Inflation Swaptions

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SOCIETE GENERALE
Corporate & Investment Banking

Summary

1. Global Picture

- Back to basics, Inflation Swaptions mechanism
- Client's needs
- What's your flavour? Inflation and Real Rate Swaptions

2. Valuation and risk management

- Available typology of pricing
- Theoretical pricing in a pure swaption market
- Pricing with liquid instrument, maths versus flows
- Risk management

3. Where do we stand, what is the next step?

- No interbank market on swaptions today
- Need a daily fixing
- New liquidity for more exotic structure

Global Picture

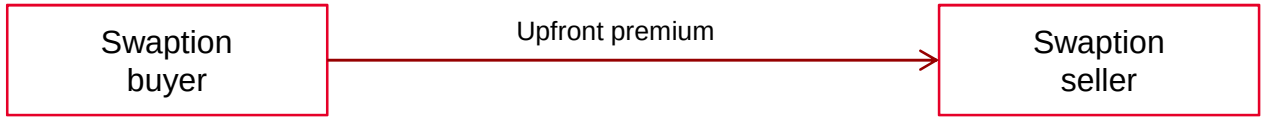
- Back to basics, Inflation Swaptions mechanism
- Client's needs
- What's your flavour? Inflation and Real Rate Swaptions

Back to basics, Inflation Swaptions mechanism

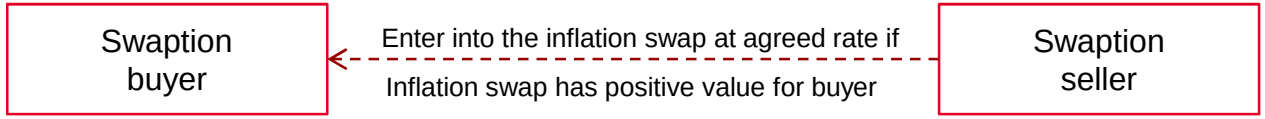
What is a swaption ?

- A payer/receiver inflation swaption gives the right to enter into a payer/receiver inflation swap with tenor $T_e - T_s$ at time T_s at a pre-specified rate k. We denote this as a T_s x ($T_e - T_s$) inflation swaption. The mechanics of an inflation swaption are given in the following figure :

- **At initiation**



- **At maturity (physical settlement)**



- Example :

  The buyer of a receiver swaption, buys the right, against a fee payment, to enter at a future date previously fixed, into an inflation receiver swap. The swap rate fixed is called the swaption strike. The tenor and the notional are fixed at the trade date of the swaption.

Back to basics, inflation Swaptions mechanism
Settlement conventions

- Two types of conventions for swaptions:
  - Swap settlement : At maturity date, we enter or not into the swap
  - Cash settlement : At maturity, we receive or we pay a PV that equals the mark-to-market
- Choice between Swap and Cash is made at the trade date
- Market is naturally Swap settlement

Back to basics, inflation Swaptions mechanism

Some observations

- The price of payer swaption decreases while it increases for a receiver one when the strike increases. We retrieve the fact that an out of the money swaption is less expensive than in the money.
- The price of a swaption (P or R) increases with the maturity of the underlying, as the volume of the flows increase with maturity. This is so called multiplicative effect of the sensitivity.
- The underlying of a swaption is the forward swap matching the terms of the swaption. For example, a 5y x 5y 3% R swaption is based on the following forward

$$R^f(t, 5y, 5y) = \frac{Sensi(t, t+10y) * R_{t,10y} - Sensi(t, t+5y) * R_{t,5y}}{Sensi(t, t+10y) - Sensi(t, t+5y)}$$

- We then observe the usual Payer/Receiver parity as on options:

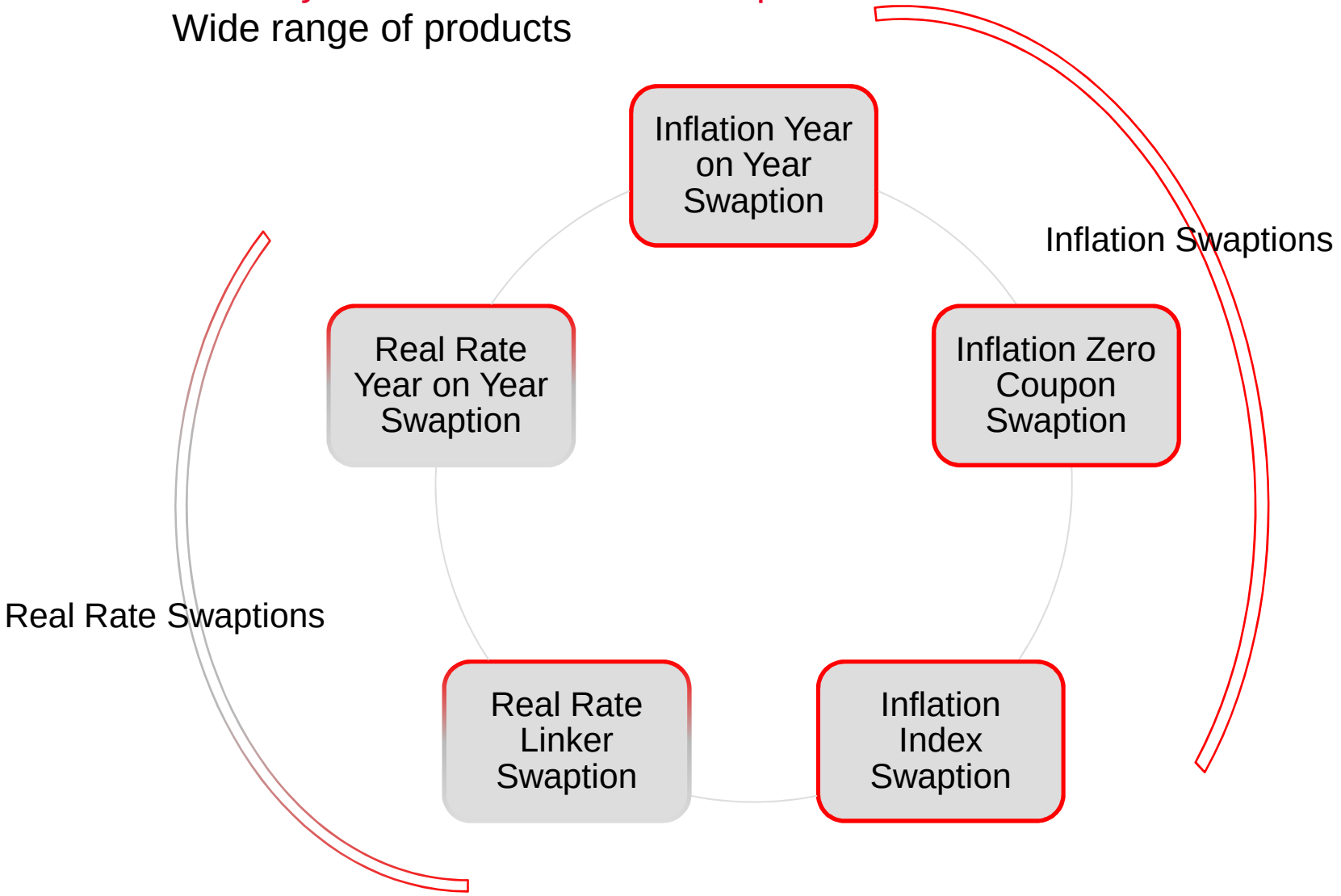
$$swaption\ P\ (K) - swaption\ R\ (K) = swap\ forward\ (K)$$

Client's needs
Buyers and Sellers

- Buyer side is the most natural way
  - Forward hedge can be done with a swaption
  - Project finance,
    - Execution is linked to several factors
    - Swaption allows its owner to hedge against the movements of the inflation swap market
  - Retail clients,
    - Uncertainty on the total amount of the note that will be raised
    - Price of the futures issues can be locked at current market levels
- Relative value and the seller : a new story
  - Buyers used to sell another strike swaption in order to have zero cost position
  - Generally this strategy is a collar swaption

