A New Framework for Correlation

- We introduce a new measure of correlation: the Mean Variance Ratio
- This simplifies the characterisation of p/l in dispersion trades…
- … and generalises seamlessly to enhanced dispersion trades and even volatility pair trades.

Overview

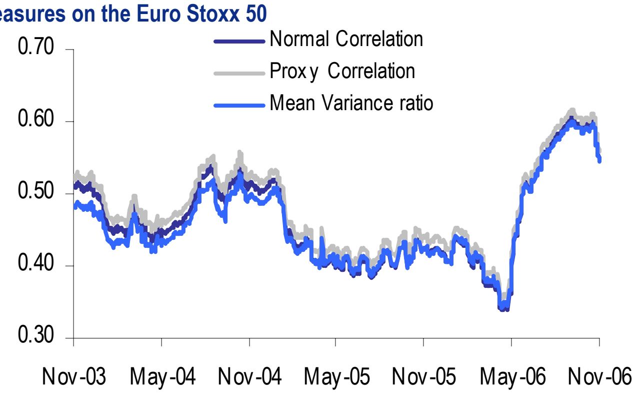
Although traditional measures of correlation are useful for characterising classic dispersion trades, drivers of p/l can be difficult to quantify. Moreover, in more general cases where the single-stocks in the basket do not comprise the index members, traditional correlation measures become less useful and increasingly meaningless.

In this note we address these issues by introducing a new measure of correlation: **the Mean Variance Ratio** (MVR). Working with this form of correlation allows us to describe dispersion trade p/l in a simple way. Unlike normal correlation measures, MVR also remains meaningful and useful when applied to generalised dispersion trades and even pairs trades.

The Mean Variance Ratio provides a unified framework for dispersion and generalised volatility pair trades which in our view is superior to traditional measures of correlation in a number of ways:

1. Dispersion trades are profitable exactly when the implied MVR exceeds that subsequently realised
2. MVR is closely related to traditional correlation measures
3. P/L is easily characterised in terms of change in MVR
4. Using the MVR makes mark-to-market straightforward
5. Results generalise to indices with reduced variance swap liquidity and even to (generalised) volatility pairs trades.

Figure 1: Mean Variance Ratio closely reflects traditional correlation measures on the Euro Stoxx 50



Source: Bloomberg ,JPMorgan

Asia-Ex Equity Derivatives Strategy
Simon Miguez, CFA +852 2800 8855
simon.miguez@jpmchase.com

European Equity Derivatives Strategy
Nicolas Granger +44 207 325 7033
nicolas.granger@jpmorgan.com

Asia-Ex Equity Derivatives Marketing

Hong Kong (US Toll Free: 866-589-1803)
Malcolm Barnett +852 2800 8857
malcolm.barnett@jpmorgan.com
Jackey Chan +852 2800 7868
jackey.Chan@jpmorgan.com
Jean-Paul Linschoten +852 2800 8854
jean-paul.linschoten@jpmorgan.com
Katherine Rose +852 2800 8852
katherine.rose@jpmorgan.com
Anna Yen, CFA +852 2800 8890
anna.yen@jpmorgan.com

New York
Bill Gulya +1 212 622 2653
william.gulya@jpmorgan.com
Kevin Andrade +1 212 622 2698
kevin.x.andrade@jpmorgan.com
Michael Romano +1 212 622 2925
michael.romano@jpmorgan.com

London
Michael Ridsdill Smith +44 207 742 8893
michael.ridsdill.smith@jpmorgan.com
Tobie Duprey +44 207 742 8064
tobie.duprey@jpmorgan.com

Simon Miguez
(852) 2800-8855
Malcolm Barnett
(852) 2800-8857

Introduction

The emergence of variance swaps has provided investors with a simple way to gain direct exposure to the volatility of an equity underlying. This has allowed for various volatility strategies to be constructed in a reasonably straightforward manner. See *Variance Swaps, Allen, Granger, Einchcomb, November 2006* for further details. In this note, we will focus our attention on relative value volatility strategies – ranging from volatility pair trades to various forms of dispersion trades**. Introducing a new characterisation of correlation allows us to view these trades under a unified framework.**

A *dispersion trade* is usually defined as a trade involving selling index variance and simultaneously buying variance of its constituents. The trade aims to benefit from the observation that the premium of implied over realised volatility is in general greater for indices than for their constituent stocks. In other words, a dispersion trade is perceived as a way to gain exposure to the fact that implied correlation tends to trade systematically above realised correlation.

Although the profitability of a classic dispersion trade is correlated with the difference between the level of implied correlation sold and that subsequently realised, the relationship is non-linear. Under certain circumstances a dispersion trade can lose, even given an apparently favourable move in correlation. In other words, **p/l is *not* a simple function of implied minus realised correlation.** See *Correlation Vehicles, Granger, Allen, May 2005* for further details.

Issues are complicated further by the fact that **a simple proxy for correlation (the square of the ratio of index volatility to average single-stock volatility)** is often used. This proxy holds well for reasonably diversified indices such as the Euro Stoxx 50, but can fail for smaller indices, indices dominated by a single large name (e.g. HSBC in the HSI), or indices where dispersion trades are typically constructed using only the largest/most liquid single names (e.g. Kospi200).

We present a new measure of correlation, in essence taking a ratio of variances rather than a squared ratio of volatilities. This was first introduced by our European team (see *European Equity Derivatives Weekly, 2 October 2006*). This new correlation measure more accurately reflects the p/l of the various forms of dispersion trade, and even volatility pairs trades, whilst retaining the simplicity of the correlation proxy. Indeed, by using this correlation measure, the various forms of dispersion trade can be seen to be profitable exactly when the level of correlation sold exceeds that subsequently realised.

Using this characterisation of correlation, **a classic dispersion trade can be likened to a correlation swap**, where the level of correlation sold (and subsequently realised) is given by our new correlation measure, but where the notional is variable depending on the average realised variance of the basket members. This is a similar situation to that using the normal dispersion measure, although by using the mean variance ratio relationship is exact, not approximate. Furthermore, this description of the p/l generalises exactly to the other flavours of dispersion trade and even to volatility pairs trades.

In Section 1, we introduce this new correlation measure, which we refer to as **mean variance ratio**. We explain how this correlation measure simplifies the calculation of the p/l for classic dispersion trades. Under this description of the p/l the "correlation notional" becomes variable, depending on the volatility realised over the course of the trade. We introduce the notions of *Target Correlation Notional* and *Realised Correlation Notional* to describe the correlation exposure of the trade.

Generalised dispersion trades are dispersion trades where the basket of single-stock variances bought does not completely represent the members of the index, on which index variance is sold. This may be either due to liquidity issues, especially in Asia where only the largest few members have reasonably liquid variance (e.g. Kospi 200), or because investors wish to buy single-stock variance where they see it as cheap, and hedge with index variance. The p/l of these *enhanced,* or *generalised dispersion trades* can be accurately described using our new correlation measure.

In Sections 3 and 4, we extend these observations to ***(generalised) volatility pairs trades,*** in which a basket of stocks with apparently cheap variances can be traded against stocks whose variances appear rich. In this context (enhanced) dispersion trades can be seen as a special case where the rich variance is that of a single index, and simple volatility pairs trades as instances where the long and short baskets contain only one member each. Although somewhat counter-intuitive, this way of thinking about, and constructing, volatility pair trades can make sense, because even in this simple case, issues resulting from the convexity of the variance swap payoff can complicate the p/l.

Simon Miguez
(852) 2800-8855
Malcolm Barnett
(852) 2800-8857

1: Dispersion Trades

A *classic dispersion trade* is constructed by selling an index variance swap and simultaneously buying variance swaps on the index constituents according to their relative weights in the index. There are two principal flavours of dispersion trade:

- **Vanilla dispersion,** where the vega notional of the index and single-stock legs is the same; and
- **Correlation-weighted dispersion,** where the vega notional of the single-stock leg is scaled (down) by the square root of the implied correlation.

A **vanilla dispersion trade** is technically dollar vega neutral at inception. However, since correlation is (almost always) less than one, a given increase in single-stock volatility would result in a somewhat smaller increase in index volatility. Hence the vanilla dispersion trade, although providing some correlation exposure, would also behave as if it was long volatility. This is indeed seen to be the case by comparing p/l of these trades with changes in volatility.

