Quantitative Finance

What is the importance of alpha, beta, rho in the SABR volatility model?

Asked 6 years, 6 months ago Modified 11 months ago Viewed 15k times

17

I just read that SABR model is a stochastic volatility model, which attempts to capture the volatility smile in derivatives markets. The name stands for "stochastic alpha, beta, rho", referring to the parameters of the model

Can anybody please help me to understand that what is the importance of alpha, beta, rho in the SABR volatility model?

options volatility pricing sabr

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asked May 16, 2018 at 4:00

user330060
291 1 2 4

5 Did you at least Google? There is a lot of material available online about this model.
– James Spencer-Lavan May 16, 2018 at 4:38

Yes, after long search i only asked.I am not a maths guy so trying to understand in simple language of its importance without complex formulas – user330060 May 16, 2018 at 5:34

4 Answers

Sorted by: Highest score (default)

24

We created the SABR model because we realized that (a) option values were nonlinear in the volatility, and (b) volatilities are stochastic. This means that if one had an option (or portfolio of options) which have positive gamma in the volatility dimension, on average we'd make money from fluctuations in the volatility, and we'd lose money with negative vol-gamma. To be fair, these gains or losses should be compensated for in the daily carry ... it's just Black Scholes in the volatility dimension. We created the model so that we wouldn't leak money due to the volatility of the volatility. It turns out that at-the-money options are nearly linear in the volatility, so there is little vol-gamma for ATM options, and much higher levels of vol-gamma for options away from the money ... so the SABR price correction is much stronger away from the money, resulting in a volatility smile. Pat

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answered Dec 24, 2020 at 20:42

Patrick S Hagan
491 3 3

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Let's relabel this as *What (TF) is SABR?*

Alpha, Beta and Rho are the point of the model. So explaining them is explaining the model.

A model of two processes

Unlike earlier models in which the volatility was modelled as a constant (Vasicek, Hull-White, etc), SABR assumes that as well as the price of the thing being stochastic, so is its volatility. That is, the volatility will also follow some stochastic path.

Thus we have two clearly related processes; the price, lets say of a forward rate (following the Wikipedia notation):

$$dF_t = \sigma_t F_t^{\beta} dW_t$$

Which just means the changes in the price are proportional to the price itself raised to power β, and a Wiener process W_t, scaled by the now time-dependent volatility σ_t.

We also have a process for the volatility σ_t:

$$d\sigma_t = \alpha\sigma_t dZ_t$$

Again, the changes in volatility are proportional to the volatility itself (so the behaviour is scale invariant) and to a second Wiener process Z_t, all scaled this time by α.

α is then the (constant) volatility of the volatility. I mean, we could model that as stochastic too but that seems like hard work.

So where is ρ?

α was the volvol, β was the power in the price relation, we are missing ρ.

Since the two processes (the price and its volatility) are very much related, the SABR model connects the two Wiener processes driving their movement by making them correlated with parameter ρ:

$$dW_t dZ_t = \rho dt$$

So changes in the two Wiener processes are correlated with ρ in time. Again, ρ is a constant.

So none of α, β or ρ are stochastic; perhaps the name should have been Stochastic Volatility, Alpha Beta Rho. But SVABR is much less catchy.

How am I going to price anything with SABR when no one quotes $\alpha/\beta/\rho$?

Ah yes. While the market does quote volatilities, it doesn't quote these parameters, so it's hard to just knit a model in Excel and wear it.

The equations we have so far model the dynamics given the parameters, so in order to get the parameters we will have to essentially solve for the parameters given some other stuff, like market prices for options which are sensitive to those parameters.

Calibrating a set of parameter values to market quotes is the subject of much effort, e.g. this blog post.

All models are finite

No model is able to magically capture all the information available, and there would be no point; a model's power is in deriving simpler truths than the information you start with. With SABR the model better recreates the dynamics of the evolution of an interest rate, but note that there are just a small, fixed number of parameters. So it cannot calibrate perfectly to a market with tens or hundreds of inputs.

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edited Dec 26, 2020 at 22:09
p.vitzliputzli
161 13

answered May 17, 2018 at 12:45
Phil H
3,699 18 20

great explanation! – 0xFEE1DEAD May 17, 2018 at 20:07

4 "what is the importance of alpha, beta, rho" that was the question but I dont see an answer here. I wanted to know what the interpretation/impact of these variables is? – emcor Nov 7, 2018 at 23:11

Excellent details and Summary. 👍🏻 – user44580 Feb 28, 2020 at 1:16

7

Unless I am missing the obvious, I do not see the question being answered? In my opinion, trying to understand in simple language what α, β, ρ mean requires an explanation what these parameters do and why it is useful.

Here is my attempt: Black (all Black Scholes formulas) assume(s) that Implied Volatility is independent of strike (constant and known). However, this is usually not the case and if you plot (quoted) IVOL and strike, you see what is called a smile or skew. SABR can be used to interpolate (and extrapolate) a vol smile.

Before talking about SABR, let us consider β separately. The CEV model does not assume a lognormal (Black) process but is more general:

$$dF = \alpha * F^{\beta} * dW$$

where α corresponds to the CEV volatility (sets the overall level of volatility), β is the CEV parameter (which determines the skew) and W is Brownian motion.

Now, in terms of what level, skew and smile actually mean or look like, I recommend to have a look at this illustration in the FX market. The level will be what is shown as the flat vol for all strikes (simple ATM only), skew is the CEV parameter. For $\beta < 1$, the vol smile is a decreasing function of the strike price. A major problem is that it is not able to produce a smile (the upward sloping wings in the FX example I linked).

That is where SABR comes in: On top of CEV, the new assumption is that volatility is not constant as it is in CEV but a stochastic process itself. Hence, σ itself is governed by an SDE, just like the forward rate (as assumed in Black and CEV). The two Brownian motions (for forward rate and vol) are correlated through correlation coefficient ρ.

How to get or set β is explained here. This answer shows how β can be estimated and what the effect and interpretation of β are.

