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## Research Note

## Trading volatility in low-rate regimes

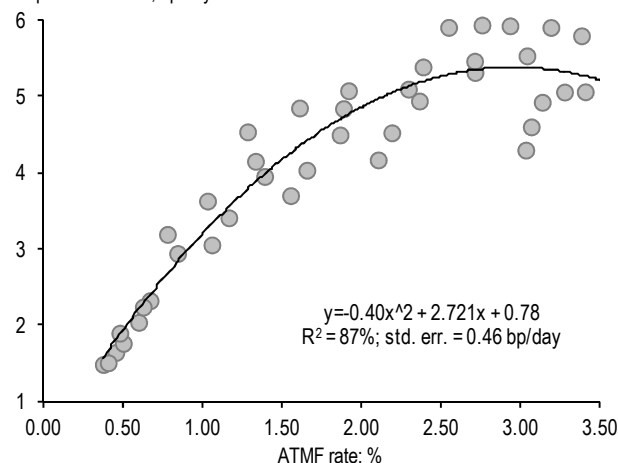
- Over the past few years, policy action across developed markets has driven yields to historically low levels
- In such a low-rate regime, normal implied volatility becomes highly directional with rates; this poses two important challenge to investors
- First, without accounting for the vol-rate relationship, regression-based fair value models for implied vols can give misleading trading signals; even if the level of yields is incorporated as a factor, nonlinearity in the vol-rate relationship causes betas to systematically deviate from backward-looking estimates when rates are trending
- Second, normal deltas are poorly behaved proxies for the “true” delta, making it difficult to construct pure volatility positions
- We develop a fair value framework that first empirically estimates the nonlinear vol-rate relationship, and then adjusts for structure-specific deviations. We also estimate the correct delta to be used in hedging volatility positions, which we refer to as “empirical deltas” to distinguish them from deltas that rely on specific distributional assumptions
- The result is a framework that allows investors to identify trading opportunities in volatility markets and monetize mispricings without incurring significant duration risk

## Overview: the “lognormal shuffle”

Over the past few years, strong policy action from the Fed has pushed rates to historically low levels. While low (and rangebound) yield environments are generally conducive to carry seeking strategies, they also present unique challenges for investors seeking to earn carry via short volatility positions. In particular, as one approaches the zero barrier, markets increasingly trade to an implied lognormal

## Exhibit 1: Normal implied volatilities are well explained as a quadratic function of ATMF rates across much of the surface...

Cross-sectional snapshot of ATMF implied volatility\* versus ATMF rate for a range of swaption structures; bp/day



\* Includes a range of structures: 3M, 6M, 1Y, 2Y, 3Y, and 5Y expiries on 1Y, 2Y, 3Y, 5Y, 7Y, 10Y, and 30Y tails; levels as of 4/2/2013.

distribution to avoid pricing in unrealistically high probabilities of negative rates.

As a result, normal implied volatility in the current regime is highly correlated with the level of rates—an effect we have frequently referred to in the past as the “lognormal shuffle.” This phenomenon can be observed by looking at the empirical relation between one (e.g., 3Mx2Y) swaption structure’s ATMF implied volatility and its forward rate over time—say, the past year. But it is also evident in just a snapshot of the ATMF vol grid at a particular point in time. In low rate regimes, the ATMF implied volatility for a range of structures spanning the vol grid can be plotted versus the corresponding forward yields, producing a relationship that is well approximated by a quadratic fit (Exhibit 1). Such a relationship has not only persisted in Japan for several years, but is now true of several

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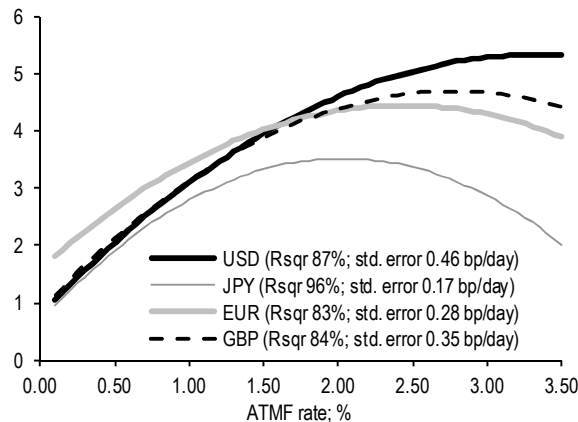
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### Exhibit 2: ...and this is true across a range of developed markets thanks to globally pervasive low yield regimes

Fitted vol-rate curve\* for ATMF implied volatility by currency; bp/day



\* ATMF implied volatility is modeled as a function of the forward rate ( $a_0 + a_1 \text{fwd} + a_2 \text{fwd}^2$ ) using the range of structures itemized in Exhibit 1; levels as of 4/2/2013.

