Salomon Brothers

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Understanding Treasury Bond Futures

Questions and Answers

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Introduction

Two of the most widely used futures contracts on U.S. Government Treasury instruments are the Treasury bond futures contract and the long-term note futures contract traded at the Chicago Board of Trade (CBOT). These contracts provide an efficient and liquid market for forward transactions in the long- and intermediate-maturity sectors of U.S. interest rates and are an indispensable tool for both hedgers and speculators. The high degree of correlation between the price movements of the Treasury futures market and the Treasury cash market makes the futures contracts useful substitutes for the cash market securities.

The U.S. T-bond futures contract represents $100,000 face amount of one of a set of deliverable cash Treasury bonds. For the bond to be acceptable for delivery, it must satisfy a requirement calling for a minimum amount of time from the delivery month to the maturity or first call date of the bond. If and when delivery takes place, the holder of the short futures position *(the short)* can choose *which* bond to deliver to the holder of the long futures position *(the long)*, and *when* during the delivery month to make delivery. The short effectively owns a delivery option that the long has sold.

The futures contracts can be used effectively without understanding all the details. Unfortunately, the contracts may seem complicated, and a lack of understanding can cause potential users to unnecessarily avoid using them. For example, although the delivery option or the uncertainty of which bond will be the most preferred for delivery can affect the price movements of the contract, only a very small percentage of futures positions go to delivery.

Through a question and answer format, this paper clarifies the workings of the futures contracts and discusses some of the questions that users may have. It does not explain all the contract details or uses,[1] and assumes some prior familiarity with the contracts. The paper focuses on some of the more interesting aspects of the contracts, and much of the discussion here will not be found in other places. The questions and answers will center around T-bond futures, and whenever there is a difference between the T-bond and T-note contracts, the appropriate comparison will be made.

The concepts relating to the futures contracts that will be discussed include: the conversion factor, the relationship to forward contracts, variation margin, the cash-and-carry trade, the implied repo rate, the cheapest-to-deliver bond, the richness or cheapness of the contract, the convergence of cash and futures, the basis, hedging with futures contracts, the embedded delivery option, and how to avoid the delivery process altogether. The summary section includes a brief description of the main points in this paper.

This paper should give readers a greater appreciation and understanding of how the contracts behave when interest rates change and as the delivery

[1] For more information on the contracts and their uses, see the Salomon Brothers publication, *Government Bond Futures — Tools for Global Risk Management and Portfolio Enhancement*, C. Parkhurst, March 1988.

month approaches. Consequently, they should be able to take greater advantage of the futures contracts' many uses and applications. A forthcoming paper, *The Salomon Brothers Delivery Option Model*, will address the delivery option in more detail.

Section I: Contract Fundamentals

1. What is the purpose of the conversion factor?

Because the T-bonds that are acceptable for delivery may have different coupons and maturities, the cash prices of the bonds can be significantly different. Therefore, the market futures price is adjusted depending on which bond is delivered. The *delivery price* received for a given bond is determined by multiplying the market futures price by a *conversion factor* (CF) for that bond. Essentially, bonds with higher cash prices have higher delivery prices. This procedure attempts to reduce the economic incentive of delivering one bond over another, thereby making all of the bonds approximately equally deliverable. This mechanism greatly increases the liquidity of the contract. The CFs for the December '90 through December '91 T-bond contracts appear in Figure 1.[2] **There is a separate conversion factor for each bond and delivery month combination.**

Figure 1. Conversion Factors

Deliverable Bonds	Dec 90	Mar 91	Jun 91	Sep 91	Dec 91
US 8.750% of 08/15/2020	1.0845	1.0841	1.0841	1.0837	1.0837
US 8.750% of 05/15/2020	1.0841	1.0841	1.0837	1.0837	1.0833
US 8.500% of 02/15/2020	1.0561	1.0557	1.0558	1.0555	1.0555
US 8.125% of 08/15/2019	1.0140	1.0137	1.0139	1.0137	1.0138
US 8.875% of 02/15/2019	1.0972	1.0968	1.0967	1.0963	1.0962
US 9.000% of 11/15/2018	1.1106	1.1105	1.1100	1.1100	1.1094
US 9.125% of 05/15/2018	1.1238	1.1237	1.1232	1.1230	1.1225
US 8.875% of 08/15/2017	1.0957	1.0952	1.0951	1.0946	1.0946
US 8.750% of 05/15/2017	1.0816	1.0816	1.0811	1.0811	1.0806
US 7.500% of 11/15/2016	0.9456	0.9460	0.9459	0.9463	0.9463
US 7.250% of 05/15/2016	0.9190	0.9194	0.9195	0.9200	0.9201
US 9.250% of 02/15/2016	1.1343	1.1336	1.1334	1.1327	1.1325
US 9.875% of 11/15/2015	1.2005	1.2001	1.1992	1.1987	1.1978
US 10.625% of 08/15/2015	1.2801	1.2789	1.2782	1.2769	1.2762
US 11.250% of 02/15/2015	1.3444	1.3429	1.3419	1.3404	1.3394
US 11.750% of 11/15/2014-09	1.3608	1.3589	1.3565	1.3545	1.3520
US 12.500% of 08/15/2014-09	1.4307	1.4278	1.4254	1.4224	1.4200
US 13.250% of 05/15/2014-09	1.4991	1.4963	1.4929	1.4899	1.4863
US 12.000% of 08/15/2013-08	1.3733	1.3705	1.3682	1.3653	1.3630
US 10.375% of 11/15/2012-07	1.2168	1.2155	1.2136	1.2122	1.2103
US 14.000% of 11/15/2011-06	1.5316	1.5277	1.5229	1.5188	N/A
US 13.875% of 05/15/2011-06	1.5120	1.5080	N/A	N/A	N/A
US 9.375% of 02/15/2006	1.1189	N/A	N/A	N/A	N/A

2. How is the conversion factor calculated?

The CF is calculated easily in three steps. First, a *rounded-term-to-maturity* is calculated by taking the time from the first day in the contract month to the bond maturity and rounding down to the nearest three-month quarter-year. For a callable bond, the first call date is used as the effective maturity for the calculation. Second, the flat price of the bond is calculated at an 8% yield (on a 30/360 basis), assuming the bond's maturity is the rounded-term-to-maturity. If the rounded-term-to-maturity is a whole multiple of six-month periods, the pricing will be on a coupon date. If the rounded-term-to-maturity has a 1/4 or 3/4 year fractional part, the first coupon will be three months after the pricing date. Third, the price is divided by 100 and rounded to four decimal places to produce the CF.

