

CHAPTER 9

Introduction to Data-Link Layer

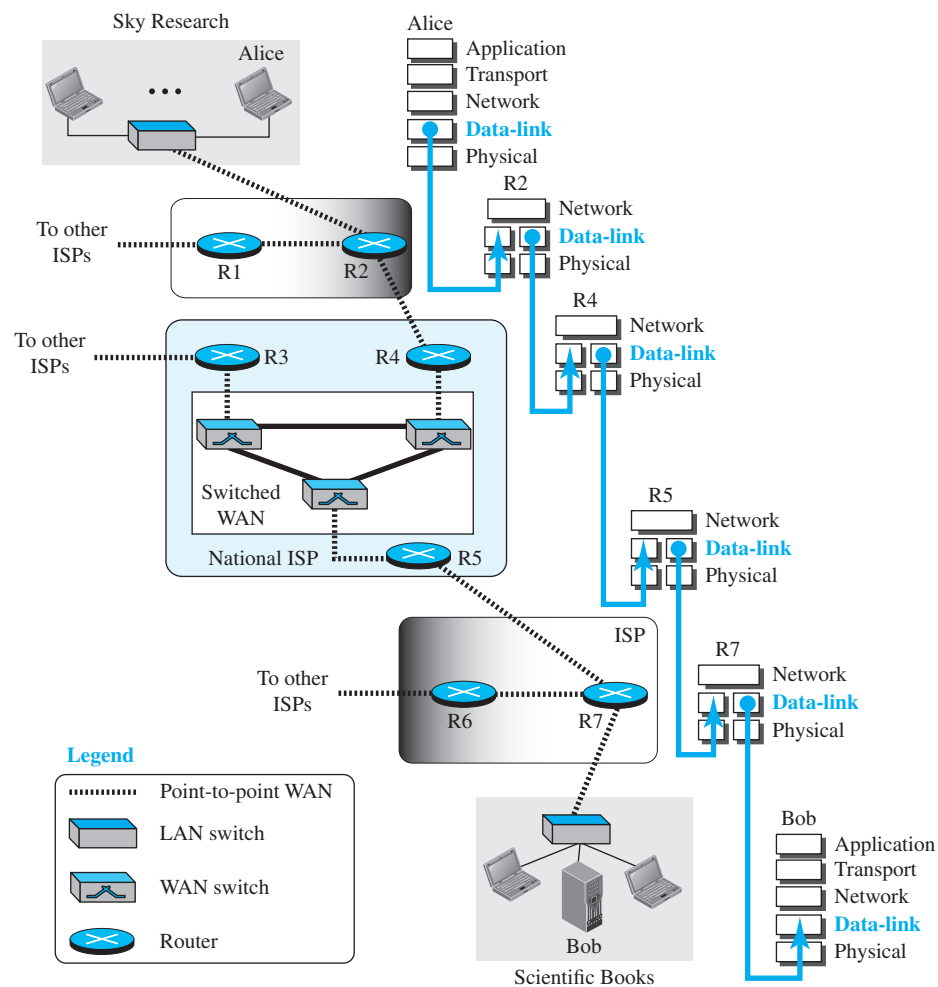
The TCP/IP protocol suite does not define any protocol in the data-link layer or physical layer. These two layers are territories of networks that when connected make up the Internet. These networks, wired or wireless, provide services to the upper three layers of the TCP/IP suite. This may give us a clue that there are several standard protocols in the market today. For this reason, we discuss the data-link layer in several chapters. This chapter is an introduction that gives the general idea and common issues in the data-link layer that relate to all networks.

- ❑ The first section introduces the data-link layer. It starts with defining the concept of links and nodes. The section then lists and briefly describes the services provided by the data-link layer. It next defines two categories of links: point-to-point and broadcast links. The section finally defines two sublayers at the data-link layer that will be elaborated on in the next few chapters.
- ❑ The second section discusses link-layer addressing. It first explains the rationale behind the existence of an addressing mechanism at the data-link layer. It then describes three types of link-layer addresses to be found in some link-layer protocols. The section discusses the Address Resolution Protocol (ARP), which maps the addresses at the network layer to addresses at the data-link layer. This protocol helps a packet at the network layer find the link-layer address of the next node for delivery of the frame that encapsulates the packet. To show how the network layer helps us to find the data-link-layer addresses, a long example is included in this section that shows what happens at each node when a packet is travelling through the Internet.

9.1 INTRODUCTION

The Internet is a combination of networks glued together by connecting devices (routers or switches). If a packet is to travel from a host to another host, it needs to pass through these networks. Figure 9.1 shows the same scenario we discussed in Chapter 3, but we are now interested in communication at the data-link layer. Communication at the data-link layer is made up of five separate logical connections between the data-link layers in the path.

Figure 9.1 *Communication at the data-link layer*



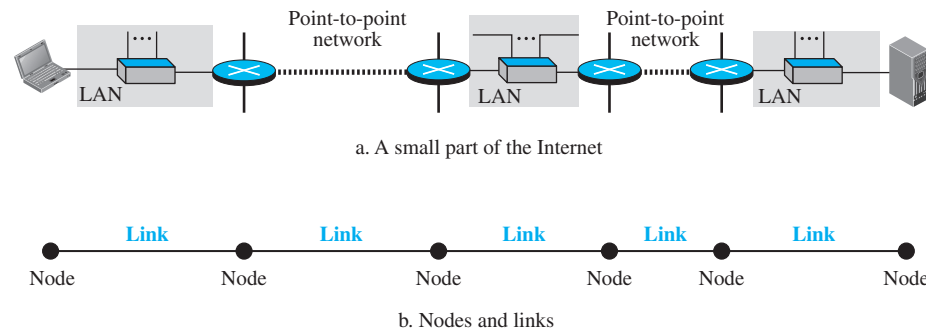
The data-link layer at Alice's computer communicates with the data-link layer at router R2. The data-link layer at router R2 communicates with the data-link layer at router R4,

and so on. Finally, the data-link layer at router R7 communicates with the data-link layer at Bob's computer. Only one data-link layer is involved at the source or the destination, but two data-link layers are involved at each router. The reason is that Alice's and Bob's computers are each connected to a single network, but each router takes input from one network and sends output to another network. Note that although switches are also involved in the data-link-layer communication, for simplicity we have not shown them in the figure.

9.1.1 Nodes and Links

Communication at the data-link layer is node-to-node. A data unit from one point in the Internet needs to pass through many networks (LANs and WANs) to reach another point. These LANs and WANs are connected by routers. It is customary to refer to the two end hosts and the routers as *nodes* and the networks in between as *links*. Figure 9.2 is a simple representation of links and nodes when the path of the data unit is only six nodes.

Figure 9.2 Nodes and Links



The first node is the source host; the last node is the destination host. The other four nodes are four routers. The first, the third, and the fifth links represent the three LANs; the second and the fourth links represent the two WANs.

9.1.2 Services

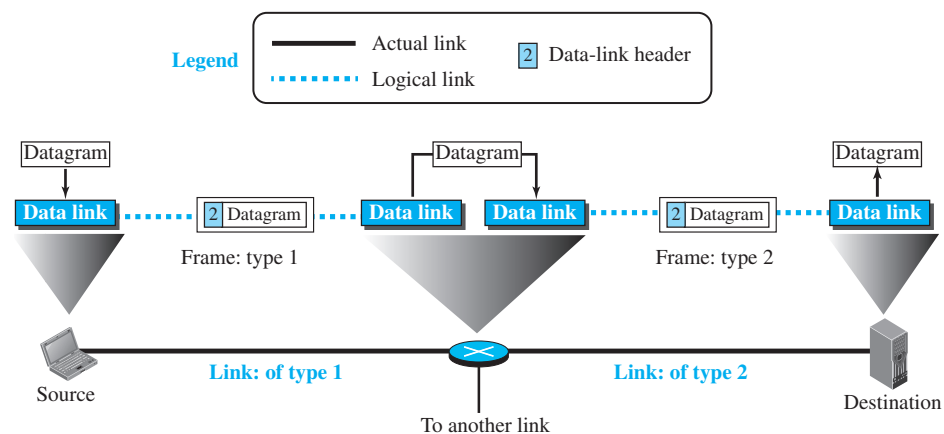
The data-link layer is located between the physical and the network layers. The data-link layer provides services to the network layer; it receives services from the physical layer. Let us discuss services provided by the data-link layer.

The duty scope of the data-link layer is node-to-node. When a packet is travelling in the Internet, the data-link layer of a node (host or router) is responsible for delivering a datagram to the next node in the path. For this purpose, the data-link layer of the sending node needs to encapsulate the datagram received from the network in a frame, and the data-link layer of the receiving node needs to decapsulate the datagram from the frame. In other words, the data-link layer of the source host needs only to

encapsulate, the data-link layer of the destination host needs to decapsulate, but each intermediate node needs to both encapsulate and decapsulate. One may ask why we need encapsulation and decapsulation at each intermediate node. The reason is that each link may be using a different protocol with a different frame format. Even if one link and the next are using the same protocol, encapsulation and decapsulation are needed because the link-layer addresses are normally different. An analogy may help in this case. Assume a person needs to travel from her home to her friend's home in another city. The traveller can use three transportation tools. She can take a taxi to go to the train station in her own city, then travel on the train from her own city to the city where her friend lives, and finally reach her friend's home using another taxi. Here we have a source node, a destination node, and two intermediate nodes. The traveller needs to get into the taxi at the source node, get out of the taxi and get into the train at the first intermediate node (train station in the city where she lives), get out of the train and get into another taxi at the second intermediate node (train station in the city where her friend lives), and finally get out of the taxi when she arrives at her destination. A kind of encapsulation occurs at the source node, encapsulation and decapsulation occur at the intermediate nodes, and decapsulation occurs at the destination node. Our traveller is the same, but she uses three transporting tools to reach the destination.

Figure 9.3 shows the encapsulation and decapsulation at the data-link layer. For simplicity, we have assumed that we have only one router between the source and destination. The datagram received by the data-link layer of the source host is encapsulated in a frame. The frame is logically transported from the source host to the router. The frame is decapsulated at the data-link layer of the router and encapsulated in another frame. The new frame is logically transported from the router to the destination host. Note that, although we have shown only two data-link layers at the router, the router actually has three data-link layers because it is connected to three physical links.

Figure 9.3 A communication with only three nodes



With the contents of the above figure in mind, we can list the services provided by a data-link layer as shown below.

Framing

Definitely, the first service provided by the data-link layer is **framing**. The data-link layer at each node needs to encapsulate the datagram (packet received from the network layer) in a **frame** before sending it to the next node. The node also needs to decapsulate the datagram from the frame received on the logical channel. Although we have shown only a header for a frame, we will see in future chapters that a frame may have both a header and a trailer. Different data-link layers have different formats for framing.

A packet at the data-link layer is normally called a *frame*.

Flow Control

Whenever we have a producer and a consumer, we need to think about flow control. If the producer produces items that cannot be consumed, accumulation of items occurs. The sending data-link layer at the end of a link is a producer of frames; the receiving data-link layer at the other end of a link is a consumer. If the rate of produced frames is higher than the rate of consumed frames, frames at the receiving end need to be buffered while waiting to be consumed (processed). Definitely, we cannot have an unlimited buffer size at the receiving side. We have two choices. The first choice is to let the receiving data-link layer drop the frames if its buffer is full. The second choice is to let the receiving data-link layer send a feedback to the sending data-link layer to ask it to stop or slow down. Different data-link-layer protocols use different strategies for flow control. Since flow control also occurs at the transport layer, with a higher degree of importance, we discuss this issue in Chapter 23 when we talk about the transport layer.

