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Can Convexity Be Exploited?[1]

We examine the extent to which investors can benefit from exposure to convexity in periods of high yield curve volatility. We find that the yield-curve dynamics are sufficiently different from those implied by a parallel shift to offset the payoff for convexity. In contrast, option strategies can be employed successfully to express views on expected volatility spikes.

INTRODUCTION

Many investors believe that convexity is a desirable portfolio attribute. Whether the market moves up or down, high-convexity portfolios will always outperform low-convexity portfolios of equal duration and yield. It can be demonstrated mathematically that if we simply model the possible changes to the yield curve as the set of all possible parallel shifts, a duration-neutral long-convexity position will outperform if the curve shifts in either direction, break even if yields remain unchanged, and never underperform.

Financial theory, however, has been built around the assumption of efficient markets that do not allow such arbitrage opportunities. Arbitrage-free yield curve models are designed around a probability distribution of yield curve changes that guarantee that no point on the yield curve has a higher expected return than any other. In such models, the fundamental view of how the yield curve might change over the near term is not in terms of a parallel shift up or down, but rather in terms of up and down movements with slightly non-parallel shapes such that the long-convexity positions will not have any expected performance advantage. The bias that is introduced into the yield curve process in order to offset the perceived convexity advantage in a parallel-shift model is sometimes known as "convexity drift".

The bottom line of the arbitrage-free assumption is that cashflow convexity, which is just a mathematical side effect of the price-yield relationship, should not have any material effect on expected returns. This is in sharp contrast with the convexity that stems from optionality, in which changes in yield can result in a different payout of cashflows. There is no denying that options have value; the arbitrage-free assumption merely requires that the premium paid for an option should equal the expected value of its eventual payoff.

Despite these theoretical arguments, many investors still hold the belief that positive convexity— of any sort—should enhance portfolio performance. A common approach for implementing a convexity exposure in a cash-only fixed-income portfolio is via a

[1] *We would like to thank Bob Fuhrman of Wellington Management for motivating this study.*

self-financing bullet-barbell trade that maintains duration neutrality.[2] This article examines the performance of this strategy over the last two decades, using data from the U.S. Treasury market. Our analysis corroborates the arbitrage-free assumption: we do not detect any significant performance advantage to the long-convexity trade, even in months with the largest magnitudes of yield curve changes.

Looking at the total returns to this trade over the past twenty years, we find them to be insignificantly different from zero on average, seemingly suggesting that convexity is correctly priced by the market. Furthermore, a closer examination reveals a directional relationship between the strategy performance and yield curve changes: the strategy performs well in a rising yield curve environment and poorly in a declining yield curve environment. We demonstrate that the directional payoff of the bullet-barbell strategy stems from the correlated nature of the three main yield curve shapes: shift, twist, and butterfly. This result is also consistent with the "convexity drift" exhibited by arbitrage-free yield curve models.

To round out our empirical study, we include an options strategy that generates positive convexity, and obtain very different results. We investigate the performance of a straddle strategy using Treasury futures options. A straddle position allows an investor to isolate his volatility bet at a specific maturity along the curve. We show this strategy generated exceptional returns whenever yields experienced large shifts, irrespective of their direction.

EMPIRICAL ANALYSIS

The Value of Convexity

To understand the argument for convexity, consider the following expression for the total return of a bond under a parallel yield shift $\Delta y_{t+\Delta t}$:

$$(1)\quad R_{t,t+\Delta t} \cong y_t \Delta t - D_t \Delta y_{t+\Delta t} + \tfrac{1}{2} C_t \Delta y_{t+\Delta t}^2$$

The bond return is equal to the sum of the carry return (as a function of the beginning-of-period yield y_t and the investment horizon Δt) and the price return. The latter is approximated by the product of duration *D* and convexity *C* with the yield change and the square of the yield change, respectively.

For two bonds or portfolios with equal duration but different convexities, we expect the higher-convexity bond to outperform in any large yield curve move. In fact, if we look at instantaneous scenarios ($\Delta t = 0$), equation (1) allows no possibility that the higher-convexity portfolio will underperform—it will always outperform unless yields are unchanged. This seeming paradox is due to the fact that equation (1) does not characterize all possible yield curve changes, it just offers a simple one-dimensional view of performance over a specific family of yield curve changes. If we are looking at two different bonds, or two different points on the yield curve, then there is always the possibility that their yields change by different amounts.

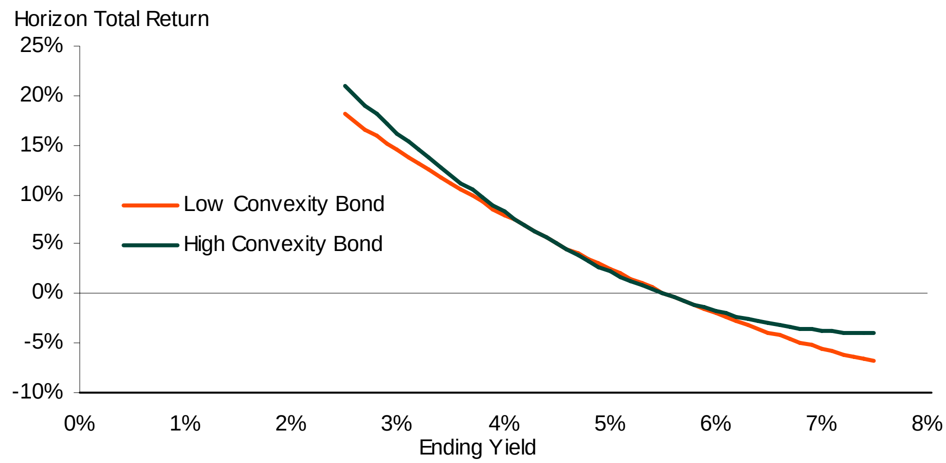
Nevertheless, the simplicity of the parallel-shift paradigm has earned it a place in our hearts and minds, and it forms the basis for many commonly held notions about bond markets. A typical illustration of the value of convexity may be found in Figure 1, which displays the returns for two hypothetical bonds over a six-month holding period as given by equation (1). Both bonds have a five-year duration but different convexities—one

[2] *All our results are equally applicable to a case in which a portfolio is structured to have a higher convexity relative to the benchmark by overweighting short and long maturity bonds while maintaining an equal duration.*

bond has a convexity of 200, while the other has a convexity of only 100. As a compensation for this, let us assume further that the higher-convexity bond yields only 4.5%, as opposed to 5.0% for the lower-convexity bond.

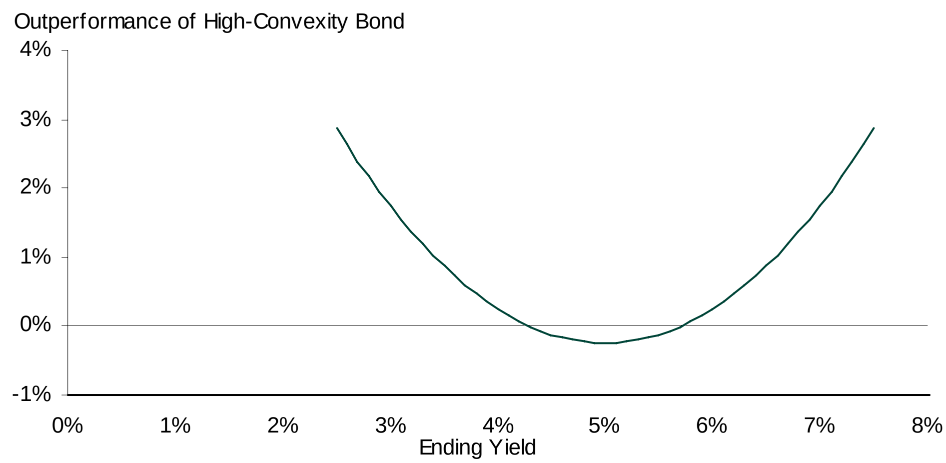
The holding period return is largely a function of the bonds' duration, and the return profiles of the two bonds are quite similar over the range of horizon yields that might be reasonably expected. However, when looking at the performance differential between the two bonds more closely (Figure 2), a clear pattern emerges. For small changes in yield, the high-convexity bond underperforms modestly due to the give-up in carry return; for larger yield changes in either direction, it outperforms handsomely.

Figure 1. Total Return as a Function of Ending Yield for High and Low Convexity Bonds



Return is computed using Equation (1) assuming a six months holding period. Bonds have a five-year duration and convexity of 100 and 200. The beginning-of-period yield for the low (high) convexity bond is 5.0% (4.5%).
Source: Lehman Brothers

Figure 2. Performance of High-Convexity Bond Relative to Low-Convexity Bond



Return is computed using Equation (1) assuming a six months holding period. Bonds have a five-year duration and convexity of 100 and 200. The beginning-of-period yield for the low (high) convexity bond is 5.0% (4.5%).
Source: Lehman Brothers

This simple example illustrates several common beliefs about convexity. First, a duration-neutral, long-convexity position should outperform whenever there is a large yield curve move in either direction. While this property obviously makes convexity valuable, we should not be able to obtain something valuable for free. A second belief is that the market tends to incorporate a "payup" for convexity. According to this view, one of the drivers of the shape of the yield curve is the value of the convexity at the long end, which causes the typical inversion of the spot curve between 20 and 30 years (termed "convexity bias").[3] This up-front cost for purchasing convexity could lead to the sort of situation depicted in Figure 2: yes, the positive-convexity position wins in any sufficiently large yield curve move, but if such a move fails to materialize, it will lose money by carrying the position at a lower yield.

Various studies have been carried out on the question of how large this payup for convexity is, and whether it is fair. Kahn and Lochoff (1990) employ a return attribution methodology to examine the premium associated with a "convexity factor". They find that convexity has not generated excess returns, and that deviations from the parallel shift assumption were significant enough to invalidate the convexity value.[4] Kang and Chen (2002) find that even for substantial yield curve changes theta and not convexity explains much more of treasury returns. They document a similar effect in all major international markets and even when for a daily frequency.

In this study, we do not attempt to directly measure the payup for convexity, but rather investigate the more fundamental belief—that positive convexity should generate outperformance in large yield curve movements. Thus, even if convexity does not represent a "free lunch", it might be a useful way to implement the view that a volatility spike is imminent and that yields are to move strongly in one direction or the other. However, even this belief is strongly rooted in the parallel shift assumption. The U-shaped profile shown in Figure 2 for the relative performance of two bond positions is based on the assumption that their yields always change by the same amounts. In fact, bond prices depend upon a term structure of interest rates. Duration and convexity may not adequately capture the differing sensitivities of the two bonds to all possible movements of the term structure. If long rates rise while short rates fall, the yield of the two bonds may change by different amounts.

Can We Exploit Convexity Using Cash Bonds?

