# Pricing Interest Rate Derivatives Post-Credit Crunch

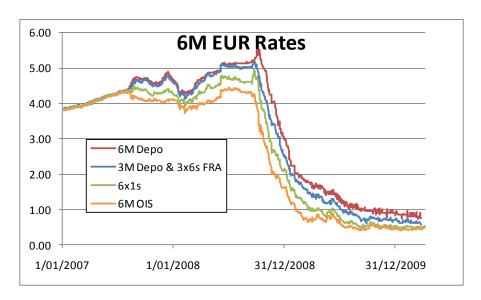
Michael Nealon SFMW Talk

## Outline

- Introduction
  - O What has happenned and why?
- Extentions to standard derivative pricing models
  - Curve Construction
  - o Black-Scholes
  - Interest Rate Models

# Post-credit crunch changes

- Post-credit crunch, rates of different compounding basis diverged
  - o These rates should be the same under standard (no-arbitrage/single curve) theory



- In light of this, the market has modified the treatment of derivatives, in terms of
  - Pricing
  - Revaluation
  - Liquidation
  - Collateralisation

# Arbitrage opportunities?

#### 3x6s trading strategy

	Buy 6M Bill	Sell 3M Bill	Sell 2 <sup>nd</sup> 3M Bill	Pay Fixed 3x6s FRA	Nett C/F
Today	-1	1			0
3M		$-\left(1+\frac{1}{4}R_{3M}\right)$	$\frac{1 + \frac{1}{2}R_{6M}}{1 + \frac{1}{4}L_{3M}}$	$\left(1 + \frac{1}{2}R_{6M}\right) \times \left(\frac{1}{1 + \frac{1}{4}F_{3x6s}} - \frac{1}{1 + \frac{1}{4}L_{3M}}\right)$	$\frac{1 + \frac{1}{2}R_{6M} - \left(1 + \frac{1}{4}R_{3M}\right)\left(1 + \frac{1}{4}F_{3x6s}\right)}{1 + \frac{1}{4}F_{3x6s}}$
6M	$1 + \frac{1}{2}R_{6M}$		$-\left(1+\frac{1}{2}R_{6M}\right)$		0

The no-arbitrage assumption implies

$$1 + \frac{1}{2}R_{6M} = \left(1 + \frac{1}{4}R_{3M}\right)\left(1 + \frac{1}{4}F_{3x6s}\right)$$

But this no longer holds: Will this strategy really produce a risk-free profit?

# Arbitrage opportunities?

#### No! It assumes:

- All cash flows are free of default risk, and
- We can always borrow at Libor FLAT in 3 months time

Historically, these issues have been considered insignificant for inter-bank lending As a result, banks could mismatch the term of their assets and liabilities

- Lend long-term to customers, and
- Fund with (cheap<sup>1</sup>) short-term inter-bank lending

The credit-crunch changed the market's perceptions, and banks are now more conservative about

- Default of other banks
- Their own future cost-of-funds (the rate at which they can borrow)

These risk are being priced into

- Money Market lending rates (eg. Libor/BBSW)
- Derivatives written on these rates

<sup>1</sup> As compared to long-term committed funding from capital markets (ie. bonds and FRNs)

# Credit versus Liquidity

Most agree the spreads are a combination of credit and liquidity issues Liquidity is used to mean:

- Funding Liquidity: Ability to pay liabilities when they are due
- Asset Liquidity: Ability to sell an asset within a required timeframe
- Systemic Liquidity: Ability to borrow funds (at any cost)

Difficult to disentangle liquidity from credit

- Distressed entities suffer from funding illiquidity (increasing the risk of default)
- During a crisis:
  - Investment contracts and migrates to safer (eg. shorter-term) opportunities
  - Funding becomes more expensive (due to lost supply as well as credit concerns)

Both Mercurio (2009) and Morini (2008 & 2009) investigated the efficacy of a **default-only** model to explain the spreads