Error Control

At the sending node, a frame in a data-link layer needs to be changed to bits, transformed to electromagnetic signals, and transmitted through the transmission media. At the receiving node, electromagnetic signals are received, transformed to bits, and put together to create a frame. Since electromagnetic signals are susceptible to error, a frame is susceptible to error. The error needs first to be detected. After detection, it needs to be either corrected at the receiver node or discarded and retransmitted by the sending node. Since error detection and correction is an issue in every layer (node-to-node or host-to-host), we have dedicated all of Chapter 10 to this issue.

Congestion Control

Although a link may be congested with frames, which may result in frame loss, most data-link-layer protocols do not directly use a congestion control to alleviate congestion, although some wide-area networks do. In general, congestion control is considered an issue in the network layer or the transport layer because of its end-to-end nature. We will discuss congestion control in the network layer and the transport layer in later chapters.

9.1.3 Two Categories of Links

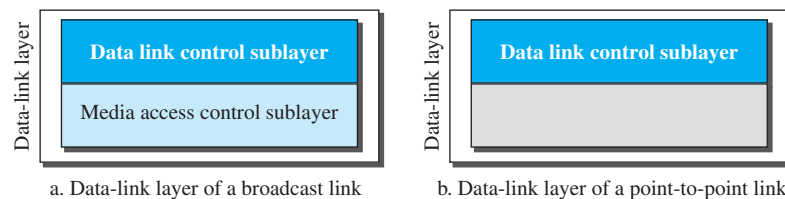
Although two nodes are physically connected by a transmission medium such as cable or air, we need to remember that the data-link layer controls how the medium is used. We can have a data-link layer that uses the whole capacity of the medium; we can also

have a data-link layer that uses only part of the capacity of the link. In other words, we can have a *point-to-point link* or a *broadcast link*. In a point-to-point link, the link is dedicated to the two devices; in a broadcast link, the link is shared between several pairs of devices. For example, when two friends use the traditional home phones to chat, they are using a point-to-point link; when the same two friends use their cellular phones, they are using a broadcast link (the air is shared among many cell phone users).

9.1.4 Two Sublayers

To better understand the functionality of and the services provided by the link layer, we can divide the data-link layer into two sublayers: **data link control (DLC)** and **media access control (MAC)**. This is not unusual because, as we will see in later chapters, LAN protocols actually use the same strategy. The data link control sublayer deals with all issues common to both point-to-point and broadcast links; the media access control sublayer deals only with issues specific to broadcast links. In other words, we separate these two types of links at the data-link layer, as shown in Figure 9.4.

Figure 9.4 Dividing the data-link layer into two sublayers



We discuss the DLC and MAC sublayers later, each in a separate chapter. In addition, we discuss the issue of error detection and correction, a duty of the data-link and other layers, also in a separate chapter.

9.2 LINK-LAYER ADDRESSING

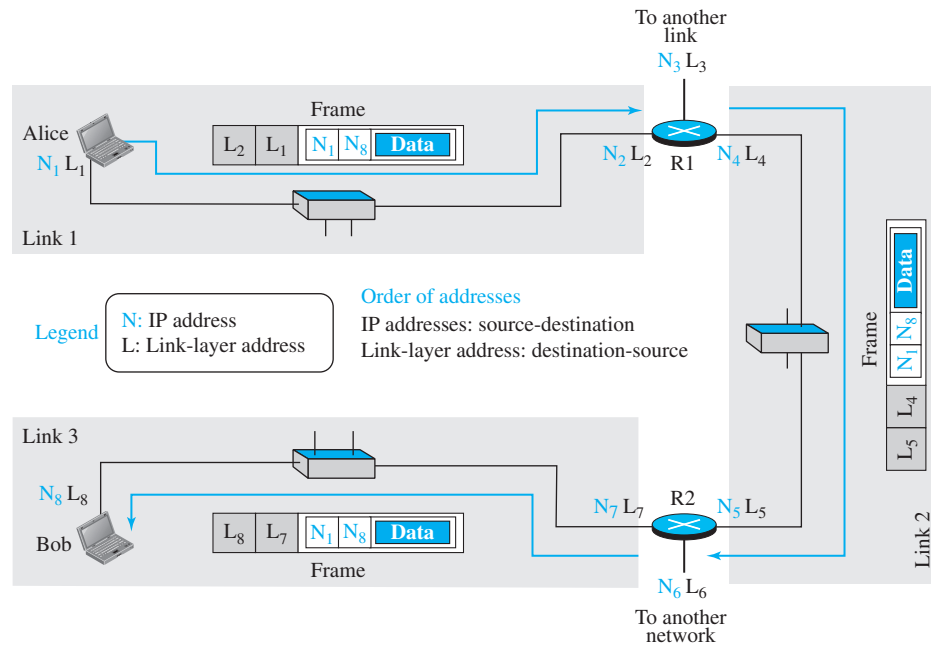
The next issue we need to discuss about the data-link layer is the link-layer addresses. In Chapter 18, we will discuss IP addresses as the identifiers at the network layer that define the exact points in the Internet where the source and destination hosts are connected. However, in a connectionless internetwork such as the Internet we cannot make a datagram reach its destination using only IP addresses. The reason is that each datagram in the Internet, from the same source host to the same destination host, may take a different path. The source and destination IP addresses define the two ends but cannot define which links the datagram should pass through.

We need to remember that the IP addresses in a datagram should not be changed. If the destination IP address in a datagram changes, the packet never reaches its destination; if the source IP address in a datagram changes, the destination host or a router can never communicate with the source if a response needs to be sent back or an error needs to be reported back to the source (see ICMP in Chapter 19).

The above discussion shows that we need another addressing mechanism in a connectionless internetwork: the link-layer addresses of the two nodes. A *link-layer address* is sometimes called a *link address*, sometimes a *physical address*, and sometimes a *MAC address*. We use these terms interchangeably in this book.

Since a link is controlled at the data-link layer, the addresses need to belong to the data-link layer. When a datagram passes from the network layer to the data-link layer, the datagram will be encapsulated in a frame and two data-link addresses are added to the frame header. These two addresses are changed every time the frame moves from one link to another. Figure 9.5 demonstrates the concept in a small internet.

Figure 9.5 IP addresses and link-layer addresses in a small internet



In the internet in Figure 9.5, we have three links and two routers. We also have shown only two hosts: Alice (source) and Bob (destination). For each host, we have shown two addresses, the IP addresses (N) and the link-layer addresses (L). Note that a router has as many pairs of addresses as the number of links the router is connected to. We have shown three frames, one in each link. Each frame carries the same datagram with the same source and destination addresses (N1 and N8), but the link-layer addresses of the frame change from link to link. In link 1, the link-layer addresses are L1 and L2. In link 2, they are L4 and L5. In link 3, they are L7 and L8. Note that the IP addresses and the link-layer addresses are not in the same order. For IP addresses, the source address comes before the destination address; for link-layer addresses, the destination address comes before the source. The datagrams and

frames are designed in this way, and we follow the design. We may raise several questions:

- ❑ If the IP address of a router does not appear in any datagram sent from a source to a destination, why do we need to assign IP addresses to routers? The answer is that in some protocols a router may act as a sender or receiver of a datagram. For example, in routing protocols we will discuss in Chapters 20 and 21, a router is a sender or a receiver of a message. The communications in these protocols are between routers.
- ❑ Why do we need more than one IP address in a router, one for each interface? The answer is that an interface is a connection of a router to a link. We will see that an IP address defines a point in the Internet at which a device is connected. A router with n interfaces is connected to the Internet at n points. This is the situation of a house at the corner of a street with two gates; each gate has the address related to the corresponding street.
- ❑ How are the source and destination IP addresses in a packet determined? The answer is that the host should know its own IP address, which becomes the source IP address in the packet. As we will discuss in Chapter 26, the application layer uses the services of DNS to find the destination address of the packet and passes it to the network layer to be inserted in the packet.
- ❑ How are the source and destination link-layer addresses determined for each link? Again, each hop (router or host) should know its own link-layer address, as we discuss later in the chapter. The destination link-layer address is determined by using the Address Resolution Protocol, which we discuss shortly.
- ❑ What is the size of link-layer addresses? The answer is that it depends on the protocol used by the link. Although we have only one IP protocol for the whole Internet, we may be using different data-link protocols in different links. This means that we can define the size of the address when we discuss different link-layer protocols.

9.2.1 Three Types of addresses

Some link-layer protocols define three types of addresses: unicast, multicast, and broadcast.

Unicast Address

Each host or each interface of a router is assigned a unicast address. Unicasting means one-to-one communication. A frame with a unicast address destination is destined only for one entity in the link.

Example 9.1

As we will see in Chapter 13, the unicast link-layer addresses in the most common LAN, Ethernet, are 48 bits (six bytes) that are presented as 12 hexadecimal digits separated by colons; for example, the following is a link-layer address of a computer.

A3:34:45:11:92:F1

Multicast Address

Some link-layer protocols define multicast addresses. Multicasting means one-to-many communication. However, the jurisdiction is local (inside the link).

Example 9.2

As we will see in Chapter 13, the multicast link-layer addresses in the most common LAN, Ethernet, are 48 bits (six bytes) that are presented as 12 hexadecimal digits separated by colons. The second digit, however, needs to be an even number in hexadecimal. The following shows a multicast address:

A2:34:45:11:92:F1

Broadcast Address

Some link-layer protocols define a broadcast address. Broadcasting means one-to-all communication. A frame with a destination broadcast address is sent to all entities in the link.

Example 9.3

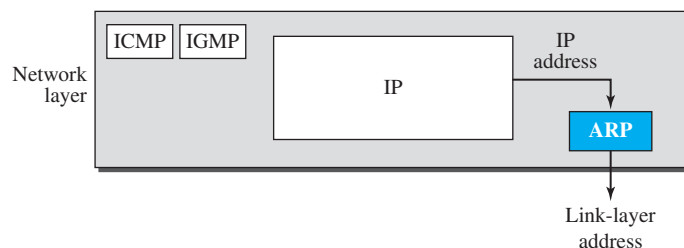
As we will see in Chapter 13, the broadcast link-layer addresses in the most common LAN, Ethernet, are 48 bits, all 1s, that are presented as 12 hexadecimal digits separated by colons. The following shows a broadcast address:

FF:FF:FF:FF:FF:FF

9.2.2 Address Resolution Protocol (ARP)

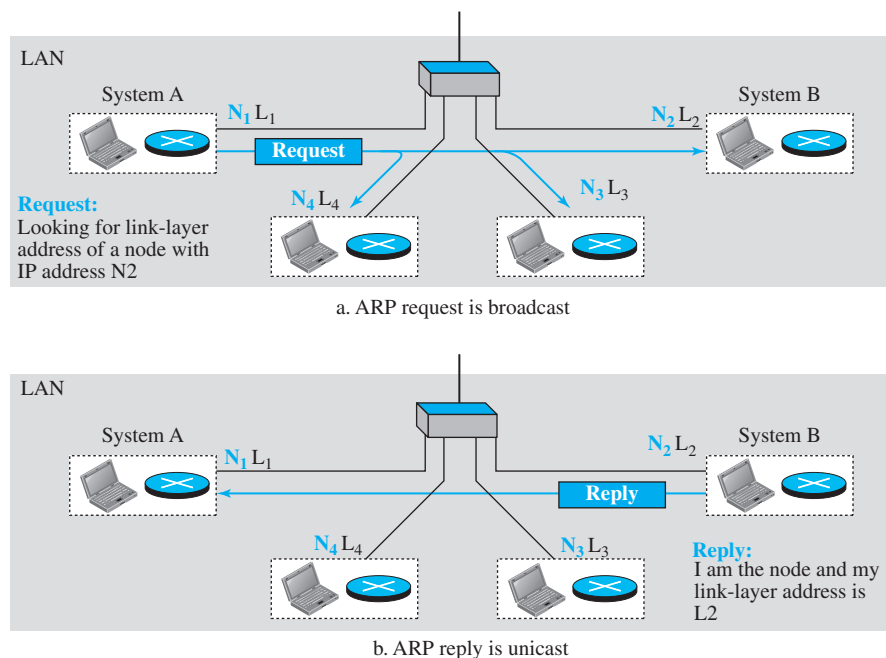
Anytime a node has an IP datagram to send to another node in a link, it has the IP address of the receiving node. The source host knows the IP address of the default router. Each router except the last one in the path gets the IP address of the next router by using its forwarding table. The last router knows the IP address of the destination host. However, the IP address of the next node is not helpful in moving a frame through a link; we need the link-layer address of the next node. This is the time when the **Address Resolution Protocol (ARP)** becomes helpful. The ARP protocol is one of the auxiliary protocols defined in the network layer, as shown in Figure 9.6. It belongs to the network layer, but we discuss it in this chapter because it maps an IP address to a logical-link address. ARP accepts an IP address from the IP protocol, maps the address to the corresponding link-layer address, and passes it to the data-link layer.