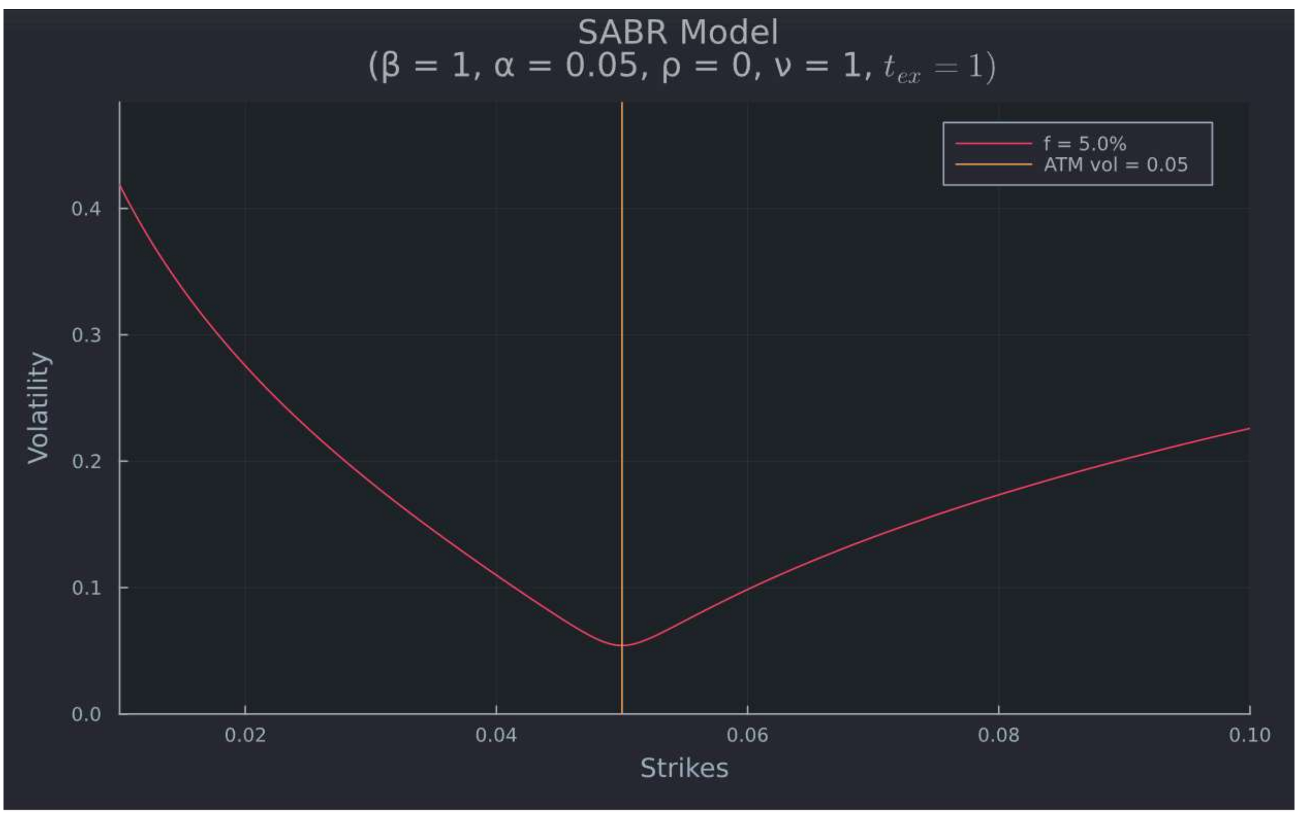
Once you have β,

- α mainly controls the overall height (like CEV),
- ρ (correlation) controls the skew (for set beta) and
- ν (vol of vol) controls the smile (not part of question but crucial) .

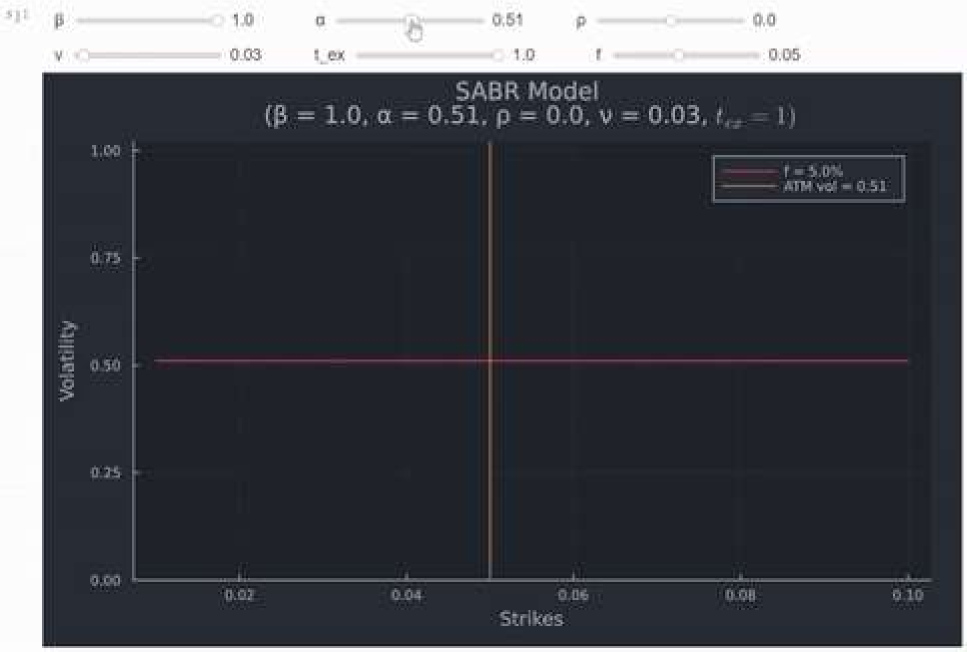
The gif below uses Julia and the formulas 2.17 onwards, starting on P.89, of Managing Smile Risk. Wilmott, 1, 84-108,.

```
# load packages
using Plots, PlotThemes, Interact, LaTeXStrings
theme(:juno)

#define inputs
β, α, ρ, ν, t_ex, f1, f2, f3, t_ex  = 1, 0.05, 0, 1, 1, 0.03, 0.05, 0.07, 1
K = 0.01:0.0001:0.1

#define the expression
function σ_b(β,α, ρ, ν, t_ex, f, K)
    A = α /(((f*K)^((1-β)/2))*(1+((1-β)^2)/24*log(2,(f/K))+ ((1-β)^4)/1920*log(4,
(f/K))))
    B = 1+(((1-β)^2)/24*(α^2/(f*K)^(1-β))+(1/4)*α*β*ρ*ν/((f*K)^((1-β)/2))+(2-
```

```
3*ρ^2)/24*ν^2)*t_ex
    z = ν/α*(f*K)^((1-β)/2)*log(f/K)
    χ_z = log((sqrt(1-2*ρ*z+z^2)+z-ρ)/(1-ρ))
    atm = α/(f^(1-β))*(1+(((1-β)^2)/24*(α^2/(f*K)^(1-β))+
(1/4)*α*β*ρ*ν/((f*K)^((1-β)/2))+(2-3*ρ^2)/24*ν^2)*t_ex)
    cond = f==K
  return cond ? atm : A*z/χ_z*B, atm
end

# define plots
plot(K,[x[1] for x in σ_b.(β,α, ρ, ν, t_ex, f, K)], size =(800,500),
margin=5Plots.mm,
                                                          title = "SABR Model
\n(β = $β, α = $α, ρ = $ρ, ν = $ν, "L"$ t_{ex}"*" = $t_ex)",
                                                          label = "f =
$(round((f*100),digits=1))%",
                                                      xlabel = "Strikes",
                                                      ylabel = "Volatility")
ylims!((0, ylims()[2]+ylims()[1]))
vline!([f], label = "ATM vol = $(round(minimum([x[2] for x in σ_b.(β,α, ρ, ν,
t_ex, f, K)]),digits = 2))")

ylims!((0,maximum(x[1] for x in σ_b.(β,α, ρ, ν, t_ex, f3, K))))
xlims!((minimum(K),maximum(K)))
```