- Each use risky generic inter-bank counterparties, and attempt to calibrate to market rates
- Simple default-only models can explain the spread between secured and unsecured lending
  - o ie. Libor vs OIS
- But basis spreads (ie. Libor vs Libor) require a more complicated model

# **Default-Only Model**

Morini (2009) models the risk that a counterparty may not remain a Libor-bank to maturity Consider an investor wanting to lend funds to a Libor-bank for 1 year, with two choices:

- 1. Committed (unsecured) funding for 12 months
- 2. Rolling funding (say, semi-annually)

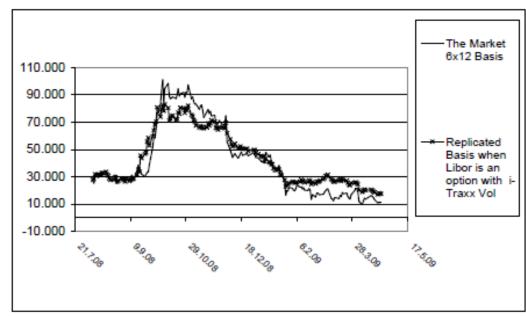
Approach (2) has an embedded option as the investor can change borrower mid-stream

• In particular, if the initial borrower's cost-of-funds exceed some threshold With Approach (1): The investor can also unwind but only at prevailing market rates and subject to liquidity

Morini (2009) models this option for EUR, calibrated to:

- EONIA OIS rates
- Euribor rates
- i-Traxx Index spread volatility

and compares it to market quoted 6x12s basis spreads



# **Basis Swap Spreads**

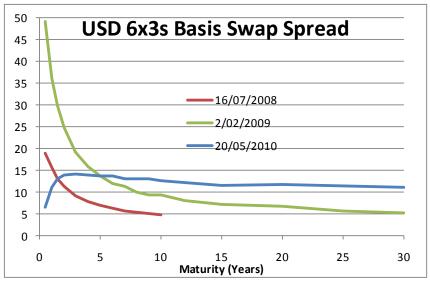
So far we have focused on the **Spot Basis** (relative spread between OIS and various Libors)

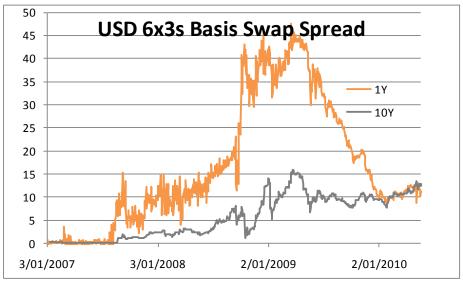
The market also trades Forward Basis via Single Currency Basis Swaps

• eg. A 10Y 6x3s exchanges Libor6M for [Libor3M + spread] for 10 years

These spreads represent (risk-neutral) expectations of future spot basis spreads

- Initially, these spreads carried only the prevailing spot basis (ie. spreads decayed to zero)
- But the market is now building in forward spot basis





# **Supply-Demand Dynamics**

There are natural buyers and sellers of basis in the market

- **Corporate lending**: Often customers require monthly repayments while inter-bank swaps are quarterly and semi-annual (and so banks accumulate 6x1s and 3x1s basis risk)
- EUR Public Sector finance: Asset swaps pay Euribor3M, while inter-bank swaps are Euribor6M

Previously many institutions held on to this risk (spreads were small and the perceived risk minimal)

 In fact, when basis spreads widened many institutions immediately made (unrealised) profits from their existing positions

Banks are now managing their basis positions more closely, due to

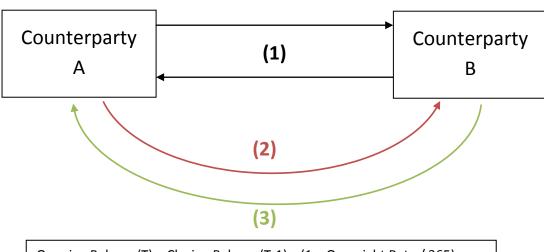
- A realisation that spreads now have volatility (up and down) and so exposure needs to be managed
- Newly imposed risk oversight

Price discovery occurs as various participates act to neutralise their basis positions