In contrast, by **correlation-weighting** the trade, we ensure that if volatility changes, but correlation remains the same, then the (expected) p/l will be zero. However, due to the scaling effect of realised volatility on the p/l this is only the case at trade inception. It is this *correlation-weighted* dispersion trade which is more common. See *Correlation Vehicles, May 2005* for more details. **From now on by dispersion trade we will always mean a correlation-weighted dispersion trade.**

In the case of a correlation-weighted dispersion trade, where we can assume that the correlation proxy holds (correlation is approximately equal to the square of the ratio of index to average single-stock volatility), then we can come close to expressing the p/l as a simple function of implied minus subsequent realised correlation. However, even assuming that the correlation proxy is accurate, there remains a second, *volatility dispersion,* term in the p/l expansion which is not related to the correlation, and can be large under some circumstances. See *Correlation Vehicles* for details.

By **characterising correlation as a ratio of variances** we can simplify the description of the p/l further, without assuming anything about the accuracy of the proxy. In this case the p/l of a dispersion trade will be just the difference between implied and realised correlation under our new measure, multiplied by the average single-stock realised variance.

$$p/l = \sum_i w_i \sigma_i^2 \left(\rho_I - \rho_H\right)$$

where ρ_I and ρ_H are implied and subsequent realised correlation and $\sum_i w_i \sigma_i^2$ is average realised single-stock variance.

Not only does this measure of correlation simplify the calculation of the p/l *and* avoid any assumptions about the accuracy of the proxy, it also generalises perfectly to enhanced dispersion trades and (generalised) volatility pairs trades. We use the term **mean variance ratio** to refer to this form of correlation measure.

The only remaining slight complication is the presence of the scaling factor of the difference between implied and realised correlation. This means that when considered as a correlation trade, the correlation notional is not fixed.

We believe that this measure of correlation is superior to the standard measure for five reasons listed below:

1. The sign of the p/l of a correlation weighted dispersion trade will always be same as that of the implied minus realised correlation over the course of the trade. This is *not* the case with the traditional correlation measure.
2. The mean variance ratio is typically very close to traditional correlation (and to the proxy).
3. Using the mean variance ratio enables us to easily express the p/l from a dispersion trade and define a *Target Correlation Notional*.
4. Correlation measured with the mean variance ratio makes marking to market of dispersion trades straightforward.
5. The mean variance ratio generalises seamlessly to enhanced dispersion trades (even using non-index members) and to generalised volatility pairs trades.

Simon Miguez
(852) 2800-8855
Malcolm Barnett
(852) 2800-8857

Why is Mean Variance Ratio the best form of correlation to use?

Suppose a dispersion trade is initiated with unit index variance notional and β of aggregate single-stock variance notional.

Let K_{Index} and K_i be index and single-stock variance swap strikes, with σ_{Index} and σ_i the subsequent realised volatility and ω_i the weight of the i[th] stock in the index.

The p/l of a dispersion trade will be given by:

$$p/l = \left(K_{Index}^2 - \sigma_{Index}^2\right) - \beta \sum_i \omega_i \left(K_i^2 - \sigma_i^2\right) \quad = \left(K_{Index}^2 - \beta \sum_i \omega_i K_i^2\right) - \left(\sigma_{Index}^2 - \beta \sum_i \omega_i \sigma_i^2\right)$$

If we set the single-stock weighting factor $\beta = \dfrac{K_{Index}^2}{\sum_i \omega_i K_i^2}$, then the first term of the p/l will be zero, leaving:

$$p/l \quad = -\left(\sigma_{Index}^2 - \frac{K_{Iindex}^2}{\sum_i \omega_i K_i^2} \sum_i \omega_i \sigma_i^2\right) = \sum_i \omega_i \sigma_i^2 \left(\frac{K_{Index}^2}{\sum_i \omega_i K_i^2} - \frac{\sigma_{Index}^2}{\sum_i \omega_i \sigma_i^2}\right)$$

$$= \sum_i \omega_i \sigma_i^2 \left(\rho_I - \rho_H\right) \quad \text{where } \rho_I = \frac{K_{Index}^2}{\sum_i \omega_i K_i^2} \quad \rho_H = \frac{\sigma_{Index}^2}{\sum_i \omega_i \sigma_i^2}.$$

The ratios ρ_I and ρ_H can be seen as proxies for implied and realised correlation respectively.

We refer to these measures of correlation as the implied and realised ***mean variance ratio (MVR)*** respectively.

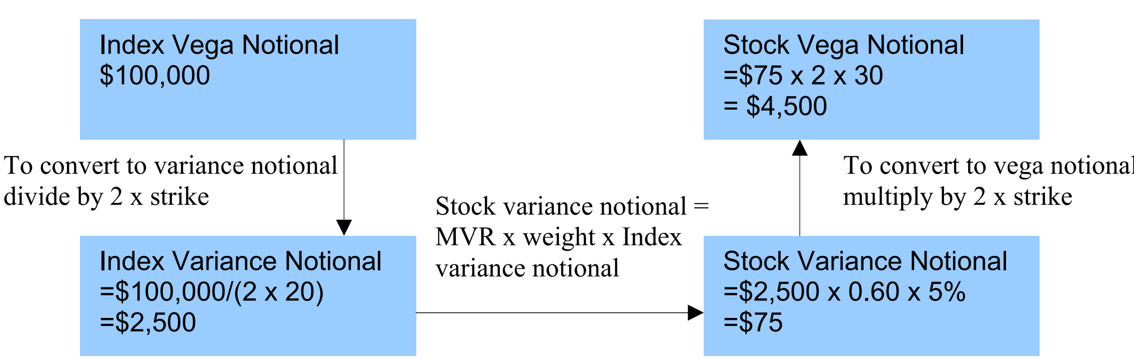
Note that our European team has previously used the term ***RMS Correlation*** for the same concept.

Weighting of Mean Variance Ratio Trades

When working with mean variance ratios, we need to work with *variance* notionals to weight dispersion trades. We first convert our required index vega notional to variance notional, then compute the single-stock variance notionals using the implied mean variance ratio and stock weights. Finally we convert single stock variance notionals back to vega notionals.

For example if we require $100K of index vega notional, with the index implied variance at 20, then the index **variance** notional will be $2,500. Assuming the implied MVR is 0.6, then a stock with weight 5% will need a variance notional of 0.6 x 0.05 x $2.5K = $75. If the implied variance of the stock is 30, then the vega notional required for that stock will be 2 x 30 x $75 = $4.5K (Figure 2). This compares to $3.873K (=$100K x 5% x sqrt(0.6)) vega notional for a traditional correlation-weighted dispersion trade.

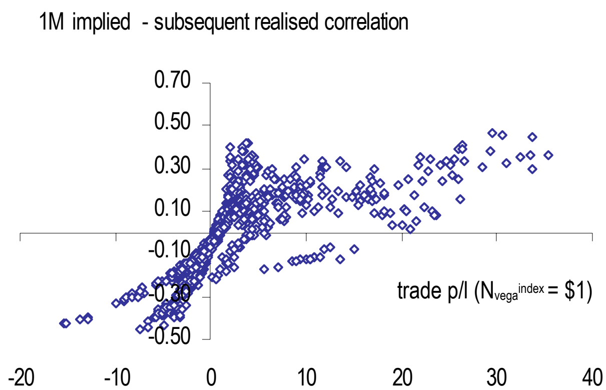
Figure 2: Calculating vega notionals for MVR dispersion trade



1. The sign of the p/l of a correlation weighted dispersion trade will always be same as that of the implied minus subsequent realised correlation over the course of the trade.

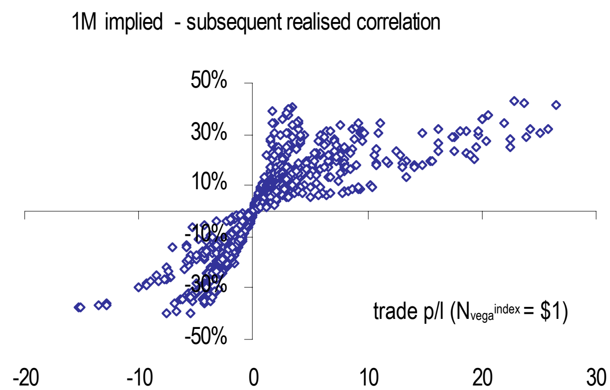
By working in terms of correlation as measured by the mean variance ratio, a (correlation weighted) dispersion trade will profit exactly when the level of implied correlation sold exceeds that subsequently realised. This is usually, but not always the case with traditional correlation. The reason for this is the use of the correlation proxy, and the volatility dispersion term present in the usual formulation of dispersion trade p/l (see *Correlation Vehicles*).

