$$P_1 = \epsilon_{\alpha\beta}\, v^\alpha w^\beta , \qquad P_2 = \epsilon_{\alpha\beta}\, w^\alpha u^\beta , \qquad P_3 = \epsilon_{\alpha\beta}\, u^\alpha v^\beta$$

$$\bar P_1 = \epsilon_{\dot\alpha\dot\beta}\, \bar v^{\dot\alpha} \bar w^{\dot\beta} , \qquad \bar P_2 = \epsilon_{\dot\alpha\dot\beta}\, \bar w^{\dot\alpha} \bar u^{\dot\beta} , \qquad \bar P_3 = \epsilon_{\dot\alpha\dot\beta}\, \bar u^{\dot\alpha} \bar v^{\dot\beta}$$

$$Q_1 = \hat X_{\alpha\dot\alpha}\, v^\alpha \bar w^{\dot\alpha} , \qquad Q_2 = \hat X_{\alpha\dot\alpha}\, w^\alpha \bar u^{\dot\alpha} , \qquad Q_3 = \hat X_{\alpha\dot\alpha}\, u^\alpha \bar v^{\dot\alpha}$$

$$\tilde Q_1 = \hat X_{\alpha\dot\alpha}\, w^\alpha \bar v^{\dot\alpha} , \qquad \tilde Q_2 = \hat X_{\alpha\dot\alpha}\, u^\alpha \bar w^{\dot\alpha} , \qquad \tilde Q_3 = \hat X_{\alpha\dot\alpha}\, v^\alpha \bar u^{\dot\alpha}$$

$$Z_1 = \hat X_{\alpha\dot\alpha}\, u^\alpha \bar u^{\dot\alpha} , \qquad Z_2 = \hat X_{\alpha\dot\alpha}\, v^\alpha \bar v^{\dot\alpha} , \qquad Z_3 = \hat X_{\alpha\dot\alpha}\, w^\alpha \bar w^{\dot\alpha}$$

Fermionic : Notes: $\hat\Theta_\alpha = \dfrac{\Theta_\alpha}{X^{1/2}}$; $\hat{\bar\Theta}_{\dot\alpha} = \dfrac{\bar\Theta_{\dot\alpha}}{X^{1/2}}$.

$$R_1 = \epsilon_{\alpha\beta}\, u^\alpha \hat\Theta^\beta , \qquad R_2 = \epsilon_{\alpha\beta}\, v^\alpha \hat\Theta^\beta , \qquad R_3 = \epsilon_{\alpha\beta}\, w^\alpha \hat\Theta^\beta$$

$$\tilde R_1 = \epsilon_{\dot\alpha\dot\beta}\, \bar u^{\dot\alpha} \hat{\bar\Theta}^{\dot\beta} , \qquad \tilde R_2 = \epsilon_{\dot\alpha\dot\beta}\, \bar v^{\dot\alpha} \hat{\bar\Theta}^{\dot\beta} , \qquad \tilde R_3 = \epsilon_{\dot\alpha\dot\beta}\, \bar w^{\dot\alpha} \hat{\bar\Theta}^{\dot\beta}$$

$$S_1 = \hat X_{\alpha\dot\alpha}\, u^\alpha \hat{\bar\Theta}^{\dot\alpha} , \qquad S_2 = \hat X_{\alpha\dot\alpha}\, v^\alpha \hat{\bar\Theta}^{\dot\alpha} , \qquad S_3 = \hat X_{\alpha\dot\alpha}\, w^\alpha \hat{\bar\Theta}^{\dot\alpha} ,$$

$$\tilde S_1 = \hat X_{\alpha\dot\alpha}\, \bar u^{\dot\alpha} \hat\Theta^\alpha , \qquad \tilde S_2 = \hat X_{\alpha\dot\alpha}\, \bar v^{\dot\alpha} \hat\Theta^\alpha , \qquad \tilde S_3 = \hat X_{\alpha\dot\alpha}\, \bar w^{\dot\alpha} \hat\Theta^\alpha .$$

$$J = \hat X_{\alpha\dot\alpha}\, \hat\Theta^\alpha \hat{\bar\Theta}^{\dot\alpha}$$

Imposing superfield conservation equations $\left(\mathcal{D}_\alpha = \dfrac{\partial}{\partial\Theta^\alpha} - 2i\,\bar\Theta^{\dot\alpha}\, \partial_{x^{\alpha\dot\alpha}} \right)$

- Need to derive $\mathcal{D}^\alpha Q_i$, $\mathcal{D}^\alpha \tilde Q_i$, $\mathcal{D}^\alpha Z_i$
  $\mathcal{D}^\alpha R_i$, $\mathcal{D}^\alpha \tilde R_i$, $\mathcal{D}^\alpha S_i$, $\mathcal{D}^\alpha \tilde S_i$
  $\mathcal{D}^\alpha J$.

