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Dynamic strategies for net income generation in a low interest rate environment

Eric Bouyé[a]*

Tao Wang[a]

[a] *The World Bank, 1225 Connecticut Ave NW, Washington DC 20433, USA*

Abstract

This paper explores some dynamic investment strategies based on interest rate derivatives to generate income. The low interest rate environment makes this question critical to public institutional investors and potentially their asset-liability management. There exists a trade-off between the potential level income generated for the medium/long term on one side, and, the uncertainty of this income and its associated mark-to-market volatility through time, on the other side. First, we develop the foundations of the Libor market model (LMM) initially calibrated using data from swaptions for USD and EUR markets. Then different types of eligible strategies for the stabilization of the net income are provided. The strategies are mostly based on interest rate derivatives – swaps, caps, floors and swaptions – , and they may be partially rule-based and/or self-financed and may allow for tactical decision. Finally, the importance of the institutional framework and the role of the governance are discussed, e.g. the impact of mark-to-market based investment rules on the degree of path-dependency of the implemented strategies.

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Keywords: Dynamic investment strategies; Asset-liability management; Derivatives strategies.

* Corresponding author. Head of Strategic Asset Allocation and Quantitative Solutions, QRA, World Bank Treasury. Tel.: +1-202-473-3066.
E-mail address: ebouye@worldbank.org.

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1. Introduction

The low interest rate environment and the likelihood of a rapid increase of interest rate levels have created new challenges for institutional investors and the asset-liability management of public institutions. Stakeholders should balance the need for income generation for the medium/long term with the mark-to-market volatility in an uncertain environment (due to monetary policy changes, central bank intervention, and economic outlook). For a set of preferences and constraints, the impact on allocable income and mark-to-market of various competing strategies can be compared using stochastic simulation and scenario analysis.

In the second section, we develop an interest rate model with a zero lower bound (ZLB) constraint. More specifically, we use a Libor market model (LMM) initially calibrated using data from swaptions for USD and EUR markets. A methodology for stochastic scenario generation that is based on a simple change of measure is also presented. In the third section, different types of eligible strategies for the stabilization of the net income are provided. The strategies are mostly based on interest rate derivatives – swaps, caps, floors and swaptions –, and they may be partially rule-based and/or self-financed and possibly include a discretionary component. Then, a dynamic comparison of the distribution of the net income and the mark-to-market of the different strategies is provided under different scenarios. The introduction of relevant indicators is also considered. Last but not least, the role of the governance is discussed, especially the impact of mark-to-market based investment rules on the degree of path-dependency of the implemented strategies.

An important point to emphasize is that this paper is not about generating additional income through exposure to other asset classes, but more about dynamically managing the interest rate exposure. It explores how strategies can be put in place to manage the uncertainty of interest rates and income generation constraints.

2. Interest rate model

Many candidate models are available. The Nelson-Siegel model has been very popular among central bankers for very valid reasons: it has a limited number of parameters to be estimated (three or four in its standard version) and the obtained parameters have a direct interpretation that allows for non-technical communication (long-term interest rates, slope and curvature). Moreover there have been new developments in the last decade: the Dynamic Nelson Siegel model (DNS) and more recently an arbitrage-free version of it (AFDNS) – see Diebold and Rudebusch (2013) for a review of the literature for this class of model. One of the drawbacks of the model is its limited capacity to model accurately the dynamics of short term interest rates, especially close to the zero bound and/or with negative interest rates. Even if this issue can be partially solved with a modified functional form - e.g. by increasing the number of parameters or by putting more weight to the short term in the optimization process, the Nelson-Siegel methodology may face collinearity of the factors.

An alternative approach is to use market models. Since the introduction of market models by Brace, Gatarek, Musiela (1997) and Jamshidian (1997), this class of models have gained increased popularity and are widely used in exotic derivatives pricing by market practitioners.

This paper mainly focuses on the “log-normal forward Libor Model”, where forward Libor rates are assumed to be log-normally distributed. Even if the lognormal distribution may not be the best fit, it has the advantage of defining interest rates which are only positive and come up with relatively tractable formulas for the derivatives instruments used for calibration. In fact LMM models based on Monte Carlo techniques provide a compromise between not to unrealistic stochastic simulations and consistency with the pricing of derivatives used for the simulated dynamic strategies. The following section specifies the LMM and highlights the steps that are required to set up the model.

2.1. Generic model specification

Let us consider a general tenor structure $0 = T_0 < T_1 < \cdots < T_n$, and define the corresponding Libor forward rates:

$$F_k(t) = \frac{1}{\tau_k}\left(\frac{P(t,T_k) - P(t,T_{k+1})}{P(t,T_{k+1})}\right), k = 0,1,...,n-1 \tag{1}$$

where:

- $P(t,T_k)$ is the zero coupon bond price with maturity T_k as observed at time t ,
- $\tau_k = T_{k+1} - T_k$ is the accrual period in years. It is usually set as either 0.25 years or 0.5 years, corresponding to the accrual period associated with observable Libor rates.

The dynamics of $F_k(t)$ (k=0,1,…,n-1) are assumed to be driven by a n-dimensional standard Brownian motion associated with physical probability measure P, that is:

$$dF_k(t) = \mu_k^P(t) F_k(t) dt + \sigma_k(t) F_k(t) dW_k^P(t), \quad 0 \le t \le T_k, k = 0,1,\cdots n-1 \tag{2}$$

where the Brownian motions are characterized by correlation

$$dW_i^P(t) dW_j^P(t) = \rho_{ij}(t) dt \tag{3}$$

As pointed out by Fries (2007), the general model can be regarded as a collection of Black Models (1976) that simultaneously evolve under a unified measure.