- In the same way as banks have always manage fixed and floating interest exposure
- Of course, this is also moderated by the market's perception of the future Spot Basis

# Changes to collateralised trades

Collateral agreements (typically via the ISDA Credit Support Annex) provide credit-risk mitigation for derivative trades



Opening Balance(T) = Closing Balance(T-1) x (1 + Overnight Rate / 365)

Margin Call(T) = Trade MTM(T) - Opening Balance(T)

Closing Balance(T) = Opening Balance(T) + Margin Call(T) = Trade MTM(T)

In practice these agreements are imperfect risk mitigators

- Margin Calls have thresholds and are not made daily
- Non-cash collateral can be posted

- Cpty A and Cpty B enter into collateralised trade
- 2. The trade becomes NPV negative for Cpty A, and posts collateral (Margin Call)
- 3. Cpty B pays Cpty A interest on the collateral (at the O/N cash rate)
- 4. If Cpty A defaults, Cpty B takes ownership of the collateral

# Changes to collateralised trades

The market is now migrating to the OIS curve for discounting trades covered by collateral agreements

The motivation is consistency with interest accrual on the collateral position

• This is reasonable (ignoring inefficiencies in the agreement)

The default of Lehman Brothers highlighted the issue

- Discrepancies in termination NPVs between various counterparties and clearing houses
- Investigation found some counterparties used OIS while others used Libor to discount

It would seem now this practice is gaining momentum in terms of

- Pricing (eg. option premiums, off-market swaps)
- Terminations
- Collateralisation (ie. margin calls)

But, revaluation (for Mark-to-Market accounting) is generally lagging

Recently LCH.Clearnet (a major clearing house) announced it was in the process of migrating to OIS curves for discounting (see Whittall (2010b))

# Curve Construction

# Pragmatic approach

The industry is not attempting to model credit/liquidity explicitly Instead, existing curve methodologies are being modified to "fit" observed market prices

- The use of multiple basis-specific curves appears to be universal
  - o ie. Build a 1M-curve, 3M-curve, 6M-curve etc for each currency
- Some early publications prescribed basis-specific discounting curves
- However, most publications now advocate a single discounting curve for all inter-bank trades
- More and more the OIS curve is being identified as the appropriate choice for discounting

#### **Floating Payment In-Arrears**

Consider a standard  $\tau$ -Libor floating payment settling in arrears. If  $T' = T + \tau$ , then pay-off is given by

$$V(T') = \tau L_{\tau}(T, T')$$

Standard industry practice is now to computed the present value as

$$V(t) = \tau L_{\tau}(t, T, T') P_D(t, T')$$

where

$$L_{\tau}(t,T,T') = \frac{1}{\tau} \left( \frac{P_{\tau}(t,T)}{P_{\tau}(t,T')} - 1 \right)$$

is the forward rate from the  $\tau$ -forward rate projection curve (defined by  $P_{\tau}(t,\cdot)$ ) and

$$P_D(t,T')$$

is the discount factor from the discounting curve

# **Curve Construction**

As always, there is a wide variety of construction methodologies in terms of

- Instrument content
- Interpolation and extrapolation techniques
- Bootstrapping procedures

#### **Objectives**

- Accurately price the spectrum of IR derivatives
- Valuations consistent with collateral margin calls and liquidation amounts
- Restrict reported exposures to liquidly traded instruments
- Minimise complexity of exposure profiles (esp. FRAs)

#### **Constraints**

System and policy limitations

The primary instruments used for bootstrapping forecast curves are:

- Cash (O/N, 1M, 3M, 6M, 12M): Floating rate reset information
- Money-Market Futures (eg. 3M Eurodollar futures): Most liquid short-term rates
- Fixed-Floating Swaps: Most liquid long-term rates
- Basis Swaps: Long-term basis information
- FRAs and Short Swaps: Short-term basis information

### **OIS Curve**

GBP and EUR: OIS quoted to 30Y (eg. ICAP)

