Market-neutral carry strategies

Harvesting carry without market risk

- Carry strategies have become one of the main pillars of risk-premia investing. As pointed out by Kolanovic (2013) in '[Systematic Strategies across Asset Classes]', an inherent feature of standard carry strategies is their long market/short volatility exposure.
- Carry trades are typically unwound during periods of falling risk appetite and liquidity elimination which results in pronounced negative skewness. In the current publication we propose a portfolio construction methodology that eliminates the market exposure from the carry portfolio.
- We show that there is an explicit link between market-neutral carry portfolios and alpha portfolios and we apply a uniform statistical approach across asset classes to neutralize the market factors exposure.
- Market-neutral carry portfolios across currencies, rates, credit, commodities and equity indices are constructed by explicitly taking into account the asset class specific features and tying the expected P&L to the carry premium.
- Significant diversification benefits arise from combining the individual asset-class carry strategies into a cross-asset carry portfolio. The market-neutral cross-asset carry portfolio exhibits positive skewness at various return horizons.
- There are also strong diversification benefits between the market-neutral carry strategy and the trend-following strategy.

Global Quantitative and Derivatives Strategy

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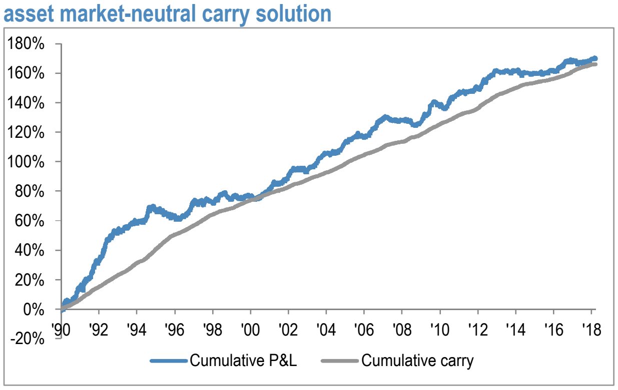
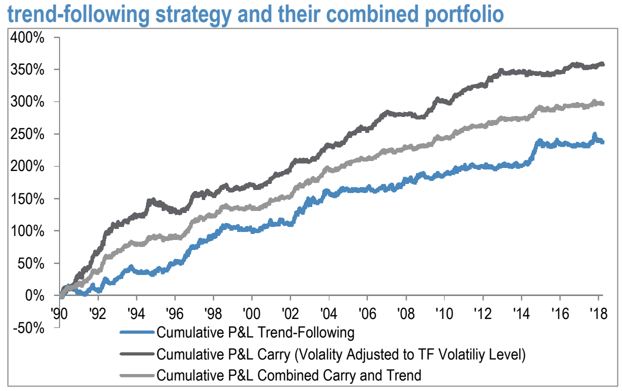
Figure 1: Cumulative P&L versus cumulative carry for the cross-asset market-neutral carry solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 2: Cross-asset market-neutral carry solution, cross-asset trend-following strategy and their combined portfolio



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 1: Performance statistics for a cross-asset carry portfolio, trend-following strategy and their combination

	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Cross-Asset Market-Neutral Carry	5.56%	4.10%	1.36	9.80%
Cross-Asset Trend-Following	8.40%	9.31%	0.9	14.74%
Combination (Vol Parity)	10.47%	6.89%	1.52	7.85%

Source: J.P. Morgan Quantitative and Derivatives Strategy

See page 57 for analyst certification and important disclosures, including non-US analyst disclosures.

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We acknowledge the contribution of Rahul Dalmia and Twinkle Mehta of J.P. Morgan India Private India Limited for data analysis in this report.

Carry as a risk-premia style: appeal, drawbacks and potential improvements

In the risk-premia primer 'Systematic Strategies across Asset Classes' Kolanovic et al. (2013) J.P. Morgan introduced the carry risk premia style as a strategy that benefits from the outperformance of higher yielding assets over lower yielding assets. It has also become standard to define carry as the return that accrues to the investor if markets remain stale (prices, yield curve etc. do not move).

Carry strategies have become one of the main pillars of risk-premia investing. Carry is perhaps the risk-premia style that most easily ticks all the boxes of requirements for a systematic strategy to be accepted as a risk premia: carry is implemented as a long/short portfolio, the source of income is well-identifiable, the profitability is persistent across asset classes and regions (both developed and EM) and the carry strategies have been well-researched and covered by both practitioners and academics.

As pointed out by Kolanovic et al. (2013), the main risk of standard carry strategies is their inherent exposure to market risk. As high-yielding assets tend to be riskier than lower-yielding assets a standard carry strategy implementation leads to an implicit long market and short volatility exposure. Carry trades are typically unwound during periods of dwindling risk appetite and liquidity elimination (Brunnermeier, Nagel, and Pedersen (2008)) which results in pronounced negative skewness (Y. Lempérière et al. (2017)).

In the current publication we aim to eliminate the market exposure from the carry strategies and design carry strategies whose performance is solely driven by the carry premium collected. The approach undertaken is to construct a statistical factor model for the residual asset's return (the return in excess of the carry, typically the spot return). Subsequently, we use the estimated model to remove (hedge) the factor exposure from the constructed carry portfolio within an asset class. The portfolio is constructed in such a way that the factor exposures of the constituents offset each other and the expected P&L is directly linked to the carry return of the portfolio.

While we have undertaken the statistical route, nothing prevents us from utilizing factor models that rely on market observable variables. For example, we can hedge a currency carry portfolio against moves in dollar and S&P500, a credit portfolio against moves in equities and volatility etc. Such an approach may be appealing to more fundamentally oriented investors and investors who want to assure that their carry portfolios are immune to risk drivers to which they are already exposed so that they are not doubling up on existing risks.

Our prior assumption is that removing the factor exposure will mitigate (and hopefully eliminate) the problem of the negative skewness that is typical for carry strategies. A further advantage of market-neutral carry strategies is that we can form an expectation for the return of the strategy (as the expected carry strategy P&L is solely driven by the carry income that can be estimated). In this way we can also compare the attractiveness of the carry strategies in the various asset classes and eventually perform tactical allocation based on the appeal of the carry return.

While hedging the factors within a carry portfolio brings natural benefits it also carries some risks. First, as we rely on models estimated on past data, we might experience a hedging error due to changing market patterns. On our side we take utmost care to mitigate this problem by monitoring the market structure for changes (using contemporaneous tests for structural breaks) and diversifying across the data-frames we estimate the historical relationships. Second, hedging the factors brings increased costs as we have to continuously adjust the hedge and also to have higher leverage in the portfolio.

In the current paper we also take a portfolio approach and we examine carry strategies across asset classes - currencies, rates, credit, commodities and equities (similar to Koijen et al. (2016) and Baz et al. (2015)). Naturally, our approach differs from the above-mentioned studies, as, in addition to the portfolio of carry strategies in different asset classes, we focus on removing the market exposure.

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Hedging the factor exposure in a carry portfolio

Capturing the carry premia in a market-neutral carry portfolio

Our goal is to isolate and capture solely the carry component. It is typically assumed that the carry is the return that accrues to the investor if the market conditions do not change. By unchanged market conditions it is generally implied that spot levels does not move, yield curves stay the same etc. We just have a passage of time and during this interval the investor can monetize a carry premium.

Let $\boldsymbol{R}_t$ represent the vector of total returns of *n* assets at time *t*, $\boldsymbol{RS}_t$ denote the corresponding vector of spot returns at time *t* and $\boldsymbol{Carry}_{t-1}$ be vector of carry premia observed at time *t-1*.[1]

$$\boldsymbol{R}_t = \boldsymbol{RS}_t + \boldsymbol{Carry}_{t-1}$$

Hence, if the spot does not move ($\boldsymbol{RS}_t = \boldsymbol{0}$), the total return is equivalent to the carry premia ($\boldsymbol{R}_t = \boldsymbol{Carry}_{t-1}$).

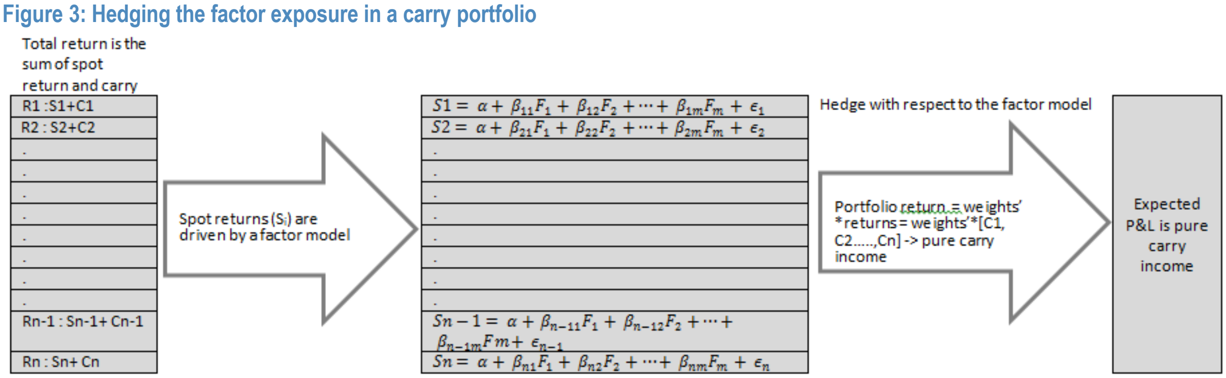
Let's assume that a factor model exists for the sport returns. $\boldsymbol{F}_t$ denotes the realization of *m* factors at time *t*, $\boldsymbol{B}$ is *nxm* matrix containing the market sensitivities and $\boldsymbol{\epsilon}_t = [\epsilon_{1,t,}\epsilon_{2,t}, \ldots\ldots, \epsilon_{n,t}]$ are the residual returns at time *t* being distributed as $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ being a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$ for $i\epsilon[1, \ldots. n]$.

$$\boldsymbol{RS}_t = \boldsymbol{a} + \boldsymbol{BF}_t + \boldsymbol{\epsilon}_t$$

Hence, if it holds $\boldsymbol{w}'\boldsymbol{a} = 0$ and $\boldsymbol{w}'\boldsymbol{B} = 0$ it follows that the expected portfolio return reflects is tied to the carry income:

$$E(\boldsymbol{w}'\boldsymbol{R}_t) = \boldsymbol{w}'\boldsymbol{Carry}_{t-1}$$

Figure 3: Hedging the factor exposure in a carry portfolio



Source: J.P. Morgan Quantitative and Derivatives Strategy

In a similar manner, the portfolio variance does not depend on the factor volatilities

$$PortVar = E(Var_{t-1}(\boldsymbol{w}'\boldsymbol{R}_t)) = Var(\boldsymbol{w}'\boldsymbol{\epsilon}_t) = \sum_{i=1}^{n} w_i^2\sigma_i^2$$

[1] As we will see later, while in many cases $\boldsymbol{RS}_t$ will be the precisely the vector of spot returns, we can also think of $\boldsymbol{RS}_t$ as the difference between the total returns $\boldsymbol{R}_t$ and the carry premia $\boldsymbol{Carry}_{t-1}$.

The above relationship also allows us easily to break down the risk contributions by assets and asset groups. In this way we can specify factor (market)-neutral carry maximization problems subject to percentage risk-contribution constraints:

$$RiskContr_i = \frac{w_i^2 \sigma_i^2}{\sum_{i=1}^{n} w_i^2 \sigma_i^2}$$

Market-neutral carry portfolio is an alpha portfolio

Below we also elaborate why in this setup the carry portfolio can be also considered an excess total return (alpha) portfolio.

Let's start by making use of the spot factor model to express the total return of the assets. In addition, let's assume that the carry returns associated with the factors at time *t* are bundled in the vector $\boldsymbol{FCarry}_t$ and $\boldsymbol{RF}_t$ is the total return vector of the factors. It follows:

$$\boldsymbol{R}_t = \boldsymbol{a} + \boldsymbol{Carry}_{t-1} - \boldsymbol{BFCarry}_t + \boldsymbol{BRF}_t + \boldsymbol{\epsilon}_t$$

We can denote as $\boldsymbol{\alpha}$ the vector of the excess total returns of the assets. In addition, let's assume that the carry measures are constant ($\boldsymbol{Carry}_t = \boldsymbol{Carry}$ and $\boldsymbol{FCarry}_t = \boldsymbol{FCarry}$).[2] Hence, we can write:

$$\boldsymbol{R}_t = \boldsymbol{\alpha} + \boldsymbol{BRF}_t + \boldsymbol{\epsilon}_t,$$

where $\boldsymbol{\alpha} = \boldsymbol{a} + \boldsymbol{Carry} - \boldsymbol{BFCarry}$. Hence, if it holds $\boldsymbol{w}'\boldsymbol{a} = 0$ and $\boldsymbol{w}'\boldsymbol{B} = 0$ it follows $\boldsymbol{w}'\boldsymbol{\alpha} = \boldsymbol{w}'\boldsymbol{Carry}$.

Hence, carry maximization under the constraint of factor neutralization can also be viewed as alpha maximization. Also note that the carry returns associated with the individual factors are not important for our optimization problem.

Another interesting point to highlight is that when the assets' carry returns reflect solely the exposures to betas and hence are solely compensation for the exposures to factors, there is no solution.

Let's assume that $\boldsymbol{Carry} = \sum_{i=1}^{m} k_i \boldsymbol{\beta}_i$, where k_i are constants and $\boldsymbol{\beta}_i$ is the vector of exposures to factor i . It follows that $\boldsymbol{w}'\boldsymbol{Carry} = \sum_{i=1}^{m} k_i \boldsymbol{w}'\boldsymbol{\beta}_i = 0$. Hence, the carry returns should reflect the assets' idiosyncratic risk or an extra premium for the carry optimization under factor neutralization to be feasible. For example, under a single factor model we can assume that carry is compensation for the market and asset specific volatility plus additional risk premium, $\boldsymbol{Carry} \sim k\boldsymbol{\beta}\sigma_m + \gamma\boldsymbol{\sigma_\epsilon} + \boldsymbol{rp}$ where $\boldsymbol{\sigma_m}$ is the market volatility, $\boldsymbol{\sigma_\epsilon}$ is the vector of asset specific volatilities, k is the compensation for per unit of market volatility and γ is the compensation for the idiosyncratic volatility across all assets and $\boldsymbol{rp}$ is the vector of additional risk premia. Then it follows $\boldsymbol{w}'\boldsymbol{Carry} \sim \boldsymbol{w}'\gamma\boldsymbol{\sigma_\epsilon} + \boldsymbol{w}'\boldsymbol{rp}$.

Carry portfolio construction approaches

In the current study we compare three portfolio construction approaches. They differ in their ease of implementation, the degree of factors-hedging and risk-management constraints.

Ranking solution

The first approach is similar to the standard established practice for constructing carry portfolios. We rank the assets on basis of their carry-to-vol ratio and we go long the assets in the top quartile of the ranked group and we go short the assets in the respective bottom quartile. We allocate the same risk to the selected assets (the individual positions are inversely scaled by volatility). Furthermore, a predetermined level of portfolio volatility is targeted at every point in time by taking into account the covariance matrix. The main advantage of such an approach is its ease of

[2] This is certainly a strong assumption over a long timeframe but a realistic one over a shorter one.

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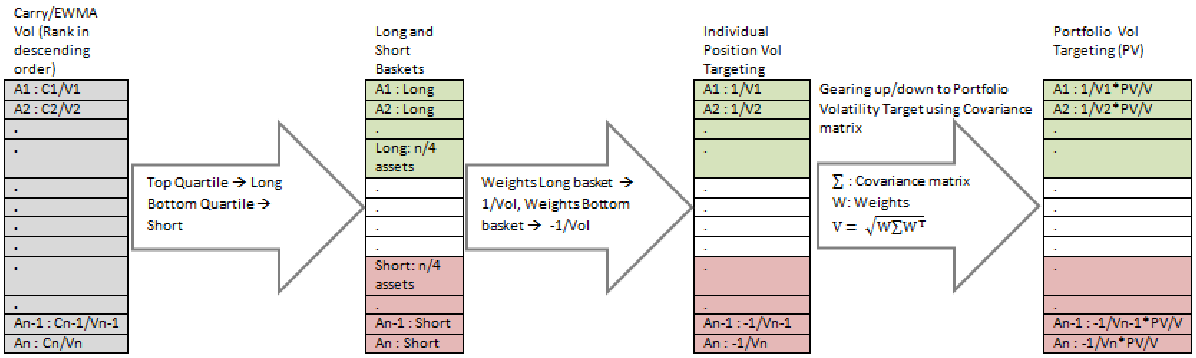
implementation. While the approach incorporates some management principles, there is no guarantee that the exposure to major risk drivers is eliminated in the long/short construction.[3] Also note that such an approach will also not result in an optimal Sharpe ratio portfolio. For constructing an optimal Sharpe ratio portfolio the investor can use the analytical results in the Appendix (at the expense of decreased control on the size of the positions and leverage).

Figure 4: Workflow of the ranking approach

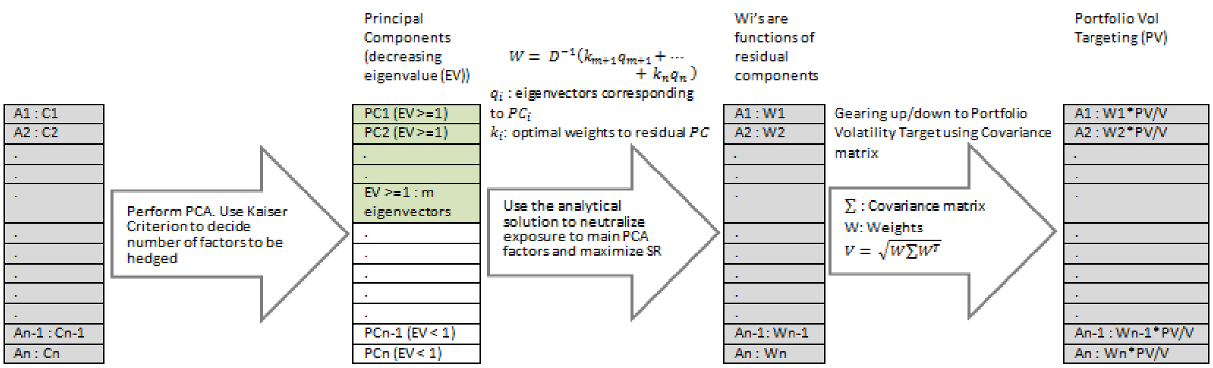


Source: J.P. Morgan Quantitative and Derivatives Strategy

Analytical market-neutral solution

The second approach relies on the analytical derivations of the optimal Sharpe ratio portfolio when a few principal components (PC) are hedged.[4] The main advantage is that the approach is completely analytical, but at the same time there are no ways to control the exposure to individual assets or groups. Additional information can be found in the Appendix, but at this stage it suffices to say that the optimal portfolio is a combination of the residual principal components (PCs) that are not being hedged.

Figure 5: Workflow of the analytical approach



Source: J.P. Morgan Quantitative and Derivatives Strategy

Comprehensive market-neutral solution

The third approach is our preferred one but also the most complex one and relies on numerical optimization. We aim to maximize the carry income while neutralizing the factor exposures. Using some of the theoretical properties of the portfolio design we impose additional constraints on the risk-contribution by asset and by groups of assets (for

[3] Note that the ranking approach will fare well if we have a single driving factor to which all markets exhibit perfect correlation. In this case the beta of an asset to the market is equal to the ratio between the volatility of the asset and the volatility of the market. Hence, as in the ranking approach the positions are inversely proportional to volatility, the resulting portfolio will be beta-neutral portfolio.

[4] Please refer to the Appendix for additional information on Principal Components Analysis (PCA).

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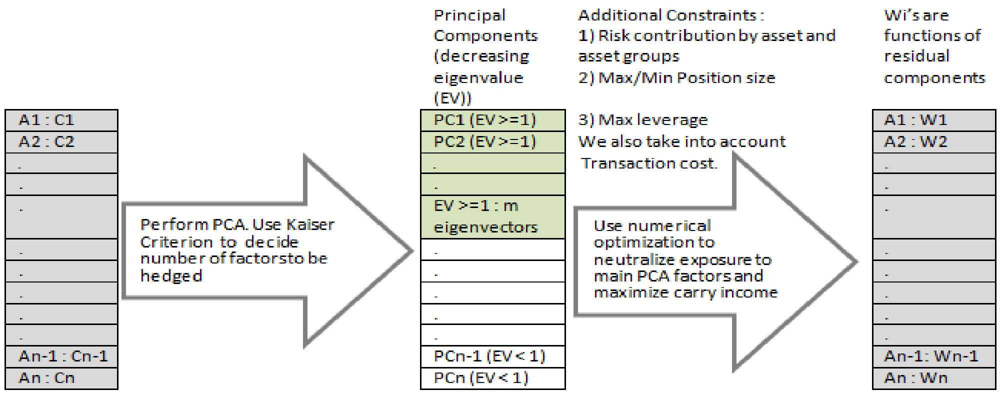
example US equity indices among all equity indices, agricultural commodities among all commodities etc). Transaction costs are also explicitly taken into account when constructing the portfolio. Further details about the approach can be found in the relevant section in the Appendix.

Figure 6: Workflow of the comprehensive approach



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 2: Advantages and disadvantages of the different portfolio construction mechanisms

<table>
<tr><th rowspan="2"></th><th rowspan="2">Ranking</th><th colspan="2">Market-neutral</th></tr>
<tr><th>Analytical</th><th>Comprehensive</th></tr>
<tr><td rowspan="2">Advantages</td><td rowspan="2">• Ease of implementation
• The solution can be close to market neutral if there is a single market factor and assets have similar betas w.r.t. the market</td><td colspan="2">• Capturing pure carry</td></tr>
<tr><td>• A quick way to achieve market-neutrality</td><td>• Ability to control the risk contribution by individual assets and groups
• We can hedge the trend-components as well
• If desired, we can even impose additional constraints</td></tr>
<tr><td rowspan="2">Disadvantages</td><td rowspan="2">• It is highly likely that the portfolio retains beta exposure
• It can be subject to sizable drawdowns and negative skewness</td><td>• No control over position sizes and risk contributions
• No trends offset</td><td></td></tr>
<tr><td colspan="2">• Potential hedging error due to changing market patterns
• Higher costs and eventually leverage</td></tr>
</table>

Source: J.P. Morgan Quantitative and Derivatives Strategy

Model-robustness considerations

The debate about overfitting has become ever more topical. In the current study we have taken an utmost care to remove the potential biases from the study.

One of the main risks in systematic strategies design is the period selection. Similarly to our trend-following paper we combine several periods. We make use of 3 months and 6 months daily returns and 1 and 2 year weekly returns to estimate volatilities, correlations and perform PCA analysis. Subsequently, our positions are an average of the model results at the various frequencies.

In addition, we apply the same methodology across all the asset classes. In the market-neutral solutions we use the Kaiser criterion in an automatic fashion to select the number the principal components to retain.[5]

Our transaction costs assumptions are quite conservative and in line with the costs for the sell-side risk-premia solutions in the market. Furthermore, to avoid problems with non-synchronous data we include a one-day delay in our backtest.

Also, in order to assure that the factors we extract from the historical data are the ones that are still relevant we use contemporaneous tests for structural breaks in our comprehensive solution.[6] If the latest observations are found to come from a different distribution that the one implied from the historical dataset used to extract the factors, we switch off the systematic strategy as the hedging algorithm is likely to fail.

Market-neutral FX carry strategy

FX carry can perhaps be considered the harbinger of all carry strategies. Borrowing in low yield currencies to invest in high yield currencies has been popular strategy not only with professional investors, but also with private individuals (for example Japanese housewives speculating on the Australian dollar or households in CEE countries taking out mortgages in Swiss Francs).

Standard FX carry implementations have been profitable over several decades but are beset with sizable drawdowns and negative skewness.[7] Typically high carry currencies are also riskier and their fortunes reverse when risk-appetite wanes. Conversely, low carry currencies such as USD, CHF and JPY are considered safe-haven and appreciate during times of risk-averseness. Subsequently the standard FX carry strategy resembles selling insurance, collecting the premium at good times and having to pay off at times when markets experience turbulence. Empirically, the premium collected at good times has been more than sufficient to offset the losses when the carry trade unwinds and we note this track record strongly refutes any academic hypothesis that interest rate differentials are compensation for expected future depreciation.

To mitigate some issues pertinent to the FX Carry trade, practitioners attempted to time the entry into and exit from the FX carry trade. Post the GFC in 2008 indicators that assess some of form of risk-appetite have become popular. Typically, they were based on extracting the momentum from volatility measures in various asset classes. Though in the beginning their performance was promising, it has dwindled in the years of QE when spikes in volatility have only been transitory. In addition, additional overlay signals like currency momentum and the momentum in the interest rate spread have been also used.[8]

Another alternative to better navigate the investments in FX carry is to analyze the interplay among carry and other risk premia styles like value and economic momentum – an approach that can resonate well with investors looking for a balanced mix between a macro-discretionary and a quantitative approach.[9]

In the current publication our goal is to remove the market exposure from the currency carry strategy at the design level and in this way to mitigate and hopefully eliminate the disadvantages linked to the negative skewness and the drawdowns associated with the strategy. In this way we also keep a pure exposure to the carry risk premia without overlaying with other risk premia styles.

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[5] Please refer to the Appendix for more information on the Kaiser criterion and some alternatives.
[6] Please see the section in the Appendix on the Mahalanobis distance.
[7] Standard FX Carry implementations are typically based on the carry-to-vol ratio. See for example, Investment Strategies No. 10: JPM FX and Commodity Barometer, and Investment Strategies No. 12: JPM Carry to Risk Primer.
[8] Please refer to Investment Strategies No. 47: Alternatives to Standard Carry and Momentum in FX.
[9] See, for example, the JP Morgan research notes by Meera Chandan: Year of the Underdog: FX carry is cheap going into 2018, When value and carry collide : Implications of improving growth for FX risk premia, Diverging Fortunes: FX value to trump carry in 2017, What has and hasn't changed in FX risk premia since US exceptionalism emerged.