A common approach to gain a convexity exposure using cash instruments is via a self-financing duration-neutral bullet-barbell strategy.[5] Bonds at the long end of the curve have more convexity per unit of duration than those with shorter maturities. As a result, a combination of a long and a short bond (a barbell position) will have greater convexity than that of a bullet bond with the same duration.

To evaluate the historical performance of such a strategy we use four On-the-run Treasuries with the following maturities: 2-, 5-, 10-, and 30-years. We track the strategy performance for two maturity combinations: a 2/10 barbell versus a 5-year bullet and a 5/30 barbell versus a 10-year bullet (hereafter "2/10–5" and "5/30–10," respectively).

To insure that our results are not affected by the liquidity premium typically associated with the On-the-run Treasuries, we also compute returns to the strategy when the underlying instruments are old OTR with similar maturities (e.g. the bonds second in

[3] Several studies present models that explain the behavior of the yield curve based on expectations of future volatility and the tradeoff between the value of convexity and theta (see for example Smit and Swart (2006)).

[4] Kahn and Lochoff (1990) use three factors to represent the returns to treasury securities: Yield, duration and convexity. They then estimate a regression of the convexity factor realizations against an intercept (alpha) and the market excess returns. In contrast to the argument in favor of convexity, they find the alpha to be insignificantly different from zero.

[5] Self-financing implies that the market value of the (long) barbell position is equal to that of the (short) bullet position. Later in this paper, we look at how to structure a long-convexity position using treasury options.

order of maturity to the current OTR series) and STRIPS. Using STRIPS magnifies the positive convexity of the barbell-bullet trade.

We examine the payoff to the bullet-barbell strategy over both monthly and daily frequencies. The dataset contains yields, durations, convexities, and returns for all bonds participating in the formation of the strategy. The monthly data spans 15 years, from January 1989 through June 2006, while the daily data begins in May 1997.

If we believe in the value of positive convexity, we expect the payoff of the bullet-barbell strategy to exhibit a U-shaped pattern similar to Figure 2. This means that the barbell position should outperform the bullet position in high-volatility periods and perhaps earn a small negative carry in a stable yield environment.

Performance of the Barbell-Bullet Strategy

We begin our analysis by looking at the strategy performance over the entire sample period (January 1989-June 2006). Figure 3 presents the average monthly return and (beginning-of-month) convexity and yield-to-maturity (YTM) by maturity and treasury instrument. Consistent with the earlier discussion, the strategy exhibits a negative carry equivalent to roughly 1 bp/month (with the exception of the 5/30–10 strategy using STRIPS where the negative carry is about 4 bp/month). The convexity of the strategy is very similar across the different treasury series, except STRIPS which exhibit much higher convexity. With respect to performance (total returns), the strategy mostly earns negative return, except for the 5/30–10 strategy using OTR. The magnitude of underperformance is roughly equal to the carry component and is not statistically significant.

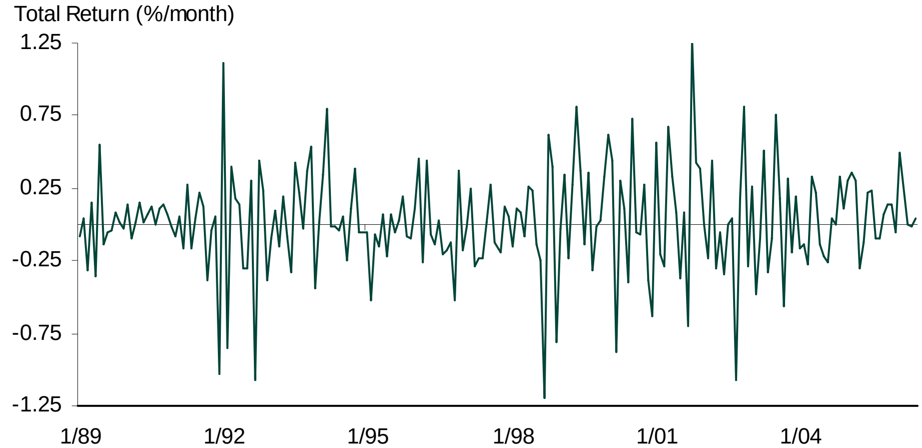
The results in the table indicate that during the entire period the strategy earned slightly negative returns. Did the strategy perform well however, in months with large yield curve movements? Based on the discussion in the previous section, we might expect the strategy to occasionally generate high positive returns in months with large yield changes and flat returns (mostly negative) otherwise. In reality we observe a very different return pattern. Figure 4 plots the return time series for the 5/30–10 Strategy. The strategy earned large negative returns in excess of 100 bp/month on several occasions and exhibited substantial volatility.

Figure 3. Performance of Long Barbell/ Short Bullet Strategy over Entire Sample Period

	Convexity	YTM (bp)	Return (bp/Month)		
			Total	Coupon	Price
		2 / 10 Barbell - 5 Bullet Strategy			
On-the-Runs (OTR)	0.10	-14.9	-2.0	-1.2	-0.8
Old OTR	0.09	-13.4	-1.5	-1.2	-0.3
Old-Old OTR	0.09	-12.7	-1.9	-1.3	-0.6
STRIPS	0.13	-12.9	-1.6	0.0	-1.6
		5 / 30 Barbell - 10 Bullet Strategy			
On-the-Runs (OTR)	0.41	-10.7	1.9	-0.9	2.8
Old OTR	0.38	-13.3	-0.7	-1.4	0.8
Old-Old OTR	0.38	-11.6	-0.3	-0.7	0.4
STRIPS	0.87	-46.7	-8.4	0.0	-8.4

Based on monthly returns, January 1989 – June 2006. Convexity and Yield-to-maturity (YTM) figures represent beginning-of-the-month values for the entire trade.
Source: Lehman Brothers

Figure 4. Performance of 5/30 Year Barbell/10-Year Bullet Strategy



Based on monthly returns, January 1989- June 2006 using on-the-run Treasuries.
Source: Lehman Brothers

What accounts for the large return variation in Figure 4? To examine this issue and the prediction about a U-shaped return pattern with respect to changes in the yield curve, we define a parallel shift of the yield curve as the average yield changes of the 2-, 5-, 10-, and 30-years On-the-run treasuries.[6] We then assign all observations into deciles based on the size of the shift in each month and compute average returns earned by each strategy for each decile.