[2] A complete list of all the conversion factors is published by the Chicago Board of Trade.

For example, consider the U.S. 10.375% bond maturing November 15, 2012, and callable starting November 15, 2007. For delivery into the December '90 T-bond futures contract, the rounded-term-to-maturity is calculated from December 1, 1990 to November 15, 2007 and is 16 and 3/4 years. A price is calculated for a bond with a coupon of 10.375%, a maturity of 16 and 3/4 years and a yield of 8%. Dividing the resulting price of 121.683 by 100 and rounding to four places gives the CF for December '90 delivery as 1.2168 for this bond. For the March '91 contract, the rounded-term-to-maturity is calculated from March 1, 1991 to November 15, 2007 and equals 16 and 1/2 years. Here, the price at an 8% yield is 121.550. Dividing by 100 and rounding to four places gives the CF for March '91 delivery as 1.2155 for this bond.

3. Does the conversion factor change over time or when yields change?

There is one conversion factor for each bond for each delivery month. For each bond/month pair, the CF does not change as the delivery month nears or as yields change. However, each bond has a different CF for different contract months. Generally speaking, a bond with a coupon above 8% has a CF above 1.0 that gets nearer to 1.0 for more distant contract months. A bond with a coupon below 8% has a CF below 1.0 that gets nearer to 1.0 for more distant contract months.[3]

4. Why are there fewer bonds deliverable for more distant contract months?

For a Treasury bond to be eligible for delivery, the CBOT T-bond futures contracts require a minimum of 15 years between the first day in the delivery month and the date of maturity or first call.[4] For more distant contract months, a bond has less time until the maturity date or first call date. When the bond no longer meets the minimum maturity requirement for a given contract month, it is not eligible for delivery for that, or any later contract month.

For example, the U.S. 9.375% bond maturing February 15, 2006 is deliverable into the December '90 T-bond contract, but not the March '91 contract.[5] On the other hand, as new 30-year bonds are issued, they immediately become eligible for delivery. In the past, the T-bond contract has had over 30 bonds eligible for delivery. Currently there are only 23.[6]

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[3] A T-bond or a T-note with a coupon near 8% may have a CF that does not strictly increase or decrease for more distant contract months. This is because of the *scalloping effect.* If the rounded term-to-maturity has a 1/4 or 3/4 year fractional part, the flat price of an 8% coupon bond or note at an 8% yield will be less than par, because the accrued interest accrues linearly without the effect of compounding. In particular, the CF for a bond or note with a coupon of 8% is equal to 1.0, for only two of the four contract months in a year. For example, the U.S. 8% note maturing August 15, 1999 has a CF for delivery into the December '90 and June '91 T-note futures contracts of 1.0. But for the March '91 and September '91 contracts the CF is .9998.

[4] The requirement for the ten-year note contracts is that the original maturity be no more than ten years and the remaining maturity be at least six and a half years.

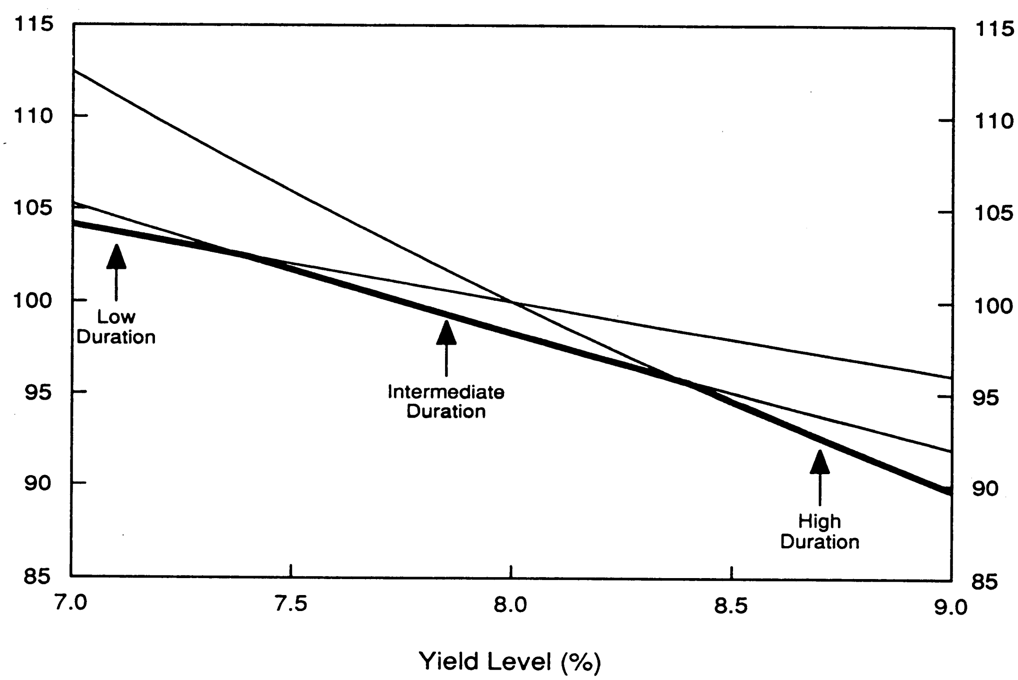
[5] After this 20-year original-issue bond is no longer eligible for delivery, only the 30-year noncallables and the 30-year callables will be left. The noncallables were not issued until February 1985 after the last callable issue of November 1984. Consequently, only one issue might become nondeliverable each time the near contract month changes. From March 1995 to December 1999, no currently existing issues will become nondeliverable. For T-notes, two issues may become nondeliverable each time the near contract month changes: an original ten-year T-note and an original seven-year T-note.

[6] The number of deliverable issues has declined mostly because the U.S. Treasury no longer issues a 20-year bond.

1. Can the price of a futures contract be determined by viewing it as a forward contract?

A simplified way to approximate the fair market price of a futures contract is to treat it as if it were a forward contract. First, the forward price of each deliverable bond is divided by the corresponding conversion factor to get the *converted forward price* of each bond.[7] **The lowest of the converted forward prices gives an approximate price for the futures contract.** This calculation is represented in Figure 2. The converted forward prices for three sample bonds are calculated at different yield levels. The lower envelope of the price-yield curves in the figure gives the approximation to the futures contract price as a function of yield level. (The vertical scale has been exaggerated to illustrate the effect.)

Figure 2. Simplified Example of Converted Forward Prices



Let us see why the *minimum converted forward price* is important.[8] If the market price of the contract were *higher* than the minimum converted forward price, then one could sell the high-priced futures contract and buy the bond with the minimum converted forward price, with the intention of later delivering with a guaranteed riskless profit. This arbitrage would force the price of the contract down until it was equal to the minimum converted forward price, at which point the arbitrage would disappear.[9]

[7] The fair forward price of a bond is the cash market price plus financing charges to some horizon minus the coupon income earned on the bond. The forward prices are divided by the CFs to place all the bonds on equal footing.

[8] The reader may think that the maximum price should be important, not the minimum, because a higher contract price means that more will be received at delivery. This is not true. The long will never pay more than the minimum for the contract because the short has the choice of which bond to deliver, and will deliver the least expensive bond, that is, the bond with the minimum converted price.

[9] Note that it is *possible* for the market price of the contract to be *lower* than that given by the minimum converted forward price. If the arbitrage described above is attempted in reverse, the holder of the long futures position would be exposed to the risk of receiving delivery of a bond other than the one sold at the outset. This delivery option that the holder of the long position is short depresses the value of the contract relative to what it would be if it were only on a single bond. The delivery option will be discussed in more detail in Sections VI and VII.

2. *What is variation margin?*

The price specified in a *forward contract* on an individual bond represents the actual cash price that will be paid upon delivery of that bond. However, the *initial* price of a *futures contract* does not indicate the actual cash that will be paid upon delivery. The cash that will be paid upon delivery into a futures contract depends on the *final* futures price as of the delivery date.[10] Even so, the *initial* price of the futures contract still represents the *effective* price.

The difference between the final futures price and the initial futures price is made by the *variation margin* process. Each day, variation margin cash flows may occur between the long and short holders of the futures contract through the exchange clearing house. If interest rates go down, the contract price will increase. The *higher* final contract price is offset by the variation margin payments that will have been paid by the holder of the short position to the holder of the long position over the time period that futures appreciated. If interest rates go up, the contract price will drop. The *lower* final contract price is offset by the variation margin payments that will have been paid by the long to the short over the time period that futures depreciated. Variation margins are calculated from the daily *settlement prices* set by the exchange.

3. *How does the variation margin affect the price of the futures contract?*

Variation margin reduces the futures price by a very small amount, that is, 1/32 or $.03125 per $100 face ($31.25 per contract) for the near contract.[11] The effect depends on the extent that short and long rates move together and on the remaining time until the delivery month.

If future interest rates were known in advance with certainty, then the price of a futures contract on a single bond would be the same as the forward price of that bond.[12] In order to understand what happens when interest rates fluctuate randomly, let us consider the variation margin flows that occur when interest rates change. If short and long yields *rise together*, the futures price will drop and the variation margin payment that the short receives from the long can be invested in the high rate environment. If short and long rates *drop together*, the futures price will rise and the variation margin payment that the long receives from the short can be invested, but in a low rate environment. A positive correlation between long and short rate movements gives a preferential advantage to the short, that is, the short invests margin flows at high rates and the long invests margin flows at low rates. To compensate for this asymmetry, the theoretical price of the futures contract is depressed relative to the price of a forward contract.

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[10] The futures price also must be adjusted by the conversion factor, of course.

[11] As a contract ages, the effect becomes progressively smaller. However, for more distant contracts the effect can be larger.

[12] See "The Relationship between Forward Prices and Futures Prices," J. Cox, J. Ingersoll, S. Ross, *Journal of Financial Economics*, Volume 9 (1981) 321-346.

Section III: The Relationship Between the Futures Contract and the Cheapest-to-Deliver Bond

1. What is the implied repo rate of a bond?

The *implied repo rate* of any of the deliverable bonds is the effective short-term investment rate that is earned by buying the bond and selling it forward by shorting futures against it. This short-term synthetic investment is referred to as a *cash-and-carry trade*. The cost of the synthetic short-term investment is the current market flat price of the bond plus the accrued interest. The effective amount received at the termination of the short-term synthetic investment is the initial futures price times the conversion factor plus the accrued interest as of the assumed termination date of the investment.[13] Any intermediate coupon income and its reinvestment must be included in the calculation. The *internal rate of return* of the cash flows of this transaction is the implied repo rate of the bond.

2. What is the cheapest-to-deliver bond?

Because the implied repo rate represents a short-term *investment rate* that the short will try to *maximize*, the bond that produces the *highest* rate is very important. This bond, which has the *highest implied repo rate*, is called the *cheapest-to-deliver bond (CTD)*; the implied repo rate for the CTD is then the implied repo rate of the futures contract. Note that the CTD will not, in general, have the lowest cash price, because the implied repo rate of a bond depends not only on the cash price, but also on the conversion factor and the coupon income for each bond.[14]

3. How does the cheapest-to-deliver bond change when interest rate levels change?

When interest rate levels change, the price of each deliverable bond changes. The *modified duration* is a percent measure of how much the value of a bond changes.[15]

The deliverable bonds can have durations that are very different from each other. As yields rise or fall, the price (and forward price) of bonds with high duration changes faster than the price of bonds with low duration. Even after the adjustment by the CF, the converted forward price of bonds with high duration changes faster than the converted forward price of bonds with low duration.[16] Consequently, because the CTD is the bond with the minimum converted forward price, **the CTD tends to be a bond of high duration at high interest rate levels, and low duration at low interest rate levels.**[17] These switches in the CTD can be seen in Figure 2. At high yield levels, the CTD is the highest-duration bond. At intermediate yield levels, the CTD is the intermediate-duration bond. At low yield levels, the CTD is the lowest-duration bond. The points at which the CTD switches are called *cusps*.

[13] Note that variation margin makes the initial futures price the effective price of the transaction.