2.2. Further specifications on volatility and correlation[1]

In order to complete the LMM model set up, one needs to specify a functional form for $\sigma_k(t)$ and $\rho_{ij}(t)$.

In general there are two popular approaches to model $\sigma_k(t)$: deterministic functions and stochastic functions. Within the deterministic forms, the simplest case is to model volatility as a piecewise constant. However it creates discontinuity of the volatility curve in each accrual interval. A more sophisticated deterministic approach is to introduce a parametric form to model the volatility. Stochastic forms can also be introduced to better capture the volatility smile, and have become more popular. A typical example are LMM-SABR[2] models. We model $\sigma_k(t)$ as piecewise constant for simplicity, that is,

$$\sigma_k(t) = \sigma_k, \forall t \tag{4}$$

In practice, the empirical correlation matrices are typically too cumbersome (with n(n-1)/2 parameters), and may contain non-intuitive entries. It is common to choose a parametric correlation structure, which is flexible enough to price many instruments, while at the same time, simple enough to implement. Several parametric forms of correlation have been proposed, a representative form is a two-parameter structure:

$$\rho_{i,j} = \rho_\infty + (1-\rho_\infty) e^{-\beta|T_i - T_j|} \tag{5}$$

where ρ_∞ represents a base correlation between arbitrary rates, and β is the exponential rate of correlation decay as $|T_i - T_j|$ increases.

This form of correlation structure enforces some desired features e.g. the correlation between 10 year forward rates and 3 year forward rates is lower than the correlation between 7 year forward rates and 4 year forward rates.

2.3. Model calibration

The next step is to obtain the model parameters through a calibration process, which is a reverse engineering work where the model parameters are reconstructed from market prices.

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1 The $\mu_k^P(t)$ are not specified here since as shown in section 2.4, by choosing an appropriate equivalent martingale measure, the drift term can be written as a function of $\sigma_k(t)$ and $\rho_{ij}(t)$.

2 SABR model is the family of stochastic volatility models. SABR stand for “stochastic alpha, beta, rho”.

Common practice is to choose a set of liquid market instruments (caps, floors, swaptions), and find the set of parameters that best replicates the observable market prices. Based on different usages of the model, practitioners are required to choose the appropriate instruments.

In our specification, we first calibrate the correlation parameters to historical rates, then use an analytical approximation formula – see Brigo and Mercurio (2006) to calibrate volatilities to ATM-swaption surface (Figure 1 and 2). The calibrated model can then be used for interest rate scenario simulations and derivatives pricing.

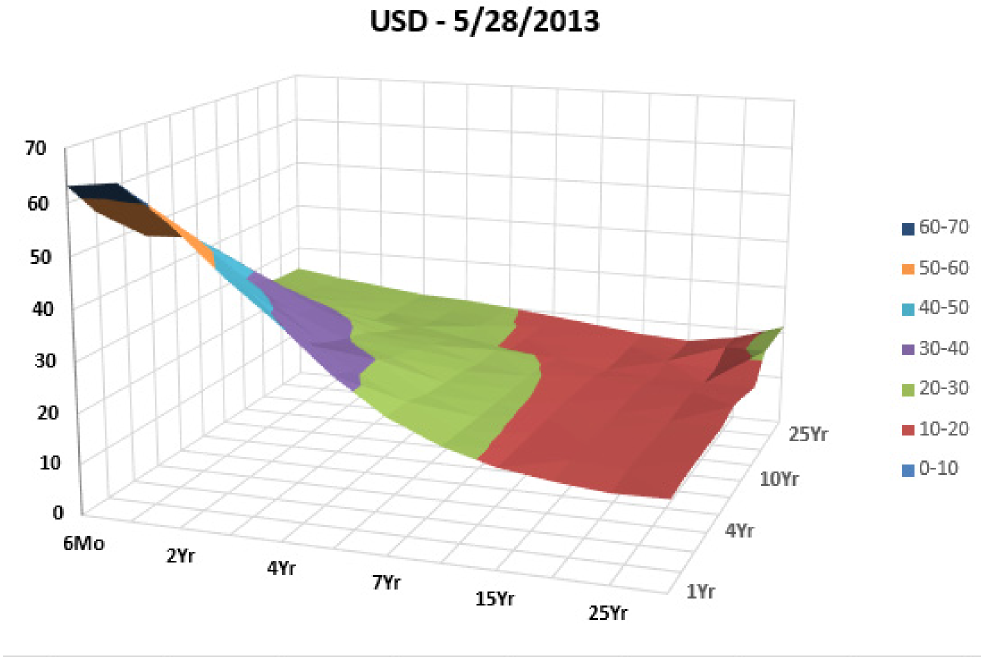

Fig. 1. Implied ATM volatility surface for USD swaptions for different expiry dates (6 months to 30 years) and different underlying swap maturities (1 to 30 years). Source Bloomberg (5/28/2013).