If convexity pays off, we should observe a U-shaped payoff pattern between the strategy performance and the magnitude of the shift in the yield curve. Stated differently, the barbell strategy would exhibit large positive gains in the extreme deciles and small negative losses in the middle deciles

Figure 5 reports monthly yield curve shifts and strategy total returns by decile for both the 2/10–5 and 5/30–10 maturity combinations. Instead of a U-shaped relationship, we observe a directional relationship between each barbell strategy return and yield curve shifts. For example, as we move from the lowest yield curve shift decile to the highest decile, the returns of the 2/10–5 strategy using OTR changed from -29.3 bp to 19.6 bp. The relation between the strategy return and the shift is almost linear. This linear relationship also holds well for all other treasuries (including STRIPS) as well as the 5/30–10 strategy.

Figure 6 depicts the relationship between the yield curve shifts and the bullet-barbell strategy returns for the 2/10–5 and 5/30–10 strategies (using OTR). As in Figure 5, the picture does not resemble a U-shape pattern, but rather a clear directional relation. The bullet-barbell strategy tends to generate positive returns when yield curve shifts up, but negative returns when yield curve shift down.[7] These results are consistent with those of Kahn and Lochoff (1990) which find that the convexity factor generates positive returns when the market excess returns are negative (e.g. when yields rise).

[6] The reason for using this specific expression will become clear in the next section.
[7] We repeated the analysis using daily data and found very similar results: the daily strategy returns were also positively correlated with the directional changes of the yield curve

Figure 5. Performance of Long Barbell/ Short Bullet Strategy by Yield Change Decile

				2 / 10 Barbell - 5 Bullet Strategy				5 / 30 Barbell - 10 Bullet Strategy			
	Yield Change Decile	N (Obs.)	Yield Chg.	OTR	Old OTR	Old-Old OTR	STRIPS	OTR	Old OTR	Old-Old OTR	STRIPS
Low	1	20	-46.7	-29.3	-24.9	-25.4	-40.6	-21.5	-32.5	-31.8	-48.3
	2	21	-30.8	-18.5	-18.4	-14.6	-22.3	-22.3	-23.0	-27.6	-57.7
	3	21	-21.0	-9.8	-9.6	-10.9	-16.7	-14.3	-12.2	-10.3	-28.0
	4	21	-12.7	-14.3	-14.0	-14.4	-12.5	1.6	3.1	2.3	-29.6
	5	21	-6.5	-7.7	-5.0	-3.5	-3.7	3.6	-2.0	-4.0	-19.5
	6	21	1.0	5.3	1.4	1.7	-4.0	-0.4	-1.1	-3.9	-1.8
	7	21	6.4	9.9	8.2	6.3	16.1	2.5	2.3	4.2	-8.6
	8	21	15.0	6.5	6.6	5.3	14.6	7.9	4.8	5.2	-18.6
	9	21	25.8	17.0	16.8	15.8	25.8	28.0	29.1	28.4	33.3
High	10	21	47.2	19.6	22.5	19.7	25.1	33.0	27.3	29.3	93.3

Based on monthly returns (bp) January 1989 -June 2006. Yield change represents the average change in yields of the 2-, 5-,10-, and 30-year on-the-runs. Performance figures for each decile reflect the mean returns across all monthly returns assigned to that decile based on the contemporaneous changes in yield.
Source: Lehman Brothers

Return Attribution

The results in the previous section indicate a clear directional relation between the performance of the strategy in response to shifts in the curve, rather than a U-shaped payoff to convexity. The barbell-bullet strategy boasts positive returns following large positive yield curve changes and, conversely, negative returns following large negative yield curve changes. What accounts for the fairly linear relation between the strategy performance and yield curve behavior?

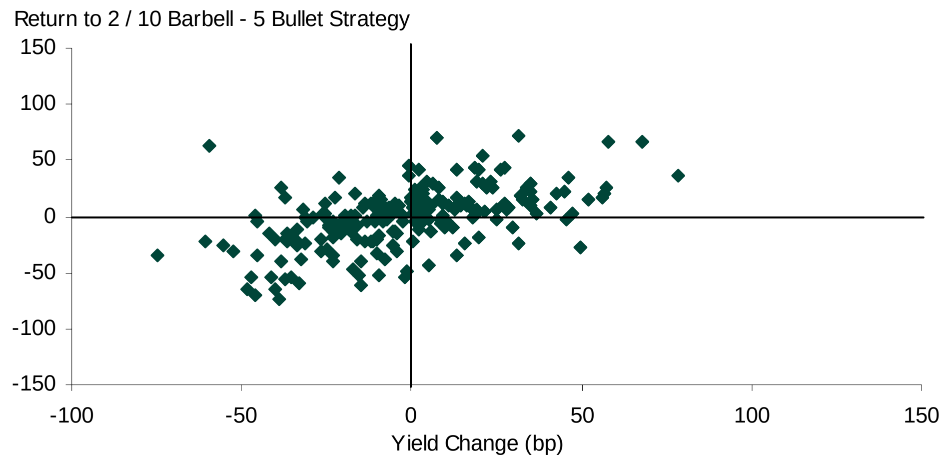
The notion of symmetrical payoffs to convexity relies on the assumption that the yield curve shifts in parallel. However, actual yield curve movements are rarely parallel. In fact, extensive research documented that three factors (shift, twist and butterfly) are able to capture the vast majority of yield curve dynamics.