What's your flavour? Inflation Swaption

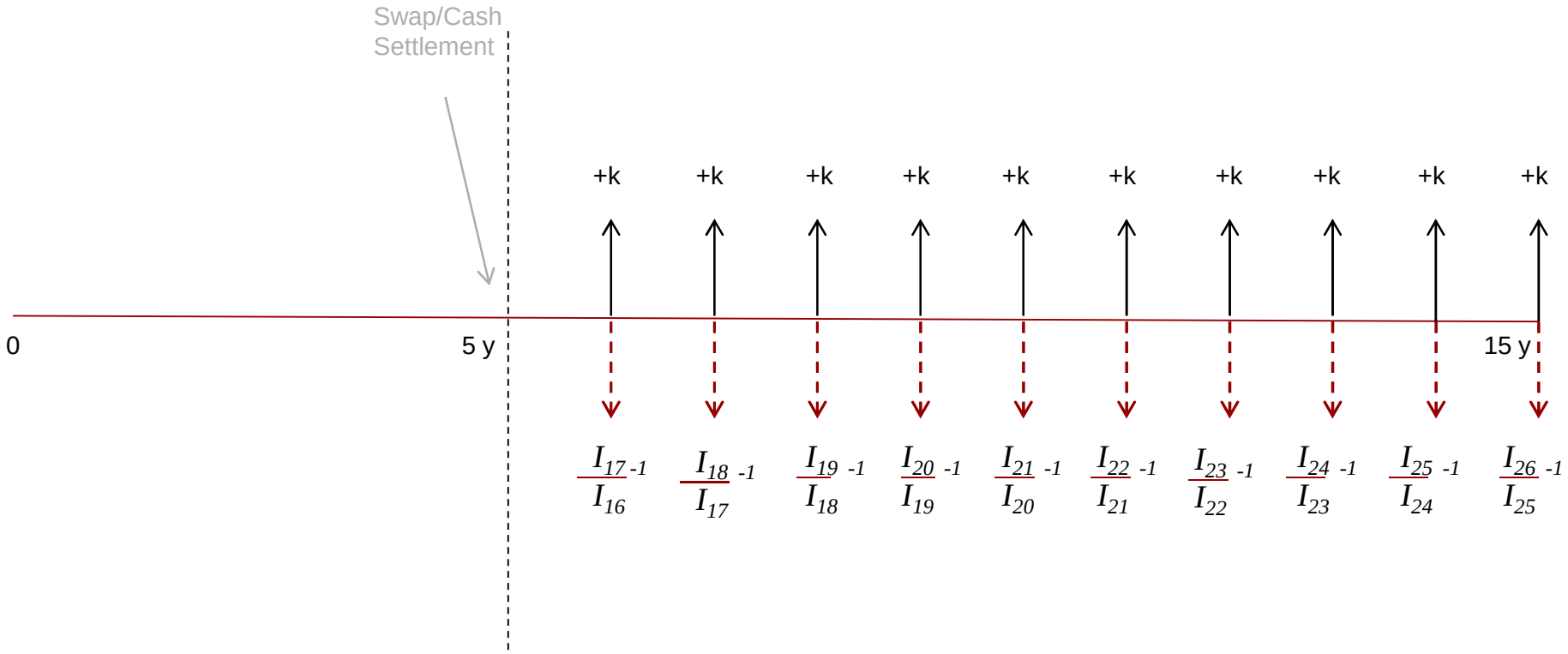
Wide range of products



What's your flavour? Inflation Swaption

Inflation year-on-year swaption

- This swaption gives the right to its owner to enter into a year on year inflation swap



What's your flavour? Inflation Swaption

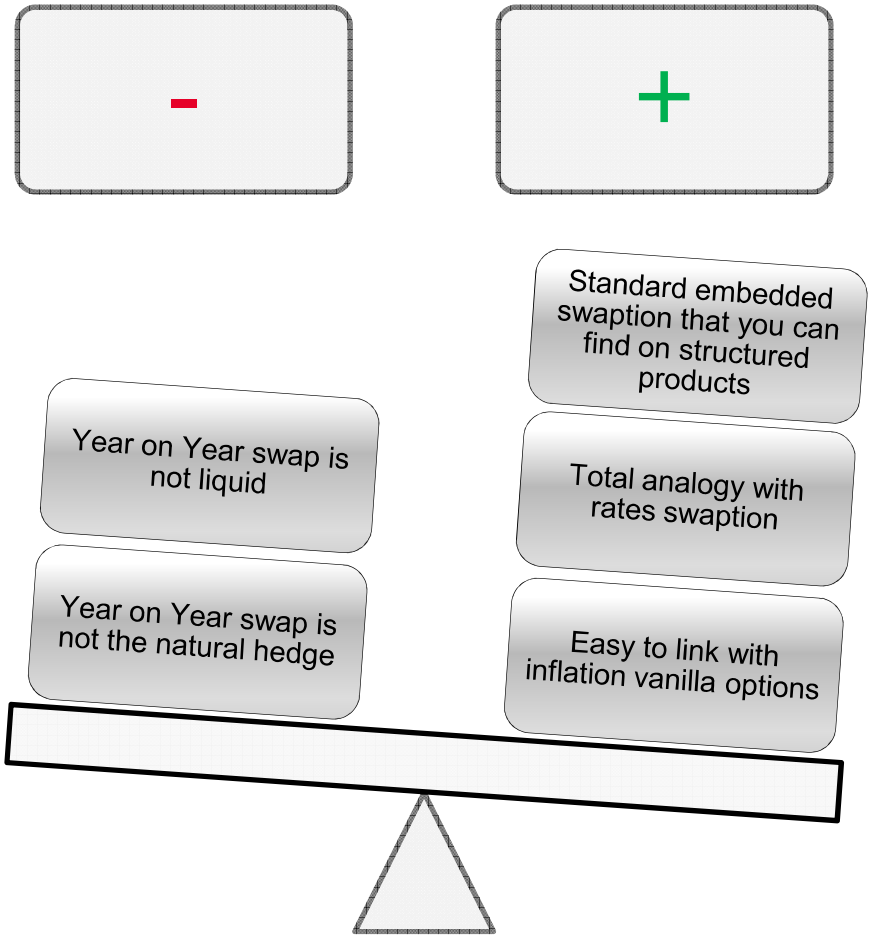
Inflation year-on-year swaption

- Term sheet

Notional:	€100,000,000
Index:	HICPxT (non revised)
Source:	First publication by Eurostat as shown on Bloomberg CPTFEMU
Start date:	1 May 2011
Option end date:	1 May 2016
Swap end date:	1 May 2026
Type:	Year-on-year
Reference month:	February
First fixing:	not yet known.
Buyer:	The right to enter into a fixed payer inflation swap starting 1 May 2016 and ending 1 May 2026 with annual flows $k - \frac{HICPxT(Feb/16 + i)}{HICPxT(Feb/15 + i - 1)}$, *where r denotes the inflation swap rate at 1 May 2016 for the February 2016 to February 2026 inflation swap.*
Seller:	upfront premium

What's your flavour? Inflation Swaption

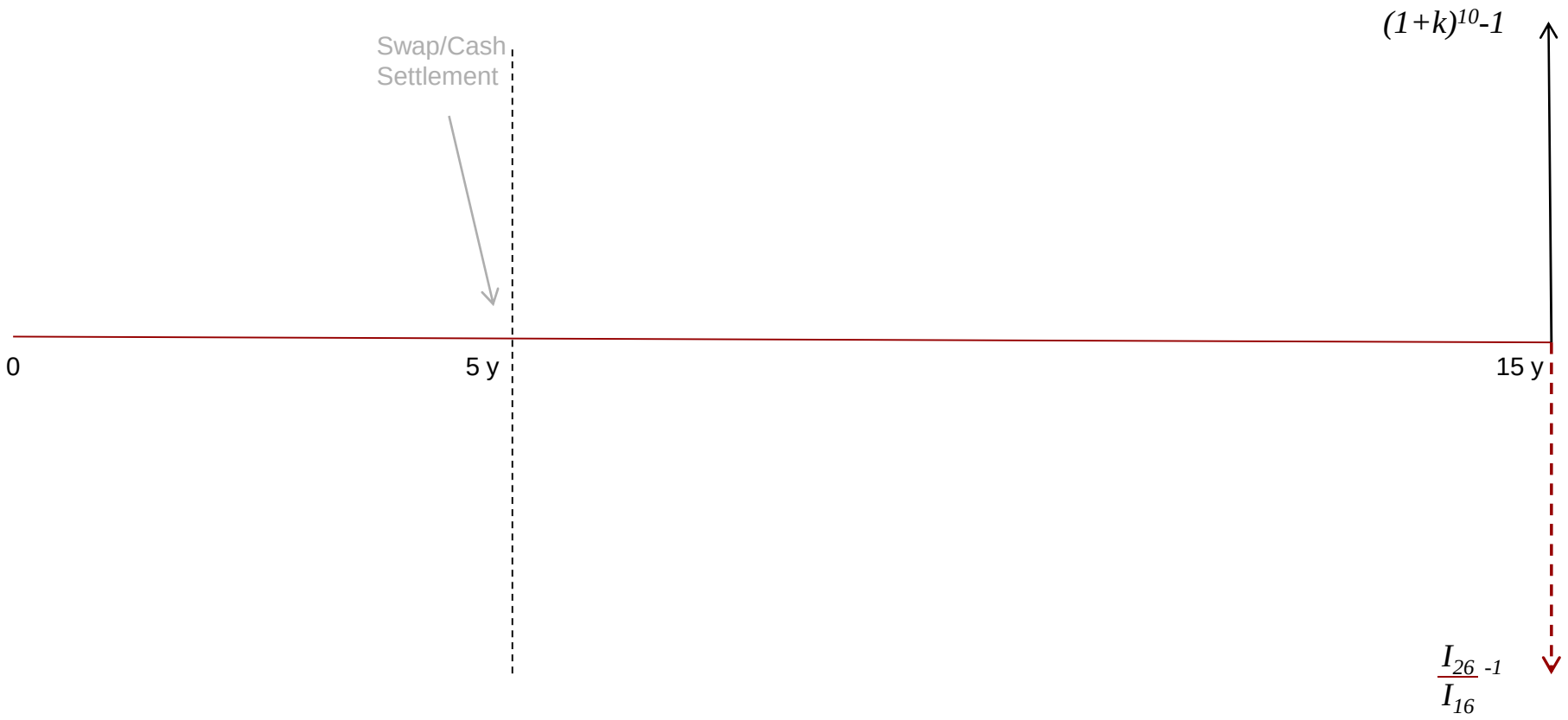
Inflation year-on-year swaption – pros and cons