Figure 3: Relationship between traditional correlation and the trade p/l



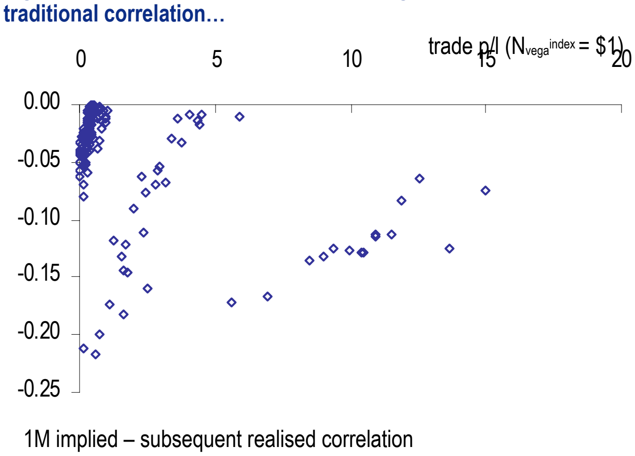
Source: JPMorgan, DAX backtest

Figure 4: Relationship between "new" correlation and the trade p/l



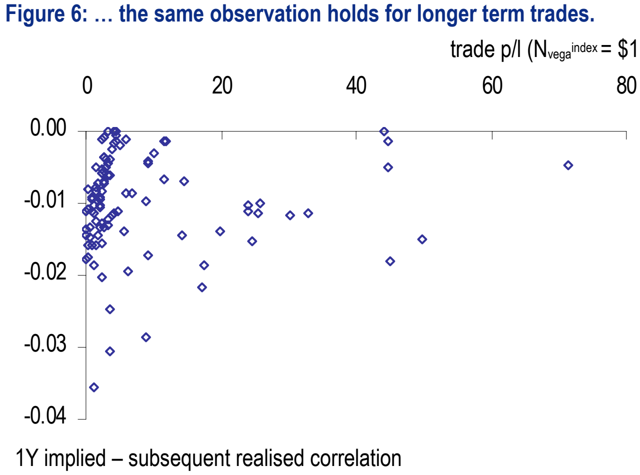
Source: JPMorgan, DAX backtest

Figure 5: Some trades fall in the wrong quadrant when we use traditional correlation…



Source: JPMorgan, DAX backtest

Figure 6: … the same observation holds for longer term trades.



Source: JPMorgan, DAX backtest

Figure 3 shows the relationship between dispersion trade p/l and traditional *correlation p/l* (measured as the difference between traditional implied and subsequent realised correlation) for 1-month maturity trades on the DAX, initiated on each business day between December 2001 and November 2006. Whilst the dispersion trades which profit are usually those where correlation moves favourably this is not always the case and some trades fall in the *wrong* quadrants. This continues to be the case, even when we consider trades of longer maturities (Figure 6).

In contrast, when correlation is measured using the mean variance ratio measure introduced above (Figure 4), the dispersion trade profits exactly when expected, in terms of the movement in correlation. I.e. in Figure 4 trades always fall in the first and third quadrants.

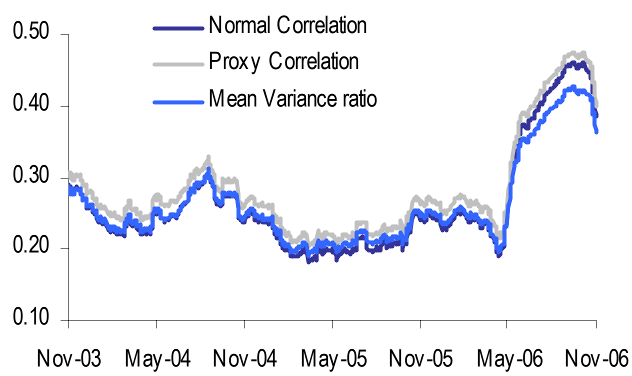
2. The mean variance ratio is typically very close to traditional correlation, especially for relatively large and well diversified indices (Figure 7, Figure 8).

As in the case of traditional correlation, the mean variance ratio will always be less than (or equal) to the correlation proxy (the ratio of index volatility squared to the square of average single-stock volatility). In fact the difference between the mean variance ratio and correlation proxy is related to the dispersion of volatilities within the index. The difference will be zero if all the volatilities of the constituent stocks are the same.

In contrast, the mean variance ratio can be either greater than or less than the traditional correlation (Figure 10). For smaller, or less diversified indices, it tends to be greater. For larger, more diversified indices (e.g. the Euro Stoxx 50) it tends to be slightly less. See *European Equity Derivatives Weekly 2nd October 2006* for details.

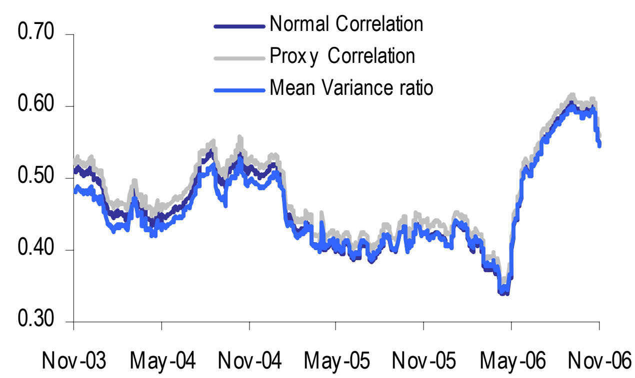
Perhaps most importantly, changes in correlation (meaning implied minus subsequent realised) are well correlated for the traditional and mean variance ratio measures of correlation. That is, if for a particular index the traditional 6-month implied correlation is greater than that realised 6-months later, then it is highly likely that the same is true for the mean variance ratio measure of correlation (Figure 9).

Figure 7: The mean variance ratio is a good proxy for correlation for FTSE…



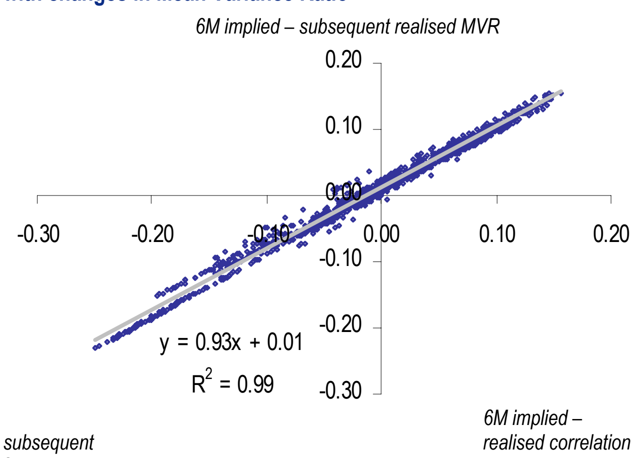
Source: Bloomberg, JPMorgan

Figure 8: … as well as for Euro Stoxx 50.



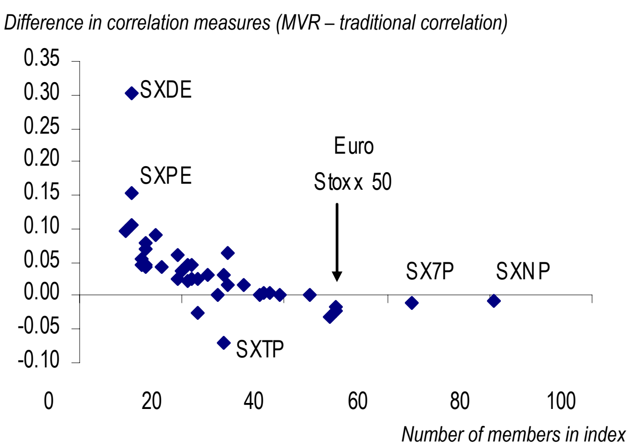
Source: Bloomberg ,JPMorgan

Figure 9: Changes in Euro Stoxx correlation are very well correlated with changes in Mean Variance Ratio



Source: JPMorgan

Figure 10: For reasonably large indices MVR and correlation are at similar levels



Source: JPMorgan

3. Unlike a correlation swap, which pays out directly on implied minus subsequent realised correlation, the exposure to correlation of a dispersion trade will vary according to the level of (average single-stock) volatility realised over the course of the trade. By using a mean variance ratio to measure correlation, the p/l of a dispersion trade can be easily expressed in terms of correlation and volatility, and a target correlation notional defined.

