

## Valuing convexity in the long end of the yield curve

### A global perspective

- We analyze the importance of convexity on long-dated fixed-income instruments and propose a fair value framework
- We compare expected return across the curve including contributions from carry, slide, and convexity. As expected, convexity is a larger contributor to the overall P&L at the ultra-long end versus shorter maturities
- Duration-neutral flatteners at the ultra-long end exhibit an option-like positive payoff (limited downside risk in a sell-off versus large upside in a rally) under parallel movements of the curve
- In the US, our framework suggests that the yield curve is frequently the cheaper source of gamma, particularly in a risk-off event; convexity earnings drive the majority of P/L for USD 30s/50s flatteners
- In EGBs, despite upward sloping yield curves, the expected returns including value of convexity are lower for 30Y+ bonds compared to 30Y ones; adjusting for expected vol does not change the results
- EGB ultra-long bonds also suffer from market segmentation and lower liquidity. However a doubling of volatility would flip the story, making ultra-long bonds very attractive
- In EUR swaps, convexity becomes an equal contributor of P&L in the 50Y sector to carry & slide. Our analysis shows that 30Y vol has to drop sharply to equate expected returns in the 30Y and 50Y sectors
- In gilts, convexity is a dominant contributor to the overall P&L at the ultra-long end of the curve. Convexity on the swap curve is close to its historical lows
- On a cross-market basis, with respect to convexity, we find ultra-long-dated instruments cheap in USD and GBP across a majority of swap structures, mixed bag in EUR, and generally expensive in JPY

---

#### Global Rates Strategy

##### Gianluca Salford<sup>AC</sup>

(44-20) 7134-1692  
gianluca.salford@jpmorgan.com  
J.P. Morgan Securities plc

##### Joshua Younger<sup>AC</sup>

(1-212) 270-1323  
joshua.d.younger@jpmorgan.com  
J.P. Morgan Securities LLC

##### Munier Salem

(1-212) 270-0317  
munier.salem@jpmorgan.com  
J.P. Morgan Securities LLC

##### Khagendra Gupta

(44-20) 7134-0486  
khagendra.x.gupta@jpmorgan.com  
J.P. Morgan Securities plc

##### Jay Barry

(1-212) 834-4951  
john.f.barry@jpmorgan.com  
J.P. Morgan Securities LLC

##### Francis Diamond

(44-20) 7134-1504  
francis.diamond@jpmorgan.com  
J.P. Morgan Securities plc

##### Aditya Chordia

(44-20) 7134-2132  
aditya.x.chordia@jpmorgan.com  
J.P. Morgan Securities plc

##### Zhan Zhao

(1-212) 834-7218  
zhan.zhao@jpmorgan.com  
J.P. Morgan Securities LLC

---

#### See page 16 for analyst certification and important disclosures.

J.P. Morgan does and seeks to do business with companies covered in its research reports. As a result, investors should be aware that the firm may have a conflict of interest that could affect the objectivity of this report. Investors should consider this report as only a single factor in making their investment decision.

## Valuing convexity: A global perspective

- 
- We analyze the importance of convexity on long-dated fixed-income instruments and propose a fair value framework
  - We compare expected return across the curve including contributions from carry, slide, and convexity. As expected, convexity is a larger contributor to the overall P&L at the ultra-long end versus shorter maturities
  - Duration-neutral flatteners at the ultra-long end exhibit an option-like positive payoff (limited downside risk in a sell-off versus large upside in a rally) under parallel movements of the curve
  - In the US, our framework suggests that the yield curve is frequently the cheaper source of gamma, particularly in a risk-off event; convexity earnings drive the majority of P/L for USD 30s/50s flatteners
  - In EGBs, despite upward sloping yield curves, the expected returns including value of convexity are lower for 30Y+ bonds compared to 30Y ones; adjusting for expected vol does not change the results
  - EGB ultra-long bonds also suffer from market segmentation and lower liquidity. However a doubling of volatility would flip the story, making ultra-long bonds very attractive
  - In EUR swaps, convexity becomes an equal contributor of P&L in the 50Y sector to carry & slide. Our analysis shows that 30Y vol has to drop sharply to equate expected returns in the 30Y and 50Y sectors
  - In gilts, convexity is a dominant contributor to the overall P&L at the ultra-long end of the curve. Convexity on the swap curve is close to its historical lows
  - On a cross-market basis, with respect to convexity, we find ultra-long-dated instruments cheap in USD and GBP across a majority of swap structures, mixed bag in EUR, and generally expensive in JPY
- 

In this note, we analyze the importance of convexity on long-dated fixed-income instruments across cash and derivatives, and propose a fair value framework. We then introduce the concept of breakeven vol and revisit the framework to compare ultra-long flatteners with long vol trades. Finally, we focus on each market separately. We find ultra-long-dated instruments cheap in USD and GBP across a majority of swap structures, mixed bag in EUR, and generally expensive in JPY.

### Thinking about 'convexity value'

Investors who typically take an agnostic view on future market moves focus on 1) carry and 2) slide, broadly speaking 1) the difference between an instrument's coupon income and its cost of funding, and 2) the impact of the slide or roll of an instrument down an unchanged curve as time passes.<sup>1</sup> This framework is not entirely

---

<sup>1</sup> Clearly this assumption is not very realistic for short-dated instruments when a central bank is active, but we believe it is generally a good starting point for the ultra-long end of the curve under most scenarios.

appropriate for long-dated instruments given that they show a more marked non-linear relationship between price and yield. Hence, there is a potentially important role for 3) value of convexity.

For some instantaneous move (“shock”) in yields,  $\Delta Y$ , we can approximate the change in a bond’s price using dollar duration (PVBP) and convexity ( $\gamma$ ):

$$\Delta P = -\Delta Y * PVBP + 0.5 (\Delta Y)^2 \gamma$$

An agnostic view on the future (e.g. one where risks towards rising and falling yields are roughly balanced and the curve, in expectation, is unchanged) for an instrument with maturity  $M$  over horizon  $T$  for an initial yield  $Y$ , cost of funding  $F$  and expected volatility  $\sigma$  would translate into an expected return:

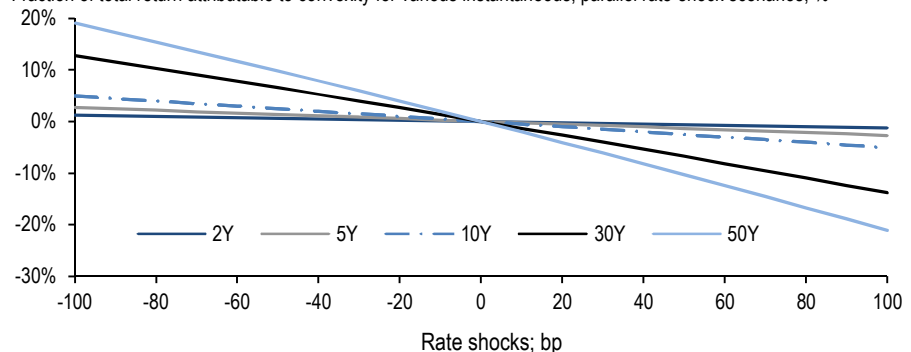
$$E(R) = \begin{array}{ll} (Y - F)T/PVBP & [carry] \\ + Y_M - Y_{M-T} & [slide] \\ + 0.5\gamma\sigma^2T/PVBP & [cvx \text{ value}] \end{array}$$

In other words, even if we make an assumption about an unchanged yield curve we incorporate expectations about volatility around this baseline scenario. The “value of convexity” is a function of 1) an instrument’s convexity (itself strongly correlated with the maturity of the instrument) and 2) expectations about future volatility of the instrument’s price or yield: the higher the expected volatility the higher the value of convexity.

These effects become a more important consideration for longer maturities, which generally have more exposure. This arises from longer-dated cashflows, the present value of which is intrinsically more volatile under changes in discount factors, leading to more variable dollar duration. We can see this by considering the P/L for various receive-fixed USD fixed/float swap positions under rate shocks. By the time rates have moved 100+ bp, more than 20% of the P/L for 50-year maturities comes from these non-linear effects, in contrast to less than 2% for 2-year (**Exhibit 1**).