USD: OIS quoted to 10Y (eg. Prebon) + [FF v Libor 3M] basis swaps to 30Y

Note: Basis swaps use an Arithmetic Average instead of Overnight Indexing

$$\frac{1}{N} \sum_{i=1}^{N} F(T_{i-1}, T_i) \neq \tau(T_0, T_N) F(T_0, T_N) = \left[ \prod_{i=1}^{N} (1 + \tau(T_{i-1}, T_i) F(T_{i-1}, T_i)) - 1 \right]$$

**AUD:** OIS quoted to 2Y – 3Y (Bloomberg has composites quotes to 5 years)

- But we need the OIS curve to 30Y 40Y
  - Note: This is one of the main hurdles for LCH.Clearnet
- Two approaches
  - Use XCCY swaps to imply AUD OIS from USD OIS curve (consistent with collateral posting)
  - Extrapolate the forward [OI v BBW3M] spread
- My thoughts:
  - If the market moves to using the OIS curve for discounting, then
  - Desks will need to trade longer dated OIS (as they accumulate revaluation exposures)
  - O Data providers (eg. ICAP) will be able quote longer maturities (due to increased volumes)
  - BUT, it is difficult to build an OIS curve without the quotes!
  - o Perhaps this will be an iterative process??

# Black-Scholes

# Standard derivative pricing

Suppose we have an asset with price S(t) (yielding a continuous dividend at  $r_D(t)$ ) with real-world dynamics

$$dS(t) = \mu_S(t)S(t)dt + \sigma_S(t)S(t)dW$$

and we wish to value a derivative written on the asset (denoted by V(t) = V(t, S(t)))

One approach is to construct a *self-financing* portfolio consisting of:

- A cash positioned  $\gamma(t)$  (borrowed/lent), and
- A position in the underlying asset  $\Delta(t)$  (long/short)

The portfolio is managed such that it replicates the value of the derivative (ie.  $V(t) = \Delta(t)S(t) + \gamma(t)$ )

The standard assumption is that the cash position accrues interest at the risk-free rate r(t), that is

$$d\gamma(t) = (r(t)\gamma(t) + r_D(t)\Delta(t)S(t))dt$$

# Standard derivative pricing

By applying Ito's Lemma, equating terms and setting  $\Delta = \frac{\partial V}{\partial S}$ , we get the standard Black-Scholes PDE

$$\frac{\partial V}{\partial t} + (r - r_D) \frac{\partial V}{\partial S} + \frac{1}{2} \sigma_S^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV$$

A few more straightforward steps (including another application of Ito's Lemma) produces

$$V(t) = E^* \left\{ e^{-\int_t^T r(u)du} V(T, S(T)) | \mathcal{F}_t \right\}$$

where the expectation is with respect to the measure  $P^*$  where

$$dS = (r - r_D)Sdt + \sigma SdW^*$$

We recognise that  $P^*$  is the risk-neutral measure where the bank-account numeraire

$$N^*(t) = B(t) = e^{\int_0^t r(u)du}$$

Changing the numeraire to  $P(t,T) = E^* \left\{ e^{-\int_t^T r(u)du} | \mathcal{F}_t \right\}$  provides a valuation under the forward measure

$$V(t) = P(t,T)E^{T}\{V(T,S(T))|\mathcal{F}_{t}\}\$$

# Pricing with collateral

Piterbarg (2010) generalises the model to account for the collateralisation of the derivative

He assumes the market lends/borrows at three distinct rates:

- $r_c(t)$ : The rate accrued by the collateral (ie. the cash rate)
- $r_R(t)$ : The reportate for the underlying asset (ie. funding secured by the asset itself)
- $r_F(t)$ : The unsecured funding rate (eg. treasury funding rate)

One would expect (due to credit concerns)

$$r_C(t) \le r_R(t) \le r_F(t)$$

The cash amount is now split into three corresponding accounts

- C(t) held as collateral against the derivative
  - o Accruing interest at the rate of  $r_{C}C(t)$
- $\Delta(t)S(t)$  borrowed/lent to fund the purchase/sale of  $\Delta(t)$  asset units
  - Accruing interest at the rate of  $r_R \Delta(t) S(t)$
- $\gamma(t) (C(t) + \Delta(t)S(t)) = V(t) C(t)$  borrowed/lent unsecured (eg. from treasury)
  - $\circ$  Accruing interest at the rate of  $r_F(V(t)-\mathcal{C}(t))$