  $\bar{\mathcal{D}}^{\dot\alpha} = \dfrac{\partial}{\partial \bar\Theta_{\dot\alpha}} \Rightarrow \bar{\mathcal{D}}^{\dot\alpha} \tilde R_i$, $\bar{\mathcal{D}}^{\dot\alpha} S_i$, $\bar{\mathcal{D}}^{\dot\alpha} J$

- Similarly, to impose conservation on Z_2 , we compute the actions of $Q_\alpha = \dfrac{\partial}{\partial\Theta^\alpha}$ and $\bar Q^{\dot\alpha} = \dfrac{\partial}{\partial\bar\Theta_{\dot\alpha}} + 2i\,\Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha} \dfrac{\partial}{\partial X^a}$ on the basis structures.

Building blocks in 4D CFT:

$$I(X) \quad I(\bar{X})$$

$$\mathcal{D}^\alpha Q_1 = \mathcal{D}^\alpha \left[\hat{X}_{\mu\dot\mu}\, v^\mu \bar\omega^{\dot\mu} \right]$$

$$= \left[-\frac{\partial}{\partial \Theta_\alpha} + 2i\, \bar\Theta_{\dot\alpha}\, \partial_{(x)}^{\dot\alpha\alpha} \right] \left[\hat{X}_{\mu\dot\mu}\, v^\mu \bar\omega^{\dot\mu} \right]$$

$$\cancel{\partial_{(x)}^{\dot\alpha\alpha} \hat{X}_{\mu\dot\mu} = (\tilde\sigma^a)^{\dot\alpha\alpha}\, \partial_{(x)a} \cdot (\sigma^b)_{\mu\dot\mu} \frac{X_b}{X}}$$
$$= \cancel{-2\, \delta^\alpha_\mu \delta}$$

$$= 2i\, \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \left[(\sigma^b)_{\mu\dot\mu} \frac{X_b}{X}\, v^\mu \bar\omega^{\dot\mu} \right]$$

$$= 2i\, \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} (\sigma^b)_{\mu\dot\mu}\, v^\mu \bar\omega^{\dot\mu}\, \underbrace{\frac{\partial}{\partial X^a}\left(\frac{X_b}{X} \right)}_{= \frac{\eta_{ab}}{X} - \frac{\hat X_a \hat X_b}{X}}$$

$$= 2i\, \frac{\bar\Theta_{\dot\alpha}}{X} (\tilde\sigma^a)^{\dot\alpha\alpha} (\sigma_a)_{\mu\dot\mu}\, v^\mu \bar\omega^{\dot\mu} - 2i\, \frac{\bar\Theta_{\dot\alpha}}{X} (\tilde\sigma^a)^{\dot\alpha\alpha} (\sigma^b)_{\mu\dot\mu}\, v^\mu \bar\omega^{\dot\mu}\, \hat X_a \hat X_b$$

$$= -4i\, \frac{\bar\Theta_{\dot\alpha}}{X}\, v^\alpha \bar\omega^{\dot\alpha} - 2i\, \frac{\bar\Theta_{\dot\alpha}}{X}\, \hat X^{\dot\alpha\alpha}\, \hat X_{\mu\dot\mu}\, v^\mu \bar\omega^{\dot\mu}$$

$$= -4i\, \frac{v^\alpha}{X^{1/2}}\, \bar R_3 - 2i\, \frac{(\hat X \cdot \hat{\bar\Theta})^\alpha}{X^{1/2}}\, Q_1$$

$$\boxed{\mathcal{D}^\alpha Q_1 = -\frac{2i}{X^{1/2}} \left[2 v^\alpha \tilde R_3 + \hat X^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Q_1 \right]}$$

$$\boxed{\begin{aligned} \mathcal{D}^\alpha Q_2 &= -\frac{2i}{X^{1/2}} \left[2 w^\alpha \tilde R_1 + \hat X^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Q_2 \right] \\ \mathcal{D}^\alpha Q_3 &= -\frac{2i}{X^{1/2}} \left[2 u^\alpha \tilde R_2 + \hat X^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Q_3 \right]. \end{aligned}}$$

$$\mathcal{D}^\alpha \tilde Q_1 = -\frac{2i}{X^{1/2}} \left[2 w^\alpha \tilde R_2 + \hat X^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} \tilde Q_1 \right]$$

$$\mathcal{D}^\alpha \tilde Q_2 = -\frac{2i}{X^{1/2}} \left[2 u^\alpha \tilde R_3 + \hat X^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} \tilde Q_2 \right]$$

$$\mathcal{D}^\alpha \tilde Q_3 = \frac{-2i}{X^{1/2}} \left[2 v^\alpha \tilde R_1 + \hat X^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} \tilde Q_3 \right]$$

$$\frac{\partial}{\partial X^a}\left(X_b \, (X^2)^{-1/2} \right) \qquad \frac{\partial}{\partial X^a} X^c X_c$$

$$= \eta_{ab} \, (X^2)^{-1/2} + X_b \left(-\tfrac{1}{2}\right) (X^2)^{-3/2} \cdot 2 \delta_a{}^c X_c$$