What's your flavour? Inflation Swaption

Inflation zero coupon swaption

- This swaption gives the right to its owner to enter into a zero coupon inflation swap



What's your flavour? Inflation Swaption

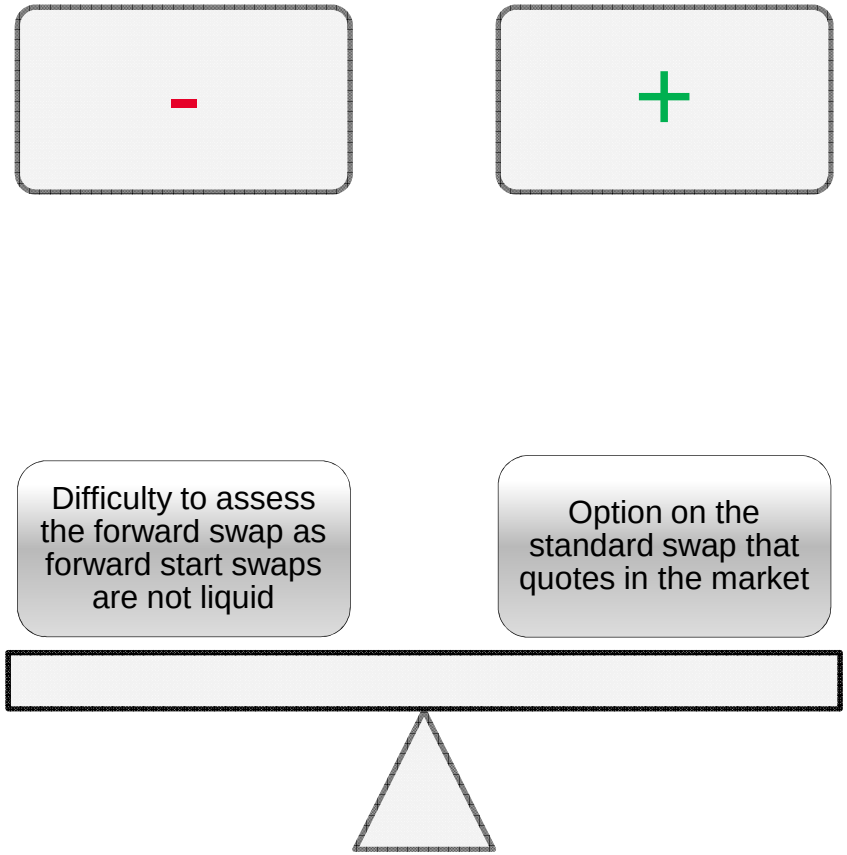
Inflation zero coupon swaption

- Term sheet

Notional:	€100,000,000
Index:	HICPxT (non revised)
Source:	First publication by Eurostat as shown on Bloomberg CPTFEMU
Start date:	1 May 2011
Option end date:	1 May 2016
Swap end date:	1 May 2026
Type:	Zero-coupon
Reference month:	February
First fixing:	not yet known.
Buyer:	The right to enter into a fixed payer inflation swap starting 1 May 2016 and ending 1 May 2026 with annual flows $(1+k)^{10} - \frac{HICPxT\,(Feb/26)}{HICPxT\,(Feb/16)}$, *where r denotes the inflation swap rate at 1 May 2016 for the February 2016 to February 2026 inflation swap.*
Seller:	upfront premium

What's your flavour? Inflation Swaption

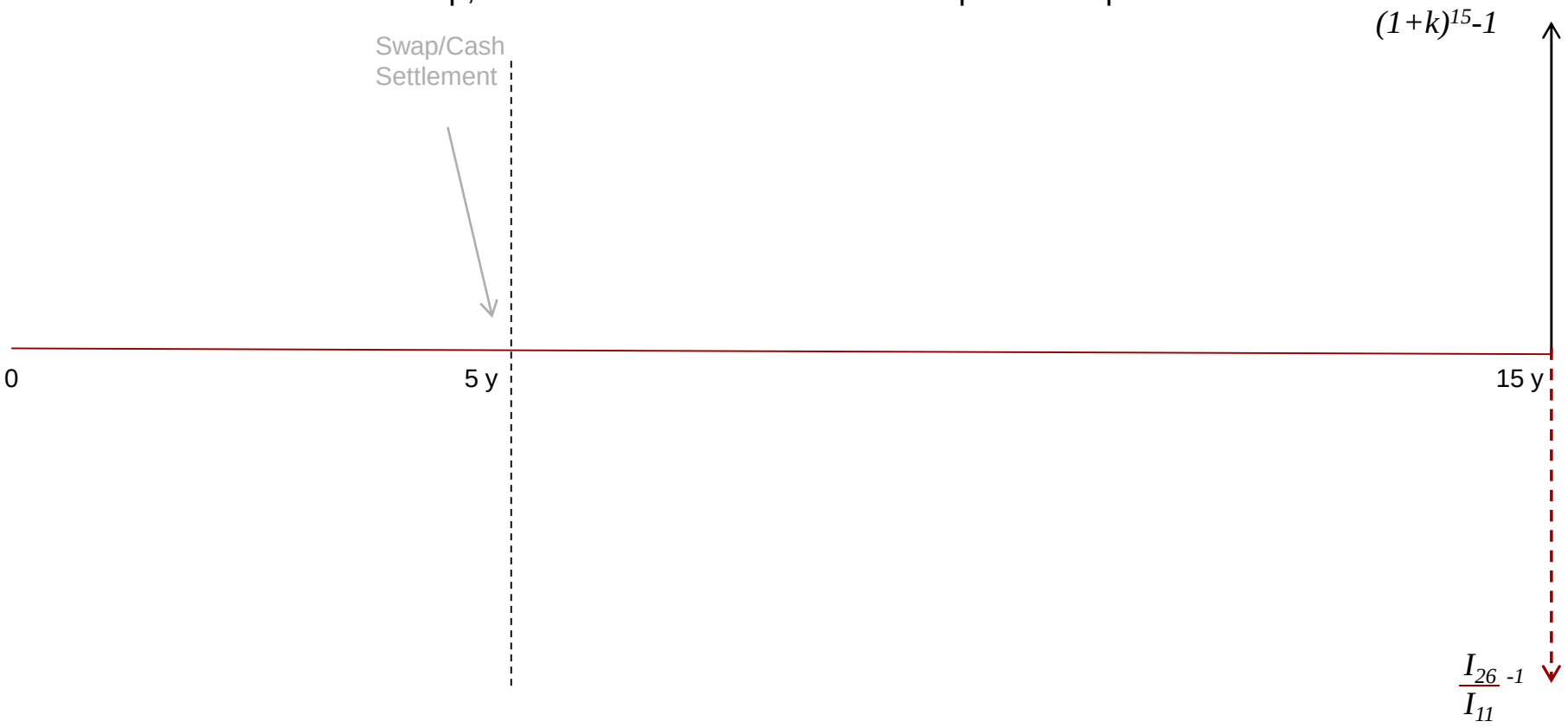
Inflation zero coupon swaption – pros and cons