Figure 9.6 Position of ARP in TCP/IP protocol suite



Anytime a host or a router needs to find the link-layer address of another host or router in its network, it sends an ARP request packet. The packet includes the link-layer and IP addresses of the sender and the IP address of the receiver. Because the sender does not know the link-layer address of the receiver, the query is broadcast over the link using the link-layer broadcast address, which we discuss for each protocol later (see Figure 9.7).

Figure 9.7 ARP operation



Every host or router on the network receives and processes the ARP request packet, but only the intended recipient recognizes its IP address and sends back an ARP response packet. The response packet contains the recipient's IP and link-layer addresses. The packet is unicast directly to the node that sent the request packet.

In Figure 9.7a, the system on the left (A) has a packet that needs to be delivered to another system (B) with IP address **N2**. System A needs to pass the packet to its data-link layer for the actual delivery, but it does not know the physical address of the recipient. It uses the services of ARP by asking the ARP protocol to send a broadcast ARP request packet to ask for the physical address of a system with an IP address of **N2**.

This packet is received by every system on the physical network, but only system B will answer it, as shown in Figure 9.7b. System B sends an ARP reply packet that includes its physical address. Now system A can send all the packets it has for this destination using the physical address it received.

Caching

A question that is often asked is this: If system A can broadcast a frame to find the link-layer address of system B, why can't system A send the datagram for system B using a broadcast frame? In other words, instead of sending one broadcast frame (ARP request), one unicast frame (ARP response), and another unicast frame (for sending the datagram), system A can encapsulate the datagram and send it to the network. System B receives it and keep it; other systems discard it.

To answer the question, we need to think about the efficiency. It is probable that system A has more than one datagram to send to system B in a short period of time. For example, if system B is supposed to receive a long e-mail or a long file, the data do not fit in one datagram.

Let us assume that there are 20 systems connected to the network (link): system A, system B, and 18 other systems. We also assume that system A has 10 datagrams to send to system B in one second.

- a. Without using ARP, system A needs to send 10 broadcast frames. Each of the 18 other systems need to receive the frames, decapsulate the frames, remove the datagram and pass it to their network-layer to find out the datagrams do not belong to them. This means processing and discarding 180 broadcast frames.
- b. Using ARP, system A needs to send only one broadcast frame. Each of the 18 other systems need to receive the frames, decapsulate the frames, remove the ARP message and pass the message to their ARP protocol to find that the frame must be discarded. This means processing and discarding only 18 (instead of 180) broadcast frames. After system B responds with its own data-link address, system A can store the link-layer address in its cache memory. The rest of the nine frames are only unicast. Since processing broadcast frames is expensive (time consuming), the first method is preferable.

Packet Format

Figure 9.8 shows the format of an ARP packet. The names of the fields are self-explanatory. The *hardware type* field defines the type of the link-layer protocol; Ethernet is given the type 1. The *protocol type* field defines the network-layer protocol: IPv4 protocol is $(0800)_{16}$. The source hardware and source protocol addresses are variable-length fields defining the link-layer and network-layer addresses of the sender. The destination hardware address and destination protocol address fields define the receiver link-layer and network-layer addresses. An ARP packet is encapsulated directly into a data-link frame. The frame needs to have a field to show that the payload belongs to the ARP and not to the network-layer datagram.

Example 9.4

A host with IP address **N1** and MAC address **L1** has a packet to send to another host with IP address **N2** and physical address **L2** (which is unknown to the first host). The two hosts are on the same network. Figure 9.9 shows the ARP request and response messages.

Figure 9.8 ARP packet

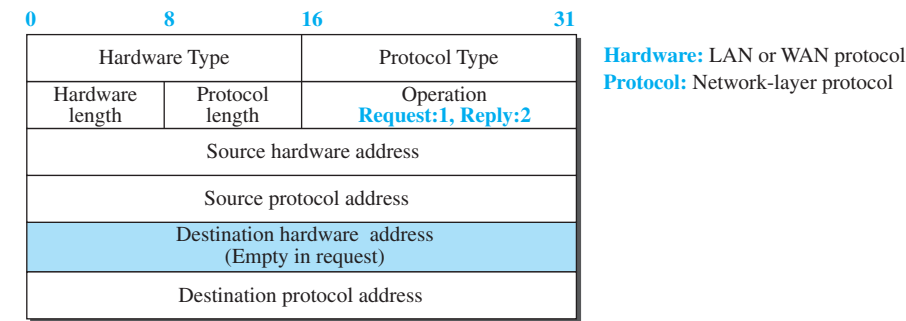
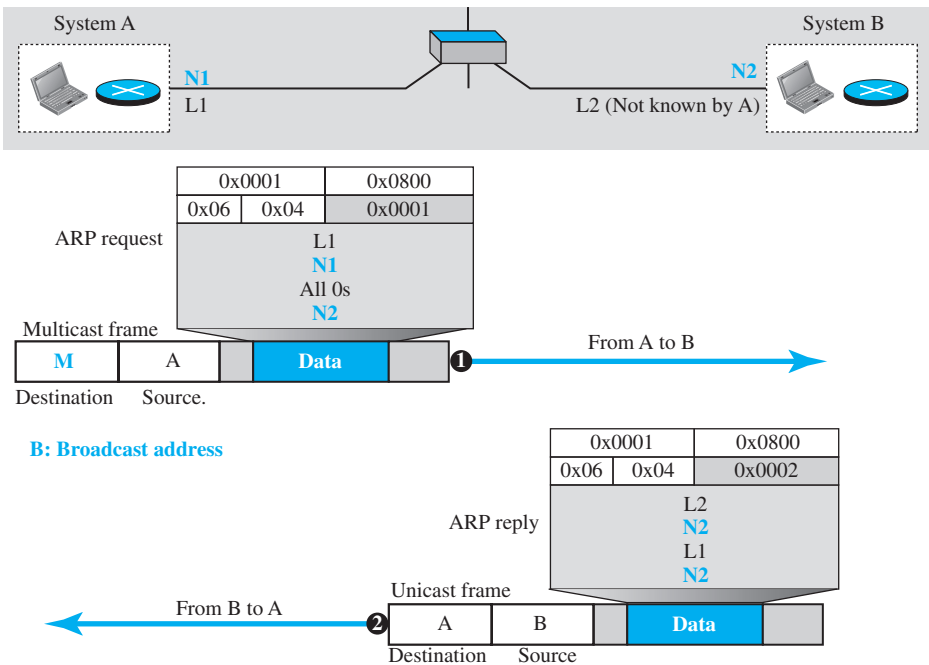


Figure 9.9 Example 9.4

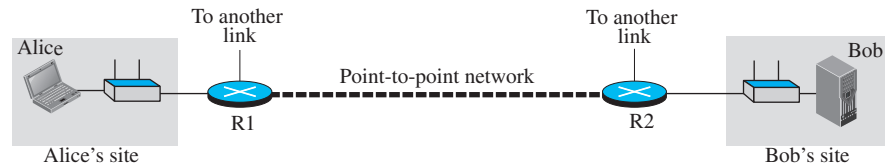


9.2.3 An Example of Communication

To show how communication is done at the data-link layer and how link-layer addresses are found, let us go through a simple example. Assume Alice needs to send a datagram to Bob, who is three nodes away in the Internet. How Alice finds the network-layer address of Bob is what we discover in Chapter 26 when we discuss DNS. For the moment, assume that Alice knows the network-layer (IP) address of Bob. In other words, Alice's host is given the data to be sent, the IP address of Bob, and the

IP address of Alice's host (each host needs to know its IP address). Figure 9.10 shows the part of the internet for our example.

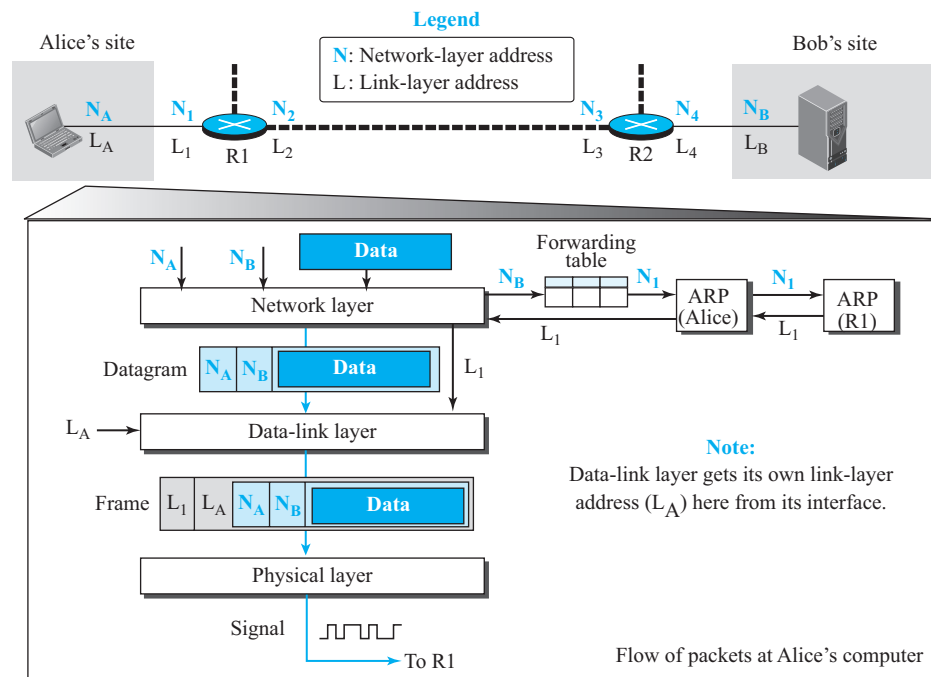
Figure 9.10 The internet for our example



Activities at Alice's Site

We will use symbolic addresses to make the figures more readable. Figure 9.11 shows what happens at Alice's site.