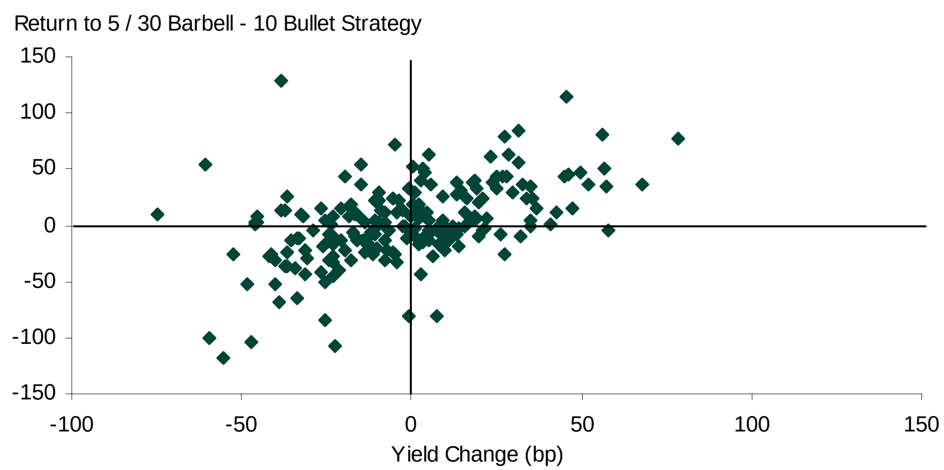
Mann and Ramanlal (1997) investigate the barbell-bullet strategy and demonstrate that once the correlations among changes in the level, slope and curvature of the yield curve are accounted for, the trade generates a directional return profile consistent with our findings.[8] To examine the relation between the yield curve re-shaping and the performance of the barbell-bullet strategy directly, we use a return-attribution methodology.

We follow the approach described in Dynkin and Hyman (1996) for its simplicity and clarity relying solely on changes in bellwether yields. It represents the price return component of total return as the return due to the passage of time and changes in the Treasury yield curve.[9] The actual changes in the yield curve are reflected using a combination of three curve movements: shift, twist and butterfly.

[8] *Mann and Ramanlal (1997) estimate the expected changes in the slope and curvature of the yield curve conditional on a shift and compute the resulting returns to a barbell-bullet strategy.*
[9] *Price returns associated with the passage of time include accretion and roll-downs returns.*

Figure 6. Performance of Long Barbell/Short Bullet Strategy Relative to Yield Changes





Based on monthly returns (bp) January 1989- June 2006. Yield change represents the average change in yields of the 2-, 5-, 10-, and 30-year On-the-runs.
Source: Lehman Brothers

Each of these components is a piecewise linear function, with prescribed values at 2-, 5-, 10, and 30-years, and a single parameter (s for shift, t for twist, and b for butterfly) that specifies the magnitude of that component in a given month. The values of the parameters s, t, and b are based on observed changes in the yields of Treasury bellwether bonds, selected at the beginning of the month, as follows.

Shift—Returns are due to a parallel yield curve shift based on the average yield changes of the 2-, 5-, 10-, and 30-year On-the-run treasuries. This type of change produces the same effect for all returns.

$$(2a) \quad s \equiv \frac{1}{4}(\Delta y_2 + \Delta y_5 + \Delta y_{10} + \Delta y_{30})$$

Twist—Returns are due to a steepening or flattening of the yield curve, centered on the 5-year. In a steepening move (t>0), the 30-year yield moves up by t/2, while the 2-year moves down by the same amount. The magnitude of the twist is computed as,

$$(2b) \quad t \equiv \Delta y_{30} - \Delta y_2$$

Butterfly—Returns are due to the middle of the curve moving in the opposite direction from the wings. This is defined as the 2- and 30-year yields moving up by the same amount b/3, while the 5-year yield moves down by twice that amount. The magnitude of the butterfly is calculated as,

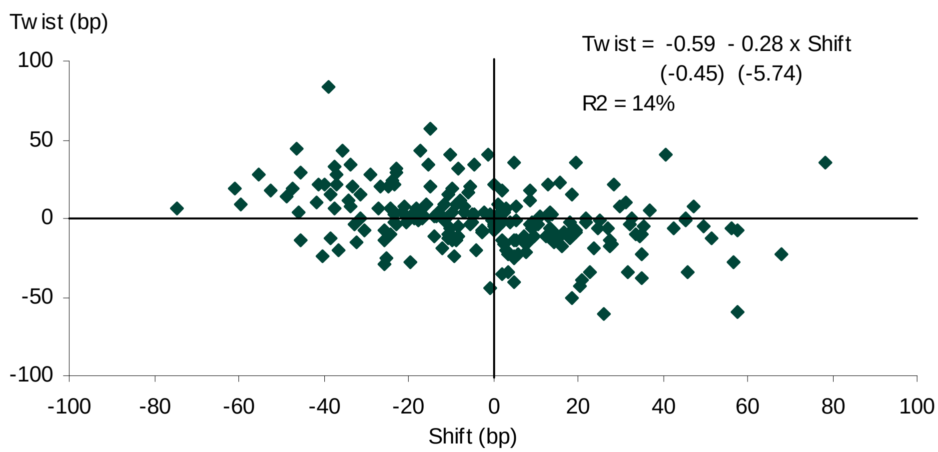
$$(2c) \quad b \equiv \frac{1}{2}(\Delta y_2 + \Delta y_{30}) - \Delta y_5$$

