# COMP9020 Problem Set 7

Kai Engelhardt

September 22, 2016

**Exercise 1** Three quantities associated with an undirected graph $G$ are its degree (the maximum degree of any of its nodes), its chromatic number $\chi(G)$ (the minimum number of colours needed to paint the nodes such that no two adjacent nodes have the same colour), and its maximum clique size $\kappa(G)$ (the number of nodes in the largest subgraph that is complete). Derive inequalities that must hold between these numbers and explain why they must hold.

**Exercise 2** Recall the complete bipartite graph $K_{m,n}$ of $m+n$ nodes. For what values of $m$ and $n$ is it planar?

Given a state machine $M = (S, s_0, \rightarrow)$, a *derived variable* is a function $f : S \longrightarrow T$ for some set $T$. Of particular interest for termination proofs are ranges $T$ that are equipped with a well-order $(T, <)$. If we can then show that there exists a derived variabe $f$ that is *strictly decreasing*, that is, $s \rightarrow s' \Rightarrow f(s') < f(s)$ for all $s \in S$, then the range of $f$ being a well-order implies that every execution is necessarily finite.

**Exercise 3** Problem 12.43 in [LLM16].

There are many more fun exercsies about graphs, for instance those in the 12.50ies. Feel free to pick any and discuss them!

## References

[LLM16] Eric Lehman, F. Thomson Leighton, and Albert R. Meyer. Mathematics for computer science. Available at https://courses.csail.mit.edu/6.042/spring16/mcs.pdf; check https://courses.csail.mit.edu/6.042 for newer versions, 2016.