Figure 9.11 Flow of packets at Alice's computer



The network layer knows it's given N_A , N_B , and the packet, but it needs to find the link-layer address of the next node. The network layer consults its routing table and tries to find which router is next (the default router in this case) for the destination N_B . As we will discuss in Chapter 18, the routing table gives N_1 , but the network layer

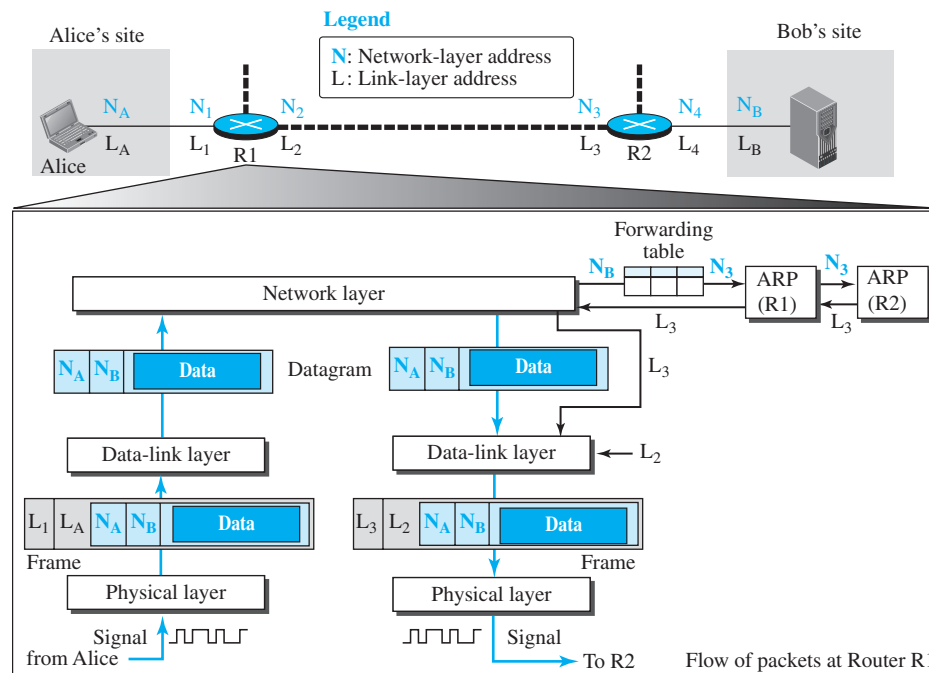
needs to find the link-layer address of router R1. It uses its ARP to find the link-layer address L_1 . The network layer can now pass the datagram with the link-layer address to the data-link layer.

The data-link layer knows its own link-layer address, L_A . It creates the frame and passes it to the physical layer, where the address is converted to signals and sent through the media.

Activities at Router R1

Now let us see what happens at Router R1. Router R1, as we know, has only three lower layers. The packet received needs to go up through these three layers and come down. Figure 9.12 shows the activities.

Figure 9.12 Flow of activities at router R1



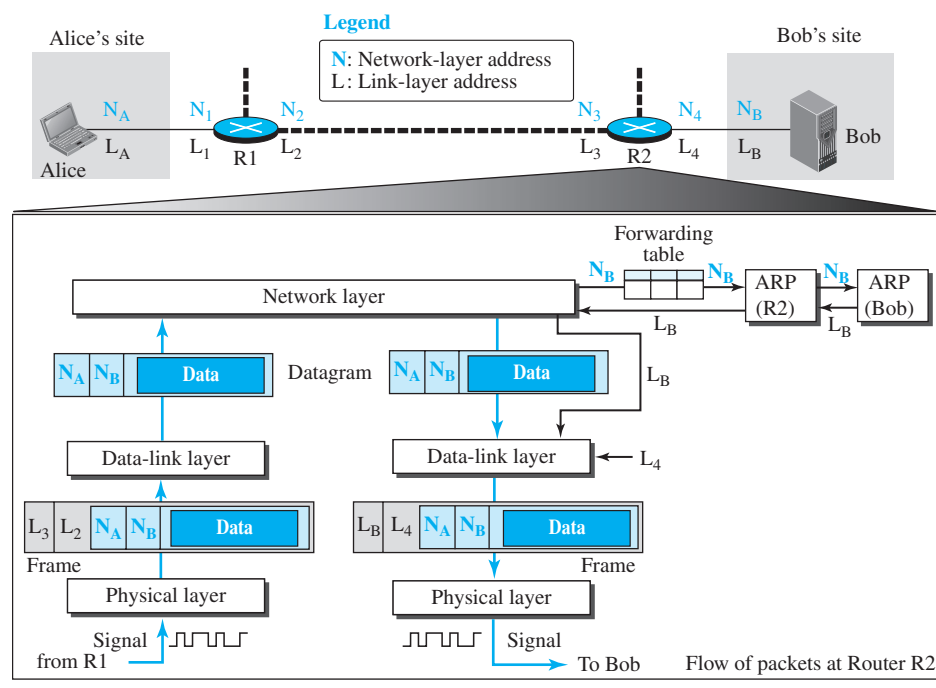
At arrival, the physical layer of the left link creates the frame and passes it to the data-link layer. The data-link layer decapsulates the datagram and passes it to the network layer. The network layer examines the network-layer address of the datagram and finds that the datagram needs to be delivered to the device with IP address N_B . The network layer consults its routing table to find out which is the next node (router) in the path to N_B . The forwarding table returns N_3 . The IP address of router R2 is in the same link with R1. The network layer now uses the ARP to find the link-layer address of this router, which comes up as L_3 . The network layer passes the datagram and L_3 to the data-link layer belonging to the link at the right side. The link layer

encapsulates the datagram, adds **L3** and **L2** (its own link-layer address), and passes the frame to the physical layer. The physical layer encodes the bits to signals and sends them through the medium to R2.

Activities at Router R2

Activities at router R2 are almost the same as in R1, as shown in Figure 9.13.

Figure 9.13 Activities at router R2.



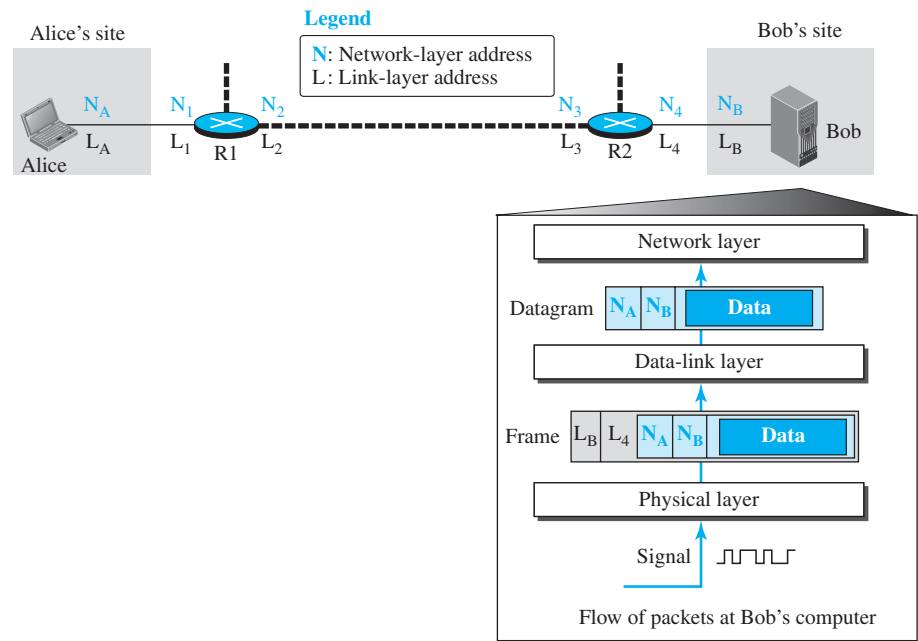
Activities at Bob's Site

Now let us see what happens at Bob's site. Figure 9.14 shows how the signals at Bob's site are changed to a message. At Bob's site there are no more addresses or mapping needed. The signal received from the link is changed to a frame. The frame is passed to the data-link layer, which decapsulates the datagram and passes it to the network layer. The network layer decapsulates the message and passes it to the transport layer.

Changes in Addresses

This example shows that the source and destination network-layer addresses, N_A and N_B , have not been changed during the whole journey. However, all four network-layer addresses of routers R1 and R2 (N_1 , N_2 , N_3 , and N_4) are needed to transfer a datagram from Alice's computer to Bob's computer.

Figure 9.14 Activities at Bob's site



9.3 END-CHAPTER MATERIALS

9.3.1 Recommended Reading

For more details about subjects discussed in this chapter, we recommend the following books. The items in brackets [...] refer to the reference list at the end of the text.

Books

Several books discuss link-layer issues. Among them we recommend [Ham 80], [Zar 02], [Ror 96], [Tan 03], [GW 04], [For 03], [KMK 04], [Sta 04], [Kes 02], [PD 03], [Kei 02], [Spu 00], [KCK 98], [Sau 98], [Izz 00], [Per 00], and [WV 00].

9.3.2 Key Terms

Address Resolution Protocol (ARP)	links
data link control (DLC)	media access control (MAC)
frame	nodes
framing	

9.3.3 Summary

The Internet is made of many hosts, networks, and connecting devices such as routers. The hosts and connecting devices are referred to as *nodes*; the networks are referred to

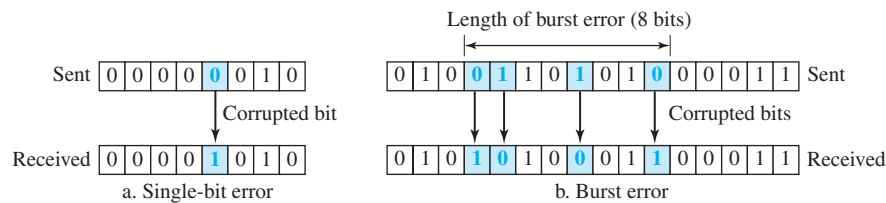
10.1 INTRODUCTION

Let us first discuss some issues related, directly or indirectly, to error detection and correction.

10.1.1 Types of Errors

Whenever bits flow from one point to another, they are subject to unpredictable changes because of **interference**. This interference can change the shape of the signal. The term **single-bit error** means that only 1 bit of a given data unit (such as a byte, character, or packet) is changed from 1 to 0 or from 0 to 1. The term **burst error** means that 2 or more bits in the data unit have changed from 1 to 0 or from 0 to 1. Figure 10.1 shows the effect of a single-bit and a burst error on a data unit.

Figure 10.1 Single-bit and burst error



A burst error is more likely to occur than a single-bit error because the duration of the noise signal is normally longer than the duration of 1 bit, which means that when noise affects data, it affects a set of bits. The number of bits affected depends on the data rate and duration of noise. For example, if we are sending data at 1 kbps, a noise of 1/100 second can affect 10 bits; if we are sending data at 1 Mbps, the same noise can affect 10,000 bits.

10.1.2 Redundancy

The central concept in detecting or correcting errors is **redundancy**. To be able to detect or correct errors, we need to send some extra bits with our data. These redundant bits are added by the sender and removed by the receiver. Their presence allows the receiver to detect or correct corrupted bits.

10.1.3 Detection versus Correction

The correction of errors is more difficult than the detection. In **error detection**, we are only looking to see if any error has occurred. The answer is a simple yes or no. We are not even interested in the number of corrupted bits. A single-bit error is the same for us as a burst error. In **error correction**, we need to know the exact number of bits that are corrupted and, more importantly, their location in the message. The number of errors and the size of the message are important factors. If we need to correct a single error in an 8-bit data unit, we need to consider eight possible error locations; if we need to correct two

errors in a data unit of the same size, we need to consider 28 (permutation of 8 by 2) possibilities. You can imagine the receiver's difficulty in finding 10 errors in a data unit of 1000 bits.

10.1.4 Coding

Redundancy is achieved through various coding schemes. The sender adds redundant bits through a process that creates a relationship between the redundant bits and the actual data bits. The receiver checks the relationships between the two sets of bits to detect errors. The ratio of redundant bits to data bits and the robustness of the process are important factors in any coding scheme.

We can divide coding schemes into two broad categories: **block coding** and **convolution coding**. In this book, we concentrate on block coding; convolution coding is more complex and beyond the scope of this book.

10.2 BLOCK CODING

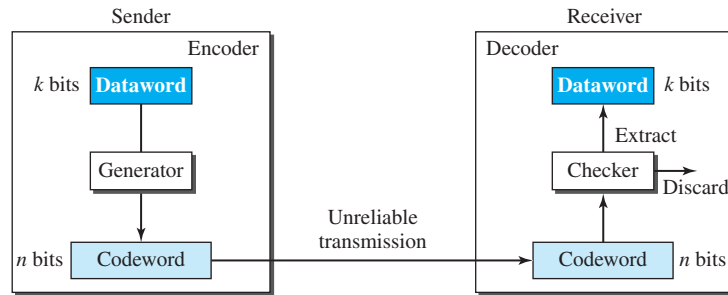
In block coding, we divide our message into blocks, each of k bits, called *datawords*. We add r redundant bits to each block to make the length $n = k + r$. The resulting n -bit blocks are called *codewords*. How the extra r bits are chosen or calculated is something we will discuss later. For the moment, it is important to know that we have a set of datawords, each of size k , and a set of codewords, each of size of n . With k bits, we can create a combination of 2^k datawords; with n bits, we can create a combination of 2^n codewords. Since $n > k$, the number of possible codewords is larger than the number of possible datawords. The block coding process is one-to-one; the same dataword is always encoded as the same codeword. This means that we have $2^n - 2^k$ codewords that are not used. We call these codewords invalid or illegal. The trick in error detection is the existence of these invalid codes, as we discuss next. If the receiver receives an invalid codeword, this indicates that the data was corrupted during transmission.