$$= \frac{\eta_{ab}}{X} - \frac{X_b \, X_a}{X^3}$$

$$= \frac{\eta_{ab}}{X} - \frac{\hat{X}_a \hat{X}_b}{X}$$

$$\begin{cases} \mathcal{D}^\alpha Z_1 = \dfrac{-2i}{X^{1/2}} \left[2u^\alpha \tilde{R}_1 + \hat{X}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Z_1 \right] \\ \mathcal{D}^\alpha Z_2 = -\dfrac{2i}{X^{1/2}} \left[2v^\alpha \tilde{R}_2 + \hat{X}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Z_2 \right] \\ \mathcal{D}^\alpha Z_3 = -\dfrac{2i}{X^{1/2}} \left[2w^\alpha \tilde{R}_3 + \hat{X}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Z_3 \right] \end{cases}$$

$$\mathcal{D}^\alpha R_1 = \left[-\frac{\partial}{\partial\Theta_\alpha} + 2i\, \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \right] \left[\frac{u^\beta \Theta_\beta}{X^{1/2}} \right]$$

$$= \left[\text{using } \frac{\partial}{\partial\Theta_\alpha} \Theta_\beta = \delta^\alpha{}_\beta \;\&\; \text{and } \frac{\partial}{\partial X^a} X^{-p} = -p\, X^{-(p+2)} X_a \right]$$

$$= -\frac{u^\alpha}{X^{1/2}} + 2i\, \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} u^\beta \Theta_\beta \left(-\tfrac{1}{2}\right) X^{-5/2} X_a$$

$$= -\frac{u^\alpha}{X^{1/2}} - \frac{i}{X^{5/2}} \bar\Theta_{\dot\alpha} X^{\dot\alpha\alpha} (u.\Theta)$$

$$= -\frac{u^\alpha}{X^{1/2}} - \frac{i}{X^{1/2}} \hat{\bar\Theta}_{\dot\alpha} \hat{X}^{\dot\alpha\alpha} R_1$$

$$\begin{cases} \mathcal{D}^\alpha R_1 = -\dfrac{u^\alpha}{X^{1/2}} - \dfrac{i}{X^{1/2}} \hat{\bar\Theta}_{\dot\alpha} \hat{X}^{\dot\alpha\alpha} R_1 \\ \mathcal{D}^\alpha R_2 = -\dfrac{v^\alpha}{X^{1/2}} - \dfrac{i}{X^{1/2}} \hat{\bar\Theta}_{\dot\alpha} \hat{X}^{\dot\alpha\alpha} R_2 \\ \mathcal{D}^\alpha R_3 = -\dfrac{w^\alpha}{X^{1/2}} - \dfrac{i}{X^{1/2}} \hat{\bar\Theta}_{\dot\alpha} \hat{X}^{\dot\alpha\alpha} R_3 \end{cases}$$

$$\mathcal{D}^\alpha \tilde{R}_1 = 2i\, \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \left[\frac{\bar u^{\dot\beta} \bar\Theta_{\dot\beta}}{X^{1/2}} \right]$$

$$= 2i\, \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \bar u^{\dot\beta} \bar\Theta_{\dot\beta} \left(-\tfrac{1}{2}\right) X^{-5/2} X_a$$

$$= -i \frac{\bar\Theta_{\dot\alpha} \bar\Theta_{\dot\beta}}{X^{5/2}} \bar u^{\dot\beta} X^{\dot\alpha\alpha}$$

$$= \frac{i}{2} \bar\Theta^2 \frac{\bar u_{\dot\alpha} X^{\dot\alpha\alpha}}{X^{5/2}} = \frac{i}{2} \hat{\bar\Theta}^2 \frac{\bar u_{\dot\alpha} \hat{X}^{\dot\alpha\alpha}}{X^{1/2}}$$

$$\begin{cases} \mathcal{D}^\alpha \tilde{R}_1 = \dfrac{i}{2} \hat{\bar\Theta}^2 \dfrac{\bar u_{\dot\alpha} \hat{X}^{\dot\alpha\alpha}}{X^{1/2}} \\ \mathcal{D}^\alpha \tilde{R}_2 = \dfrac{i}{2} \hat{\bar\Theta}^2 \dfrac{\bar v_{\dot\alpha} \hat{X}^{\dot\alpha\alpha}}{X^{1/2}} \\ \mathcal{D}^\alpha \tilde{R}_3 = \dfrac{i}{2} \hat{\bar\Theta}^2 \dfrac{\bar w_{\dot\alpha} \hat{X}^{\dot\alpha\alpha}}{X^{1/2}} \end{cases}$$