**Exhibit 1: Convexity, meaning non-linear contributions to P/L, is a more important consideration in longer maturities, particularly under large moves in rates**

Fraction of total return attributable to convexity for various instantaneous, parallel rate-shock scenarios; %



Source: J.P. Morgan

The framework described above allows us to compare long-dated instruments with different characteristics. 1) In its simplest form, one could choose instruments based on the highest expected return adjusting for their different duration. 2) Alternatively, an argument can be made for adjusting the expected return also for expected

volatility, in an extension of the risk-adjusted-carry framework. In practice, realized and implied volatilities show relatively modest variability in long maturities (**Exhibit 2**), so disregarding the volatility adjustment does not have material implications.

#### Exhibit 2: Realized and implied volatilities have shown relatively modest variability in long maturities

Carry, slide, value of convexity, 3M realized bp vol and risk adjusted carry and slide for 30Y+ French bonds;

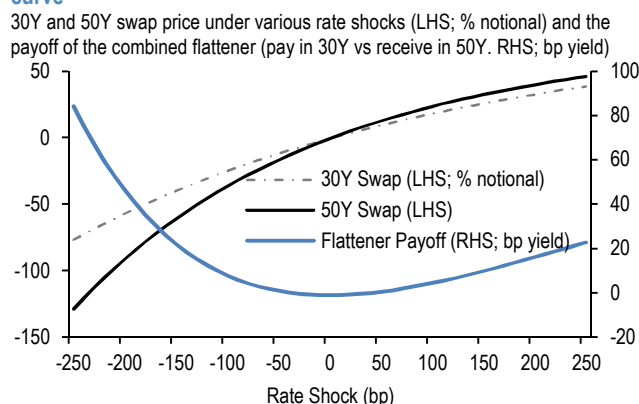
OAT maturity	Carry	Slide	Convexity	Expected return	3M realised vol.	RAC
25-May-48	10.6	2.0	3.3	15.9	3.0	0.33
25-Apr-55	10.7	1.0	3.6	15.4	3.0	0.32
25-Apr-60	9.9	0.2	4.1	14.2	3.0	0.30
25-May-66	7.9	-1.0	4.9	11.8	3.0	0.25

Source: J.P. Morgan

### An options-based framework for pricing convexity

**In pricing the convexity value of a particular duration-weighted curve position, we prefer to think in terms of expected payoff under various rate shocks.** For example, consider a 30s/50s flattener in two high-quality sovereign bonds (i.e., ignoring credit risk). At inception, by construction, we're exposed to an equal dollar-duration in both the long (50Y) and short (30Y) bond positions. Thus under a small parallel shift in the curve, our total price return should be roughly zero. But as that parallel shift in size grows, the two bonds behave differently. The longer leg has approximately 20 years of additional cashflows that contract in PV terms more dramatically in a sell-off. Thus, in a parallel sell-off the dollar-duration of the longer tenor contracts more rapidly than that of the shorter tenor, and its price falls at a slower rate, leaving the trade with a positive P/L. In a rally, the opposite occurs: the long-dated cashflows in the 50Y leg rise in PV more rapidly, duration on that bond extends more rapidly, and its price rises faster than the short leg, with the structure again enjoying a positive P/L. **In other words, duration-neutral flatteners with fixed hedge ratios will have positive P/L if the curve shifts in parallel in either direction. The resulting P/L diagram looks quite a bit like a straddle (Exhibit 3).**

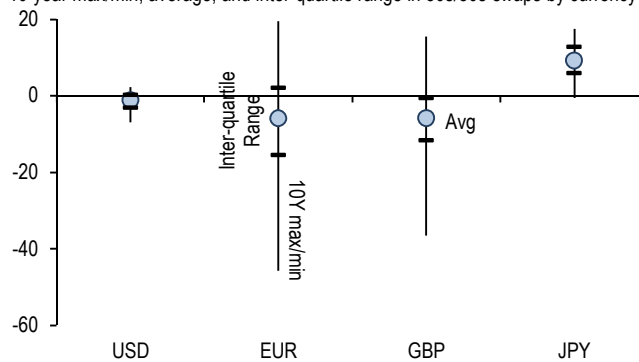
#### Exhibit 3: Duration-neutral flatteners at the ultra-long end will have an straddle-like positive payoff under a parallel movement of the curve



Note: Rate shocks are a parallel shift in the curve.  
Source: J.P. Morgan

#### Exhibit 4: Because 30s/50s has a positively convex payoff, it typically trades inverted across most currencies

10-year max/min, average, and inter-quartile range in 30s/50s swaps by currency



Source: J.P. Morgan

Of course, there is no free lunch. In principle, one should have to pay for this positive convexity with negative carry/roll. **This biases the 30s/50s yield curve flatter, or often inverted, whereby one pays the higher rate in a flattener with positive convexity.** And in fact we find this is generally true across currencies for the vast majority of the period for which we have reliable data (**Exhibit 4**).

**The question then becomes one of comparison.** Focusing on the other side of the trade (i.e., a steepener), given the range and likelihood of different outcomes for moves in rates, **we need to determine whether a short position in 50s versus 30s compensates us appropriately via carry for taking on the risk of losses if rates move sufficiently far in either direction.** For this we need an estimate of this distribution of potential outcomes, from which we can derive an expected value for the convexity payoff for comparison to the *ex-ante* carry on the trade.

**For this we turn to the options market** (see also discussion in [An option by any other name](#), J. Younger et al., 2/3/17). This is convenient in that the level of implied volatility—and secondarily the slope of the skew—provide a very straightforward gauge of the market's 'view' of the likelihood of various outcomes. Such a framework is also attractive because it gives us a sense of relative value. **In other words, we can determine whether options or ultra-long dated flatteners are a cheaper source of gamma.** And in principle, we can explicitly monetize any divergence between the two by trading one instrument versus the other—e.g., initiating a flattener in 30s/50s swaps and selling ATMF straddles in 30-year tails against it.

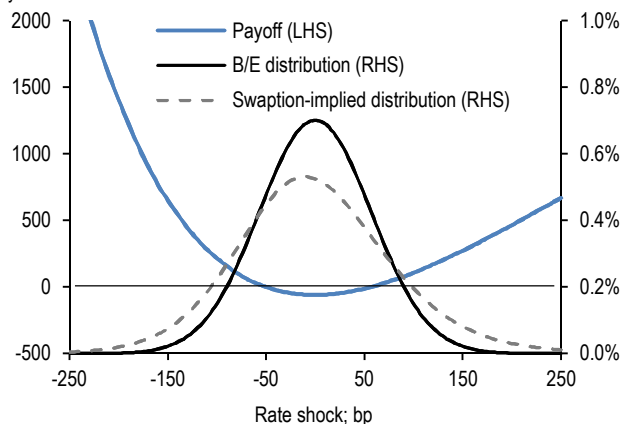
**Specifically, we extract an implied distribution from ATMF and OTM pricing for each expiry.** This is then multiplied with the payoff profile of an aged flattener at fixed coupon—primarily to incorporate carry costs—to estimate an expected return. **Since, as discussed above, these trades tend to be located in longer-dated swaps, we use 1Yx30Y swaptions for a 1-year horizon and assume parallel shifts in rates.**

**Alternatively, we can solve for the implied volatility priced into the long end of the yield curve.** This can be accomplished by making a specific assumption for the terminal distribution of rates—in our case a normal distribution, again with parallel shifts in the curve—and solving for zero expected payoff over a given horizon. In other words, we are estimating the level of normal daily volatility in rates that is sufficient to offset the carry costs on a given curve trade.

**Examples of both distributions as well as the payoff profile of a 30s/50s flattener over a 1-year horizon are shown in Exhibit 5.** Both measures can be interpreted as a relative value signal for curve convexity trades versus swaptions. When the expected payoff on a flattener using an implied distribution extracted from swaption pricing is positive, the curve trade is the cheaper source of long gamma exposure. The same can also be said when the level of volatility priced into the curve is less than that implied by ATMF swaptions. **Looking historically, we can get a sense of whether the long end of the yield curve has typically been cheaper or richer as a source of long gamma exposure relative to swaptions, and in fact find that in USD this is frequently the case (Exhibit 6).**

**Exhibit 5: The width of the breakeven distribution of terminal rates on long-end flatteners over a given horizon can differ—and is frequently narrower—than that implied by co-expiry swaptions**

Payoff profile\* (LHS; bp), breakeven implied distribution† (RHS; bp of notional), and swaption-implied distribution\*\* (RHS; unitless) for a 30s/50s flattener held over a 1-year horizon



\* Net P/L for a spot 30s/50s flattener under parallel shifts in rate, with coupons equal to the 1-year forward rates.