10.2.1 Error Detection

How can errors be detected by using block coding? If the following two conditions are met, the receiver can detect a change in the original codeword.

1. The receiver has (or can find) a list of valid codewords.
2. The original codeword has changed to an invalid one.

Figure 10.2 shows the role of block coding in error detection. The sender creates codewords out of datawords by using a generator that applies the rules and procedures of encoding (discussed later). Each codeword sent to the receiver may change during transmission. If the received codeword is the same as one of the valid codewords, the word is accepted; the corresponding dataword is extracted for use. If the received codeword is not valid, it is discarded. However, if the codeword is corrupted during transmission but the received word still matches a valid codeword, the error remains undetected.

Figure 10.2 Process of error detection in block coding**Example 10.1**

Let us assume that $k = 2$ and $n = 3$. Table 10.1 shows the list of datawords and codewords. Later, we will see how to derive a codeword from a dataword.

Table 10.1 A code for error detection in Example 10.1

Dataword	Codeword	Dataword	Codeword
00	000	10	101
01	011	11	110

Assume the sender encodes the dataword 01 as 011 and sends it to the receiver. Consider the following cases:

1. The receiver receives 011. It is a valid codeword. The receiver extracts the dataword 01 from it.
2. The codeword is corrupted during transmission, and 111 is received (the leftmost bit is corrupted). This is not a valid codeword and is discarded.
3. The codeword is corrupted during transmission, and 000 is received (the right two bits are corrupted). This is a valid codeword. The receiver incorrectly extracts the dataword 00. Two corrupted bits have made the error undetectable.

An error-detecting code can detect only the types of errors for which it is designed; other types of errors may remain undetected.

Hamming Distance

One of the central concepts in coding for error control is the idea of the Hamming distance. The **Hamming distance** between two words (of the same size) is the number of differences between the corresponding bits. We show the Hamming distance between two words x and y as $d(x, y)$. We may wonder why Hamming distance is important for error detection. The reason is that the Hamming distance between the received codeword and the sent codeword is the number of bits that are corrupted during transmission. For example, if the codeword 00000 is sent and 01101 is received, 3 bits are in error and the Hamming distance between the two is $d(00000, 01101) = 3$. In other words, if the Hamming

distance between the sent and the received codeword is not zero, the codeword has been corrupted during transmission.

The Hamming distance can easily be found if we apply the XOR operation (\oplus) on the two words and count the number of 1s in the result. Note that the Hamming distance is a value greater than or equal to zero.

The Hamming distance between two words is the number of differences between corresponding bits.

Example 10.2

Let us find the Hamming distance between two pairs of words.

1. The Hamming distance $d(000, 011)$ is 2 because $(000 \oplus 011)$ is 011 (two 1s).
2. The Hamming distance $d(10101, 11110)$ is 3 because $(10101 \oplus 11110)$ is 01011 (three 1s).

Minimum Hamming Distance for Error Detection

In a set of codewords, the **minimum Hamming distance** is the smallest Hamming distance between all possible pairs of codewords. Now let us find the minimum Hamming distance in a code if we want to be able to detect up to s errors. If s errors occur during transmission, the Hamming distance between the sent codeword and received codeword is s . If our system is to detect up to s errors, the minimum distance between the valid codes must be $(s + 1)$, so that the received codeword does not match a valid codeword. In other words, if the minimum distance between all valid codewords is $(s + 1)$, the received codeword cannot be erroneously mistaken for another codeword. The error will be detected. We need to clarify a point here: Although a code with $d_{\min} = s + 1$ may be able to detect more than s errors in some special cases, only s or fewer errors are guaranteed to be detected.

To guarantee the detection of up to s errors in all cases, the minimum Hamming distance in a block code must be $d_{\min} = s + 1$.

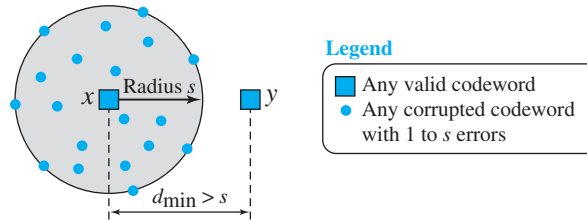
We can look at this criteria geometrically. Let us assume that the sent codeword x is at the center of a circle with radius s . All received codewords that are created by 0 to s errors are points inside the circle or on the perimeter of the circle. All other valid codewords must be outside the circle, as shown in Figure 10.3. This means that d_{\min} must be an integer greater than s or $d_{\min} = s + 1$.

Example 10.3

The minimum Hamming distance for our first code scheme (Table 10.1) is 2. This code guarantees detection of only a single error. For example, if the third codeword (101) is sent and one error occurs, the received codeword does not match any valid codeword. If two errors occur, however, the received codeword may match a valid codeword and the errors are not detected.

Example 10.4

A code scheme has a Hamming distance $d_{\min} = 4$. This code guarantees the detection of up to three errors ($d = s + 1$ or $s = 3$).

Figure 10.3 Geometric concept explaining d_{\min} in error detection

Linear Block Codes

Almost all block codes used today belong to a subset of block codes called **linear block codes**. The use of nonlinear block codes for error detection and correction is not as widespread because their structure makes theoretical analysis and implementation difficult. We therefore concentrate on linear block codes. The formal definition of linear block codes requires the knowledge of abstract algebra (particularly Galois fields), which is beyond the scope of this book. We therefore give an informal definition. For our purposes, a linear block code is a code in which the exclusive OR (addition modulo-2) of two valid codewords creates another valid codeword.

Example 10.5

The code in Table 10.1 is a linear block code because the result of XORing any codeword with any other codeword is a valid codeword. For example, the XORing of the second and third codewords creates the fourth one.

Minimum Distance for Linear Block Codes

It is simple to find the minimum Hamming distance for a linear block code. The minimum Hamming distance is the number of 1s in the nonzero valid codeword with the smallest number of 1s.

Example 10.6

In our first code (Table 10.1), the numbers of 1s in the nonzero codewords are 2, 2, and 2. So the minimum Hamming distance is $d_{\min} = 2$.

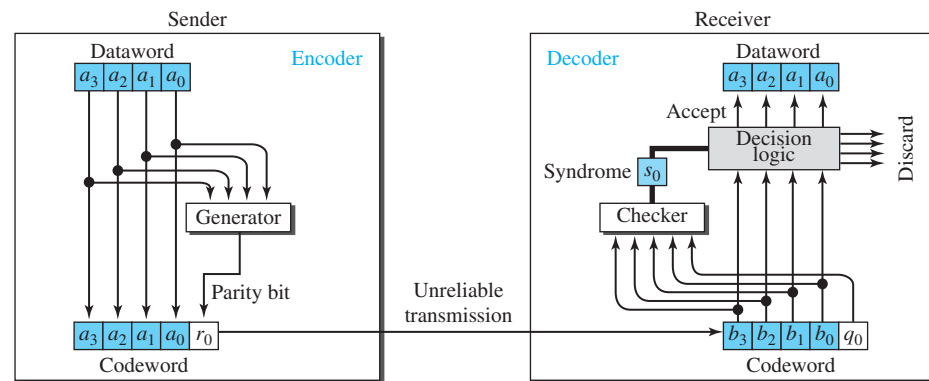
Parity-Check Code

Perhaps the most familiar error-detecting code is the **parity-check code**. This code is a linear block code. In this code, a k -bit dataword is changed to an n -bit codeword where $n = k + 1$. The extra bit, called the *parity bit*, is selected to make the total number of 1s in the codeword even. Although some implementations specify an odd number of 1s, we discuss the even case. The minimum Hamming distance for this category is $d_{\min} = 2$, which means that the code is a single-bit error-detecting code. Our first code (Table 10.1) is a parity-check code ($k = 2$ and $n = 3$). The code in Table 10.2 is also a parity-check code with $k = 4$ and $n = 5$.

Table 10.2 Simple parity-check code $C(5, 4)$

Dataword	Codeword	Dataword	Codeword
0000	00000	1000	10001
0001	00011	1001	10010
0010	00101	1010	10100
0011	00110	1011	10111
0100	01001	1100	11000
0101	01010	1101	11011
0110	01100	1110	11101
0111	01111	1111	11110

Figure 10.4 shows a possible structure of an encoder (at the sender) and a decoder (at the receiver).

Figure 10.4 Encoder and decoder for simple parity-check code

The calculation is done in **modular arithmetic** (see Appendix E). The encoder uses a generator that takes a copy of a 4-bit dataword (a_0, a_1, a_2 , and a_3) and generates a parity bit r_0 . The dataword bits and the parity bit create the 5-bit codeword. The parity bit that is added makes the number of 1s in the codeword even. This is normally done by adding the 4 bits of the dataword (modulo-2); the result is the parity bit. In other words,

$$r_0 = a_3 + a_2 + a_1 + a_0 \quad (\text{modulo-2})$$

If the number of 1s is even, the result is 0; if the number of 1s is odd, the result is 1. In both cases, the total number of 1s in the codeword is even.

The sender sends the codeword, which may be corrupted during transmission. The receiver receives a 5-bit word. The checker at the receiver does the same thing as the generator in the sender with one exception: The addition is done over all 5 bits. The result,

which is called the **syndrome**, is just 1 bit. The syndrome is 0 when the number of 1s in the received codeword is even; otherwise, it is 1.

$$s_0 = b_3 + b_2 + b_1 + b_0 + q_0 \quad (\text{modulo-2})$$

The syndrome is passed to the decision logic analyzer. If the syndrome is 0, there is no detectable error in the received codeword; the data portion of the received codeword is accepted as the dataword; if the syndrome is 1, the data portion of the received codeword is discarded. The dataword is not created.

Example 10.7

Let us look at some transmission scenarios. Assume the sender sends the dataword 1011. The codeword created from this dataword is 10111, which is sent to the receiver. We examine five cases:

1. No error occurs; the received codeword is 10111. The syndrome is 0. The dataword 1011 is created.
2. One single-bit error changes a_1 . The received codeword is 10011. The syndrome is 1. No dataword is created.
3. One single-bit error changes r_0 . The received codeword is 10110. The syndrome is 1. No dataword is created. Note that although none of the dataword bits are corrupted, no dataword is created because the code is not sophisticated enough to show the position of the corrupted bit.
4. An error changes r_0 and a second error changes a_3 . The received codeword is 00110. The syndrome is 0. The dataword 0011 is created at the receiver. Note that here the dataword is wrongly created due to the syndrome value. The simple parity-check decoder cannot detect an even number of errors. The errors cancel each other out and give the syndrome a value of 0.
5. Three bits— a_3 , a_2 , and a_1 —are changed by errors. The received codeword is 01011. The syndrome is 1. The dataword is not created. This shows that the simple parity check, guaranteed to detect one single error, can also find any odd number of errors.

A parity-check code can detect an odd number of errors.