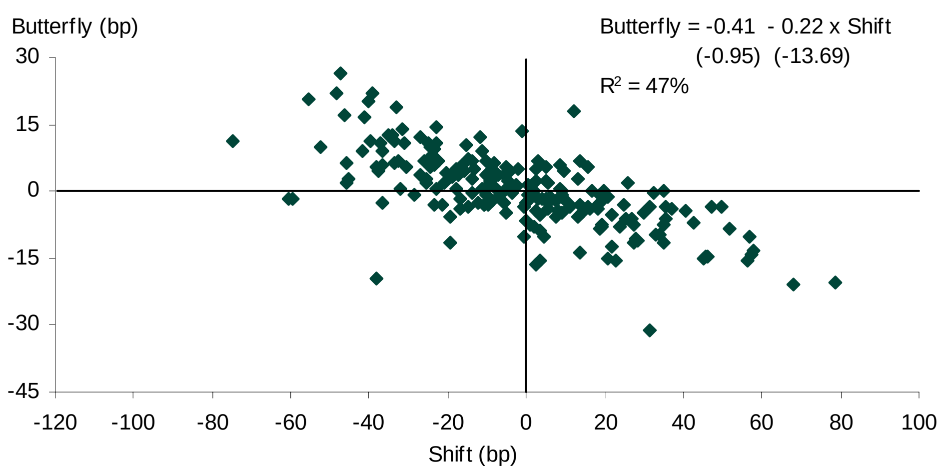
Any residual return not explained by the three curve moves is termed shape return.

During the sample period, yields declined on average by 2 bp/month, while the mean slope and butterfly realizations were practically zero. However, changes in yield curve level, slope, and curvature exhibited fairly high correlation as Figure 7 illustrates.

The scatter-plots show a clear negative relation between yield curve shifts and twist / butterfly realizations. When the yield curve shifts up, it tends to flatten and become more humped (e.g., the yield on the 5-year treasury increases by more then the corresponding 2- and 30-year yields). The opposite holds when the yield curve shifts down. A regression analysis suggests that accounting for the shift explains 14% and 47% of the variation in twist and butterfly realizations and it is highly statistically significant.

Figure 7. Change in Yield Curve Slope and Curvature as a Function of Shift





January 1989-June 2006. Shift, twist, and butterfly realizations computed based on Equations (2a) – (2c).
Source: Lehman Brothers

How do these patterns affect the performance of the barbell-bullet strategy? Figure 8 reports return attribution results for the 2/10–5 and 5/30–10 trades using data since January 1996 (earlier attribution returns are unavailable). As before, figures are reported separately by yield change deciles (e.g., shift).

The table displays the strategy price return broken down to its various components as well as the total and coupon returns. Since the strategy is duration neutral, the shift return essentially represents the convexity payoff. This allows us to easily ascertain, what part of the strategy price return is due to the convexity exposure and what part follows from non-parallel changes of the yield curve.

The results in Figure 8 indicate that the convexity premium is quite small—2-4 bp/month—even for the two extreme deciles. The strategy performance is largely driven by the butterfly return. For example, looking at the top and bottom deciles, the 2/10–5 strategy total return was -38.9 bp/month and 21.4 bp/month, of which -31.4 bp/month and 24.3 bp/month, respectively, can be attributed to the butterfly shape.

Figure 8. Return Attribution of Barbell- Bullet Strategy Performance

	Yield Change	Shape				Return Components						
	Decile	Shift	Twist	Butterfly	Convexity	Total	Coupon	Price	Time	Shift	Twist	Butterfly
Panel A: 2/10 Barbell - 5 Bullet Strategy												
Low	1	-43.9	19.0	13.0	10.5	-38.9	-1.3	-37.7	-2.1	2.7	-9.7	-31.4
	2	-27.2	2.8	8.5	9.9	-12.2	-1.2	-10.9	0.5	0.9	-1.0	-17.9
	3	-18.0	2.3	1.4	10.0	-10.0	-1.4	-8.5	0.6	0.6	-1.6	-6.2
	4	-10.1	-2.0	2.8	10.7	-7.3	-1.6	-5.7	-0.4	0.5	-0.2	-4.0
	5	-3.7	2.9	1.1	10.2	-5.0	-1.1	-3.9	1.4	-0.2	-0.8	-4.4
	6	2.3	-11.3	-3.8	10.4	11.1	-0.5	11.6	-0.4	0.3	3.6	9.3
	7	8.1	-1.7	-0.7	10.4	0.0	-0.7	0.7	0.4	-0.4	0.4	1.9
	8	15.4	-6.2	-3.5	10.4	9.1	-0.6	9.8	1.9	-0.1	1.4	10.4
	9	25.3	-6.2	-9.1	10.3	17.6	-1.1	18.6	2.5	-1.1	2.0	22.4
High	10	46.7	-6.7	-10.8	10.7	21.4	-1.1	22.5	1.5	-0.8	0.8	24.3
Panel B: 5/30 Barbell - 10 Bullet Strategy												
Low	1	-43.9	19.0	13.0	49.2	-31.4	-1.8	-29.6	-6.3	4.9	-11.2	-17.2
	2	-27.2	2.8	8.5	42.5	-33.3	-0.4	-33.0	-5.2	2.7	-0.2	-9.7
	3	-18.0	2.3	1.4	43.8	-5.6	0.2	-5.7	-5.5	2.1	-0.9	-4.0
	4	-10.1	-2.0	2.8	45.1	0.6	-1.3	1.9	-2.2	-0.8	-0.5	-1.9
	5	-3.7	2.9	1.1	46.8	-2.7	0.8	-3.5	-4.1	0.2	-1.2	-4.5
	6	2.3	-11.3	-3.8	47.8	13.1	-1.0	14.2	-2.5	-0.2	3.7	5.6
	7	8.1	-1.7	-0.7	45.4	3.8	-0.5	4.4	-3.4	1.3	-0.1	1.0
	8	15.4	-6.2	-3.5	47.1	13.8	-1.0	14.8	-3.9	1.3	1.1	5.8
	9	25.3	-6.2	-9.1	45.8	35.6	-0.3	35.9	-4.6	2.1	1.7	13.5
High	10	46.7	-6.7	-10.8	47.3	34.5	-1.6	36.1	1.8	2.3	0.3	11.0