† A normal distribution of terminal rate shifts centered on zero with standard deviation fit to produce a probability-weighted payoff of zero assuming parallel shifts in rates.

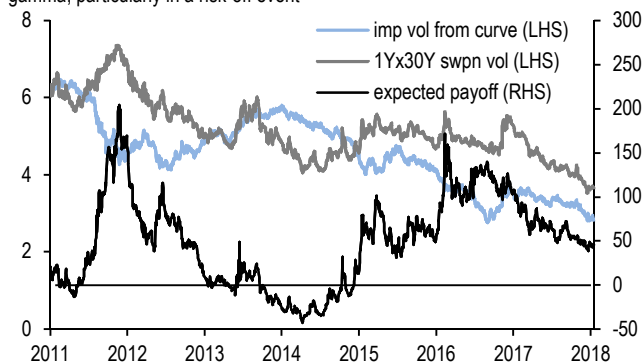
\*\* The terminal distribution of rate shifts implied by ATM and OTM 1Yx30Y swaption pricing.

Note: All data as 1/31/18.

Source: J.P. Morgan

**Exhibit 6: Our framework suggests the yield curve is frequently the cheaper source of gamma, particularly in a risk-off event**

Our framework suggests the yield curve is frequently the cheaper source of gamma, particularly in a risk-off event



Source: J.P. Morgan

## Ultra-long end analysis by jurisdiction

Until now the framework has been very theoretical and generic. We present sections on ultra-long instruments in different DM markets to allow a richer description of the idiosyncrasies of each market.

### US Treasuries

The U.S. Treasury does not issue debt obligations beyond 30 years in maturity, making a long-end flattener impossible to execute in USD government bond space. That said, shortly after the 2016 presidential election, Treasury Secretary Steven Mnuchin reintroduced the idea of a 50-year “ultra-long bond” issue.

Charged with assessing the viability of ultra-long issuance at the May 2017 refunding, the Treasury Borrowing Advisory Committee (TBAC) concluded that there was little evidence of sustainable or strong demand for an ultra-long maturity but noted that if Treasury wished to add another maturity point in response to excessive funding needs, it could consider reintroducing the 20-year and/or further investigation of a 50-year zero-coupon bond. Moreover, the Treasury Secretary has taken this to heart, as he commented during the fall that “we’ve done a bunch of research and reached out and at least as of now, we don’t see a lot of demand for [ultra-long bonds]...If we could issue ultra-long bonds at the same yield as 30-year bonds, it makes a lot of sense for us to extend the duration...If it turns out there’s a big premium to issue ultra-long bonds, there’s no reason for us to do that.” Clearly TBAC’s work on ultra-long issuance, as well as our own client survey on the topic, found that there was no significant demand for an ultra-long bond (see [US Treasury Market Daily](#), 5/11/17).



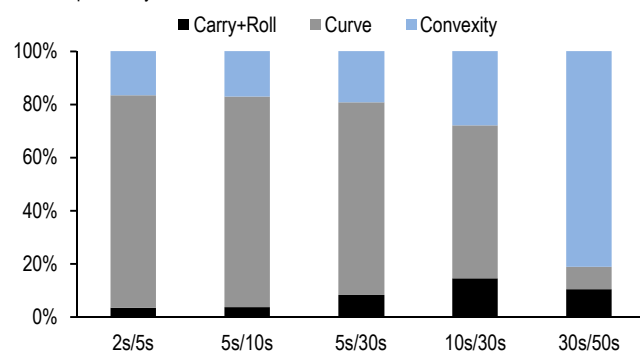
Looking ahead, we project a \$765bn deficit in FY18 and \$1022bn in FY19, and we expect net issuance of Treasury debt to the public to total \$1.424tn this year, with \$900bn allocated to coupon securities and \$524bn allocated to T-bills. Given the sheer size of financing that will be done over the coming years, Treasury announced \$1bn increases to 30-year bond auction sizes at the last week's quarterly refunding, and we project 30-year auction sizes to increase by another \$3bn/month by the end of 2018. This will leave monthly auction sizes close to the maximums recommended by the Primary Dealer Auction Size Survey, and we would not be surprised if Treasury discussed adding another long-duration maturity point along the curve at some point in the future. However, given the work done by TBAC, we think most signs point toward a 20-year rather than an ultra-long security.

## US swaps

As highlighted early last year ([An option by any other name](#), J. Younger et al., 2/3/17), the flatness and stability of the long end of the USD swap curve allows us to construct flatteners whose P/L is often primarily driven by (positive) convexity (**Exhibit 7**). Using the framework described above, we found many of these long-end structures were substantially underpriced relative to the swaptions market, a situation that has persisted since the start of 2015 (see Exhibit 6). We found the relative value framework presented above allowed us to trade long-end flatteners versus 1Yx30Y ATM swaption straddles with a favorable hit rate, especially for long-dated *forward* curve structures (56% for 30s/50s, and 86% for 25Y/20Yx5Y). The cheapness of these flatteners—relative to swaptions but also to one another—has remained remarkably stable even as the overall level of volatility has ground lower, and then recently resurged, over the past year. We continue to see the most value in forward curve pairs, in particular 20Y/40Yx10Y (**Exhibit 8**), which often come with not only high expected convexity value, but also positive *ex-ante* roll.

**Exhibit 7: Convexity earnings drive the majority of P/L for USD 30s/50s flatteners, thanks to a relatively stable long-end curve and high intrinsic convexity in longer-dated swaps**

Fraction of the variance in P/L\* for curve trades by pair held over a 1-year horizon over the past ten years; %

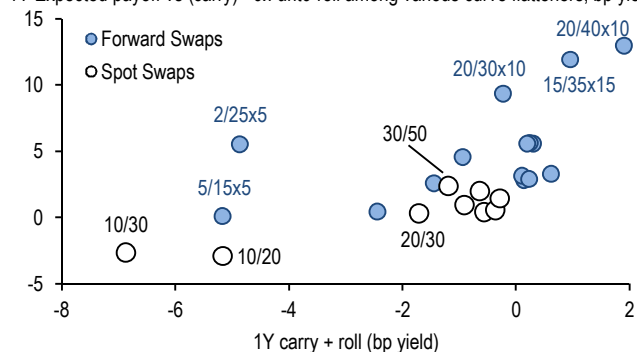


\* P/L calculated for 1-year forward flatteners initiated daily and held to spot. Carry is based on the ex-ante difference between spot and forward, curve is the change in the spot curve over the trade horizon, and convexity is the remainder versus total trade P/L. Data from 1/3/10-2/5/18.

Source: J.P. Morgan

**Exhibit 8: Long-dated forward swap flatteners feature the most attractive payoff, with some structure featuring both substantial convexity value and positive roll**

1Y Expected payoff vs (carry)+ *ex-ante* roll among various curve flatteners; bp yield



\* Expected payoff is the net P/L of holding flattener for one year under the probability-weighted average of various parallel rate shifts, using a terminal distribution of rates from ATM and OTM 1Yx30Y swaptions. This payoff thus includes carry, roll and the fair value of the flattener's net convexity. See [An option by any other name](#), J. Younger et al, 2/3/17. Data as of 2/5/18.

Source: J.P. Morgan

## Euro area government bonds

The market for 30Y+ Euro area government bonds has blossomed in the past few years as DMOs exploited the record-low level of interest rates (**Exhibit 9**). Still, the asset class remains relatively small (only 2.5% of total government conventional bond outstanding).

