US Rates Viewpoint

Understanding Principal Components

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The virtues of PCAs… and some shortcomings

Principal components analysis (PCA) is a commonly used tool in financial econometrics. It allows researchers to reduce the dimensionality of a problem, and to understand the main forms of movement in a dataset. In the rates volatility space, PCA analysis is one of the main tools used understand the dynamic of the grid and RV opportunities.

There are, however, some problems with the application of PCA to the volatility grid, including: (1) how to incorporate the directionality between the volatility grid and the yield curve; (2) how to make the model responsive to changing market conditions while minimizing the estimation errors; and the more generic interpretability problem of PCA in complex problems, i.e. (3) how to interpret the principal components of the vol grid.

The first two were the subject of Lower for longer volatility, where we introduced the principal components of the yield curve in the framework, and used a clustering approach to add more granularity to the PCA framework and make the it more reactive to market conditions.

A new twist to an old approach

Here we introduce a new twist to an old approach to improve the interpretability of the principal components and perhaps add to the usefulness of these types of frameworks in risk type of contexts. The new twist is to merge the two step process introduced in Lower for longer volatility (where we perform the PCA step on each of the clusters of the grid) into one step. To do this we used a hierarchical PCA (HPCA) methodology.

With this framework we are able to assign to the first three principal components of the volatility grid a clear interpretation: the first principal component (HPC1) corresponds to a broader up/down move on the grid, the second (HPC2) corresponds to a gamma vs vega rotation, and the third (HPC3) to a left side vs right side rotation.

When we move on to higher order PCs, however, the implications start to be even more interesting, particularly from a risk perspective. When we look at components 4 and 5, we start seeing much of the HPCs loadings populated by zero, and therefore isolating modes of rotation that are intrinsic to the different segments of the grid.

The ability to decompose the grid dynamic into main orthogonal rotations modes allows the projection of risk along these eigenvectors, and the fact that we start seeing these eigenvectors populated by zero under HPCA (not in PCA) helps in a parsimonious decomposition of risk.

The virtues of PCAs …

Principal components analysis (PCA) is a commonly used tool in financial econometrics. It allows researchers to reduce the dimensionality of a problem, and to understand the main forms of movement in a dataset. The first three principal components of the yield curve, for example, are easy to associate with the overall level of rates, the shape of the curve, and the convexity of the curve, and allow the practitioner to reduced dimensionality of the yield curve from a large number of highly correlated time-series for different tenors into 3 orthogonal principal components.

In the rates volatility space, PCA analysis is one of the main tools used understand the dynamic of the grid and relative value (RV) opportunities. In the standard approach one picks the first few principal components of the vol grid to apply the inverse PCA transform on only those eigenvectors (reducing the dimensionality of the problem) to estimate a model volatility grid. One can then calculate Z-Scores for the spreads between the actual vol grid and the PCA estimation to understand RV on the grid.

This approach is particularly suited for the volatility grid, because the information content on the grid is significantly lower than its number of tenors. Out of an 11x7 volatility grid (see Table 1) there are only a small subset that are actively traded, and the rest are extrapolated out of these. The extrapolation process contains information in itself, but only a limited amount. A process of dimensional reduction like PCA is ideality suited in this kind of applications therefore.

Table 1: Implied US volatility grid as of 18 August

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	16.1	18.0	23.0	33.6	46.2	55.8	74.0
3M	19.3	23.0	29.1	40.3	52.1	61.7	81.0
6M	20.6	24.3	30.5	41.3	50.9	60.0	76.4
1Y	23.9	28.0	34.3	44.5	53.2	60.6	70.1
2Y	35.8	41.0	40.5	50.8	56.6	62.0	68.1
3Y	48.2	47.8	50.1	54.8	59.3	62.5	66.4
4Y	51.3	53.7	54.9	57.9	60.4	62.7	64.7
5Y	55.5	57.2	57.5	59.6	61.1	62.5	62.6
10Y	61.9	62.2	61.9	61.1	61.4	60.7	55.9
15Y	59.6	59.6	59.3	57.9	58.2	57.9	52.5
30Y	50.4	52.7	52.0	51.1	50.9	50.4	52.1