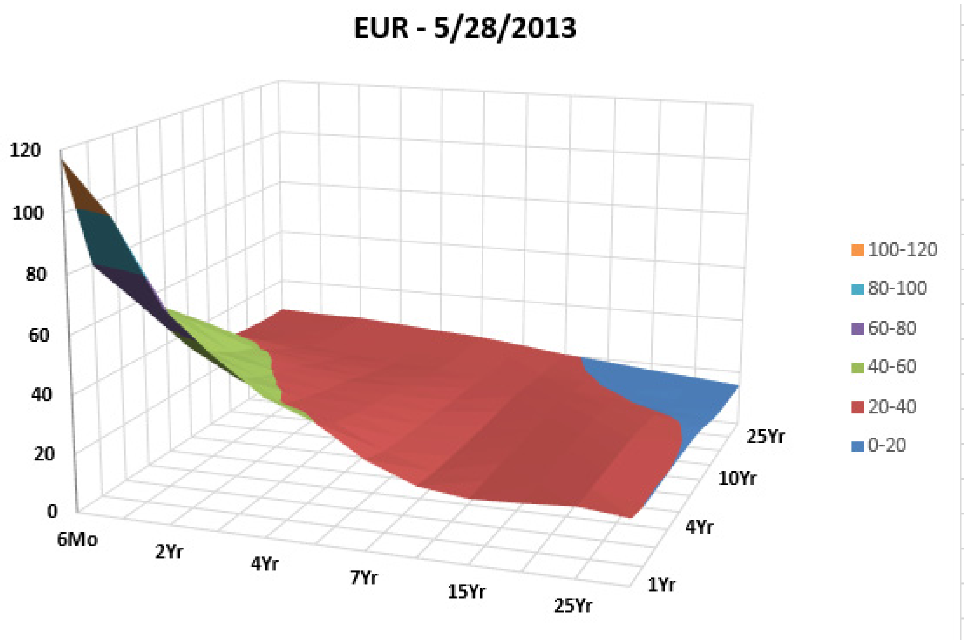

Fig. 2. Implied ATM volatility surface for EUR swaptions for different expiry dates (6 months to 30 years) and different underlying swap maturities (1 to 30 years). Source Bloomberg (5/28/2013).

2.4. Monte-Carlo Simulation

Final step is to simulate forward Libor rates. We begin by choosing a numeraire and the corresponding measure under which the $F_k(t)$ processes are simulated. We choose the discretely rolled over bond account $B(t)$ as the numeraire:

$$B(t) = P(t, T_{m(t)+1}) \prod_{i=0}^{m(t)} (1 + \tau_i F_i(T_i)) \tag{6}$$

where $m(t) = \max\{i : T_i \le t\}$. The corresponding equivalent martingale measure Q^B is known as the spot measure.

As Fries (2007) showed, under the spot measure, the process defined by equation (2) can be rewritten as [3]:

$$dF_k(t) = \mu_k^{Q^B}(t) F_k(t) dt + \sigma_k(t) F_k(t) dW_k^{Q^B}(t), \quad 0 \le t \le T_k, k = 0, 1, \cdots n-1 \tag{7}$$

where drift is given by[4]:

$$\mu_k^{Q_B}(t) = \sigma_k(t) \sum_{i=m(t)}^{k} \frac{\tau_i \rho_{k,i} \sigma_i(t) F_i(t)}{1 + \tau_i F_i(t)} \tag{8}$$

Normally distributed random numbers that drives the Brownian motion based on correlation structure $\rho_{ij}(t)$ can then be generated, the simulation result would be a three dimension cube $F_i(t_j, npath)$. Some simulated rates are

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3 Based on Girsanov's Theorem, change of measure is equivalent to change of drift.

4 $P(t, T_k) / B(t) = \prod_{i=m(t)+1}^{k-1} (1 + \tau_i F_i(t))^{-1} \prod_{i=0}^{m(t)} (1 + \tau_i F_i(T_i))^{-1}$ is a martingale under spot measure leads to the fact that the process has zero drift, from where the $\mu_k^{Q_B}(t)$ term can be derived. For details please see Fries (2007).

shown in Figure 3 and 4.