Our approach relies on identifying market driving factors in a statistical way, but fundamental factors can also be used. We plan to further elaborate on this thematic in a separate publication focused on currency risk premia.

The mechanics of the market-neutral FX carry strategy

Let $F_{i,t,T}$ denotes the forward FX rate for a currency pair $CCY1/CCY2$ – how many units of $CCY2$ for one unit of $CCY1$ at time *t* for a contract expiring in *T* days.

It is well known that by no-arbitrage arguments:

$$F_{i,t,T} = FX_{i,t}e^{(r_{CCY2,t}-r_{CCY1,t})T/365} \quad ,$$

where $r_{CCY1,t}$ is the annualized interest rate for currency $CCY1$ at time *t* with tenor *T* and $r_{CCY2,t}$ is the annualized rate for currency $CCY2$ at time *t* with tenor *T*. Iterating forward:

$$F_{i,t+1,T-1} = FX_{i,t+1}e^{(r_{CCY2,t+1}-r_{CCY1,t+1})(T-1)/365}$$

Typically we refer to carry as the difference between the interest rate of $CCY1$ and $CCY2$, i.e. $FXCarry_{i,t} = r_{CCY1,t} - r_{CCY2,t}$. Let's denote the daily return for the forward contract *i* at time *t* as $R_{i,t}$, the daily spot return for currency *i* at time *t* as $RS_{i,t}$ and the change in carry as $dFXCarry_{i,t} = FXCarry_{i,t} - FXCarry_{i,t-1}$. It follows that for the daily return for the forward contract *i* at time *t+1* it holds:

$$R_{i,t+1} = RS_{i,t+1} + FXCarry_{i,t}/365 - dFXCarry_{i,t+1}(T-1)/365$$

Let's assume that a factor model exists for the vector of spot returns $\boldsymbol{RS}_t$ of all the currencies in our universe:

$$\boldsymbol{RS}_t = \boldsymbol{a} + \boldsymbol{BF}_t + \boldsymbol{\epsilon}_t$$

As usual $\boldsymbol{F}_t$ denotes the factors at time *t*, $\boldsymbol{B}$ is matrix containing the market sensitivities to the factors and $\boldsymbol{\epsilon}_t = [\epsilon_{1,t,}\epsilon_{2,t}, \ldots\ldots, \epsilon_{n,t}]$ are the residual spot returns having the standard distributional assumptions $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ being a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$ for $i\epsilon[1, \ldots. n]$.

Hence, if it holds $\boldsymbol{w'a} = 0$, $\boldsymbol{w'B} = 0$ and $E(dFXCarry_{i,t+1}) = 0$, we arrive at:[10]

$$E(\boldsymbol{w'R}_t) = \boldsymbol{w'FXCarry}_{t-1}/365$$

Under the assumption that $\boldsymbol{\epsilon}_t$ and $\boldsymbol{dFXCarry}_t$ are uncorrelated and if $Cov(\boldsymbol{dFXCarry}_t) = \boldsymbol{\Omega}$, it follows:

$$Var(\boldsymbol{w'R}_t) = Var\left(\boldsymbol{w'\epsilon}_t + \boldsymbol{w'}\frac{\boldsymbol{dFXCarry}_t(T-1)}{365}\right) = \sum_{i=1}^{n} w_i^2\sigma_i^2 + \left(\frac{T-1}{365}\right)^2 \boldsymbol{w'\Omega w}$$

For the currency markets we analyze the volatility of the changes in carry are negligible in comparison to the volatility of the residual returns and hence impact of the volatility of the changes in the carry on total portfolio risk can be safely ignored. Such an empirical effect can be attributed to the relative stability of interest rate policies managed by the Central Banks.

[10] Our universe consists of forwards that at inception have a maturity of 1 month. The assumption $E(dFXCarry_{i,t}) = 0$ is equivalent to the assumption that the term structure of the spreads between the yields of CC1 and CC2 is flat between 0 and 1 month. Given the average maturity of the forwards in the backtest 2 weeks (maximum 1 month) we consider such an assumption realistic to us. t-tests have also not rejected the hypothesis that the average change in the spread is zero.

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If an investor is concerned by the change in carry effect, we can specify a factor model for the difference between the return of the FX forward and the carry ($R_{i,t+1} - \frac{FXCarry_{i,t}}{365}$) instead of specifying a factor model for the sport returns.

Empirical results

Our tradable assets consist of 1 month FX forwards of G10 currencies versus USD.

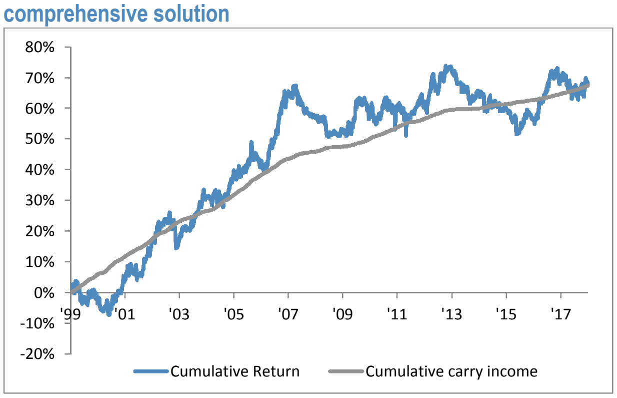
Table 3: Universe of FX Forwards

EURUSD	GBPUSD	USDJPY	USDSEK	USDNOK	AUDUSD	NZDUSD	USDCAD	USDCHF

Source: J.P. Morgan Quantitative and Derivatives Strategy

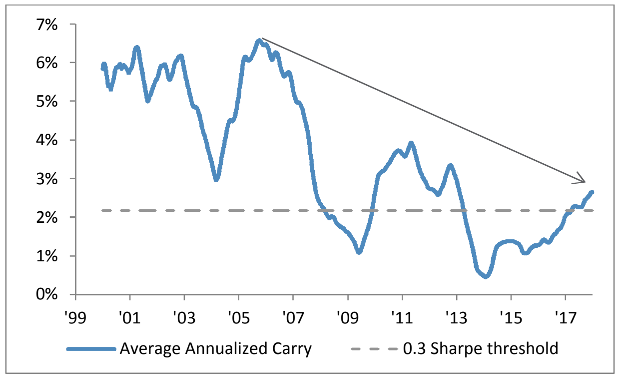
Below we show the backtest results for comprehensive approach along with the carry income that has been harvested by the strategy at every point in time.

Figure 7: Cumulative return and carry income of the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 8: Average annualized carry of the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 4: Performance statistics of the G10 carry comprehensive solution

Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
3.59%	7.25%	0.50	22%

Source: J.P. Morgan Quantitative and Derivatives Strategy

It also well-known that the carry income harvested by the G10 carry strategies has compressed post the GFC. The main culprits have been convergence in the monetary policies and adoption of the QE policies, which have compressed the spread between the high yielders and low yielders and increased correlation.

After 2008 on a very few occasions (namely the period from 2010 to mid-2013) has the carry income (after costs) been sufficient to break above a hurdle level corresponding to 0.3 Sharpe ratio[11]. As of recently the carry income has increased and it remains to be seen whether this will be a sustainable development.

In a market-neutral carry implementation carry is the main driver of return. In that respect investing in a strategy with an expected Sharpe ratio below a certain threshold is a subjective call. In our opinion a Sharpe ratio of 0.3 is reasonable minimum level that a market-neutral carry strategy should aim to achieve.

[11] At annualized volatility of 7.25%, the carry income needed to achieve a Sharpe of 0.3 is 2.17% per year.

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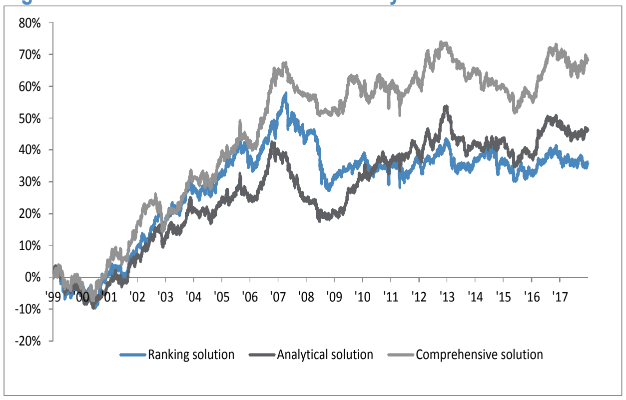
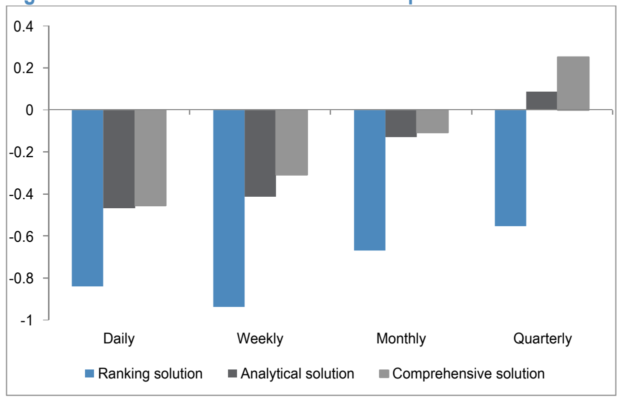
The lack of carry to be captured is shared problem among the different portfolio construction approaches. The comprehensive approach fares much better in a backtest in comparison to the ranking and analytical ones. Both the analytical and comprehensive approach exhibit smaller negative skewness than the ranking one and the return distribution is almost symmetric at the monthly and quarterly frequencies for those two approaches. In contrast, the ranking approach has pronounced negative skewness at all frequencies and the biggest drawdown.

Figure 9: Cumulative returns various carry solutions



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 10: Skewness at different return frequencies



Source: J.P. Morgan Quantitative and Derivatives Strategy

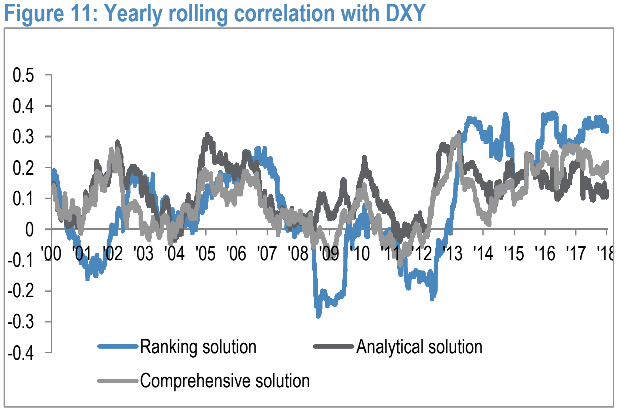
Table 5: Comparative performance statistics

	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Ranking solution	1.91%	7.29%	0.26	30.69%
Analytical solution	2.43%	6.41%	0.38	25.05%
Comprehensive solution	3.59%	7.25%	0.50	22.37%

Source: J.P. Morgan Quantitative and Derivatives Strategy

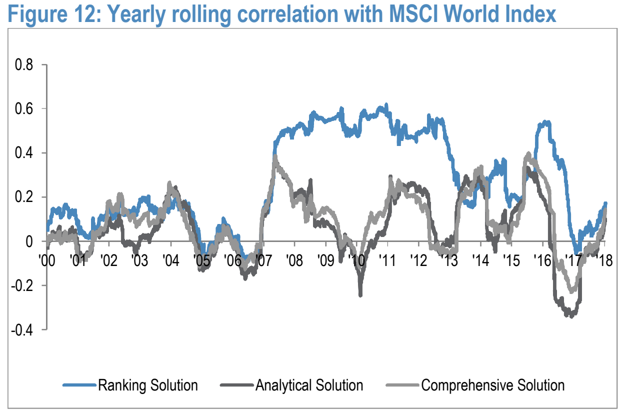
Additional insights can be gathered from the analysis of the correlations of the carry strategies' returns with the returns of markets that exhibit certain behaviors during crisis periods. In sell-offs equity markets are falling, the dollar typically strengthens and carry strategies suffer a drawdown as risk appetite wanes.

Figure 11: Yearly rolling correlation with DXY



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 12: Yearly rolling correlation with MSCI World Index



Source: J.P. Morgan Quantitative and Derivatives Strategy

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Table 6: Correlation statistics of the FX carry solutions with DXY and MSCI World

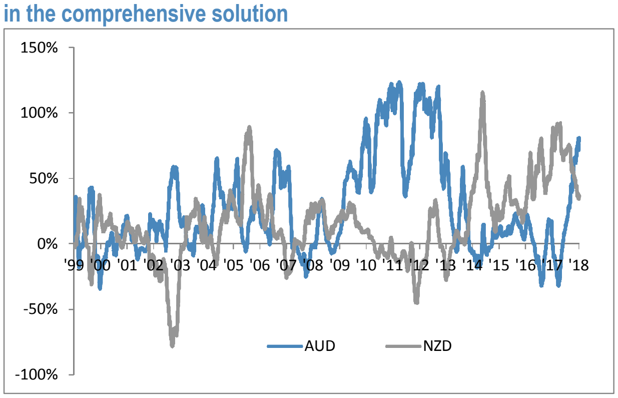
	DXY			MSCI World		
	Average	Max	Min	Average	Max	Min
Ranking solution	0.09	0.38	-0.28	0.27	0.62	-0.09
Analytical solution	0.13	0.31	-0.06	0.06	0.38	-0.34
Comprehensive solution	0.09	0.30	-0.12	0.08	0.40	-0.23

Source: J.P. Morgan Quantitative and Derivatives Strategy

Above we have shown the correlations of the returns of the various portfolio approaches with the returns of the DXY and the MSCI World Index. The ranking solution can exhibit a highly negative correlation with DXY at times of crisis, while the market-neutral implementations almost do not exhibit negative correlation to DXY. In a similar vein, the ranking solution has a positive correlation to the equity market on average, while the correlation of the market-neutral approaches to equity market is close to zero.

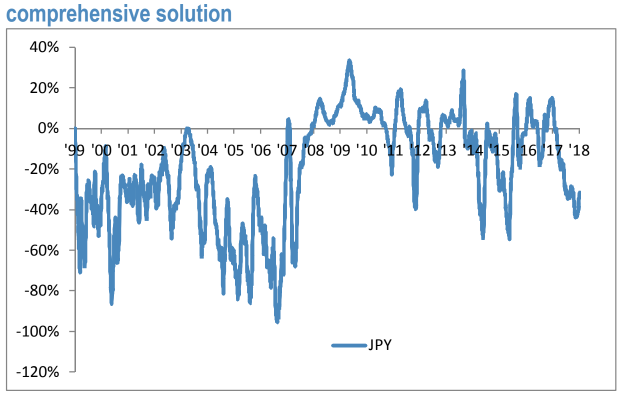
An insight into why market-neutral implementations are different than the standard one can also be gathered from analyzing the positions of the comprehensive solution. A standard implementation would have strictly kept a long position in AUD and NZD, while the comprehensive one can on occasions have a relative-value position between the two. Similarly, a standard carry implementation would have had a short position in JPY, while the comprehensive solution might occasionally take a positive position.

Figure 13: Positions in AUD and NZD as a percentage of capital in the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 14: Position in JPY as a percentage of capital in the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Rates roll-down strategy

Risk-averse investors require a compensation for holding a long-term bond over rolling a short maturity one. Hence, the yield of a long-term bond can be decomposed into the expected short-term rate over the life of the bond plus a risk (term) premium. As the additional term premium for every unit of holding period has to be positive it is easy to see why the holder of a long-term bond monetizes the term-premium as we get closer to maturity.

It is difficult to disentangle the term premium from the expected short-term rate. An upward sloping curve can reflect expectations for future increases in rates or/and a term premium that increases with maturity. A downward sloping one indicates expectations for rates cuts or/and a term premium that falls with maturity.

Hence, fixed income carry strategies have typically been structured to benefit from the shape of the yield curve directly. When the curve is upward sloping, rates rolls down on the curve with maturity approaching and this translates into excess positive returns for investors with long positions. Conversely, when the yield curve is downward sloping rates increase when we get closer to maturity and that leads to positive excess returns for the investors with short positions.

A popular portfolio construction approach within fixed income carry strategies has been to construct a duration neutral portfolio. In this way the portfolio is protected if the underlying rates move by the same amount. Naturally, the rates in different countries can exhibit different volatilities and additional adjustments have to be made, i.e. the different rates are not bound to move by the same amount. Also, there might be additional factors driving the dynamics of a cross-section of rates from different countries. As in the seminal paper of Litterman and Scheinkman (1991) we use **P**rincipal **C**omponent **A**nalysis (PCA) to extract the factors.

The mechanics of the market-neutral rates roll-down strategy

Our tradable assets consist of forward starting swaps. As the swap is forward starting, there is no carry in the traditional sense as the rate of the floating leg has still not been fixed. If we assume the swap curve does not move between two consecutive dates there is a P&L effect that solely arises due to the roll-down on the curve. If the curve is upwards sloping, the roll-down will be positive. If the curve is downward sloping, the roll-down will be negative.

Let $FSwRate_{i,t,T}$ be the rate at time t for a swap i starting in T days and $FSwRate_{i,t+1,T-1}$ be the swap rate at time $t+1$ for a swap i starting in $T-1$ days. Furthermore, let $D_{i,t}$ denote the duration at time t for the swap i starting in T days. It follows that return of the swap i at time t can be represented as:

$$R_{i,t+1} = -D_{i,t} * \left(FSwRate_{i,t+1,T-1} - FSwRate_{i,t,T}\right)$$

We can make use of the implied forward rate at time t for a swap with maturity $T-1$ (the implied forward rate from the current curve at time t for as swap that will start one day sooner) and express the swaps return as:

$$R_{i,t+1} = -D_{i,t} * \left(FSwRate_{i,t+1,T-1} - FSwRate_{i,t,T-1}\right) - D_{i,t} * \left(FSwRate_{i,t,T-1} - FSwRate_{i,t,T}\right)$$

Our carry measure will be the difference between the implied tomorrow's forward rate and the current one adjusted for duration: $RollDown_{i,t} = -D_{i,t}\left(FSwRate_{i,t,T-1} - FSwRate_{i,t,T}\right)$. Let's denote the difference between the realized and the implied forward rate as $\Delta_{i,t+1} = FSwRate_{i,t+1,T-1} - FSwRate_{i,t,T-1}$. Subsequently, let's bundle the carry measures of n assets in our universe in the vector, their respective deviations between the realized and the implied forward rates and the duration measures into the following vectors:

$$\boldsymbol{RollDown}_t = \left[RollDown_{1,t}, \quad RollDown_{2,t}, \dots \dots \dots, RollDown_{n,t}\right]'$$

$$\boldsymbol{D}_t = \left[D_{1,t}, \quad D_{2,t}, \dots . \dots, D_{n,t}\right]'$$

$$\Delta_t = \left[\Delta_{1,t}, \quad \Delta_{2,t}, \dots \dots \dots \Delta_{n,t}\right]'$$

If $\boldsymbol{w}$ is the vector of portfolio weights, the portfolio return can be represented as:

$$\boldsymbol{w}'\boldsymbol{R}_{t+1} = -\boldsymbol{w}'(\boldsymbol{D}_t \circ \Delta_{t+1}) + \boldsymbol{w}'\boldsymbol{RollDown}_t \quad ,$$

where $\circ$ is the Hadamard product (elementwise) multiplication.

Let's assume that common factor structure (with m factors) exists for the deviations of the actual realized swap rates from their implied values based on the curve observed on the previous day:

$$\Delta_t = \boldsymbol{a} + \boldsymbol{BF}_t + \boldsymbol{\epsilon}_t$$

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As it has already been discussed $\boldsymbol{F}_t$ denotes the factors at time t, $\boldsymbol{B}$ is matrix containing the market sensitivities to the factors and $\boldsymbol{\epsilon}_t = [\epsilon_{1,t}, \epsilon_{2,t}, \ldots\ldots, \epsilon_{n,t}]$ are the residual returns having the standard distributional assumptions $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ being a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$ for $i \epsilon [1, \ldots . n]$.

Hence, if it holds $\boldsymbol{w}'(\boldsymbol{D}_t \circ \boldsymbol{a}) = \boldsymbol{0}$ and $\boldsymbol{w}'(\boldsymbol{D}_t \circ \boldsymbol{B}) = 0$ it follows:

$$\boldsymbol{E}(\boldsymbol{w}'\boldsymbol{R}_t) = \boldsymbol{w}'\boldsymbol{RollDown}_{t-1}$$

$$\boldsymbol{Var}(\boldsymbol{w}'\boldsymbol{R}_t) = \boldsymbol{Var}(\boldsymbol{w}'\boldsymbol{\epsilon}_t) = \sum_{i=1}^{n} w_i^2 \sigma_i^2$$

Empirical results

Our tradable assets consist of forward starting in either 3 or 6 months swaps with maturity of the underlying swap of either 9 years and 9 months (for the swaps starting in 3 months) or 9 years and a half (for the swaps starting in 6 months).

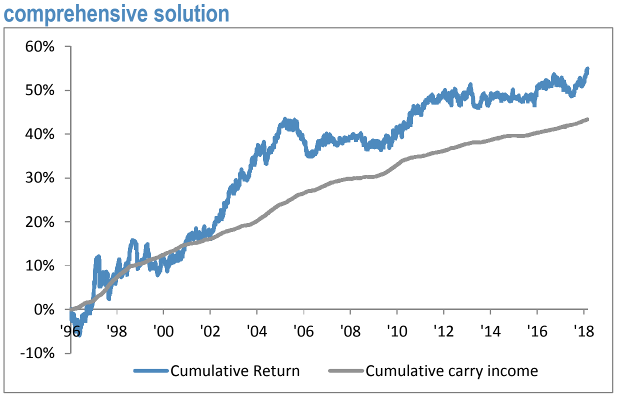
Table 7: Universe of forward starting swaps

EUR	GBP	USD	JPY	AUD	DKK	SEK	CHF	CAD

Source: J.P. Morgan Quantitative and Derivatives Strategy

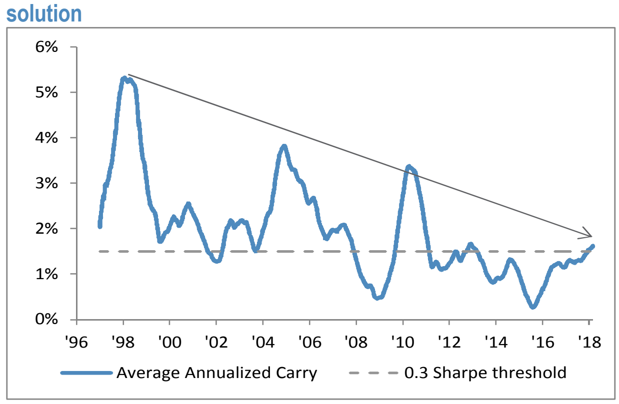
Below we show the backtest results for comprehensive approach with respect to the various underlyings in our universe.

Figure 15: Cumulative return and carry income for the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 16: Average annualized carry for the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 8: Performance statistics of the G10 carry strategy

Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
2.47%	5.01%	0.49	9.89%

Source: J.P. Morgan Quantitative and Derivatives Strategy

The P&L of the strategy outperforms the collected carry income mainly in the period 2002-2006 when curves were steepening. The carry of the rates strategy compressed heading into the GFC (as curves were flattening and even inverting), recovered after the GFC as curves steepened but then compressed again as monetary policies converged across the globe.

In a backtest the analytical approach outperforms in terms of Sharpe ratio, while the comprehensive one has been the laggard as of recently. The ranking approach has also performed strongly in recent years. Note that the ranking

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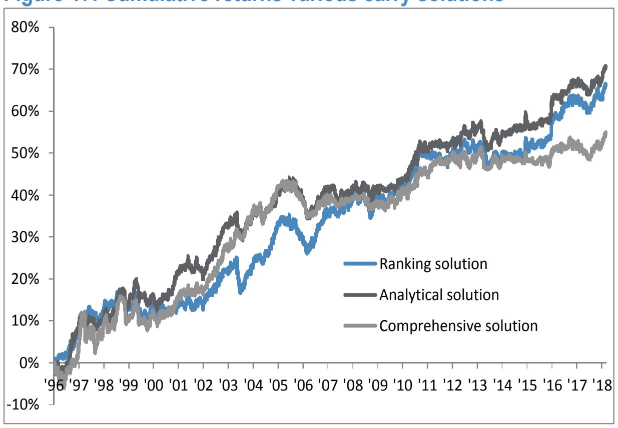
approach will fare well if we have a single driving factor to which all markets exhibit a perfect correlation[12]. In a similar setup the market-neutral approaches might falsely overhedge and incur additional costs.

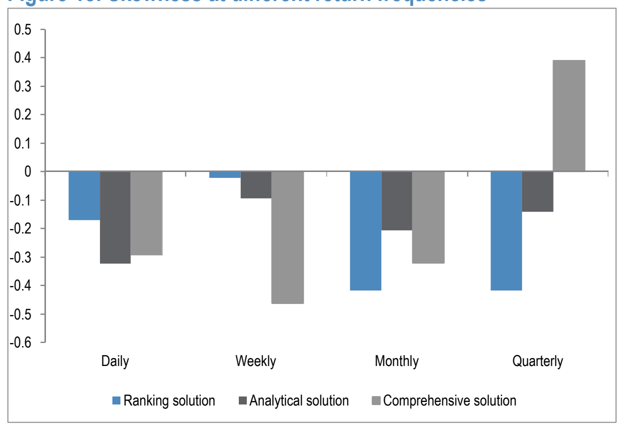
Another potential explanation is the strong trends that have been present in the fixed income markets. As we have discussed the trends are offset within the comprehensive approach but not in the market analytical. It has been observed during the QE years that markets with the most attractive carry (steep curves) have also exhibited stronger trends which might explain the underperformance of the comprehensive approach.

