

# Solutions to COMP9020 Problems

## Week 1

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**Solution to Exercise 1** See pages 23–24 of the textbook.

**Solution to Exercise 2** We define:

$A$	$B$	$C$	$A ? B : C$
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	T
T	T	T	T

**Solution to Exercise 3** We claim that

$$A ? B : C \Leftrightarrow (A \wedge B) \vee (\neg A \wedge C) \quad (1)$$

provides an alternative characterisation in propositional logic of  $A ? B : C$ .

**Proof:**

$A$	$B$	$C$	$A \wedge B$	$\neg A \wedge C$	$(A \wedge B) \vee (\neg A \wedge C)$
F	F	F	F	F	F
F	F	T	F	T	T
F	T	F	F	F	F
F	T	T	F	T	T
T	F	F	F	F	F
T	F	T	F	F	F
T	T	F	T	F	T
T	T	T	T	F	T

Comparing the last column with our answer to the previous exercise we conclude that the alternative characterisation is correct. ■

**Solution to Exercise 4** To express  $A ? B : C$  using NAND only we recall that

$$A \text{ NAND } B \Leftrightarrow \neg(A \wedge B) \quad (2)$$

$$\neg A \Leftrightarrow A \text{ NAND } A \quad (3)$$

and transform the right-hand-side of (1) via a series of equivalences into the desired form.

$$\begin{aligned}
 A ? B : C &\Leftrightarrow (A \wedge B) \vee (\neg A \wedge C) && (1) \\
 &\Leftrightarrow \neg\neg(A \wedge B) \vee \neg\neg(\neg A \wedge C) && \text{double negation} \\
 &\Leftrightarrow \neg(A \text{ NAND } B) \vee \neg(\neg A \text{ NAND } C) && (2) \\
 &\Leftrightarrow \neg((A \text{ NAND } B) \wedge (\neg A \text{ NAND } C)) && \text{de Morgan} \\
 &\Leftrightarrow (A \text{ NAND } B) \text{ NAND } (\neg A \text{ NAND } C) && (2) \\
 &\Leftrightarrow (A \text{ NAND } B) \text{ NAND } ((A \text{ NAND } A) \text{ NAND } C) && (3)
 \end{aligned}$$

A circuit diagram for this is:

