

Unsupervised Learning

COMP9417 Machine Learning and Data Mining

May 2, 2017

Acknowledgements

Material derived from slides for the book
"Elements of Statistical Learning (2nd Ed.)" by T. Hastie,
R. Tibshirani & J. Friedman. Springer (2009)
<http://statweb.stanford.edu/~tibs/ElemStatLearn/>

Material derived from slides for the book
"Machine Learning: A Probabilistic Perspective" by P. Murphy
MIT Press (2012)
<http://www.cs.ubc.ca/~murphyk/MLbook>

Material derived from slides for the book
"Machine Learning" by P. Flach
Cambridge University Press (2012)
<http://cs.bris.ac.uk/~flach/mlbook>

Material derived from slides for the book
"Bayesian Reasoning and Machine Learning" by D. Barber
Cambridge University Press (2012)
<http://www.cs.ucl.ac.uk/staff/d.barber/brml>

Material derived from figures for the book
"Python Data Science Handbook" by J. VanderPlas
O'Reilly Media (2017)
<http://shop.oreilly.com/product/0636920034919.do>

Material derived from slides for the course
"Machine Learning" by A. Srinivasan

BITS Pilani, Goa, India (2016)

Contents

① Aims

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② Introduction

Supervised vs. Unsupervised Learning

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So: *unsupervised learning* methods address the problem of assigning instances to classes *given only observations about the instances*, i.e., without being given class “labels” for instances by a “teacher”.

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- unfortunately, often the class is not a known feature

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- to learn generative models for images, text, video, speech, etc.

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③ Dimensionality Reduction

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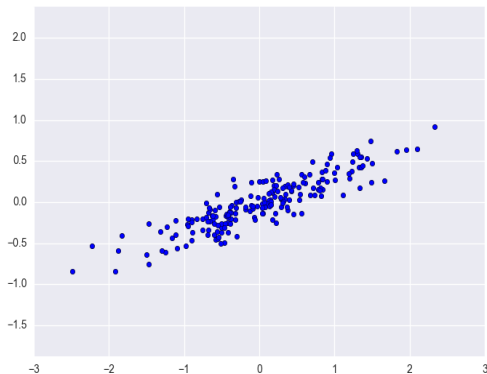
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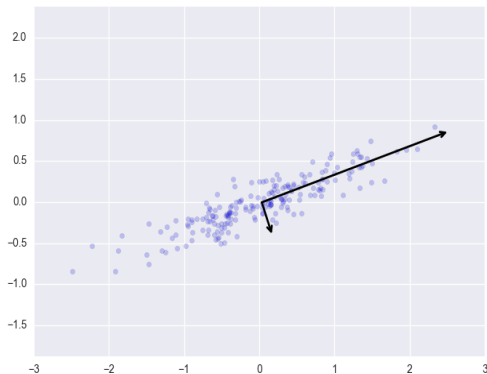
This suggests an approach: look into using the variance-covariance matrix we recall from correlation and regression

PCA Example



PCA looks for linear combinations of the original features. This dataset of 200 points seems to show such a relationship between two feature dimensions.

PCA Example



PCA finds two new features on which the original data can be projected, rotated and scaled. These explain respectively 0.75 and 0.02 of the variance.

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- 5 sort columns of \mathbf{S} in decreasing order (decreasing variance)
- 6 remove columns of \mathbf{S} below some minimum threshold



PCA Example



By rejecting the second component we reduce the dimensionality by 50% while preserving much of the original variance, seen by plotting the inverse transform of this component along with the original data.

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- many other methods use essentially the same matrix decomposition idea, such as finding “topics” in text using Latent Semantic Analysis (next slide), finding hidden “sub-groups” for recommender systems, and so on

Finding Topics in Text

$$\begin{array}{cccc}
 \left[\begin{array}{c} \tilde{\mathbf{X}} \end{array} \right] & = & \left[\begin{array}{c} \mathbf{U}_k \end{array} \right] & \left[\begin{array}{c} s_1 \quad s_2 \quad \dots \quad s_k \end{array} \right] \left[\begin{array}{c} \mathbf{V}_k^T \end{array} \right] \\
 t \times d & & t \times k & k \times k \quad k \times d
 \end{array}$$

Latent Semantic Analysis (LSA) factorizes a word count matrix for t terms from d documents using the **Singular Value Decomposition (SVD)** to find a number $k < d$ of **topics**. Here the diagonal matrix \mathbf{S} has the topic “strength” sorted in decreasing order.

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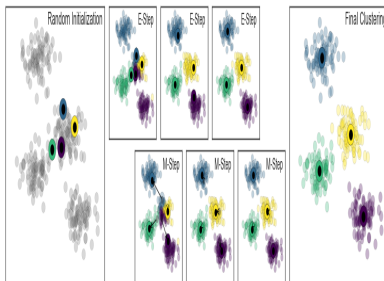
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- algorithm applies matrix operations to translate, rotate and scale
- many alternatives, e.g., Random Projections, Independent Component Analysis, etc.



Contents

④ Clustering

k -Means Clustering Example



Data.

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⑤ Summary

Summary: Unsupervised Learning

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 - learning is designed to optimize function of reward
- active learning
 - learning system acts to generate its own examples

Note: unsupervised learning an increasingly active research area, particularly in neural nets, e.g., Yann LeCun: “Unsupervised Learning: The Next Frontier in AI”

<https://www.youtube.com/watch?v=XTbL0jVF-y4>