Figure 17: Cumulative returns various carry solutions



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 18: Skewness at different return frequencies



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 9: Comparative performance statistics

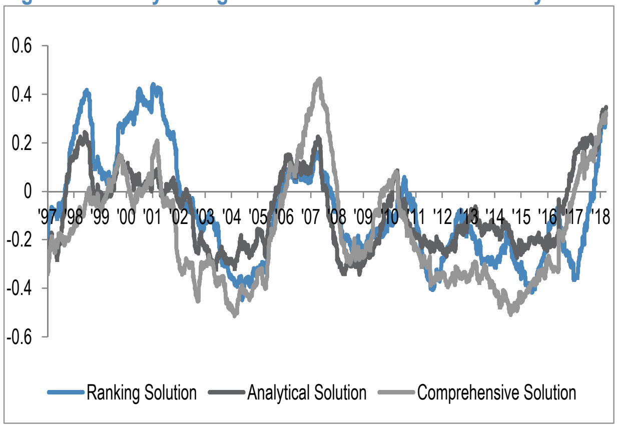
	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Ranking solution	2.97%	5.05%	0.59	9.35%
Analytical solution	3.17%	4.56%	0.70	9.79%
Comprehensive solution	2.47%	5.01%	0.49	9.89%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Interestingly neither of the market-neutral approaches has been successful at reducing the negative skewness of the rates carry strategy. The comprehensive approach has a positive skewness at the quarterly return horizon but negative skewness at the rest of the return horizons.

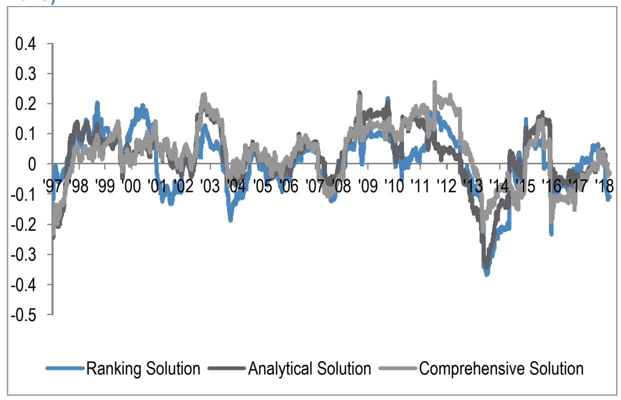
[12] In this case the beta of an asset to the market is equal to the ratio between the volatility of the asset and the volatility of the market. Hence, as in the ranking approach the positions are inversely proportional to volatility, the resulting portfolio will be beta-neutral portfolio.

Figure 19: Yearly rolling correlation with the 10Y Treasury rate



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 20: Yearly rolling correlation with SRVIX (VIX before June 2013)



Source: J.P. Morgan Quantitative and Derivatives Strategy

With respect to correlation all the approaches show similar features. The correlations to the changes in 10Y Treasury rate are slightly negative, but that can be a consequence of the downward trend in 10Y Treasury rate and the positive performance of the strategies over the backtest period. All the approaches have almost no sensitivity to volatility (represented by the VIX before 2013 and the SRVIX after 2013).

Table 10: Correlation statistics of the rates carry solutions with the 10Y Treasury rate and SRVIX

	10Y US Treasury rate			SRVIX (VIX before June 2013)		
	Average	Max	Min	Average	Max	Min
Ranking solution	-0.16	0.15	-0.37	0.01	0.22	-0.37
Analytical solution	-0.13	0.12	-0.35	0.03	0.24	-0.35
Comprehensive solution	-0.10	0.05	-0.22	0.03	0.27	-0.23

Source: J.P. Morgan Quantitative and Derivatives Strategy

Credit market-neutral carry strategy

The credit risk premium is perhaps the one that is easiest to comprehend. The investor should be compensated for selling protection against default over a certain period. As time passes the investor monetizes part of the protection premium (the default can happen over a shorter period of time).

The standard implementations of credit carry strategies typically involve selling protection on riskier debt and holding protection on the debt with higher quality. It is evident that such an implementation is nothing more than exposure to credit risk via a long short-portfolio. The more advanced implementations consider the carry-to-vol ratios (for example, see Investment Strategies No. 36: Carry-to-Risk Credit Indices) and are in fact similar to our ranking solution.

Below we demonstrate how to implement the market neutral-approaches within a universe of credit default-swap indices.

The mechanics of the market-neutral credit carry strategy

The credit carry strategy is in many aspects similar to the rates one. The main difference is the explicit presence of a premium collected that comes in addition to the roll-down effect on the credit curve.

The daily return from selling protection (assuming the implied change in the credit duration between two days is small enough to be neglected) can be represented as:

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$$CR_{i,t+1} = \frac{Coupon_i}{365} - CD_{i,t} * \left(Spread_{i,t+1,T-1} - Spread_{i,t,T}\right),$$

where $CR_{i,t+1}$ is the daily return for selling protection on credit default swap *i* at time *t+1* (when the remaining maturity is *T-1*), $Coupon_i$ is the yearly coupon the protection buyer pays for protection on asset *i* (which is fixed at inception), $CD_{i,t}$ is the credit duration at time *t* and $Spread_{i,t,T}$ denotes the market CDS spread at time *t* for asset *i* with time to maturity *T*.

Let $Spread_{i,t,T-1}$ denote the implied spread at time *t* for protection on asset *i* with time to maturity *T-1* (that will be the spread tomorrow if the CDS curve does not move). Then we can represent the daily return as:

$$CR_{i,t+1} = \frac{Coupon_i}{365} - CD_{i,t} * \left(Spread_{i,t+1,T-1} - Spread_{i,t,T-1}\right) - CD_{i,t} * \left(Spread_{i,t,T-1} - Spread_{i,t,T}\right)$$

Let $\Delta_{i,t+1}$ denote the difference between the realized spread at time *t+1* and the implied spread at time *t* (both for time to maturity *T-1*) and let $Carry_{i,t}$ denote the carry of asset *i* at time *t*:

$$\Delta_{i,t+1} = Spread_{i,t+1,T-1} - Spread_{i,t,T-1}$$

$$Carry_{i,t} = Coupon_i - CD_{i,t}\left(Spread_{i,t,T-1} - Spread_{i,t,T}\right)$$

$$CR_{i,t+1,T-1} = Carry_{i,t} - CD_{i,t}\Delta_{i,t+1}$$

Let's bundle together the observations for credit default indices return, the carry, duration and the difference between the realized and implied spreads into the following vectors:

$$\boldsymbol{CR}_t = \left[CR_{1,t}, \quad CR_{2,t}, \dots \dots \dots, CR_{n,t}\right]'$$

$$\boldsymbol{Carry}_t = \left[Carry_{1,t}, \quad Carry_{2,t}, \dots \dots \dots, Carry_{n,t}\right]'$$

$$\boldsymbol{CD}_t = \left[CD_{1,t,T}, \quad CD_{2,t,T}, \dots \dots, CD_{n,t,T}\right]'$$

$$\Delta_t = \left[\Delta_{1,t}, \quad \Delta_{2,t}, \dots \dots \dots \Delta_{n,t}\right]'$$

It follows that the portfolio return is $\boldsymbol{w}'\boldsymbol{CR}_{t+1} = -\boldsymbol{w}'(\boldsymbol{CD}_t \circ \Delta_{t+1}) + \boldsymbol{w}'\boldsymbol{Carry}_t$, where $\circ$ is the Hadamard product (elementwise) multiplication.

Let's assume that common factor structure (with *m* factors) exists for the deviations of the actual realized credit spreads from their implied values based on the curve observed on the previous day:

$$\Delta_t = \boldsymbol{a} + \boldsymbol{B}\boldsymbol{F}_t + \boldsymbol{\epsilon}_t$$

The standard assumptions hold: $\boldsymbol{F}_t$ denotes the factors at time *t*, $\boldsymbol{B}$ is matrix containing the market sensitivities to the factors and $\boldsymbol{\epsilon}_t = \left[\epsilon_{1,t,}\epsilon_{2,t}, \dots\dots, \epsilon_{n,t}\right]$ are the residual returns having the standard distributional assumptions $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ being a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$ for $i \epsilon [1, \dots . n]$.

Hence, if it holds $\boldsymbol{w}'(\boldsymbol{CD}_t \circ \boldsymbol{a}) = \boldsymbol{0}$ and $\boldsymbol{w}'(\boldsymbol{CD}_t \circ \boldsymbol{B}) = 0$, it follows that the portfolio return and variance can be explained as:

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$$E(\boldsymbol{w}'\boldsymbol{CR}_t) = \boldsymbol{w}'\boldsymbol{Carry}_{t-1}$$

$$Var(\boldsymbol{w}'\boldsymbol{CR}_t) = Var(\boldsymbol{w}'\boldsymbol{\epsilon}_t) = \sum_{i=1}^{n} w_i^2\sigma_i^2$$

Empirical results

Our tradable assets consist of trackers on the total return performance from selling protection on the following credit-default indices (net of rolling costs):

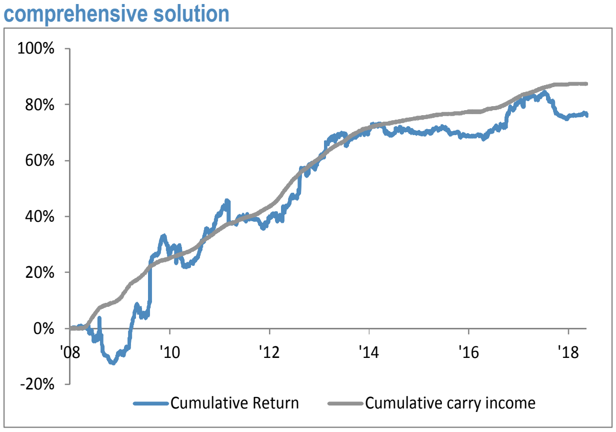
Table 11: Universe of credit-default indices

Europe	USA
ITraxx Main *(3Y,5Y,10Y)*	CDX IG *(3Y,5Y,10Y)*
ITraxx Crossover *(5Y)*	CDX HY *(5Y)*

Source: J.P. Morgan Quantitative and Derivatives Strategy

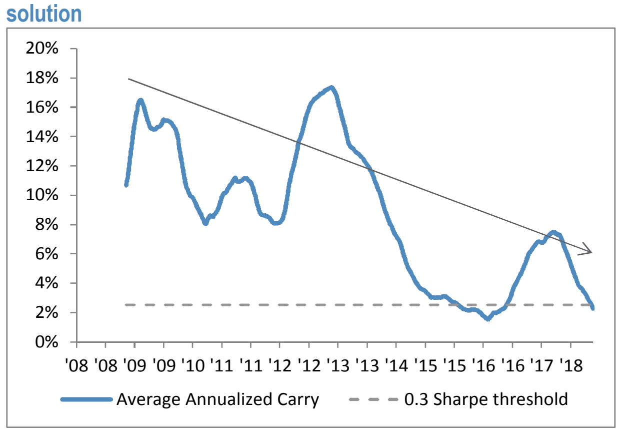
Below we show the backtest results for comprehensive approach.

Figure 21: Cumulative return and carry income for the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 22: Average annualized carry for the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 12: Performance statistics of the credit market-neutral strategy

Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
7.29%	8.44%	0.86	16.25%

The comprehensive market-neutral credit carry strategy achieves an attractive Sharpe ratio of 0.86 after transaction costs and the P&L of the strategy closely tracks the embedded carry. As with the rest of the strategies the carry income embedded in the strategy decreased post the GFC, but stayed relatively attractive until 2014.

An analysis of the actual positions reveals that, in the European credit markets, the comprehensive strategy has kept predominantly long positions in ITraxx Main 3Y and 5Y, short position in ITraxx Main 10Y and both a long and a short position in Itraxx Crossover 5Y. In the US, the position in CDX IG 5Y has been predominantly long, the position in CDX IG 10Y mainly short, while the positions in CDX IG 3Y and CDX HY 5Y have been both long and short.

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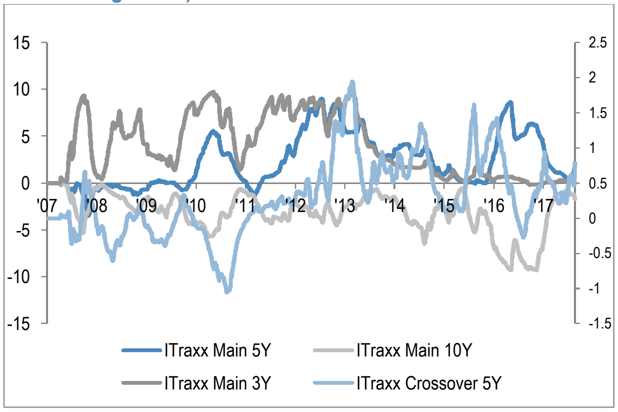
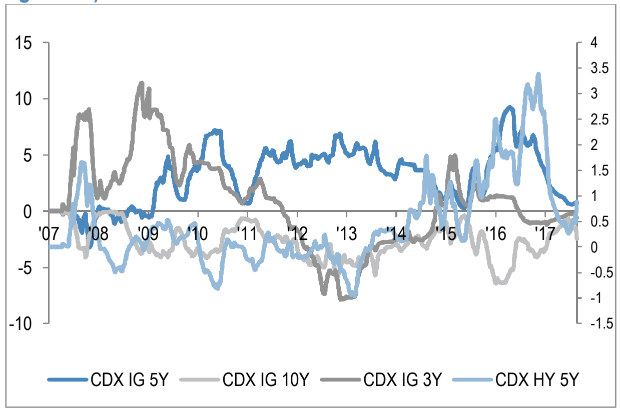
Figure 23: Positions in European credit indices in $ notionals for $1 dollar capital and 10% target annualized vol (ITraxx Crossover 5Y on the right axis)



Source: J.P. Morgan Quantitative and Derivatives Strategy

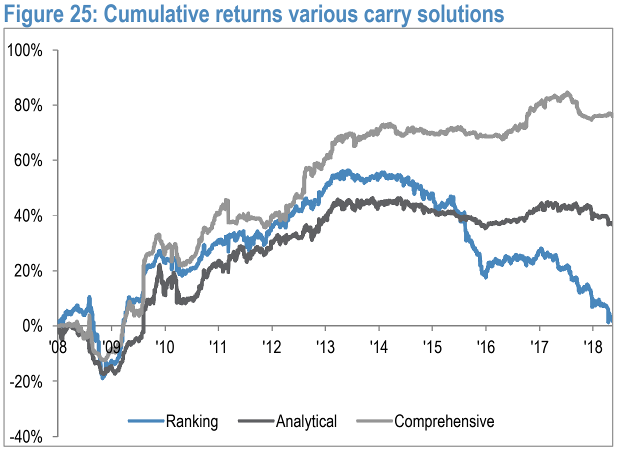
Figure 24: Positions in US credit indices in $ notionals for $1 dollar capital and 10% target annualized vol (CDX HY 5Y on the right axis)



Source: J.P. Morgan Quantitative and Derivatives Strategy

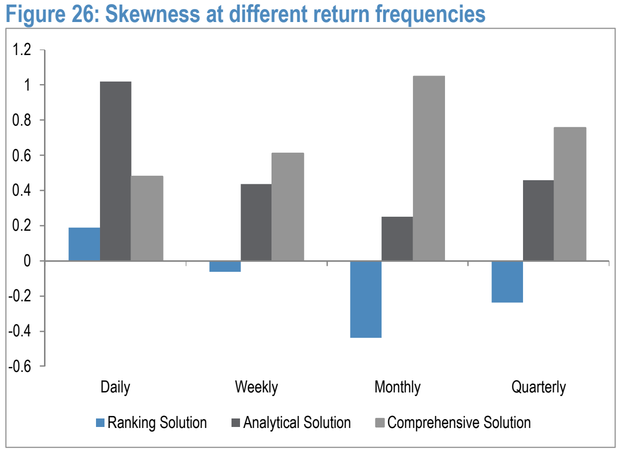
The market-neutral approaches significantly outperform the ranking approach and we note offer much more appealing risk-return profile. The same conclusion can be drawn from the analysis of the skewness of the returns at different horizon. The skewness of the returns of the market-neural approaches is positive at every return horizon.

Figure 25: Cumulative returns various carry solutions



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 26: Skewness at different return frequencies



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 13: Comparative performance statistics

	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Ranking solution	0.18%	9.34%	0.02	55.17%
Analytical solution	3.53%	8.70%	0.41	18.0%
Comprehensive solution	7.29%	8.44%	0.86	16.25%

Source: J.P. Morgan Quantitative and Derivatives Strategy

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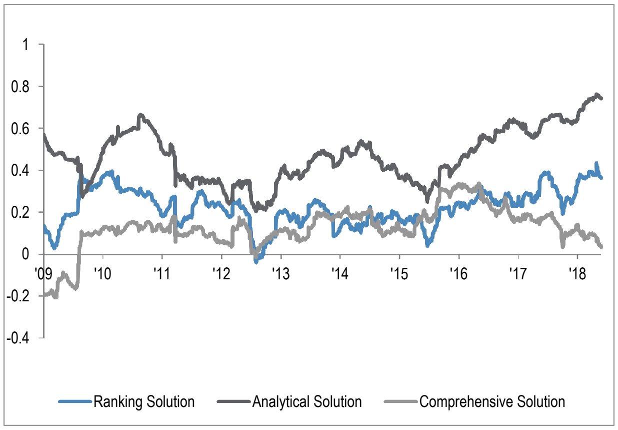
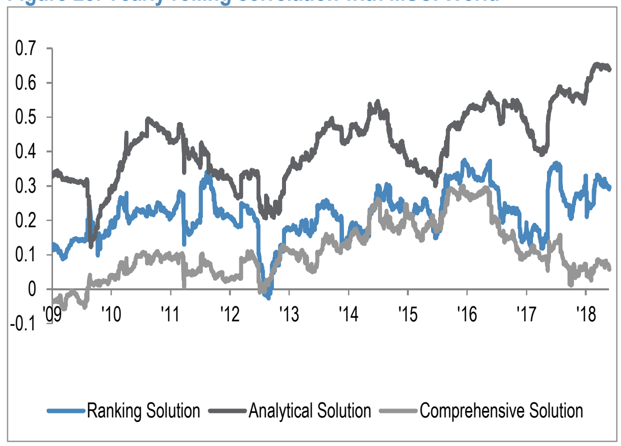
Figure 27: Yearly rolling correlation with an aggregate credit index



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 28: Yearly rolling correlation with MSCI World



Source: J.P. Morgan Quantitative and Derivatives Strategy

For the purpose of analyzing the correlation of the credit carry strategy with the broader market, we have constructed a proxy of the credit market that is an average of the returns of the assets in our universe scaled by their one-year rolling historical volatility. We have also analyzed the correlation with respect to the equity market. The correlations of the returns of the comprehensive solution to both credit and equity market are relatively contained though they remain positive on average. Somewhat surprisingly the correlation of the returns of the analytical approach to both the credit and the equity markets has been relatively high.

Table 14: Correlation statistics of the credit carry solutions with an aggregate credit index and MSCI World

	Aggregate credit index			MSCI World		
	Average	Max	Min	Average	Max	Min
Ranking solution	0.22	0.44	-0.04	0.22	0.38	-0.03
Analytical solution	0.47	0.76	0.20	0.42	0.70	0.15
Comprehensive solution	0.13	0.34	-0.21	0.11	0.30	-0.06

Source: J.P. Morgan Quantitative and Derivatives Strategy

Commodity market-neutral carry strategy

Commodities carry strategies benefit from the shape of the commodities futures curve. Various economic/financial theories have been put forward to decipher the shape of the commodities futures curve.

Starting from the theory of storage (Kaldor (1939)) an upward-sloping (positive slope) commodities futures curve (contango) reflects a high current level of inventories. The convenience yield of holding the commodity is low and does not compensate the financing and storage costs. Conversely, a downward-sloping (negative slope) futures curve (backwardation) arises when supply is limited and the convenience yield exceeds the financing cost and storage costs.

Alternatively, the hedging hypothesis first put forward by Keynes (1930) postulates that producers who sell production forward have to compensate the speculators for taking a long position. This naturally results in backwardation with the spot being above the futures prices. Cootner (1960) extended the hedging hypothesis by allowing for consumers to hedge in the market. If the hedging activity of consumers for a particular commodity outweighs that of producers, then the futures price would be set above the expected future spot price leading to an upward sloping curve (contango).

The various explanations for the shape of the commodities future curve have received their fair share of empirical support in the academic world. But more importantly, for investment practitioners commodity carry strategies based on the slide on the futures curve have become a well-established systematic strategy (for an overview please refer to Investment Strategies no. 54: Profiting from slide in commodity curves or the J.P. Morgan risk premia primer). In both the scenarios of contango or backwardation and under the assumption that the spot price does not change, the investor can profit from the shape of the curve. If the curve is in backwardation, a long position in the future will benefit from the convergence of the futures to the spot price. Conversely, if the curve is in contango, a short position will be profitable.

Below we present a commodities futures carry strategy whereby the common risk factors in the commodities markets are neutralized. In addition, we also explicitly take into account the curvature of the commodities futures curve and incorporate the seasonality effect.

The mechanics of commodities futures carry strategy

Taking into account the curvature of the curve

Let's first start by assuming that a spot reference exists for a commodity market i. If $F_{i,t,T}$ denotes the future price at time t for a commodity i with T days to maturity, $r_{i,t,T}$ is the annualized financing rate and $c_{i,t,T}$ is the annualized convenience yield (after storage costs), the non-arbitrage condition implies:

$$F_{i,t,T} = S_{i,t} e^{(r_{i,t,T} - c_{i,t,T})T/365}$$

Hence, we can consider the difference between the convenience yield and the funding rate as our carry measure:

$$Carry_{i,t} = c_{i,t,T} - r_{i,t,T}.$$

Similarly to the case of FX, we can iterate the future pricing equation forward $F_{i,t+1,T-1} = S_{i,t+1} e^{(r_{i,t+1,T-1} - c_{i,t+1,T-1})(T-1)/365}$. If we denote the change in carry as $dCarry_{i,t+1} = Carry_{i,t+1} - Carry_{i,t}$, it follows that the future's return $R_{i,t+1}$ at time *t+1* for future *i* can be represented as:

$$R_{i,t+1} = RS_{i,t+1} + \frac{Carry_{i,t}}{365} - \frac{dCarry_{i,t+1}(T-1)}{365} ,$$

where $RS_{i,t+1}$ denotes the spot return at time *t+1* for future *i*.

If the commodity's futures curve were linear, we would not expect changes in the carry measure, i.e. $\mathrm{E_t}(\mathrm{dCarry_{i,t+1}}) = 0$. In reality, commodities futures curves exhibit curvature and a precise calculation of the daily carry (curve slide) requires taking into account curvature of the futures curve. [13]

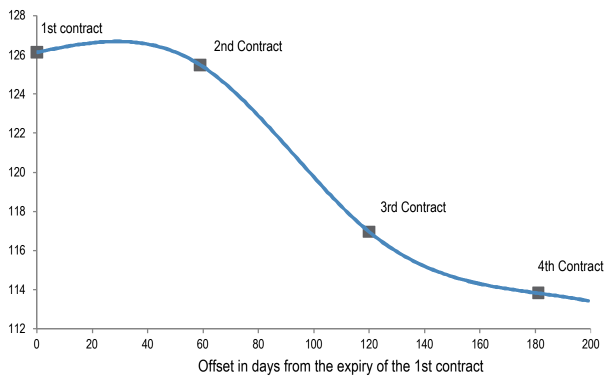
Let's assume that the curve for commodity i at time t is summarized by a function $f_{i,t}(T)$ where T is the time to maturity. Under the assumption the functional form of the curve does not change between two consecutive time points, i.e. $f_{i,t}(T) = f_{i,t+1}(T)$ and using the Taylor expansion, it follows:

[13] Note the implied sign for the change in carry $\mathrm{dCarry_{i,t+1}}$ depending on the shape of the curve. If the curve is downward sloping (backwardation), the curve is typically convex and the carry (slide) increases with maturity approaching. Hence, the expected future change in carry is positive and the carry collected on the day is smaller than what is implied by a linear model assumption. Conversely, if the curve is upward sloping (contango), the curve is typically concave and the carry/slide becomes more negative for shorter maturities. In this case the expected change in the carry is negative and negative carry incurred on the day is less negative than the value implied by the linear model.

$$ImpledCarry_{i,t} = \ln\left(f_{i,t}(T-1)\right) - \ln\left(f_{i,t}(T)\right) = -\frac{1}{F_{i,t,T}}\frac{\partial f_{i,t}(T)}{\partial T} - \frac{1}{2}\left(\frac{\partial f_{i,t}(T)}{\partial T}\right)^2 \frac{1}{\left(F_{i,t,T}\right)^2} + \frac{1}{2}\frac{1}{F_{i,t,T}}\frac{\partial^2 f_{i,t}(T)}{\partial T^2}$$

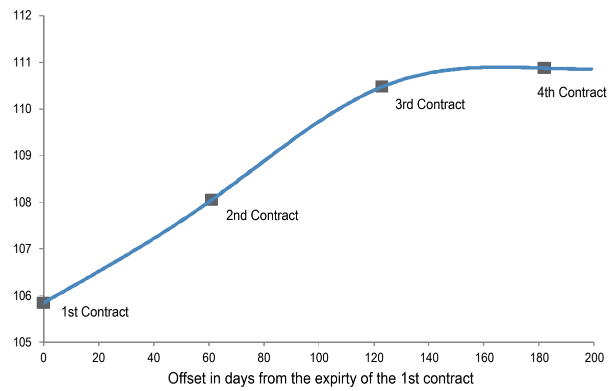
To calculate the implied carry taking into account the curvature, we make use of a spline that passes through the first four generic futures (for more details about the spline methodology, please see the Appendix).

Figure 29: Spline fitted to the live cattle futures curve of on the 5th February, 2018



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 30: Spline fitted to the live cattle futures curve on the 21st August, 2018



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 15: Comparison carry estimates for the 2nd and 3rd contract for live cattle on the 5th February, 2018

Carry Estimate	Linear Fit	Spline
2nd contract	3.2%	25.7%
3rd contract	22.2%	36.7%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 16: Comparison carry estimates for the 2nd and 3rd contract for live cattle on the 21st August, 2018

Carry Estimate	Linear Fit	Spline
2nd contract	-12.2%	-14.2%
3rd contract	-6.5%	-8.1%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Factor structure

Let's use $F^*_{i,t,T-1} = f_{i,t}(T-1)$ to denote the interpolated curve value for a future of maturity *T-1* at time *t*. It follows:

$$R_{i,t+1} = \ln(F_{i,t+1,T-1}) - \ln(F^*_{i,t,T-1}) + \ln(F^*_{i,t,T-1}) - \ln(F_{i,t,T}) = \ln(F_{i,t+1,T-1}) - \ln(F^*_{i,t,T-1}) + ImpliedCarry_{i,t}$$

Let $\Delta_{i,t} = \ln(F_{i,t+1,T-1}) - \ln\left(F^*_{i,t,T-1}\right)$ denotes the percentage difference between the realized value of the future at time *t+1* with remaining maturity of *T-1* and the implied value based on the curve at time *t*.[14]

[14] If we experience a proportional shift $\Delta_{i,t} = \ln(F_{i,t+1,T-1}) - \ln(F^*_{i,t,T-1}) = \ln\left(\frac{S_{i,t+1}}{S_{i,t}}\right)$. Hence the futures return can be disentangled in the spot return and the implied carry return $R_{i,t+1} = RS_{i,t+1} + ImpliedCarry_{i,t}$, where $RS_{i,t+1}$ is the spot return at time *t+1*. If the curve moves in a parallel fashion, the carry will measure will not change (as slope, curvature etc. remain the same) but in this case the percentage difference between the future and its implied value and the spot return will differ: $\Delta_{i,t} = \ln(F_{i,t+1,T-1}) - \ln(F^*_{i,t,T-1}) \neq \ln(\frac{S_{i,t+1}}{S_{i,t}})$.

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If we assume that a factor model exists for the vector of percentage differences between the realized value and the implied values of the futures in our universe, $\Delta_t = [\Delta_{1,t}, \quad \Delta_{2,t}, \dots \dots \dots, \Delta_{n,t}]'$, it follows:

$$\Delta_t = \boldsymbol{a} + \boldsymbol{B}\boldsymbol{F}_t + \boldsymbol{\epsilon}_t$$

The standard assumptions hold: $\boldsymbol{F}_t$ denotes the factors at time *t*, $\boldsymbol{B}$ is matrix containing the sensitivities to the factors and $\boldsymbol{\epsilon}_t = [\epsilon_{1,t,}\epsilon_{2,t}, \dots \dots, \epsilon_{n,t}]$ are the residuals being distributed as $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$ for $i \epsilon [1, \dots . n]$.

If in addition for the vector of portfolio weights $\boldsymbol{w}$ holds $\boldsymbol{w}'\boldsymbol{a} = \boldsymbol{0}$ and $\boldsymbol{w}'\boldsymbol{B} = \boldsymbol{0}$ and under standard assumptions for $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ being a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$, it follows:

$$E(\boldsymbol{w}'\boldsymbol{R}_t) = \boldsymbol{w}'\boldsymbol{ImpliedCarry}_{t-1},$$

where $\boldsymbol{R}_t = [R_{1,t}, \quad R_{2,t}, \dots \dots \dots, R_{n,t}]'$ contains the returns at time *t* of the futures in our universe. Furthermore, the variance of the portfolio is a function of the weights $\boldsymbol{w}$ and the variance of the idiosyncratic noise $\boldsymbol{\epsilon}_t$:

$$Var(\boldsymbol{w}'\boldsymbol{R}_t) = \boldsymbol{Var}(\boldsymbol{w}'\boldsymbol{\epsilon}_t) = \sum_{i=1}^{n} w_i^2 \sigma_i^2$$

Seasonality

It is well-known that commodities futures prices exhibit seasonal patterns. For example, demand for energy related commodities is expected to peak at certain times, the expected arrival of new crop in certain months affects the inventories of some agricultural commodities etc. The commodities futures curve reflects the market expectation of those seasonal patterns. Subsequently, the carry measure might need to be stripped from the seasonality effects.

We have opted to remove the seasonality effects using a statistical technique[15]. In particular, we are using a statistical package called 'Prophet' that is a freeware and developed at Facebook. More details can be found in the Appendix but it suffices to say that approach uses a flexible econometric specification allowing us to remove the seasonality at the annual frequency.

Below we show an example of estimating the seasonal component from a second generic heating oil contract (HO2 Comdty). As seasonality might be stochastic (Hevia et. al. (2018)), we use a rolling 5 year estimation. It is evident that there has been a strong seasonal component picking up in March and in August.[16] The seasonal effect seems to have decreased as of recently.

[15] Simpler methods based on moving averages might be employed as well.

[16] In March, the first contract will be March and the second April and the seasonal component will be positive as we expect the first contract to trade higher than the first due to the colder weather in March (switch from winter to summer). In August the first contract is in August and the second in September and the seasonal component will be negative – the first contract is linked to the summer time and the second to the autumn and we expect to the second contract to trade higher.

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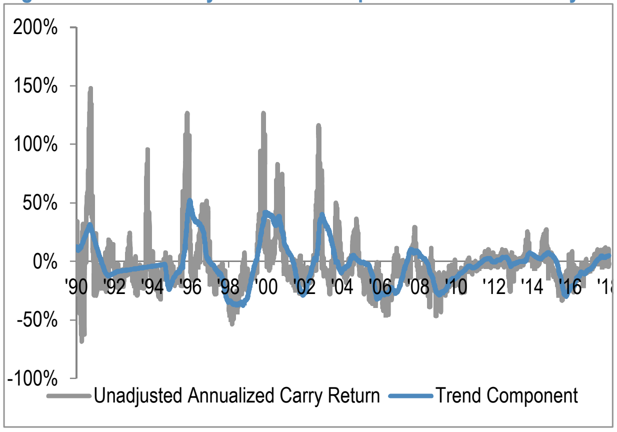
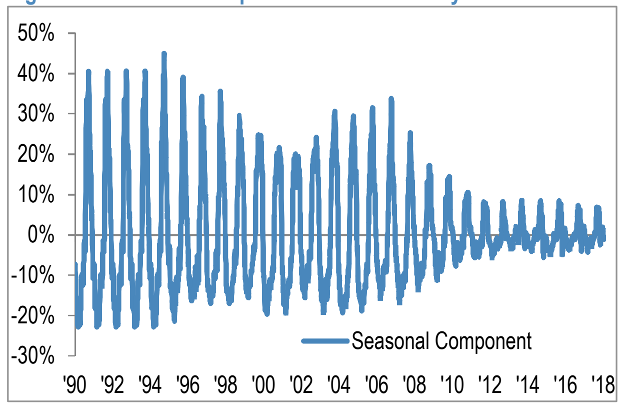
Figure 31: Actual carry and trend component for HO2 Comdty



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 32: Seasonal component for HO2 Comdty



Source: J.P. Morgan Quantitative and Derivatives Strategy

Empirical results

Data universe and modelling considerations

Our universe consists of the first, second and third generic commodities futures covering the energy, metals and agricultural sectors[17]. We have also considered the spread between the second and the third contract[18].

Table 17: Universe commodities futures

Energy	Metals	Agriculturals
Brent Crude (CO1 Comdty, CO2 Comdty, CO3 Comdty, spread CO2/CO3 Comdty)	*Gold* (GC1 Comdty, GC2 Comdty, GC3 Comdty, spread GC2/GC3 Comdty)	*Corn* (C 1 Comdty, C 2 Comdty, C 3 Comdty, spread C 2/C 3 Comdty)
WTI Crude (CL1 Comdty, CL2 Comdty, CL3 Comdty, spread CL2/CL3 Comdty)	*Silver* (SI1 Comdty, SI2 Comdty, SI3 Comdty, spread SI2/SI3 Comdty)	*Soybean* (S 1 Comdty, S 2 Comdty, S 3 Comdty, spread S 2/S 3 Comdty)
Heating Oil (HO1 Comdty, HO2 Comdty, HO3 Comdty, spread HO2/HO3 Comdty)	*Copper* (HG1 Comdty, HG2 Comdty, HG3 Comdty, spread HG2/HG3 Comdty)	*Soybean Oil* (BO1 Comdty, BO2 Comdty, BO3 Comdty, spread BO2/BO3 Comdty)
Gasoline (XB1 Comdty, XB2 Comdty, XB3 Comdty, spread XB2/XB3 Comdty)	*Aluminum* (LA1 Comdty, LA2 Comdty, LA3 Comdty, spread LA2/LA3 Comdty)	*Cotton* (CT1 Comdty, CT2 Comdty, CT3 Comdty, spread CT2/CT3 Comdty)
Natural Gas (NG1 Comdty, NG2 Comdty, NG3 Comdty, spread NG2/NG3 Comdty)	*Nickel* (LN1 Comdty, LN2 Comdty, LN3 Comdty, spread LN2/LN3 Comdty)	*Coffee* (KC1 Comdty, KC2 Comdty, KC3 Comdty, spread KC2/KC3 Comdty)
	Zinc (LX1 Comdty, LX2 Comdty, LX3 Comdty, spread LX2/LX3 Comdty)	*Sugar* (SB1 Comdty, SB2 Comdty, SB3 Comdty, spread SB2/SB3 Comdty)
		Wheat (W 1 Comdty, W 2 Comdty, W 3 Comdty, spread W2/W3 Comdty)
		Live Cattle (LC1 Comdty, LC2 Comdty, LC3 Comdty, spread LC2/LC3 Comdty)
		Lean Hogs (LH1 Comdty, LH2 Comdty, LH3 Comdty, spread LH2/LH3 Comdty)
		Red Winter Wheat (KW1 Comdty, KW2 Comdty, KW3 Comdty, spread KW2/KW3 Comdty)
		Soybean Meal (SM1 Comdty, SM2 Comdty, SM3 Comdty, spread SM2/SM3 Comdty)

[17] While the carry for the first generic contract cannot be estimated taking into account the curvature of the commodity curve (as the spline at the end points has a zero second derivative), we have kept the first generic contracts in our backtests as they are the most liquid.
[18] As mentioned earlier, the estimate of the carry for the first contract does not incorporate the curvature and subsequently we have focused on the spread between the second and the third contract.

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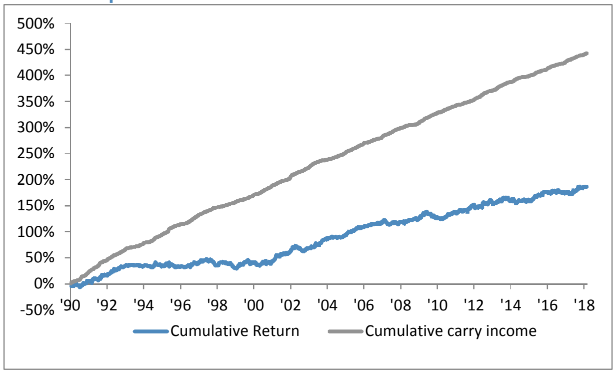
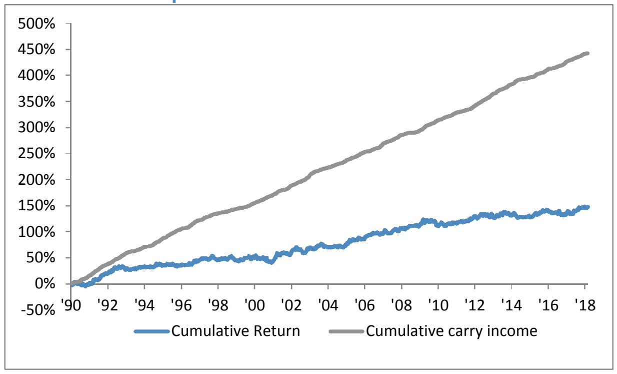
Below we show the backtest results for comprehensive approach with respect to the various underlyings in our universe.

Figure 33: Backtest results for the first generic contracts based on the comprehensive solution



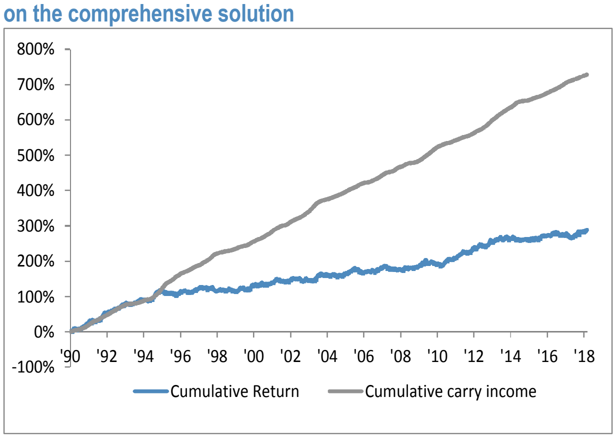
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 34: Backtest results for the second generic contracts based on the comprehensive solution



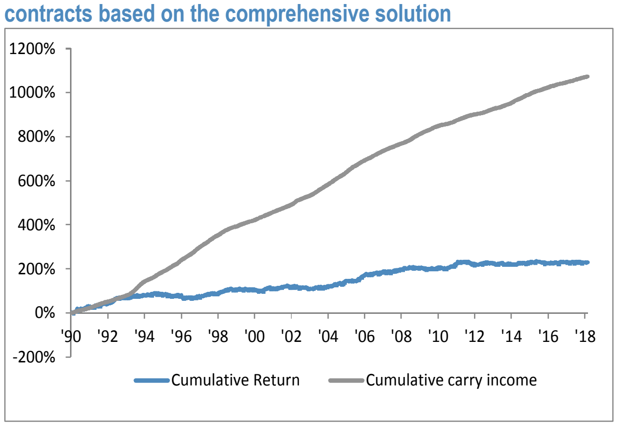
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 35: Backtest results for the third generic contracts based on the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 36: Backtest results for the spread second/third generic contracts based on the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 18: Performance statistics for the comprehensive approach

Underlying	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
1st Generic Contacts	6.58%	7.98%	0.82	19%
2ndt Generic Contacts	5.22%	8.01%	0.65	15%
3rd Generic Contracts	10.17%	9.46%	1.07	18%
Spread 2/3 Generic Contacts	8.13%	10.31%	0.79	24%

Source: J.P. Morgan Quantitative and Derivatives Strategy

The performance of the commodity carry strategy is quite consistent among the various implementations. In fact there, are only two commodities with negative Sharpe ratios in the backtest - zinc and sugar. The average Sharpe ratios per market are relatively small as evident from the table below, but the average correlation among the individual markets P&Ls is close to zero (-0.02) pointing to strong diversification benefits.

Table 19: Average Sharpe ratio per commodity group

Commodities Group	Energy	Metals	Agriculturals
Average Sharpe	0.23	0.17	0.18

Source: J.P. Morgan Quantitative and Derivatives Strategy

The empirical results above show that there is an attractive carry to be captured in commodities, but the results also highlight that it is difficult to monetize it. Below we present several potential explanations as to why the full carry component cannot be extracted.

First, in contrast to other asset classes, in the case of commodities the carry cannot be measured precisely and known with certainty in advance. As we have already discussed seasonality and curvature both play importance. Seasonality might be stochastic and the changing patterns not always might be captured with historical data. The seasonal adjustment works on average (we have both positive and negative seasonality adjustments). Let's assume that a commodity typically is in contango, but occasionally goes into backwardation due to seasonal reasons. As the seasonal adjustment (trend-extraction) works on average, we will adjust both the observations during the contango and backwardation periods rather than just the observation in the backwardation period. In contango, we will tend to overestimate the carry (less negative than in reality). Similarly in backwardation, the seasonal component will not be removed fully and the carry will be overestimated (more positive). Given that contango is the more common shape, we might expect carry to be typically overestimated on occasions.

Second, our backtests assume trading with one day delay to assure that there are no issues with non-synchronous prices and to be in turn more realistic. Therefore, the strategy returns are dependent on the two-day lagged value of the carry rather than the one day lagged value. Speculative activity can lead to compression of the carry (the positive becomes less positive and the negative less negative). In an ideal world, we will need to hedge the changes in the carry measure to assure that this impact is not observed in reality (though that will be challenging as the positive and the negative carry measures will be moving in opposite directions). To confirm that hypothesis, we performed a regression of the change in carry on the lagged value of the carry measure using data for the second generic contracts and we found highly significant negative beta coefficients. The magnitude of the coefficients cannot itself justify the size of the discrepancy we observe (for example we find that the maximum correction is for metals where the carry decreases/increases by 7% in case of positive/negative carry), though the significance of the estimated coefficients certainly confirms the presence of the effect discussed above.

$$d(Carry_{i,t}) = \alpha_i + \beta_i Carry_{i,t-1} + \epsilon_{i,t}$$

Table 20: Beta coefficients and t-stats in a regression of the change in the carry on the lagged value of carry

Commodity Group	Energy	Metals	Agriculturals
Average Beta coefficient	-0.03	-0.07	-0.01
Average t-statistics	-10.48	-14.39	-6.72

Source: J.P. Morgan Quantitative and Derivatives Strategy

Last but not the least, high carry measures (in absolute value sense) typically reflect market-specific demand/supply imbalances which are often idiosyncratic. For example, a supply shock (bad crop in a certain agricultural commodity) can lead to pronounced backwardation, high convenience yield and high carry for a certain commodity. Gradually the supply chains might start adjusting, which will lead to normalization of the carry and decrease in convenience yield. As such moves are mainly idiosyncratic they are difficult to hedge in a systematic way (the price will move up and down in an uncorrelated to other commodities way). The normalization will also imply that only a part of the initially calculated carry will be captured. Given that the typical shape of the curve is contango, we will also expect that, in the case of backwardation, the overestimation of the carry measure will be bigger (when the 'fear premium' of short supply reaches extreme levels).

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To analyze the presence of any differential impact we regressed the futures return ($R_{i,t}$) on the product of the actual carry ($Carry_{i,t-1}$) and a dummy variable reflecting whether the carry is positive ($D_{1,i}$) or negative ($D_{2,i}$):

$$R_{i,t} = \alpha_i + \beta_{1,i} D_{1,i} Carry_{i,t-1} + \beta_{2,i} D_{2,i} ImpledCarry_{i,t-1} + \epsilon_{i,t}$$

$$D_{1,i} = 1 \text{ if } Carry_{i,t-1} > 0,\ D_{1,i} = 0 \text{ otherwise}$$

$$D_{2,i} = 1 \text{ if } Carry_{i,t-1} < 0,\ D_{2,i} = 0 \text{ otherwise}$$

In standard circumstances would expect both $\beta_{1,i}$ and $\beta_{2,i}$ to be close 1 (and α_i to be the average return net of carry). Below we present the results in the case of the second generic contracts.

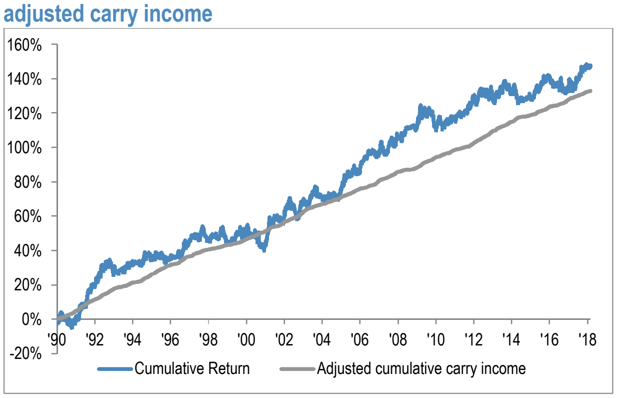
Table 21: Beta coefficients and t-stats in a regression of the change in the carry on the lagged value of carry

	Energy	Metals	Agriculturals	Average among commodities groups
β_1	0.01	-0.25	0.41	0.06
β_2	0.64	0.67	0.34	0.55

Source: J.P. Morgan Quantitative and Derivatives Strategy

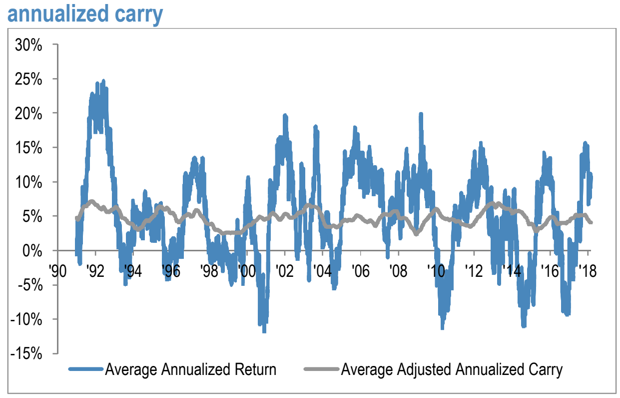
The results confirm differential impact and on average negative carry can be monetized to a much greater extent than a positive one. There are also differences among commodity groups with negative carry being captured well within the energy and metals groups and positive carry within agriculturals. The average coefficient between the two cases of positive and negative carry occurrence will around 0.3[19]. Below we show the adjusted carry income by a coefficient of 0.3 versus the cumulative P&L for the second generic contracts:

Figure 37: Backtest results for the second generic contracts with adjusted carry income



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 38: Average annualized return versus average adjusted annualized carry



Source: J.P. Morgan Quantitative and Derivatives Strategy

Aggregated track-record and comparison to other approaches

As we have already mentioned there are strong diversification benefits among the P&Ls of the individual assets (correlation is close to 0). But there are also attractive diversification benefits among the track records with different underlyings (1st, 2nd, 3rd contract and the spread 2nd-3rd contract). The average correlation among the track records based on different underlyings is 0.3 and the aggregate backtest results in an attractive risk-return ratio of 1.24 with confined drawdown.

[19] We obtain a coefficient of a similar magnitude in a regression of the future's return on the lagged carry measure.

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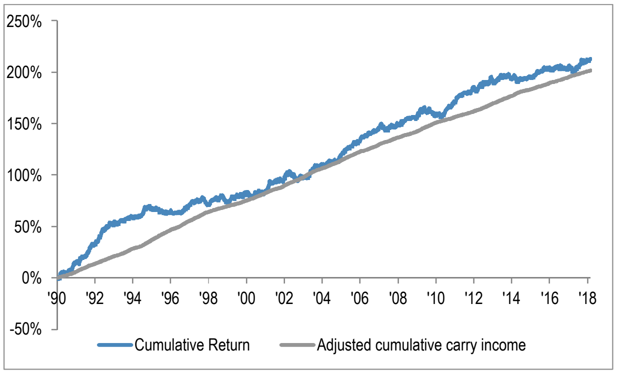
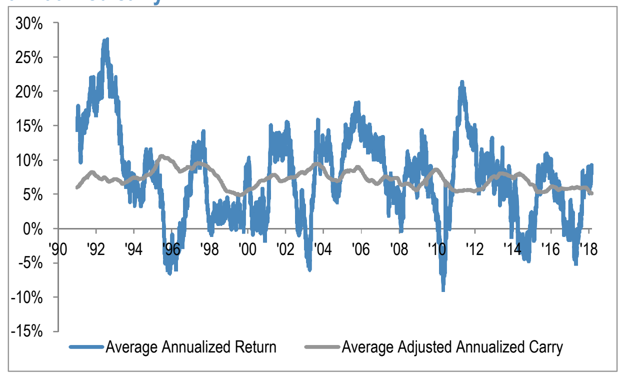
Figure 39: Aggregated track record versus the adjusted carry income



Source: J.P. Morgan Quantitative and Derivatives Strategy

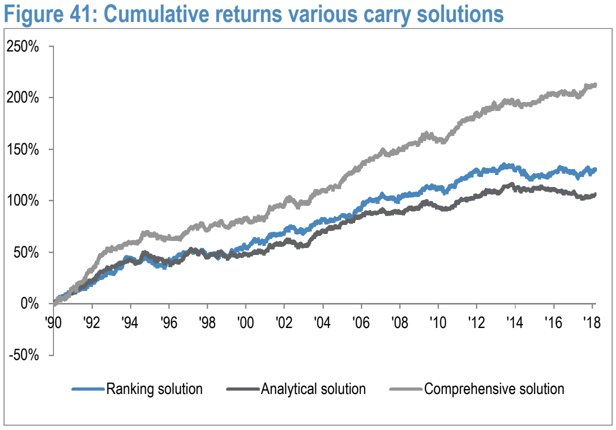
Figure 40: Average annualized return versus average adjusted annualized carry



Source: J.P. Morgan Quantitative and Derivatives Strategy

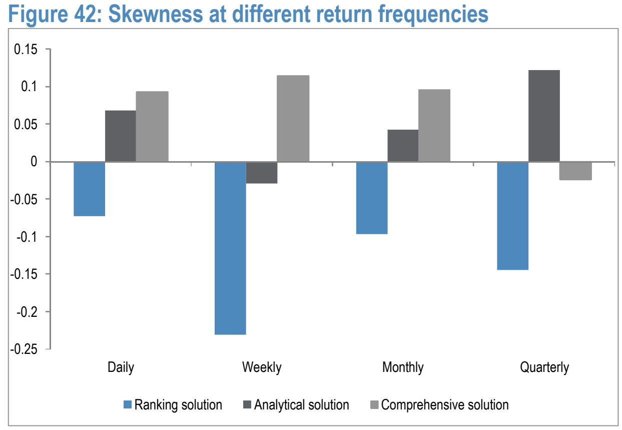
As a whole the commodity carry strategy provides appealing risk/return profile. The comprehensive approach strongly outperforms the analytical and the ranking one. Nevertheless, both the ranking and the analytical approaches deliver an attractive Sharpe ratio of 0.82. The market-neutral approaches have the merit of positive skewness, while the ranking one has exhibits negative skewness at all frequencies.