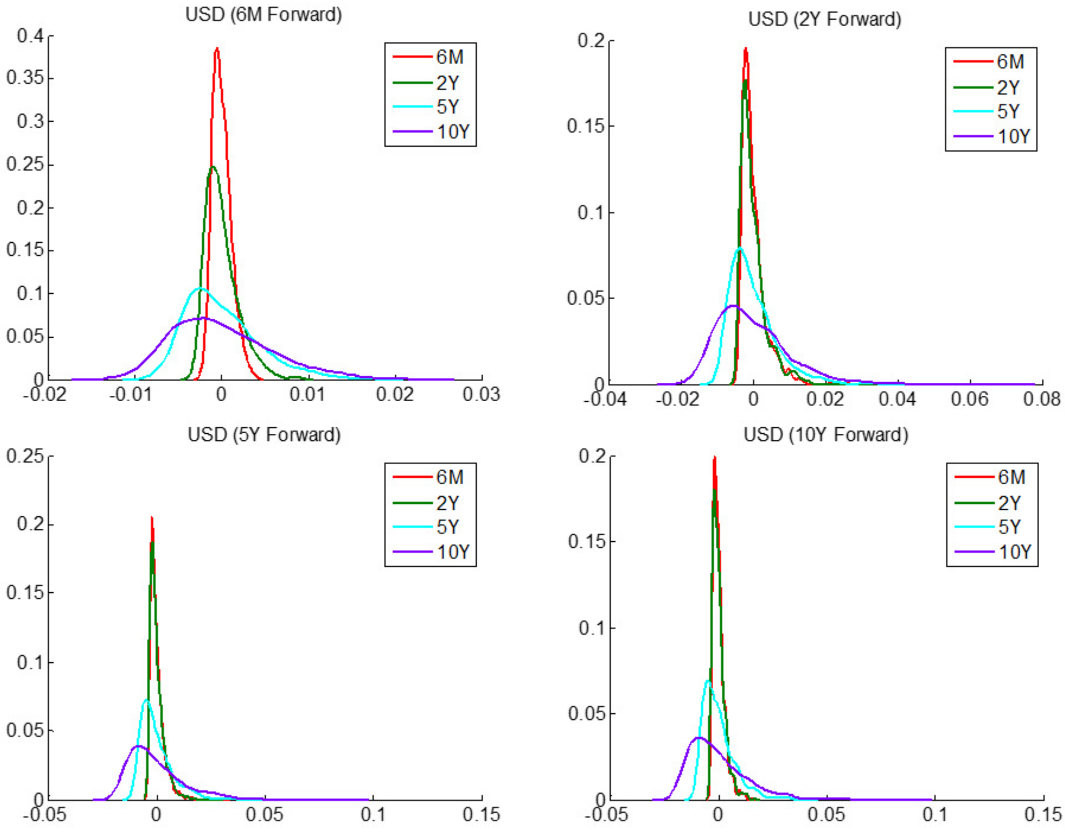

Fig. 3. Simulated USD interest rates for a model calibrated with USD swaptions volatilities on 5/28/2013 for different expiry and maturity dates (6 months, 2-year, 5-year, 10-year).

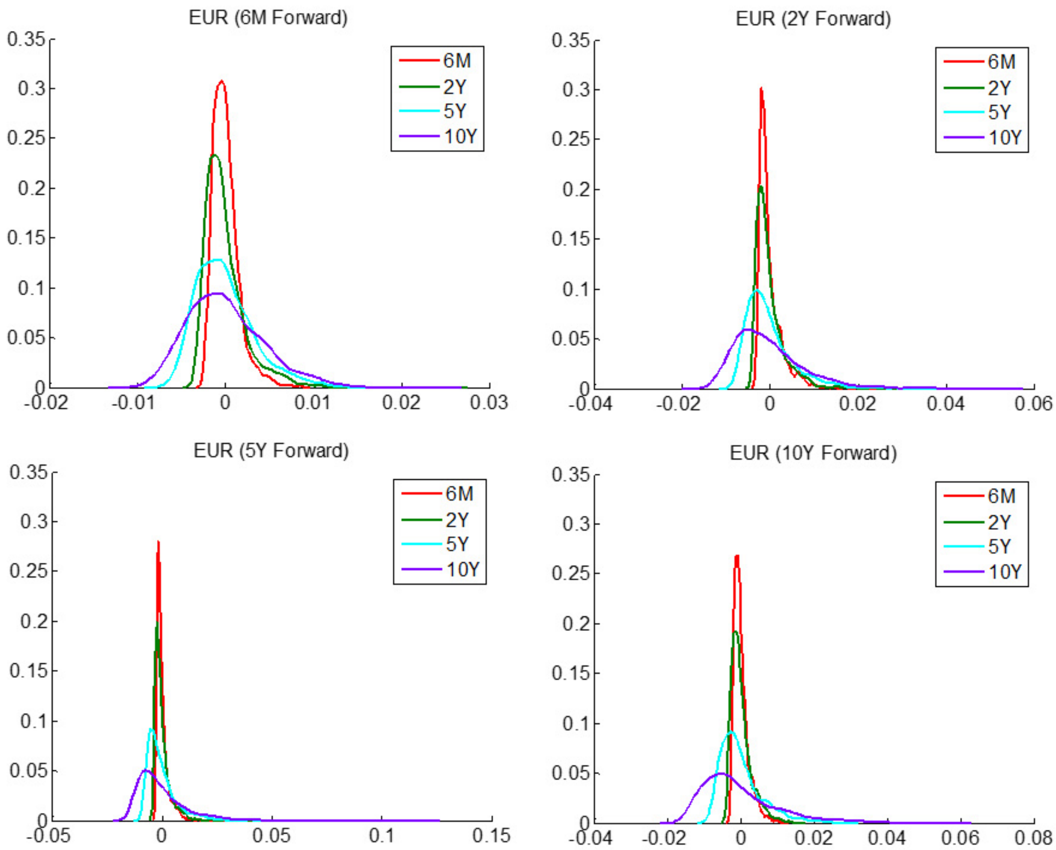

Fig. 4. Simulated EUR interest rates for a model calibrated with EUR swaptions volatilities on 5/28/2013 for different expiry and maturity dates (6 months, 2-year, 5-year, 10-year).

3. Strategy design

In a low interest rate environment, the institutional investor who has to generate income faces a tradeoff between (i) locking medium/long-term interest rates at current low levels but still benefiting from the term premium and (ii) waiting for an interest rate increase that will more than compensate the decrease of the coupon induced by the shortening of the investment horizon as time passes. The investor choice depends on her risk preference, typically:

(i) stability of the income through time: depending on the cash-flow needs, stakeholders may accept more or less volatility in the coming years;

(ii) minimum income (or even maximum income in some cases) that is necessary to cover cash-flow needs e.g. interest rate paid to deposits for central banks, transfers to Ministry of Finance, or other types of liabilities.