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$$\mathcal{D}^\alpha S_1 = \left[-\frac{\partial}{\partial \Theta_\alpha} + 2i \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \right] \left[\hat X_{\beta\dot\beta} u^\beta \frac{\bar\Theta^{\dot\beta}}{X^{1/2}} \right]$$

$$= 2i \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} u^\beta \bar\Theta^{\dot\beta} \frac{\partial}{\partial X^a} \left[\frac{X_b}{X^{3/2}} \right] (\sigma^b)_{\beta\dot\beta}$$

$$= 2i \bar\Theta_{\dot\alpha} \bar\Theta^{\dot\beta} (\tilde\sigma^a)^{\dot\alpha\alpha} u^\beta (\sigma^b)_{\beta\dot\beta} \left[\frac{\eta_{ab}}{X^{3/2}} - \frac{3}{2} X^{-3/2} \hat X_a \hat X_b \right]$$

$$= \frac{2i \bar\Theta_{\dot\alpha} \bar\Theta^{\dot\beta}}{X^{3/2}} (-2) \delta^\alpha{}_\beta \delta^{\dot\alpha}{}_{\dot\beta} u^\beta - \frac{3i}{X^{3/2}} \bar\Theta_{\dot\alpha} \bar\Theta_{\dot\beta} u_\beta \hat X^{\dot\alpha\alpha} \hat X^{\dot\beta\beta}$$

$$= -\frac{4i \bar\Theta_{\dot\alpha} \bar\Theta^{\dot\alpha} u^\alpha}{X^{3/2}} - \frac{3i}{2} \frac{1}{X^{3/2}} \bar\Theta^2 u^\beta \hat X^{\dot\alpha\alpha} \hat X_{\dot\alpha\beta}$$

$$= -\frac{4i \hat{\bar\Theta}^2 u^\alpha}{X^{1/2}} + \frac{3i}{2} \frac{\hat{\bar\Theta}^2}{X^{1/2}} u^\alpha$$

$$= -\frac{5i}{2} \frac{\hat{\bar\Theta}^2 u^\alpha}{X^{1/2}}.$$

$$\begin{cases} \mathcal{D}^\alpha S_1 = -\dfrac{5i}{2} \dfrac{\hat{\bar\Theta}^2 u^\alpha}{X^{1/2}} \\ \mathcal{D}^\alpha S_2 = -\dfrac{5i}{2} \dfrac{\hat{\bar\Theta}^2 v^\alpha}{X^{1/2}} \\ \mathcal{D}^\alpha S_3 = -\dfrac{5i}{2} \dfrac{\hat{\bar\Theta}^2 w^\alpha}{X^{1/2}} \end{cases}$$

$$\mathcal{D}^\alpha \tilde S_1 = \left[-\frac{\partial}{\partial \Theta_\alpha} + 2i \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \right] \left[\frac{X^{\dot\beta\beta} \bar u_{\dot\beta} \Theta_\beta}{X^{3/2}} \right]$$

$$= -\frac{X^{\dot\beta\alpha} \bar u_{\dot\beta}}{X^{3/2}} + 2i \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} (\tilde\sigma^b)^{\dot\beta\beta} \Theta_\beta \bar u_{\dot\beta} \frac{\partial}{\partial X^a} \left[\frac{X_b}{X^{3/2}} \right]$$

$$= -\frac{\hat X^{\dot\beta\alpha} \bar u_{\dot\beta}}{X^{1/2}} + 2i \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} (\tilde\sigma^b)^{\dot\beta\beta} \Theta_\beta \bar u_{\dot\beta} \left[\frac{\eta_{ab}}{X^{3/2}} - \frac{3}{2} X^{-3/2} \hat X_a \hat X_b \right]$$

$$\begin{cases} \mathcal{D}^\alpha \tilde S_1 = -\dfrac{1}{X^{1/2}} \left[\hat X^{\dot\alpha\alpha} \bar u_{\dot\alpha} - 4i \hat{\bar\Theta}^\alpha \tilde R_1 + 3i \hat{\bar\Theta}_{\dot\alpha} \hat X^{\dot\alpha\alpha} \tilde S_1 \right] \\ \mathcal{D}^\alpha \tilde S_2 = -\dfrac{1}{X^{1/2}} \left[\hat X^{\dot\alpha\alpha} \bar v_{\dot\alpha} - 4i \hat{\bar\Theta}^\alpha \tilde R_2 + 3i \hat{\bar\Theta}_{\dot\alpha} \hat X^{\dot\alpha\alpha} \tilde S_2 \right] \\ \mathcal{D}^\alpha \tilde S_3 = -\dfrac{1}{X^{1/2}} \left[\hat X^{\dot\alpha\alpha} \bar w_{\dot\alpha} - 4i \hat{\bar\Theta}^\alpha \tilde R_3 + 3i \hat{\bar\Theta}_{\dot\alpha} \hat X^{\dot\alpha\alpha} \tilde S_3 \right]. \end{cases}$$

$$\begin{cases} \mathcal{D}^\alpha Z_1 = -\dfrac{2i}{X^{1/2}} \left[2u^\alpha \tilde{R}_1 + \hat{x}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Z_1 \right] \\ \mathcal{D}^\alpha Z_2 = -\dfrac{2i}{X^{1/2}} \left[2v^\alpha \tilde{R}_2 + \hat{x}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Z_2 \right] \\ \mathcal{D}^\alpha Z_3 = -\dfrac{2i}{X^{1/2}} \left[2w^\alpha \tilde{R}_3 + \hat{x}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} Z_3 \right] \end{cases}$$