Figure 41: Cumulative returns various carry solutions



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 42: Skewness at different return frequencies



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 22: Comparative performance statistics

Strategy	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Comprehensive approach	7.52%	6.07%	1.24	10.7%
Ranking approach	4.62%	5.62%	0.82	15.02%
Analytical approach	3.77%	4.62%	0.82	14.39%

Source: J.P. Morgan Quantitative and Derivatives Strategy

All the portfolio solutions have negligible correlation to both the BCOM and the MSCI World Index. On occasions the ranking solution can exhibit higher correlation but still on average it displays almost no correlation to the broad commodity and equity benchmarks.

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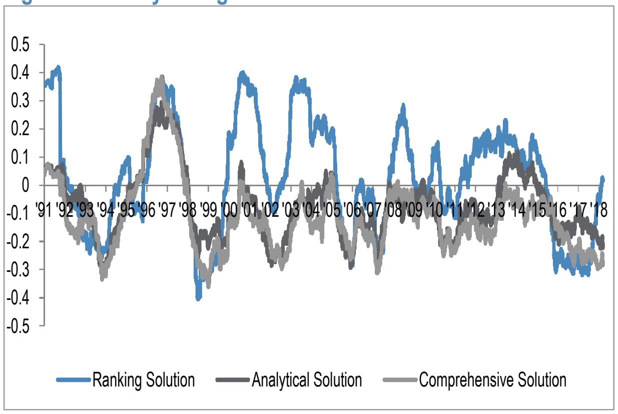
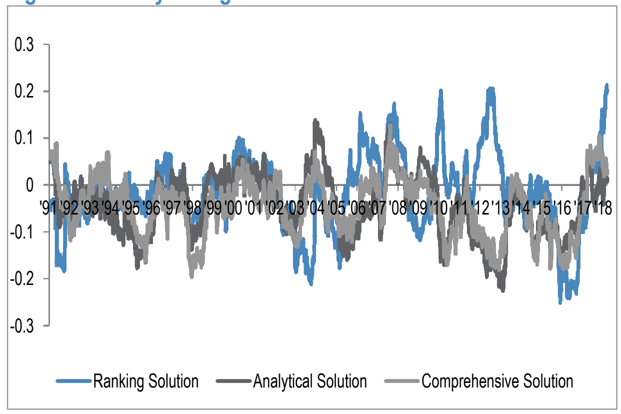
Figure 43: Yearly rolling correlation with BCOM



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 44: Yearly rolling correlation with MSCI World



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 23: Correlation statistics of the commodity carry solutions with an aggregate credit index and MSCI World

	BCOM			MSCI World		
	Average	Max	Min	Average	Max	Min
Ranking solution	0.028	0.419	-0.406	-0.017	0.213	-0.251
Analytical solution	-0.076	0.295	-0.317	-0.043	0.138	-0.226
Comprehensive solution	-0.111	0.385	-0.362	-0.046	0.128	-0.196

Source: J.P. Morgan Quantitative and Derivatives Strategy

Equity index futures market-neutral carry strategy

In the subsection below we show that (perhaps somewhat contrary to the common perception) the standard definition of carry as the return that accrues if the spot does not move does not naturally extend to the case of equity futures. As we illustrate, the interplay between dividends, spot returns and the financing rate is more evolved in reality and attention to detail should be paid, especially if we aim to neutralize the factor exposure.

We propose two approaches for designing a carry strategy based on equity futures. The first one relies solely on observed data but comes at the cost of being considered a statistical arbitrage model. The second one relies on inputs that are derived from the market data via statistical techniques but bears a similarity to an equity income style investing.

The mechanics of equity indices futures carry strategy

Factor model for spot returns

Let's assume that a dividend is paid during the life of the future and for simplicity let's further assume the dividend is paid just before maturity and is known with certainty. If we use $F_{i,t,T}$ to denote the future price at time t for an index i with T days to maturity, $r_{i,t,T}$ to be the annualized financing rate at time t for a future on the index i with T days to maturity and $D_{i,t+T}$ to be the dividend to be paid for index i at the maturity $t+T$.[20] The standard non-arbitrage condition implies:

[20] As we discuss later the financing rate consists of two components: the relevant interest rate for the maturity of interest and the borrow rate. Both of those are important determinants of the sign and the magnitude of the carry.

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$$F_{i,t,T} = S_{i,t}\left(1+\frac{r_{i,t,T}T}{365}\right) - S_{i,t}(D_{i,t+T}\ \ /S_{i,t}) \sim S_{i,t}e^{(r_{i,t,T}-d_{i,t})T/365},$$

where $d_{i,t} = (D_{i,t+T}\ \ /S_{i,t})/(T/365)$.

At maturity the futures price converges to the spot. Hence we can write:

$$\frac{F_{i,t+T,0}}{F_{i,t,T}} = \frac{S_{i,t+T}}{F_{i,t,T}} = \frac{S_{i,t+T}}{S_{i,t+T-1}}\frac{S_{i,t+T-1}}{S_{i,t+T-2}}\dots\dots.\frac{S_{i,t+1}}{S_{i,t}}\frac{S_{i,t}}{F_{i,t,T}}$$

It follows that the (log) cumulative return of the future between time *t* and maturity *t+T* is:

$$R_{i,t+T,t} = RS_{i,t+T} + RS_{i,t+T-1} + \cdots \dots. RS_{i,t+1} + \ln\left(\frac{S_{i,t}}{F_{i,t,T}}\right) = \sum_{s=t+1}^{t+T} RS_{i,s} + (d_{i,t} - r_{i,t,T})T/365,$$

where $RS_{i,t}$ is the spot return at time *t*.

First, let's consider the case when there is **no dividend payment**. As dividends are paid infrequently and typically concentrated in particular months of the year, it is much more likely that there are no dividend payments through the life of the future.[21] Hence, the daily carry for a future *i* is the negative of the daily financing rate, $-r_{i,t,T}/365$ and an equity carry futures strategy will be similar to a fixed income carry strategy (with the notable difference of the important role of the borrow rate in addition the interest rate).

Second, let's assume that there **is a dividend payment.** It has become a standard concept to think of the carry measure as the return that accrues if the spot does not move. Note that the assumption that the spot does not move through the life of the equity future is quite restrictive when there is a dividend payment. In reality, a spot price move is more or less guaranteed to happen once the dividend is paid (as the price before the payment accrues the expected dividend and after the dividend payment the price drops with the amount of the dividend). In the case of equity futures we should rather think of the cumulative return over a certain period being zero, i.e. $R_{i,T,t} = \sum_{s=t+1}^{t+T} RS_{i,s} = 0$. In fact, we can easily incorporate the framework of Gordon's model (1956) with no growth in dividends. Under the assumption of a constant dividend rate the model implies that the price grows with the dividend yield until the payment of the dividend at which point it drops to the initial value. The price does not change between two dividend payment dates (but it will move between any other set of dates).

Hence, in case of a payment of dividends during the life of the future, we can assume that the futures daily carry between time t and maturity t+T is the difference between the daily dividend yield and the daily financing rate, i.e. $((d_{i,t} - r_{i,t,T})/365)$ and aim to hedge the spot moves among the assets.

$$\boldsymbol{Carry}_{t+T,t} = \left[ln\left(\frac{S_{1,t}}{F_{1,t,T}}\right),\quad ln\left(\frac{S_{2,t}}{F_{2,t,T}}\right), \dots\dots\dots, \ln\left(\frac{S_{n,t}}{F_{n,t,T}}\right)\right]'$$

Let's assume that the same factor structure exists for the spot returns $\boldsymbol{RS}_s$ at every point *s=t+1,…,.t+T.*

$$\boldsymbol{RS}_s = \boldsymbol{a} + \boldsymbol{BF}_s + \boldsymbol{\epsilon}_s$$

As usual $\boldsymbol{F}_s$ denotes the factors at time *s*, $\boldsymbol{B}$ is the matrix containing the sensitivities to the factors and $\boldsymbol{\epsilon}_t = [\epsilon_{1,s,}\epsilon_{2,s}, \dots\dots, \epsilon_{n,s}]$ are the residuals being distributed as $\boldsymbol{\epsilon}_s \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$ for $i\epsilon[1, \dots. n]$.

Hence, if we impose $\boldsymbol{w'a} = \boldsymbol{0}$ and $\boldsymbol{w'B} = \boldsymbol{0}$ for every $i = 1, \dots., m$ (m is the number of factors) it follows:

[21] Note that we consider only first generic future contracts with maturities of 1 or 3 months.

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$$\boldsymbol{E}(\boldsymbol{w}'\boldsymbol{R}_{t+T,t}) = \boldsymbol{w}'(\boldsymbol{Carry}_{t+T,t}),$$

where $\boldsymbol{R}_{t+T,t}$ contains cumulative returns of the futures in our universe between time *t* and maturity *t+T*.

We would like to emphasize that the result above heavily relies on the existence of factor structure for spot returns. As we have already mentioned, on the day the dividend is paid we expect a more sizable (negative) return in the index with dividend payment. Hence, we can have situations when one index return de-correlates from the rest simply due to fact that a dividend is paid and hence finding a reliable factor model for the spot returns can be challenging. Therefore, we consider such an approach to have a strong empirical/statistical focus.

Being able to capture the dividend by placing the position at the time the dividend is paid can be quite profitable if the strategy is applied to several markets and the dividend payments are spread through the year. For example, if we can capture the dividend of EuroStoxx50 by hedging its spot move with the spot moves of other markets and then move to capture the dividend of S&P500 by being able to hedge out the S&P500 spot move, we will be able to obtain a return consisting of the sum of the two dividend yields. Hence, such an approach can be thought of as a statistical arbitrage one and its profitability will be a contender among the challenges of the Efficient Market Hypothesis (EMH, Fama (1965)).

Factor model for implied total returns

Alternatively, we can look for a factor model in the hypothetical returns the markets would have posted if there were no dividend payments. Let's use the upper asterix symbol * to denote the market variable if there were no dividend payment. Note that, in this case, we assume the dividend is ploughed back and taxed at the equilibrium tax rate implied by the market. We will refer to this type of returns as implied net total returns as reflected in the future's returns. As the dividend is paid at *t+T* it follows $RS_{i,t+s}$=$RS^{*}_{i,t+s}$ for $s=1,..,T-1$.

We have already seen that the standard non-arbitrage condition implies:

$$F_{i,t,T} = S_{i,t}e^{r_{i,t,T}T/365} - D_{i,t+T} \sim S_{i,t}\left(1 + \frac{r_{i,t,T}T}{365}\right) - S_{i,t}(D_{i,t+T} \ /S_{i,t}) \sim S_{i,t}e^{(r_{i,t,T}-d_{i,t})T/365},$$

where $d_{i,t} = (D_{i,t+T} \ /S_{i,t})/(\mathrm{T}/365)$.

Iterating forward in time, it follows $F_{i,t+1,T-1} = S_{i,t+1}e^{(r_{i,t+1,T-1}-d_{i,t+1})(T-1)/365}$ with $d_{i,t+1} = (D_{i,t+T} \ /S_{i,t+1})/((\mathrm{T}-1)/365)$.

Hence, if $R_{i,s+1}$ denotes the future's return between time *s* and *s+1* and *s=t,…,.t+T-2* then it holds:

$$R_{i,s+1} = RS_{i,s+1} - \frac{r_{i,s,T-(s-t)}(T-(s-t))}{365} + \frac{r_{i,s+1,T-(s-t)-1}(T-(s-t)-1)}{365} - \frac{D_{i,t+T}}{S_{i,s+1}} + \frac{D_{i,t+T}}{S_{i,s}}$$

Hence, under the assumption that the financing rate and the prices do not vary much between two dates:

$$R_{i,s+1} \sim RS_{i,s+1} - \frac{r_{i,s,T-(s-t)}}{365} = RS^{*}_{i,s+1} - \frac{r_{i,s,T-(s-t)}}{365}$$

The above relationship holds for any *s* between *t* and *t+T-2*. The future's return at the maturity date when the dividend is paid requires a special attention.

At the maturity *t+T* date we can express the spot price return for asset *i* as:

$$RS_{i,t+T} = \ln\left(\frac{S_{i,t+T}}{S_{i,t+T-1}}\right) = \ln\left(\frac{S^{*}_{i,t+T}-D_{i,t+T}}{S_{i,t+T-1}}\right) = \ln\left(1 + \frac{S^{*}_{i,t+T}-S^{*}_{i,t+T-1}-D_{i,t+T}}{S^{*}_{i,t+T-1}}\right) \sim \frac{S^{*}_{i,t+T}-S^{*}_{i,t+T-1}}{S^{*}_{i,t+T-1}} - \frac{D_{i,t+T}}{S^{*}_{i,t+T-1}} = RS^{*}_{i,t+T} - \frac{D_{i,t+T}}{S_{i,t+T-1}}$$

The corresponding return of the future can be expressed as:

$$R_{i,t+T} = RS_{i,t+T} - \frac{r_{i,t+T-1,1}}{365} \quad + \frac{D_{i,t+T}}{S_{i,t+T-1}} = RS^*_{i,t+T} - \frac{r_{i,t+T-1,1}}{365}$$

Hence, we can think of the futures return as the implied total return of the market adjusted for the cost of financing. Therefore, in this case, we can consider hedging the implied total returns (which can be defined as the sum of the futures returns and the financing rates). If we hedge the implied total returns we will be capturing the (negative of) financing rate and hence we will be long assets with the most negative (least positive) and short assets with the least negative (most positive) financing rate.

There are few practical obstacles to such an approach. First, the financing rate (and in particular the borrow rate) is not directly observable. Second, when we backtested a strategy along those lines using a proprietary dataset for the borrow rates we found that the carry income that can be captured is insufficient to compensate costs, especially in the current low yield environment.

We can take the analysis above a step further and attempt to model the expected implied total returns directly. As we have already mentioned in a Gordon's model (1956) without dividend growth the required rate of return is equal to the dividend yield. Taking this logic on board leads to the familiar setup:

$$E(R_{i,t}) = RS^*_{i,t} - \frac{r_{i,t,T}}{365} = \frac{E(DivYield_{i,t,t+T})}{365} - \frac{r_{i,t,T}}{365},$$

with $E(DivYield_{i,t,t+T})$ denoting the expected dividend yield for asset *i* between *t* and *t+T*.

Therefore, we can define our carry measure as the expected dividend yield above the financing rate:

$$\boldsymbol{Carry}_t = \left[E(R_{1,t}),\ E(R_{2,t}), \dots \dots \dots, E(R_{n,t})\right]'$$

Subsequently, we can consider hedging the deviations between the realized and the expected returns. Let's denote $\Delta_{i,t} = R_{i,t} - E(R_{i,t}) = R_{i,t} - Carry_{i,t}$ and bundle the observations for the deviations between the realized and the expected returns at time *t* for all the assets in our universe into the vector $\Delta_t = \left[\Delta_{1,t}, \quad \Delta_{2,t}, \dots \dots \dots, \Delta_{n,t}\right]'$. Those deviations can be thought of for example as the dividend growth rate or as an idiosyncratic return.

Let's assume that a factor model exists for Δ_t:

$$\Delta_t = \boldsymbol{a} + \boldsymbol{B}\boldsymbol{F}_t + \boldsymbol{\epsilon}_t,$$

The standard assumptions hold: $\boldsymbol{F}_t$ denotes the factors at time *t*, $\boldsymbol{B}$ is the matrix containing the sensitivities to the factors and $\boldsymbol{\epsilon}_t = \left[\epsilon_{1,t,}\epsilon_{2,t}, \dots.., \epsilon_{n,t}\right]$ are the residuals being distributed as $\boldsymbol{\epsilon}_t \sim N(0, \boldsymbol{D})$ with $\boldsymbol{D}$ a diagonal matrix with elements $\boldsymbol{D}_{i,i} = \sigma_i^2$ for $i\epsilon[1, \dots. n]$.

The difference between the expected dividend yield and the financing rate is not directly observable except for the maturities when a dividend will be paid during the time the future is the first generic contract. Therefore, we have decided to filter this quantity using the Prophet package with a similar specification to the one used in the case of the commodities carry. We extract the trend component subject to annual seasonal adjustment. Below we illustrate numerically and graphically why such an approach is sound.

Let's assume a dividend yield of 3%, monthly expiries and a funding rate of 1% (note that below we always refer to annualized figures). Then we would expect the average annualized observed value of the equity carry to be 2% (the difference between the dividend yield and the funding rate). We can also arrive at the same number as follows: during the maturity month we observe equity carry of 3%*12-1%, while in the remaining 11 months we observe the negative of the funding rate (-1%). The average value will be indeed (3%*12-1%-1%*11)/12= 2%.

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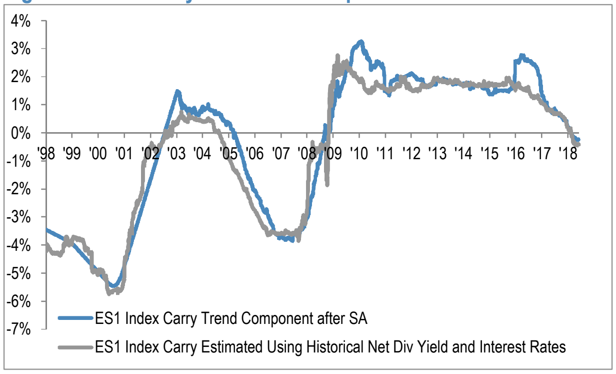
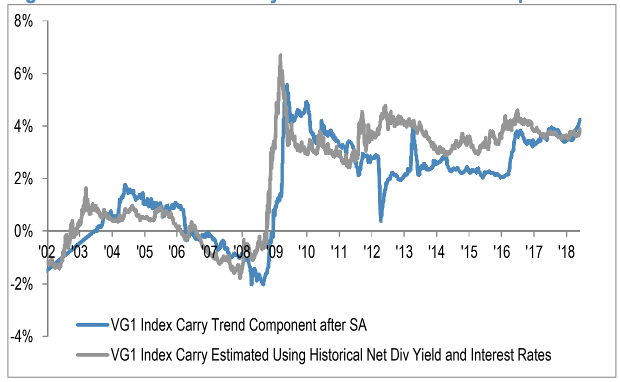
As a robust check we have compared our carry estimates obtained via filtering from the observed ratio of the spot and the future price $\left(\ln\left(S_{i,t}/F_{i,t,T}\right)\right)$ to the ones obtained using the historical net dividend yield and the actual interest rate. As evident below the results are quite similar, though on occasions important discrepancies might be present.

Figure 45: SPX carry estimation comparison



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 46: EuroStoxx50 carry estimation methods comparison



Source: J.P. Morgan Quantitative and Derivatives Strategy

We consider using the extracted carry signal from the market a far better alternative than using the historical net dividend yield and interest rate assumptions. The historical dividend yield will be a backward looking estimator while the market data will reflect the expectations. The tax assumptions used to calculate the net dividend yield might be different to the assumptions in the future's prices. Last, but not least, not taking into account the borrow rate might in turn have important implications.

Empirical results

Data universe and modelling considerations

Our universe consists of the first generic futures for equity indices in Europe, Americas and Asia.

Table 24: Universe equity futures

Europe	Americas	Asia
CF1 Index (CAC 40)	ES1 Index (S&P 500)	NH1 Index (Nikkei 225)
GX1 Index (DAX)	NQ1 Index (Nasdaq 100)	TP1 Index (Topix)
VG1 Index (EuroStoxx 50)	PT1 Index (S&P/TSX 60)	XP1 Index (ASX SPI 200)
Z 1 Index (FTSE 100)		HI1 Index (Hang Seng)
SM1 Index (SMI)		
IB1 Index (IBEX 35)		
ST1 Index (FTSE MIB)		
QC1 Index (OMXS 30)		

Source: J.P. Morgan Quantitative and Derivatives Strategy

As our universe spans different geographical regions, there might be a strong impact of non-synchronous prices upon the measurement of returns and subsequently upon estimation of the factor structure. In order to mitigate that effect in the case of equity futures effect we have solely relied on weekly returns and used 1-year and 2-year periods to estimate the factor structure.

For the calculation of the carry measure (based on the ratio between the spot and the future price) we have used contemporaneous tick data. Furthermore, as DAX is a total return index, we have used the relevant first generic future (GX1 Index) only for the backtests based on the assumption for a factor model for the spot returns.

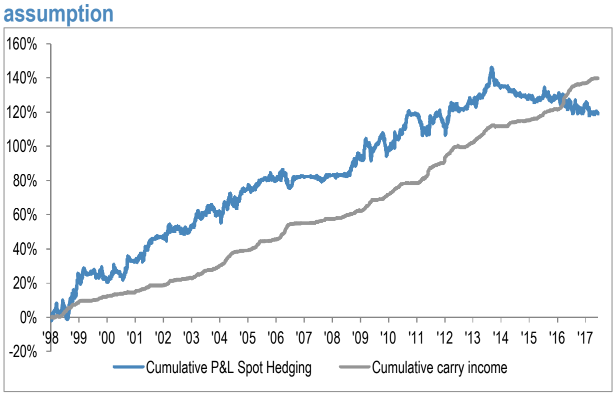
As one geographical group can dominate the rest in terms of size of the dividends similarly to the case for commodities, we have imposed additional risk limits to the individual geographical groups (the risk contribution of a group cannot exceed 40%).

Factor model for spot returns

First, let's consider the assumption that a factor model for spot returns exists. In this case the strategy will tend to have long positions in the markets for which dividends are being paid out during the maturity of the future[22]. Subsequently, any positive/negative surprises will directly impact the P&L of the strategy. Conversely, the short positions are expected to be markets with high financing rates.

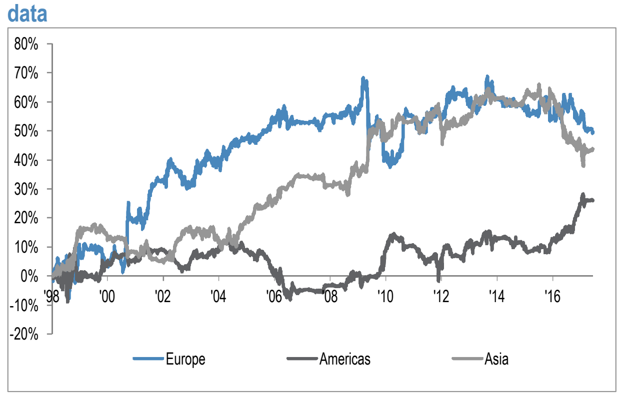
The model achieves a Sharpe ratio above 0.5 under conservative transaction costs assumptions which is respectable given the statistical nature of the model. Most of the losses have been incurred in the recent years. As we have discussed before on occasions the hedging can break down, especially in situations when the sentiment towards a style gathers even stronger momentum (for example investors are getting more and more bullish than in the past and invest in growth stocks with small dividend payouts). In such situations the breakdown of the factor structure can work in both directions depending on the positioning of the model, either adding to or subtracting from the strategy's profits.

Figure 47: Cumulative P&L and carry under spot factor model assumption



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 48: Average annualized carry and P&L based on yearly data



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 25: Performance statistics spot factor model approach

Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
6.12%	11.08%	0.55	28%

Source: J.P. Morgan Quantitative and Derivatives Strategy

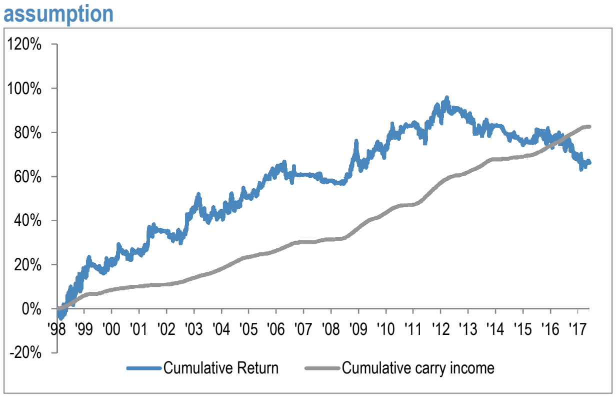
Historically most of the profits have been generated in Europe and Asia though the profits in two regions have recently faltered. The positioning of the model is quite dynamic and in terms of notional exposure the model has been long in the Americas and Europe at the expense of Asia.

Factor model for implied total returns

The carry strategy under the assumption for a factor model for the implied returns generates a Sharpe ratio of 0.3. A lower Sharpe in comparison to the carry strategy based on the factor model for spot returns is expected as the latter approach aims to capture additional inefficiencies.

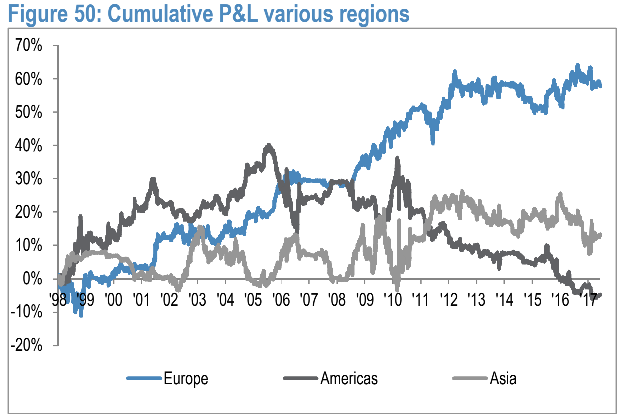
[22] Abstracting from the additional constraints imposed for factor neutrality.