If N_{Index} represents the variance notional of the index variance swap, then p/l of a dispersion trade will be given by:

$$p/l = N_{Index} \times \sum_i \omega_i \sigma_i^2 \left(\rho_I - \rho_H\right) \text{ where } \rho_I \text{ and } \rho_H \text{ are the implied and realised mean variance ratios.}$$

This means, that give-or-take a factor of 100, the per-correlation-point p/l will be the product of the index variance notional and the average single-stock realised variance. This quantity is called the ***realised correlation notional (RCN).*** By definition, the RCN cannot be known until trade maturity, but can be estimated by the *target correlation notional.*

Investors entering into a dispersion trade will want some idea of their exposure to correlation. Whilst, as we have seen above, the actual correlation exposure is not fixed, it is possible to estimate it. The ***target (or implied) correlation notional (TCN)*** is defined as the *expected* p/l per point change in correlation. Hence the TCN is the expected value of the RCN, and is easily seen to be the product of the index variance notional and the average single-stock constituent implied variance.

Since the target correlation notional is not fixed, it will change over the course of the trade as realised variance is accrued and implied variance changes. If volatility increases the TCN will increase. If volatility decreases TCN will decrease. The ***vega sensitivity of the TCN*** is defined to be the average change in TCN for a 1 vega change in all basket constituent volatilities. This turns out to be twice the weighted average of the mark-to-market values of the single-stock variance swaps. In particular, not only does the TCN (exposure to correlation) increase with volatility, but so does the rate of change on the TCN. Thus TCN will be larger *and* increase more quickly if volatility is higher: TCN is *convex* in volatility.

Target and Realised Correlation Notional

Suppose that N_{Index} is the index variance notional for a correlation weighted dispersion trade (using MVR).

The p/l of the trade is given by: $N_{Index} \times \sum_i \omega_i \sigma_i^2 \left(\rho_I - \rho_H\right)$ where $\rho_I = \dfrac{K_{Index}^2}{\sum_i \omega_i K_i^2}$ and $\rho_H = \dfrac{\sigma_{Index}^2}{\sum_i \omega_i \sigma_i^2}$

We will now measure correlation in whole number units (e.g. 0.40 will be represented as a correlation of 40).

By defining $RCN = N_{Index} \times \dfrac{\sum_i \omega_i \sigma_i^2}{100}$ dispersion p/l is given by: $p/l = RCN \times \left(\rho_I - \rho_H\right)$

Given the above definition of RCN, we can define the target (or implied) correlation notional TCN by:

$$TCN = N_{Index} \times \frac{\sum_i \omega_i K_i^2}{100} \text{ giving the expected p/l per point change in correlation.}$$

The *vega sensitivity of the TCN* is the average change in TCN for a 1 vega change in all single stock variance strikes:

$$\textit{vega sensitivity of the } TCN = \frac{1}{2}\left[\frac{\sum_i \omega_i \left(K_i + 1\right)^2}{100} - \frac{\sum_i \omega_i \left(K_i - 1\right)^2}{100}\right] = \frac{2\sum_i \omega_i K_i}{100}$$

4. Working with correlation, as measured by the mean variance ratio, allows the mark-to-market of a dispersion trade to be easily expressed.

By using mean variance ratio to measure correlation, p/l of dispersion trades can be calculated in four easy stages:

1. **Compute the mark-to-market of the index and constituent single-stock variance swaps** – easy, since variance is additive.
2. **Compute the average of the single-stock mark-to-market variances**. This will give the mark-to-market for the target correlation notional
3. **Compute the mark-to-market correlation** (the ratio of mark-to-market index variance to the average single-stock mark-to-market variance).
4. **Compute the mark-to-market p/l** (mark-to-market TCN multiplied by implied minus MTM correlation)

Dispersion trade mark-to-market

Consider a dispersion trade (index variance notional N_{index}) of maturity *n* days and we want to mark-to-market after *m* days.

Let $\sigma^2_{i,m}$ be the annualised variance of asset *i* realised between the inception of the trade and day *m* and let $K^2_{i,m}$ be the strike (squared) on day *m* of a variance swap on asset *i* expiring on *n* (the trade expiry).

Then the mark-to-market value of a variance swap on asset i is given by $\tilde{\sigma}^2_{i,m} = \frac{m}{n}\sigma^2_{i,m} + \frac{n-m}{n}K^2_{i,m}$

We then calculate a mark-to-market Target Correlation Notional as $TCN_m = N_{Index} \times \sum_i \omega_i \tilde{\sigma}^2_{i,m}$

…and a *mark-to-market realised correlation* as $\tilde{\rho}_m = \frac{\tilde{\sigma}^2_{Index}}{\sum_i \omega_i \tilde{\sigma}^2_{i,m}}$.

We can then give the mark-to-market p/l of the dispersion trade after *m* days as:

$$p/l = N_{Index} \times \sum_i \omega_i \tilde{\sigma}^2_{i,m}\left(\rho_I - \tilde{\rho}_m\right) \quad = TCN_m \times \left(\rho_I - \tilde{\rho}_m\right)$$ … or in full as:

mark-to-market = $$N_{Index} \times \sum_i \omega_i \left(\frac{m}{n}\sigma^2_{i,m} + \frac{n-m}{n}K^2_{i,m}\right)\left(\frac{K^2_{Index}}{\sum_i \omega_i K_i^2} - \frac{\frac{m}{n}\sigma^2_{Index,m} + \frac{n-m}{n}K^2_{Index,m}}{\sum_i \omega_i \left(\frac{m}{n}\sigma^2_{i,m} + \frac{n-m}{n}K^2_{i,m}\right)}\right)$$

Note that we have arrived at the formulation of the mark-to-market p/l by assuming it makes sense to mark to market both the correlation and the target notional. We can verify that this is indeed the case by directly computing the mark-to-market of the constituent variance swaps and calculating the MTM p/l of the dispersion trade considered as a weighted spread trade. Rearranging gives the above equation, showing that our method is consistent.

5. By using the mean variance ratio instead of traditional correlation measures, virtually all of the above discussion generalises seamlessly to enhanced dispersion trades (even using non-index members) and to generalised volatility pairs trades. These trades and their relationship with classic dispersion will be discussed in detail in the following sections.

Example 1 below summarises all the concepts introduced.

Example 1:
Assume an equally-weighted index consisting of two stocks A and B. We initiate a 1-year dispersion trade on the index according to our convention. The *vega* notional of the index trade is assumed to be \$100K .

At inception, we have $K_{Index} = 20.3$, $K_A = 25.75$, $K_B = 34.5$ and $\omega_A = \omega_B = 50\%$

Hence,
$$\rho_I = \frac{K_{Index}^2}{\sum_i \omega_i K_i^2} = \frac{20.3^2}{0.5 \times (25.75^2 + 34.5^2)} = 0.4475$$

(This gives the beta adjustment for the single-stock variance notionals and the MVR implied correlation).

The index *variance* notional will be $N_{Index} = \dfrac{\$100K}{2 \times 20.3} = \$2,463$

The variance notionals of the constituents will be the same for both stocks since they both have the same weight:

$$N_A = N_B = N_{Index} \times 50\% \times 0.4475 = \$548 .$$

Then, target correlation notional
$$TCN = N_{Index} \times \frac{\sum_i \omega_i K_i^2}{100} = \$2,463 \times \frac{0.5 \times (25.75^2 + 34.5^2)}{100} = \$22,824$$

This sets the *expected* correlation notional at \$22,824 per correlation point.

Vega sensitivity of TCN is:
$$N_{Index} \times \frac{2\sum_i \omega_i K_i}{100} = \$2,463 \times \frac{2 \times 0.5 \times (25.75 + 34.5)}{100} = \$1,484$$

I.e. if average single-stock variance changes by 1 pt, exposure to correlation will, on average, change by \$1,484 per pt.

At expiry after 1 year, realised volatilities are: $\sigma_{Index} = 18.31$, $\sigma_A = 23.51$, $\sigma_B = 34.99$.