Adding a few lines similar to [this answer]() makes the chart interactive.



Changing the level of correlation makes the smile "rotate" around the ATM point, and if $\rho < 0$, volatility is lower for higher (ITM) strike prices and vice versa (vol increases on the left-hand side of the figure).

Therefore, it is possible to fit the entire vol curve nicely with this model. One side remark, it will NOT fit quoted vols as it is a general best fit around all points. If matching quoted vols is a desired feature, one could for example combine piecewise linear within quoted spectrum and SABR for extrapolation.

Edit:

I followed the notation on P.13 of the original paper which states that

> The three parameters α, ρ and ν have different effects on the curve: the parameter α mainly controls the overall height of the curve, changing the correlation ρ controls the curve's skew, and changing the vol of vol ν controls how much smile the curve exhibits.

The other answer (by the way, refering to the answer above me is misleading because you can sort the answers in several ways) uses the notation from Wikipedia it seems. Wikipedia defines α as σ and ν as α, which is quite misleading, given the choice of parameters in the original paper. Also, the authors (see the paper on P.8) chose the name "stochastic-αβρ model", which has become known as the SABR model because they make α (the volatility) a stochastic process. That's also something the other answer clearly missed it seems.

2 +1. And congrats on being number 1 on QSE second year running. Are you gonna go for the "golden hattrick" ? :) That hasn't been done since...Manchester United...? :D – Jan Stuller Dec 26, 2022 at 6:50

It has only been done by ManU, hasn't it? Thanks, although Football teaches you to never think its over until the very end. With regards to SE, I plan to add another hobby next year and I think next year might be busier at work, with new EBA guidelines for IRRB and CSRBB and so forth. Nonetheless, it is good fun and keeps my looking at stuff, so who knows... – AKdemy Dec 27, 2022 at 14:12

Yep, only ManU in the Premier league. In other major sports leagues: Chicago Bulls between 90-93 in the NBA, and a few hockey teams in the NHL (Toronto 47-49, Montreal 56-60, 76-79, New York Islanders 80-83). And then maybe AKdemy on QSE? :D (but worth saying that sooo many teams won two consecutive titles, it is the third one that proves elusive :D). Anyway, keep going, QSE is in short supply of good contributors. PS: maybe sometime in 23 we can grab a coffee if you're Lon-based (I am mostly WFH, but can ping you when I am in the office...). – Jan Stuller Dec 29, 2022 at 11:14

@AKdemy thanks, the answer above you says "α was the volvol", while you call Nu as volvol, I see this very often in literature, which is which? Isn't Alpha IV ATM, being the smile level and Nu Vol of vol? – Skittles Dec 12, 2023 at 20:34

1 @AKdemy thanks, makes sense, to address the mistake in the first answer, I saw it in literature a few times already – Skittles Dec 13, 2023 at 22:00

2

The are implied parameters. You basically do a parametric dimension reduction by implying them across a range of observed prices, checking the errors, and then you might interpolate or even cautiously extrapolate. In reality, desks will have their own spreads but you can generate a volatility surface as a baseline from these parameters.

If I remember correctly, there might be some issues with this parametrization in terms of stability depending on what you are doing.

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answered Feb 29, 2020 at 19:45

safetyduck
236 1 8