Figure 49: Cumulative P&L and carry under TR factor model assumption



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 50: Cumulative P&L various regions



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 26: Performance statistics factor model for implied total returns approach

Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
3.39%	11.08%	0.31	33%

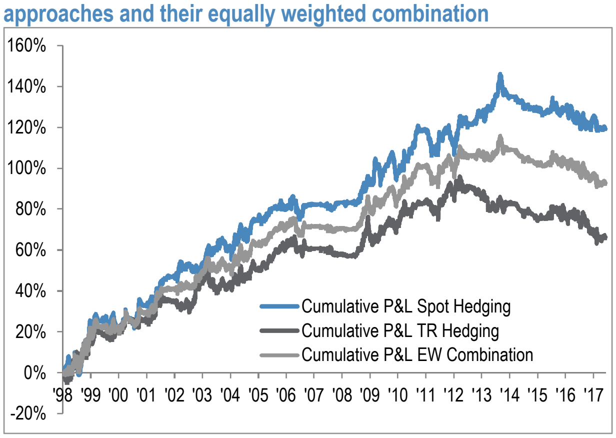
Source: J.P. Morgan Quantitative and Derivatives Strategy

Similarly to the strategy based on the spot factor model, historically most of the profits have been generated in Europe and Asia though the profits in two regions have recently faltered. The positioning of the model is less dynamic as the carry inputs are more stable and in terms of notional exposure the model has been long in the Americas and Europe at the expense of Asia.

Comparison to other approaches and styles

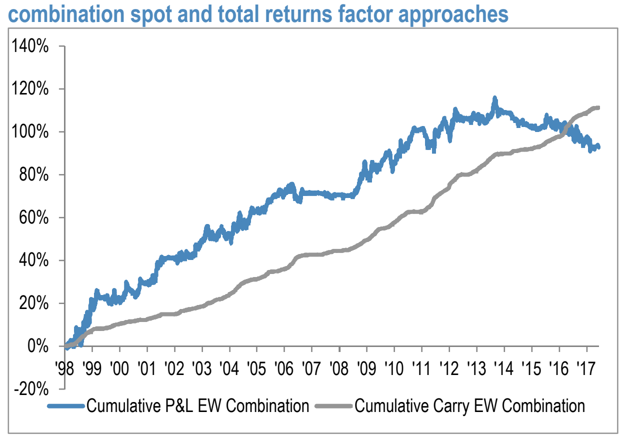
The strategies based on the spot and total returns factor approaches have a correlation of 0.5. While the correlation is not muted, there are still diversification benefits of combining the two approaches. The combined strategy will have a Sharpe ratio closer to the spot factor approach, but a more attractive drawdown measure.

Figure 51: Cumulative P&Ls for the spot and total returns factor approaches and their equally weighted combination



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 52: Cumulative P&L and carry equally-weighted combination spot and total returns factor approaches



Source: J.P. Morgan Quantitative and Derivatives Strategy

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Table 27: Performance statistics equally-weighted combination spot and total returns factor approaches

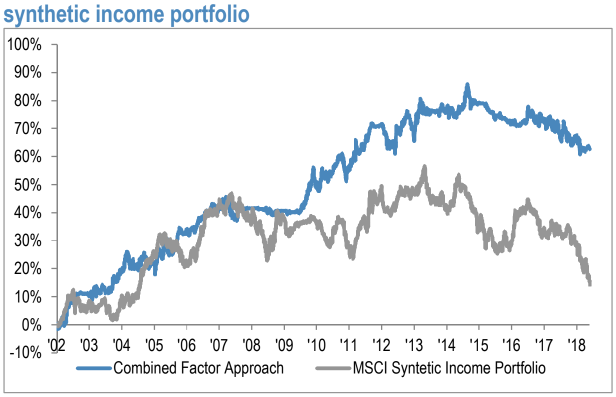
Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
4.76%	9.63%	0.49	25.23%

Source: J.P. Morgan Quantitative and Derivatives Strategy

In its essence our approach bears close similarly to the equity income risk premia style (see for example the J.P. Morgan primer on Equity Risk Premia Strategies). The equity risk premia income style investing is based on investments in individual stocks (rather than indices) and long-short portfolios are constructed with long positions in the stocks with high dividends and short positions in the stocks with smaller dividends. Note that our approach, while similar, can be different from equity income risk premia investing as factor exposures are taken into account and long positions in the assets with the higher dividends are not always guaranteed.[23] Nevertheless, the performance of our approach will be linked to the equity income style coming in and out -of-vogue.

Below we have shown the performance of a portfolio consisting of the MSCI World High Dividend Yield Net Total Return Index (M1WDHDVD Index) beta-hedged by the MSCI World Net Total Return Index (NDDUWI Index).[24] The correlation between the synthetic income portfolio and the market-neutral carry approach is low, around 0.16. Nevertheless, it is evident that recently the equity income risk premia style has been under pressure and we note a factor hedging approach would have helped to mitigate some of the losses.

Figure 53: Cumulative P&Ls factor hedging approach and a synthetic income portfolio



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 28: Performance statistics combined factor approach and MSCI synthetic portfolio

	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Combined Factor Approach	3.81%	8.80%	0.43	25.23%
MSCI Synthetic Portfolio	0.87%	10.16%	0.09	42.4%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 29: Performance statistics combined factor approach and MSCI synthetic portfolio

	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Combined Factor Approach	3.81%	8.80%	0.43	25.23%
MSCI Synthetic Portfolio	0.87%	10.16%	0.09	42.4%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Despite its recent underperformance the comprehensive solution significantly outperforms the ranking and the analytical approaches. It is also the only solution that attains positive skewness at the majority of return horizons. In terms of correlations, all the solutions display no significant correlation to either the MSCI World or the VIX.

[23] For example, under the assumption of one factor market model if one asset has a market beta of 1 and a dividend yield of 2% and another asset has a beta of 2 and a dividend yield of 3%, our approach will be long in asset 1 and short in asset 2, with the position in asset 2 being half of the position in asset 1.
[24] We beta-hedge using one year of data and we have assumed to transaction costs linked to the hedging.

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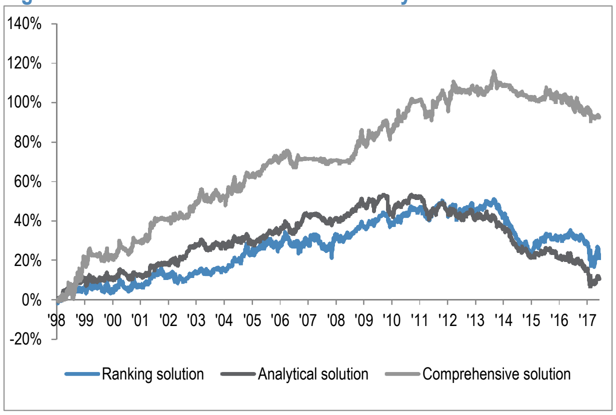
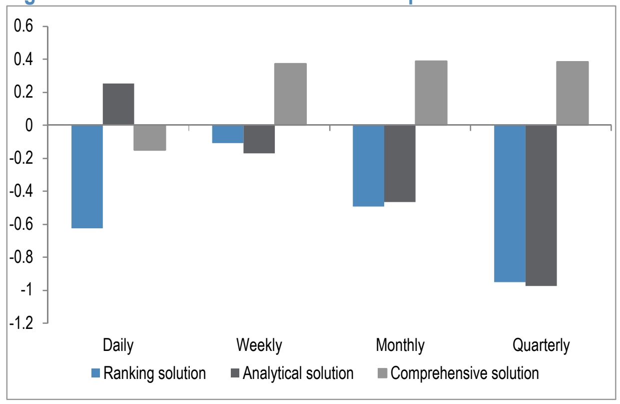
Figure 54: Cumulative returns various carry solutions



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 55: Skewness at different return frequencies



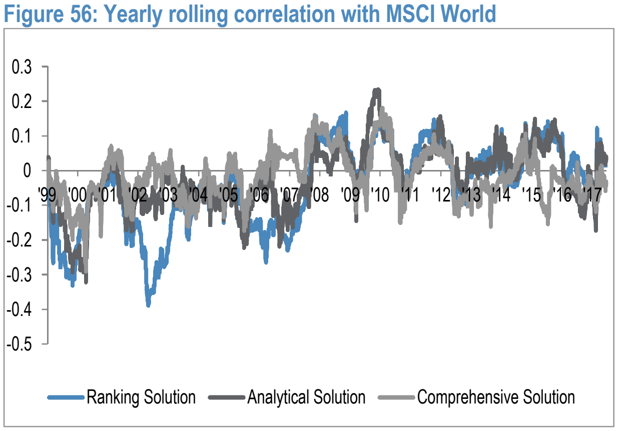
Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 30: Comparative performance statistics

Strategy	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Comprehensive approach	4.76%	9.63%	0.49	25.23%
Ranking approach	1.14%	9.60%	0.12	34.59%
Analytical approach	0.57%	7.49%	0.08	46.11%

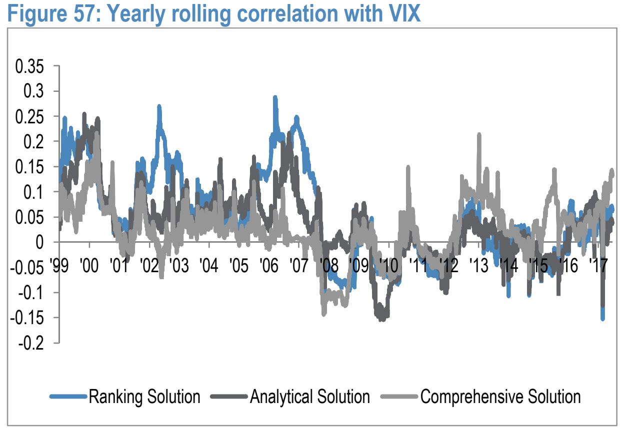
Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 56: Yearly rolling correlation with MSCI World



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 57: Yearly rolling correlation with VIX



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 31: Correlation statistics of the equity carry solutions with MSCI World and VIX

	MSCI World			VIX		
	Average	Max	Min	Average	Max	Min
Ranking solution	-0.04	0.17	-0.39	0.05	0.29	-0.15
Analytical solution	-0.02	0.23	-0.32	0.04	0.25	-0.15
Comprehensive solution	-0.02	0.18	-0.29	0.02	0.22	-0.14

Source: J.P. Morgan Quantitative and Derivatives Strategy

Combined portfolio of carry strategies

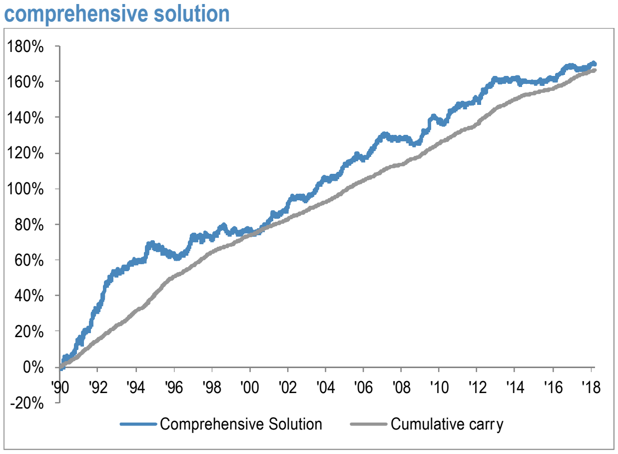
We note that the real benefit of the market-neutral carry strategies becomes even more evident when we aggregate the track records of the carry strategies in the individual asset classes. The table below displays the correlations among the returns of the various asset classes based on the comprehensive approach. It is evident that the tracks of the individual asset classes are virtually uncorrelated. The strong diversification among the individual sleeves is indeed something that we expect from the market-neutral approach. The strategy return in every individual asset should consist of a deterministic carry component and a noise component. We would expect the noise components among the individual asset class strategies to be uncorrelated and hence a strong diversification benefit is expected.

Table 32: Correlation matrix of the carry strategies asset-class returns in the comprehensive approach implementation

	FX	Rates	Credit	Equity	Commodity
FX	1.00				
Rates	0.05	1.00			
Credit	0.02	0.01	1.00		
Equity	0.02	0.00	-0.04	1.00	
Commodity	0.00	0.00	0.00	0.01	1.00

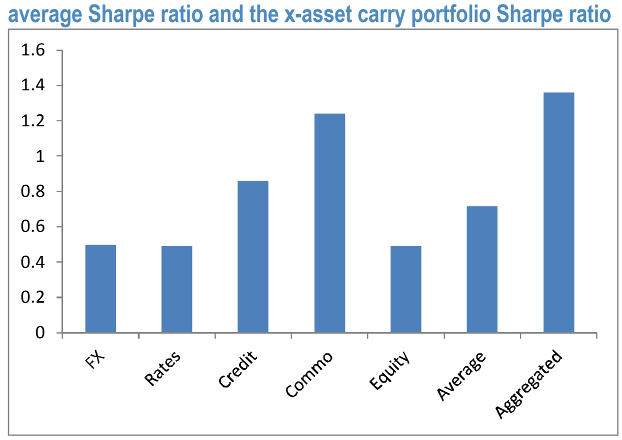
Source: J.P. Morgan estimates, Bloomberg.

Figure 58: Cumulative P&L versus cumulative carry for the comprehensive solution



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 59: Sharpe ratios in various asset classes versus the average Sharpe ratio and the x-asset carry portfolio Sharpe ratio



Source: J.P. Morgan Quantitative and Derivatives Strategy

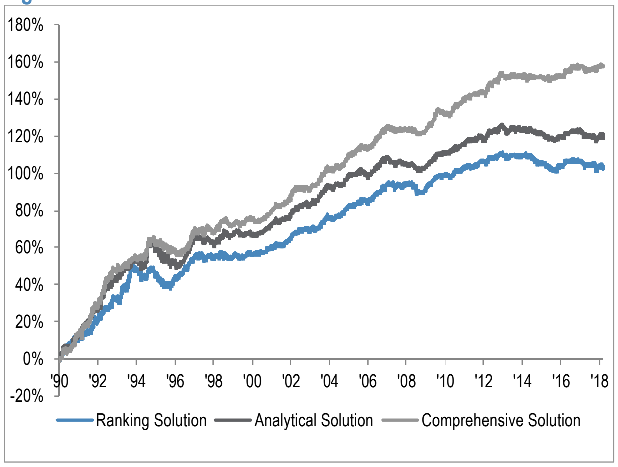
Table 33: Performance statistics aggregated carry portfolio comprehensive approach

Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
5.6%	4.1%	1.36	9.8%

Source: J.P. Morgan Quantitative and Derivatives Strategy

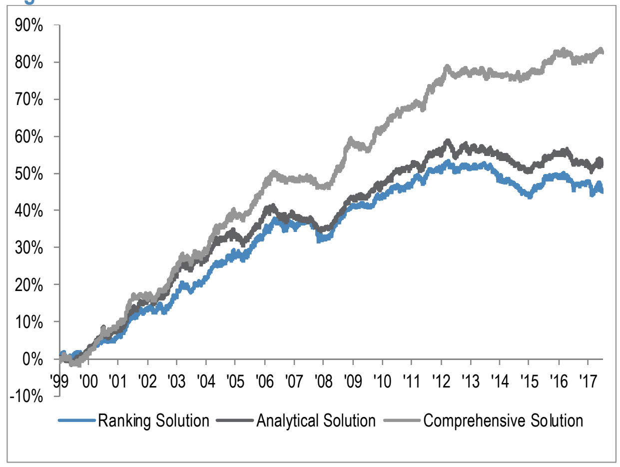
The aggregated carry portfolio delivers a Sharpe ratio of 1.36, an improvement of almost 90% upon the average asset class carry strategy Sharpe ratio. The strategy has a relatively sizable drawdown but it has been incurred during the time when the commodity carry strategy has been the sole component of the aggregate carry portfolio.

Figure 60: Cumulative P&L since 1990



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 61: Cumulative P&L since 2000



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 34: Performance statistics various solutions

	Since 1990			Since 2000		
	Ranking solution	Analytical solution	Comprehensive solution	Ranking solution	Analytical solution	Comprehensive solution
Annualized Return	3.62%	4.22%	5.56%	2.43%	2.82%	4.44%
Annualized Volatility	4.10%	4.10%	4.10%	3.02%	3.02%	3.02%
Sharpe	0.88	1.03	1.36	0.81	0.94	1.47
Maximum Drawdown	11.7%	15.9%	9.8%	9.8%	8.7%	4.3%

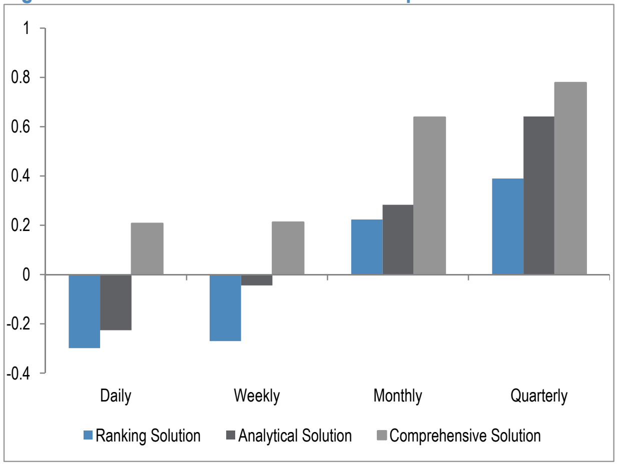
Source: J.P. Morgan Quantitative and Derivatives Strategy

Indeed, analyzing separately the performance before and after 2000 for all the solutions we find much more attractive drawdown characteristics for the latter period where diversification among tracks in all asset classes is present. The Sharpe ratio of the comprehensive solution is increased to 1.47 and the drawdown corresponds to 1.4 times the volatility. The Sharpe ratios of ranking and the analytical approaches are sub 1.0. Nonetheless, those portfolio solutions also benefit from diversification though their performance has faded away in the years of QE.

The comprehensive solution has also been found to provide more stable returns through various macro-economic regimes. Making use of the JP Morgan QMI (Quant Macro Index) we have analyzed the performance in periods of recovery, expansion, slowdown and contraction. In particular, the performance of the comprehensive solution has been found much more resilient during the periods of slowdown and contraction.

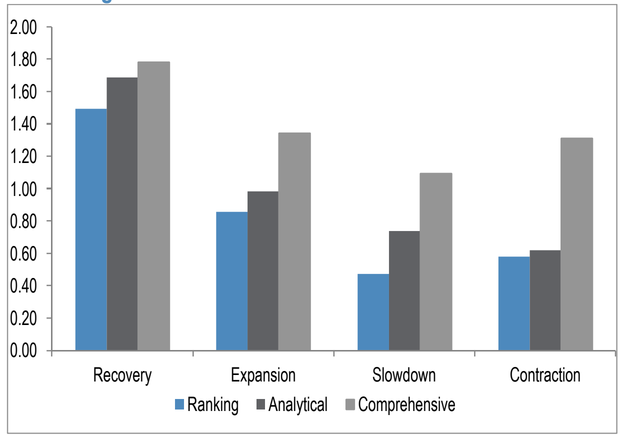
The inspection of returns' skewness shows that the comprehensive solution can deliver positive skewness at every of the analyzed return horizons. The ranking and the market-neutral solution are also characterized by positive skewness but only at the monthly and the quarterly frequency.

Figure 62: Skewness at different return frequencies



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 63: Sharpe ratios for the comprehensive solution in various regimes based on the QMI indicator



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 35: Performance statistics various solutions

	Since 1990			Since 2000		
	Ranking solution	Analytical solution	Comprehensive solution	Ranking solution	Analytical solution	Comprehensive solution
Recovery	1.49	1.69	1.78	1.83	1.87	2.33
Expansion	0.85	0.98	1.34	0.52	0.56	0.99
Slowdown	0.47	0.74	1.09	0.41	0.61	1.27
Contraction	0.58	0.62	1.31	0.10	0.38	1.47

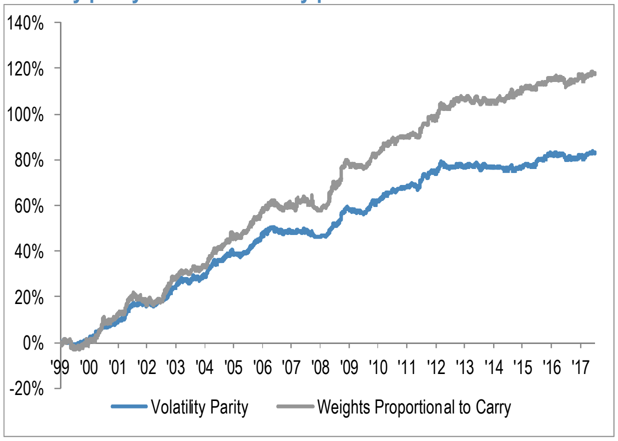
Source: J.P. Morgan Quantitative and Derivatives Strategy

Can we benefit from yield rotation?

The aggregated cross-asset carry portfolio has been constructed using the volatility parity concept. As the carry component should be the main driver of the market-neutral carry strategy returns we can try to overweight the asset classes offering higher carry income at the expense of those with a lower carry income. At first sight such an approach should be profitable as we have seen that the carry income in some asset classes, like commodities, has traditionally been very attractive.

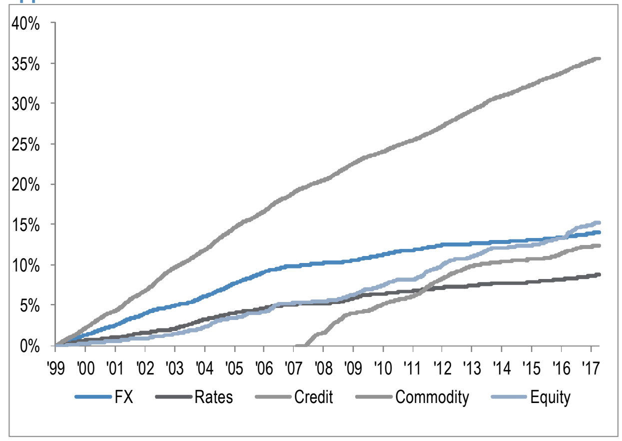
Below we show the comparison between the volatility parity approach and the yield rotation approach in which the allocation to every strategy depend on the ratio between latest carry return embedded in the strategy and the average carry across all asset classes (with adjustment for the differing volatility in every asset class). While the yield rotation approach outperforms in terms of absolute return the two approaches are quite similar when performance is measured via the Sharpe ratio. The main reason is that overweighting some asset classes at the expense of others the diversification embedded in the aggregated strategy is impacted. While we can achieve a higher return by yield rotation the decrease in diversification increases the portfolio volatility and the overall Sharpe ratio is almost unchanged.

Figure 64: Comparison of the yield rotation approach to a volatility parity cross-asset carry portfolio



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 65:Cumulative carry harvested by the comprehensive approach in various asset classes



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 36: Performance statistics volatility parity and yield rotation cross-asset carry portfolios

	Annualized Return	Annualized Volatility	Sharpe	Maximum Drawdown
Volatility Parity	4.44%	3.02%	1.47	4.30%
Weights proportional to the carry yield	6.32%	4.32%	1.46	7.00%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Diversification benefits with trend-following

The market-neutral carry portfolios are constructed on the basis of statistical models. As we have discussed, one of the important steps in the comprehensive approach is the neutralization of the idiosyncratic spot returns (represented by the vector **a** in our equations). We can also think of those idiosyncratic spot returns as asset specific trends beyond the general trends in the asset class.

As a consequence if the relationship between those asset specific trends changes our hedging algorithm might encounter difficulties. But at the same trend-following system might benefit from the stronger trends in one group of assets at the expense of others.

Furthermore, despite the statistical nature of our approach the hedging might be challenged, especially when we transition from a macro-economic regime of strong growth into one of a slowdown or contraction. At such turning points high carry assets come under pressure and the previous statistical relationship breaks (as upside and downside betas are not always symmetric). It is well-known that trend-following has delivered strong returns in market sell-off environments.

Therefore, a priori we expect strong diversification benefits between the market neutral carry portfolio and the trend-following approach. Below we show the results of mixing the returns of the comprehensive carry portfolio with trend-following strategies as described in our publication "Designing robust trend-following system : Behind the scenes of trend-following".

The table below shows the correlation between the comprehensive carry approach and the trend-following solution. The correlations are relatively low re-emphasizing the expectations for strong diversification benefits.

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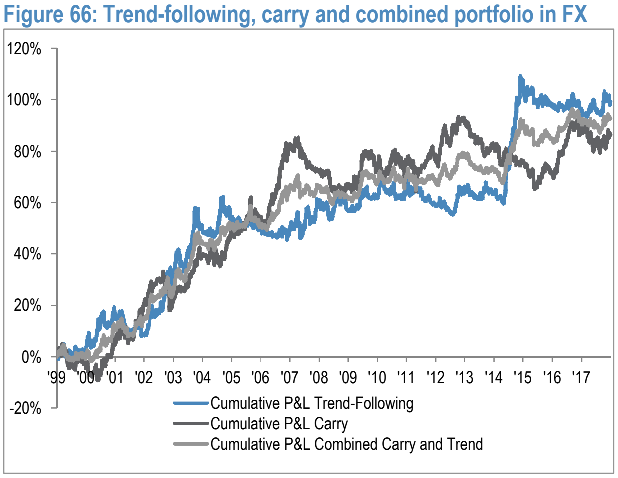
Table 37: Correlations between the returns of comprehensive carry portfolio and the trend-following system

	FX	Rates	Commodities	Equities	Combined
Correlation	0.06	0.16	0.1	-0.02	0.09

Source: J.P. Morgan Quantitative and Derivatives Strategy

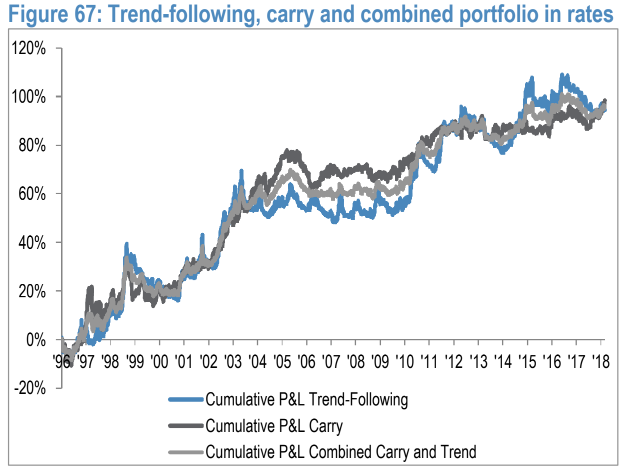
Indeed, both in FX and rates the combined Sharpe ratio improves upon the individual ones but the more significant benefits are evident in the much better controlled drawdowns, especially in the case of FX.