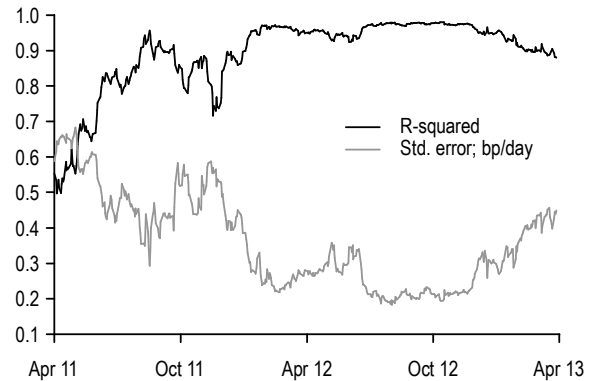
major currencies given the prevalence of extremely accommodative monetary policy across much of the developed world (**Exhibit 2**). In the US in particular, historical goodness of fit statistics show that the simple quadratic cross-sectional relationship above has been a good description of the directionality of volatility with the level of rates for much of the past two years (**Exhibit 3**). This is especially true since early 2012, when the Fed extended its rate guidance; since then, the vol-rate relationship has not only persisted, but has also been relatively stable, as seen in **Exhibit 4**.

In general, such stability is unusual. While the existence of a vol-rate relationship is to be expected in low yield regimes, the vol-rate curve itself can evolve over time. This too is highlighted in Exhibit 4, which also shows the vol-rate curve in JPY for three points in time. Thus, today's vol-versus-rate fitted curve is not set in stone, and can vary over time. Therefore, it must be conceptualized as a dynamic relationship for valuation and hedging purposes.

The practical implications for investors are two-fold. First, **establishing fair value becomes difficult**. Any reasonable fair value framework must account for not only a dependency on rates, but must also recognize the potential for the entire vol-rate relationship to evolve. *Indeed, a key part of addressing the question of fair value for volatility boils down to estimating the "fair value" for the entire vol-versus-rate cross-sectional*

### Exhibit 3: A strong vol-rate relationship has prevailed in the US since early 2012

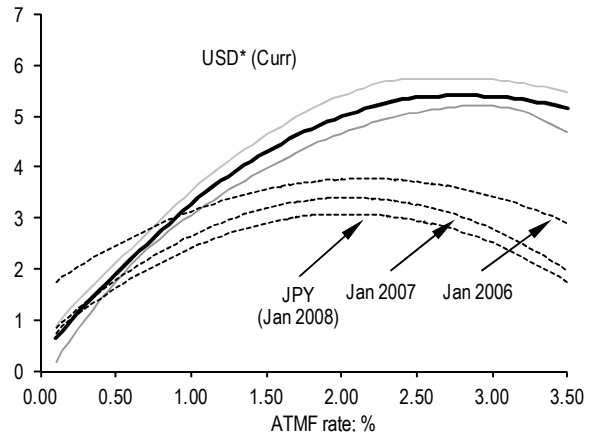
Historical goodness of fit statistics for USD cross-sectional vol-rate curve\*



\*ATMF implied volatility is modeled as a function of the forward rate ( $a_0 + a_1 \text{fwd} + a_2 \text{fwd}^2$ ) using the range of structures itemized in Exhibit 1.

### Exhibit 4: While the vol-rate curve has been stable in the US over the past year, such stability is not guaranteed—vol-rate curves can evolve over time, as highlighted by the Japanese experience

Evolution of the vol-rate curve, USD and JPY swaptions; bp/day



\* 1-year median (black) and 10th/90th percentile range for the vol-rate fitted curve for USD swaptions.

Note: ATM implied volatility is modeled as a function of the forward rate ( $a_0 + a_1 \text{fwd} + a_2 \text{fwd}^2$ ) using the range of structures itemized in Exhibit 1.

*quadratic curve*. Not doing this can lead one astray; for example, **Exhibit 5** highlights the limitations of more traditional regression-based fair value models for 3Yx10Y ATMF implied volatility. Over the past few years this sector could be modeled via a linear regression using a combination of forward rate, inflation expectations, empirical mortgage market convexity, and spec investor positioning as independent factors. However, despite a significant statistical fit, and despite the inclusion of the level of

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the forward rate as a factor, the residual of this is clearly not mean reverting. Thus, such a framework would have shown 3Yx10Y volatility to be consistently cheap even as vols kept trending lower, missing the fact that falling volatility was a consequence of trending rates and the steeper vol-rate slope at lower yields. In other words, backward-looking regression models fail when the vol-rate curve exhibits strong nonlinearity (as is the case in interest rate markets across several major currencies; Exhibit 2) and rates are trending, since forward-looking betas between implied and rates systematically deviate from those estimated using a backward-looking regression framework.