10.3 CYCLIC CODES

Cyclic codes are special linear block codes with one extra property. In a **cyclic code**, if a codeword is cyclically shifted (rotated), the result is another codeword. For example, if 1011000 is a codeword and we cyclically left-shift, then 0110001 is also a codeword. In this case, if we call the bits in the first word a_0 to a_6 , and the bits in the second word b_0 to b_6 , we can shift the bits by using the following:

$$b_1 = a_0 \quad b_2 = a_1 \quad b_3 = a_2 \quad b_4 = a_3 \quad b_5 = a_4 \quad b_6 = a_5 \quad b_0 = a_6$$

In the rightmost equation, the last bit of the first word is wrapped around and becomes the first bit of the second word.

10.3.1 Cyclic Redundancy Check

We can create cyclic codes to correct errors. However, the theoretical background required is beyond the scope of this book. In this section, we simply discuss a subset of

cyclic codes called the **cyclic redundancy check (CRC)**, which is used in networks such as LANs and WANs.

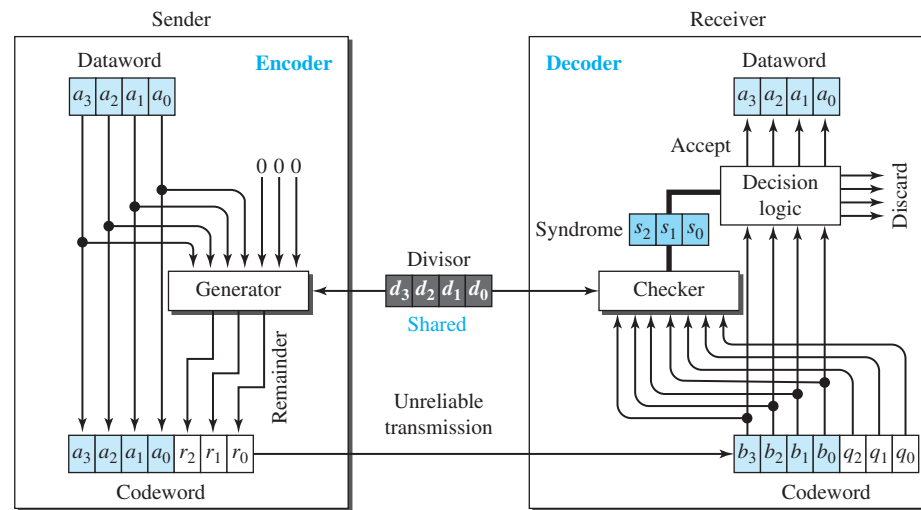
Table 10.3 shows an example of a CRC code. We can see both the linear and cyclic properties of this code.

Table 10.3 A CRC code with $C(7, 4)$

Dataword	Codeword	Dataword	Codeword
0000	0000000	1000	1000101
0001	0001011	1001	1001110
0010	0010110	1010	1010011
0011	0011101	1011	1011000
0100	0100111	1100	1100010
0101	0101100	1101	1101001
0110	0110001	1110	1110100
0111	0111010	1111	1111111

Figure 10.5 shows one possible design for the encoder and decoder.

Figure 10.5 CRC encoder and decoder



In the encoder, the dataword has k bits (4 here); the codeword has n bits (7 here). The size of the dataword is augmented by adding $n - k$ (3 here) 0s to the right-hand side of the word. The n -bit result is fed into the generator. The generator uses a divisor of size $n - k + 1$ (4 here), predefined and agreed upon. The generator divides the augmented dataword by the divisor (modulo-2 division). The quotient of the division is discarded; the remainder ($r_2r_1r_0$) is appended to the dataword to create the codeword.

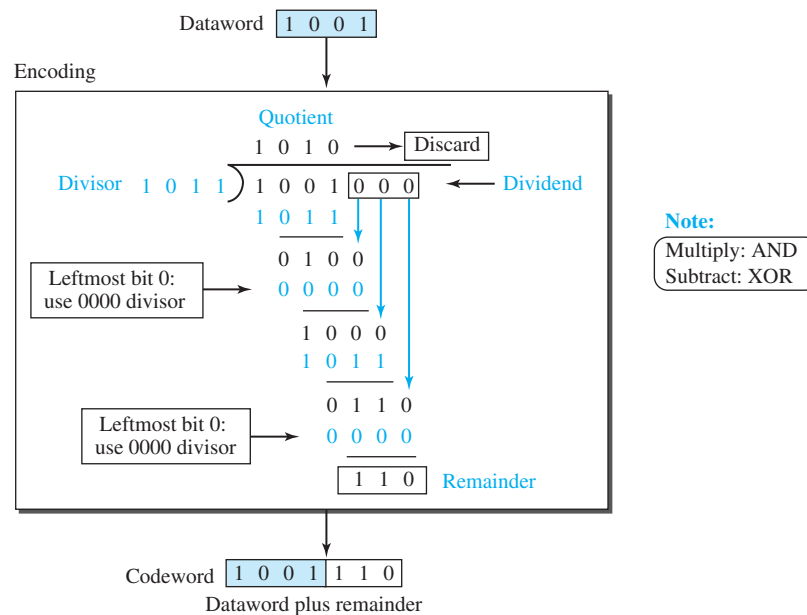
The decoder receives the codeword (possibly corrupted in transition). A copy of all n bits is fed to the checker, which is a replica of the generator. The remainder produced

by the checker is a syndrome of $n - k$ (3 here) bits, which is fed to the decision logic analyzer. The analyzer has a simple function. If the syndrome bits are all 0s, the 4 leftmost bits of the codeword are accepted as the dataword (interpreted as no error); otherwise, the 4 bits are discarded (error).

Encoder

Let us take a closer look at the encoder. The encoder takes a dataword and augments it with $n - k$ number of 0s. It then divides the augmented dataword by the divisor, as shown in Figure 10.6.

Figure 10.6 Division in CRC encoder



The process of modulo-2 binary division is the same as the familiar division process we use for decimal numbers. However, addition and subtraction in this case are the same; we use the XOR operation to do both.

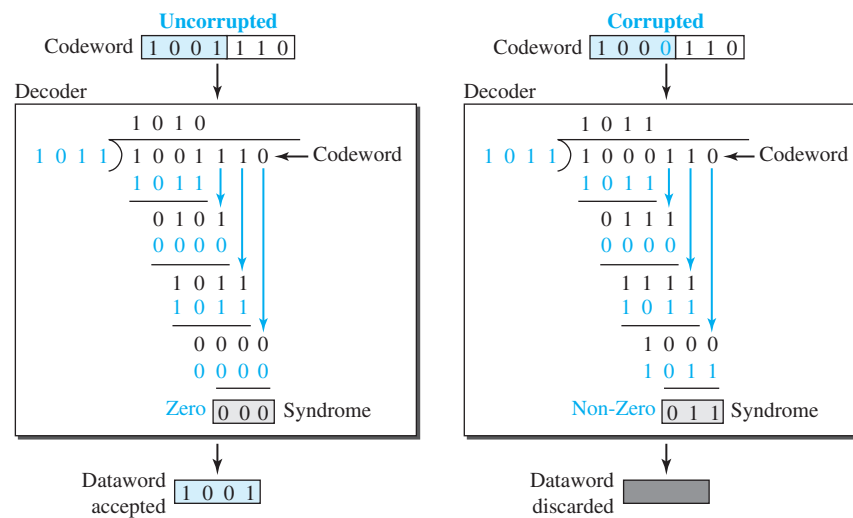
As in decimal division, the process is done step by step. In each step, a copy of the divisor is XORed with the 4 bits of the dividend. The result of the XOR operation (remainder) is 3 bits (in this case), which is used for the next step after 1 extra bit is pulled down to make it 4 bits long. There is one important point we need to remember in this type of division. If the leftmost bit of the dividend (or the part used in each step) is 0, the step cannot use the regular divisor; we need to use an all-0s divisor.

When there are no bits left to pull down, we have a result. The 3-bit remainder forms the **check bits** (r_2 , r_1 , and r_0). They are appended to the dataword to create the codeword.

Decoder

The codeword can change during transmission. The decoder does the same division process as the encoder. The remainder of the division is the syndrome. If the syndrome is all 0s, there is no error with a high probability; the dataword is separated from the received codeword and accepted. Otherwise, everything is discarded. Figure 10.7 shows two cases: The left-hand figure shows the value of the syndrome when no error has occurred; the syndrome is 000. The right-hand part of the figure shows the case in which there is a single error. The syndrome is not all 0s (it is 011).

Figure 10.7 Division in the CRC decoder for two cases



Divisor

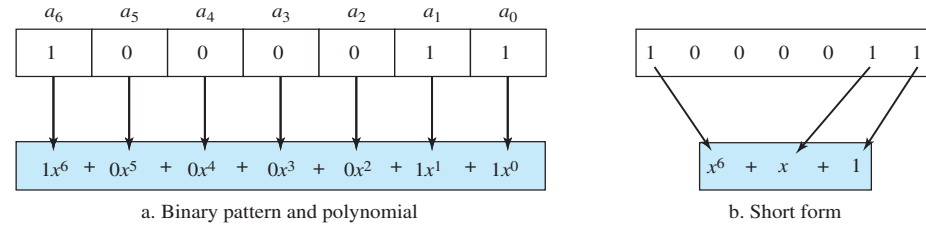
We may be wondering how the divisor 1011 is chosen. This depends on the expectation we have from the code. We will show some standard divisors later in the chapter (Table 10.4) after we discuss polynomials.

10.3.2 Polynomials

A better way to understand cyclic codes and how they can be analyzed is to represent them as polynomials. Again, this section is optional.

A pattern of 0s and 1s can be represented as a **polynomial** with coefficients of 0 and 1. The power of each term shows the position of the bit; the coefficient shows the value of the bit. Figure 10.8 shows a binary pattern and its polynomial representation. In Figure 10.8a we show how to translate a binary pattern into a polynomial; in Figure 10.8b we show how the polynomial can be shortened by removing all terms with zero coefficients and replacing x^1 by x and x^0 by 1.

Figure 10.8 shows one immediate benefit; a 7-bit pattern can be replaced by three terms. The benefit is even more conspicuous when we have a polynomial such as

Figure 10.8 A polynomial to represent a binary word

$x^{23} + x^3 + 1$. Here the bit pattern is 24 bits in length (three 1s and twenty-one 0s) while the polynomial is just three terms.

Degree of a Polynomial

The degree of a polynomial is the highest power in the polynomial. For example, the degree of the polynomial $x^6 + x + 1$ is 6. Note that the degree of a polynomial is 1 less than the number of bits in the pattern. The bit pattern in this case has 7 bits.

Adding and Subtracting Polynomials

Adding and subtracting polynomials in mathematics are done by adding or subtracting the coefficients of terms with the same power. In our case, the coefficients are only 0 and 1, and adding is in modulo-2. This has two consequences. First, addition and subtraction are the same. Second, adding or subtracting is done by combining terms and deleting pairs of identical terms. For example, adding $x^5 + x^4 + x^2$ and $x^6 + x^4 + x^2$ gives just $x^6 + x^5$. The terms x^4 and x^2 are deleted. However, note that if we add, for example, three polynomials and we get x^2 three times, we delete a pair of them and keep the third.

Multiplying or Dividing Terms

In this arithmetic, multiplying a term by another term is very simple; we just add the powers. For example, $x^3 \times x^4$ is x^7 . For dividing, we just subtract the power of the second term from the power of the first. For example, x^5/x^2 is x^3 .

Multiplying Two Polynomials

Multiplying a polynomial by another is done term by term. Each term of the first polynomial must be multiplied by all terms of the second. The result, of course, is then simplified, and pairs of equal terms are deleted. The following is an example:

$$\begin{aligned}(x^5 + x^3 + x^2 + x)(x^2 + x + 1) &= x^7 + x^6 + x^5 + x^5 + x^4 + x^3 + x^4 + x^3 + x^2 + x^2 + x \\ &= x^7 + x^6 + x^3 + x\end{aligned}$$

Dividing One Polynomial by Another

Division of polynomials is conceptually the same as the binary division we discussed for an encoder. We divide the first term of the dividend by the first term of the divisor to get the first term of the quotient. We multiply the term in the quotient by the divisor and

subtract the result from the dividend. We repeat the process until the dividend degree is less than the divisor degree. We will show an example of division later in this chapter.