Based on the risk preference of the investor, there is an infinite range of strategies that can be accomplished through derivatives or cash instruments. Let us list some of them depending on the degree of uncertainty:

- **Interest rate swaps [5] and combined strategies**. An interest rate swap is an agreement to exchange a floating payment that is linked to short-term interest rates (Libor 3-month or 6-month) for a fixed payment. It allows insuring a fixed coupon for the maturity of the swap. In case of increase of interest rates, the mark-to-market would become negative. As a consequence, it creates an incentive to hold to maturity the position and may leave little room to implement a dynamic strategy to avoid realized losses. There is obviously a potential opportunity cost associated with this type of strategy as it depends on market conditions at the time the trade is put in place.

  A well-known example of combined strategy is the *ladder* of swaps. For example, a 5-year ladder consists in 5 swaps with respective maturities of 1-year, 2-year, …, 5-year with an equally weighted notional (for each swap 20% of the total notional). Then, the strategy is rolled automatically. In one year, the 1-year swap matures and the strategy enters into a new 5-year swap for 20% of the total notional. This type of strategy allows smoothing market conditions through time (Figure 5 and 6).

- **Swaptions and associated strategies**. A swaption[6] is an option that gives its owner the right (not the obligation) to enter into an underlying swap. There exist two types of swaptions: (i) a *receiver* swaption grants the owner the right to enter into a swap where she pays a floating rate and receives a fixed coupon; (ii) a *payer* swaption grants the owner the right to enter into a swap where she pays a fixed coupon and receives a floating rate. To make the strategies comparable, we will focus on the case when the investor can pay a floating rate on a notional amount. Under this constraint, the investor can only be long receiver swaptions and/or short payer swaptions.

- **Interest rate caps and floors[7] and their combinations**. An interest rate cap is a series of european interest rate call options that give the buyer the right to receive payments equal to the positive difference between the underlying floating rate and the strike price. An interest rate floor is a series of european interest rate put options that give the buyer the right to receive payments equal to the positive difference between the strike price and the underlying floating rate.

5 For derivatives instruments, it is assumed that the investor can pay a variable rate for a given notional and will receive a payoff.
6 The discussion here focuses on european swaptions i.e. options that give the right to the owner to enter a swap on the expiration date. The analysis could be extended to swaptions that give the right to enter a swap on any date (american swaptions), or on multiple specified dates (bermudean swaptions).
7 There exist no-arbitrage relations between Caps/Floors and swaptions through the correlation structure of interest rates – see Longstaff, Santa-Clara and Schwartz (2001).

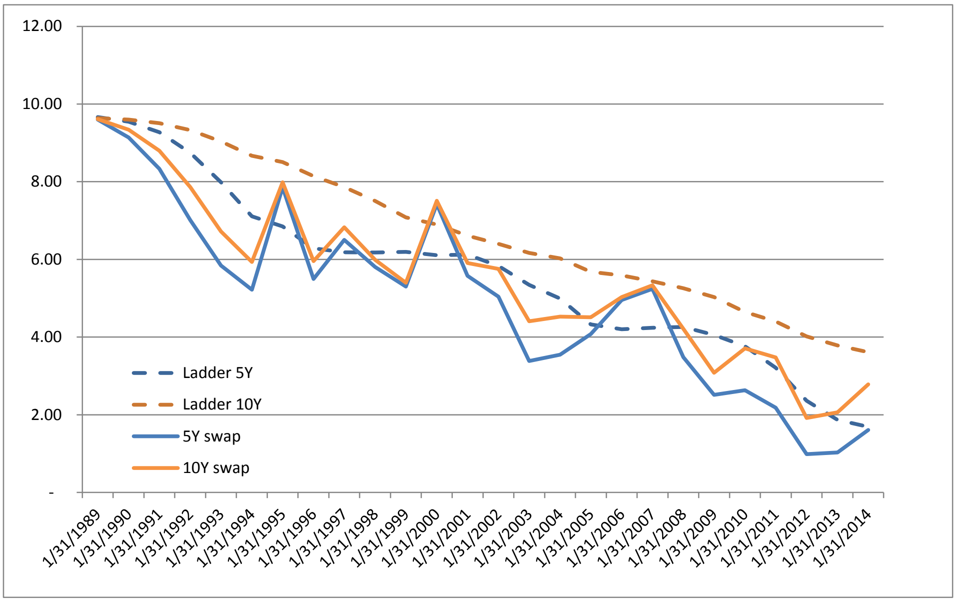

Fig. 5. Smoothing effect for USD 5-year and USD 10-year ladder strategies. The dotted lines correspond to the coupon income that is generated under the systematic ladder strategies. The solid lines represent the 5-year and 10-year USD swap rates. The dataset starts in January 1989.