Second, **having identified a sector as rich or cheap, monetizing mispricings requires removing directionality with rates.** For example, were one to systematically sell short-dated straddles and hedge daily using normal deltas over the past year, returns would have been highly correlated to moves in rates, with this phenomenon being especially pronounced for shorter tails which are closer to the zero boundary and thus exhibit a greater degree of directionality (**Exhibit 6**). Thus, whereas historically selling straddles was an uncorrelated source of carry, the directionality of implied with rates means that short volatility positions can amplify duration risk in portfolios that are typically long the market. As a consequence, in addition to examining hedging frequency in seeking the optimal volatility trading strategy, low-rate regimes force investors to confront a more fundamental question: what is the appropriate delta to use to produce non-directional returns?

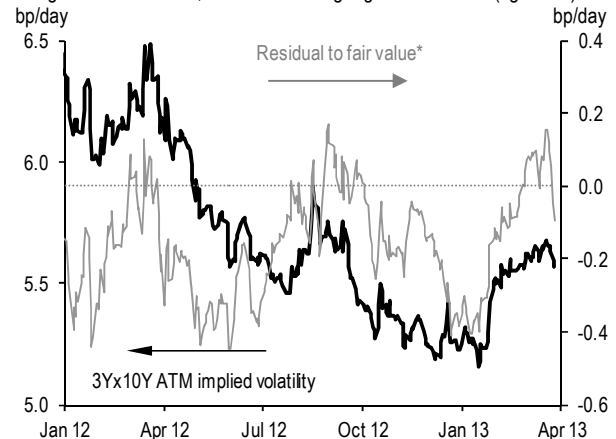
In this Research Note, we examine these important practical considerations for trading interest rate volatility in the current environment.

## Fair value modeling

We begin with a new framework for estimating fair value in a low-rate environment. As we noted earlier, estimating the “fair value” for the entire vol-rate curve is a key first step. But how does one model an entire curve? Quite simply, by modeling the value of the vol-rate curve at selected points. First, we introduce some terminology: we will refer to the value of the vol-rate curve at a given forward rate (say, 2%) as the *fixed-*

### Exhibit 5: Traditional fair value models consistently signaled buying opportunities in some sectors, even as implied drifted steadily lower

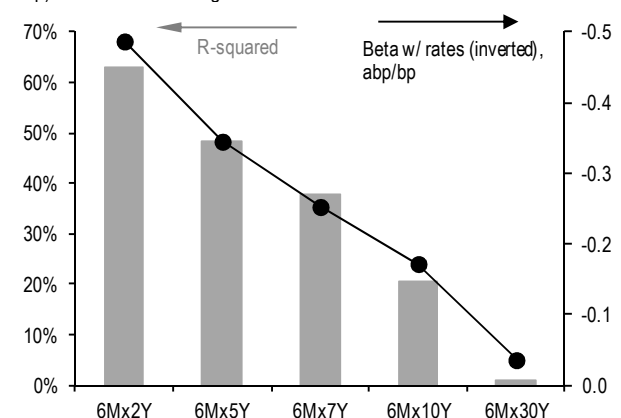
3Yx10Y ATM implied volatility (left side) and residual to fair value estimated using a more traditional, backward-looking regression model\* (right side)



\* Based on a rolling 1-year regression of ATM implied volatility versus forward rate, 5Y inflation swap yield, empirical mortgage market convexity (see *US Fixed Income Markets Weekly*, 2/1/2013 for details), and the absolute value of net longs by speculative investors (based on CFTC data); the average R-squared and standard error over the period considered were 75% and 0.17 bp/day, respectively.

### Exhibit 6: Systematically selling volatility since early 2012 would have added significant duration risk, particularly in short and intermediate tails

Statistics from an 18-month regression of 2-week short straddle returns\* (in abp) versus 2-week changes in ATM rate for various structures



\* Returns from the J.P. Morgan Volatility Index, which assumes options are re-structed on the first day of the month and delta-hedged daily using normal deltas with zero transaction costs.

*forward volatility* at 2%. Then, by empirically modeling the fair value of fixed forward volatility at select forward rate values (say 0.25%, 0.5%, 1%, 1.5%, 2% and 3%), one effectively models the entire vol-rate curve since every other fixed forward volatility can be interpolated.

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Our empirical model for fixed-forward volatility is estimated as follows. For every day in the desired historical period, we estimate a quadratic cross-sectional fit of ATMF implied volatilities versus forward rates, and use the coefficients to calculate the fixed-forward volatility corresponding to a given forward rate (e.g., roughly 2.20% for 6Mx10Y swaptions). These fixed-forward volatilities (for a range of forward rates) are then modeled using observable market variables representing the traditional drivers of volatility, including: market depth, Fed purchases of long-term securities (which tend to dampen volatility), MBS market moneyness (as a proxy for options-based hedging demand from MBS portfolio managers and servicers), and the slope and shape of the yield curve. The results make intuitive sense: market depth and Fed purchases matter more for the front-end (lower forward rates), and implieds across the grid are relatively insensitive to mortgage market moneyness (consistent with relatively low mortgage market aggregate convexity needs in recent years). A summary of our models for fixed forward volatilities for a range of forward rates is presented in **Exhibit 7**.