### Exhibit 9: The universe of 30Y+ EGBs is growing

Statistics on 30Y+ EGBs excluding private placements:

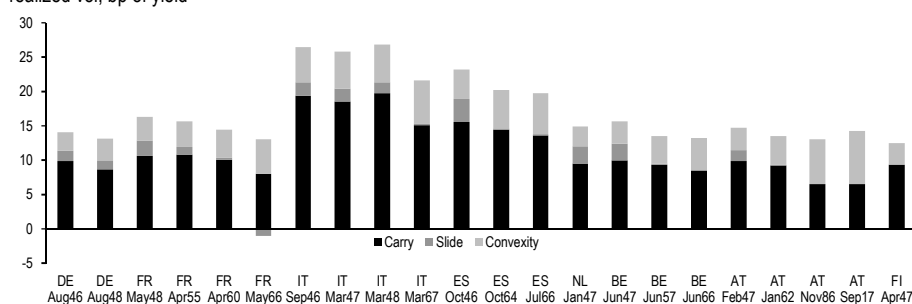
Country	Maturity	Year of issuance	Current size (€bn)
Austria	26-Jan-62	2012	3.8
Austria	02-Nov-86	2016	2.5
Austria	20-Sep-17	2017	3.5
Belgium	22-Jun-57	2017	3.0
Belgium	22-Jun-66	2016	3.0
France	25-Apr-55	2005	14.9
France	25-Apr-60	2010	13.1
France	25-May-66	2016	9.1
Italy	01-Mar-67	2016	6.6
Spain	31-Oct-64	2014	1.0
Spain	30-Jul-66	2016	7.6
Total size			68.1

Source: National DMOs, J.P. Morgan

We use the framework described above to assess fair value for high-convexity bonds (**Exhibit 10**).<sup>2</sup> Given the very low level of realised volatility, the contribution from value of convexity for expected returns is quite modest at the moment. On the other hand, carry is the most important component as the contribution of slide is negligible.

### Exhibit 10: Currently the value of convexity is not as important as carry for all 30Y+ EGBs

Decomposition of 3M expected return for 30Y+ EGBs as per calculations above; value of convexity based on 3M realized vol; bp of yield



Source: J.P. Morgan

**Based on this framework 30Y bonds offer better value than longer-dated one (Exhibits 11 and 12),** despite optically upward sloping yield curves. Ultra-long bonds' lack of appeal compounded by: 1) Market segmentation as ECB QE purchases that do not extend beyond the 31Y maturity; 2) Liquidity: ultra-long bonds' trading volumes are typically much lower and bid-offer spreads (in bp) wider than for shorter maturities.

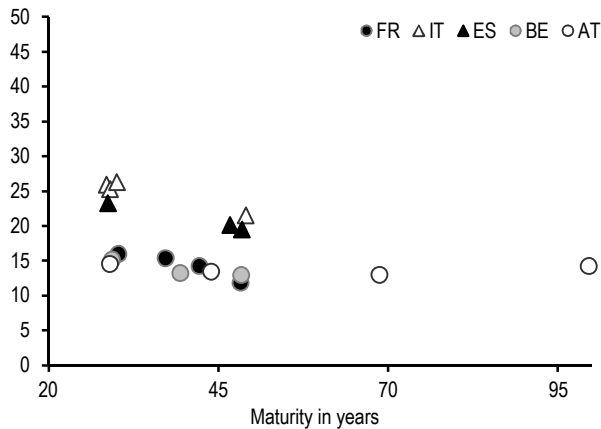
<sup>2</sup> As discussed above, the analysis does not change if the expected return is adjusted by realised volatility.



However, it is interesting to highlight that, **even in a fairly low vol environment, the 70Y and 100Y RAGBs offer a higher expected return than some 50Y core bonds.**

**Exhibit 11: 30Y bonds look more attractive than longer maturity ones at current levels of volatility**

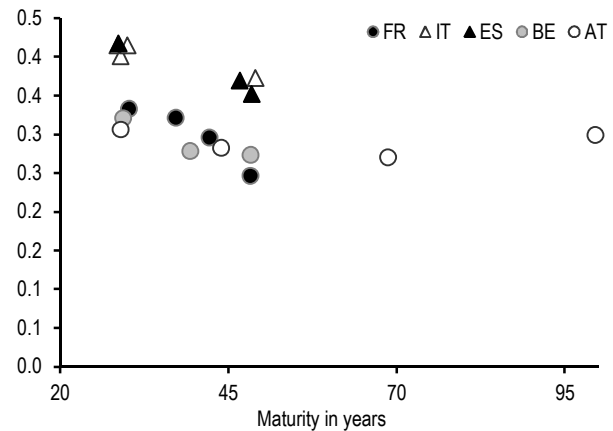
3M expected return vs. bond maturity; value of convexity based on 3M realized vol; bp of yield



Source: J.P. Morgan

**Exhibit 12: On a risk-adjusted basis the picture is very similar**

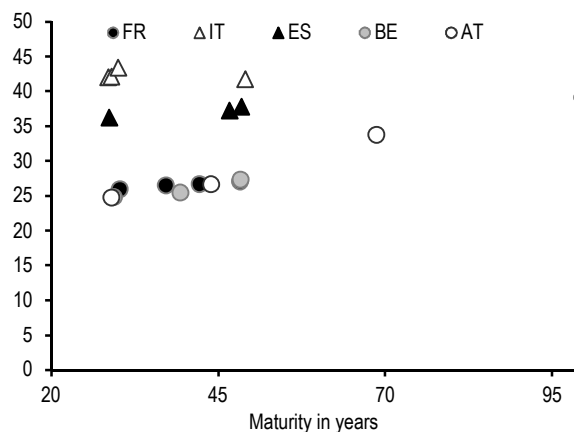
Annualised 3M expected return divided by annualised expected volatility vs. bond maturity; value of convexity based on 3M realized vol;



The attractiveness of ultra-long securities is crucially dependent on volatility assumption. **Exhibit 13** shows that a doubling of expected volatility would change the picture significantly: with the exception of Italy, the expected return curves would become upward sloping, i.e. very long-dated instruments would become more appealing than short dated ones.

**Exhibit 13: A doubling of expected volatility would lift expected returns, especially at the back end of the curve**

3M expected return vs. bond maturity; value of convexity based on 3M realized vol<sup>2</sup>; bp of yield



Source: J.P. Morgan

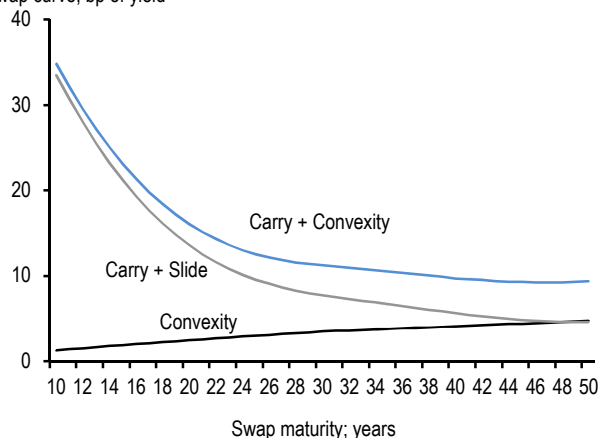
## Euro area swaps

In line with the dynamic seen on the bond curve, the value of convexity on the swap curve increases almost monotonically. In the front/intermediate sector, carry & slide are the dominant source of expected return. However, the contribution of convexity

increases as we move further along the curve. In terms of ratio, it rises significantly versus carry as we move further out (also aided by a sharp decline in carry & slide in the ultra-long end of the curve; **Exhibit 14**).

**Exhibit 14: At the long end of the swap curve, the contribution from value of convexity increases compared to carry**

Ex-ante 1Y carry and slide and value of convexity at various points on the EUR swap curve; bp of yield



Source: J.P. Morgan

**Exhibit 15: In EUR swaps the value of convexity is currently towards the bottom end of its range, in line with volatility**

Statistics on 1Y value of convexity at various points on the EUR swap curve; since Euro inception; bp of yield

	10Y	20Y	30Y	40Y	50Y
Current	1.2	2.3	3.3	3.9	4.5
High	7.6	17.9	31.4	39.2	45.9
Low	1.1	1.8	2.3	2.8	3.1
Average	2.5	4.4	6.1	7.4	8.6
SD	1.0	2.2	3.8	4.8	5.7
Z-score	-1.3	-0.9	-0.7	-0.8	-0.7
%ile	1%	3%	15%	13%	16%

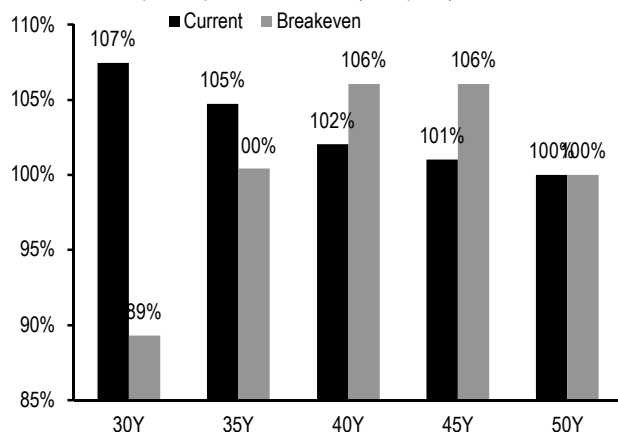
From a historical perspective, the current value of convexity is close to the bottom of its long-term ranges. In **Exhibit 15**, we compare the convexity component at various points on the long end of the EUR swap curve to its history since Euro inception. The highs were reached during the Lehman period, when volatility had skyrocketed. However, current historical low levels of volatility have resulted in a low value of convexity.