[14] Many alternate, though incorrect, ways have been suggested to determine the CTD. Among these are: lowest price, highest yield, and lowest basis. The correct and generally accepted way is the maximum implied repo rate calculation as described above. If term repo rates are available and are different for each issue, then the converted forward price calculation of Section II.1 should be used. In addition, during the delivery month when immediate delivery is allowed, the implied repo rate for very small time horizons would not be meaningful. If and when delivery is ever elected, the CTD is the bond with the *minimum converted cash price.*

[15] For instance, with a modified duration of ten years, a 100-basis-point yield move produces a 10% change in (full) price. For more details on modified duration, see the Salomon Brothers publication *Understanding Duration and Volatility*, September 1985.

[16] Because the conversion factor is calculated at only an 8% yield level, it does not compensate for *changes* in modified duration among the bonds as interest rates fluctuate away from 8%.

[17] For more details on how the contracts behave in different yield curve environments, see the Salomon Brothers publication, *Treasury Futures in a Low Interest Rate Environment — Changes in Deliverability,* V. Haghani, R. Stavis, May 1986.

4. How does the futures contract behave when interest rate levels change?

The individual bonds that are deliverable into the futures contract all exhibit positive convexity. That is, the modified duration increases when interest rates fall, and the modified duration decreases when interest rates rise.[18] In a wide range around 8%, the futures contract price does not exhibit the same degree of positive convexity as any of the deliverable bonds. When *rates are high*, the contract behaves more and more like a *high-duration* bond, and when *rates are low*, the contract behaves more and more like a *low-duration* bond. In fact, the convexity of the futures contract can be zero or even negative.[19] Negative convexity indicates that modified duration falls as interest rates fall.

5. Can the Treasury bond futures contract be thought of as a 20-year 8% coupon bond?

Because the basket of deliverables for the T-bond contract has maturities ranging from 15 years to 30 years, and the conversion factor mechanism normalizes the contract to an 8% coupon, it is often said that the contract looks like a 20-year 8% bond. However, the futures contract can exhibit negative convexity, whereas all the bonds in the deliverable basket and even a hypothetical 20-year 8% bond exhibit positive convexity. Consequently, this is not an appropriate way to look at the contract. When interest rates change, the CTD can have a maturity of as low as 15 years with a relatively low duration and as great as 30 years with a relatively large duration. **Even if the price sensitivities are sometimes similar between the futures contract and the current CTD, the convexities can be very different.**

6. How does the cheapest-to-deliver bond change when interest rate spreads between bonds change?

The CTD can also change if the yield spreads between bonds change, ever if the overall yield levels do not. At the current rate levels (of about 9%), the CTD for December '90 delivery is the U.S. 7.5% maturing November 15, 2016. At a rate level about 50 basis points lower, there is a cusp at which the high-coupon callables become CTD. If the yields of the high-coupon callables were to *increase* relative to the 7.5s, the callables would become relatively cheap, and the cusp or rate level at which the CTD switches from the 7.5s to the callables would move to a higher level. In other words, if the yields of the callables increase, then overall rates would not have to drop by as much as 50 basis points for them to become CTD. If the yields of the high-coupon callables *decrease*, then overall rate would have to drop even more than 50 basis points for them to become CTD. The yield spreads among the deliverable issues (or shape of the bon yield curve) are what determine the location of the cusps or the overall yield levels at which the CTD switches.

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[18] For more details on convexity see the Salomon Brothers publication *Convexity of Fixed-Income Securities*, October 1985.

[19] To compensate for this underperformance (too short a duration when rates fall, and too long a duration when rates rise), the market should price the contract lower than a theoretical forward price. The amount that the contract price is depressed reflects the option that the long position has effectively sold to the short.

Section IV: The Relationship Between the Futures and Cash Markets

1. How can one tell whether a futures contract is rich or cheap?

If the implied repo rate is greater than the market repo rate, futures are rich and should be sold. For example, if the implied repo rate of the contract is greater than the market repo rate of the CTD, a low-risk arbitrage profit can be earned by buying the CTD bond financed in the repo market and selling it forward by shorting the rich futures contract. This cash-and-carry trade earns the difference between the implied (high) repo rate and the market (low) repo rate.[20]

If the implied repo rate is less than the market repo rate, futures may be cheap and (perhaps) should be bought. That is, a holder of actual T-bonds can improve returns by selling the bonds and investing the cash proceeds for a short-term horizon, and effectively purchasing the bond forward by buying the cheap futures contract.[21] However, because the short delivery option embedded in the futures contract depresses the value of the futures contract, **the price of the futures contract may incorrectly appear cheap if the delivery option is not properly accounted for.**

2. What is "convergence" between the futures and cash markets?

If and when delivery is made, a *convergence* takes place between the cash price of the CTD and the market futures price.[22] At convergence the price of the futures contract would be the price of the CTD divided by its conversion factor. The gap between cash and futures is called the *basis* and has a value of the bond price (B) minus the conversion factor (CF) times the futures price: $Basis = B - CF \times F$. As the delivery month approaches, the basis (for the CTD) gradually becomes smaller.[23] When the yield curve is upwardly sloping, forward (and thereby futures) prices are lower than cash prices, and convergence of futures to cash will occur from below (that is, the basis will be positive and approach zero). Conversely, when the yield curve is downwardly sloping, forward (and thereby futures) prices are higher than cash prices and convergence of futures to cash will occur from above (that is, the basis will be negative and approach zero).

3. Does a cash-and-carry trade earn the implied repo rate of the bond or the current yield of the bond?

A long basis position (long bond and short futures) taken through to delivery (a cash-and-carry trade) earns the implied repo rate of the bond rather than the current yield of the bond (coupon divided by full price).[24] A cash bond held by itself would earn its current yield (in the absence of price change), reflecting the yields at the long end of the curve. The implied repo rate reflects rates on the short end of the curve, because the basis trade is hedged with respect to changes in the long end.

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[20] The purchase of a bond in a cash-and-carry trade can be financed in the *repo market* by lending it out through *a repurchase agreement.* The financing rate paid on this transaction is the *market repo rate*, which can depend on the supply and demand of the given bond and the financing term. Because little capital is required for this trade, the implied repo rate will almost never be above the market repo rate.

[21] If the cash bonds are not already owned, buying the contract and shorting the cash bond is often difficult.