Such an empirical model for fixed forward volatilities takes us much of the way towards estimating fair value—a first guess for the fair value of (say) 6Mx10Y swaption volatility would simply be the fair value for fixed-forward volatility at the current 6Mx10Y ATMF rate. But this is not good enough. Even given a robust model for fixed-forward volatilities, historical data clearly shows that **ATMF implied vols for specific structures may systematically trade above or below the fixed-forward volatility** that is consistent with the broader vol-rate curve (**Exhibit 8**). While using the fixed-forward volatility accounts for the directionality of implieds with rates that characterizes the vol surface at a broad level, it does not distinguish between swaption structures that may have similar forward rates. In other words, it does not account for the smaller—but systematic—biases that may exist at a structure-specific level between actual ATMF implied volatility and the corresponding fixed-forward volatility. We refer to this bias for a given swaption as its fixed-forward offset, which may be thought of as a structure-specific adjustment to the initial fair value estimate. We model this offset separately for each

#### Exhibit 7: Fixed-forward implied volatility can be modeled using observable market variables

Regression coefficients from modeling fixed-forward implied volatility at different forward rates, as a function of several independent variables; as of 4/2/2013

Forward Rate (%)	Fixed-fwd Vol. (bp/day)	Local Slope*	Regression Betas				
			Market Depth†	Fed Purch.**	Mtge mkt \$ness‡	Curve Slope***	Curve Shape****
0.50	2.04	2.29	0.00065	-0.00008	0.25	0.19	0.021
1.00	3.09	1.89	-0.00005	0.00004	0.22	-0.34	0.041
1.50	3.93	1.49	-0.00052	-0.00003	0.20	-0.59	0.056
2.00	4.58	1.09	-0.00075	-0.00030	0.18	-0.57	0.065
2.50	5.03	0.70	-0.00076	-0.00077	0.17	-0.26	0.068
3.00	5.28	0.30	-0.00053	-0.00144	0.15	0.32	0.067
3.50	5.33	-0.10	-0.00008	-0.00230	0.14	1.18	0.059

\* (Local Slope) =  $2 \cdot a_2 \cdot \text{fwd} + a_1$  where  $a_1$  and  $a_2$  are the linear and quadratic coefficients from a daily fit to the vol-rate curve.

† Market depth is calculated as the average size of the top three bids and offers, in \$mn, for the on-the-run 10-year Treasury note, averaged between 8:30 a.m. and 10:30 a.m. daily

\*\* Monthly trailing Fed purchases of agency MBS and long-term Treasuries, in \$bn

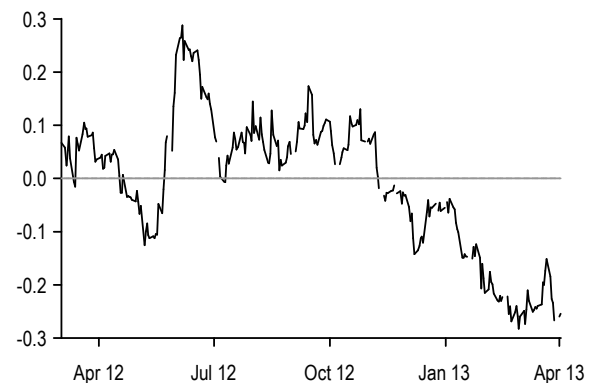
‡ Difference between the J.P. Morgan MBS Index WAC and the 30Y current coupon (%).

\*\*\* Spot 2s/30s curve (%).

\*\*\*\* Curve shape using 10Y minus 0.5 x (5Y+30Y) swap yields as a proxy (bp).

#### Exhibit 8: Fixed-forward offsets do not necessarily mean revert towards zero...

Fixed-forward offset\* for 6Mx10Y swaptions; bp/day



\* The fixed-forward offset is defined as the actual ATMF implied volatility for a given swaption structure minus the fixed-forward implied volatility at its forward rate. Fixed-forward implied volatility is estimated from the fitted vol-rate curve at the structures' ATMF rate.

structure, which is illustrated schematically in **Exhibit 9**.

In short expiries, we find that fixed-forward offsets are well-described by three market variables, including: the spot 2s/10s swap curve, spec investor duration positioning, and the level of fixed-forward volatility for that sector (**Exhibit 10**). Combining such a model for fixed-forward offsets with our broader fair value framework for fixed-forward volatilities, we can calculate fair values for ATMF implied volatility for specific points on the vol surface.