Shifting

A binary pattern is often shifted a number of bits to the right or left. Shifting to the left means adding extra 0s as rightmost bits; shifting to the right means deleting some rightmost bits. Shifting to the left is accomplished by multiplying each term of the polynomial by x^m , where m is the number of shifted bits; shifting to the right is accomplished by dividing each term of the polynomial by x^m . The following shows shifting to the left and to the right. Note that we do not have negative powers in the polynomial representation.

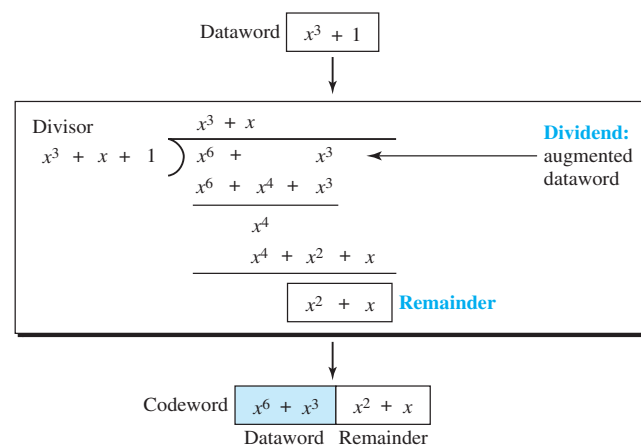
Shifting left 3 bits: 10011 becomes 10011000	$x^4 + x + 1$ becomes $x^7 + x^4 + x^3$
Shifting right 3 bits: 10011 becomes 10	$x^4 + x + 1$ becomes x

When we augmented the dataword in the encoder of Figure 10.6, we actually shifted the bits to the left. Also note that when we concatenate two bit patterns, we shift the first polynomial to the left and then add the second polynomial.

10.3.3 Cyclic Code Encoder Using Polynomials

Now that we have discussed operations on polynomials, we show the creation of a codeword from a dataword. Figure 10.9 is the polynomial version of Figure 10.6. We can see that the process is shorter. The dataword 1001 is represented as $x^3 + 1$. The divisor 1011 is represented as $x^3 + x + 1$. To find the augmented dataword, we have left-shifted the dataword 3 bits (multiplying by x^3). The result is $x^6 + x^3$. Division is straightforward. We divide the first term of the dividend, x^6 , by the first term of the divisor, x^3 . The first term of the quotient is then x^6/x^3 , or x^3 . Then we multiply x^3 by the divisor and subtract (according to our previous definition of subtraction) the result from the dividend. The result is x^4 , with a degree greater than the divisor's degree; we continue to divide until the degree of the remainder is less than the degree of the divisor.

Figure 10.9 CRC division using polynomials



It can be seen that the polynomial representation can easily simplify the operation of division in this case, because the two steps involving all-0s divisors are not needed here. (Of course, one could argue that the all-0s divisor step can also be eliminated in binary division.) In a polynomial representation, the divisor is normally referred to as the *generator polynomial* $t(x)$.

The divisor in a cyclic code is normally called the *generator polynomial* or simply the *generator*.

10.3.4 Cyclic Code Analysis

We can analyze a cyclic code to find its capabilities by using polynomials. We define the following, where $f(x)$ is a polynomial with binary coefficients.

Dataword: $d(x)$ **Codeword:** $c(x)$ **Generator:** $g(x)$ **Syndrome:** $s(x)$ **Error:** $e(x)$

If $s(x)$ is not zero, then one or more bits is corrupted. However, if $s(x)$ is zero, either no bit is corrupted or the decoder failed to detect any errors. (Note that \div means divide).

In a cyclic code,

1. If $s(x) \div 0$, one or more bits is corrupted.
2. If $s(x) = 0$, either
 - a. No bit is corrupted, or
 - b. Some bits are corrupted, but the decoder failed to detect them.

In our analysis we want to find the criteria that must be imposed on the generator, $g(x)$ to detect the type of error we especially want to be detected. Let us first find the relationship among the sent codeword, error, received codeword, and the generator. We can say

$$\text{Received codeword} = c(x) + e(x)$$

In other words, the received codeword is the sum of the sent codeword and the error. The receiver divides the received codeword by $g(x)$ to get the syndrome. We can write this as

$$\frac{\text{Received codeword}}{g(x)} = \frac{c(x)}{g(x)} + \frac{e(x)}{g(x)}$$

The first term at the right-hand side of the equality has a remainder of zero (according to the definition of codeword). So the syndrome is actually the remainder of the second term on the right-hand side. If this term does not have a remainder (syndrome = 0), either $e(x)$ is 0 or $e(x)$ is divisible by $g(x)$. We do not have to worry about the first case (there is no error); the second case is very important. Those errors that are divisible by $g(x)$ are not caught.

In a cyclic code, those $e(x)$ errors that are divisible by $g(x)$ are not caught.

Let us show some specific errors and see how they can be caught by a well-designed $g(x)$.

Single-Bit Error

What should the structure of $g(x)$ be to guarantee the detection of a single-bit error? A single-bit error is $e(x) = x^i$, where i is the position of the bit. If a single-bit error is caught, then x^i is not divisible by $g(x)$. (Note that when we say *not divisible*, we mean that there is a remainder.) If $g(x)$ has at least two terms (which is normally the case) and the coefficient of x^0 is not zero (the rightmost bit is 1), then $e(x)$ cannot be divided by $g(x)$.

If the generator has more than one term and the coefficient of x^0 is 1, all single-bit errors can be caught.

Example 10.8

Which of the following $g(x)$ values guarantees that a single-bit error is caught? For each case, what is the error that cannot be caught?

- $x + 1$
- x^3
- 1

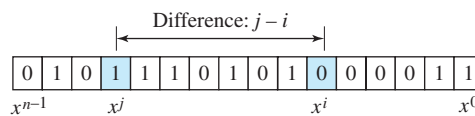
Solution

- No x^i can be divisible by $x + 1$. In other words, $x^i/(x + 1)$ always has a remainder. So the syndrome is nonzero. Any single-bit error can be caught.
- If i is equal to or greater than 3, x^i is divisible by $g(x)$. The remainder of x^i/x^3 is zero, and the receiver is fooled into believing that there is no error, although there might be one. Note that in this case, the corrupted bit must be in position 4 or above. All single-bit errors in positions 1 to 3 are caught.
- All values of i make x^i divisible by $g(x)$. No single-bit error can be caught. In addition, this $g(x)$ is useless because it means the codeword is just the dataword augmented with $n - k$ zeros.

Two Isolated Single-Bit Errors

Now imagine there are two single-bit isolated errors. Under what conditions can this type of error be caught? We can show this type of error as $e(x) = x^j + x^i$. The values of i and j define the positions of the errors, and the difference $j - i$ defines the distance between the two errors, as shown in Figure 10.10.

Figure 10.10 Representation of two isolated single-bit errors using polynomials



We can write $e(x) = x^i(x^{j-i} + 1)$. If $g(x)$ has more than one term and one term is x^0 , it cannot divide x^i , as we saw in the previous section. So if $g(x)$ is to divide $e(x)$, it must divide $x^{j-i} + 1$. In other words, $g(x)$ must not divide $x^t + 1$, where t is between 0 and $n - 1$. However, $t = 0$ is meaningless and $t = 1$ is needed, as we will see later. This means t should be between 2 and $n - 1$.

If a generator cannot divide $x^t + 1$ (t between 0 and $n - 1$), then all isolated double errors can be detected.

Example 10.9

Find the status of the following generators related to two isolated, single-bit errors.

- a. $x + 1$
- b. $x^4 + 1$
- c. $x^7 + x^6 + 1$
- d. $x^{15} + x^{14} + 1$

Solution

- a. This is a very poor choice for a generator. Any two errors next to each other cannot be detected.
- b. This generator cannot detect two errors that are four positions apart. The two errors can be anywhere, but if their distance is 4, they remain undetected.
- c. This is a good choice for this purpose.
- d. This polynomial cannot divide any error of type $x^t + 1$ if t is less than 32,768. This means that a codeword with two isolated errors that are next to each other or up to 32,768 bits apart can be detected by this generator.

Odd Numbers of Errors

A generator with a factor of $x + 1$ can catch all odd numbers of errors. This means that we need to make $x + 1$ a factor of any generator. Note that we are not saying that the generator itself should be $x + 1$; we are saying that it should have a factor of $x + 1$. If it is only $x + 1$, it cannot catch the two adjacent isolated errors (see the previous section). For example, $x^4 + x^2 + x + 1$ can catch all odd-numbered errors since it can be written as a product of the two polynomials $x + 1$ and $x^3 + x^2 + 1$.

A generator that contains a factor of $x + 1$ can detect all odd-numbered errors.

Burst Errors

Now let us extend our analysis to the burst error, which is the most important of all. A burst error is of the form $e(x) = (x^j + \dots + x^i)$. Note the difference between a burst error and two isolated single-bit errors. The first can have two terms or more; the second can only have two terms. We can factor out x^i and write the error as $x^i(x^{j-i} + \dots + 1)$. If our generator can detect a single error (minimum condition for a generator), then it cannot divide x^i . What we should worry about are those generators that divide $x^{j-i} + \dots + 1$. In other words, the remainder of $(x^{j-i} + \dots + 1)/(x^r + \dots + 1)$ must not be zero. Note that the denominator is the generator polynomial. We can have three cases:

1. If $j - i < r$, the remainder can never be zero. We can write $j - i = L - 1$, where L is the length of the error. So $L - 1 < r$ or $L < r + 1$ or $L \nmid r$. This means all burst errors with length smaller than or equal to the number of check bits r will be detected.
2. In some rare cases, if $j - i = r$, or $L = r + 1$, the syndrome is 0 and the error is undetected. It can be proved that in these cases, the probability of undetected burst error of length $r + 1$ is $(1/2)^{r-1}$. For example, if our generator is $x^{14} + x^3 + 1$, in which $r = 14$, a burst error of length $L = 15$ can slip by undetected with the probability of $(1/2)^{14-1}$ or almost 1 in 10,000.
3. In some rare cases, if $j - i > r$, or $L > r + 1$, the syndrome is 0 and the error is undetected. It can be proved that in these cases, the probability of undetected burst error of length greater than $r + 1$ is $(1/2)^r$. For example, if our generator is $x^{14} + x^3 + 1$, in which $r = 14$, a burst error of length greater than 15 can slip by undetected with the probability of $(1/2)^{14}$ or almost 1 in 16,000 cases.

- ☐ All burst errors with $L \leq r$ will be detected.
- ☐ All burst errors with $L = r + 1$ will be detected with probability $1 - (1/2)^{r-1}$.
- ☐ All burst errors with $L > r + 1$ will be detected with probability $1 - (1/2)^r$.

Example 10.10

Find the suitability of the following generators in relation to burst errors of different lengths.

- a. $x^6 + 1$
- b. $x^{18} + x^7 + x + 1$
- c. $x^{32} + x^{23} + x^7 + 1$

Solution

- a. This generator can detect all burst errors with a length less than or equal to 6 bits; 3 out of 100 burst errors with length 7 will slip by; 16 out of 1000 burst errors of length 8 or more will slip by.
- b. This generator can detect all burst errors with a length less than or equal to 18 bits; 8 out of 1 million burst errors with length 19 will slip by; 4 out of 1 million burst errors of length 20 or more will slip by.
- c. This generator can detect all burst errors with a length less than or equal to 32 bits; 5 out of 10 billion burst errors with length 33 will slip by; 3 out of 10 billion burst errors of length 34 or more will slip by.

Summary

We can summarize the criteria for a good polynomial generator:

A good polynomial generator needs to have the following characteristics:

1. It should have at least two terms.
2. The coefficient of the term x^0 should be 1.
3. It should not divide $x^t + 1$, for t between 2 and $n - 1$.
4. It should have the factor $x + 1$.