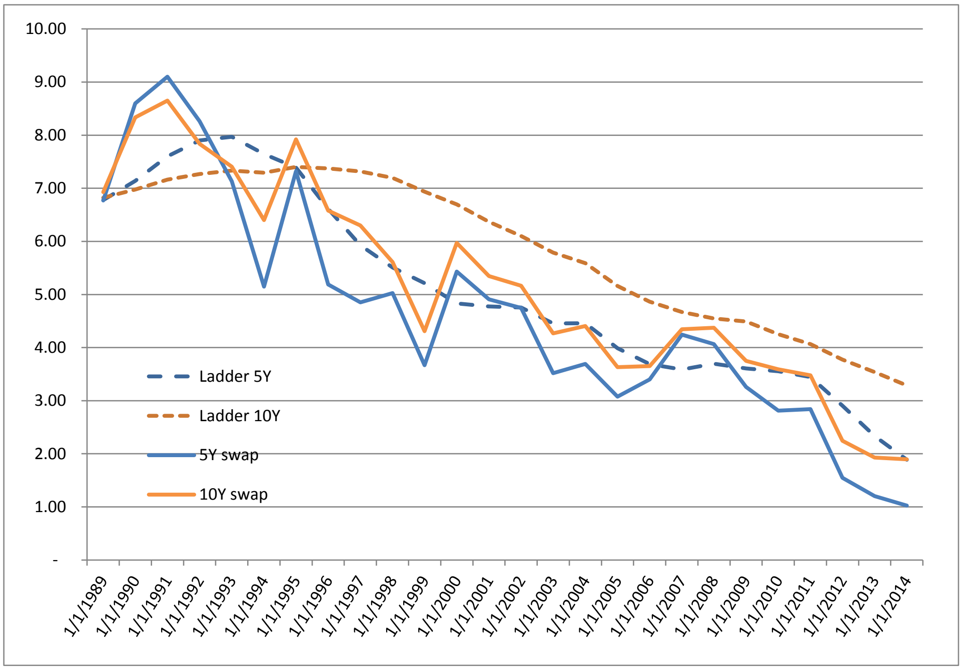

Fig. 6. Smoothing effect for EUR 5-year and EUR 10-year ladder strategies. The dotted lines correspond to the coupon income that is generated under the systematic ladder strategies. The solid lines represent the 5-year and 10-year EUR swap rates. The dataset starts in January 1989.

- Short-term interest rate exposure (3m or 6m Libor). In this paper, it is considered as the "wait-and-see" strategy. It does not benefit from a potential term premium. By definition, the average path of this strategy corresponds to forward rates implied by the spot curve.

In fact, the choice of the dynamic strategy is a direct consequence of the preferences of the investor. What is the typical utility function of the public investor? It often has a very high risk aversion – if not infinite – close to the level of wealth that one wants to be protected (e.g. guarantee of the capital, minimum income level). A simple way to achieve this objective is to enter into a receiver swaption. Figures 7 and 8 below provide the different level of premium the investor has to pay to get the option of locking in a minimum income in n years (expiry date of the swaption) for a period of m years (maturity of the swaption).

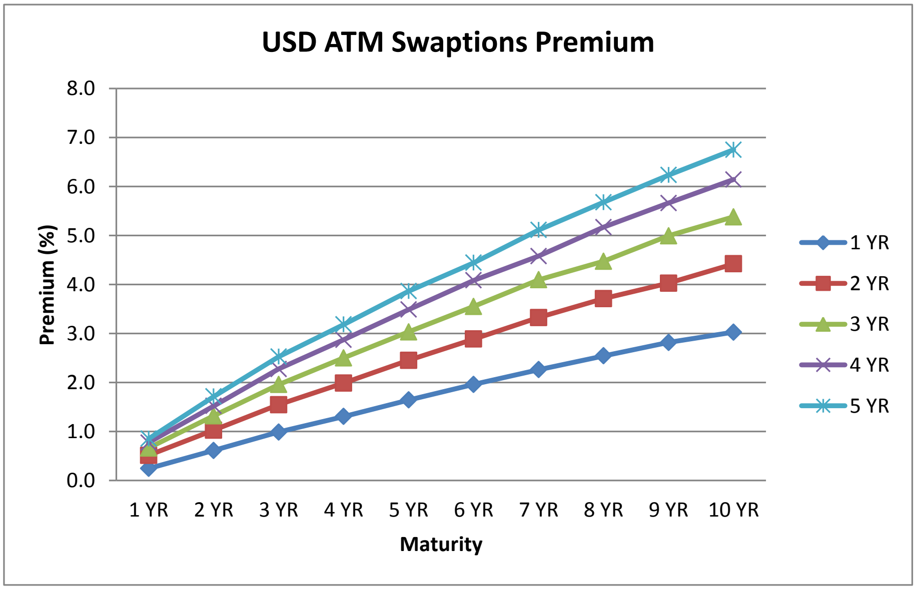

Fig. 7. USD ATM Swaptions Premium (as of market conditions on 10/24/2014) for 1 to 5 year expiry dates.

The longer the maturity during which a minimum income is locked, the higher the premium. Also, the further the expiry of the option, the more the investor will have to pay. All the numbers are reported for At-The-Money (ATM) options i.e. the strike equals the forward rates. For example, if the investor wants to get the option to lock a minimum income in USD of [2.68%] (ATM forward) for 5 years starting in 2 years, she will have to pay a 2.45% premium.

Also some of the public investors are willing to renounce to the highest part of the upside to finance the guarantee of a minimum income by selling a payer swaption. The combination of buying a receiver swaption and selling a payer one is a swaption collar strategy that has been popular among pension funds for the last decade. A special case is a zero-cost collar strategy where the strikes of the payer and receiver are chosen so that the cost of the receiver swaption equals the gain from the payer swaption. Table 1 below provides the higher strike that can be reached for the payer swaption in a zero-cost collar strategy, assuming that the lower strike (receiver) equals the corresponding maturity spot rates. For example, if one wants to guarantee a minimum income of 1.70% for 5 years starting in 3 years (noted 3Y5Y), this can be financed at the cost of renouncing at the potential gains coming from 5y interest rate swap being above 4.62% in 3 years.