Based on monthly returns (bp) January 1996- June 2006. Return component figures for each decile reflect the mean component returns across all monthly returns assigned to that decile based on the respective yield curve shift. Each decile contains 12-13 observations.
Source: Lehman Brothers

As mentioned earlier, when the yield curve shifts up, it becomes more humped (e.g., a negative butterfly return) and vice versa. Since the net sensitivity of the 2/10–5 position to the butterfly return is negative, the strategy boasts positive returns when yields rise and negative returns when yields decline.[10] Similarly, the magnitude of the butterfly return for the 2/10–5 trade is larger (in absolute terms) than the corresponding figure for the 5/30–10 trade for any decile, since the net exposure to any butterfly realization is larger.

Overall, the results indicate that the effect of the non-parallel reshaping of the yield curve dominates the convexity payoff, which we estimate to be quite small. As such, the barbell-bullet trade is unsuitable for investors who wish to express a view on large move of the yield curve in either direction.

Is this behaviour of the yield curve and consequently the barbell-bullet strategies a function of our choice of maturities or time period? Perhaps, other parts of the yield curve would behave differently? To answer these questions we use the STRIPS data we compiled to construct "local" barbell/bullet positions with long maturities. We examine the performance of 20/25–22, 25/30–27 strategies constructed using STRIPS. Although in a few sub-periods we find some rewards to convexity (e.g. in months assigned to one of the two extreme yield change deciles, the strategy generated large positive returns), in others - we do not. We, therefore, cannot conclude that these rewards are to be expected

Does Options' Convexity Pay?

The barbell-bullet strategy failed to exploit convexity and generate exceptional returns in periods of large yield changes. Its failure reflected the different sensitivities, each of the strategy constituents' exhibit to the main yield curve shapes (shift, twist, and butterfly) and the high degree of correlation among them. One alternative for investors who are not restricted to cash-only securities in their portfolios is to construct a straddle strategy using treasury options.

A straddle is a delta-hedged option position that consists of a call option and a put option on the same underlying, with the same strike price. Using a straddle neutralizes the risk of a non-parallel shift of the yield curve and allows the investor to isolate his volatility position at a specific maturity along the curve.

We evaluate the historical performance of a straddle using options on the 10-year CBOT treasury futures that, in general, are the most liquid. To form the straddle, we first identify the front futures contract for each month.[11] Once the front future contract is identified, we use its price to determine the at-the-money (ATM) call and put options. The ATM options are defined as those with a strike price within $0.5 of the underlying future price. We use ATM options since they exhibit the highest gamma as well as the most liquidity (which insures high-quality pricing data). The exact positions in the put and call options are determined based on the ratio of their deltas.[12]

Based on available data, we compute the straddle performance between January 1999 and May 2006. Several months are discarded because we can not find ATM call and put options with the same strike price or we lack subsequent month prices for the options selected. Overall, we have a total of 81 observations.

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[10] *Recall that the sensitivity for maturities of 2-, 5-, and 10-years is b/3, -2/3b, and -1/6b, respectively with intermediate sensitivities that are piecewise linear. This implies a negative net sensitivity for the 2/10-5 position.*
[11] *The front futures contract is determined based on the following schedule: June contract for January- March; September contract for April- June; December contract for July -September; March contract for October- December.*
[12] *For example, if the call option delta equals 0.6, and the put option delta is -0.4, then we would long 0.67 share of the call and long one share of the put. The resulting straddle would be delta neutral.*