What's your flavour? Inflation Swaption

Inflation Index zero coupon swaption

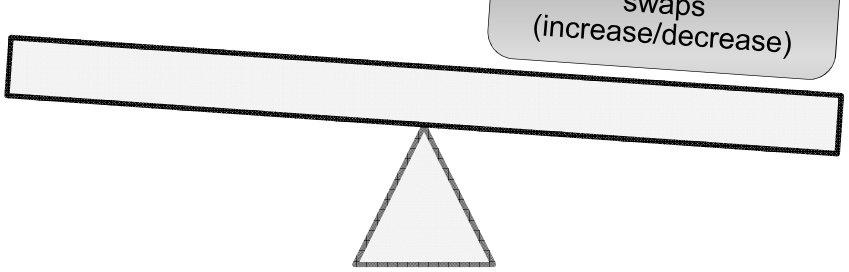
- This swaption gives the right to its owner to enter into a zero coupon inflation swap, but now it is not a forward start swap, because the start date of the swap is in the past



What's your flavour? Inflation Swaption

Inflation Index zero coupon swaption – pros and cons

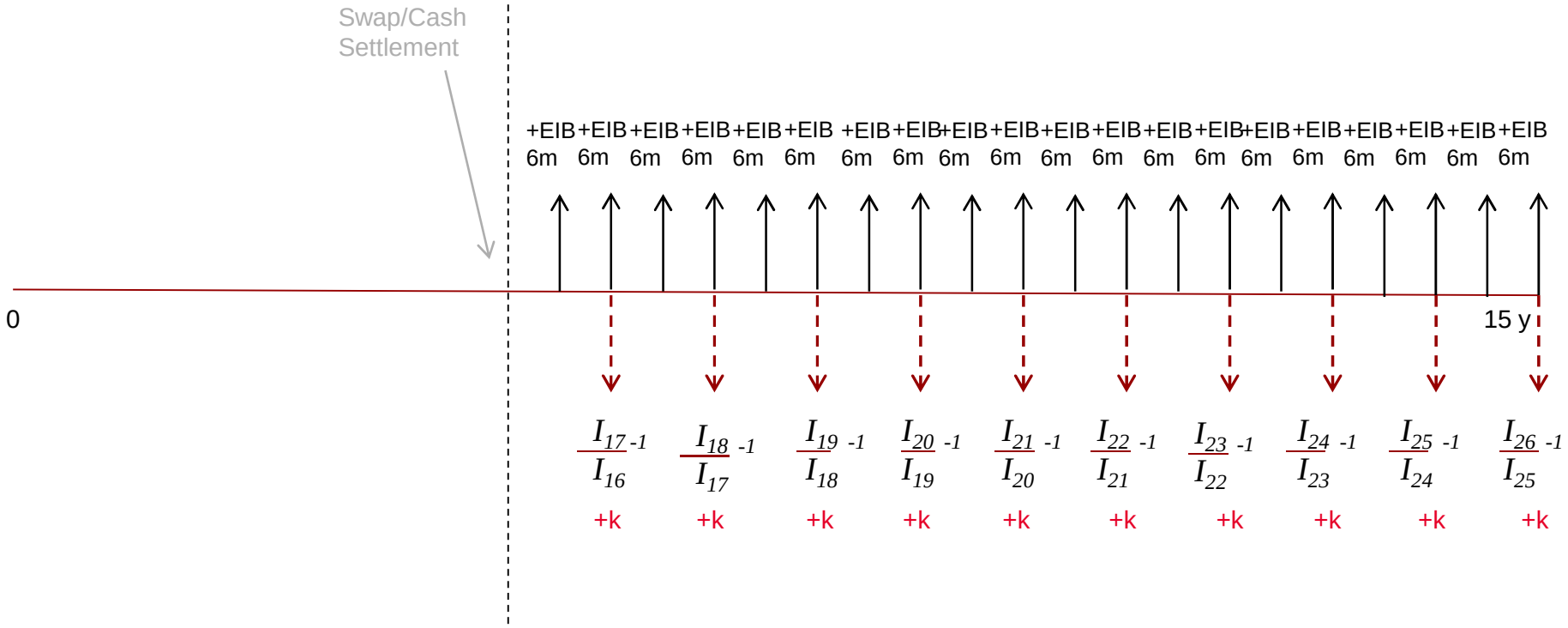
-	+
	Delta/Gamma hedge is straight forward using just one standard inflation swap
	Natural instrument for offering flexibility on zc swaps (increase/decrease)



What's your flavour? Real Rate Swaption

Real Rate year-on-year swaption

- This swaption gives the right to its owner to enter into a Real Rate year-on-year swap



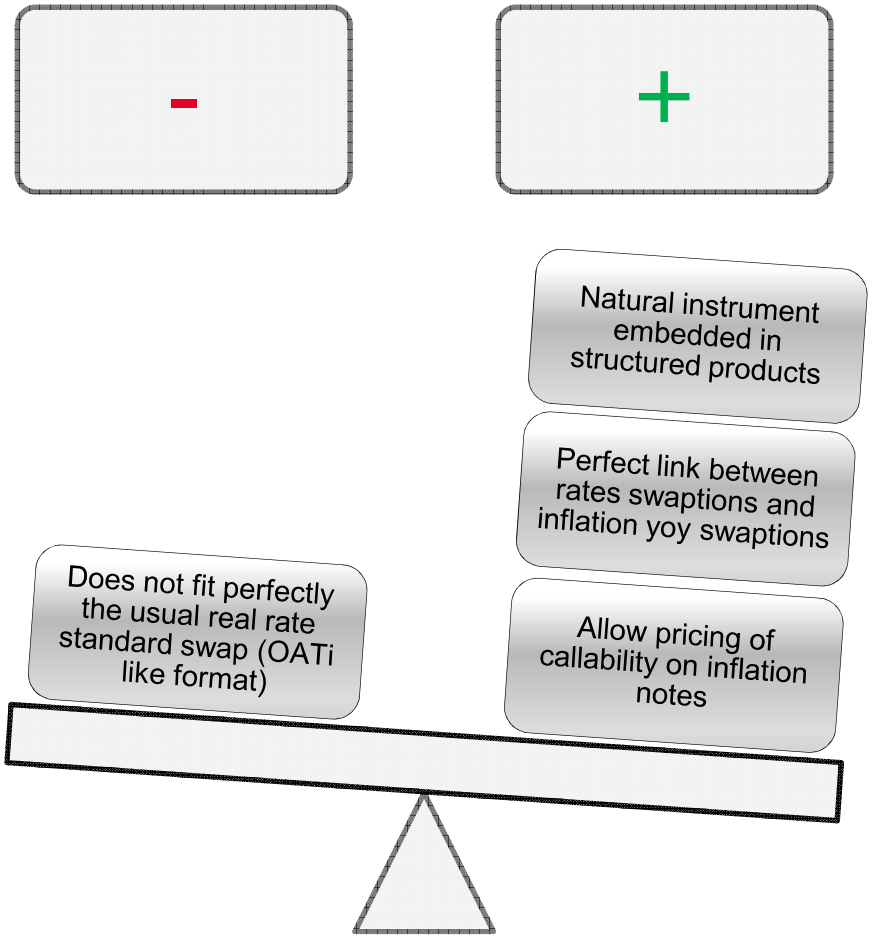
What's your flavour? Real Rate Swaption

Real Rate year-on-year swaption

Notional:	€100,000,000
Index:	HICPxT (non revised)
Source:	First publication by Eurostat as shown on Bloomberg CPTFEMU
Start date:	1 May 2011
Option end date:	1 May 2016
Swap end date:	1 May 2026
Type:	Real rate
Reference month:	February
First fixing:	not yet known.
Buyer:	The right to enter into a fixed payer inflation swap starting 1 May 2016 and ending 1 May 2026 with annual flows $Euribor\ 6m - \frac{HICPxT\ (Feb/16 + i)}{HICPxT\ (Feb/15 + i - 1)}$, *where r denotes the inflation real swap rate at 1 May 2016 for the February 2016 to February 2026 inflation swap.*
Seller:	upfront premium