[22] If delivery is not optimum, then the value of the delivery option will prevent complete convergence from taking place.

[23] A position is *long the basis* if it is long a cash bond and short futures contracts in proportion to the bond's conversion factor. A cash-and-carry trade is initiated by a long basis position.

[24] If the CTD changes before the cash-and-carry trade is unwound, the old CTD can be swapped for the new CTD. Because this will increase the return earned, **the implied repo rate represents a lower bound on the rate the cash-and-carry trade will earn.**

If long rates are higher than short rates, the question naturally arises as to how the cash-and-carry trade only earns the lower short-term rate rather than the higher long-term rate. Remember that for an upwardly sloping yield curve, futures prices converge to cash from below, so a long basis position will suffer a loss either due to an increase in the price of the short futures position or a decrease in the price of the long cash position. This narrowing of the basis has the required effect of reducing a higher current yield to the lower short-term rate. A similar argument applies to the inverted yield curve case. **In any event, the basis trade will earn a rate appropriate to the short end of the yield curve, not the long end.**

4. How does one weight the basis trade?

The cash-and-carry trade locks in the implied repo rate for a bond held through delivery into a futures contract if it is properly weighted. The appropriate weighting is the conversion factor (CF). For each $100,000 face amount of the cash bond held long, *1 x CF* of futures contracts must be shorted.[25]

Let us see why this is true. If the futures price moves while the trade is on, variation margin will flow from the long to the short in the amount of: *(initial futures price - final futures price)* for *each* futures contract.[26] If the number of contracts shorted is weighted by the conversion factor, then the cumulative variation margin received by the short will be *(initial futures price - final futures price) x CF*. The amount received by the short upon delivery into *each* futures contract is the *(final futures price) x CF*. Adding the final delivery price to the cumulative variation margin produces the *effective* delivery price of *(initial futures price) x CF*, which is what would be expected based on the initial futures price. Weighting the trade with the conversion factor makes the *initial futures price the effective delivery price*. The initial price is locked in, even though the final futures price is subject to interest rate changes.

5. What is "buying back the tail"?

A cash-and-carry trade can be unwound by closing out the cash and futures position separately or by delivering the cash bond into the short futures position. If delivery is chosen, the bonds must be delivered into the contract *one-for-one*, even though the position was conversion-factor-weighted. With a conversion factor of *1.2*, the number of futures contracts shorted against $100 million face of bonds is 1,200. The $100 million face of bonds would be delivered into 1,000 of the contracts, and the other 200 contracts must be bought back at the time of delivery. This is called *buying back the tail*.[27] The transaction to adjust the cash and futures also could be performed in the cash market.

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[25] This weighting ignores any interest that may be earned on the variation margin flows. When the futures price rises, the positive variation margin can be invested to the horizon of the hedge, making the overall price sensitivity of the contract higher than it would be if there were no variation margin. Consequently, the number of contracts can be reduced by the present value factor to the hedge horizon.

[26] If the initial futures price is higher (lower) than the final futures price, this quantity is positive (negative).

[27] The process of buying back contracts only involves closing out the excess short futures position and does not require any expenditure of funds. To lock in the original implied repo rate of the bond, the tail should be bought back when the futures settlement price used for delivery is set. If the conversion factor were less than one, then the additional contracts would have to be *sold* at the time of delivery.

Section V: Hedging with Futures Contracts

1. How does one hedge with a futures contract?

When a bond is hedged, two numbers must be calculated: the price sensitivity of the bond and the price sensitivity of the futures contract. The price sensitivity is measured by the *dollar value of a basis point (DVO1).*[28] The hedge ratio is merely the ratio of the price sensitivity of the bond to be hedged to the price sensitivity of the futures contract. The more price-sensitive the bond is compared to the futures contract, the more futures contracts are necessary for the hedge.

Let us consider hedging a bond as of the end of a delivery month. The DVO1 of the bond as of the end of the month is the change in price divided by the change in yield. Under the assumption that the futures contract tracks the CTD, the DV01 of the contract is the DVO1 of the CTD as of the end of the month divided by the conversion factor of the CTD.[29] (An example is worked out in Section V.4.)

2. Does the futures contract ever track the cheapest-to-deliver bond?

Assuming that the futures contract tracks the CTD makes the calculation of the price sensitivity of the contract easier, but the potential changes in the CTD can prevent the tracking from being perfect.[30] However, in two cases the futures contract does actually track the CTD. First, when the volatility is very small, the CTD will be known in advance with relative certainty. Because the probability is low that the CTD will change, the price sensitivity of the contract can be calculated from the current CTD. Second, when yield levels are significantly above or below 8%, there is a dominant CTD and such a slight chance that the CTD will change, that the futures price sensitivity will again be very close to that of the CTD.[31]

3. How much does the cash price of the benchmark Treasury 30-year bond move when the futures price moves one point?

Although the cash price of the benchmark bond does not usually move one-for-one with the T-bond futures contract when interest rates change, it is easy to figure out how much it does change. Consider the U.S. 7.5% bond maturing November 15, 2016, which is currently the CTD. It has a forward DV01 of $.0880 per $100 at a yield of 8.92%, and its CF for December '90 delivery is .9456. Assuming that the December '90 T-bond futures contract tracks the CTD, it has a DVO1 of (.0880/.9456), or *$.0931*. A *1.0* point drop in futures price means that yields have risen (1/.0931), or *10.7* basis points. The 30-year benchmark is the U.S. 8.75% bond maturing August 15, 2020, which has a DV01 of $.1035 per $100 at a yield of 8.85%. Consequently, a 10.7 basis point rise in yield will make the price of the 30-year benchmark drop by (10.7 x .1035), or *1.1* points, instead of the 1.0 point that futures dropped.

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[28] The DVO1 of a 30-year bond near par has a value of about $0.10 per $100 face, or $1,000 per million. A 100-basis-point move would make the price of the bond change by about $10 per $100 face. Dividing the DV01 per million by the bond full price gives the modified duration.

[29] The statement that the futures contract tracks the CTD means that the futures contract price moves with the *forward* market price of the bond, not the cash market price. The forward DV01 of the CTD is divided by the CF, just as the forward price of the CTD is divided by the CF (see footnote 7 on page 4). The forward DVO1 of the CTD should be calculated with the same yield curve movement as was assumed for the bond to be hedged.

