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The new and improved Treasury par curve model

- The Treasury par curve is a fundamental component of all our US interest analysis. It is used as a pricing input for various spread product, helps in identifying relative value along the Treasury curve, and also aids in accurately estimating slide
- We make enhancements to the methodology for our par curve: the new model shows a smaller overall yield error and demonstrates better yield error distribution with same number of knots
- The enhanced par curve is used in Treasury analytics reports available on JPMM. Historical par curve data as well as yield errors for individual Treasury securities can be found in our DataQuery analytical tool

Fixed Income Strategy

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See page 6 for analyst certification and important disclosures.

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Background

The Treasury par yield curve is one of the most fundamental constructs in financial markets. Its uses include estimation of richness/cheapness of specific Treasury securities, estimation of the yield slide, and as an input for pricing various spread products. In this note, we discuss the methodology we use to construct our par curve as well as recent changes we have made to improve the stability and robustness of this curve.

Parametric Discounting Curve

Construction of a yield curve typically involves three steps. **First**, we specify the exact basket of Treasury securities used in the estimation. **Second**, we specify a parametric model for the curve. **Finally**, we estimate model parameters by fitting a discount factor function to observed market prices of securities in the basket. Treasury notes and bonds make semi-annual interest rate payments with principal paid back at maturity. We model bond B_i's cash flow schedule via $C_i(t_j), i = 1, \dots, n_i$, where $C_i(t_j)$ denotes its cash flow amount at time t_j. If we use $d(t_j)$ to denote the discounting factor for each cash flow payment at time t_j, then B_i's theoretical price at settlement is

$$\widehat{P_i} = \sum_{j=1}^{n_i} d(t_j) C_i(t_j)$$

In constructing our yield curve, we aim to build a curve of relatively uniform liquidity; so, we filter out Treasury securities that have features which could distort the smoothness of our curve. We exclude the following securities:

1. On-the-runs, olds, and double-olds in the 2-year, 3-year, 5-year, 7-year, and 10-year sectors, as these securities are actively traded and tend to exhibit a liquidity premium. We exclude current 30-year bonds, but include the old and double-old issues, since fewer securities exist at the very long end.

2. Treasury securities trading significantly special in the repo market. We exclude securities with 6-month repo rates that are 25bp or more below 6-month GC repo rates. Issues trading special in repo will exhibit a liquidity premium and could distort curve construction.

3. Securities with less than 1 year to maturity. T-bills and short coupons are excluded as they are owned primarily by money market funds and other short-end investors, and tend to exhibit different supply/demand characteristics than securities that are longer maturity in nature. Instead, we use GC repo rates to model the short end, which results in more stable forward rate and carry calculations.

4. Bonds with embedded options, floating-rate and zero-coupon structures (TIPS, FRNs, STRIPS).

Once a discounting curve is properly constructed, we can compare the theoretical price $\widehat{P_i}$ with its dirty price P_i to find the price error, $e_i = \widehat{P_i} - P_i$, which provides a measure to quantify the relative value of the bond.

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We model the discounting factor $d(t)$ via a cubic spline function. It is fully specified by 9 model parameters $x_1, \dots, x_9$ and 6 knots $t_4, \dots, t_9$ with below functional form:

$$d(t) = \begin{cases} \textit{Based on linear interpolation of GC repo rates}, & t < 0.5 \\ x_0 + \sum_{i=1}^{3} \frac{x_i (t-0.5)^i 1_{t>0.5}}{10^i} + \sum_{i=4}^{9} \frac{x_i (t-t_i)^3 1_{t>t_i}}{10^3}, & t \geq 0.5 \end{cases}$$

Where

$$1_{t>t_i} = \begin{cases} 0, & t < t_i \\ 1, & t \geq t_i \end{cases}$$

Knots $t_i, i = 4, \dots, 9$ are determined such that there is roughly the same number of bonds between neighboring knots. The short end of the discounting curve is controlled by the GC repo market (O/N, 1m, 3m and 6m) with the constraint that:

$$d(0.5) = x_0 = \textit{discounting factor from 6m GC repo}$$

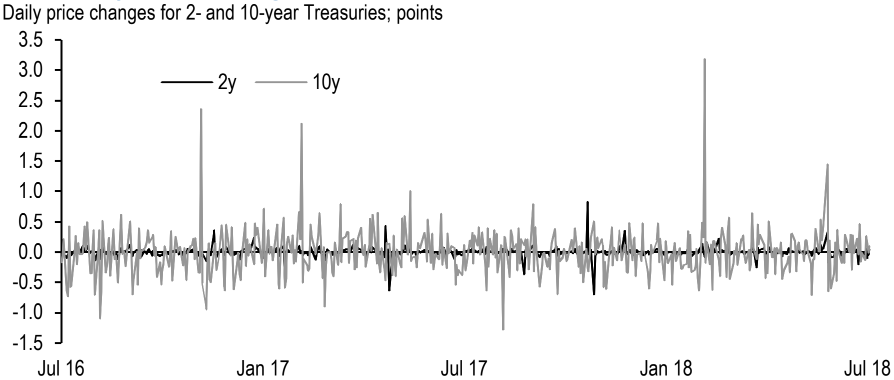
With the functional form of $d(t)$ specified, we next define the optimization problem. In our previous par curve model, the optimization problem was formulated to minimize the sum of the squared price errors of all participating bonds,