Hence,
$$\rho_H = \frac{\sigma_{Index}^2}{\sum_i \omega_i \sigma_i^2} = \frac{18.31^2}{0.5 \times (23.51^2 + 34.99^2)} = 0.3775$$

and
$$RCN = \$2,463 \times \frac{0.5 \times (23.51^2 + 34.99^2)}{100} = \$21,884$$

The p/l of the trade is,
$$p/l = 100 \times RCN \times (\rho_I - \rho_H) = \$21,884 \times (44.47 - 37.75) = \$147,069$$

And we can verify that:

$$p/l = N_{Index} \times (20.3^2 - 18.31^2) - N_A \times (25.75^2 - 23.51^2) - N_B \times (34.5^2 - 34.99^2) = \$147,069$$

2: Generalised Dispersion Trades

Index dispersion trades usually consist of selling the variance of an index against buying the variance of its constituents (according to their weights in the index). However generalised-dispersion trades can be constructed using only a subset of the stocks in the index, or even using stocks outside the index. There are two principal reasons for not using the entire index:

1. **Liquidity issues**: For some indices liquidity factors make it impossible to trade variance swaps on all the index members. This is a common issue especially with Asian indices, where dispersion trades on the Kospi200, Hang Seng, HSCEI, TWSE and ASX200 are usually set up by buying variance on only the top 10-15 most liquid names in the index.

2. **Relative value of volatility**: One key reason for trading dispersion is that the premium of implied over realised volatility tends to be greater for indices than for single stocks, given investors' bias to be long index volatility to protect their portfolio. When trading the volatility of an index against that of its members, this is expressed by the fact that implied correlation tends to trade rich to that realised. However, the potential profitability of a dispersion trade can be enhanced by choosing to buy single-stock variances which offer particularly good value. These stocks could form a subset of the index, or may even be chosen from outside the index. Setting up a relative value trade by trading a basket of rich looking single-stock volatilities against a basket of cheap looking single-stock volatilities would be a way to generalise this idea, which we will expand upon in the following section.

The Mean Variance Ratio generalises directly to Generalised Dispersion Trades

Suppose a generalised dispersion trade is initiated with an arbitrary basket of single-stock variances. We hold unit index variance notional and β of total single-stock variance notional.

The p/l will be given by:

$$\begin{aligned} p/l &= \left(K_{Index}^2 - \sigma_{Index}^2\right) - \beta\sum_i \omega_i\left(K_i^2 - \sigma_i^2\right) \\ &= \left(K_{Index}^2 - \beta\sum_i \omega_i K_i^2\right) - \left(\sigma_{Index}^2 - \beta\sum_i \omega_i \sigma_i^2\right) \end{aligned}$$

and if we set $\beta = \frac{K_{Index}^2}{\sum_i \omega_i K_i^2}$, the implied mean variance ratio, then the first term of the p/l will be zero, leaving:

$$p/l = \sum_i \omega_i \sigma_i^2 \left(\rho_I - \rho_H\right)$$

where $\rho_I = \frac{K_{Index}^2}{\sum_i \omega_i K_i^2}$ $\rho_H = \frac{\sigma_{Index}^2}{\sum_i \omega_i \sigma_i^2}$.

Where ρ_I and ρ_H are respectively the implied and realised mean variance ratios.

We can analogously define the Target correlation notional (TCN) to be

$$TCN = N_{Index} \times \frac{\sum_i \omega_i K_i^2}{100}$$

giving the expected p/l per point change in MVR over the lifetime of the trade.

Simon Miguez
(852) 2800-8855
Malcolm Barnett
(852) 2800-8857

Identical to the case of classic dispersion trades, the p/l of a generalised dispersion trade can be simply expressed as the product of the difference between the implied and realised mean variance ratios with the average variance of the single-stock basket realised over the trade. In the same way, we can also define the target correlation notional (TCN) to give the expected exposure to changes in the mean variance ratio.

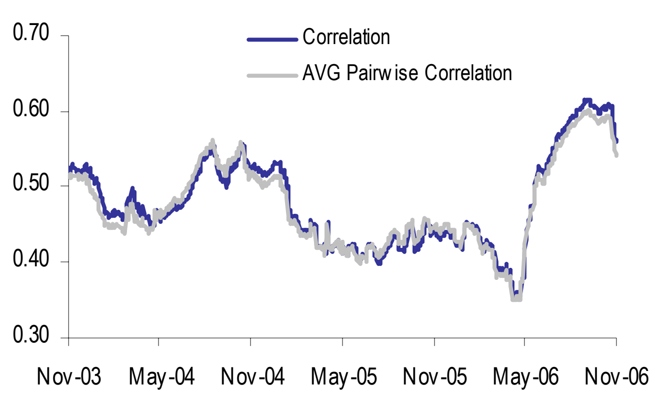
In the case of generalised dispersion trades, using the mean variance ratio is preferable to using the traditional correlation measure in our view. Firstly, this traditional measure neglects the tracking error that appears when replicating an index with only a subset of constituents, or a basket containing stocks from outside the index. Secondly, the fundamental relationship $\sum_{i,j} \omega_i \omega_j \sigma_i \sigma_j \rho_{index} = \sum_{i,j} \omega_i \omega_j \sigma_i \sigma_j \rho_{i,j}$ fails to hold when the basket of stocks does not accurately reflect the index constituents. Hence the traditional correlation calculated using only the selected basket members can be somewhat meaningless and a poor estimator of true index correlation.

Moreover, when using the traditional correlation measure, the only way to express the resulting dispersion p/l in a reasonable tractable form involves assuming that the correlation proxy holds. We already know that this proxy breaks down for small or undiversified indices, and errors from using this proxy will further increase if the basket of single stocks fails to accurately represent the index members.

When working in terms of traditional correlation, a generalised dispersion trade in Asia suffers from these two disadvantages. Firstly, indices tend to have “heavy” members (eg HSBC in HSI, Samsung in KOSPI, PetroChina in HSCEI) with large weights in the index. This means that even if the index has a relatively large number of members, the correlation proxy will not usually be accurate. Secondly, due to liquidity issues, in Asia dispersion trades are typically constructed using basket of only 10 to 15 names. Thus, not only does the use of the correlation proxy run the risk of achieving a poor approximation of traditional correlation, but traditional correlation itself is not a good arbiter of p/l.

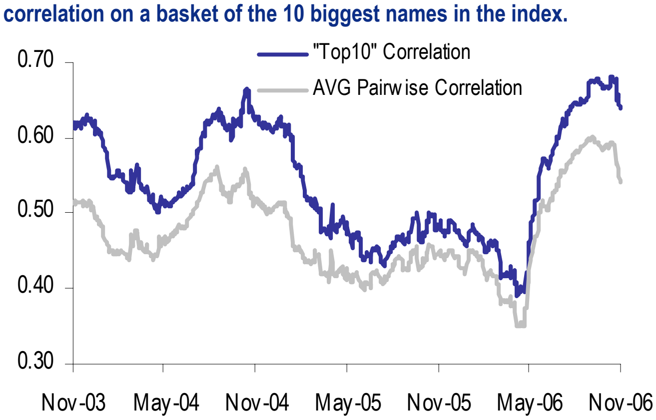
The above argument suggests that the traditional correlation measure is only useful and truly meaningful in a classic dispersion trade where index variance is traded against the full basket of constituent variance. Even in this case the mean variance ratio, which it closely mirrors, is easier to work with. In the case of generalised dispersion trades where the proxy breaks down, and traditional correlation is in any case no longer appropriate, the mean variance ratio continues to be both useful and meaningful and all properties associated with this measure (characterisation of p/l, mark-to-market, target correlation notional, etc.) generalise directly from the classic dispersion case.

Figure 11: Traditional Correlation is good estimator of the average pairwise correlation for the Euro Stoxx 50 …



Source: JPMorgan

Figure 12: … but this does not hold very well when we calculate the correlation on a basket of the 10 biggest names in the index.



Source: JPMorgan

Simon Miguez
(852) 2800-8855
Malcolm Barnett
(852) 2800-8857

Example 2: Constructing a generalised dispersion trade on the KOSPI using the Mean Variance Ratio convention.