[30] A better hedge is obtained if the price sensitivity of the futures contract is calculated taking into account the delivery option.

[31] With the current set of deliverable bonds and yield relationships, the relevant limits of rates are 7.00% and 9.50%. At low rates the CTD will be the low-duration bond, that is, the U.S. 13.875% maturing May 15, 2011 and callable starting May 15, 2006. At high rates the CTD will be the high duration bond, that is, the U.S. 7.25% maturing May 15, 2016.

4. Does the conversion factor weighting produce the same hedge ratio as the DVO1 weighting?

If the bond to be hedged is the CTD, then the conversion factor weighting produces the same hedge ratio as the DV01 weighting. The DVO1 of each contract is the forward DVO1 of the CTD *divided* by the CF. If the number of contracts held is 1.0 *multiplied* by the CF, the CF cancels, and the futures position will have the same price sensitivity as the cash CTD position. The result is that the hedged position will be insensitive to interest rate movements.

However, if the bond to be hedged is not the CTD, the conversion-factor-weighted hedge will be different from the DV01 weighted hedge.[32] For example, consider hedging $100 million face of the U.S. 11.75% bond maturing in November 15, 2014 and callable starting November 15, 2009, with the December '90 futures contract. Because the CF for this bond is 1.3608 for December '90 delivery, conversion factor weighting would give *1,361* contracts. The DVO1 of the futures contract, as calculated above, is *$.0931*. The 11.75s have a DVO1 of *$.1103* (taking into account the call option) at a yield-to-call of 9.01%. Because the futures contract is less sensitive than the 11.75s, the appropriate number of contracts is (.1103/.0931) x 1,000 or *1,185*, instead of 1,361.

5. What is the modified duration of a futures contract?

There is a temptation to talk about a modified duration for a futures contract even though the price of the contract does not reflect any current expenditure of funds. Technically, because the actual cost of entering into a futures contract is zero and the variation flows are not known in advance, the duration calculation is not defined. Consequently, **the calculation for duration of a futures contract should not be performed.** However, an acceptable calculation for modified duration may be used in a portfolio context.

The calculation of the *modified duration of a portfolio* that includes futures contracts is easily modified. The price sensitivity is adjusted by the aggregate price sensitivity of the futures contracts. The full price of the portfolio is not adjusted at all, because the cost of the futures contracts is zero. The portfolio modified duration is the portfolio price sensitivity divided by the portfolio market value.

* * *

[32] Note that the conversion factor weighting locks in the implied repo rate of the cash-and-carry trade. This hedge is not adjusted if the market moves, and can earn an even higher return if the CTD switches. The DV01 hedge may require adjusting if the market moves, but in general, the return will vary less than for the conversion-factor-weighted hedge. Essentially, if the bond's modified duration is less than (greater than) the modified duration of the CTD, then conversion factor weighting will require a greater (lesser) number of futures contracts.

Section VI: Simple Questions Concerning the Delivery Option

1. What is the delivery option?

As yield levels change, the CTD can change, and the holder of the short futures position has the choice of which bond to deliver and when to deliver during the delivery month. This option is called the *delivery option.* The holder of the long futures position is short this same option.[33] The potential changes in the CTD are what gives the option value. The short can deliver a bond that is effectively cheaper than the bond that was CTD before the switch.

The delivery option has four separate features: the quality option, the timing option, the last-week option, and the afternoon wild-card option. The *quality option* represents the choice that the short has as to which instrument to deliver. The *timing option* reflects that delivery can be made at any time during the delivery month. The *afternoon wild-card option* reflects the value of electing delivery in the afternoon after the futures market has closed and the cash market still trades. This opportunity exists each day that the futures contract trades during the delivery month. On the eighth business day before the end of the delivery month, futures stop trading and delivery must be made by month's end at the last futures settlement price before it went off the board. This is the *last-week option.*

2. How does one avoid the delivery process altogether?

The utility of the futures contract does not require that delivery ever be made. In fact, very few open contract positions ever go to delivery. The easiest way to avoid delivery is to close out the position by two business days before the delivery month.[34] When the yield curve is upwardly sloping, delivery would not ordinarily be made at the beginning of the month (because of positive carry). But to avoid delivery it would be best to close out any open long positions before the beginning of the month, when the liquidity is greatest.

3. When does the delivery option have greatest value?

The delivery option is most important when the contract is near a *cusp* — that is, when there is great likelihood that the CTD will change sometime before the end of the delivery month. The closer it is to a cusp or the greater the volatility, the greater the possibility of changes in the CTD and the greater the value of the option. If the contract is far from a cusp and one bond will be CTD for a large range of yield curve levels, then the value of the option is small.

[33] The delivery option is an option to exchange the CTD for the new CTD. If the cusp is at prices above the market, then the option can be viewed as a *call* on the market. If the cusp is at prices below the market, then the option can be viewed as a *put* on the market. When there is more than one cusp, the option can be viewed as either a call or put.

[34] The delivery process actually takes three consecutive business days. It can start as early as two business days before the delivery month and must end by the last business day in the delivery month: 1) Intention Day: The short's clearing member notifies the exchange clearing house by 8:00PM (CST) of the intention to make delivery. 2) Notice Day: The long positions are selected (first in, first out), and the long's clearing member is notified in early morning. The short specifies by 2:00PM the issue that will be delivered. 3) Delivery Day: T-bonds must be ready for delivery by 10:00AM, and money and securities are exchanged by 1:00PM.

