

# COMP9020 Problem Set 3

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**Exercise 1** Here are some declarative sentences.

1. Statements can be true, false, or undecided.
2. The truth of some statements is hard to decide.
3. Hard statements are only considered true when they have been proved and false when their negation has been proved.
4.  $\mathbf{P} = \mathbf{NP}$  is a hard to decide statement.
5. Only professors prove hard statements.
6. I have proved  $\mathbf{P} \neq \mathbf{NP}$ .
7. I'm not a professor.
8. Exactly one of these 8 statements is false.

Model the above as a propositional formula. Is it satisfiable? If so, what are the satisfying assignments?

**Exercise 2** Prove by induction on  $n \in \mathbb{N}$  that  $\sum_{i=0}^n i^3 = \frac{n^2(n+1)^2}{4}$ .

**Exercise 3** Let us define the factorial function  $! : \mathbb{N} \rightarrow \mathbb{N}$  by

$$0! = 1 \text{ and } (n+1)! = (n+1) \cdot (n!) .$$

Prove by that  $n! \geq 2^{n-1}$ , for  $n \in \mathbb{N}$ ,

1. using the WOP,
2. using induction.

Problems from the textbook: 2.2, 2.4, 2.5, 2.9, 2.15. I also recommend to go through the problem section of chapter 3 to see whether there's anything you find hard to do.