What's your flavour? Real Rate Swaption

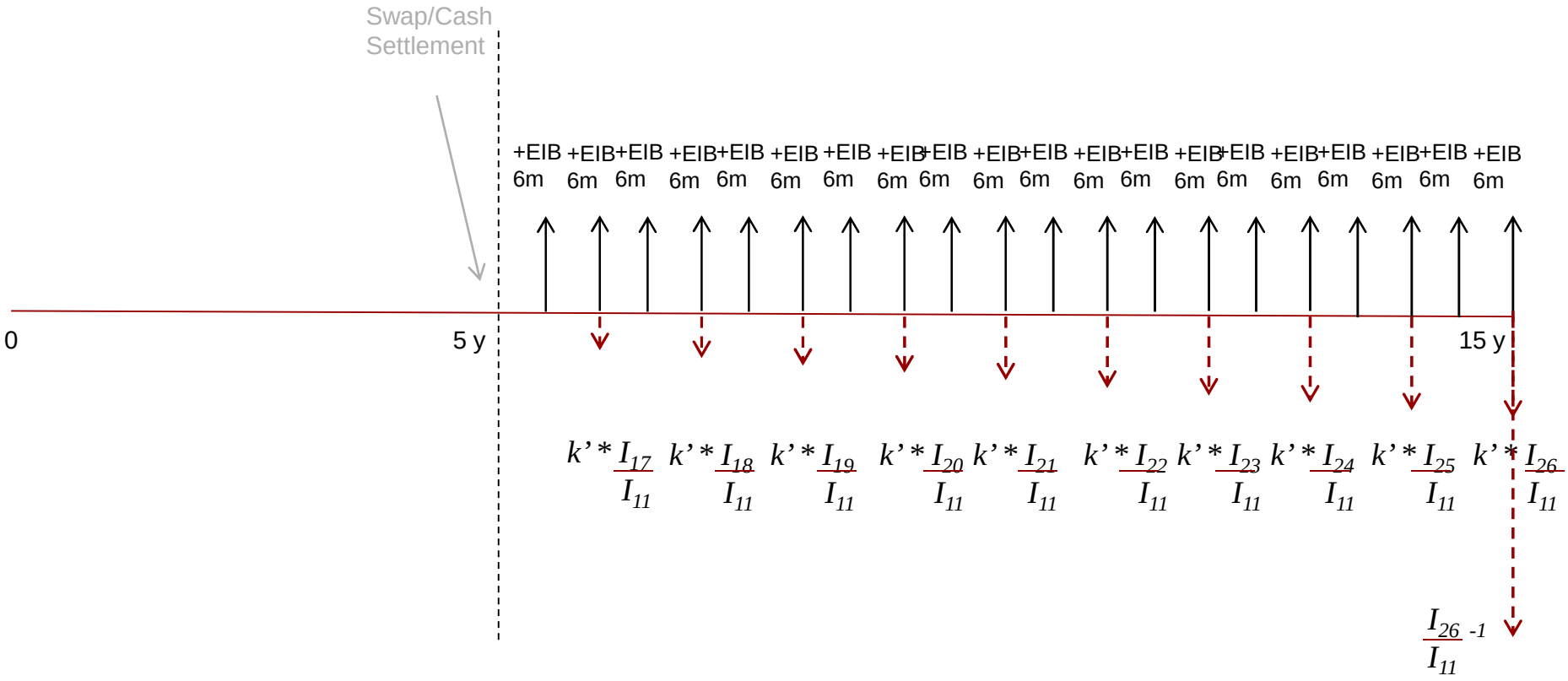
Real Rate year-on-year swaption



What's your flavour? Real Rate Swaption

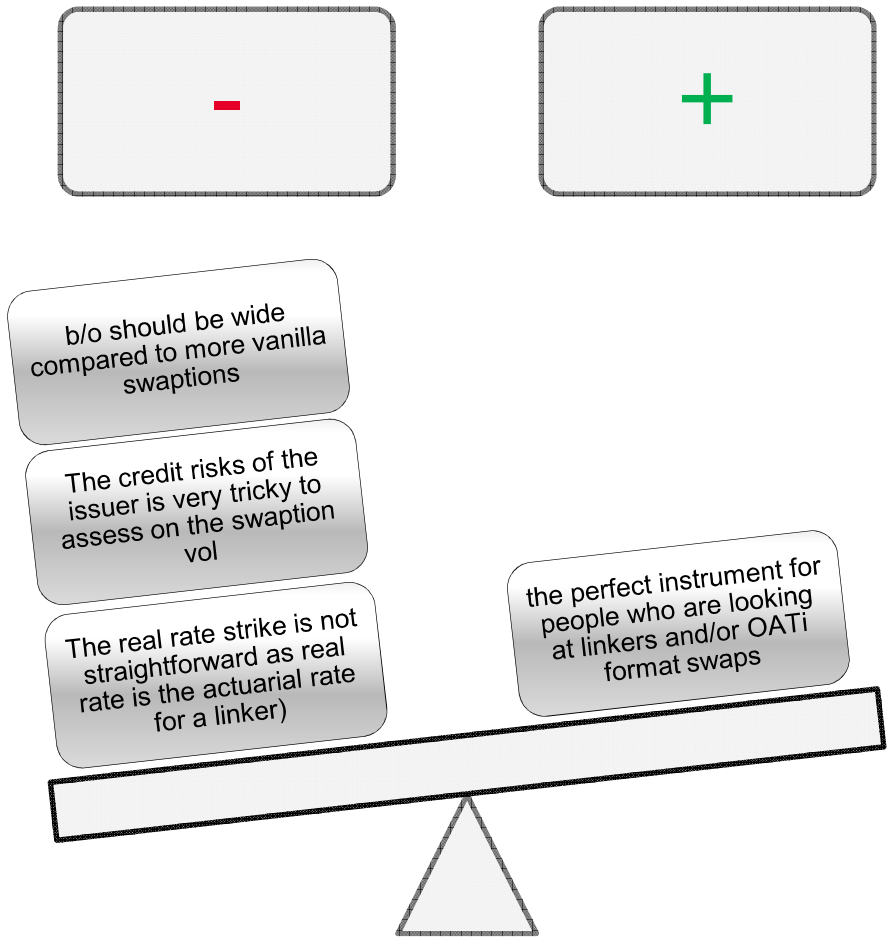
Real rate OATi format swaption

- This swaption gives the right to its owner to enter into a Real Rate OATi like swap



What's your flavour? Real Rate Swaption

Real rate OATi format swaption



Valuation and risk management

- Available typology of pricing
- Theoretical pricing in a pure swaption market
- Pricing with liquid instrument, maths versus flow
- Risk management

Available typology of pricing
The possible market evolution

- Characteristics
  - Expiries from 1M to 30y
  - Flavoured tenors: 1y to 5y , 10y, 30y
  - Quotes – cents (Be careful of CSA issue in non-synchron flows : ZC, Currency CSA must be taken into account too)

- Swaptions market is not liquid
  - Recovering the liquidity of natural instruments (Caps/Floors on inflation)
  - Nevertheless, necessity of challenging the projection with a pure swaption vision
  - Then Cap & Floor vol projection vs Swaption vol basis evolve with the natural flows (Wedge history on IRD)

Theoretical pricing in a pure swaption market

Basics for swaption pricing

- It is crucial to consider swaption as a pure instrument
- This approach will avoid any misleading

- Pure swaption approach can be done in 3 differents frameworks
  - Normal Filter for the Swap rate dynamic
  - Lognormal Filter for the Swap rate
  - Stochastic Volatility for the Swap rate

Theoretical pricing in a pure swaption market
The Normal Filter

- The swap rate is assumed to follow

$$\begin{aligned} dS_{T_0,\delta}(t) &= \sigma_n dW_t^{Q_{bpv}} \\ S_{T_0,\delta}(0) &= F \end{aligned}$$

- The price of the swaption is :

$$\pi_t = bpv(t)\sigma_n\sqrt{T}\left[\frac{S_{T_0,\delta}(0)-K}{\sigma_n\sqrt{T}}N\left(\frac{(S_{T_0,\delta}(0)-K)}{\sigma_n\sqrt{T}}\right)+\phi\left(\frac{S_{T_0,\delta}(0)-K}{\sigma_n\sqrt{T}}\right)\right]$$

With N the normal CDF and ϕ the normal density : $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$

For ATM we have

$$\pi_t = bpv(t)\sigma_n\frac{\sqrt{T}}{\sqrt{2\pi}}$$

Theoretical pricing in a pure swaption market

The Black Filter

- The swap rate is assumed to follow

$$\begin{aligned} dS_{T_0,\delta}(t) &= S_{T_0,\delta}(t)\sigma_B dW_t^{Q_{bpv}} \\ S_{T_0,\delta}(0) &= F \end{aligned}$$

