COMP9020 Problem Set 6

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(I noticed that at MIT, they've switched over to the Fall term. The textbook for that term is a version of [LLM] from 2010. I'll keep referring to the spring 2016 version linked to from the subject outline.)

Exercise 1 (4.16) Describe a total injective function [$= 1$ out], [≤ 1 in] from $\mathbb{R} \longrightarrow \mathbb{R}$ that is not a bijection.

Exercise 2 (4.21) Let $R \subseteq A \times B$ be a binary relation. Each of the following formulas expresses the fact that R has a familiar relational "arrow" property such as being surjective or being a function.

Identify the relational property expressed by each of the following relational expressions. Explain your reasoning.

1. $R^{-1}; R \subseteq \mathrm{Id}_B$
2. $R; R^{-1} \subseteq \mathrm{Id}_A$
3. $R; R^{-1} \supseteq \mathrm{Id}_A$
4. $R^{-1}; R \supseteq \mathrm{Id}_B$

Exercise 3 If a directed graph has 6 nodes, what is the largest number of edges it can have? Generalise to n nodes.

Exercise 4 Repeat the previous exercise for undirected graphs.

Exercise 5 If a directed graph of n nodes is acyclic, what is the largest possible number of edges?

The textbook defines the *distance*, $\mathrm{dist}(u, v)$, in a graph from vertex u to vertex v as the length of a shortest path from u to v and states that it satisfies the triangle inequation common among all mathematical distance functions:

$$\mathrm{dist}(u, v) \leq \mathrm{dist}(u, x) + \mathrm{dist}(x, v)$$

for all vertices u, v, x with equality holding iff x is on a shortest path from u to v.

Exercise 6 Prove both directions of this claimed equavalence.

I also like Problem 10.5 in the textbook but it's a bit of work.