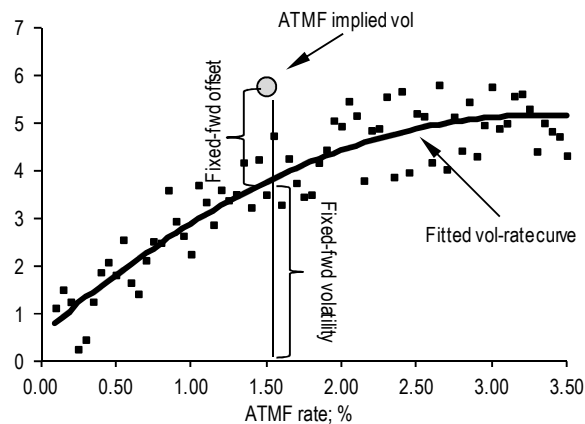


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### Exhibit 9: ...compelling us to adopt a two-stage fair value framework, in which fixed-forward volatility and fixed-forward offsets are modeled separately

A schematic overview of our two-stage fair value framework for ATMF implied volatilities in which we define the fixed-forward volatility and offset; bp/day



Note: The vol-rate curve is a quadratic fit to ATMF implied volatilities versus forward rates as in Exhibit 1. The level and shape of the curve and points in this chart are indicative, and do not reflect market data.

Our fair value framework is thus a two-stage process. In order to estimate the fair value for 6Mx10Y swaption implied volatility we first estimate the fair value of the fixed-forward volatility at its ATM rate—this accounts for the broader vol-rate relationship. We then estimate the fair value for the fixed-forward offset, which is the structure-specific adjustment applicable for 6Mx10Y swaptions. Putting those two together yields a fair value for 6Mx10Y swaption implied volatility. We summarize this procedure, along with model betas, goodness of fit statistics, etc. for both stages, in Exhibit 10.

We can also use a similar procedure to model longer expiries—the first stage of the process would be identical, but we choose different explanatory variables for the fixed-forward offset (**Exhibit 11**). In the case of 3Yx5Y, for example, we find that inflation expectations, Fed policy, and the level of fixed-forward volatility collectively explain the historical fixed-forward offset. Moreover, comparing 6Mx10Y to 3Yx5Y also serves to highlight the importance of a two-stage framework; despite having similar ATMF rates, their fair values are considerably different, thanks to different drivers for their fixed-forward offsets.

### Exhibit 10: An example of our fair value framework for the gamma sector...

Fair value model for 6Mx10Y ATMF implied volatility, levels as of 4/2/2013

Structure	6Mx10Y		
ATMF (%)	2.19		
Phase 1 - Modeling the fixed-fwd implied vol*			
Factor	Coefficient	T-stat	Curr Value
Intercept	5.75		
Market depth (\$mn)	-0.0008	-5.4	230
Fed purchases (\$bn)	-0.0005	-0.9	98
Mtge market moneyness (%)	0.18	2.4	2.09
2s/30s curve (%)	-0.49	-10.7	2.59
5s/10s/30s fly (bp)	0.07	11.1	4.1
R-squared	87%		
Std error (bp/day)	0.09		
Fixed-fwd vol fair value (bp/day)		4.91	

Phase 2 - Modeling the structure-specific fixed-fwd offset**			
Factor	Coefficient	T-stat	Curr Value
Intercept	-0.21	2.6	
2s/10s swap curve (%)	-0.53	-14.2	1.59
Spec investor duration positions	0.05	9.9	0.70
Fixed-fwd vol (bp/day)	0.18	2.7	4.78
R-squared	72%		
Std error (bp/day)	0.07		
Fixed-fwd offset fair value (bp/day)			-0.17
6Mx10Y ATMF implied vol. fair value (bp/day)			4.74
Current Value (bp/day)			4.53
Mispricing (bp/day)			-0.22

\* The fixed-forward volatility is defined as the level of the fitted vol-rate curve at fixed ATM rate using that day's close. It is modeled via a 1-year regression against market depth, 1-month trailing Fed purchases, mortgage market moneyness (Index WAC minus current coupon yield), curve slope (spot 2s/30s) and curve shape (5s/10s/30s equal-wtd butterfly).

\*\* The structure-specific fixed-forward offset is defined as ATMF implied volatility minus the level of the fitted-vol-rate curve at the ATM rate evaluated daily. It is modeled via a 1-year regression against the 2s/10s swap yield curve, speculative investor duration positioning inferred from the absolute value of the J.P. Morgan Treasury Positions Index, and the level of fixed-forward volatility.

### Introducing empirical deltas

Given a view on the richness or cheapness of a given sector, the problem then becomes how to most effectively isolate and monetize this mispricing. Since the fair value for a given swaption structure's normal implied volatility will change as rates move, doing so requires hedging against the systematic variation of normal implied volatility with rates. This involves adding a rate-hedge overlay to a "normal-delta-hedged" swaption volatility position.