Suppose that on 1 June 2006 we construct a Top-10 Kospi generalised dispersion trade as follows:

Sell a 6-month Kospi200 variance swap with $100K of vega notional, with strike 21.2

Buy a basket of 6-month variance swaps on the top 10 names in the Kospi200 (see below for the weightings)

The index *variance notional* will be $100K / (2 x 21.2) = $2,359

The mean variance ratio (beta adjustment) is given by dividing index variance by average single-stock variance:

Mean Variance Ratio (β) = $\rho_I = \dfrac{K_{Index}^2}{\sum_i \omega_i K_i^2} = \dfrac{21.2^2}{911.6} = 0.493$,

The single-stock variance notionals are then calculated by multiplying the index variance notional by the stock's weight in its basket and finally by the mean variance ratio. For example, in the case of Samsung Electronics which represents 15.1% of the index and 34.4% of the top 10 basket, the variance notional will be given by multiplying index variance notional ($2,359) by the basket weight (34.4%) and finally by the MRV (0.493). Therefore the variance notional will be $2,359 x 0.343 x 0.493 = $399.

As a final step we must calculate the single-stock *vega* notionals given by multiplying the variance notionals by twice the strike. In the case of Samsung with variance notional $399 and variance strike 26.506 the vega notional will be $21,152.

Target Correlation Notional $TCN = N_{Index} \times \dfrac{\sum_i \omega_i K_i^2}{100} = \$21{,}505$

Ticker	Name	Wgt in index	Wgt in basket	Variance swap strike	Variance notional	Vega notional	Realised volatility on December 1st 2006
KOSPI2	KOSPI 200	-	-	21.2	$2,359	$100,000	16.3
005930 KS	Samsung Electronics	15.1%	34.3%	26.5	$399	$21,152	20.2
005490 KS	Posco	4.4%	10.0%	32.5	$117	$7,586	20.2
015760 KS	Korea Electric Power	4.4%	9.9%	30.0	$115	$6,870	14.6
060000 KS	Kookmin Bank	4.0%	9.1%	38.0	$106	$8,085	28.8
017670 KS	Sk Telecom	3.0%	6.9%	27.7	$80	$4,418	26.3
055550 KS	Shinhan Financial	2.9%	6.7%	28.8	$77	$4,458	18.9
053000 KS	Woori Finance	2.9%	6.5%	32.3	$76	$4,893	25.9
000660 KS	Hynix Semico	2.7%	6.2%	41.2	$72	$5,902	31.5
005380 KS	Hyundai Motor	2.4%	5.5%	29.6	$64	$3,795	25.1
030200 KS	Kt Corp	2.2%	5.0%	18.0	$58	$2,076	14.9

At expiry we compute $\rho_H = \dfrac{\sigma_{Index}^2}{\sum_i \omega_i \sigma_i^2} = 0.526$ and $RCN = N_{Index} \times \dfrac{\sum_i \omega_i \sigma_i^2}{100} = \$11{,}946$

The p/l of the trade is, $p/l = 100 \times RCN \times (\rho_I - \rho_H) = \$11{,}946 \times (49.3 - 52.6) = -\$39{,}422$

That, is the trade lost 3.3 'MVR-correlation points' which was multiplied by the RCN of c.$12K to give a loss of $39,422.

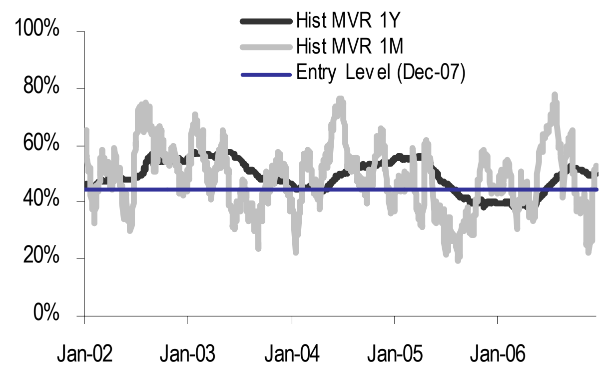
Below we give an overview of our newly introduced parameters for the Hang Seng and KOPSI200.

Table 1: Indicative Top10 basket of HSI members Dec-07 variance.

Ticker	Name	Weight in Index	Vega Notional	Indicative Variance Level
HSI Index	Hang Seng Index	-	-$100,000	16.10%
5 HK Equity	Hsbc Holdings Plc	24.65%	$16,676	15.00%
941 HK Equity	China Mobile Ltd	19.89%	$5,936	34.00%
13 HK Equity	Hutchison Whampoa Ltd	4.88%	$2,514	19.70%
883 HK Equity	Cnooc Ltd	4.56%	$1,641	28.20%
16 HK Equity	Sun Hung Kai Properties	3.46%	$1,603	21.90%
1 HK Equity	Cheung Kong Holdings Ltd	3.31%	$1,534	21.90%
11 HK Equity	Hang Seng Bank Ltd	3.08%	$2,562	12.20%
2388 HK Equity	Boc Hong Kong Holdings Ltd	3.07%	$1,520	20.50%
939 HK Equity	China Construction Bank-H	2.16%	$777	28.20%
2 HK Equity	Clp Holdings Ltd	1.94%	$1,458	13.50%

Implied MVR (Current)	44.75
Realised 1Y MVR (Current)	49.89
Realised 1M MVR (Current)	52.90
Adjustment Coefficient	0.45
Vega Notional (index Leg)	$100,000
Variance Notional (index Leg)	$3,106
Target Correlation Notional	$17,989
TCN Vega Sensitivity	$1,429

Figure 13: HSI implied vs. realised MVR correlation



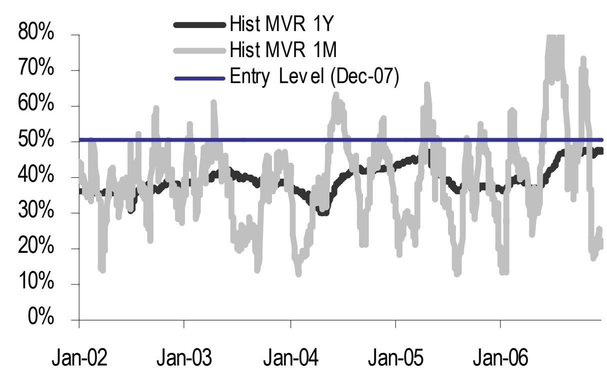
Source: Bloomberg, JPMorgan, , cob 11 Dec 06

Table 2: Indicative Top10 basket of KOSPI members Dec-07 variance.

Ticker	Name	Weight in Index	Vega Notional	Indicative Variance Level
KOSPI2 Index	Kospi 200 Index	-	-$100,000	19.00%
005930 KS Equity	Samsung Electronics Co Ltd	8.76%	$13,617	25.20%
005490 KS Equity	Posco	5.94%	$3,573	25.90%
015760 KS Equity	Korea Electric Power Corp	5.77%	$4,034	22.50%
060000 KS Equity	Kookmin Bank	4.85%	$2,861	30.50%
017670 KS Equity	Sk Telecom	4.16%	$2,097	29.80%
055550 KS Equity	Shinhan Financial Group Ltd	2.88%	$2,324	26.80%
053000 KS Equity	Woori Finance Holdings Co	2.48%	$2,169	28.40%
000660 KS Equity	Hynix Semiconductor Inc	2.14%	$1,800	34.10%
005380 KS Equity	Hyundai Motor Co	2.04%	$2,076	26.40%
030200 KS Equity	Kt Corp	1.97%	$2,446	19.90%

Implied MVR (Current)	50.79
Realised 1Y MVR (Current)	47.57
Realised 1M MVR (Current)	21.19
Adjustment Coefficient	0.51
Vega Notional (index Leg)	$100,000
Variance Notional (index Leg)	$2,632
Target Correlation Notional	$18,705
TCN Vega Sensitivity	$1,429

Figure 14: KOSPI implied vs. realised MVR correlation



Source: Bloomberg, JPMorgan, , cob 11 Dec 06

3: Generalised Volatility Pair Trades.

A generalised volatility pair trade can be seen as a trade where an investor sells a basket of variance swaps on single names with overvalued implied volatilities whilst buying a basket of variance swaps on single names with undervalued implied volatilities. Various motivations could be behind such a trade amongst which we can highlight the following:

- **Carry trade:** in a similar way to how index volatility trades rich relative to single-stock volatility, some single stock volatilities (systematically) trade rich in comparison to the volatilities of other stocks. This positive 'carry' can be reflected by the implied mean variance ratio trading at a premium to the historical mean variance ratio. A generalised volatility pair trade can be used to exploit this discrepancy in a analogous manner to (generalised) dispersion trades.