Source: BofA Global Research

Table 2: Segmentation of the volatility grid

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	1	1	1	2	2	2	2
3M	1	1	1	2	2	2	2
6M	1	1	1	2	2	2	2
1Y	1	1	1	2	2	2	2
2Y	3	3	3	4	4	4	4
3Y	3	3	3	4	4	4	4
4Y	3	3	3	4	4	4	4
5Y	3	3	3	5	5	5	5
10Y	3	3	3	5	5	5	5
15Y	3	3	3	5	5	5	5
30Y	3	3	3	5	5	5	5

Source: BofA Global Research

… and some shortcomings

For more complex problems, the interpretation of the main principal components, the ones that explain the largest portion of the variance of the data set, becomes more difficult. While that interpretation is relatively straightforward on the yield curve, for example, that is not the case already on the volatility grid.

Other issues with the application of PCA to the grid, include: (1) how to incorporate the directionality between the volatility grid and the yield curve in the framework; or (2) how to make the model responsive to changing market conditions while minimizing the estimation errors.

The last two were the subject of Lower for longer volatility, where we introduced the principal components of the yield curve in the framework, and used a clustering framework to add more granularity to the PCA approach on the grid and make it more reactive to market conditions. Here we introduce a new twist to an old approach to improve the interpretability of the principal components and perhaps add to the usefulness of these types of frameworks in risk type of contexts.

The new twist is to merge the two step process introduced in Lower for longer volatility (where we perform the PCA step on each of the clusters of the grid) into one step. To do this we used a hierarchical PCA (HPCA) methodology.

A new twist to an old approach

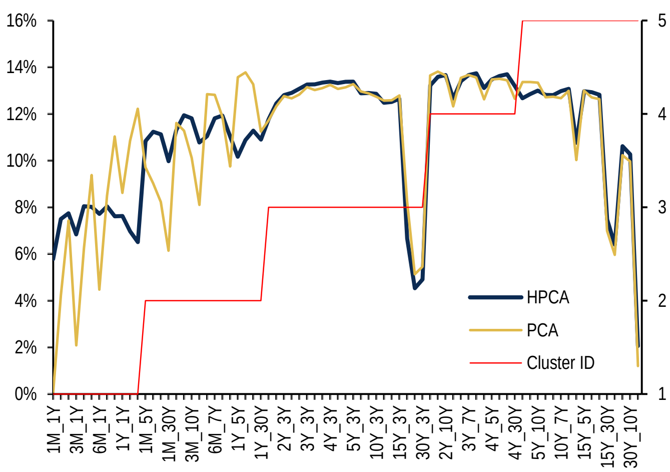
In a recent pre-print article[1] the authors discuss an alternative approach to PCA that makes use of natural segmentations in the data. One of the applications discussed for this HPCA (Hierarchical PCA) approach is in the analysis of equity index data, which is naturally segmented into sectors.

Under HPCA the correlation matrix of assets returns is modified to reflect the assumption that the residuals of the regression of each asset on the first factor of each segment (or sector) are uncorrelated across segments.

The volatility grid seems to be particularly suited for this type of analysis, given its natural segmentation. Indeed it was this realization that led to the clustering PCA framework mentioned above. A clear advantage of HPCA relative to clustering PCA approach is that while in the latter the PCs for each cluster are in an equal footing, in a HPCA framework they are sorted by their variance explained.

Here we chose to use a static segmentation of the vol grid (see Table 2) into upper left corner (ULC – cluster 1), upper right corner (URC – cluster 2), lower left corner (LLC – cluster 3), intermediate vega (IV – cluster 4) and lower right corner (LRC – cluster 5). However, we could have just as easily used a dynamic segmentation as we did in Lower for longer volatility, where we also discuss some of the drivers for the observed segmentation of the volatility grid.

