names default with it, and the "dimension" of the risk that needs to be tracked thus explodes.

As it turns out, however, one can do better than listing default scenarios. Our "JTDn" measure generalizes the standard JTD1, by averaging the P&L impact of each name over all possible default scenarios. Of course, it is JPMorgan 🛑

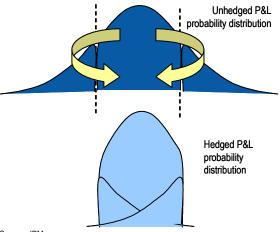
Figure 18: Robustness of ranking

Percent of ranked names that match in two solutions. For comparison, the match between two random solutions is also shown.



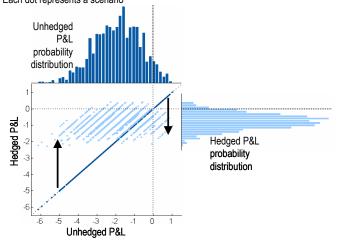
Source: JPMorgan

Figure 19: Reshaping the distribution



Source: JPMorgan

Figure 20: Hedged vs. unhedged P&L distributions at 5 defaults Each dot represents a scenario



Source: JPMorgan

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<sup>&</sup>lt;sup>6</sup> We say "could", because the effect need not appear in 5-default scenarios. But it has to appear somewhere (there is no free lunch).

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out of the question to enumerate all possible default scenarios, but we can make an estimate, based on a Monte Carlo simulation. The more scenarios we simulate, the more accurate this JTDn estimate will be, but we really need it to be effective for the size of Monte Carlo analysis that is feasible. So it is reassuring that repeating the simulation comes out with very similar rankings of JTDns across names. It is also reassuring that the natural default hedging trades that fall out of these rankings transform risks in the anticipated way.

The framework we have developed recognizes that any measurement of default risk must take a "view" on default probabilities and the correlation of defaults. However, such a view does not typically encompass all possibilities, and it may be more accurate to describe it vaguely than to try to press it into a particular family of probability distributions. We show how it is possible to operate by dividing book default scenarios into likely and unlikely, as opposed to, say, casting one's view as a Gaussian copula with correlation equal to some value.



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