- **Relative values trades:** Investors may have views on the relative values of single-stock volatilities over and above that given by the mean variance ratio. For instance they may have fundamental reasons for believing that one set of stocks will be more volatile than another set, or use a metric other than variance ratios to establish the richness or cheapness of single-stock variances – e.g. a cross-sectional regression model. In either case use of the implied mean variance ratio to weight the trade can help to remove overall sensitivity to volatility, and to provide a simple characterisation of the resulting p/l.

By using mean variance ratio – defined as the ratio of mean variance of the short variance basket to the mean variance of the long variance basket – these generalised volatility pair trades can be treated under the same framework as generalised dispersion trades. That is

1. The variance notional of the long variance leg is weighted by the implied mean variance ratio;
2. The expected exposure to the change in the mean variance ratio is given by the Target Correlation Notional;
3. The final p/l is proportional to the realised variance of the long basket and difference between implied and subsequent realised MVR.

This allows us to deal with generalised volatility pair trades in the same way as we did for generalised dispersion trades. In fact, in the special case where the short variance basket consists of a single index variance swap, the generalised volatility pair trade becomes a generalised dispersion trade. In the more specific situation where the long variance basket contains the (weighted) variances of the constituents of the index in the short variance basket, then we have a classic dispersion trade. In this way classic dispersion, generalised dispersion and volatility pair trades can all be brought under a single framework.

Simon Miguez
(852) 2800-8855
Malcolm Barnett
(852) 2800-8857

Generalised Volatility Pair Trades

Suppose we have two baskets of variance swaps:

a *short basket* consisting of variance swaps on assets $j=1,\dots m$ with variance strikes $K_1,\dots K_m$ and weights $\omega_1,\dots \omega_m$

a *long basket* consisting of variance swaps on assets $i=1,\dots .n$ with variance strikes $K'_1,\dots K'_n$ and weights $\omega'_1,\dots \omega'_n$

We assume the weights attributed to each of the variance swaps are such that the weights of each basket add to 1.

The p/l of the variance spread trade (in terms of variance notionals) is then given by:

$$p/l = \sum_j \omega_j \left(K_j^2 - \sigma_j^2\right) - \beta \sum_i \omega'_i \left(K_i'^2 - \sigma_i^2\right).$$

If we set the beta adjustment factor to $\beta = \frac{\sum_j \omega_j K_j^2}{\sum_i \omega'_i K_i'^2}$ (the implied mean variance ratio) then the p/l can be simplified to:

$$p/l = \sum_i \omega_i \sigma_i^2 \left(\rho_I - \rho_H\right)$$

where $\rho_I = \frac{\sum_j \omega_j K_j^2}{\sum_i \omega'_i K_i'^2}$ and $\rho_H = \frac{\sum_j \omega_j \sigma_j^2}{\sum_i \omega'_i \sigma_i'^2}$ are respectively the implied and realised mean variance ratios.

Again, if we define N_{Short} to be the variance notional of the short basket, we can define our ***target correlation notional***:

$$TCN = N_{Short} \times \frac{\sum_i \omega'_i K_i'^2}{100}.$$

Note: This generalisation illustrates our motivation to label $\frac{\sum_j \omega_j K_j^2}{\sum_i \omega'_i K_i'^2}$ *as the mean variance ratio. The RMS correlation introduced in section 1 is indeed only a particular case of that ratio.*

Simon Miguez
(852) 2800-8855
Malcolm Barnett
(852) 2800-8857

4: Simple Volatility Pair Trades

Volatility pair trades are a class of trades in which an investor takes a view on the relative value of the volatility of two assets. Typically these trades will involve either two indices, or two stocks belonging to the same sector, or even a stock against its sector index when variance swaps are available on the latter.

Volatility pair trades are typically quoted as a spread of volatilities corresponding to the difference between the strikes of the variance swaps on the two assets. It is tempting to think that the resulting p/l of this trade will be directly proportional to the difference between the spread of implied volatilities (variance swap strikes) and the spread of realised volatilities at the expiry of the trade. This could be indeed achieved if the trade was put on with (linear) volatility swaps.

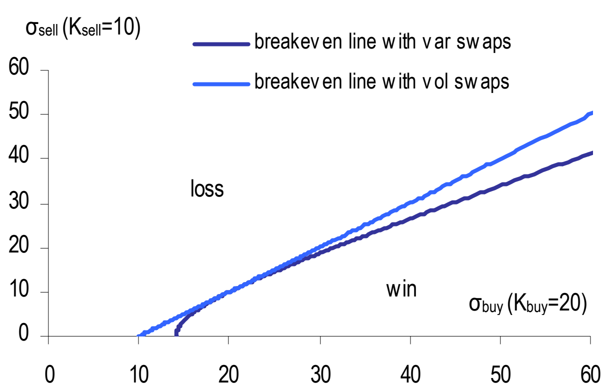
However, the p/l of a variance swap is non-linear, and this can have somewhat counter-intuitive consequences for volatility pair trades. See example below. Variance swap p/l can be broken down into a linear term in volatility (that will be proportional to the difference between the strike of the variance swap and the realised volatility at expiry) and a convexity term (that will be proportional to the square of the difference between the strike of the variance swap and the realised volatility at expiry). The convexity of the p/l will be particularly felt in a large move in volatility relative to the strike (see Figure 15). In the case of a small move from the strike, the p/l of the trade *will* be approximately linear in volatility.

Example: Suppose we enter into a volatility pair trade, short a variance swap with strike 10 and long a variance swap with strike 20. At expiry volatility has increased and the 10-strike variance swap has realised 20 and the 20-strike variance swap has realised 31. Note that we were correct on movement in the volatility spread: volatility on the asset on which we are short variance has increased by 10, less than the 11 vega increase on the long volatility asset. We may therefore naively expect to make a p/l proportional to the 1 vega difference in volatility spreads.

However if we compute the p/l of the variance swaps we find this is not the case. Assuming unit vega notional on each leg, the short leg has made $(10^2 - 20^2)/(2\text{x}10) = -15$; whereas the long leg has made $(31^2 - 20^2)/(2\text{x}20) = 14$. Thus even though volatility has apparently moved in our favour, the convexity of the variance swap payout means that the pairs trade will have *lost* about a vega.

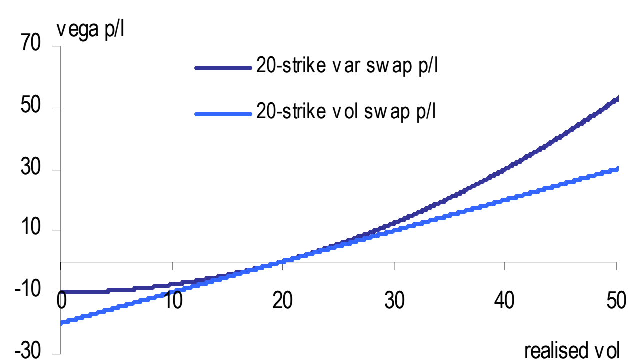
In practice, in a volatility pair trade, the convexity on one leg will often at least partially offset the convexity on the other leg leaving a function close to linear in the spread of volatilities. However, if the volatilities of the assets move in opposite directions the convexity can become more apparent. Also, as in the example above, when the strikes of the two variance swaps traded are far apart, the difference in convexity terms for similar sized moves can be noticeable.

Figure 15: Convexity appears in a volatility pair trade...



Source: JPMorgan

Figure 16: ...because they use variance swaps.



Source: JPMorgan

Figure 15 shows that a combined small move in volatility on the two names will result in a p/l close to linear in spread. However combined large moves in volatility will involve convexity in the p/l. As a result the realised spread could be bigger than the implied spread and yet the trade could lose. Conversely the trade could win in spite of the realised spread being smaller than the implied spread (in Figure 15, all the points located between the two breakeven lines will comply with this condition). These relatively extreme scenarios may be unlikely, but investors should be aware of them as they show that even relatively simple volatility pairs trades can produce unexpected results due the convex payout of the variance swap.

We therefore suggest an extension of the results and conventions introduced previously to the case of volatility pair trades. These trades are clearly a special case of the generalised volatility pair trades discussed in the section above, with only one variance swap underlying per long and short basket. By allowing investors to trade the ratio (rather than spread) of volatilities, they will offer investors an alternative way to look at these kind of trades. Following the same framework as previously we can show that the p/l's of these types of trades are directly proportional to the difference between the realised ratio of variances (or the ratio of realised variance) and the implied ratio of variances.