Chart 1: Comparison of first PC (PC1) and HPC (HPC1)



Source: BofA Global Research

Table 3: Loadings for HPC1

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	6%	7%	8%	11%	11%	11%	10%
3M	7%	8%	8%	11%	12%	12%	11%
6M	8%	8%	8%	11%	12%	12%	11%
1Y	8%	7%	7%	10%	11%	11%	11%
2Y	12%	12%	13%	13%	14%	14%	13%
3Y	13%	13%	13%	13%	14%	14%	13%
4Y	13%	13%	13%	13%	14%	14%	13%
5Y	13%	13%	13%	13%	13%	13%	13%
10Y	13%	13%	13%	13%	13%	13%	11%
15Y	12%	13%	13%	13%	13%	13%	7%
30Y	7%	5%	5%	6%	11%	10%	2%

Source: BofA Global Research

In Chart 1 we compare the first PC of the vol grid (PC1), with the first HPC (HPC1), along with an identification of the different tenors (x-axis) with the corresponding segments (red line on RHS). There is little difference between PC1 and HPC1, with perhaps slightly more noise in the PC1. Interestingly, both PC1 and HPC1 seem to suggest that the 30y1y, 30y2y and 30y3y tenors are perhaps better placed in cluster 5 (LRC).

When we move on to higher order PCs and HPCs, however, the differences start to look more striking, and these differences are quite significant in the identification of the meaning of these higher order components. Indeed Chart 2 shows that the second principal component of the volatility grid corresponds clearly to steepening and flattening moves in the term structure of volatility (long the gamma sector of clusters 1 and 2, and short longer date expiries in clusters 3, 4 and 5.)

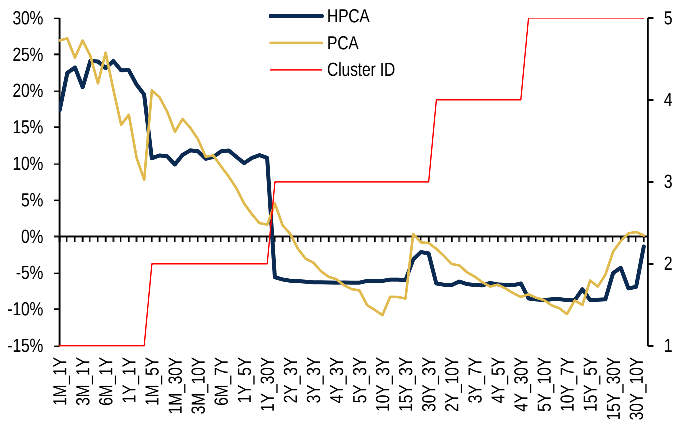
[1] https://arxiv.org/abs/1910.02310

The third PC and HPC seem to correspond to the curvature of the volatility grid across underlyings (long the left side of the grid of clusters 1 and 3, and short the right side of the grid of sectors 2, 4 and 5). Again here, the HPC3 seems to be much less noisy than the PC3.

These are interesting conclusions, because we are able to assign to the first three principal components of the volatility grid the same clear interpretation as we have noted above for the first three principal components of the yield curve; the first principal component corresponds to a broader up/down move on the grid, the second corresponds to a gamma vs vega rotation, and the third to a left side vs right side rotation.