**Exhibit 16** compares current volatility of various points on the ultra-long end of the curve with that of the 50Y point. We also present the expected ratio of volatility, which would equate the expected return of these points to that of the 50Y. Currently, volatility is downward sloping at the long end as realised and implied vol decrease with maturity. However, given that the expected return (carry plus slide plus value of convexity) is higher in the 30Y sector than in 50Y, an almost 20% decline in 30Y vol would be required to reduce the 30Y value of convexity to a breakeven level for the expected return. The difference between current and breakeven vol in 35Y, 40Y and 45Y is not as marked.

Going forward, we forecast an increase in implieds as we approach the ECB hiking cycle and yields rise (although the increase vs. spot is expected to be more pronounced in the 5-10Y sector), which would push the value of convexity higher as well. **Exhibit 17** shows the sensitivity of the EUR swap curve value of convexity for various scenarios of EUR volatility; a doubling of volatility quadruples the value of convexity. Another conclusion from our analysis is that, assuming a parallel yield curve and volatility movement scenario, long-end flatteners are still likely to be profitable in a rising yield environment given higher convexity whose impact gets accentuated with rising volatility.

**Exhibit 16: A sharp drop in 30Y vol is required to equate expected returns in the 30Y and 50Y sectors**

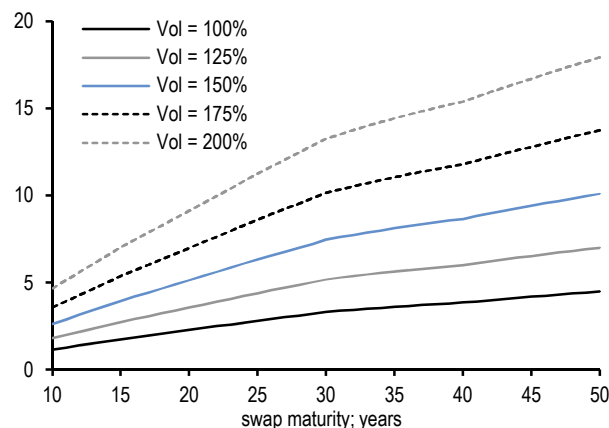
Current ratio of implied volatility of various points versus 50Y and breakeven level of the ratio that equate expected 50Y returns (1Y expiries);



Source: J.P. Morgan

**Exhibit 17: Doubling of volatility quadruples the value of convexity. Thus, in a scenario where yield and volatility move up in parallel, long end flatteners will likely be profitable**

1Y value of convexity at various points on the swap curve under different assumptions of the volatility surface (expressed as a ratio of current levels); bp of yield



**UK**

In the UK, the ultra-long sector (i.e. 30Y+) has been a key component of the gilt market. The UK DMO first issued ultra-long end gilts in 2005 (4Q55) with around £130bn of 30Y+ gilts currently outstanding, some 12% of the total market (**Exhibit 18**). Demand for ultra-long-end bonds stems from an active domestic investor base, namely pension funds, LDI and some insurance companies, for hedging long-dated liabilities with the DMO supplying 30Y+ gilts both via auction and syndications.

This hedging demand is also evident in GBP swap space as end investors seek better cash-flow matches, use swaps as overlay positions and take active swap spread views. As a result, identifying the 'fair value of convexity' in UK gilts and swaps can be useful to evaluate the impact of these idiosyncratic factors on the ultra-long end of the curve.

**Exhibit 18: The ultra-long end is an important sector of the gilt market**

Statistics for UK conventional gilt market by sector\*

	No bonds	Outstanding ex. DMO; £bn	Proportion of total outstanding	Wt. modified duration; years	Wt. 10Y equivalent duration; years
<15Y	25	681	65%	4.5	0.5
15-30Y	11	244	23%	16.7	1.9
30Y>	7	131	12%	25.3	2.8

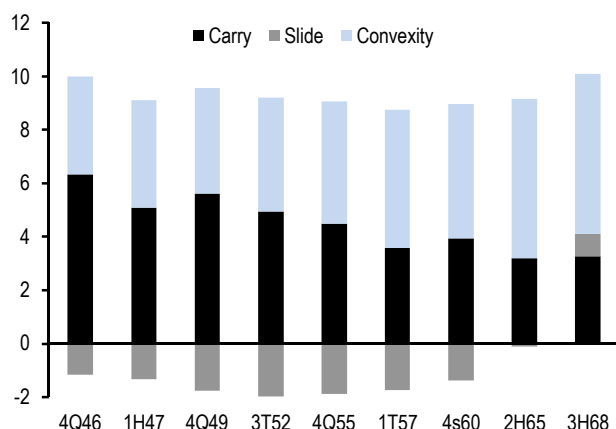
\*Outstanding amounts excluded DMO holdings

Source: J.P. Morgan

**Exhibit 19** shows the expected returns from carry, slide and convexity using the framework outlined previously. For 30Y gilts, the expected contribution from convexity is lower than the carry component whereas the opposite is true for 40Y and 50Y gilts.

**Exhibit 19: In gilt space, ultra-long end bonds expected return is more a function of convexity rather than carry, whilst the opposite is the case for 30Y gilts**

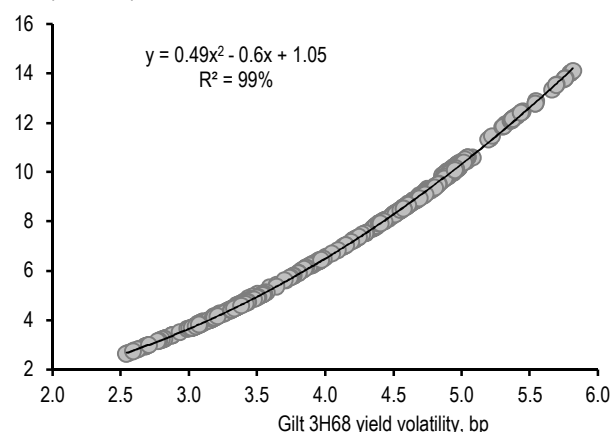
Contributions to return from carry, slide and convexity over 1Y holding horizon; bp



Source: J.P. Morgan

**Exhibit 20: The value of convexity increases with expected volatility**

1Y value of convexity for 3H68 regressed vs. expected daily yield volatility for gilt 3H68; past 2Y; bp



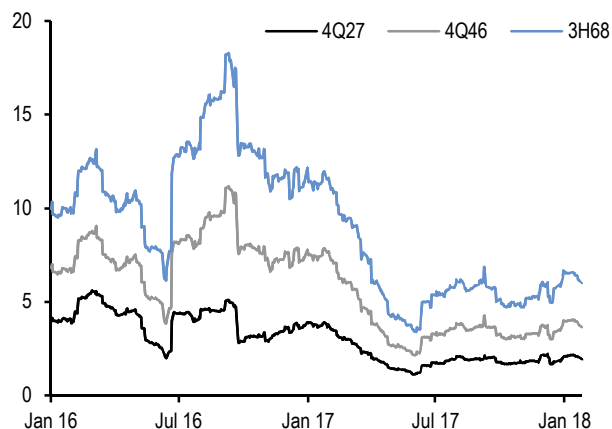
The value of convexity depends on the expected future volatility (**Exhibit 20**). On a historical basis, the value of convexity changes over time across various sectors of the curve, and we can see that in cash gilt space, the increase in historical volatility in mid-2016 drove a spike higher in convexity values (**Exhibit 21**).

We can perform a similar analysis in GBP swap space to determine the value of convexity and note that convexity is towards the bottom end of long-run historical ranges, with 10Y and 20Y relatively lower than 30Y, 40Y and 50Y (**Exhibit 22**). Historically, the highest levels in the value of convexity were seen during the Lehman period when yield volatility significantly increased with the current low levels of convexity broadly reflecting low levels of volatility.