Figure 66: Trend-following, carry and combined portfolio in FX



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 67: Trend-following, carry and combined portfolio in rates



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 38: Performance statistics for trend-following, carry and combined portfolios in FX and rates

	FX			Rates		
	Trend-Following	Carry	Combination	Trend-Following	Carry	Combination
Annualized Return	5.22%	3.59%	4.88%	4.25%	2.47%	4.32%
Annualized Volatility	9.16%	7.25%	6.69%	8.97%	5.01%	6.84%
Sharpe	0.57	0.50	0.73	0.47	0.49	0.63
Maximum Drawdown	16.82%	22.37%	11.88%	23.60%	9.89%	16.09%

Source: J.P. Morgan Quantitative and Derivatives Strategy

Similar results have been found in the case of commodities and equities. The Sharpe ratio is improved (especially in the case of equities) and the drawdown is better controlled.

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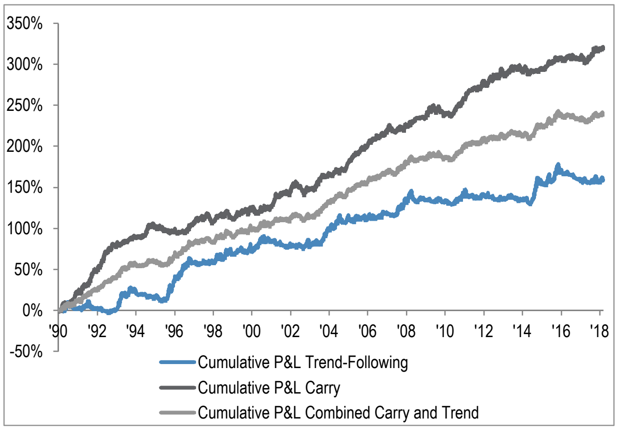
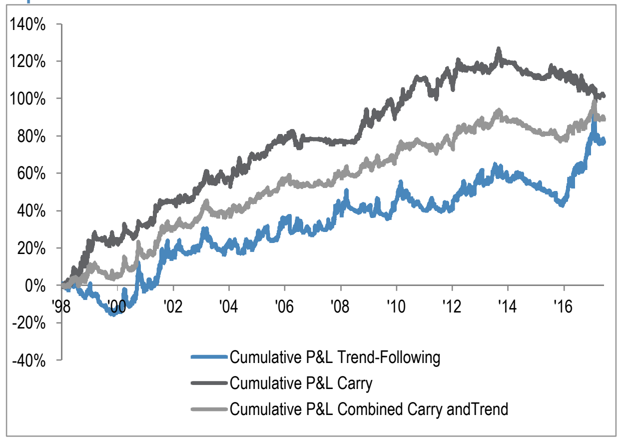
Figure 68: Trend-following, carry and combined portfolio in commodities



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 69: Trend-following, carry and combined portfolio in Equities



Source: J.P. Morgan Quantitative and Derivatives Strategy

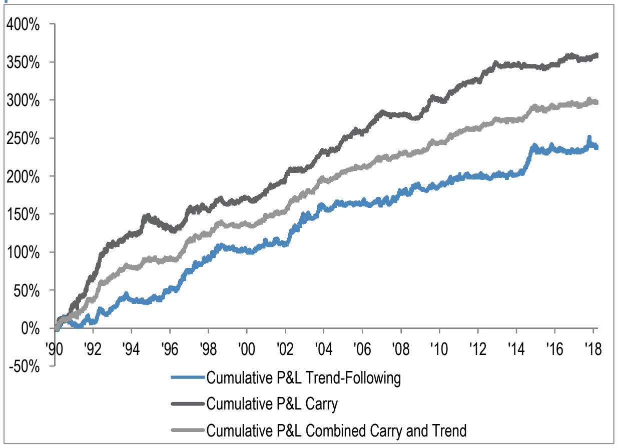
Table 39: Performance statistics for trend-following, carry and combined portfolios in commodities and equities

	Commodities			Equities		
	Trend-Following	Carry	Combination	Trend-Following	Carry	Combination
Annualized Return	5.61%	7.52%	8.45%	3.96%	4.76%	4.58%
Annualized Volatility	9.15%	6.07%	6.80%	10.53%	9.63%	7.34%
Sharpe	0.64	1.24	1.38	0.38	0.49	0.62
Maximum Drawdown	23.09%	10.07%	14.40%	22.62%	25.23%	17.21%

Source: J.P. Morgan Quantitative and Derivatives Strategy

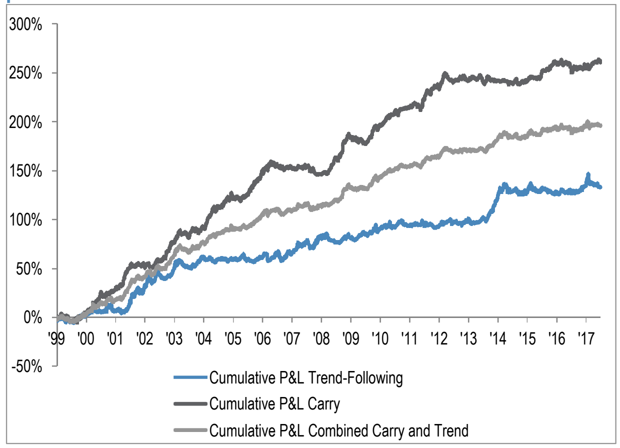
The diversification benefits naturally transfer from the individual asset classes to the combined cross-asset portfolios. While the Sharpe ratio of the combined trend-following and carry portfolio is not a sizable improvement upon the Sharpe ratio of the cross-asset carry portfolio, the drawdown of the combined trend/following carry portfolio is much attractive and the results show that analyzing the returns since 2000 the drawdown is just marginally higher than the corresponding volatility. Such an empirical result strongly supports our conjecture that there exist strong diversification benefits between the market-neutral carry portfolio and the trend-following portfolio.

Figure 70: Cross-asset trend-following, carry and combined portfolio



Source: J.P. Morgan Quantitative and Derivatives Strategy

Figure 71: Cross-asset trend-following, carry and combined portfolio since 2000



Source: J.P. Morgan Quantitative and Derivatives Strategy

Table 40: Performance statistics for cross-asset trend-following, carry and combined portfolios

	Whole history			Since 2000		
	Trend-Following	Carry	Combination	Trend-Following	Carry	Combination
Annualized Return	8.40%	5.56%	10.47%	7.18%	4.44%	10.54%
Annualized Volatility	9.31%	4.10%	6.89%	9.52%	3.02%	7.11%
Sharpe	0.90	1.36	1.52	0.75	1.47	1.48
Maximum Drawdown	14.74%	9.8%	7.85%	14.74%	4.3%	8.21%

Source: J.P. Morgan Quantitative and Derivatives Strategy

References:

Baltas, N. (2017), "Optimising Cross-Asset Carry", Factor Investing, Elsevier & ISTE Press, 2017 (Forthcoming). working paper, available at SSRN: https://ssrn.com/abstract=2968677

Baz, J.; Granger, N.; and Harvey, C.R.; Roux, N.L. and Rattray, S. (2015), "Dissecting Investment Strategies in the Cross Section and Time Series" working paper, available at SRN: https://ssrn.com/abstract=2695101 or http://dx.doi.org/10.2139/ssrn.2695101

Brunnermeier, M.K.; Nagel, S.; and Pedersen, L.H. (2008), "Carry trades and currency crashes", NBER Macroeconomics Annual.

Braeken, J.; and Assen, M. A. L. M.V. (2017), "An empirical Kaiser criterion", Psychological Methods, *22*(3), 450-466.

Chandan, M. (2016), "Diverging fortunes: FX value to trump carry in 2017.", J.P. Morgan Research Paper: https://www.jpmm.com/research/content/GPS-2191028-0

Chandan, M. (2017), "When value and carry collide : Implications of improving growth for FX risk premia", J.P. Morgan Research Paper: https://www.jpmm.com/research/content/GPS-2304871-0

Chandan, M. (2017), "Year of the Underdog: FX carry is cheap going into 2018 ", J.P. Morgan Research Paper: https://www.jpmm.com/research/content/GPS-2510268-0

Chandan, M. (2018), "It's not that EASI: What has and hasn't changed in FX risk premia since US exceptionalism emerged", J.P. Morgan Research Paper: https://www.jpmm.com/research/content/GPS-2785748-0

Cootner, P.H. (1960), "Returns to Speculators: Telser versus Keynes", The Journal of Political Economy, 68(4), pp 396–404.

Eastment, H.T.; and Krzanowski, W.J. (1982), "Cross-validatory choice of the number of components from a principal component analysis". Technometrics, 24, 73–77.

Fama, E. (1965), "The Behavior of Stock Market Prices". Journal of Business. 38: 34–105. doi:10.1086/294743.

Gordon, M.J. (1959), "Dividends, Earnings and Stock Prices". Review of Economics and Statistics. The MIT Press. 41 (2): 99–105.

Gordon, M.J.; Shapiro, E. (1956), "Capital Equipment Analysis: The Required Rate of Profit," Management Science, 3(1) (October 1956) 102-110. Reprinted in Management of Corporate Capital, Glencoe, Ill.: Free Press of, 1959.

Guttman, L. (1954), "Some necessary conditions for common factor analysis". Psychometrika, 19, 149 –161.

Harris, R. J. (2001), "A primer of multivariate statistics (3rd ed.)". Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Hastie, T.; and Tibshirani, R. (1987), "Generalized additive models: some applications", Journal of the American Statistical Association 82(398), 371–386.

Hevia, C.; Petrella, I.; Sola,M. (2016), "Risk Premia and Seasonality in Commodity Futures" working paper, available at Online Library: https://onlinelibrary.wiley.com/doi/abs/10.1002/jae.2631

Kaiser, H. F. (1960), "The application of electronic computers to factor analysis". Educational and Psychological Measurement, 20, 141–151.

Kaldor, N. (1939), "Speculation and economic stability", Review of Economic Studies 7, 1-27.

Keynes, M. (1930), "A Treatise on Money, II: The Applied Theory of Money", Edition Macmillan and Co.

Koijen, R.S. J.; Moskowitz, T.J.; Pedersen, L.H.; and Vrugt, E.B., (2016) "Carry. Fama-Miller". Working paper, available at SSRN: https://ssrn.com/abstract=2298565 or http://dx.doi.org/10.2139/ssrn.2298565

Kolanovic, M.; and Wei, Z. (2015), "Systematic Strategies Across Asset Classes Risk Factor Approach to Investing and Portfolio Management", J.P. Morgan Research Paper: https://www.jpmm.com/research/content/GPS-1277971-0

Kolanovic, M.; and Wei, Z. (2015), "Equity Risk Premia Strategies: Risk Factor Approach to Portfolio Management", J.P. Morgan Research Paper: https://www.jpmm.com/research/content/GPS-1486191-0

Lempérière, Y.; Deremble, C.; Nguyen, T.T.; Seager, P.; Potters, M.; and Bouchaud, J.P. (2017), "Risk premia: asymmetric tail risks and excess returns", Quantitative Finance, 17:1, 1-14, DOI: 10.1080/14697688.2016.1183035

Litterman, R.B. and Scheinkman, J. (1991), "Common Factors Affecting Bond Returns", The Journal of Fixed Income Summer 1991, 1 (1) 54-61; DOI: https://doi.org/10.3905/jfi.1991.692347

Lobo, M.S.; Fazel, M.; and Boyd, S. (2006), "Portfolio optimization with linear and fixed transaction"

Mertens, B.; Fearn, T.; and Thompson, M. (1995), "The efficient cross validation of principal components applied to principal component regression". Statist. Comput., 5, 227–235.

Morrissey, J.; and Spector, B. (2015), "Portfolio Optimization using the NAG Library".

MOSEK Optimizer API for Python 8.1.0.63, available at: https://docs.mosek.com/8.1/pythonapi.pdf

Normand, J. (2004), "Investment Strategies no. 10: JP Morgan's FX and Commodity Barometer", J.P. Morgan Research Paper : https://www.jpmm.com/research/content/MMRC-209537-1

Normand, J.; and Ghia, K. (2008), "Investment Strategies no. 47: Alternatives to standard carry and momentum in FX", J.P. Morgan Research Paper : https://www.jpmm.com/research/content/MMRC-228981-1

Ribeiro, R. (2009), "Investment Strategies no. 54: Profiting from slide in commodity curves", J.P. Morgan Research Paper : https://www.jpmm.com/research/content/GPS-285504-0

Taylor, S.J.; and Lehmam, B. (2017), "Forecasting at Scale". working paper, available at PeerJPreprints: https://peerj.com/preprints/3190.pdf

Tzotchev, D.; Kolanovic, M.; Lau, A.; Krishnamachari, R.T.; and Silvestrini, D. (2015), " Designing robust trend-following system : Behind the scenes of trend-following", J.P. Morgan Research Paper: https://www.jpmm.com/research/content/GPS-2570400-0

Wahid, O. (2005), "Investment Strategies no. 12: JPMorgan Emerging Market Carry-to-Risk Model", J.P. Morgan Research Paper : https://www.jpmm.com/research/content/MMRC-228981-1

Wold, S. (1978), "Cross-validatory estimation of the number of components in factor and principal components models. *Technometrics"*, **20**, 397–405.

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Appendix

Carry strategy Sharpe ratio optimization

General case

Let's assume that there are n assets with carry returns $\boldsymbol{c} = [c_1, c_2, \dots, c_n]$ and covariance matrix $\boldsymbol{\Sigma}$. We aim to maximize the following function:

$$\max \mathrm{f}(\boldsymbol{w}) = \frac{\boldsymbol{w'c}}{\sqrt{\boldsymbol{w'\Sigma w}}} \ ,$$

where $\boldsymbol{w}$ are the allocations to the individual assets. Note that $\mathrm{f}(\boldsymbol{w})$ is homogenous function of degree 0 - if w is a solution then $k\boldsymbol{w}$ where k is a positive constant is also a solution. Hence if $\boldsymbol{w}^*$ is a solution, there exists k for which holds $k(\boldsymbol{w}^*)'c = 1$ and $k\boldsymbol{w}^*$ is a solution to the optimization problem as well. It follows that we can impose that $\boldsymbol{w'c} = 1$ and solve a variance minimization problem subject to a constraint:

$$\min \mathrm{g}(\boldsymbol{w}) = \boldsymbol{w'\Sigma w} \quad \text{s.t. } \boldsymbol{w'c} = 1$$

Making use of the method of Lagrange multipliers we arrive at:

$$\min \mathrm{L}(\boldsymbol{w}, \lambda) = \boldsymbol{w'\Sigma w} + \lambda(\boldsymbol{w'c} - 1)$$

Proceeding further $\frac{\partial \mathrm{L}(\boldsymbol{w},\lambda)}{\partial \boldsymbol{w}} = \boldsymbol{2\Sigma w} + \lambda \boldsymbol{c} = 0$ and $\boldsymbol{w} = -\frac{1}{2}\lambda \boldsymbol{\Sigma^{-1}c}$. Multiply both sides by the transpose of the carry vector ($\boldsymbol{c'}$) we obtain $1 = -\frac{1}{2}\lambda \boldsymbol{c'\Sigma^{-1}c}$ and hence $\lambda = -\frac{2}{\boldsymbol{c'\Sigma^{-1}c}}$. It follows that $\boldsymbol{w} = \frac{\boldsymbol{\Sigma^{-1}c}}{\boldsymbol{c'\Sigma^{-1}c}}$. Hence, $\boldsymbol{c'\Sigma^{-1}c}$ is a normalization constant and $\boldsymbol{w} \sim \boldsymbol{\Sigma^{-1}c}$. Furthermore, note that the optimal solution implies that the portfolio variance is $PortVar = \boldsymbol{w'\Sigma w} = \frac{1}{\boldsymbol{c'\Sigma^{-1}c}}$ and the portfolio Sharpe ratio is $SR = \sqrt{\boldsymbol{c'\Sigma^{-1}c}}$.

Uncorrelated assets

Let's assume that assets are uncorrelated and $\boldsymbol{\Sigma} = diag([\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2])$. It follows $\frac{w_i}{w_j} = \frac{c_i/\sigma_i^2}{c_j/\sigma_j^2}$.

Note that the allocation among assets depends on the individual carry-to-variance ratios. We can make an analogy with many other systematic strategies where the position is equal to the ratio between the signal and the volatility. In this case the signal is the carry-to-vol ratio $signal_i = (c_i/\sigma_i\ )$ and $position_i = (signal_i/\sigma_i\ )$.

Furthermore, if $\boldsymbol{\Sigma}$ is diagonal it follows that $SR = \sqrt{\sum_{i=1}^{n} c_i^2/\sigma_i^2}$.

Carry strategy Sharpe ratio optimization with a single market factor neutralisation

Similarly to the previous section, let's assume that there are n assets with carry returns $\boldsymbol{c} = [c_1, c_2, \dots, c_n]$ and covariance matrix $\boldsymbol{\Sigma}$. Furthermore, let's gather the assets betas with respect to the market factor into the vector $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_n]$. We aim to maximize the following function:

$$\max \mathrm{f}(\boldsymbol{w}) = \frac{\boldsymbol{w'c}}{\sqrt{\boldsymbol{w'\Sigma w}}},$$

where $\boldsymbol{w}$ are the allocations to the individual assets. We have already shown that $\mathrm{f}(\boldsymbol{w})$ is homogenous function of degree 0 - if $\boldsymbol{w}$ is a solution then $k\boldsymbol{w}$ where $k > 0$ is also a solution. Hence if $\boldsymbol{w}^*$ is a solution, there exists k for which holds $k(\boldsymbol{w}^*)'\boldsymbol{c} = 1$ and $k\boldsymbol{w}^*$ is a solution to the optimization problem as well. It follows that we can impose that $\boldsymbol{w'c} = 1$ and solve a variance minimization problem subject to a constraint:

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$$\min \mathrm{g}(\boldsymbol{w}) = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \quad \text{s.t.}\, \boldsymbol{w}'\boldsymbol{c} = 1 \text{ and } \boldsymbol{w}'\boldsymbol{\beta} = 0.$$

Note that when the carry is proportional to the market exposure, i.e. $\boldsymbol{\beta} = k\boldsymbol{c}$ there is no solution.

Making use of the method of Lagrange multipliers we arrive at:

$$\min \mathrm{L}(\boldsymbol{w}, \lambda) = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} + \lambda_1(\boldsymbol{w}'\boldsymbol{c} - 1) + \lambda_2(\boldsymbol{w}'\boldsymbol{\beta})$$

Proceeding further $\frac{\partial \mathrm{L}(\boldsymbol{w},\lambda)}{\partial \boldsymbol{w}} = 2\boldsymbol{\Sigma}\boldsymbol{w} + \lambda_1\boldsymbol{c} + \lambda_2\boldsymbol{\beta} = 0$ and $\boldsymbol{w} = -\frac{1}{2}\boldsymbol{\Sigma}^{-1}(\lambda_1\boldsymbol{c} + \lambda_2\boldsymbol{\beta})$. Multiply both sides by the transpose of the carry vector ($\boldsymbol{c}'$) we obtain $1 = -\frac{1}{2}\boldsymbol{c}'\boldsymbol{\Sigma}^{-1}(\lambda_1\boldsymbol{c} + \lambda_2\boldsymbol{\beta})$. Multiply both sides by the transpose of the exposures vector ($\boldsymbol{\beta}'$) we obtain $0 = -\frac{1}{2}\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}(\lambda_1\boldsymbol{c} + \lambda_2\boldsymbol{\beta})$. Expanding further those two equations we arrive at:

$$\boldsymbol{w} = \frac{1}{\boldsymbol{c}'\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{c} - \frac{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{c}}{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}}\boldsymbol{\beta}\right)}\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{c} - \frac{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{c}}{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}}\boldsymbol{\beta}\right) \sim \boldsymbol{\Sigma}^{-1}\left(\boldsymbol{c} - \frac{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{c}}{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}}\boldsymbol{\beta}\right)$$

Note that once we have a neutralization of a market factor, the weights are not proportional just to carry but to the difference between the carry and beta vector modified by an adjustment ($\frac{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{c}}{\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}}$). The adjustment can be thought of as the ratio between the portfolio carry and the portfolio beta of a portfolio with weights $\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}$.

Let's describe the returns generating process for an asset i as $R_{it} = a_i + \beta_i R_{mt} + \varepsilon_{it}$, where R_{mt} is the market factor return at time, $\sigma_m^2 = Var(R_{mt})$, $\varepsilon_{it} \sim NID(0, \sigma_{\varepsilon i}^2)$, $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots., \varepsilon_{nt}]$ and $Cov(\boldsymbol{\varepsilon}_t) = \boldsymbol{\Omega} = diag([\sigma_{\varepsilon 1}^2, \sigma_{\varepsilon 2}^2, \ldots., \sigma_{\varepsilon n}^2])$. It follows that:

$$\boldsymbol{\Sigma} = \boldsymbol{Cov}(\boldsymbol{R}_t) = \boldsymbol{\beta}\boldsymbol{\beta}'\sigma_m^2 + \boldsymbol{\Omega}$$

We can proceed further and write $\frac{1}{\sigma_m^2}\boldsymbol{\Sigma} = \boldsymbol{\beta}\boldsymbol{\beta}' + \boldsymbol{D}$, where $\boldsymbol{D} = diag([\sigma_{\varepsilon 1}^2/\sigma_m^2, \sigma_{\varepsilon 2}^2/\sigma_m^2, \ldots., \sigma_{\varepsilon n}^2/\sigma_m^2])$.

Making use of the Sherman–Morrison formula we can express:

$\boldsymbol{\Sigma}^{-1} = (1/\sigma_m^2)(\boldsymbol{\beta}\boldsymbol{\beta}' + \boldsymbol{D})^{-1} = (1/\sigma_m^2)\left(\boldsymbol{D}^{-1} - \frac{\boldsymbol{D}^{-1}\boldsymbol{\beta}\boldsymbol{\beta}'\boldsymbol{D}^{-1}}{1+\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{\beta}}\right)$. Subsequently we can show that $\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta} = (1/\sigma_m^2)\left(\frac{\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{\beta}}{1+\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{\beta}}\right)$ and $\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{c} = (1/\sigma_m^2)\left(\frac{\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{c}}{1+\boldsymbol{\beta}'\boldsymbol{D}^{-1}\boldsymbol{\beta}}\right)$. It follows that:

$$\boldsymbol{w} \sim \boldsymbol{\Sigma}^{-1}\left(\boldsymbol{c} - \frac{\sum_{i=1}^n c_i\beta_i \ /\sigma_{\varepsilon i}^2}{\sum_{i=1}^n \beta_i^2/\sigma_{\varepsilon i}^2}\boldsymbol{\beta}\right)$$

Let's denote the correction term in front of the $\boldsymbol{\beta}$ vector as $\omega = \frac{\sum_{i=1}^n c_i\beta_i \ /\sigma_{\varepsilon i}^2}{\sum_{i=1}^n \beta_i^2/\sigma_{\varepsilon i}^2}$. The correction term can be viewed as the ratio between the carry and the beta of a portfolio with individual weights equal to $\beta_i \ /\sigma_{\varepsilon i}^2$.

Note that the volatility of the market factor σ_m is not relevant for the optimal solution (it does matter though for the calculation of the individual betas). As discussed before when the carry is proportional to the market exposure, i.e. $\boldsymbol{c} = k\boldsymbol{\beta}$ there is no solution. Carry should contain either compensation for idiosyncratic risk or additional risk premium.

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Carry strategy Sharpe ratio optimization with PCA-based factors neutralization

The time-series of the principal components are given by the matrix $\boldsymbol{F} = \boldsymbol{XQ}$, where $\boldsymbol{X}$ contains the normalized returns (derived from the returns vector $\boldsymbol{R}$) and $\boldsymbol{Q}$ contains the eigenvectors. $\boldsymbol{F}$ and $\boldsymbol{X}$ are both Txn matrices where T is the number of observations.

As $\boldsymbol{Q}' = \boldsymbol{Q}^{-1}$ it follows that $\boldsymbol{X} = \boldsymbol{FQ}'$. $Cov(\boldsymbol{X}) = Corr(\boldsymbol{R}) = \boldsymbol{Q} \quad \boldsymbol{\Lambda Q}'$ where $\boldsymbol{\Lambda}$ is a diagonal matrix containing the eigenvalues $\lambda_1, \lambda_2, \dots \dots, \lambda_n$ of $Corr(\boldsymbol{R})$.