$$\mathcal{D}^\alpha R_1 = -\frac{\partial}{\partial\Theta_\alpha} \epsilon_{\beta\gamma} u^\beta \hat{\Theta}^\gamma = -\left(\frac{\partial}{\partial\Theta_\alpha}\Theta_\beta\right)\frac{1}{X^{1/2}} u^\beta = -\frac{\delta^\alpha{}_\beta u^\beta}{X^{1/2}}$$

$$= -\frac{u^\alpha}{X^{1/2}}$$

$$\boxed{\begin{aligned} \mathcal{D}^\alpha R_1 &= -\frac{u^\alpha}{X^{1/2}} \\ \mathcal{D}^\alpha R_2 &= -\frac{v^\alpha}{X^{1/2}} \\ \mathcal{D}^\alpha R_3 &= -\frac{w^\alpha}{X^{1/2}} \end{aligned}}$$

$$\mathcal{D}^\alpha \tilde{R}_i = 0.$$

$$\mathcal{D}^\alpha S_1 = \left[-\frac{\partial}{\partial\Theta_\alpha} + 2i\bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial x^a} \right] \left[(\sigma^b)_{\beta\dot\beta} \frac{X_b}{X^{3/2}} u^\beta \bar\Theta^{\dot\beta} \right]$$

$$= 2i\, \bar\Theta_{\dot\alpha} (\hat\sigma^a)^{\dot\alpha\alpha} (\hat\sigma^b)^{\dot\beta\beta} u_\beta \bar\Theta_{\dot\beta} \frac{\partial}{\partial X^a} \left[\frac{X_b}{X^{3/2}} \right]$$

$$\mathcal{D}^\alpha J = \mathcal{D}^\alpha \left(\hat{X}_{\beta\dot\beta}\, \hat\Theta^\beta \hat{\bar\Theta}^{\dot\beta} \right)$$

$$= \left[-\frac{\partial}{\partial\Theta_\alpha} + 2i\, \bar\Theta_{\dot\alpha} (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \right] \left[\frac{1}{X^2} (\sigma^b)_{\beta\dot\beta}\, X_b\, \Theta^\beta \bar\Theta^{\dot\beta} \right]$$

$$= -\frac{1}{X^{1/2}} \hat{X}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} - 2i\, \bar\Theta_{\dot\alpha} \bar\Theta^{\dot\beta} (\tilde\sigma^a)^{\dot\alpha\alpha} (\sigma^b)_{\beta\dot\beta}\, \Theta^\beta \frac{\partial}{\partial X^a} \left[\frac{X_b}{X^2} \right]$$

$$= -\frac{1}{X^{1/2}} \hat{X}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} - i\, \bar\Theta^2 (\tilde\sigma^a)^{\dot\alpha\alpha} (\sigma^b)_{\beta\dot\alpha}\, \Theta^\beta \left[\frac{\eta_{ab}}{X^2} - 2 \frac{\hat{X}_a \hat{X}_b}{X^2} \right]$$

$$= -\frac{1}{X^{1/2}} \left[\hat{X}^{\dot\alpha\alpha} \hat{\bar\Theta}_{\dot\alpha} - 2i\, \hat{\bar\Theta}^2 \hat\Theta^\alpha \right]$$

$$\bar{\mathcal{D}}^{\dot\alpha} \tilde{R}_1 = \bar{\mathcal{D}}^{\dot\alpha} \left[\frac{\bar u^{\dot\beta} \bar\Theta_{\dot\beta}}{X^{1/2}} \right] = \left(\frac{\partial}{\partial\bar\Theta_{\dot\alpha}} \bar\Theta_{\dot\beta} \right) \frac{\bar u^{\dot\beta}}{X^{1/2}} = \frac{\bar u^{\dot\alpha}}{X^{1/2}} .$$

$$\begin{cases} \bar{\mathcal{D}}^{\dot\alpha} \tilde{R}_1 = \dfrac{\bar u^{\dot\alpha}}{X^{1/2}} \\[2ex] \bar{\mathcal{D}}^{\dot\alpha} \tilde{R}_2 = \dfrac{\bar v^{\dot\alpha}}{X^{1/2}} \\[2ex] \bar{\mathcal{D}}^{\dot\alpha} \tilde{R}_3 = \dfrac{\bar w^{\dot\alpha}}{X^{1/2}} \end{cases}$$

$$\bar{\mathcal{D}}^{\dot\alpha} S_1 = \frac{\partial}{\partial\bar\Theta_{\dot\alpha}} \left[\frac{X_{\beta\dot\beta}\, u^\beta \bar\Theta^{\dot\beta}}{X^{3/2}} \right] = -\frac{X_\beta{}^{\dot\alpha}\, u^\beta}{X^{3/2}}$$

