COMP9020 Problem Set 5

Kai Engelhardt

September 6, 2016

1 Numbers

Exercise 1 Prove the basic properties of gcd carefully.

1. $\gcd(m, m) = m$
2. $\gcd(m, 0) = m$
3. $\gcd(m, n) = \gcd(n, m)$
4. $\gcd(m + n, n) = \gcd(m, n)$

Exercise 2 Prove that $\gcd(m, n) \cdot \mathrm{lcm}(m, n) = m \cdot n$

2 Sets

Exercise 3 (LLM) Problem 4.6

3 Relations

Exercise 4 Define relation R on $\mathrm{pow}(U)$ for some set U where ARB iff $|A \cap B| \geq 1$. When is R transitive?

Exercise 5 Define $R \subseteq \mathbb{R} \times \mathbb{R}$ where $(a, b) \in R$ iff $a \in [b - 0.5, b + 0.5]$. Is R a (a) partial order? (b) total order? (c) equivalence relation?

Exercise 6 Define a relation $R \subseteq \mathbb{R} \times \mathbb{R}$ where $(a, b) \in R$ iff either $a \leq b - 0.5$ or $a = b$. Show that R is a partial order, but not a total order.

Exercise 7 Consider the relation on the natural numbers defined by xRy iff x and y both have a prime divisor (which need not be a common divisor) that is strictly smaller than both x and y.

Which of the basic properties (reflexive, anti-reflexive, symmetric, anti-symmetric, transitive) does this relation satisfy? For each property explain briefly why it is satisfied or provide a counterexample if it is not satisfied.

Exercise 8 (LLM) Problem 4.28.