$$\min_{x_1 \dots x_9} \sum_{i=1}^{N} e_i^2$$

This becomes an ordinary least square problem with a closed-form analytical solution. However, formulating the optimization problem in this way creates some inefficiency. Bonds with different maturities and coupons have different durations, such that the daily price changes of bonds are not comparable. **Exhibit 1** shows daily price changes for 2- and 10-year Treasuries over the last two years, and it is apparent that longer-duration securities display greater price volatility.

Exhibit 1: Given that longer-duration bonds exhibit greater price volatility than shorter-duration bonds, and that each bond in our legacy model carried an equal weight, our revised model seeks to minimize yield errors along the curve

Daily price changes for 2- and 10-year Treasuries; points



Source: J.P. Morgan

In the original least square formulation under our legacy model, each bond had an equal weight in the price domain, which may not accurately reflect market movements, as bonds with lower duration tend to have smaller price errors and larger yield errors. Therefore, in our new model, we revise the objective function to

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minimize the sum of squared yield errors (e_i^Y) . We use a weighted least square objective function by introducing a weight $w_i = \frac{1}{P_i D_i}$ to adjust bond B_i's price error, e_i, for modified duration D_i and price P_i. The revised objective becomes:

$$\min_{x_1 \dots x_9} \sum_{i=1}^{N} (e_i^Y)^2$$

Where

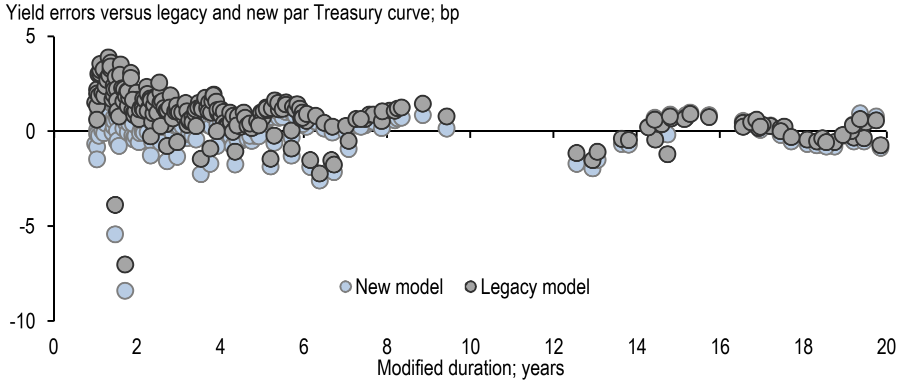
$$e_i^Y = e_i w_i$$

Results from enhanced par curve model

The modifications made to our Treasury par curve improve the fit in a number of ways. **First**, **Exhibit 2** shows that yield errors are for the most part lower in the new par curve model when compared with the legacy model, especially at the front end.

Exhibit 2: Yield errors at the front end are saller in the new par curve model than under the legacy model

Yield errors versus legacy and new par Treasury curve; bp



Source: J.P. Morgan

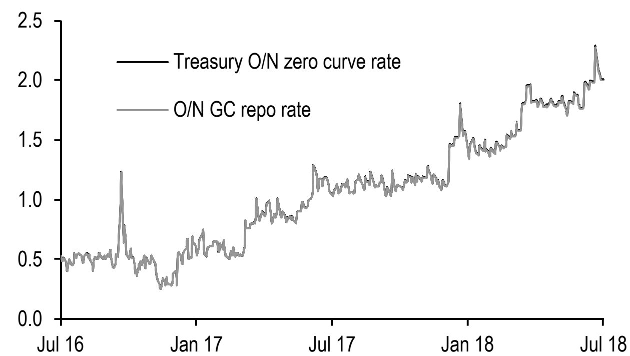
Exhibit 3: The aggregate error of our updated par curve is more stable than in the legacy version of the model

Root Mean Square Error of off-the run Treasury curve; bp

Source: J.P. Morgan

Exhibit 4: The short end of the Treasury zero curve closely tracks the O/N repo rate

1-day Treasury zero rate versus O/N GC repo rate; %



Source: J.P. Morgan

Second, Exhibit 3 shows the Root Mean Square Error (RMSE) as a measure of the aggregate yield error of the new par curve. The error of the par curve is relatively stable. **Third**, short end more closely tracks observable market rates: **Exhibit 4** shows that the Treasury zero rate largely tracks the GC repo rate.

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We have rolled out these changes, and the enhanced par curve is used in the "Treasury Carry, Roll, and Relative Value Report" which can be found in our *US Cash Interest Rate Products Analytics Package* on JPMM. Historical par curve data as well as yield errors for individual Treasury securities can be found in our DataQuery analytical tool.

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