$$\begin{cases} \bar{\mathcal{D}}^{\dot\alpha} S_1 = \dfrac{\hat{X}^{\dot\alpha\alpha} u_\alpha}{X^{1/2}} \\[2ex] \bar{\mathcal{D}}^{\dot\alpha} S_2 = \dfrac{\hat{X}^{\dot\alpha\alpha} v_\alpha}{X^{1/2}} \\[2ex] \bar{\mathcal{D}}^{\dot\alpha} S_3 = \dfrac{\hat{X}^{\dot\alpha\alpha} w_\alpha}{X^{1/2}} \end{cases}$$

$$\bar{\mathcal{D}}^{\dot\alpha} J = \frac{\partial}{\partial\bar\Theta_{\dot\alpha}} \left[\frac{X_{\beta\dot\beta}\, \Theta^\beta \bar\Theta^{\dot\beta}}{X^2} \right] = \frac{X_\beta{}^{\dot\alpha}\, \Theta^\beta}{X^2} = -\frac{\hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha}{X^{1/2}} .$$

$$Q^\alpha = -\frac{\partial}{\partial \Theta_\alpha}$$

The only non-vanishing ones are just

$$Q^\alpha R_i \,, \quad Q^\alpha \tilde{S}_i \,, \quad Q^\alpha J$$

$$\begin{cases} Q^\alpha R_1 = -\dfrac{u^\alpha}{X^{1/2}} \\ Q^\alpha R_2 = -\dfrac{v^\alpha}{X^{1/2}} \\ Q^\alpha R_3 = -\dfrac{w^\alpha}{X^{1/2}} \end{cases}$$

$$\begin{cases} Q^\alpha \tilde{S}_1 = -\dfrac{\hat{X}^{\dot{\alpha}\alpha}\,\bar{u}_{\dot{\alpha}}}{X^{1/2}} \\ Q^\alpha \tilde{S}_2 = -\dfrac{\hat{X}^{\dot{\alpha}\alpha}\,\bar{v}_{\dot{\alpha}}}{X^{1/2}} \\ Q^\alpha \tilde{S}_3 = -\dfrac{\hat{X}^{\dot{\alpha}\alpha}\,\bar{w}_{\dot{\alpha}}}{X^{1/2}} \end{cases}$$

$$Q^\alpha J = -\frac{\hat{X}^{\dot{\alpha}\alpha}\,\hat{\bar{\Theta}}_{\dot{\alpha}}}{X^{1/2}} \,.$$

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$$Q_\alpha = \cancel{\frac{\partial}{\partial \theta^\alpha}}$$

$$\overline{Q}^{\dot\alpha} = \frac{\partial}{\partial \overline{\Theta}_{\dot\alpha}} + 2i\, \Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a}$$

We then compute

$$\overline{Q}^{\dot\alpha} Q_i \,, \quad \overline{Q}^{\dot\alpha} \tilde{Q}_i \,, \quad \overline{Q}^{\dot\alpha} Z_i$$

Fermionic : $\overline{Q}^{\dot\alpha} R_i$, $\overline{Q}^{\dot\alpha} \tilde{R}_i$

$\overline{Q}^{\dot\alpha} S_i$, $\overline{Q}^{\dot\alpha} \tilde{S}_i$

$\overline{Q}^{\dot\alpha} J$

$$\overline{Q}^{\dot\alpha} Q_1 = \left[2i\, \Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \right] \left[\frac{X_b}{X} (\sigma^b)_{\mu\dot\mu}\, v^\mu\, \overline{\omega}^{\dot\mu} \right]$$

$$= 2i\, \Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha} (\sigma^b)_{\mu\dot\mu}\, v^\mu\, \overline{\omega}^{\dot\mu} \left[\frac{\eta_{ab}}{X} - \frac{\hat{X}_a \hat{X}_b}{X} \right]$$

$$= -4i \frac{R_2\, \overline{\omega}^{\dot\alpha}}{X^{1/2}} - 2i \frac{\hat{X}^{\dot\alpha\alpha}\, \hat\Theta_\alpha\, Q_1}{X^{1/2}}$$

$$\left\{ \begin{aligned} \overline{Q}^{\dot\alpha} Q_1 &= -\frac{2i}{X^{1/2}} \left[2\overline{\omega}^{\dot\alpha} R_2 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha Q_1 \right] \\ \overline{Q}^{\dot\alpha} Q_2 &= -\frac{2i}{X^{1/2}} \left[2\overline{u}^{\dot\alpha} R_3 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha Q_2 \right] \\ \overline{Q}^{\dot\alpha} Q_3 &= -\frac{2i}{X^{1/2}} \left[2\overline{v}^{\dot\alpha} R_1 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha Q_3 \right] \end{aligned} \right.$$

$$\left\{ \begin{aligned} \overline{Q}^{\dot\alpha} \tilde{Q}_1 &= -\frac{2i}{X^{1/2}} \left[2\overline{v}^{\dot\alpha} R_3 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha \tilde{Q}_1 \right] \\ \overline{Q}^{\dot\alpha} \tilde{Q}_2 &= -\frac{2i}{X^{1/2}} \left[2\overline{\omega}^{\dot\alpha} R_1 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha \tilde{Q}_2 \right] \\ \overline{Q}^{\dot\alpha} \tilde{Q}_3 &= -\frac{2i}{X^{1/2}} \left[2\overline{u}^{\dot\alpha} R_2 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha \tilde{Q}_3 \right] \end{aligned} \right.$$