The value of convexity derived from swaps is currently higher than that for cash bonds. Looking specifically at the 50Y sector, we note that whilst the value of convexity derived from the two markets shows a positive beta over the past two years, **convexity as measured in the gilt and swap markets can differ in magnitude over time (Exhibit 23)**. This likely reflects: 1) different volatility assumptions in swaps and cash gilts used to calculate the value of convexity, 2) coupon effects from comparing a par swap vs. a fixed coupon bond, and 3) maturity effects from using a fixed maturity gilt and a rolling 50Y swap in our analysis.

**Exhibit 21: The value of convexity in gilts spiked around the 2016 EU referendum before tailing off and recently has been slowly rising from historical lows**

1Y value of convexity for gilts 4Q27, 4Q46 and 3H68; bp of yield



Source: J.P. Morgan

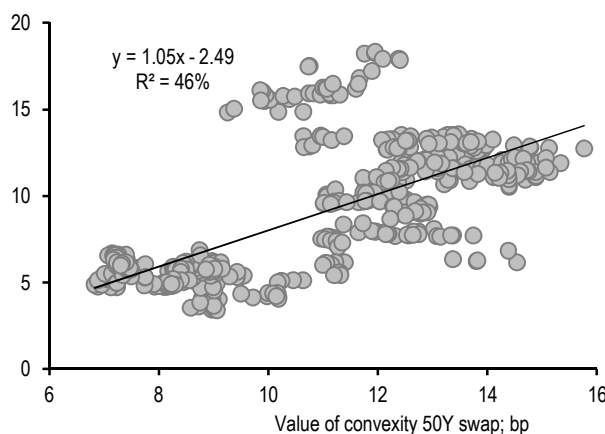
**Exhibit 22: GBP swap value of convexity is towards the bottom end of long-run historical ranges**

Statistics on the 1Y value of convexity for various points on the GBP swap curve since Jan99; bp of yield

	10Y	20Y	30Y	40Y	50Y
Current	2.0	3.5	4.9	6.1	7.3
High	6.6	10.9	14.6	18.1	20.9
Low	1.3	2.0	2.4	2.9	3.4
Average	3.1	4.9	6.2	7.6	8.8
SD	1.1	1.8	2.5	3.1	3.7
Z-score	-1.0	-0.8	-0.5	-0.5	-0.4
%ile	21%	29%	39%	40%	40%

**Exhibit 23: The value of convexity derived from 50Y swaps and 50Y gilts is positively correlated but has not always moved in lockstep**

1Y value of convexity for 3H68 regressed against value of convexity for 50Y swaps; since Jan16; bp



Source: J.P. Morgan

In principle, **the idea of positive convexity and the expected return profile of 50Y vs. 30Y bonds and swaps should be reflected by an inverted 30s/50s curve**, as investors need to pay for positive convexity with negative carry and slide. Empirically, the 30s/50s curve in both gilts and GBP swaps has indeed been inverted on average over the past ten years (**Exhibit 24**) with idiosyncratic drivers (Lehman's episode, BoE QE, long end supply/demand dynamics) resulting in a large trading range, particularly in par gilt space.

Currently, the 30s/50s curve is at (for swaps) and close to (for par gilts) its flattest levels since 2009. We can attempt to identify whether this curve over or under compensates for the value of convexity. Intuitively, given volatility in 50Y yields is only modestly higher over recent months and is well below historical highs, we think that the current curve flatness is *overcompensating* for the benefit of convexity. One

way to empirically demonstrate this is to model the 30s/50s par gilt curve as a function of the level of 50 yields and our derived value of convexity (**Exhibit 25**). **This regression model implies the 30s/50s curve is some 5bp too flat**, although we recognise that this model is based on a backward-looking volatility measure in deriving a value of convexity rather than a forward looking implied measure. Idiosyncratic flow drivers and recent increased pension-fund hedging demand likely explain this relative flatness of 30s/50s.

**Exhibit 24: 30s/50s gilt and GBP swap curves on average have been inverted, reflecting the positive value of convexity**

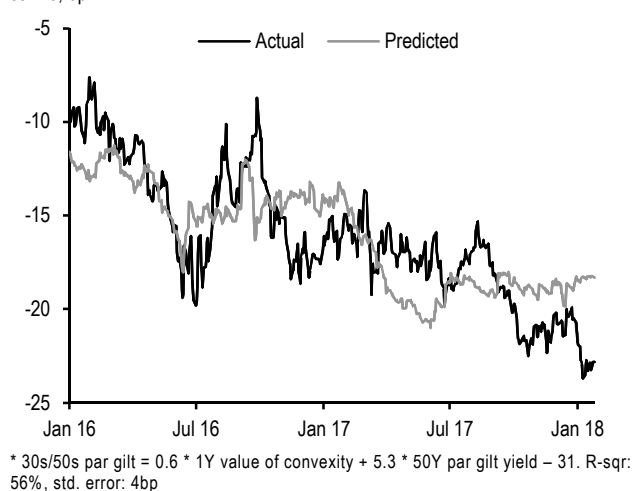
30s/50s swaps and 30s/50s par gilt curves statistics over past 5Y and past 10Y; bp

	Past 10Y		Past 5Y	
	Par Gilt	Swap	Par Gilt	Swap
Max	10	16	6	10
Min	-62	-36	-26	-17
Average	-11	-6	-11	-7

Source: J.P. Morgan

**Exhibit 25: The 30s/50s par gilt curve looks too flat when adjusted for the level of yields and the implied value of convexity**

Actual 30s/50s par gilt curve and regression model 30/50s predicted curve\*; since Jan16, bp



## Cross-Market Swaps

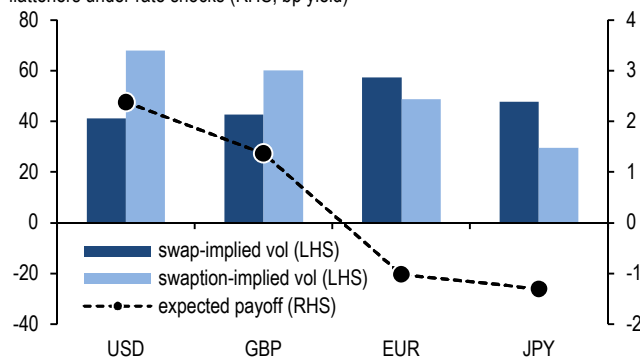
As a final exercise, our convexity fair-value framework allows a relatively clean comparison of swap curve flatteners across DM currencies. Focusing our attention on the canonical 30s/50s flattener, for instance, we find convexity value substantially underpriced in both USD and GBP curves: the breakeven volatility required for these flatteners to price “fairly” (the “*swap-implied volatility*”) is substantially lower than that implied from the swaptions market (**Exhibit 26**). As a result our expected payout under rate shocks is positive. In contrast, the EUR and JPY curves require higher breakeven volatility than what’s priced into their respective swaptions markets, and thus their expected payoff is negative.

This relative value across DM markets holds for a broad variety of flattener structures (**Exhibit 27**). USD and GBP structures consistently enjoy higher convexity value and higher expected payoffs. That said, opportunities for underpriced convexity exist for swap flatteners in all four currencies we explored, where forward-starting structures consistently performed the best. Curiously enough, for USD and GBP, 20Y/40Yx10Y was the clear winner, and in EUR this structure was neck-and-neck with 15Y/35Yx15Y.