# Pricing with collateral

With this generalisation, the Black-Scholes PDE becomes

$$\frac{\partial V}{\partial t} + (r_R - r_D) \frac{\partial V}{\partial S} + \frac{1}{2} \sigma_S^2 S^2 \frac{\partial^2 V}{\partial S^2} = r_F V - (r_F - r_C) C$$

and the derivative's value can be expressed (via Ito's Lemma) as both

$$V(t) = E^* \left\{ e^{-\int_t^T r_F(u)du} V\left(T, S(T)\right) + \int_t^T e^{-\int_t^u r_F(v)dv} \left(r_F(u) - r_C(u)\right) C(u) du \mid \mathcal{F}_t \right\}$$

and

$$V(t) = E^* \left\{ e^{-\int_t^T r_C(u)du} V\left(T, S(T)\right) + \int_t^T e^{-\int_t^u r_C(v)dv} \left(r_F(u) - r_C(u)\right) \left(V(u) - C(u)\right) du \,|\mathcal{F}_t \right\}$$

where the expectation is with respect to the measure  $P^*$  where

$$dS = (r_R - r_D)Sdt + \sigma SdW^*$$

# Derivative pricing with and without collateral

There are two cases of specific interest:

• Full collateralisation (CSA): C(t) = V(t)

$$V_{CSA}(t) = E^* \left\{ e^{-\int_t^T r_C(u)du} V(T, S(T)) | \mathcal{F}_t \right\}$$
$$= P_C(t, T) E^T \left\{ V(T, S(T)) | \mathcal{F}_t \right\}$$

where  $\boldsymbol{E}^T$  is the expectation under the forward measure defined by the numeraire

$$P_C(t,T) = E^* \left\{ e^{-\int_t^T r_C(u)du} | \mathcal{F}_t \right\}$$

• No collateralisation (No CSA): C(t) = 0

$$V_{No\ CSA}(t) = E^* \left\{ e^{-\int_t^T r_F(u) du} V(T, S(T)) | \mathcal{F}_t \right\}$$
$$= P_F(t, T) \tilde{E}^T \left\{ V(T, S(T)) | \mathcal{F}_t \right\}$$

where  $\tilde{E}^T$  is the expectation under the *risky* forward measure defined by the numeraire

$$P_F(t,T) = E^* \left\{ e^{-\int_t^T r_F(u)du} | \mathcal{F}_t \right\}$$

# Forward prices and convexity

Consider a forward contract with forward price K (ie. V(T) = S(T) - K)

Depending on whether or not collateral is posted the value of the contract is either

$$V_{CSA}(t) = P_C(t, T)E^T\{S(T) - K|\mathcal{F}_t\}$$

or

$$V_{No\ CSA}(t) = P_F(t, T)\tilde{E}^T\{S(T) - K|\mathcal{F}_t\}$$

Note that the expectation is different (in addition to the discounting)

Hence the fair-market forward prices are also different

$$F_{CSA}(t,T) = E^{T}\{S(T)|\mathcal{F}_{t}\} \neq \tilde{E}^{T}\{S(T)|\mathcal{F}_{t}\} = F_{No\ CSA}(t,T)$$

But the market only quotes one set of forward prices (generally based on collateralised trading)

Therefore a convexity adjustment is required for uncollateralised trades

- This is conceptually similar to the standard futures convexity adjustment
- However, it differs as no discounting is applied to margin calls on futures