$$\left\{ \begin{aligned} \overline{Q}^{\dot\alpha} Z_1 &= -\frac{2i}{X^{1/2}} \left[2\overline{u}^{\dot\alpha} R_1 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha Z_1 \right] \\ \overline{Q}^{\dot\alpha} Z_2 &= -\frac{2i}{X^{1/2}} \left[2\overline{v}^{\dot\alpha} R_2 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha Z_2 \right] \\ \overline{Q}^{\dot\alpha} Z_3 &= -\frac{2i}{X^{1/2}} \left[2\overline{\omega}^{\dot\alpha} R_3 + \hat{X}^{\dot\alpha\alpha} \hat\Theta_\alpha Z_3 \right] \end{aligned} \right.$$

$$\bar{Q}^{\dot\alpha} R_1 = \left[\frac{\partial}{\partial \bar\Theta_{\dot\alpha}} + 2i\, \Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \right] \left[\frac{u^\beta \Theta_\beta}{X^{1/2}} \right]$$

$$= 2i\, \Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha}\, u^\beta \Theta_\beta \left(-\tfrac{1}{2}\right) X^{-3/2}\, \hat{X}_a$$

$$= -\frac{i}{2}\, \Theta^2 u_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha}\, X^{-3/2}\, \hat{X}_a$$

$$= -\frac{i}{2}\, \frac{\hat\Theta^2 u_\alpha \hat{X}^{\dot\alpha\alpha}}{X^{1/2}} .$$

$$\begin{cases} \bar{Q}^{\dot\alpha} R_1 = -\frac{i}{2} \frac{1}{X^{1/2}} \hat\Theta^2 u_\alpha \hat{X}^{\dot\alpha\alpha} \\ \bar{Q}^{\dot\alpha} R_2 = -\frac{i}{2} \frac{1}{X^{1/2}} \hat\Theta^2 v_\alpha \hat{X}^{\dot\alpha\alpha} \\ \bar{Q}^{\dot\alpha} R_3 = -\frac{i}{2} \frac{1}{X^{1/2}} \hat\Theta^2 w_\alpha \hat{X}^{\dot\alpha\alpha} \end{cases}$$

$$\bar{Q}^{\dot\alpha} \tilde{R}_1 = \left[\frac{\partial}{\partial \bar\Theta_{\dot\alpha}} + 2i\, \Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha} \frac{\partial}{\partial X^a} \right] \left[\frac{\bar{u}^{\dot\beta} \bar\Theta_{\dot\beta}}{X^{1/2}} \right]$$

$$= \frac{\bar{u}^{\dot\alpha}}{X^{1/2}} + 2i\, \Theta_\alpha (\tilde\sigma^a)^{\dot\alpha\alpha}\, \bar{u}^{\dot\beta} \bar\Theta_{\dot\beta} \left(-\tfrac{1}{2}\right) X^{-3/2}\, \hat{X}_a$$

$$= \frac{\bar{u}^{\dot\alpha}}{X^{1/2}} - i\, \frac{\hat\Theta_\alpha\, \bar{u}^{\dot\beta} \hat{\bar\Theta}_{\dot\beta}\, \hat{X}^{\dot\alpha\alpha}}{X^{1/2}}$$

$$= \frac{1}{X^{1/2}} \left[\bar{u}^{\dot\alpha} - i\, \hat\Theta_\alpha \hat{X}^{\dot\alpha\alpha} \tilde{R}_1 \right]$$

$$\begin{cases} \bar{Q}^{\dot\alpha} \tilde{R}_1 = \frac{1}{X^{1/2}} \left[\bar{u}^{\dot\alpha} - i \hat\Theta_\alpha \hat{X}^{\dot\alpha\alpha} \tilde{R}_1 \right] \\ \bar{Q}^{\dot\alpha} \tilde{R}_2 = \frac{1}{X^{1/2}} \left[\bar{v}^{\dot\alpha} - i \hat\Theta_\alpha \hat{X}^{\dot\alpha\alpha} \tilde{R}_2 \right] \\ \bar{Q}^{\dot\alpha} \tilde{R}_3 = \frac{1}{X^{1/2}} \left[\bar{w}^{\dot\alpha} - i \hat\Theta_\alpha \hat{X}^{\dot\alpha\alpha} \tilde{R}_3 \right] \end{cases}$$

$$\bar Q^{\dot\alpha} S_1 = \left[\frac{\partial}{\partial\bar\theta_{\dot\alpha}} + 2i\,\theta_\alpha(\tilde\sigma^a)^{\dot\alpha\alpha}\frac{\partial}{\partial x^a}\right]\left[(\sigma^b)_{\beta\dot\beta}\, u^\beta\, \bar\theta^{\dot\beta}\,\frac{x_b}{x^{3/2}}\right]$$