**Exhibit 26: Under our framework, the 30s/50s swap curve is presently a cheap source of vol in both USD and GBP, while it appears relatively expensive in EUR and JPY**

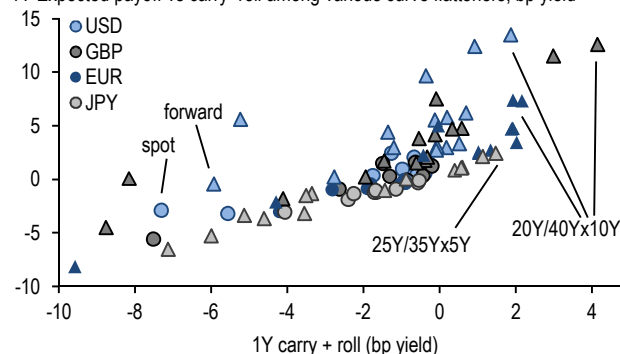
Breakeven vol, e.g. "swap-implied vol" from 30s/50s curve flatteners vs. implied vol for 1Yx30Y swaptions (LHS; abp), compared to the expected payoff of these flatteners under rate shocks (RHS; bp yield)



Source: J.P. Morgan

**Exhibit 27: Across markets, long forward-dated flatteners enjoy a markedly higher payoff than spot structures from both a convexity and carry/roll perspective; with USD and GBP most attractive**

1Y Expected payoff vs carry+roll among various curve flatteners; bp yield



\* Expected payoff is the net P/L of holding flattener for one year under the probability-weighted average of various parallel rate shifts, using a terminal distribution of rates from ATMF and OTM 1Yx30Y swaptions. This payoff thus includes carry, roll and the fair value of the flattener's net convexity. See [An option by any other name](#), J. Younger et al, 2/3/17. Data as of 2/5/18. Source: J.P. Morgan

## Disclosures

**Analyst Certification:** The research analyst(s) denoted by an “AC” on the cover of this report certifies (or, where multiple research analysts are primarily responsible for this report, the research analyst denoted by an “AC” on the cover or within the document individually certifies, with respect to each security or issuer that the research analyst covers in this research) that: (1) all of the views expressed in this report accurately reflect his or her personal views about any and all of the subject securities or issuers; and (2) no part of any of the research analyst's compensation was, is, or will be directly or indirectly related to the specific recommendations or views expressed by the research analyst(s) in this report. For all Korea-based research analysts listed on the front cover, they also certify, as per KOFIA requirements, that their analysis was made in good faith and that the views reflect their own opinion, without undue influence or intervention.

**Company-Specific Disclosures:** Important disclosures, including price charts and credit opinion history tables, are available for compendium reports and all J.P. Morgan–covered companies by visiting <https://www.jpmm.com/research/disclosures>, calling 1-800-477-0406, or e-mailing [research.disclosure.inquiries@jpmorgan.com](mailto:research.disclosure.inquiries@jpmorgan.com) with your request. J.P. Morgan’s Strategy, Technical, and Quantitative Research teams may screen companies not covered by J.P. Morgan. For important disclosures for these companies, please call 1-800-477-0406 or e-mail [research.disclosure.inquiries@jpmorgan.com](mailto:research.disclosure.inquiries@jpmorgan.com).

**Analysts' Compensation:** The research analysts responsible for the preparation of this report receive compensation based upon various factors, including the quality and accuracy of research, client feedback, competitive factors, and overall firm revenues.

## Other Disclosures

J.P. Morgan (“JPM”) is the global brand name for J.P. Morgan Securities LLC (“JPMS”) and its affiliates worldwide. J.P. Morgan Cazenove is a marketing name for the U.K. investment banking businesses and EMEA cash equities and equity research businesses of JPMorgan Chase & Co. and its subsidiaries.

**Options related research:** If the information contained herein regards options related research, such information is available only to persons who have received the proper option risk disclosure documents. For a copy of the Option Clearing Corporation's Characteristics and Risks of Standardized Options, please contact your J.P. Morgan Representative or visit the OCC's website at <https://www.theocc.com/components/docs/riskstoc.pdf>

## Legal Entities Disclosures

**U.S.:** JPMS is a member of NYSE, FINRA, SIPC and the NFA. JPMorgan Chase Bank, N.A. is a member of FDIC. **U.K.:** JPMorgan Chase N.A., London Branch, is authorised by the Prudential Regulation Authority and is subject to regulation by the Financial Conduct Authority and to limited regulation by the Prudential Regulation Authority. Details about the extent of our regulation by the Prudential Regulation Authority are available from J.P. Morgan on request. J.P. Morgan Securities plc (JPMS plc) is a member of the London Stock Exchange and is authorised by the Prudential Regulation Authority and regulated by the Financial Conduct Authority and the Prudential Regulation Authority. Registered in England & Wales No. 2711006. Registered Office 25 Bank Street, London, E14 5JP. **South Africa:** J.P. Morgan Equities South Africa Proprietary Limited is a member of the Johannesburg Securities Exchange and is regulated by the Financial Services Board. **Hong Kong:** J.P. Morgan Securities (Asia Pacific) Limited (CE number AAJ321) is regulated by the Hong Kong Monetary Authority and the Securities and Futures Commission in Hong Kong and/or J.P. Morgan Broking (Hong Kong) Limited (CE number AAB027) is regulated by the Securities and Futures Commission in Hong Kong. **Korea:** This material is issued and distributed in Korea by or through J.P. Morgan Securities (Far East) Limited, Seoul Branch, which is a member of the Korea Exchange(KRX) and is regulated by the Financial Services Commission (FSC) and the Financial Supervisory Service (FSS). **Australia:** J.P. Morgan Australia Limited (JPMAL) (ABN 52 002 888 011/AFS Licence No: 238188) is regulated by ASIC and J.P. Morgan Securities Australia Limited (JPMSAL) (ABN 61 003 245 234/AFS Licence No: 238066) is regulated by ASIC and is a Market, Clearing and Settlement Participant of ASX Limited and CHI-X. **Taiwan:** J.P. Morgan Securities (Taiwan) Limited is a participant of the Taiwan Stock Exchange (company-type) and regulated by the Taiwan Securities and Futures Bureau. **India:** J.P. Morgan India Private Limited (Corporate Identity Number - U67120MH1992FTC068724), having its registered office at J.P. Morgan Tower, Off. C.S.T. Road, Kalina, Santacruz - East, Mumbai – 400098, is registered with Securities and Exchange Board of India (SEBI) as a ‘Research Analyst’ having registration number INH0000001873. J.P. Morgan India Private Limited is also registered with SEBI as a member of the National Stock Exchange of India Limited (SEBI Registration Number - INB 230675231/INF 230675231/INE 230675231), the Bombay Stock Exchange Limited (SEBI Registration Number - INB 010675237/INF 010675237) and as a Merchant Banker (SEBI Registration Number - MB/INM000002970). Telephone: 91-22-6157 3000, Facsimile: 91-22-6157 3990 and Website: [www.jpmpil.com](http://www.jpmpil.com). For non local research reports, this material is not distributed in India by J.P. Morgan India Private Limited. **Thailand:** This material is issued and distributed in Thailand by JPMorgan Securities (Thailand) Ltd., which is a member of the Stock Exchange of Thailand and is regulated by the Ministry of Finance and the Securities and Exchange Commission and its registered address is 3rd Floor, 20 North Sathorn Road, Silom, Bangrak, Bangkok 10500. **Indonesia:** PT J.P. Morgan Securities Indonesia is a member of the Indonesia Stock Exchange and is regulated by the OJK a.k.a. BAPEPAM LK. **Philippines:** J.P. Morgan Securities Philippines Inc. is a Trading Participant of the Philippine Stock Exchange and a member of the Securities Clearing Corporation of the Philippines and the Securities Investor Protection Fund. It is regulated by the Securities and Exchange Commission. **Brazil:** Banco J.P. Morgan S.A. is regulated by the Comissao de Valores Mobiliarios (CVM) and by the Central Bank of Brazil. **Mexico:** J.P. Morgan Casa de Bolsa, S.A. de C.V., J.P. Morgan Grupo Financiero is a member of the Mexican Stock Exchange and authorized to act as a broker dealer by the National Banking and Securities Exchange Commission. **Singapore:** This material is issued and distributed in Singapore by or through J.P. Morgan Securities Singapore Private Limited (JPMS) [MCI (P) 202/03/2017 and Co. Reg. No.: 199405335R], which is a member of the Singapore Exchange Securities Trading Limited and/or JPMorgan Chase Bank, N.A., Singapore branch (JPMCB Singapore) [MCI (P) 059/09/2017], both of which are regulated by the Monetary Authority of Singapore. This material is issued and distributed in Singapore only to accredited investors, expert investors and institutional investors, as defined in Section 4A of the Securities and Futures Act, Cap. 289 (SFA). This material is not intended to be issued or distributed to any retail investors or any other investors that do not fall into the classes of “accredited investors,” “expert investors” or “institutional investors,” as defined under Section 4A of the SFA. Recipients of this document are to contact JPMS or JPMCB Singapore in respect of any matters arising from, or in connection with, the document. **Japan:** JPMorgan Securities Japan Co., Ltd. and JPMorgan Chase Bank, N.A., Tokyo Branch are regulated by the Financial Services Agency in Japan. **Malaysia:** This material is issued and distributed in Malaysia by JPMorgan Securities (Malaysia) Sdn

Bhd (18146-X) which is a Participating Organization of Bursa Malaysia Berhad and a holder of Capital Markets Services License issued by the Securities Commission in Malaysia. **Pakistan:** J. P. Morgan Pakistan Broking (Pvt.) Ltd is a member of the Karachi Stock Exchange and regulated by the Securities and Exchange Commission of Pakistan. **Saudi Arabia:** J.P. Morgan Saudi Arabia Ltd. is authorized by the Capital Market Authority of the Kingdom of Saudi Arabia (CMA) to carry out dealing as an agent, arranging, advising and custody, with respect to securities business under licence number 35-07079 and its registered address is at 8th Floor, Al-Faisaliyah Tower, King Fahad Road, P.O. Box 51907, Riyadh 11553, Kingdom of Saudi Arabia. **Dubai:** JPMorgan Chase Bank, N.A., Dubai Branch is regulated by the Dubai Financial Services Authority (DFSA) and its registered address is Dubai International Financial Centre - Building 3, Level 7, PO Box 506551, Dubai, UAE.

