

COMP9020 Problem Set 9

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Exercise 1 A footy panel needs to be formed, representing each of the three codes (league, union, aerial pingpong) by at least one member. How many different panels can be formed in either of the following situations?

1. Each code shortlists 5 panel members from its ranks. The panel size is 3.
2. Each code shortlists 4 panel members from its ranks but A. Johns (league) refuses to serve with D. Campese (union). The panel size is 3.
3. Each code shortlists 5 panel members from its ranks. The panel size is 5.
4. Each code shortlists 4 panel members from its ranks but A. Johns (league) refuses to serve with D. Campese (union). The panel size is 5.

The remaining exercises are taken from [\[RW03\]](#).

Exercise 2 Let $\Sigma = \{a, b, c, d, e\}$. How many elements are there in each of the following sets?

1. Σ^k , for $k \in \mathbb{N}$
2. $\{ w \in \Sigma^3 \mid \text{no letter in } w \text{ is used more than once} \}$
3. $\{ w \in \Sigma^4 \mid \text{the letter } c \text{ occurs in } w \text{ exactly once} \}$
4. $\{ w \in \Sigma^4 \mid \text{the letter } c \text{ occurs in } w \text{ at least once} \}$

Exercise 3 1. In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are next to each other?

2. In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are not next to each other?
3. In how many ways can the letters a, b, c, d, e, f be arranged so that the letters a and b are next to each other but a and c are not?

Exercise 4 Among 200 people, 150 either swim or jog or both. If 85 swim and 60 swim and jog, how many jog?

Exercise 5 Twelve identical letters are to be placed into 4 mailboxes.

1. In how many ways can this be done?
2. How many ways are possible if each mailbox must receive at least 2 letters?

Exercise 6 A 4-letter word is selected at random from Σ^4 , where $\Sigma = \{a, b, c, d, e\}$.

1. What is the probability that the letters in the word are distinct?

2. What is the probability that there are no vowels in the word?
3. What is the probability that the word begins with a vowel?

Exercise 7 A black die and a red die are tossed. What is the probability that

1. the number on the red die is bigger than the number on the black die?
2. the number on the red die is twice the number on the black die?

From β eta #75:

Exercise 8 (Kid Trouble) Lisa and Milhouse have two kids, one of which is a girl. Assume that the probability of each gender is $\frac{1}{2}$. What is the probability that the other kid is also a girl?

References

[RW03] Kenneth A. Ross and Charles R. B. Wright. *Discrete Mathematics*. Prentice Hall, fifth edition, 2003.