Let's assume that we impose that the portfolio is neutral to PCA factors *1….s*. If we denote the volatility of asset *i* as σ_i and construct the diagonal matrix $\boldsymbol{D} = diag[\sigma_1, \sigma_2, \dots \dots, \sigma_n]$, it follows $\boldsymbol{\Sigma} = Cov(\boldsymbol{R}) = \boldsymbol{D}Corr(\boldsymbol{R})\boldsymbol{D}$ and $PortVar = \boldsymbol{w}'\boldsymbol{\Sigma w} = \boldsymbol{w}'\boldsymbol{D}Corr(\boldsymbol{R})\boldsymbol{Dw} = \boldsymbol{w}'\boldsymbol{DQ} \quad \boldsymbol{\Lambda Q}'\boldsymbol{Dw}.$

Let $\boldsymbol{Q} = [\boldsymbol{Q_1}, \boldsymbol{Q_2}]$ where $\boldsymbol{Q_1}$ contains the first *s* PCA vectors to which we want to be immune and hence we want to impose that $\boldsymbol{w}'\boldsymbol{DQ_1} = 0$. It follows that

$$PortVar = \boldsymbol{w}'\boldsymbol{\Sigma w} = \boldsymbol{w}'\boldsymbol{D}[\boldsymbol{Q_1}, \boldsymbol{Q_2}] \quad \boldsymbol{\Lambda}[\boldsymbol{Q_1}, \boldsymbol{Q_2}]'\boldsymbol{Dw} = [\boldsymbol{0}, \boldsymbol{w}'\boldsymbol{DQ_2}]\boldsymbol{\Lambda}\begin{bmatrix}\boldsymbol{Q}_1' \\ \boldsymbol{Q}_2'\end{bmatrix}\boldsymbol{Dw}$$

$$=[\boldsymbol{0}, \boldsymbol{w}'\boldsymbol{DQ_2}]\begin{bmatrix}\boldsymbol{\Lambda_1} & \\ & \boldsymbol{\Lambda_2}\end{bmatrix}\begin{bmatrix}\boldsymbol{0}' \\ \boldsymbol{Q}_2'\boldsymbol{Dw}\end{bmatrix} = [\boldsymbol{0}, \boldsymbol{w}'\boldsymbol{DQ_2}\,\boldsymbol{\Lambda_2}]\begin{bmatrix}\boldsymbol{0}' \\ \boldsymbol{Q}_2'\boldsymbol{Dw}\end{bmatrix} = \boldsymbol{w}'\boldsymbol{DQ_2}\,\boldsymbol{\Lambda_2}\,\boldsymbol{Q}_2'\boldsymbol{Dw}$$

Naturally the variance of the portfolio is driven only by the exposure of the assets to the remaining PCA and their volatilities.

It is evident that any of the vectors in $\boldsymbol{Q_2}$ is a solution (as the vectors in $\boldsymbol{Q_2}$ are orthogonal to $\boldsymbol{Q_1}$).

Hence, if $k_{s+1}, k_{s+2} \dots \dots, k_n$ are parameters to be calculated, from $\boldsymbol{w}'\boldsymbol{DQ_1} = 0$ we can write that for the portfolio weights it should hold:

$$\boldsymbol{D} \quad \boldsymbol{w} = k_{s+1}\boldsymbol{q}_{s+1} + k_{s+2}\boldsymbol{q}_{s+2} + \cdots \dots k_n\,\boldsymbol{q}_n$$

$$\boldsymbol{w} = \boldsymbol{D}^{-1}(k_{s+1}\boldsymbol{q}_{s+1} + k_{s+2}\boldsymbol{q}_{s+2} + \cdots \dots k_n\,\boldsymbol{q}_n)$$

Subsequently, algebraic simplifications can show $PortVar = \sum_{i=s+1}^{n} k_i^2\,\lambda_i$.

Note that we can solve the Sharpe optimization problem again via the Lagrange method. We will have to minimize PortVar subject to the constraint $\mathrm{w}'\mathrm{c} = 1$ and find the parameters $k_{s+1}, k_{s+2} \dots \dots, k_n$.

A more elegant way is to consider the non-hedged principal components (eigenvectors) as portfolios. Those portfolios will be uncorrelated. In the first section we have shown that the Sharpe ratio optimization for uncorrelated assets implies that $\frac{k_i}{k_j} = \frac{CI_i/\sigma_{qi}^2}{CI_j/\sigma_{qj}^2}$ where CI_i denotes the carry income of the portfolio constructed from the eigenvector q_i and σ_{qi}^2 its variance.

Note that CI_i=$\boldsymbol{c}'\boldsymbol{D}^{-1}q_i$ and σ_{qi}^2= λ_i . It follows that $\frac{k_i}{k_j} = \frac{\boldsymbol{c}'\boldsymbol{D}^{-1}q_i\,/\lambda_i}{\boldsymbol{c}'\boldsymbol{D}^{-1}q_j\,/\lambda_j}$.

Subsequently, fixing one of the k_s=1, we can calculate the remaining parameters as:

$$\boldsymbol{v}_s = \boldsymbol{D}^{-1}\boldsymbol{q}_s = [\frac{q_{s,1}}{\sigma_1}, \frac{q_{s,2}}{\sigma_2}, \dots\dots, \frac{q_{s,n}}{\sigma_n}]'$$

$$k_s = (\boldsymbol{c}'\boldsymbol{D}^{-1}\boldsymbol{q}_s)/\lambda_s = (\boldsymbol{c}'\boldsymbol{v}_s)/\lambda_s$$

$$\boldsymbol{w} = ((\boldsymbol{c}'\boldsymbol{v}_{s+1})\boldsymbol{v}_{s+1}/\lambda_{s+1} + (\boldsymbol{c}'\boldsymbol{v}_{s+2})\boldsymbol{v}_{s+2}/\lambda_{s+2} + \cdots \dots (\boldsymbol{c}'\boldsymbol{v}_n)\boldsymbol{v}_n/\lambda_n)$$

Numerical optimization in the comprehensive approach

In the comprehensive approach we apply a numerical optimization that maximizes the carry income subject to the factors neutrality constraints and additional risk contribution controls by individual assets and asset groups. We also take into account the transaction costs.

As in our previous research piece on trend-following strategies, there are two types of costs: running costs and execution costs. The running costs (RC) are linked to the size of the position and the execution costs (EC) are related to the change in the position. Let's denote the costs as $\boldsymbol{RC} = [RC_1, \quad RC_2, \dots, RC_n]'$ and $\boldsymbol{EC} = [EC_1, \quad EC_2, \dots, EC_n]'$.

Let's assume that we hold the positions for *hp* days and assume that the factor model structure is $\boldsymbol{RS}_t = \boldsymbol{a} + \boldsymbol{BF}_t + \boldsymbol{\epsilon}_t$ with $\boldsymbol{\epsilon}_t \sim NID(0, \sigma_{i,t}^2)$. We will use $\sigma_{i,t}^2$ to denote the estimate of σ_i^2 at time *t*. Furthermore, we might want to constrain the risk allocated to various groups of assets (we can think of equities in different geographic regions, commodities in different groups etc). Let's assume that the sets $Set_1, \ Set_2, \dots\dots, Set_q$, contain the indices of group 1, group 2,.... , group q of assets and the associated risk contribution budgets are $RCLimit_1, RCLimit_2 \dots. RCLimit_q$. In our case $RCLimit_1 = RCLimit_2 = \dots\dots = RCLimit_q$.

If we denote the portfolio volatility target by VolTarget and if $\boldsymbol{Carry}_t$ contains carry premia for the individual assets at time *t* , then our optimization problem can be specified as :

$$\max(hp * (\boldsymbol{w}_t'\boldsymbol{Carry}_t - \boldsymbol{abs}(\boldsymbol{w}_t')\boldsymbol{RC}) \quad - \left(\boldsymbol{abs}\left(\boldsymbol{w}_t' - \boldsymbol{w}_{t-hp}'\right)\right)\boldsymbol{EC} \quad)$$

s.t.

$$\boldsymbol{w}_t'\boldsymbol{a} = 0$$

$$\boldsymbol{w}_t'\boldsymbol{B} = \boldsymbol{0}$$

$$\sqrt{\boldsymbol{w}_t'\boldsymbol{\Sigma}\boldsymbol{w}_t} \le V\text{olTarget}$$

$$\sum_{i=1}^{n} abs(w_{i,t}) \quad < Max\text{Leverage}$$

$$\sum_{i=1}^{i\epsilon Set_1} w_{i,t}^2\sigma_{i,t}^2 \quad < RCLimit_1 * (V\text{olTarget})^2$$

$$\sum_{i=1}^{i\epsilon Set_2} w_{i,t}^2\sigma_{i,t}^2 \quad < RCLimit_2 * (V\text{olTarget})^2$$

.

.

$$\sum_{i=1}^{i\epsilon Set_q} w_{i,t}^2 \sigma_{i,t}^2 \quad < RCLimit_q * (V\text{olTarget})^2$$

$$abs(w_{i,t}) < MaxPositionSize_i$$

$MaxPositionSize_i$ is linked to the max risk budget of the group and the number of assets within the group. If for example $i\epsilon Set_1$, then $MaxPositionSize_i = min\left(\frac{Max\text{Leverage}}{\text{n}}, k\sqrt{(\frac{RCLimit_1}{N_{Set_1}}) * (V\text{olTarget})^2}/vol_{i,t}\right)$. In our case we have chosen $k = 2$.

We introduce additional variables to handle the transaction costs constraints. Let $\boldsymbol{z}_t$ =abs($\boldsymbol{w}_t$) and $\boldsymbol{q}_t$ =abs($\boldsymbol{w}_t' - \boldsymbol{w}_{t-hp}'$). Hence $\boldsymbol{z}_t \geq abs(\boldsymbol{w}_t)$ and $\boldsymbol{q}_t \geq abs(\boldsymbol{w}_t' - \boldsymbol{w}_{t-hp}')$

$$\max(hp * (\boldsymbol{w}_t'\boldsymbol{Carry}_{t-1} - \boldsymbol{z}_t'\boldsymbol{RC}_t) - \boldsymbol{q}_t'\boldsymbol{EC}_t)$$

At maximum $\boldsymbol{z}_t = abs(\boldsymbol{w}_t)$ and $\boldsymbol{q}_t = abs(\boldsymbol{w}_t' - \boldsymbol{w}_{t-hp}')$.

Principal Component Analysis (PCA)

Principal Component Analysis (PCA) is a statistical procedure for dimensionality reduction and decomposes each variable into a linear combination of orthogonal factors called principal components. The principal components are ordered such that the first component has the highest variance and the second principal component by design has zero correlation to the first principal component and has the second highest variance, and so on.

Eigenvectors and eigenvalues are vectors and scalars associated to square matrices. Together they provide the eigen-decomposition of a matrix. An eigenvector of the matrix A is a vector u that satisfies the following equation:

$$\mathbf{Aq} = \lambda\mathbf{q} \; ,$$

where λ is a scalar called the eigenvalue associated to the eigenvector $\boldsymbol{q}$.

Let's assume that we have N time series each with T observations of (typically normalized) returns $\boldsymbol{R} = [r_1, \dots, r_N]$. Our aim is to find a weighting matrix $\boldsymbol{W} = [\boldsymbol{w}_1, \dots, \boldsymbol{w}_N] = (w_{ij})_{N\times N}$ so that we can express the factors as:

$$\boldsymbol{F} = [\boldsymbol{f}_1, \dots, \boldsymbol{f}_N] = \boldsymbol{R} \times \boldsymbol{W} = [\boldsymbol{Rw}_1, \dots, \boldsymbol{Rw}_N]$$

are uncorrelated sources of systematic risks ($Cov(\boldsymbol{F})$ is diagonal). Suppose the weighting matrix $\boldsymbol{W}$ is invertible and $\boldsymbol{V} = \boldsymbol{W}^{-1}$, then the returns could be represented as a linear combination of the factors:

$$\boldsymbol{R} = \boldsymbol{F} \times \boldsymbol{V} = [\boldsymbol{Fv}_1, \dots, \boldsymbol{Fv}_N]$$

In addition we would like the first factor to extract a as much of the variance of the original data as possible, and that the second factor extracts as much of the variance left unexplained by the first factor, and so on for the remaining factors.

It can be shown (for example by the method of Lagrangian multipliers as in Harris (2001)) that the optimal solution to this problem is equating the matrix $\boldsymbol{W}$ to the eigenvectors of the covariance matrix of $\boldsymbol{R}$. The principal components of the returns are then created by eigenvector rotations, the "rotated" factors are uncorrelated and the variance of principal components corresponds to the eigenvalues of the factor covariance matrix $\sum = Cov(\boldsymbol{R}) = (\sigma_{ij})_{N\times N}$.

More concretely, suppose the eigen-decomposition of $\sum$ is given by

$$\sum \ = \ \boldsymbol{Q\Lambda Q^T}$$

where $\boldsymbol{Q} = [\boldsymbol{q}_1, \dots, \boldsymbol{q}_N]$ are the eigen-vectors so that $\boldsymbol{QQ^T} = \boldsymbol{I}$ (identity matrix) and $\boldsymbol{\Lambda} = diag(\lambda_{1,\dots,}\lambda_N)$ is a diagonal matrix of eigenvalues (usually ranked by descending order). Since the covariance matrix $\sum$ is positive semidefinite, the diagonal elements of $\boldsymbol{\Lambda}$ are non-negative.

By construction, the covariance matrix of the "principal components" $\boldsymbol{F} = \boldsymbol{R} \times \boldsymbol{Q} = (\boldsymbol{Rq}_1, \dots, \boldsymbol{Rq}_N)$ is given by $Cov(\boldsymbol{F}) = \ \boldsymbol{Q^T\Lambda Q} = \boldsymbol{\Lambda}$. As a result, the principal components are linear combinations of original data, are uncorrelated and their variances are given by the diagonal elements of matrix $\boldsymbol{\Lambda}$ **-**the eigenvalues.

We have provided further mathematical details with respect to the PCA analysis in our primer on systematic cross asset strategies. We have also applied PCA a set of cross-asset risk premia to isolate the main principal components. In our primer on Big Data and AI Strategies, we have demonstrated the use of PCA in conjunction with supervised learning technique to make return forecast and have also illustrated the use of PCA to explore the USDJPY implied volatility surface.

Selecting the number of principal components

Perhaps the most important decision in PCA is the selection the appropriate number of principal components to be retained. There are various methods for deciding how many principal components should be retained so that the real factor structure is represented. In our empirical work we have made use of Kaiser-Guttman rule and we have applied it uniformly across all asset classes and estimation periods.

Kaiser-Guttman criterion

The Kaiser-Guttman rule (also shortly referred to as just the Kaiser rule) suggests retaining all components with eigenvalues greater than 1.0. The justification for such a criterion is that each retained factor should account for at least as much variability as explained by a single variable.

The main critique with respect to the Kaiser rule is that the optimality of the eigenvalue criterion applies to the population correlation matrix. In practice, the population correlation matrix is not available and the model will not hold exactly. When used in conjunction with the sample correlation matrix, the Kaiser rule will tend to overestimate the number of factors. We are not particularly concerned by the overestimation of the number of factors as we consider over-hedging a smaller problem than under-hedging.

Empirical Kaiser Criterion

Empirical Kaiser Criterion (Braeken, 2017) makes use of the known sampling behavior of eigenvalues under the assumption that there is no factor structure (all the markets are uncorrelated in the population and all eigenvalues of the correlation matrix are hence 1). The assumption of no factor structure in the population model is also referred to as the null model.

Under the null model eigenvalues at the sample level show random variation, with typically about the first half of the eigenvalues above and the latter half below 1 under the null model. The distribution of the sample eigenvalues under null model follows asymptotically Marčenko-Pastur distribution.

The Empirical Kaiser Criterion (Braeken, 2017) starts by setting the first reference eigenvalue to the asymptotic maximum sample eigenvalue under the null model. This first reference value $l_1^{EKC} \ = \ l_{up} = \ (1 + \sqrt{\gamma})^2$ is a function of the variables-to-sample-size ratio γ in the dataset (as given in the Marčenko-Pastur distribution). Subsequent eigenvalues takes into account the serial nature of eigenvalues by means of a proportional empirical correction of the first reference value. Finally, the observed eigenvalue should exceed 1.

Empirical Kaiser Criterion is a powerful and promising factor retention method, because it is based on distribution theory of eigenvalues, shows good performance, is easily visualized and computed

Cross Validation:

The number of principal components to be used is decided by the importance of each component. In K Fold cross validation, the data is divided into k subsets, such that one of the k subsets is used as the validation test and the remaining k-1 subsets are used to build out a new model and this procedure is repeated for each subsets, with an estimation of the Predicted Residual Sum of Squares (PRESS) . The number of components with the lowest Predicted Residual Sum of Squares (PRESS) should be chosen for the PCA model. Cross Validation methods are not based on the eigenvalues of the sample covariance matrix but on the predictive ability of different PC models. Main drawback of cross validation technique is its computational cost.

Wold's (1978) cross-validation scheme provides both a way to calculate PRESS by a specific leave-out pattern and a criterion for the selection of the number of components. The number of PCs, is successively taken as 1, 2, . . . , and so on, until overall prediction is no longer significantly improved by the addition of extra terms (PCs). The number of PCs to be retained, m, is then taken to be the minimum number necessary for adequate prediction.

Cross-validation of Principal Components is computationally expensive for large data sets. Mertens et al. (1995) describe efficient algorithms for cross-validation, with applications to principal component regression.

Seasonality adjustment with Prophet forecasting model

Prophet is an open source library designed at Facebook. It is based on an additive regression model with three main components: trend, seasonality and holidays. They are combined in the following equation:

$$y(t) = g(t) + s(t) + h(t) + \epsilon_t$$

$g(t)$: **trend** function, models non-periodic changes

$s(t)$: **seasonality** function, models periodic changes (weekly, yearly)

$h(t)$: **holiday** function, models effects of user-provided holidays

ϵ_t: **error** term that accounts for any unusual changes not accommodated by the model

The specification is similar to a **G**eneralized **A**dditive **M**odel (**GAM**) (Hastie & Tibshirani), a class of regression models with non-linear smoothers applied to regressors. Using time as regressor, Prophet fits several linear and non-linear functions of time as components. More details can be found in Taylor, S. and Lehmam, B. (2017), "Forecasting at Scale".

Trend is modelled by fitting a piece wise linear curve over the non-periodic part of the time series. The trend model can be represented as:

$$g(t) = kt + m,$$

where k the growth rate and m an offset parameter.

The model also allows for a varying rate of growth. The trend changes in the growth are incorporated by explicitly defining change points where the growth rate is allowed to change. Suppose there are S change points at times s_j, $j = 1, \ldots, S$, then the vector of rate adjustments is defined as $\delta \in \mathbb{R}^S$, where δ_j is the change in rate that occurs at time s_j. The rate at any time t is the base rate k, plus all of the adjustments up to that point: $k + \sum_{j:t>s_j} \delta_j$. This is represented more cleanly by defining a vector $a(t) \epsilon \{0,1\}^S$ such that

$$a_j(t) = \begin{cases} 1, & \text{if } t \geq s_j \\ 0, & \text{otherwise} \end{cases}$$

The rate at time t is then $k + a(t)^T\delta$. The adjustment for the offset parameter m at change point j is computed as

$$\gamma_j = \left(s_j - m - \sum_{l<j} \gamma_l\right)\left(1 - \frac{k + \sum_{l<j} \delta_l}{k + \sum_{l\leq j} \delta_l}\right)$$

The piecewise linear growth model can then be represented as:

$$g(t) = (k + a(t)^T\delta)t + (m + a(t)^T\gamma)$$

Seasonality is often present in asset prices series. Our goal is to isolate and remove these effects from our time series. Prophet fits these effects by specifying seasonality models that are periodic functions of t and relies on Fourier series to provide a flexible model. Seasonal effects $s(t)$ are approximated by the following function:

$$s(t) = \sum_{n=1}^{N}\left(a_n \cos\left(\frac{2\pi nt}{P}\right) + b_n \sin\left(\frac{2\pi nt}{P}\right)\right)$$

where P is the regular period (e.g. $P = 365.25$ for yearly data or $P = 7$ for weekly data). Modeling seasonality requires estimating parameters $[a_1, b_1, \ldots, a_N, b_N]$ for a given N. The Fourier order N defines whether high frequency changes are allowed to be modelled, i.e. increasing N allows for fitting seasonality patterns that change more quickly, albeit with the increased risk of overfitting.

There can also be holidays and events which provide somewhat predictable shocks to asset prices and often do not follow a periodic pattern. Prophet allows providing a custom list of past and future events and incorporating this list into the model is straightforward by assuming that the effects of holidays are independent. For each holiday i, let D_i be the set of past and future dates for that holiday. An indicator function representing whether time t is during holiday i, and assign each holiday a parameter κ_i which is the corresponding change in forecast. Thus,

$$h(t) = Z(t)\kappa$$

where $Z(t) = [1(t \epsilon D_1), \ldots, 1(t \epsilon D_L)]$ is a matrix of regressors.

Cubic Spline Interpolation

A cubic spline is a piecewise cubic polynomial function which is twice continuously differentiable at each point known as knot points or nodes.

Given $(x_i, y_i)_{i=0}^n$, a function $S(x)$ is a cubic spline interpolant on $[a, b]$ if

$$S \epsilon C^{k-1}[a, b]$$

$a = t_0 < t_1 < \ldots\ldots < t_n = b$ and

$$S(x) = \begin{cases} S_0(x) = a_0(x - x_0)^3 + b_0(x - x_0)^2 + c_0(x - x_0) + d_0, & t_0 \leq x \leq t_1 \\ \vdots \\ S_{n-1}(x) = a_{n-1}(x - x_{n-1})^3 + b_{n-1}(x - x_{n-1})^2 + c_{n-1}(x - x_{n-1}) + d_{n-1}, & t_{n-1} \leq x \leq t_n \end{cases}$$

which satisfies

$$S(x) \epsilon\, C^2[t_0, t_1] : \left.\begin{matrix} S_{i-1}(x_i) = S_i(x_i) \\ S'_{i-1}(x_i) = S'_i(x_i) \\ S''_{i-1}(x_i) = S''_i(x_i) \end{matrix}\right\} , i = 1,2\ldots\ldots, n-1 \tag{1}$$

$$S(x_i) = y_i, i = 0,1\ldots\ldots\ldots, n \tag{2}$$

$$S''(t_0) = 0 \tag{3}$$

$$S''(t_n) = 0 \tag{4}$$

The 1st set of $3(n-1)$ constraints, (1) requires that the spline function joins the knot points perfectly and that the first and second derivative constraints match the adjacent splines. The requirement that it is a cubic spline interpolant and passes through the observed data points gives us the 2nd set of $n+1$ constraints, (2). Finally, the last two constraints, (3) and (4) are end point or boundary constraints25 that set the derivatives equal to zero at both ends. Thus, the algebraic system consists of $4n$ equations and $4n$, $(a_i, b_i, c_i, d_i)_{i=0}^{n-1}$ unknowns.

Solving for Cubic spline

Let $h_j = x_{j+1} - x_j$ then

$$S_{i+1}(x_{i+1}) = d_{i+1} = S_i(x_{i+1}) = a_i(x_{i+1} - x_i)^3 + b_i(x_{i+1} - x_i)^2 + c_i(x_{i+1} - x_i) + d_i \tag{5}$$

$$\Rightarrow d_{i+1} = a_i h_i^3 + b_i h_i^2 + c_i h_i + d_i\ , i = 0,1\ldots\ldots, n \tag{6}$$

Similarly, from continuity of first derivative and second derivative at nodes, we have, for $i = 0,1,\ldots\ldots, n$

$$c_{i+1} = 3a_i h_i^2 + 2b_i h_i + c_i \tag{7}$$

[25] The boundary conditions used here define a natural cubic spline. Other boundary conditions can also be used to define different subtypes of cubic spline.

$$b_{i+1} = 6a_i h_i + 2b_i \tag{8}$$

Solving for (6), (7) and (8) together with (3) and (4), leads to a simpler tridiagonal system which can be further solved to give the coefficients of the polynomials.

Mahalanobis distance

The Mahalanobis distance is a measure of the distance between a (multidimensional) point P and a distribution D.

Given a distribution with mean vector $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3, \ldots, \mu_N)'$ and covariance matrix Σ, the squared Mahalanobis distance of an observation $\mathbf{R} = (R_1, R_2, R_3, \ldots, R_N)^{T}$ is defined as:

$$D_M^2(\mathbf{R}) = (\boldsymbol{R} - \boldsymbol{\mu})' \textstyle\sum^{-1} (\boldsymbol{R} - \boldsymbol{\mu})$$

$D_M^2(\mathbf{R})$ has a chi-square distribution with degree of freedom equal to the number of variables, $D_M^2(\mathbf{R}) \sim \chi_N^2$.

The chi-squared distribution with N degrees of freedom is the distribution of a sum of the squares of *N* independent standard normal random variables.

Sum of independent chi-squared variables is also chi-squared distributed. If $\{Y_i\}_{i=1}^k$ are independent chi-squared variables with $\{n_i\}_{i=1}^k$ degrees of freedom, respectively then $X = Y_1 + \cdots + Y_k$ is chi-squared distributed with $n_1 + \cdots + n_k$ degrees of freedom.

When using the factor model based on PCA for hedging it is very important to check whether the latest observations are coming from the same distribution as the observations used for the PCA. We are using an observation of the asset returns (net of carry) across all the markets and estimated covariance matrix of these asset returns to calculate Mahalanobis distance.

For a single observation of returns across all the assets at time *t* we can calculate the Mahalanobis distance, set a confidence level and check if a single observation is an outlier using chi-square test. But the nature of financial markets implies that will often encounter exceedances and we will have to close/open positions rather frequently.

To mitigate that problem we can calculate Mahalanobis distance for the each for the last *s* observations and then add up those *s* Mahalanobis distances. Abstracting from any potential time-series dependence in the returns and by the additivity property of chi-square variables, sum of the *s* Mahalanobis distances will also follow a chi-squared distribution with degree of freedom equal to *s*N*. We can test this group statistic using chi-square test and decide whether the recent s observations are jointly an outlier.

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