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### Exhibit 11: ...as well as the vega sector

Fair value model for 3Yx5Y ATM implied volatility, levels as of 4/2/2013

Structure	3Yx5Y		
ATMF (%)	2.38		
Phase 1 - Modeling the fixed-fwd implied vol*			
Factor	Coefficient	T-stat	Curr Value
Intercept	5.62		
Market depth (\$mn)	-0.0008	-5.8	230
Fed purchases (\$bn)	-0.0006	-1.4	98
Mtge market moneyness (%)	0.17	2.5	2.09
2s/30s curve (%)	-0.36	-8.6	2.59
5s/10s/30s fly (bp)	0.07	12.3	4.1
R-squared	88%		
Std error (bp/day)	0.09		
Fixed-fwd vol fair value (bp/day)		5.08	
Phase 2 - Modeling the structure-specific fixed-fwd offset**			
Factor	Coefficient	T-stat	Curr Value
Intercept	1.41	6.5	
3Y inflation swap yield (%)	0.07	1.5	2.37
Fed expectations	0.92	11.0	0.62
Fixed-fwd vol (bp/day)	-0.38	-11.1	4.94
R-squared	64%		
Std error (bp/day)	0.10		
Fixed-fwd offset fair value (bp/day)		0.26	
3Yx5Y ATMF implied vol. fair value (bp/day)		5.33	
Current Value (bp/day)		5.39	
Mispricing (bp/day)		0.06	

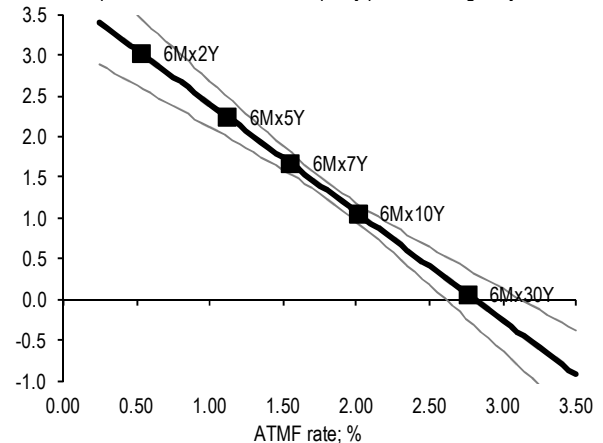
\* Same as Exhibit 10.

\*\* Same definition and regression period as Exhibit 10, using a different set of explanatory variables: 3-year inflation swap yield as a proxy for long-term inflation hedge demand, forward looking market expectations for Fed policy (using 3Y forward 3M OIS minus spot 3M OIS as a proxy), and the level of fixed-forward volatility.

In principle, adding a rate-hedge to a delta-hedged swaption is equivalent to merely adjusting the delta—i.e., one needs to delta hedge using a different delta from the normal delta. Using lognormal deltas could work in some sectors such as the upper left, for instance, but this is not a general enough framework since not all sectors are equally directional (**Exhibit 12**). Indeed, moving to a lognormal delta in longer tails, where the implied distribution is much closer to normal, introduces greater (and opposite) directionality. This motivates the need for a framework that leverages normal models, but incorporates an empirical correction to adjust for the directionality of implied normal volatility with rates. Such an “empirical delta” approach should naturally interpolate between sectors of the curve which are quite directional (e.g., shorter expiries on 2Y tails) and those that are less so (e.g.,

### Exhibit 12: Front end volatility is highly directional with rates

1-year average (black line) and +/- 1 std. deviation bands (light grey lines) of the local slope of the vol-rate relation\*; bp/day per 1% change in yield



\* ATMF implied volatility is modeled as a function of the forward rate ( $a_0 + a_1 \text{fwd} + a_2 \text{fwd}^2$ ) using the range of structures itemized in Exhibit 1. Filled squares indicate benchmark structures.

30Y tails and longer expiries). A key advantage of such an approach is that it is fundamentally non-parametric, and therefore does not rely on any particular distributional assumption.

How do we estimate such an empirical delta? We begin with the fitted vol-rate curve, which is estimated daily from ATMF implied volatilities and forward rates across a range of structures. The local slope of this fitted curve gives us the systematic change in ATMF normal implied volatility that should be expected based on a change in rates; knowing this and the vega of the swaption, we may calculate the empirical adjustment needed to the normal deltas. The result is a modified delta (which we will refer to as the *empirical delta*) which naturally corrects for the normal vol versus rate correlation to the extent that it exists in each sector of the vol surface. Also by recalculating this adjustment daily, this framework also accounts for the potential dynamics of the vol-rate curve, given that its shape can vary over time (as was the case in Japan; Exhibit 4).