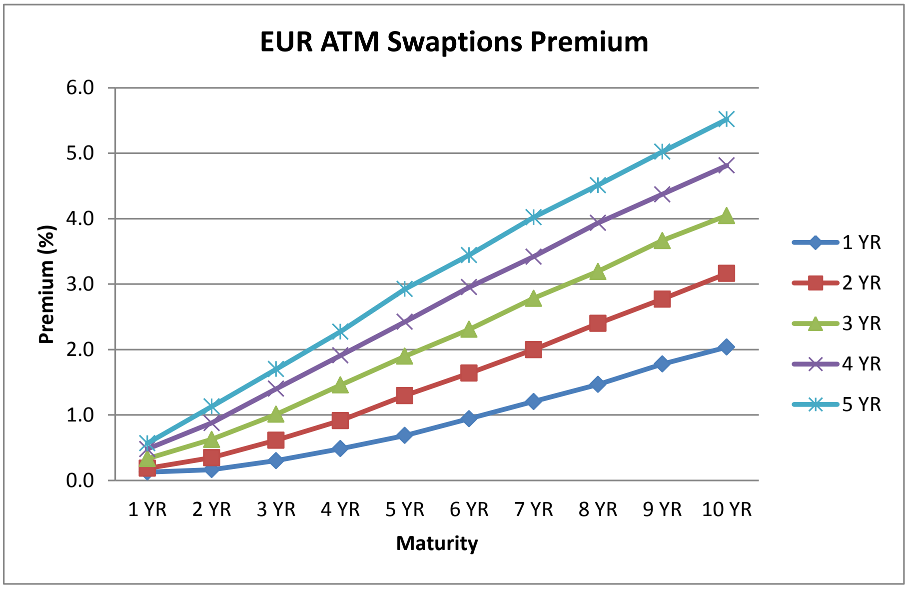

Fig. 8. EUR ATM Swaptions Premium (as of market conditions on 10/24/2014) for 1 to 5 year expiry dates.

Table 1. Higher reachable strike for a zero-cost swaption collar strategy (USD swaptions, 10/24/2014).

Expiry / Maturity	1 YR	2 YR	3 YR	4 YR	5 YR	7 YR	10 YR
Spot	*0.31*	*0.65*	*1.07*	*1.41*	*1.70*	*2.10*	*2.48*
1 YR	2.36	2.68	2.78	2.84	2.89	3.01	3.12
2 YR	4.87	4.51	4.31	4.02	3.96	3.80	3.75
3 YR	5.73	5.67	5.23	4.82	4.62	4.36	4.17
4 YR	6.78	6.55	6.20	5.46	5.15	4.66	4.44
5 YR	7.14	7.08	6.63	5.87	5.52	5.02	4.61

Interestingly, this type of zero-cost collar strategy is relatively stable through time. For the purpose of comparison with the ladder strategies above, historical values of receiver and payer strikes have been computed for the last 14 years (Figure 9). The strikes have been chosen so that the probability of the receiver swaption to be exercised is 1/6.