- The price of the swaption is :

$$\begin{aligned} \pi_t &= bpv(t)\left[S_{T_0,\delta}(0)N(d_1) - KN(d_2)\right] \\ d_{1,2} &= \frac{\ln\left(\frac{S_{T_0,\delta}(0)}{K}\right) \pm \frac{v^2}{2}}{v} \end{aligned}$$

- For ATM we have :

$$\pi_t = bpv(t)S_{T_0,\delta}(0)\left[N\left(\frac{\sigma_B\sqrt{T}}{2}\right) - N\left(-\frac{\sigma_B\sqrt{T}}{2}\right)\right]$$

Theoretical pricing in a pure swaption market
The SABR Filter

- The swap rate is assumed to follow

$$\begin{aligned} dS_{T_0,\delta}(t) &= (S_{T_0,\delta}(t))^{\beta}\,\sigma(t)\,dW_{1t}^{Q_{bpv}} \\ d\sigma(t) &= \sigma(t)\,\nu dW_{2t} \\ S_{T_0,\delta}(0) &= F \\ \sigma(0) &= \sigma_0 \\ \left\langle dW_{1t}^{Q_{bpv}}, dW_{2t} \right\rangle &= \rho dt \end{aligned}$$

- The price of the swaption is :

$$\pi_t = bpv(t)\text{Black}\left(K, S_{T_0,\delta}(0), \sigma^{imp}\left(K, S_{T_0,\delta}(0)\right), T\right)$$

$$\sigma^{imp}(K,F) = \frac{\alpha}{FK^{\frac{1-\beta}{2}}\left[1+\frac{(1-\beta)^2}{24}\ln^2\left(\frac{F}{K}\right)+\frac{(1-\beta)^4}{1920}\ln^4\left(\frac{F}{K}\right)\right]}\frac{z}{x(z)}\left\{1+\left[\frac{(1-\beta)^2\alpha^2}{24(FK)^{1-\beta}}+\frac{\rho\beta\nu\alpha}{4(FK)^{\frac{1-\beta}{2}}}+\nu^2\frac{2-3\rho^2}{24}\right]T_x\right\}$$

where

$$z := \frac{\nu}{\alpha}(FK)^{\frac{1-\beta}{2}}\ln\left(\frac{F}{K}\right)$$

and

$$x(z) := \ln\left\{\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right\}$$

Pricing with liquid instruments, maths vs flows

Our Pricing Philosophy for Exotics

- For benchmarking exotics, the more intuitive approach is to look at a model that has an intuitive hedge with a vanilla plain basic:
- Price of a derivative is the price of its hedge
- We must uncharge models from explaining a product, thinking price means thinking hedge, so costs
- We are looking for models that have the ability to measure properly exotic risks, leaving boundable and measurable exotics residual risks.
- YoY Swap rate is seen as a basket on YoY
- Projection is done on Cap/Floor smile + Correlation risk
- The smiling process is based on an intuitive replication argument

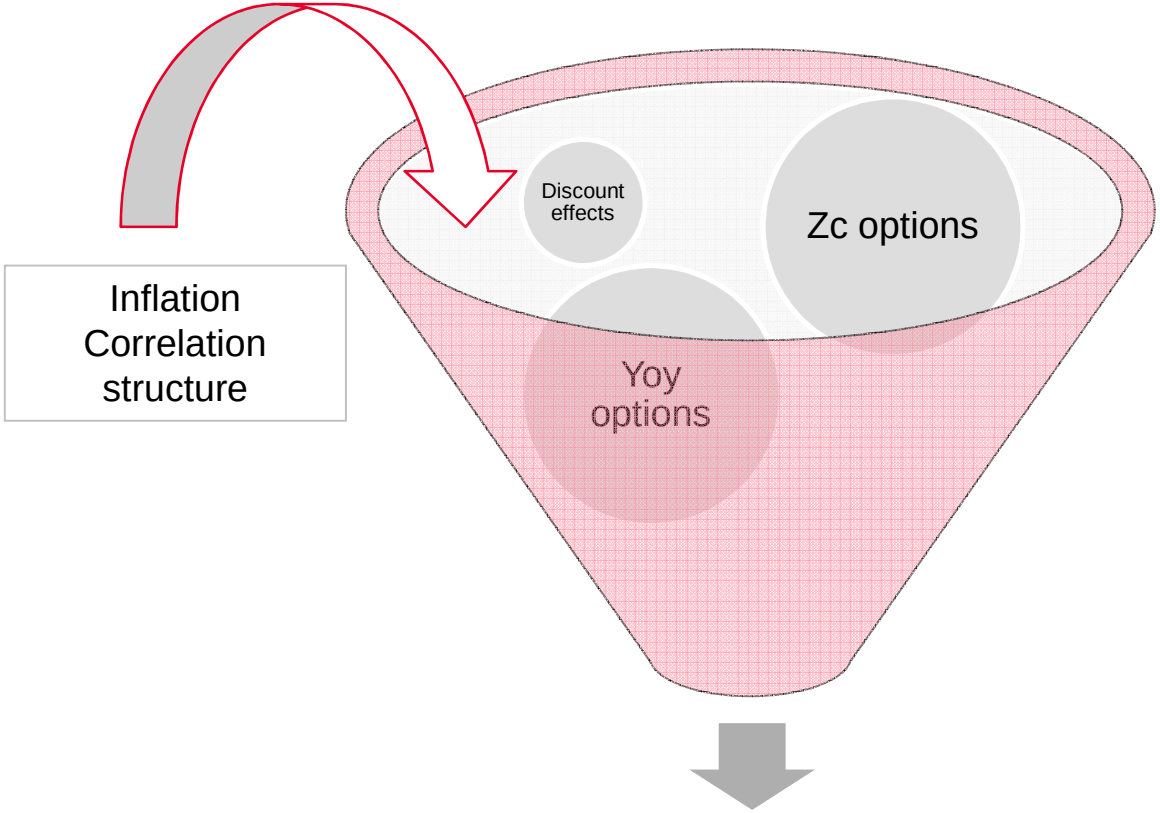
Pricing with liquid instruments, maths vs flows

Our Pricing Philosophy for Exotics

- Components of this framework :
  1. Yoy smile ➔ available by smiling methodology
  2. Convexified Law ➔ Convexity on each Yoy (Sensi Vol ATM Euribor + Yoy + Correlation)
  3. Autocorrelation structure ➔ Flexible structure is needed
  4. Pricing & hedging procedure are then mixed up.

