COMP9020 Problem Set 11

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Exercise 1 Problem 17.2 in [LLM16].

Exercise 2 Consider an urn containing three red and eight black marbles; draw two without replacement. We write b_1 — black on the first draw, b_2 — black on the second draw etc.

Find the probabilities

1. both red:
$$P(r_1 \& r_2)$$
2. both black:
$$P(b_1 \& b_2)$$
3. one red, one black:
$$P(r_1 \& b_2) + P(b_1 \& r_2)$$

Exercise 3 As in the previous, but perform the draws *with* replacement.

Exercise 4 Suppose you're writing code for a computer game where values in the integer interval 0..59 are to be generated at random, using simulated dice rolls. The faces of an n-sided are labelled with the numbers $0..n-1$. Rolling such a die should return each of the possible values with probability $\frac{1}{n}$. Determine the distributions when the values are created using

1. one 60-sided die,
2. two 30-sided dice,
3. three 20-sided dice,
4. four 15-sided dice,
5. five 12-sided dice,
6. six 10-sided dice,
7. ten 6-sided dice,
8. twelve 5-sided dice,
9. fifteen 4-sided dice,
10. twenty 3-sided dice, or
11. thirty 2-sided dice.

It is recommended to treat this as a programming exercise and to graph the distributions using any one of the usual suspects, e.g., gnuplot.

Exercise 5 Two dice, one black and one red, are tossed. Let b and r denote their outcomes, while s is the sum. Calculate:

1. $P(r \geq 5 | s = 9)$
2. $P(r \geq 5 | s \geq 9)$

Exercise 6 Problem 17.5 in [LLM16].

Exercise 7 You have applied to join a chess club and been told that to qualify you must play three games against X, winning two games in a row. "Who gets the white pieces?" you ask and are told you and X alternate and you get to decide whether to start with white or with black.

Knowing that the probability of beating X is better with the white pieces (first-move advantage) should you choose white or black for the first game?

Use probabilites to model the problem and prove your answer answer correct.

References

[LLM16] Eric Lehman, F. Thomson Leighton, and Albert R. Meyer. Mathematics for computer science. Available at https://courses.csail.mit.edu/6.042/spring16/mcs.pdf; check https://courses.csail.mit.edu/6.042 for newer versions, 2016.