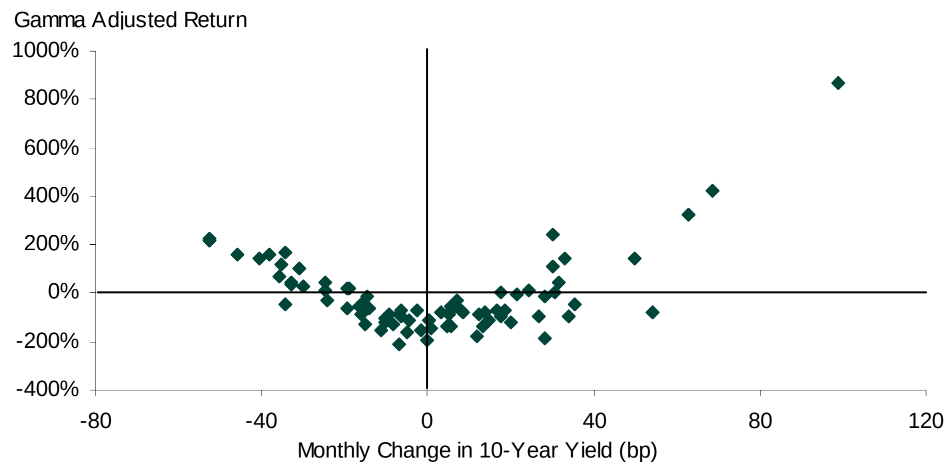
Figure 10 plots the straddle returns against changes in the underlying 10-year Treasury par yield. To neutralize difference in performance due to variation in the strategy over time, Figure 10 presents gamma-adjusted returns (defined as the straddle return divided by straddle gamma). Examining Figure 10 reveals a clear "smile" effect. The straddle posts large positive returns following large movements in the 10-year yield in either direction. When the 10-year movement is small (less than 20 bp), the straddle has negative returns because realized volatilities are not enough to offset the theta decay of the straddle. The straddle returns are symmetrical with respect to yield movements in either direction, although large positive yield moves (larger than 60 bp) were more common than large negative yield moves during the period.

To complete the analysis we re-examine the relation between the straddle performance and changes in the 10-year yield over daily frequency between January 2003 and December 2005. The results are largely unchanged with a similar "U-shape" relation between the straddle returns and changes in 10-year yield. Not surprisingly, we find that daily straddle returns can be well approximated by using the straddle sensitivities to changes in volatility (vega), underlying future contract (gamma) and passage of time (theta).

Comparing the straddle actual returns with those predicted using the straddle vega and gamma sensitivities indicates that the two account for 96% of the straddle performance on average. The t-statistics for the gamma and vega factors are 58 and 120, respectively, indicating that Vega accounts for a larger fraction of the strategy return variation.[13]

It is important to note that our results do not suggest that being "long" volatility as a strategy is in general beneficial. For example, Goodman and Ho (1997) conclude that over the period 1991- 1995, investors have been adequately compensated for selling volatility in both the treasury-options and mortgages markets.

Figure 10. Straddle Performance (Gamma Adjusted) as a Function of Monthly Change in 10-Year Yield



Source: Based on monthly returns per one unit of gamma (beginning-of-the-month) during January 1999–May 2006.
Source: Lehman Brothers

[13] *If the analysis is repeated using only days in which the 10-year yield changed more than 10 bp, gamma proves relatively more important. The t-statistics for the gamma and vega factors are now 31.86 and 56.35, respectively. The two factors combined explain 99% of the straddle P&L.*

CONCLUSION

This paper explores ways convexity can be exploited in the U.S. Treasury market. We show that the traditional approach using a long bullet–short barbell strategy does not generate the prototypical positive-convexity return profile illustrated in Figures 1 and 2. In contrast, an options straddle position exhibits a return profile that fall exactly in line with these expectations—negative returns due to theta decay when yields remain stable, and positive returns in large yield curve movements in either direction. What conclusions can we draw from these results?

A possible conclusion is that a fundamental distinction needs to be drawn between cashflow convexity and option convexity. Stated simply, there is a payoff only to option convexity, but not to cashflow convexity. The convexity that is shared by all fixed-cashflow securities as a natural result of the price-yield function does not lend itself to expressing views on convexity or, more precisely, on volatility. A view that a large yield change is imminent can be expressed efficiently using the derivatives markets—for example, by purchasing option straddles as described in this paper.

The problem with cashflow convexity is that the convexity of a security is largely a function of its maturity. To arrive at a position with an appreciable convexity exposure using cash bonds, one will necessarily take an even larger exposure to non-parallel shifts in the yield curve. Essentially, one can view our bullet-barbell results as an apparent paradox – if this strategy is truly positive convexity, why do we not achieve the expected return profile? As with most apparent paradoxes, the solution lies in pointing out a mistaken assumption. In this case, we have assumed that by making the strategy duration-neutral, we have cancelled out all the first-order exposures of the position, thereby isolating a clean exposure to the second-order convexity effect. However, while duration-neutral on the whole, this neutralizes only the exposure to parallel shifts in the curve. However, the key rate duration profile of such trades shows huge exposures to non-parallel changes in the shape of the curve. Our return attribution analysis shows that these non-parallel effects swamp the pure convexity return component that we have been trying to capture.

By contrast, the option straddle strategy that we have examined is comprised of two instruments, a put and a call, with the same underlying futures contract. When we make such a strategy delta-neutral, we truly neutralize the first-order risk exposure and leave a clean exposure to the second order effect.

Based on this analysis, it might be more precise to say that the difference between the two types of convexity is not that investors are compensated for one but not the other, but rather that derivatives markets offer a much richer set of instruments with which to express a specific view on convexity precisely. Investors must realize that the inability to precisely isolate a positive-convexity view with no extraneous exposures is yet another limitation that can be traced to the cash-only constraint. In previous work,[14] we have argued for the inclusion of futures in portfolios, as the cash-only constraint prevents managers from taking a long duration view without also taking a flattening view. Our current investigation can be used as the basis for a similar recommendation vis-à-vis options: to enable the cleanest possible expression of a convexity view, option strategies should be added to a manager's arsenal.

[14] *Cost of the No-Leverage Constraint in Duration Timing, Lehman Brothers, Global Relative Value, October 2002.*

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