Pricing with liquid instruments, maths vs flows

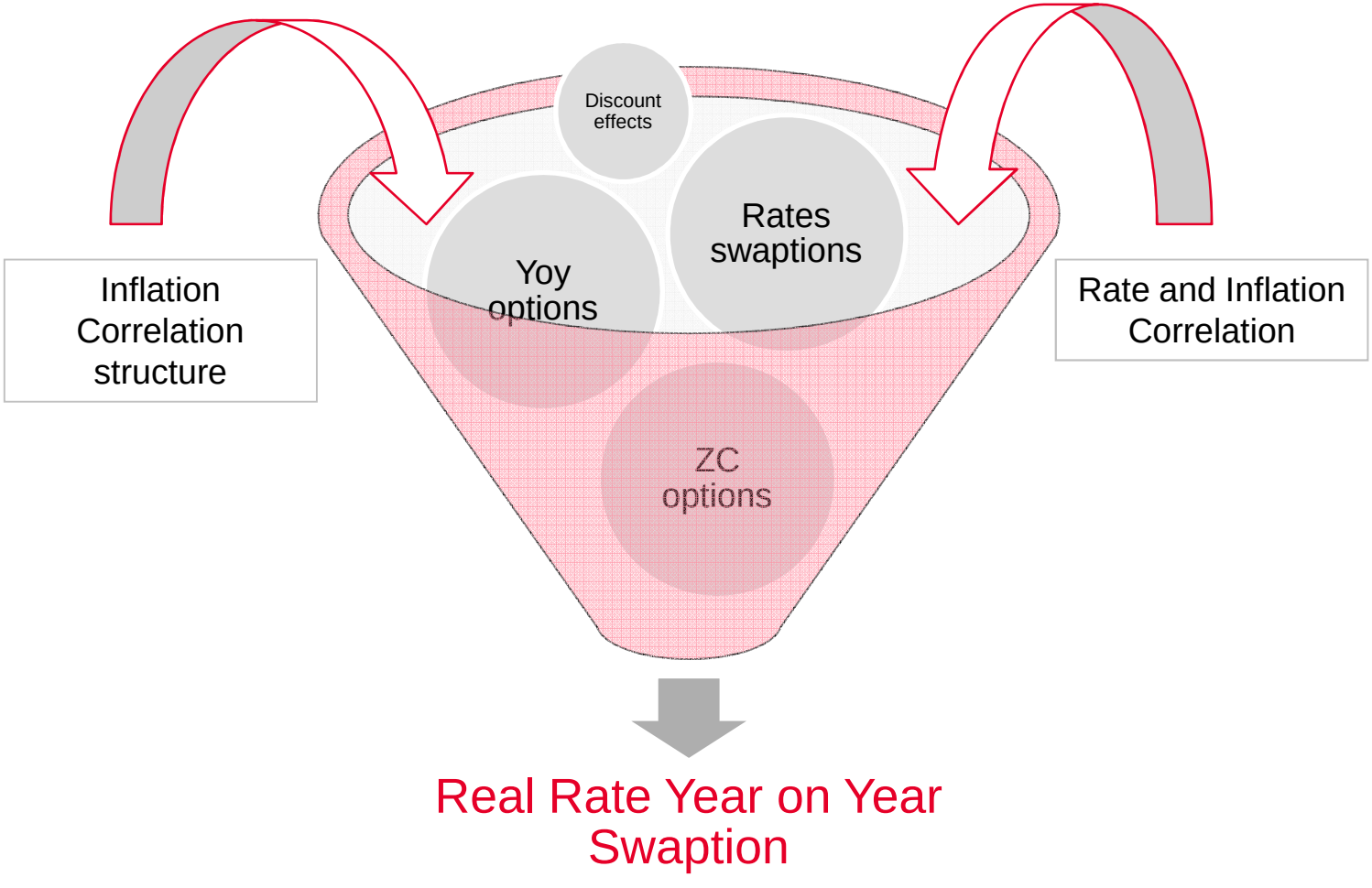
Pricing of Inflation year-on-year swaption



Inflation Year on Year Swaption

Pricing with liquid instruments, maths vs flows

Pricing of Real Rate year-on-year swaption



Pricing with liquid instruments, maths vs flows

Wedge is a correlation trade/exposure

- Wedge is a correlation trade/exposure: Cap/floor straddle vol vs corresponding swaption vol (2x4 vs 2y2y , 5x10 vs 5y5y)
- A proxy for the implied correlation of short forward rates
  - Cap = portfolio of caplets, ie portfolio of options on short forward rates: The change in short rates correlation has no impact on the price of a cap
  - Swaption = an option on a portfolio of short forward rates: correlation of short forward rates matters
  - An increase in correlation will increase the swaption vol with respect to cap/floor vol and vice versa
- Wedges are very sensitive to market flows as it can be describe as a pure relative value trade

Pricing with liquid instruments, maths vs flows

Analogy to the yoy and zero coupon option market

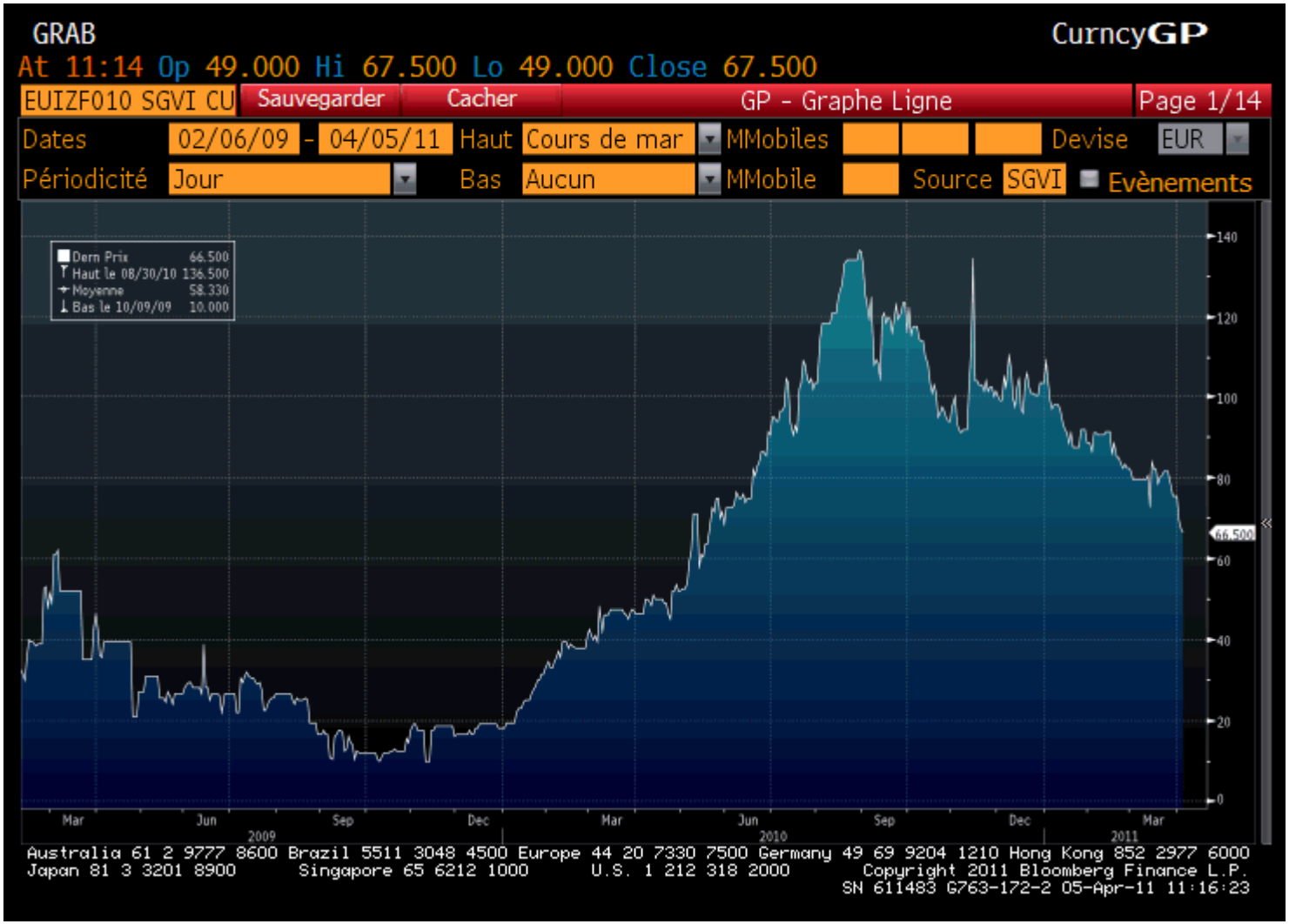
- Inflation volatility market started with year on year options
- Flows were only on structured products (yoy format)
- Players were implying zero coupon volatility thanks to models
- In 2007 embedded floors on linkers asset swaps started to be quoted
- Dynamic on both markets was really similar especially during 2008 crisis
- With massive buying flows on zero coupon options in 2010 (Insurance company on 10y 0% floor zc), joint dynamic is changing
- Then a proper market on each kind of option is emerging
- But it is always important to keep in mind the implied correlation parameters between the two markets