#### Country and Region Specific Disclosures

**U.K. and European Economic Area (EEA):** Unless specified to the contrary, issued and approved for distribution in the U.K. and the EEA by JPMS plc. Investment research issued by JPMS plc has been prepared in accordance with JPMS plc's policies for managing conflicts of interest arising as a result of publication and distribution of investment research. Many European regulators require a firm to establish, implement and maintain such a policy. Further information about J.P. Morgan's conflict of interest policy and a description of the effective internal organisations and administrative arrangements set up for the prevention and avoidance of conflicts of interest is set out at the following link <https://www.jpmorgan.com/jpmpdf/1320742677360.pdf>. This report has been issued in the U.K. only to persons of a kind described in Article 19 (5), 38, 47 and 49 of the Financial Services and Markets Act 2000 (Financial Promotion) Order 2005 (all such persons being referred to as "relevant persons"). This document must not be acted on or relied on by persons who are not relevant persons. Any investment or investment activity to which this document relates is only available to relevant persons and will be engaged in only with relevant persons. In other EEA countries, the report has been issued to persons regarded as professional investors (or equivalent) in their home jurisdiction. **Australia:** This material is issued and distributed by JPMSAL in Australia to "wholesale clients" only. This material does not take into account the specific investment objectives, financial situation or particular needs of the recipient. The recipient of this material must not distribute it to any third party or outside Australia without the prior written consent of JPMSAL. For the purposes of this paragraph the term "wholesale client" has the meaning given in section 761G of the Corporations Act 2001. **Germany:** This material is distributed in Germany by J.P. Morgan Securities plc, Frankfurt Branch which is regulated by the Bundesanstalt für Finanzdienstleistungsaufsicht. **Hong Kong:** The 1% ownership disclosure as of the previous month end satisfies the requirements under Paragraph 16.5(a) of the Hong Kong Code of Conduct for Persons Licensed by or Registered with the Securities and Futures Commission. (For research published within the first ten days of the month, the disclosure may be based on the month end data from two months prior.) J.P. Morgan Broking (Hong Kong) Limited is the liquidity provider/market maker for derivative warrants, callable bull bear contracts and stock options listed on the Stock Exchange of Hong Kong Limited. An updated list can be found on HKEx website: <http://www.hkex.com.hk>. **Korea:** This report may have been edited or contributed to from time to time by affiliates of J.P. Morgan Securities (Far East) Limited, Seoul Branch. **Singapore:** As at the date of this report, JPMS is a designated market maker for certain structured warrants listed on the Singapore Exchange where the underlying securities may be the securities discussed in this report. Arising from its role as designated market maker for such structured warrants, JPMS may conduct hedging activities in respect of such underlying securities and hold or have an interest in such underlying securities as a result. The updated list of structured warrants for which JPMS acts as designated market maker may be found on the website of the Singapore Exchange Limited: <http://www.sgx.com>. In addition, JPMS and/or its affiliates may also have an interest or holding in any of the securities discussed in this report – please see the Important Disclosures section above. For securities where the holding is 1% or greater, the holding may be found in the Important Disclosures section above. For all other securities mentioned in this report, JPMS and/or its affiliates may have a holding of less than 1% in such securities and may trade them in ways different from those discussed in this report. Employees of JPMS and/or its affiliates not involved in the preparation of this report may have investments in the securities (or derivatives of such securities) mentioned in this report and may trade them in ways different from those discussed in this report. **Taiwan:** This material is issued and distributed in Taiwan by J.P. Morgan Securities (Taiwan) Limited. According to Paragraph 2, Article 7-1 of Operational Regulations Governing Securities Firms Recommending Trades in Securities to Customers (as amended or supplemented) and/or other applicable laws or regulations, please note that the recipient of this material is not permitted to engage in any activities in connection with the material which may give rise to conflicts of interests, unless otherwise disclosed in the "Important Disclosures" in this material. **India:** For private circulation only, not for sale. **Pakistan:** For private circulation only, not for sale. **New Zealand:** This material is issued and distributed by JPMSAL in New Zealand only to persons whose principal business is the investment of money or who, in the course of and for the purposes of their business, habitually invest money. JPMSAL does not issue or distribute this material to members of "the public" as determined in accordance with section 3 of the Securities Act 1978. The recipient of this material must not distribute it to any third party or outside New Zealand without the prior written consent of JPMSAL. **Canada:** The information contained herein is not, and under no circumstances is to be construed as, a prospectus, an advertisement, a public offering, an offer to sell securities described herein, or solicitation of an offer to buy securities described herein, in Canada or any province or territory thereof. Any offer or sale of the securities described herein in Canada will be made only under an exemption from the requirements to file a prospectus with the relevant Canadian securities regulators and only by a dealer properly registered under applicable securities laws or, alternatively, pursuant to an exemption from the dealer registration requirement in the relevant province or territory of Canada in which such offer or sale is made. The information contained herein is under no circumstances to be construed as investment advice in any province or territory of Canada and is not tailored to the needs of the recipient. To the extent that the information contained herein references securities of an issuer incorporated, formed or created under the laws of Canada or a province or territory of Canada, any trades in such securities must be conducted through a dealer registered in Canada. No securities commission or similar regulatory authority in Canada has reviewed or in any way passed judgment upon these materials, the information contained herein or the merits of the securities described herein, and any representation to the contrary is an offence. **Dubai:** This report has been issued to persons regarded as professional clients as defined under the DFSA rules. **Brazil:** Ombudsman J.P. Morgan: 0800-7700847 / [ouvidoria.jp.morgan@jpmorgan.com](mailto:ouvidoria.jp.morgan@jpmorgan.com).

**General:** Additional information is available upon request. Information has been obtained from sources believed to be reliable but JPMorgan Chase & Co. or its affiliates and/or subsidiaries (collectively J.P. Morgan) do not warrant its completeness or accuracy except with respect to any disclosures relative to JPMS and/or its affiliates and the analyst's involvement with the issuer that is the subject of the research. All pricing is indicative as of the close of market for the securities discussed, unless otherwise stated. Opinions and estimates constitute our judgment as of the date of this material and are subject to change without notice. Past performance is not indicative of future results. This material is not intended as an offer or solicitation for the purchase or sale of any financial instrument. The opinions and recommendations herein do not take into account individual client circumstances, objectives, or needs and are not intended as recommendations of particular securities, financial instruments or strategies to particular clients. The recipient of this report must make its own independent decisions regarding any securities or financial instruments mentioned herein. JPMS distributes in the U.S. research published by non-U.S. affiliates and accepts responsibility for its contents. Periodic updates may be provided on companies/industries based on company specific developments or announcements, market conditions or any other publicly available information. Clients should contact analysts and execute transactions through a J.P. Morgan subsidiary or affiliate in their home jurisdiction unless governing law permits otherwise.

Gianluca Salford  
(44-20) 7134-1692  
gianluca.salford@jpmorgan.com

Joshua Younger  
(1-212) 270-1323  
joshua.d.younger@jpmorgan.com

Global Fixed Income Strategy  
Valuing convexity in the long end of the yield curve  
09 February 2018

J.P.Morgan

"Other Disclosures" last revised January 01, 2018.

---

**Copyright 2018 JPMorgan Chase & Co. All rights reserved. This report or any portion hereof may not be reprinted, sold or redistributed without the written consent of J.P. Morgan.**