4. How does the delivery option affect the richness or cheapness of the futures contract?

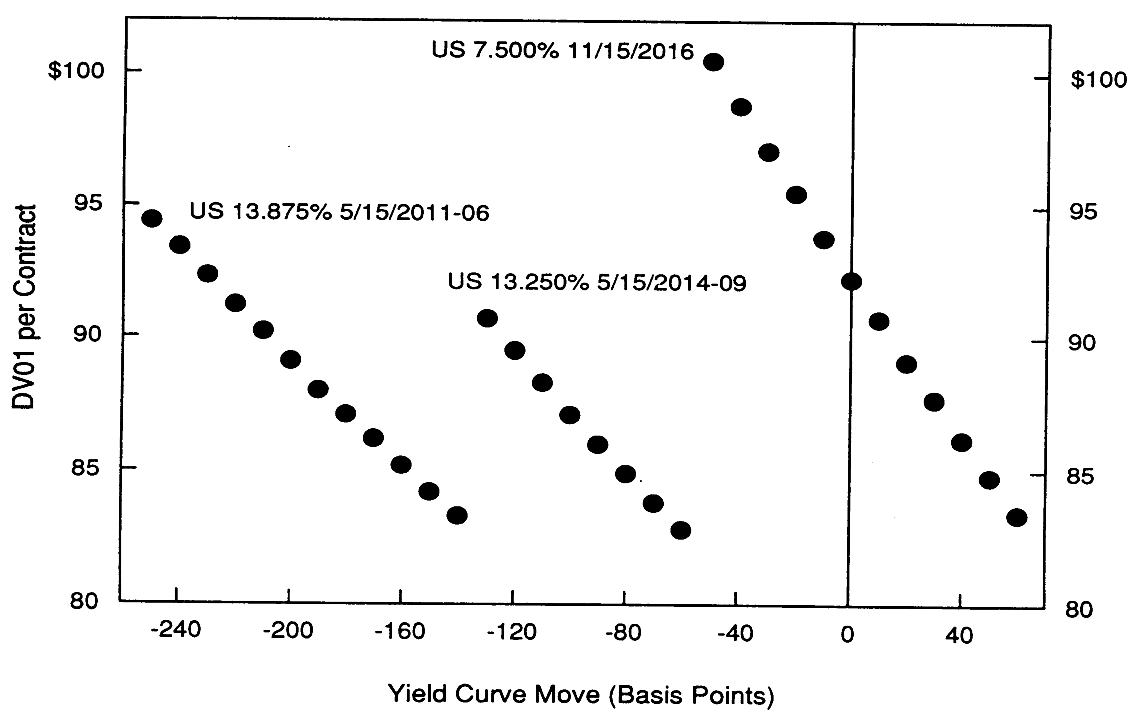
Any determination of the richness or cheapness of the contract must correctly take into account the value of the option. Because the value of the option depresses the value of the contract, ignoring it may result in a determination of a market futures price that appears low, or equivalently, an implied repo rate that appears low. **An implied repo rate that is low with respect to market rates does not automatically indicate that futures are cheap.** To calculate whether the futures are in fact cheap, it is necessary to adjust the value of the futures contract for the delivery option.

5. How does the delivery option affect the price sensitivity of the futures contract?

As yield levels change, the CTD can change. Because the futures contract *tends* to track the CTD, the DVO1 of the contract will also change as yields change. As yields drop, the prices of all the deliverables rise, and so does their price sensitivity. However, as yields drop, the CTD can change to a bond that appreciates more slowly. At the point that the CTD switches (the cusp), the price sensitivity of the futures contract would drop dramatically (assuming it tracks the CTD). Similarly, as yields increase, the price sensitivity would jump at each switch of the CTD.

If the futures contract were to track the CTD perfectly, the resulting profile of the price sensitivity of the futures contract would be very ragged, with discrete jumps at each cusp (see Figure 3).

Figure 3. Futures Price Sensitivity (Tracking the Cheapest-to-Deliver)
October 1, 1990: December T-bond Futures Price = 89-12



The DV01 of the December '90 T-bond futures contract is plotted at 10-basis-point intervals. The current yield-level (of about 9%) is indicated by the vertical line at the "0" point of the horizontal axis. There is a cusp at -50 basis points, at which point the CTD switches from the 7.5s to the 13.25s and the DV01 drops from $101 to $82 per contract. A second cusp is at -130 basis points, at which point the CTD switches from the 13.25s to the 13.875s and the DV01 drops from $91 to $83 per contract. However, this method of approximating the price sensitivity of the contract ignores the delivery option. When the effect of the delivery option is modeled, a smooth DVO1 curve is obtained. The option adjusts the value of the futures contract during the CTD transitions, thereby making the futures contract price-sensitivity curve smooth.[35]

Section VII: Advanced Questions Concerning the Delivery Option

1. How does one determine whether delivery should be made?

As long as the futures are still trading, it is possible to close out a futures position without delivering. If delivery is desired, the CTD is the bond that maximizes the delivery profit to the short: $CF \times F - B$.[36] When it is efficient to deliver the CTD during the delivery month, the basis will be reduced to zero. This means that the futures price will be the CTD price divided by its conversion factor. This is **proper convergence**, and it **only occurs if there is no time value of the delivery option left and it is therefore optimum to deliver**. If there is any time value remaining to the delivery option, delivery should be postponed.[37]

2. If delivery does occur, will it be at the beginning or the end of the delivery month?

When the yield curve is upwardly sloping, forward prices as of the end of the delivery month are less than as of the beginning of the delivery month, and there will be a preference for end-of-month delivery.[38] The implied repo rate calculation is correspondingly made to the end of the month and represents a lower bound on the short-term investment rate that the cash-and-carry trade will earn. An inversion in the yield curve would induce early delivery because reinvestment at the higher short-term interest rates to the end of the delivery month would result in a greater return. If delivery is still made at the end of the month, the calculated implied repo rate will be earned as expected.

If the yield curve is initially inverted, forward prices as of the beginning of the delivery month are less than as of the end of the delivery month, and there will be a preference for delivery at the beginning of the month.[39] The implied repo rate calculation is correspondingly made to the beginning of the month and represents a lower bound on the short-term investment rate that the cash-and-carry trade will earn. A steepening in the yield curve would promote late delivery and a resulting greater return, because delivering prematurely would forfeit the newly positive carry as well as

[35] See the forthcoming paper, *The Salomon Brothers Delivery Option Model.*

[36] The profit is not affected by the accrued interest of the bond, because conceptually the bond is settled in the cash and futures markets on the same day. In reality, one business day of carry would be necessary to guarantee possession of the cash bond in time for the early morning deadline on delivery day.

[37] When futures stop trading on the eighth business day before the end of the delivery month, convergence may not take place because the last week option can still have value. In addition, because the futures price can no longer change during the last week, any cash market move would also prevent convergence.

[38] It is advantageous to delay delivery for as long as possible because of the positive carry associated with an upwardly sloping yield curve.