# Collateralised Interest Rate Derivatives

# Interpretation of projection curves

It is now clear how to interpret forward rates taken from the projection curves

The multi-curve valuation of a (proto-typical) FRA, fixing at T and settling at T'=T+ au is

$$V_{\text{FRA}}(t) = \tau(L_{\tau}(t, T) - K)P(t, T')$$

where

- $L_{\tau}(t,T) = \frac{1}{\tau} \left( \frac{P_{\tau}(t,T)}{P_{\tau}(t,T')} 1 \right)$  is the forward rate off the  $\tau$  projection curve, and
- P(t,T') is the discount factor of the discounting curve (hopefully the OIS curve)

But

$$V_{\text{FRA}}(t) = P(t, T')E^{T'}\left\{\tau(L_{\tau}(T, T) - K)|\mathcal{F}_{t}\right\}$$

SO

$$L_{\tau}(t,T) = E^{T'}\{L_{\tau}(T,T)|\mathcal{F}_t\}$$

Note:  $L_{\tau}(t,T)$  is a martingale under the T'-forward measure (by the tower property of expectations)

# FRA Pricing

Standard-market FRA contracts settle in-advance according to one of two conventions:

$$V_{\text{BBA}}(T) = \frac{\tau(L_{\tau}(T, T) - K)}{1 + \tau L_{\tau}(T, T)}$$

and

$$V_{\text{ABA}}(T) = \frac{1}{1 + \tau K} - \frac{1}{1 + \tau L_{\tau}(T, T)}$$

In the one-curve world no convexity adjustment was required

However, in the multi-curve environment, the fair-market rate in both cases is

$$K^{*}(t,T) = \frac{1}{\tau} \left( \frac{1}{E^{T} \left\{ \frac{1}{1 + \tau L_{\tau}(T,T)} \right\}} - 1 \right) \neq L_{\tau}(t,T)$$

Mercurio (2010) derives an approximation for the correction (assuming shifted-lognormal processes)

He argues that under "reasonable" conditions the convexity is small, even for long maturities

# **Caplet Pricing**

Consider a caplet on  $\tau$ -Libor with pay-off

$$V_{\text{Caplet}}(T') = \tau [L_{\tau}(T,T) - K]^+$$

The caplet's value at t (under the T'-forward measure) is

$$V_{\text{Caplet}}(t) = \tau P(t, T) E^{T'} \{ [L_{\tau}(T, T) - K]^{+} | \mathcal{F}_{t} \} |$$

But as  $L_{\tau}$  is martingale under this measure, we can assume its dynamics to be of the form

$$dL_{\tau}(t,T) = \sigma_{\tau}L_{\tau}(t,T)dW^{T'}$$

where  $W^{T'}$  is a Brownian motion under forward measure, which leads to (using the standard approach)

$$V_{\text{Caplet}}(t) = \tau P(t, T) \text{Black}(L_{\tau}(t, T), K, \sigma_{\tau} \sqrt{T - t})$$

So we can price the caplet as per usual except the forward rate is taken from the projection curve and discount factor is taken from the discount curve.

But from where do we find  $\sigma_{\tau}$ ?

- We have market observable Caps/Floors for  $\tau = 3M$
- What about  $\tau = 1M$  or 6M?
- Standard volatility conversion algorithms assume single-curve dynamics

# Swaption pricing

Now consider a payer swaption paying at time  $T_0$ 

$$V_{\text{Swaption}}(T_0) = A_{\tau}^{T_0, T_N}(T_0) [S_{\tau}^{T_0, T_N}(T_0) - K]^{+}$$

where

$$A_{\tau}^{T_0, T_N}(t) = \sum_{i=1}^{N} \tau_i P(t, T_i)$$

is the level function and

$$S_{\tau}^{T_0, T_N}(t) = \frac{\sum_{i=1}^{N} \tau_i P(t, T_i) L_{\tau}(t, T_i)}{A_{\tau}^{T_0, T_N}(t)}$$

is the fair swap rate.