### Backtesting empirical deltas

It is also clearly essential to backtest this framework to confirm that empirical deltas effectively remove duration risk in practice. This is particularly important given that one key assumption underlying our empirical delta framework is that implied volatilities for structures with the same ATMF rate have the same

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### Exhibit 13: Hedging short straddles with empirical deltas significantly reduces the volatility of returns and more effectively removes directionality with rates

Return statistics for short straddle positions\* over the past 18 months

Structure	ATMF	Local Slope	Current Delta			Std. deviation of 2w returns (abp)			R-sqr w.r.t. rates†			Beta w.r.t. rates†		
			Normal	Lognormal	Empirical**	Normal	Lognormal	Empirical**	Normal	Lognormal	Empirical**	Normal	Lognormal	Empirical**
3Mx2Y	0.45	2.33	-0.23	0.01	-0.08	4.7	3.3	3.2	50%	0%	1%	-0.48	0.01	-0.05
3Mx5Y	1.05	1.85	-0.16	0.02	-0.05	7.1	5.7	5.7	31%	2%	1%	-0.37	0.07	-0.06
3Mx7Y	1.55	1.45	-0.15	0.00	-0.06	7.0	6.2	6.0	23%	4%	1%	-0.29	0.10	-0.05
3Mx10Y	2.10	1.01	-0.11	0.02	-0.05	7.5	7.3	6.9	12%	6%	1%	-0.20	0.15	-0.05
3Mx30Y	3.03	0.28	-0.08	0.01	-0.06	8.9	10.5	8.9	0%	14%	0%	-0.03	0.25	-0.01
6Mx2Y	0.50	2.29	-0.26	0.06	-0.05	4.8	3.1	2.9	63%	2%	0%	-0.49	0.06	-0.02
6Mx5Y	1.16	1.76	-0.19	0.08	-0.03	5.6	4.1	3.9	48%	9%	1%	-0.34	0.11	-0.03
6Mx7Y	1.65	1.37	-0.17	0.05	-0.05	5.1	4.4	3.9	37%	19%	0%	-0.25	0.16	-0.02
6Mx10Y	2.19	0.95	-0.13	0.06	-0.05	5.1	5.4	4.5	20%	26%	1%	-0.17	0.20	-0.02
6Mx30Y	3.06	0.25	-0.10	0.04	-0.08	5.7	7.7	5.7	1%	33%	0%	-0.04	0.27	-0.01
1Yx2Y	0.62	2.19	-0.28	0.17	-0.01	4.8	2.9	2.7	67%	18%	1%	-0.42	0.14	0.03
1Yx5Y	1.39	1.58	-0.21	0.15	-0.01	4.7	3.6	3.0	58%	31%	0%	-0.28	0.16	0.00
1Yx7Y	1.86	1.21	-0.19	0.12	-0.04	4.1	3.8	2.9	50%	44%	0%	-0.21	0.19	-0.01
1Yx10Y	2.36	0.81	-0.15	0.11	-0.05	3.6	4.6	3.0	28%	54%	1%	-0.14	0.24	-0.02
1Yx30Y	3.13	0.19	-0.14	0.06	-0.11	3.8	6.1	3.7	2%	55%	1%	-0.04	0.28	-0.02
2Yx2Y	1.03	1.87	-0.31	0.32	0.02	4.8	3.9	2.8	68%	52%	6%	-0.30	0.22	0.05
2Yx5Y	1.88	1.19	-0.23	0.22	-0.03	4.1	3.9	2.6	61%	58%	1%	-0.21	0.20	-0.01
2Yx7Y	2.29	0.86	-0.21	0.19	-0.06	3.4	4.1	2.5	53%	66%	5%	-0.16	0.22	-0.03
2Yx10Y	2.71	0.53	-0.18	0.17	-0.09	2.8	4.5	2.3	35%	73%	9%	-0.11	0.24	-0.04
2Yx30Y	3.27	0.08	-0.18	0.09	-0.17	2.7	5.3	2.6	5%	71%	3%	-0.04	0.27	-0.03

\* Options are re-struck on the first day of the month, and returns assume daily delta rebalancing with zero transaction costs.

\*\* (Empirical Delta) = (Normal Delta) – (Local Slope) x Vega / (100 x PVBP) where the local slope is equal to  $2 \cdot a_2 \cdot \text{strike} + a_1$  ( $a_1$  and  $a_2$  are the linear and quadratic coefficients to a daily fit of the vol-rate relation), the deltas are defined as bp of notional per bp of yield divided by PVBP, the local slope is bp/day per 1% change in yield, vega is bp of notional per bp/day, and the PVBP is for the underlying forward swap, all re-estimated daily. In the few cases where the local slope is negative, we impose a floor at 0 to ensure that empirical deltas converge to normal deltas in the high rate limit.