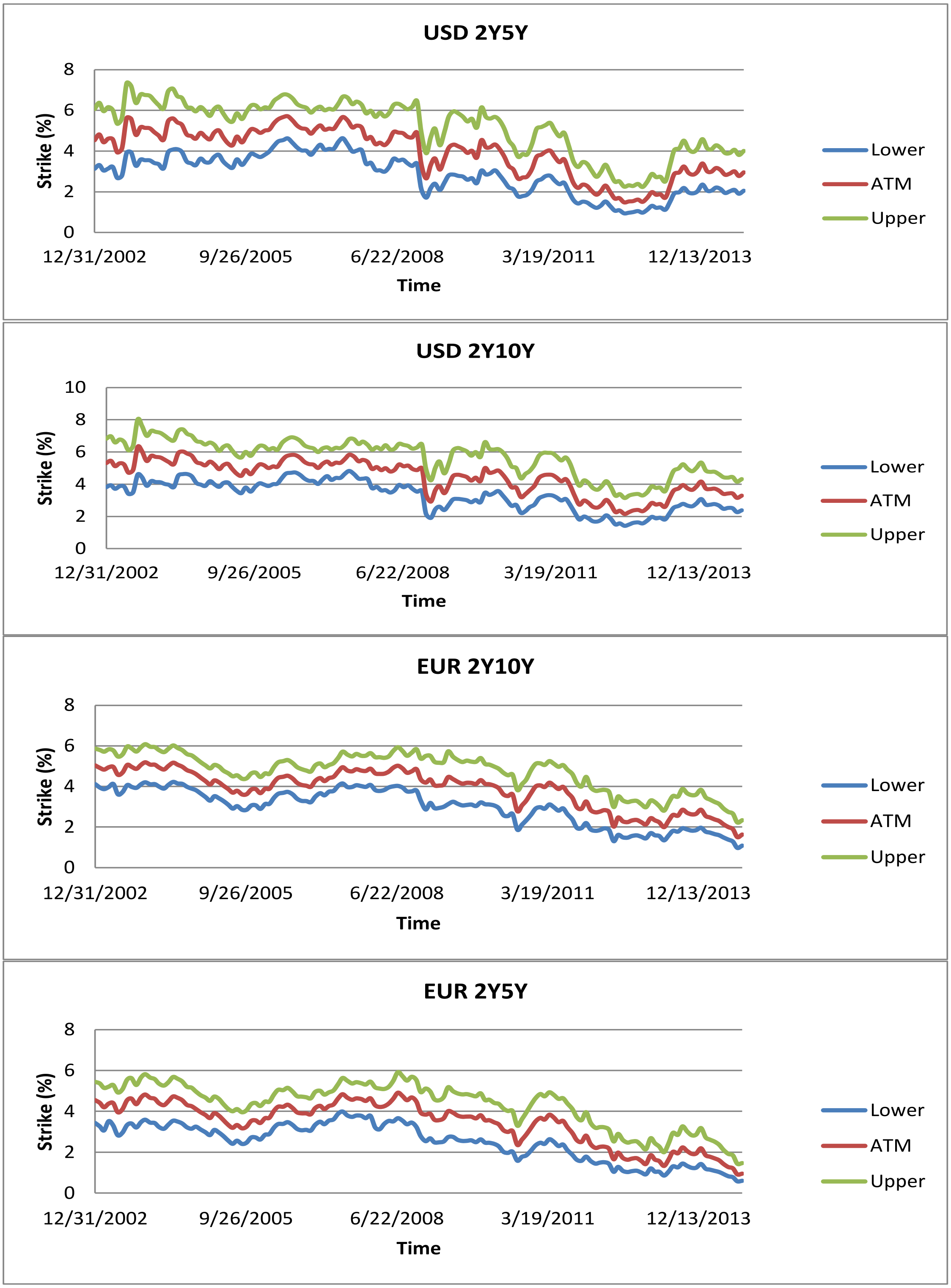

Fig. 9.Strikes for 2Y5Y and 2Y10Y collar swaption strategies in EUR and USD

4. Framework and governance

The investment framework and the governance associated with dynamic strategies are critical for their implementation. As often in standard investment processes, a clear separation between strategic and tactical decisions is useful. Strategic benchmark may be defined either through a reference strategy that is systematic or with a stream of income objectives for each year, for example aligned to specific liabilities. Tactical decisions may be defined within an ex-ante risk budget.

Second, the mark-to-market may trigger investment decisions. For example, in case of high MtM, the stakeholder may want to liquidate at least partially the holdings to benefit from strategically asymmetric economic environment. It is the case of long duration holdings that have been invested years before interest rates have reached close to zero levels. This type of liquidation rules could obviously be very efficient if interest rates exhibit mean reversion in the medium-term as it allows for the ‘crystallization’ of gains. On the contrary, the investor may have to liquidate in case of significant negative MtM, because of stop-loss type of rules or liquidity constraints – due to collateral posting on derivatives for example. If positive or negative MtM triggers investment decisions, then the strategy becomes path-dependent i.e. the distribution of outcomes of the strategy depends on the trajectory of underlying assets.

As usual, the decision variables have to be properly chosen. Here are a list of potential measures that may be of interest to practitioners:

- Minimum income/Expected Income generated by different strategies within certain horizon
- Probability of negative return/Conditional Value-at-Risk
- Potential Future Exposure/Expected Positive Exposure

All these measures can be obtained using Monte Carlo simulations of the LMM model presented in Section 2. Figure 10 below gives some intuition on the income distribution characters associated with different strategies.

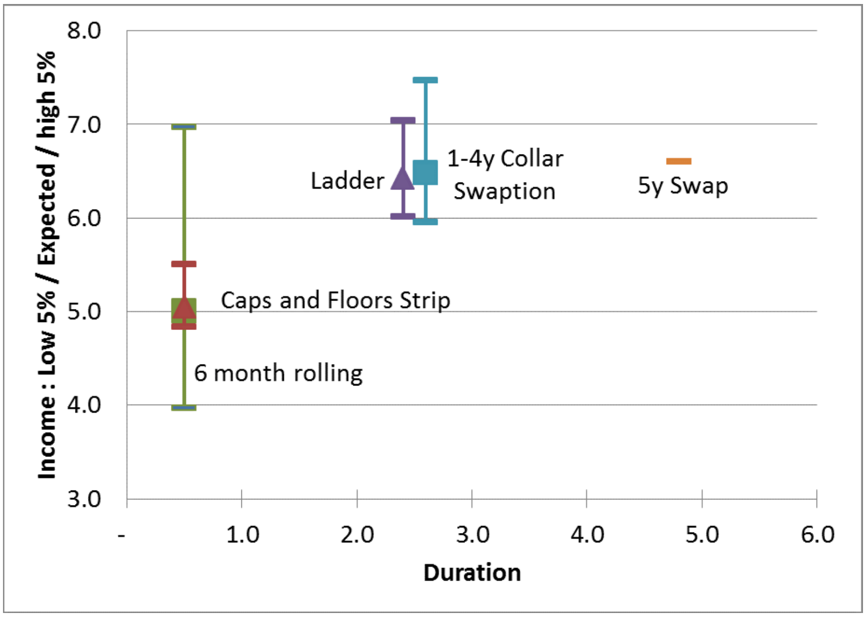

Fig. 10. Duration and ex-ante income simulations (average first 2 years) associated with strategies

5. Conclusion

This short paper presented some strategies to generate income with interest rate derivatives. We explored different types of strategies, with a focus on the collar swaption strategy as it can be a good candidate given the risk preference of public institutional investors. A natural extension is to study different set of rules when the realized underlying swap rate is between the payer and receiver strikes at the time of expiry of the swaptions (no exercise). An interesting topic would consist in linking the strategy to the monetary policy in zero lower bound environments.

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