$$= (\tilde\sigma^b)^{\dot\alpha\beta} u_\beta \frac{x_b}{x^{3/2}} + 2i\,\theta_\alpha(\tilde\sigma^a)^{\dot\alpha\alpha}(\sigma^b)_{\beta\dot\beta}\, u^\beta\bar\theta^{\dot\beta}\times$$

$$\times\left[\frac{\eta_{ab}}{x^{3/2}} - \frac{3}{2}\frac{\hat x_a \hat x_b}{x^{3/2}}\right]$$

$$= \frac{u_\beta\,\hat x^{\dot\alpha\beta}}{x^{1/2}} - 4i\,\frac{\hat\theta_\beta u^\beta\hat{\bar\theta}^{\dot\alpha}}{x^{1/2}} - 3i\,\frac{\hat\theta_\alpha u^\beta\hat{\bar\theta}^{\dot\beta}\hat x^{\dot\alpha\alpha}\hat x_{\beta\dot\beta}}{x^{1/2}}$$

$$= \frac{u_\beta\,\hat x^{\dot\alpha\beta}}{x^{1/2}} + 4i\,\frac{\hat{\bar\theta}^{\dot\alpha}R_1}{x^{1/2}} - 3i\,\frac{\hat\theta_\alpha\hat x^{\dot\alpha\alpha}S_1}{x^{1/2}}$$

$$\begin{cases} \bar Q^{\dot\alpha} S_1 = \dfrac{1}{x^{1/2}}\left[u_\beta\hat x^{\dot\alpha\beta} + 4i\,\hat{\bar\theta}^{\dot\alpha}R_1 - 3i\,\hat\theta_\alpha\hat x^{\dot\alpha\alpha}S_1\right] \\ \bar Q^{\dot\alpha} S_2 = \dfrac{1}{x^{1/2}}\left[v_\beta\hat x^{\dot\alpha\beta} + 4i\,\hat{\bar\theta}^{\dot\alpha}R_2 - 3i\,\hat\theta_\alpha\hat x^{\dot\alpha\alpha}S_2\right] \\ \bar Q^{\dot\alpha} S_3 = \dfrac{1}{x^{1/2}}\left[w_\beta\hat x^{\dot\alpha\beta} + 4i\,\hat{\bar\theta}^{\dot\alpha}R_3 - 3i\,\hat\theta_\alpha\hat x^{\dot\alpha\alpha}S_3\right] \end{cases}$$

$$\bar Q^{\dot\alpha}\tilde S_1 = \left[\frac{\partial}{\partial\bar\theta_{\dot\alpha}} + 2i\,\theta_\alpha(\tilde\sigma^a)^{\dot\alpha\alpha}\frac{\partial}{\partial x^a}\right]\left[(\sigma^b)_{\beta\dot\beta}\,\bar u^{\dot\beta}\,\theta^\beta\,\frac{x_b}{x^{3/2}}\right]$$

$$= 2i\,\theta_\alpha(\tilde\sigma^a)^{\dot\alpha\alpha}(\sigma^b)_{\beta\dot\beta}\,\bar u^{\dot\beta}\theta^\beta\left[\frac{\eta_{ab}}{x^{3/2}} - \frac{3}{2}\frac{\hat x_a\hat x_b}{x^{3/2}}\right]$$

$$= -4i\,\frac{\theta_\alpha\bar u^{\dot\alpha}\theta^\alpha}{x^{3/2}} - 3i\,\frac{\hat\theta_\alpha\hat x^{\dot\alpha\alpha}\hat x_{\beta\dot\beta}\bar u^{\dot\beta}\hat\theta^\beta}{x^{1/2}}$$

$$= 4i\,\frac{\hat\theta^2\bar u^{\dot\alpha}}{x^{1/2}} - 3i\,\frac{\hat\theta_\alpha\hat x^{\dot\alpha\alpha}\tilde S_1}{x^{1/2}} = \frac{5i}{2}\,\frac{\hat\theta^2\bar u^{\dot\alpha}}{x^{1/2}}.$$

$$\begin{cases} \bar Q^{\dot\alpha}\tilde S_1 = \dfrac{5i}{2}\dfrac{\hat\theta^2\bar u^{\dot\alpha}}{x^{1/2}} \\ \bar Q^{\dot\alpha}\tilde S_2 = \dfrac{5i}{2}\dfrac{\hat\theta^2\bar v^{\dot\alpha}}{x^{1/2}} \\ \bar Q^{\dot\alpha}\tilde S_3 = \dfrac{5i}{2}\dfrac{\hat\theta^2\bar w^{\dot\alpha}}{x^{1/2}}. \end{cases}$$

$$\bar Q^{\dot\alpha} J = -\frac{1}{x^{1/2}}\left[\hat x^{\dot\alpha\alpha}\hat\theta_\alpha - 2i\,\hat\theta^2\hat{\bar\theta}^{\dot\alpha}\right].$$