Pricing with liquid instruments, maths vs flows

Analogy to the yoy and zero coupon option market

Pricing with liquid instruments, maths vs flows

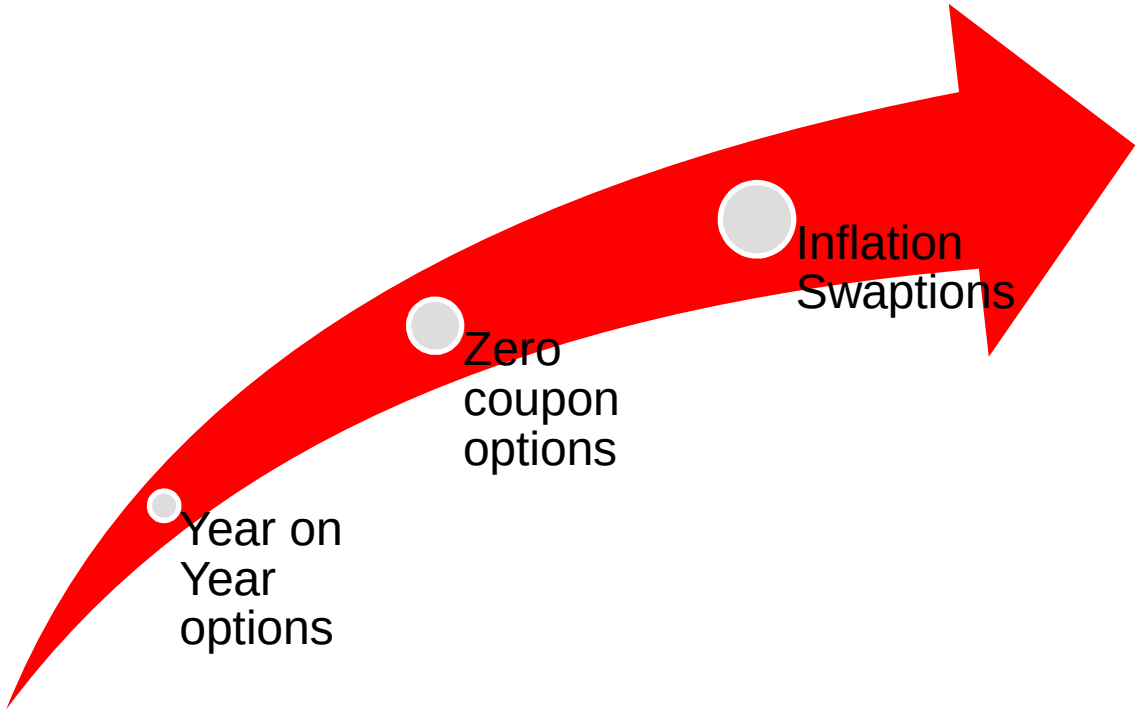
Analogy to the yoy and zero coupon option market



Pricing with liquid instruments, maths vs flows

Analogy to the yoy and zero coupon option market

- Then for swaptions, this representation is just useful for a start point to develop the market
- It is important to not be stuck with this representation as the flows on the underlying could generate a basis between the Cap/Floor smile and pure swaption smile
- As conclusion Maths could stay behind flows, the rule is done by offer and demand in illiquid market.



Risk Management
Toolbox for Risk management

For a chosen model greeks are given

- **Delta**

- Strike influence :Delta in absolute value is increasing is this sens : OTM ➔ ITM
- Line Delta (tenor) Line Delta is moving proportionally to the sensi of the swap (Line delta, with respect to the underlying maturity)
- Column Delta same than standard underlying as equity

- **Vega**

- Vol is stored in a cube (Maturity, Tenor , Strike)
- Vega is an increasing function of the vol
- Strike influence : Maximal ATM of the underlying
- Line Vega is moving proportionally to the sensi of the swap
- Colum Vega as a belly shape with the maturity (sensi effect, the variance is not compensated by the discounting effect of the Zero-coupon, so for a given maturity vega is decreasing)

- **Gamma**

- Strike influence : Maximal ATM as classical results
- Line Gamma is moving proportionally to the sensi of the swap
- Column Gamma same than standard underlying : decreasing with maturity

Where do we stand, what is the next step?

- No interbank market on swaptions today
- Need a daily fixing
- New liquidity for more exotic structure

No interbank market on swaptions today

- Brokers are developing platform for quoting swaptions
- Players have to quote it on an interbank base in order to generate a market
- The transparency of this market is the key word for a good development

Need a daily fixing

- Proposed format
  - Which contributors: Banks are selected by ISDAFIX for interest rate swaps (IRS)
  - Prices quoted: where each dealer would quote the mid-market inflation swaps
  - Rates reported are made public
- Year-on-year versus zero-coupon rate

  Year-on-year inflation swaps could easily be combined with IRS rates, but they are less liquid than zero-coupons
- Interpolated versus monthly inflation indices

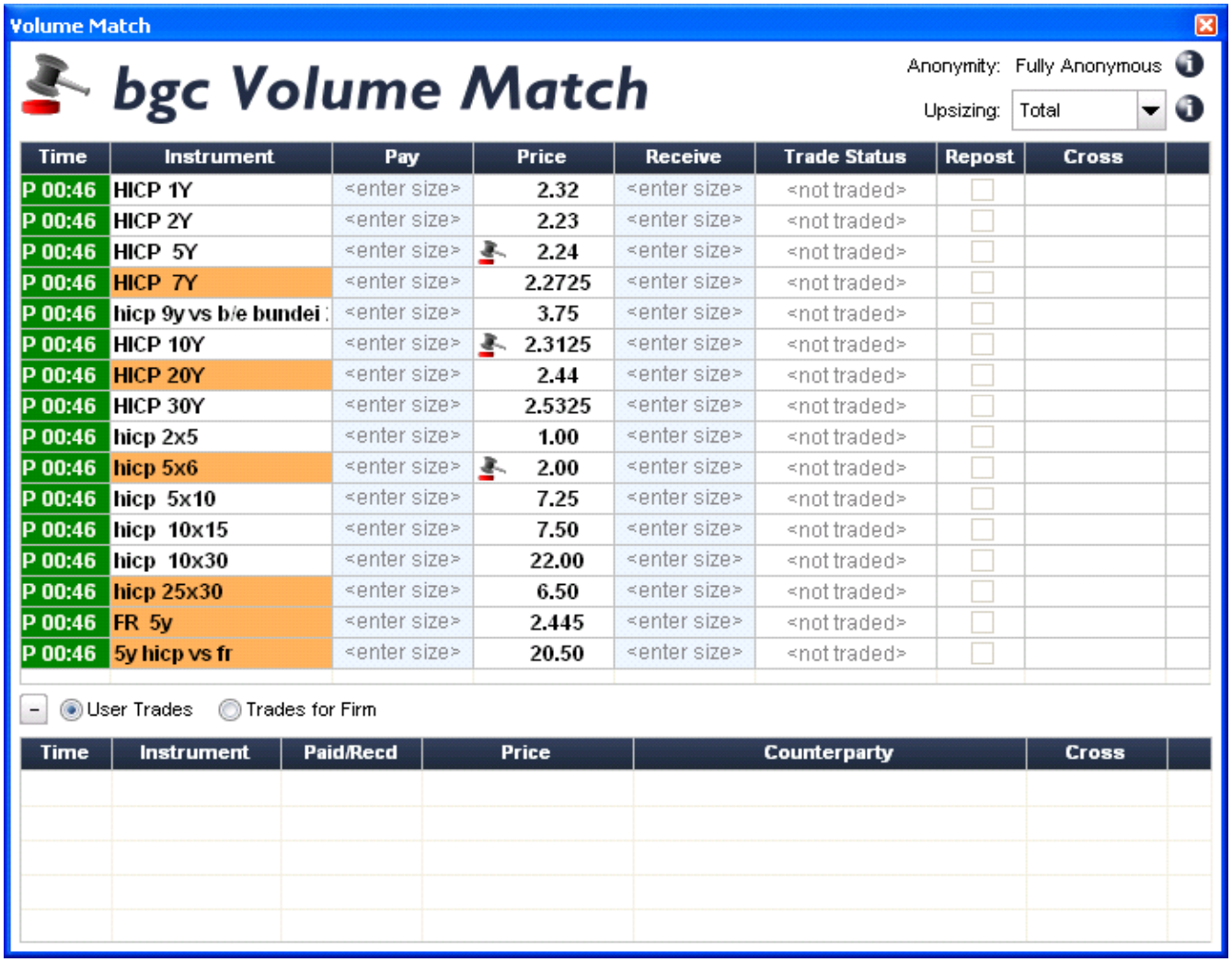
  Different market conventions: French and US inflation swaps trade on an interpolated basis while euro zone and UK trade on a monthly fixing
- Real versus inflation rate

  Real rates can be computed directly from the IRS and zero-coupon inflation swap fixing
- What maturities should be contributed

  Specified by each panel contributors based on which maturities are relevant to their market

Need a daily fixing

- Trading platform can provide daily mid market level



Need a daily fixing

- Short-term benefits
  - Attract more participants to the inflation derivatives market
  - Increase volumes and liquidity of zero-coupon inflation swaps
  - More comfort for investors holding inflation derivatives
  - Use of the fixing for swap terminations and cash-settled options
  - Best execution to clients thanks to benchmarking the price of inflation derivative transactions

- Long-term benefits
  - Proper setting for exotic products
  - Increase client demand for exotic products
  - 5/10 year fixing could be used as the underlying benchmark for new cash-settled futures contracts
  - More exposure to inflation expectations for users in long-term inflation futures contracts and tighter bid/ask spreads

New liquidity for more exotic structure

- Pricing callability easily
- Offering flexibility on zero coupon swaps
- Creation of a new range of products: CMS on inflation