Chart 2: Comparison of PC2 and HPC2

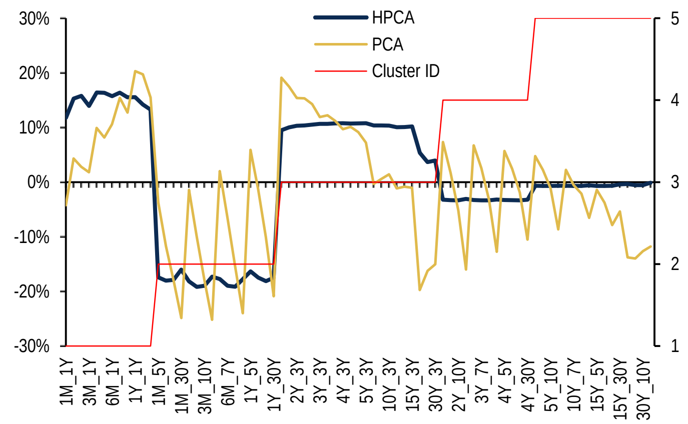
Gamma vs. vega rotation



Source: BofA Global Research

Chart 3: Comparison of PC3 and HPC3

Left side vs. right side rotation



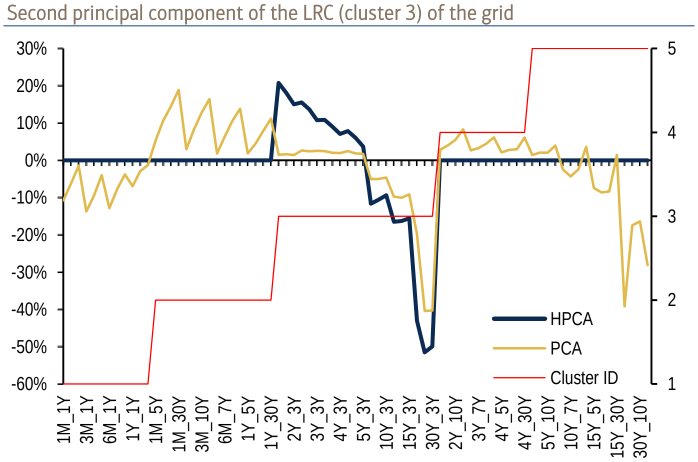
Source: BofA Global Research

When we move on to higher order HPCs, however, the implications start to be even more interesting, particularly from a risk perspective. When we look at components 4 and 5, we start seeing much of the HPCs loading populated by zero, and therefore isolating modes of rotation that are intrinsic to the different segments of the grid.

The fourth principal component, shown in Chart 4, reveals that the fourth most significant rotation on the volatility grid is a rotation intrinsic to the LLC (cluster 3 in Table 2). Indeed, when we display the HPC4 loadings in matrix format (see Table 4) we see clearly how HPC4 corresponds to a rotation in the LLC of the grid between intermediate expiries and longer dated expiries, or the second principal component of the LLC of the grid.

Chart 4: Comparison of PC4 and HPC4

Second principal component of the LRC (cluster 3) of the grid



Source: BofA Global Research

Table 4: The fourth HPC (HPC4) corresponds to a rotation in the LLC of the grid, between intermediate expiries and longer dated expiries

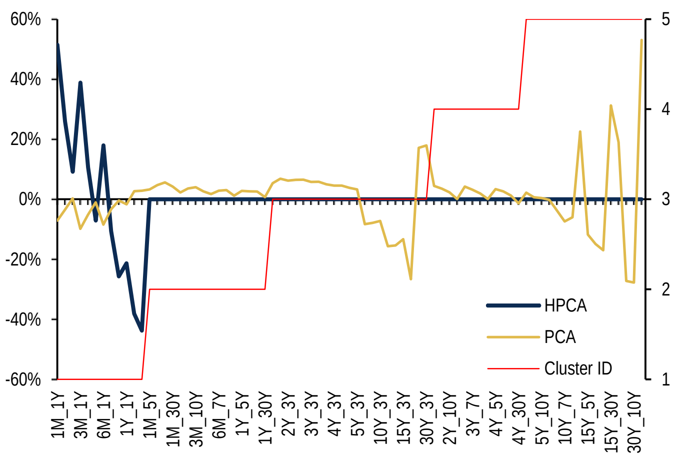
	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	0%	0%	0%	0%	0%	0%	0%
3M	0%	0%	0%	0%	0%	0%	0%
6M	0%	0%	0%	0%	0%	0%	0%
1Y	0%	0%	0%	0%	0%	0%	0%
2Y	21%	18%	15%	0%	0%	0%	0%
3Y	16%	14%	11%	0%	0%	0%	0%
4Y	11%	9%	7%	0%	0%	0%	0%
5Y	8%	6%	4%	0%	0%	0%	0%
10Y	-12%	-11%	-9%	0%	0%	0%	0%
15Y	-17%	-16%	-16%	0%	0%	0%	0%
30Y	-43%	-51%	-50%	0%	0%	0%	0%