Quoting a volatility pair trade with a level corresponding to a ratio of variances rather than a spread of volatilities may prove less intuitive. In addition, it is not clear whether it is spreads or ratios one should seek to trade. Which are insensitive to the overall direction of volatility? Looking at the data, it seems that the truth is somewhere in between. However, we believe that using the mean variance ratio to trade the ratio of variances is an interesting alternative to standard pairs trades, and allows us to avoid the occurrence of an unexpected p/l at the expiry of the trade.

Example continued: In the example above the variance ratio sold was $10^2/20^2 = 0.25$. The subsequent realised variance ratio was $20^2/31^2 = 0.416$. Since the ratio increased, this means that the trade will have lost. The loss will be equal to the difference in ratios multiplied by the realised variance of the long variance swap. In this case, this gives a p/l of 31^2 x (0.25 – 0.416) = -159.5 times the short variance swap notional. This is equates to a loss of 8 times the vega notional of the short.

Weighting a simple volatility pair trade

A simple volatility pairs trade will have p/l of the form:

$$p/l = \left(K_{sell}^2 - \sigma_{sell}^2\right) - \beta\left(K_{buy}^2 - \sigma_{buy}^2\right).$$

This is easily transformed into $p/l = \sigma_{buy}^2\left(\rho_I - \rho_H\right)$

by fixing the beta adjustment factor to be $\beta = \frac{K_{sell}^2}{K_{buy}^2}$ where $\rho_I = \frac{K_{sell}^2}{K_{buy}^2}$ and $\rho_H = \frac{\sigma_{sell}^2}{\sigma_{buy}^2}$.

Note that since all the ratios are positive the p/l of the trade as demonstrated above, will be positive exactly when the ratio of realised volatilities is less than the ratio of implied variances sold (even taking into account the convexity of the variance swap pay off).

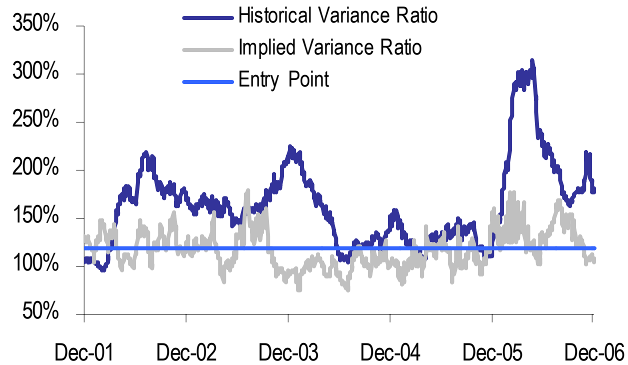
Alternatively we can also express the profit function of the trade so that it will be proportional to the difference between the ratio of implied volatilities and the ratio of realised volatilities as shown below:

$$\sigma_{buy}^2\left(\rho_I - \rho_H\right) = \sigma_{buy}^2\left(\frac{K_{sell}^2}{K_{buy}^2} - \frac{\sigma_{sell}^2}{\sigma_{buy}^2}\right) = \sigma_{buy}^2\left(\frac{K_{sell}}{K_{buy}} + \frac{\sigma_{sell}}{\sigma_{buy}}\right)\left(\frac{K_{sell}}{K_{buy}} - \frac{\sigma_{sell}}{\sigma_{buy}}\right) = f(\sigma_{sell}, \sigma_{buy})\left(\frac{K_{sell}}{K_{buy}} - \frac{\sigma_{sell}}{\sigma_{buy}}\right)$$

where the first term is obviously positive and a scaling factor of the p/l depending on the realised volatility of the two names involved in the trade.

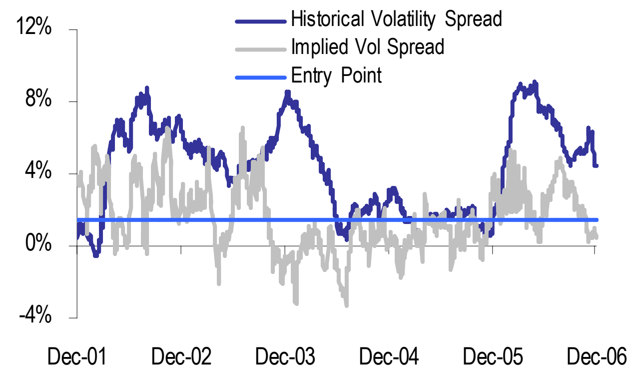
Trade Example: sell HSI 6-Month variance and buy Nikkei 6-Month variance

Figure 17: 6M implied variance ratio looks attractive at current levels…



Source: JPMorgan, cob 11 Dec 06

Figure 18: … so does the spread of 6M implied volatilities.



Source: JPMorgan, cob 11 Dec 06

1. Volatility pair trade quoted in terms of volatility spread

On 1 February 2006, we sell a 6-month HSI variance swap for a vega notional of $100K and buy a 6-month Nikkei variance swap also for a vega notional of $100K. Expiry is set on 28 July 2006 (*n*=126 observations) and the variance swap strikes are 16.2 for HSI and 18.5 for NKY.

P/L from the HSI position: $p/l_{HSI} = \$100K \times \left(\frac{K_{HSI}^2 - \sigma_{HSI}^2}{2K_{HSI}} \right)$

P/L from the NKY position: $p/l_{NKY} = \$100K \times \left(\frac{\sigma_{NKY}^2 - K_{NKY}^2}{2K_{NKY}} \right)$

At expiry, on 28 July 2006, the realised volatilities are as follows: $\sigma_{HSI} = 17.8$, $\sigma_{NKY} = 21.4$

The final p/l of the trade for a vega notional invested on both legs of $100K is therefore given by:

$$p/l = \$100K \times \left[\left(\frac{16.2^2 - 17.8^2}{2 \times 16.2} \right) + \left(\frac{21.4^2 - 18.5^2}{2 \times 18.5} \right) \right] = \$144{,}830$$

In terms of volatility spread, the p/l is (16.2 – 17.8) + (21.4 – 18.5) = -1.6 + 2.9 = 1.3 vegas.

Thus, thinking in terms of 'volatility p/l' and 'convexity p/l' the volatility p/l equates to $130,000 and the extra $14,830 can be though of as an extra 'convexity p/l' resulting from the non-linearity of the variance swap payoffs.

Hence investors need to be aware when putting on volatility pairs trades with variance swaps that the convexity can sometimes have a significant effect on the eventual p/l. In the above case the convexity of the payoff accounts for around 13% of the trade p/l. As discussed above, in extreme situations, this convexity can actually make an expected (in linear terms) profit into a loss and vice versa.

2. Volatility pair trade in terms of mean variance ratio

We consider the same trade as above: long 6-month Nikkei variance and short 6-month Hang Seng Variance.

The variance strikes are 18.5 for the Nikkei and 16.2 for the Hang Seng.

Suppose we wish to trade $100K of vega on the short (HSI) leg of the trade. This equates to a variance notional of $100K / (2 x 16.2) = $3,086.

The mean variance ratio will be $\frac{K_{sell}^2}{K_{buy}^2} = \frac{16.2^2}{18.5^2} = 0.767$

The variance notional for the short leg will be $N_{\text{var}iance} \times \frac{K_{sell}^2}{K_{buy}^2} = \$3086 \times 0.767 = \$2366$

At expiry we observe the following realised volatilities: $\sigma_{HSI} = 17.8$, $\sigma_{NKY} = 21.4$

If the variance notional on the variance swap we are selling is $N_{\text{var}iance}$, we then structured the trade such that the variance notional of the variance swap we are buying is $N_{\text{var}iance} \times \frac{K_{sell}^2}{K_{buy}^2}$ and the p/l of the trade at expiry is written as:

$$p/l \quad = N_{\text{var}iance} \times \sigma_{buy}^2 \times \left(\frac{K_{sell}^2}{K_{buy}^2} - \frac{\sigma_{sell}^2}{\sigma_{buy}^2} \right)$$

$$= N_{\text{var}iance} \times 21.4^2 \times \left(\frac{16.2^2}{18.5^2} - \frac{17.8^2}{21.4^2} \right) = \frac{N_{\text{var}iance} \times 21.4^2}{100} \times (76.7 - 69.2)$$

Since the variance notional of the short HSI leg is $3,086, the p/l is equal to $105,995

We also attain the same result using the traditional p/l formula:

$$p/l = \$3{,}086 \times (16.2^2 - 17.8^2) - \$3{,}086 \times \frac{16.2^2}{18.5^2} \times (18.5^2 - 20.4^2) = \$105{,}995$$