

Economic growth and comparative development

Presentation of a paper

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The Interaction between the Quantity and Quality of Children By: Gary S. Becker and H. Gregg Lewis

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- **Other papers:** Some authors have argued that the negative relation between quantity and quality often observed is a consequence of a low substitution elasticity in a family's utility function between parents' consumption or level of living and that of the children. (Duesenberry 1960; Willis 1969)

- **This paper:** different but it makes equally special assumptions about the substitution between **quantity** and **quality** in the utility function and in household production. (Becker, Lewis. p. S279)

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Utility function

$$U = U(n, q, y)$$

s.t

$$I = nq\pi + y\pi_y$$

Where n is the number of children, q the quality of the children (assumed to be the same for all the children), y the rate of consumption of all other commodities, I the total income, π the price of nq and π_y the price of y .

First order conditions

$$MU_n = \lambda q \pi = \lambda p_n$$

$$MU_q = \lambda n \pi = \lambda p_q$$

$$MU_y = \lambda p_y = \lambda p_y$$

MU's are the marginal utilities, p's are the marginal costs or shadow price of children with respect to number p_n is positively related to q , the level of quality (p_q) is positively related to n , the number of children.

Interpretations

- An increase in quality is more expensive if there are more children.
- An increase in quantity is more expensive if the children are of higher quality.

The implication that this may have in price effects and income haven't been explored.

Income effects Income elasticity's of demand for the number (n) of children that has a quality and for all other commodities (y) be Υ_n , Υ_q and Υ_y respectively. This elasticity's are derived by the changes in income while holding prices constant. The appropriate prices for this purpose are the shadow prices (wich are the marginal rates of sustitution in the utility function) p_n , p_q , and p_y .

For all of this **Income** may be:

$$R = np_n + qp_q + yp_y$$

$$R = I + nq\pi$$

Which is the total expenditure on the number of children, their quality and y , calculated at the shadow prices.

The **mean value** of the true **income elasticities** is:

$$1 = \frac{np_n}{R} \gamma_n + \frac{qp_q}{R} \gamma_q + \frac{yp_y}{R} \gamma_y$$

The **weighet mean** of the observed income elasticities is

$$\frac{I}{R} = \frac{I}{I + nq\pi}$$

This is less than a unity: that is:

$$1 > \frac{I}{R} = \frac{I}{I + nq\pi}$$

$$1 > \frac{I}{R} = \frac{np_n}{R} \Upsilon_n + \frac{qp_q}{R} \Upsilon_q + \frac{yp_y}{R} \Upsilon_y +$$

Where all the Υ 's are fixed. All of this means that on the average, the observed elasticities are smaller than the true elasticities on the ratio $\frac{I}{R}$

If l increase, n , q and y increase. If n and q increase, then the shadow prices p_n and p_q rise.

- 1 Money income increase - wage rates increase = price effect
- 2 In ratio terms the increase is less in real income than in money income because the cost of producing commodities in the household increase by the rise in the price.

- The true income elasticity with respect to quality (Υ_q) is larger than that with respect to quantity.
- The observed elasticity for quantity (Υ_n) may be negative even though the true elasticity (Υ_n) is not.
- Υ_q declines as income I rises

Price effects The budget constraint is:

$$p_n = n\pi_n + nq\pi + q\pi_q + yp_y$$

So that the shadow prices or marginal costs are now

$$p_n = \pi_n + q\pi$$

$$p_n = \pi_q + n\pi p_y = \pi_y$$

These shadow prices for n and q each contain a "fixed" component: π .

The component π in child costs consist of costs that depend on quantity but not in quality.

Examples: Prenatal child costs, such as maternity care.

Interpretations

- A reduction in the shadow prices of quality

$$p_q = p i_q + n \pi$$

. This induces a substitution in favor of quality.

- This results in a **fall of quantity** and an **increase in quality**.
- Also, an increase in the education of parents

$$\pi_q$$

make a **fall of quantity** and an **increase in quality**. This is also an increase in the shadow price of quantity

$$(p_n = \pi_n + q_\pi)$$

- ① Quantity and quality are closely related because the shadow price of quality depends on quantity and the shadow price of quantity depends on quality.
- ② The income compensated elasticity of quantity with respect to equal percentage changes in

$$\pi_n, \pi_q \text{ and } \pi$$

tends to be greater than the elasticity for quality.

- ③ **Example:** An increase in women's wage rates reduces the number of children by a much bigger percentage than the quality of children.

- The observed price elasticity of quantity exceeds that of quality, just the opposite of our conclusion for observed income elasticity's.
- This reversal is not only unexpected, but it also gives a consistent interpretation to literature existence.