Source: BofA Global Research

The fact that the HPC4 loadings are isolated in cluster 3, and zero in the remaining partitions, is significant from a risk perspective. The ability to decompose the grid dynamic into main rotations modes allows the projection of risk along these eigenvectors, and the fact that we start seeing these eigenvectors populated by zero under HPCA (not in PCA) helps in a parsimonious decomposition of risk.

Similarly, the fifth HPC corresponds to a rotation of in the ULC of the volatility grid (cluster 1 in Table 2), or the second principal component of the ULC cluster.

Chart 5: Comparison of PC5 and HPC5



Source: BofA Global Research

Table 5: HPC5 loadings corresponds to second PC of the ULC

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	51%	26%	9%	0%	0%	0%	0%
3M	39%	11%	-7%	0%	0%	0%	0%
6M	18%	-11%	-26%	0%	0%	0%	0%
1Y	-21%	-38%	-44%	0%	0%	0%	0%
2Y	0%	0%	0%	0%	0%	0%	0%
3Y	0%	0%	0%	0%	0%	0%	0%
4Y	0%	0%	0%	0%	0%	0%	0%
5Y	0%	0%	0%	0%	0%	0%	0%
10Y	0%	0%	0%	0%	0%	0%	0%
15Y	0%	0%	0%	0%	0%	0%	0%
30Y	0%	0%	0%	0%	0%	0%	0%

Source: BofA Global Research

Below we show also the matrices for HPC6 and HPC7, which seem to corresponds to the second principal components of the LRC and URC, respectively. It is interesting how the idiosyncratic dynamic of the URC of the grid, which is the focus of much of the programmatic flow, is relegated to seventh place in terms of terms of significance in the grid HPCA.

Table 6: HPC6 loadings corresponds to second PC of the LRC

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	0%	0%	0%	0%	0%	0%	0%
3M	0%	0%	0%	0%	0%	0%	0%
6M	0%	0%	0%	0%	0%	0%	0%
1Y	0%	0%	0%	0%	0%	0%	0%
2Y	0%	0%	0%	0%	0%	0%	0%
3Y	0%	0%	0%	0%	0%	0%	0%
4Y	0%	0%	0%	0%	0%	0%	0%
5Y	0%	0%	0%	8%	7%	6%	2%
10Y	0%	0%	0%	7%	8%	7%	-29%
15Y	0%	0%	0%	8%	9%	8%	-45%
30Y	0%	0%	0%	-36%	5%	7%	-73%

Source: BofA Global Research

Table 7: HPC7 loadings corresponds to second PC of the URC

	1Y	2Y	3Y	5Y	7Y	10Y	30Y
1M	0%	0%	0%	6%	-11%	-26%	-45%
3M	0%	0%	0%	20%	3%	-16%	-36%
6M	0%	0%	0%	32%	17%	-2%	-24%
1Y	0%	0%	0%	44%	32%	15%	-10%
2Y	0%	0%	0%	0%	0%	0%	0%
3Y	0%	0%	0%	0%	0%	0%	0%
4Y	0%	0%	0%	0%	0%	0%	0%
5Y	0%	0%	0%	0%	0%	0%	0%
10Y	0%	0%	0%	0%	0%	0%	0%
15Y	0%	0%	0%	0%	0%	0%	0%
30Y	0%	0%	0%	0%	0%	0%	0%

Source: BofA Global Research

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