The value of the swaption under the swap measure (defined by the numeraire  $A_{\tau}^{T_0,T_N}$ ) is

$$V_{\text{Swaption}}(t) = A_{\tau}^{T_0, T_N}(t) E^{T_0, T_N} \left\{ \left[ S_{\tau}^{T_0, T_N}(T_0) - K \right]^+ | \mathcal{F}_t \right\}$$

But  $S_{ au}^{T_0,T_N}$  is martingale under this measure, we get

$$V_{\text{Swaption}}(t) = A_{\tau}^{T_0, T_N}(t) \text{Black}\left(S_{\tau}^{T_0, T_N}(t), K, \sigma_{\tau}^{T_0, T_N} \sqrt{T_0 - t}\right)$$

for some volatility  $\sigma_{\tau}^{T_0,T_N}$  (again, of unspecificed origin)

# Libor Market Model

In general, we need to model

- $L_{\tau}(t, T_k)$  the forward Libor (FRA) rate possibly for multiple  $\tau$
- $P(t, T_k)$  the risk-free bond price

Under the (risk-free)  $T_{k+1}$ -forward measure, we have three martingales

- $L_{\tau}(t, T_k) := E^{T_{k+1}}\{L_{\tau}(T_k, T_k) | \mathcal{F}_t\}$  (by the tower-property of expectations)
- $F_{\tau}(t, T_k) \coloneqq \frac{P(t, T_k) P(t, T_{k+1})}{\tau P(t, T_{k+1})}$  (as  $P(t, T_{k+1})$  is the numeraire)
- $S_{\tau}(t, T_k) \equiv L_{\tau}(t, T_k) F_{\tau}(t, T_k)$  (difference of two martingales)

Mercurio (2009 & 2010) suggests modelling any two of these martingales

He favours modelling  $F_{\tau}(t, T_k)$  and  $S_{\tau}(t, T_k)$  to guarantee for t that

$$0 < F_{\tau}(t, T_k) < L_{\tau}(t, T_k)$$

# **Spread Dynamics**

Mercurio (2010) suggests modelling the spread with 1-factor dynamics,

$$S_{\tau}(t, T_k) = S_{\tau}(0, T_k) M_{\tau}(t)$$

where  $M_{\tau}(t)$  is a martingale.

In his model:

- The martingales  $M_{\tau}(t)$  are independent of the discounting forwards
  - So that spreads are martingales under all forward and swap measures
- $M_{\tau_1}(t)$  is correlated with  $M_{\tau_2}(t)$  to capture cross-basis relationships
  - o But this does not guarantee:  $L_{ au_1}(t,T_k) < L_{ au_2}(t,T_k)$  for all t and  $au_1 < au_2$

He also derives formulae for Cap/Floor and Swaptions under these models

Facilitating the pricing of options on non-standards tenors

How should all these parameters (volatilities and correlations) be calibrated?

- Mercurio (2010) suggests using additional degrees of freedom to fit:
  - The Cap/Floor smile, and/or
  - CMS spreads
- Still need to calibrate volatility and correlation of non-standard tenors (eg.  $\tau$  = 1M or 6M)

# Derivative pricing with and without collateral

So in summary the approach to pricing and revaluation is:

- Trades fully collateralised with cash: (eg. standard inter-bank swaps and options)
  - Discount with OIS curve
  - Forecast with basis-specific curves (bootstrapped with OIS curve for discounting)
  - o First Order: Correct moneyness using basis-specific curves (static-basis model)
  - o [H] Second Order: Correct volatility (model spread dynamics)
  - o [H] Apply a convexity adjustment to In-advance instruments
- Unsecured Trades: (eg. trades with non-bank customers, notes and hedges with exotic coupons)
  - As above except,
  - Discount with a treasury (cost-of-funds) curve
  - o [H] Apply a convexity adjustment to forwards for the miss-matched numeraire
  - [H] Apply CVA (adjusting for counterparty default risk)
     Burgard & Kjar (2010) and Fries (2010) look at credit and funding simultaneously
- Partially secured Trades: (eq. trades with non-cash collateral, or thresholds on margining)
  - As above except,
  - o [H] Discount with repo curve appropriate to collateral type (eg. Bond Repo rates)
  - [H] Apply CVA (simulating collateralisation rules, eg Assefa et al 2009)

[H] A lot of this hard!

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