COMP9020 Problem Set 4

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Exercise 1 For each of the following, prove whether it is a valid equivalence of propositional logic.

$$A \wedge \neg A \Leftrightarrow \neg(A \vee \neg A) \tag{1}$$
$$(A \wedge B) \vee C \Leftrightarrow (A \vee C) \wedge (B \vee C) \tag{2}$$
$$(A \wedge A) \Leftrightarrow A \tag{3}$$
$$(A \Rightarrow B) \Leftrightarrow (\neg B \vee A) \tag{4}$$

Exercise 2 Let $\phi_1 = (p \Rightarrow (q \wedge r))$, $\phi_2 = (s \Rightarrow (q \wedge p))$, and $\phi = \phi_1 \vee \phi_2$.

1. Draw Karnaugh maps[1] for the three formulae, ϕ_1, ϕ_2, and ϕ.
2. Read off a minimal DNF for ϕ.
3. Give a minimal CNF for $\neg\phi_1$. (*Hint:* de Morgan.)

Exercise 3 Suppose Portia puts a portrait of herself in one of two caskets and places the following inscriptions on the caskets:

Gold casket: The portrait is not in this casket.

Silver casket: Exactly one of these inscriptions is true.

Portia tells her suitor to pick a casket that contains the portrait.

Formalise the problem in propositional logic and justify your answer to the question "Which casket should the suitor choose?" with a proof.

Exercise 4 Consider the following proof attempt.

> "**Claim:** all people in Tasmania are of the same age.
> **Proof:** by induction on the number n of people in Tasmania. The statement $S(n)$ is that in any group of n Tasmanians, everybody in that group has the same age.
> Base case $n = 1$. In any group of just one person, all are of the same age.
> For the inductive step, let G be any group of Tasmanians of $k + 1$ people. Let $p, q \in G$ be different persons. It suffices to prove that p and q are of the same age. By the inductive hypothesis, both $G \setminus \{P\}$ and $G \setminus \{Q\}$ which are of size k must consist of people of the same age. Let $r \in G$ be someone else. Then r is in both of these sets. Hence r, p, and q must be of the same age."

Where is the mistake in this proof?

[1] That's the grid-based technique for finding minimal DNF of propositional formulae of up to 4 variables.

Exercise 5 Prove or disprove that $(S \cup T) \times (U \cup V) = (S \times U) \cup (T \times V)$ holds for all sets S, T, U, and V.

Exercise 6 Recall the party trick with phone numbers:

> Write down your mobile number. Now re-arrange its digits however you want and write down the reulsting (different) number. Subtract the smaller of these two numbers from the larger. Add the digits of the resuting number up. Repeat this step until there's exactly one digit left.

Prove that that digit must be 9.