† Both beta and R-squared are based on a 18-month regression of 2-week total returns on the short straddle position (in abp) against 2-week changes in the ATMF rate for a given structure.

local slope with respect to their underlying forward rate. The results confirm that empirical deltas are clearly the preferred strategy for delta-hedging in the current regime. To demonstrate this, in **Exhibit 13** we show statistics for returns on short delta-hedged straddles (rebalanced daily) based on three different deltas—normal, lognormal, as well as our empirical delta. In addition to effectively removing directionality with rates across a wide range of structures, empirical deltas furthermore significantly reduce the variance of returns on short and intermediate tails. This is particularly important for longer expiries on short and intermediate tails, whose greater vega risk makes normal-delta-hedged returns more correlated with rates than shorter expiries with the same ATMF rate.

Our framework also has intuitive limiting behavior, producing results that are closer to what one would obtain using lognormal deltas at the very front of the curve, and closer to normal delta results in the lower-right sector of the vol surface. All in all, using empirical deltas efficiently removes directionality with rates across a wide range of structures, allowing investors to better isolate and monetize volatility

mispricings without incurring effective duration risk. Therefore, going forward we will look to use empirical deltas for delta-hedging purposes in our volatility trades as long as the current low-rate regime persists.

### Backtesting our framework for fair value

We can also pair these empirical deltas with fair value estimates as discussed above to verify the effectiveness of our proposed fair value framework. In **Exhibit 14**, we show total returns from a simple trading strategy: every day within the backtest period, an investor buys/sells straddles in each sector if the implied volatility differs from our fair value estimate by at least one standard error (which, since we use a two-stage approach, is a composite standard error from the two stages). Each trade, if initiated, is held for exactly two weeks. This is admittedly rather conservative, since we arbitrarily close out the trades after a fixed holding period. We are furthermore limited to a relatively short amount of history over which to test our approach; this, however, is inevitable given that the premise underlying our framework (low yield levels across the entire curve and a strong vol-rate relationship across

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## Exhibit 14: Trading strategies using our fair value and empirical delta framework have been profitable over the past six months

Results from a simple rules-based trading strategy\* over the past 6 months

Structure	# of trades			Average returns (bp of notional)			10th/90th inter-quartile range (bp of not.)		
	Short	Long	Total	Short	Long	Total	Short	Long	Total
6Mx2Y	25	7	32	0.7	1.2	0.8	-1.3 to 1.9	0 to 2.5	-1.2 to 2.3
6Mx5Y	21	3	24	3.9	5.5	4.1	-4.6 to 9.5	3 to 8.1	-3.8 to 9.3
6Mx10Y	17	13	30	11.4	-8.6	2.7	-1 to 22	-26 to 6	-17 to 22
6Mx30Y	27	7	34	15.3	14.4	15.1	-15 to 50	-18 to 42	-22 to 51

\* We assume a rules-based trading strategy as follows: sell/buy straddles in each sector, delta hedging daily with empirical deltas and assuming no transaction costs, if the current ATM implied volatility has a zscore of +/-1 relative to fair value. Total returns are calculated assuming options are re-structured on the first day of each month with a holding period of 2-weeks. We consider overlapping trades in calculating return statistics.

much of the swaption volatility surface) has been true only recently.

Looking at the past six months, it appears that such a strategy would have been profitable overall, with positive overall returns across a range of structures. This is even more true of short positions across a range of structures, while trades involving long volatility positions on 10-year tails in our backtesting did not perform as well. This is perhaps a consequence of changes in Fed policy (QE 3 and 3.5) over the period considered—i.e., long vol trades that were initiated when cheapness in that sector was indicated could have suffered from subsequent Fed stimulus and its depressive effects on volatility. Considering this, and the fact that overall returns were positive across a range of structures, we believe the results of our backtest are sufficiently compelling to validate our fair value framework.

## Conclusion

In this paper, we have addressed two important issues confronting portfolio managers who utilize volatility trading strategies in the current low yield environment. Both these issues arise from the fact that normal implied volatility has become correlated with forward rates. This makes it challenging to: (i) develop views on the fair values for swaptions in various sectors, and (ii) monetize volatility mispricings without taking implicit duration risk.

To address the former, we have introduced a new, two-stage framework in which we first model the vol-rate curve itself (the *fixed-forward volatility*), and then independently estimate a correction (the *fixed-forward offset*) to account for structure-specific drivers and

flows which can lead to systematic deviations from the broader vol-rate curve. We also introduce *empirical deltas*, calculated using a nonparametric approach, that efficiently remove the directionality of volatility positions with rates. Empirical deltas do not require any *a priori* distributional assumption, and naturally blend between lognormal and normal distributions as necessary.

We have also backtested our framework. One limitation is the relatively short amount of history over which we have backtested our approach; however, this is inevitable given that the premise underlying our framework (low yield levels across the entire curve and a strong vol-rate relationship across much of the swaption volatility surface) is only true recently. That said, the results are promising: our empirical deltas successfully remove the directionality of vol returns across a wide range of structures, and also produces intuitive limiting behavior in sectors that are strongly lognormal or normal. In addition, our valuation framework appears reasonably successful in generating trading signals.



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