[39] It may be advantageous to make delivery as soon as possible because of the negative carry associated with a downwardly sloping yield curve.

any remaining time value of the delivery option. If delivery is still made at the beginning of the month, the calculated implied repo rate will be earned as expected.

3. Are all bonds equally deliverable at 8%?

The conversion factor mechanism attempts to make all bonds equally deliverable, so that there is no economic incentive of delivering one bond over another. If all the deliverable bonds were priced at an 8% yield (yield-to-maturity for the noncallables and yield-to-call for the callables), then all the bonds would, in fact, be equally deliverable. However, as the yield curve changes level and shape, there will be an economic incentive for the short to deliver some bonds over others.

The fact that the CTD can change is what gives the delivery option its value. If the conversion factor mechanism were not as good as it is, then there would be only one bond that would dominate, and the CTD would not switch. As a consequence, the delivery option would be worth less, but the futures contracts would be less liquid. The liquidity of the contracts is increased when delivery is not totally restricted to one issue.

4. Can the basis ever become negative during the delivery month?

The basis $(B - CF \times F)$ cannot become negative during the delivery month (for any of the deliverable bonds) because arbitrage would force it back to zero. If futures became too rich, the basis would become negative *temporarily*. But then the contract could be shorted against a long position in the CTD, and the bond could be delivered *immediately* into the contract without any price risk. The profit in this trade would force the basis back to zero. As long as the option has value, the price of the contract is depressed relative to cash. This keeps the basis positive during the delivery month until efficient delivery forces convergence and the basis becomes equal to zero. Before the delivery month the basis can be negative, because delivery is not yet possible.

5. When would the afternoon wild-card option get exercised?

The afternoon wild-card option reflects that delivery can be elected in the afternoon after futures stop trading and the cash market is still open.[40] Unlike most options, the afternoon option has to be exercised to extract its value, which is only attained after the futures market closes. If it is exercised, any remaining option value and any positive carry are forfeited.[41] Because the futures market will adjust to the cash market move the next day, anyway, the cash market must move substantially to induce afternoon delivery.[42]

To understand the afternoon option, consider a long basis position in the CTD that has a conversion factor of *1.2*. If the cash market drops after futures closes, the *tail* can be *bought* in the *cash* market so that enough bonds will be owned to make delivery into the futures position. This *purchase* will be at a *low* price with respect to the futures contract close, upon which delivery is based. For this to be profitable, the cash market drop must be large enough to compensate for the lost time value of the option. If the cash market were to rise in the afternoon, it would not be advantageous to make afternoon delivery with the CF of the CTD equal to 1.2.

[40] A second aspect is that the delivered instrument does not have to be declared until the following day (Notice Day).

[41] An inverted yield curve with its tendency for early delivery will increase the value of the afternoon wild-card option, because early delivery would not induce a loss of positive carry.

[42] Because there has been very little cash market volatility recently after the futures market closes, the afternoon option has been exercised infrequently for the T-bond or T-note contracts. Because of low volatility and the short time allowed for exercise, very little value can be attributed to the afternoon option. However, there has been some recent afternoon delivery for the U.S. Treasury five-year note contract.

Again, consider a long basis position, but assume the CTD has a conversion factor of *0.9*. If the cash market rises after the futures market closes, the *tail* can be *sold* in the *cash* market so that only the needed amount of bonds will be owned for delivery into the futures position. This *sale* will be at a *high* price with respect to the futures market close, upon which delivery is based. For this to be profitable, the cash market rise must be large enough to compensate for the lost time value of the option. If the cash market were to drop in the afternoon, it would not be advantageous to make afternoon delivery with the CF of the CTD equal to 0.9.

6. What happens during the last week after futures stop trading?

Futures stop trading on the eighth business day before the end of the delivery month. At that time, unless it is optimal to deliver immediately, the futures price will still be depressed relative to cash because of the remaining time value associated with the delivery option. During the last week, any outstanding contracts must go to delivery before the end of the month, and will be made at an *old* delivery price.

Because futures no longer move with the cash market, a long basis position should be adjusted so that the futures and cash will be weighted one-for-one. If rates rise or fall during the last week and the CTD does not change, the long basis position will have locked in the last futures settlement price. If the CTD changes, then the cash position can be sold at an effectively higher price by selling the old CTD and buying the same face amount of the new CTD to deliver into the contract. Because there can be large variations in the price sensitivities of the deliverable bonds, this last-week option can have significant value even though the time to expiration is short.

Summary

Probably the single most important source of confusion for users of futures contracts is that any one of a basket of eligible Treasury securities can be delivered by the holder of the short position to settle the futures contract. This paper has explored how this affects the valuation of the futures contract, as well as the behavior of the contract as the market environment changes. To summarize:

- In order to adjust for coupon differences among the deliverable bonds, the market futures price is multiplied by a conversion factor (CF) to determine the delivery price.

- The most important feature distinguishing a futures contract from a forward contract is variation margin. This affects the mechanics as well as the valuation of the contract.

- The bond with the highest implied repo rate is called the cheapest-to-deliver (CTD) bond.

- Changes in the CTD with changes in the level of interest rates cause the futures contracts to price off low-duration bonds when rates are low, and high-duration bonds when rates are high.

- The resulting futures price sensitivity makes the contract behavior unlike a hypothetical 8% 20-year bond.

- If the implied repo rate of the CTD is low, futures are cheap and can be substituted for cash instruments. If the implied repo rate is high, then futures are rich and can be sold to augment cash market returns.

• In order to hedge with futures contracts, the relative price sensitivity to interest rates of the cash instrument and the contract has to be determined. When there is little likelihood that the CTD will change, its price sensitivity can be used to approximate the price sensitivity of the contract.

• The choice that the holder of the short position has as to which bond to deliver and when is called the delivery option. This option can lower the price of the futures contract and has the most value when the contract is near a cusp.

• A contract that appears cheap may not be, unless the delivery option is properly accounted for.

• By closing out any open futures positions before the delivery month, the delivery process can be avoided completely.

While this paper has explored qualitatively how the delivery option affects the contract's behavior as interest rates change and the delivery month nears, it has not discussed how the impact of the delivery option can be quantified. A general framework for studying the delivery option in more detail is presented in the forthcoming paper, *The Salomon Brothers Delivery Option Model*.

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