Standard Polynomials

Some standard polynomials used by popular protocols for CRC generation are shown in Table 10.4 along with the corresponding bit pattern.

Table 10.4 Standard polynomials

Name	Polynomial	Used in
CRC-8	$x^8 + x^2 + x + 1$ 100000111	ATM header
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^2 + 1$ 11000110101	ATM AAL
CRC-16	$x^{16} + x^{12} + x^5 + 1$ 10001000000100001	HDLC
CRC-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$ 100000100110000010001110110110111	LANs

10.3.5 Advantages of Cyclic Codes

We have seen that cyclic codes have a very good performance in detecting single-bit errors, double errors, an odd number of errors, and burst errors. They can easily be implemented in hardware and software. They are especially fast when implemented in hardware. This has made cyclic codes a good candidate for many networks.

10.3.6 Other Cyclic Codes

The cyclic codes we have discussed in this section are very simple. The check bits and syndromes can be calculated by simple algebra. There are, however, more powerful polynomials that are based on abstract algebra involving Galois fields. These are beyond the scope of this book. One of the most interesting of these codes is the **Reed-Solomon code** used today for both detection and correction.

10.3.7 Hardware Implementation

One of the advantages of a cyclic code is that the encoder and decoder can easily and cheaply be implemented in hardware by using a handful of electronic devices. Also, a hardware implementation increases the rate of check bit and syndrome bit calculation. In this section, we try to show, step by step, the process. The section, however, is optional and does not affect the understanding of the rest of the chapter.

Divisor

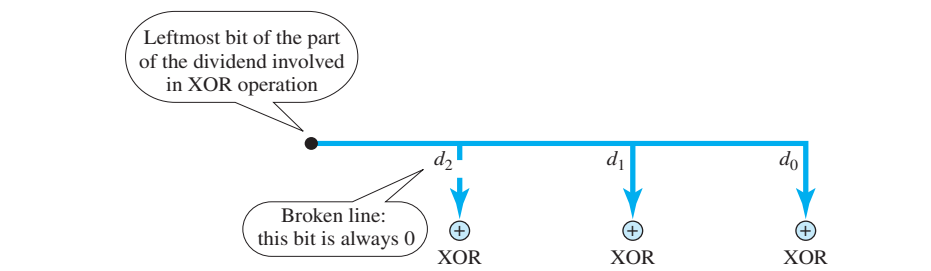
Let us first consider the divisor. We need to note the following points:

1. The divisor is repeatedly XORed with part of the dividend.
2. The divisor has $n - k + 1$ bits which either are predefined or are all 0s. In other words, the bits do not change from one dataword to another. In our previous example, the divisor bits were either 1011 or 0000. The choice was based on the leftmost bit of the part of the augmented data bits that are active in the XOR operation.

3. A close look shows that only $n - k$ bits of the divisor are needed in the XOR operation. The leftmost bit is not needed because the result of the operation is always 0, no matter what the value of this bit. The reason is that the inputs to this XOR operation are either both 0s or both 1s. In our previous example, only 3 bits, not 4, are actually used in the XOR operation.

Using these points, we can make a fixed (hardwired) divisor that can be used for a cyclic code if we know the divisor pattern. Figure 10.11 shows such a design for our previous example. We have also shown the XOR devices used for the operation.

Figure 10.11 Hardwired design of the divisor in CRC



Note that if the leftmost bit of the part of the dividend to be used in this step is 1, the divisor bits ($d_2d_1d_0$) are 011; if the leftmost bit is 0, the divisor bits are 000. The design provides the right choice based on the leftmost bit.

Augmented Dataword

In our paper-and-pencil division process in Figure 10.6, we show the augmented dataword as fixed in position with the divisor bits shifting to the right, 1 bit in each step. The divisor bits are aligned with the appropriate part of the augmented dataword. Now that our divisor is fixed, we need instead to shift the bits of the augmented dataword to the left (opposite direction) to align the divisor bits with the appropriate part. There is no need to store the augmented dataword bits.

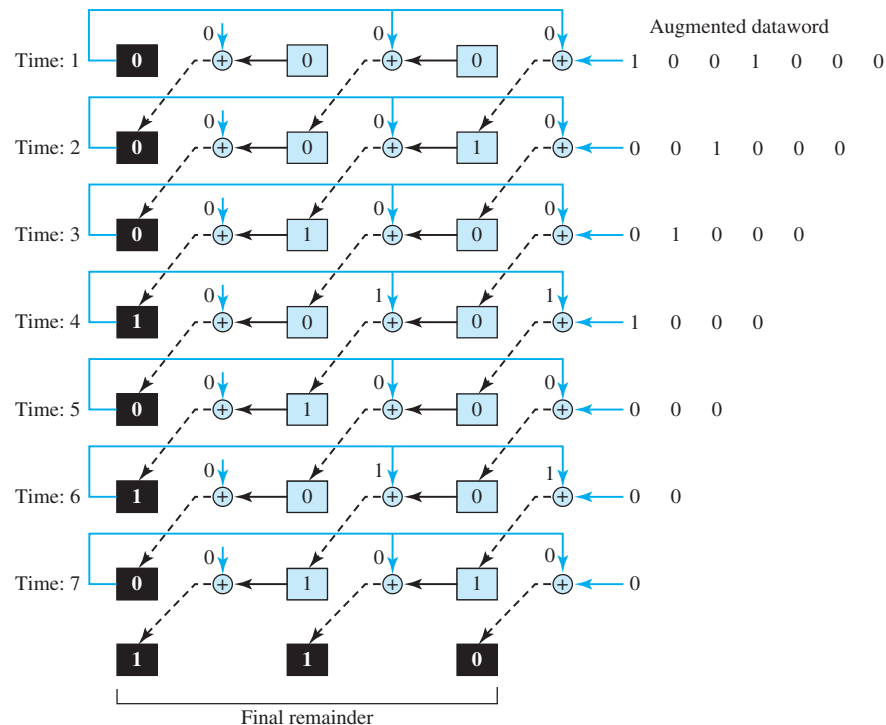
Remainder

In our previous example, the remainder is 3 bits ($n - k$ bits in general) in length. We can use three **registers** (single-bit storage devices) to hold these bits. To find the final remainder of the division, we need to modify our division process. The following is the step-by-step process that can be used to simulate the division process in hardware (or even in software).

1. We assume that the remainder is originally all 0s (000 in our example).
2. At each time click (arrival of 1 bit from an augmented dataword), we repeat the following two actions:
 - a. We use the leftmost bit to make a decision about the divisor (011 or 000).
 - b. The other 2 bits of the remainder and the next bit from the augmented dataword (total of 3 bits) are XORED with the 3-bit divisor to create the next remainder.

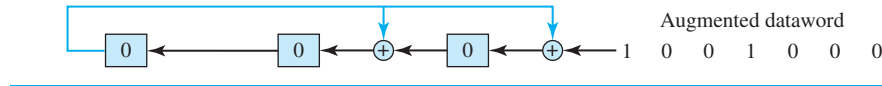
Figure 10.12 shows this simulator, but note that this is not the final design; there will be more improvements.

Figure 10.12 Simulation of division in CRC encoder



At each clock tick, shown as different times, one of the bits from the augmented dataword is used in the XOR process. If we look carefully at the design, we have seven steps here, while in the paper-and-pencil method we had only four steps. The first three steps have been added here to make each step equal and to make the design for each step the same. Steps 1, 2, and 3 push the first 3 bits to the remainder registers; steps 4, 5, 6, and 7 match the paper-and-pencil design. Note that the values in the remainder register in steps 4 to 7 exactly match the values in the paper-and-pencil design. The final remainder is also the same.

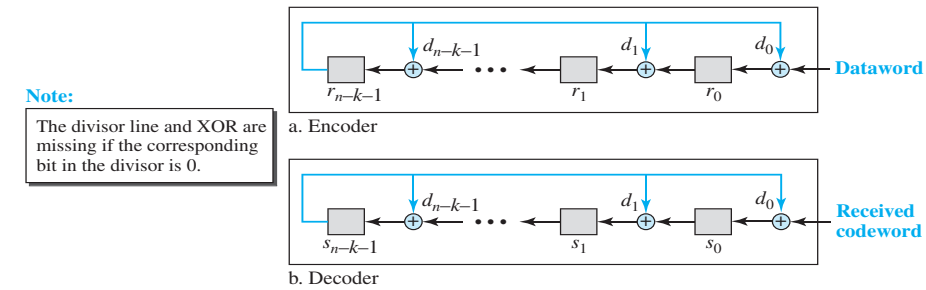
The above design is for demonstration purposes only. It needs simplification to be practical. First, we do not need to keep the intermediate values of the remainder bits; we need only the final bits. We therefore need only 3 registers instead of 24. After the XOR operations, we do not need the bit values of the previous remainder. Also, we do not need 21 XOR devices; two are enough because the output of an XOR operation in which one of the bits is 0 is simply the value of the other bit. This other bit can be used as the output. With these two modifications, the design becomes tremendously simpler and less expensive, as shown in Figure 10.13.

Figure 10.13 The CRC encoder design using shift registers

We need, however, to make the registers shift registers. A 1-bit shift register holds a bit for a duration of one clock time. At a time click, the shift register accepts the bit at its input port, stores the new bit, and displays it on the output port. The content and the output remain the same until the next input arrives. When we connect several 1-bit shift registers together, it looks as if the contents of the register are shifting.

General Design

A general design for the encoder and decoder is shown in Figure 10.14.

Figure 10.14 General design of encoder and decoder of a CRC code

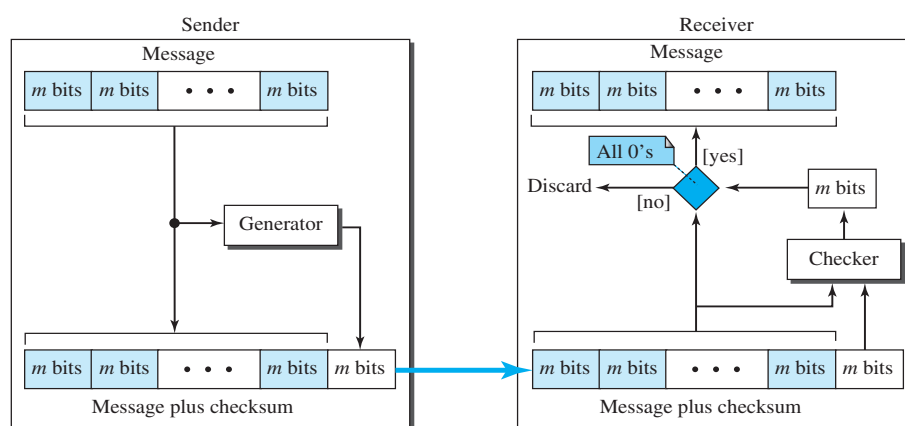
Note that we have $n - k$ 1-bit shift registers in both the encoder and decoder. We have up to $n - k$ XOR devices, but the divisors normally have several 0s in their pattern, which reduces the number of devices. Also note that, instead of augmented datawords, we show the dataword itself as the input because after the bits in the dataword are all fed into the encoder, the extra bits, which all are 0s, do not have any effect on the rightmost XOR. Of course, the process needs to be continued for another $n - k$ steps before the check bits are ready. This fact is one of the criticisms of this design. Better schemes have been designed to eliminate this waiting time (the check bits are ready after k steps), but we leave this as a research topic for the reader. In the decoder, however, the entire codeword must be fed to the decoder before the syndrome is ready.

10.4 CHECKSUM

Checksum is an error-detecting technique that can be applied to a message of any length. In the Internet, the checksum technique is mostly used at the network and transport layer rather than the data-link layer. However, to make our discussion of error-detecting techniques complete, we discuss the checksum in this chapter.

At the source, the message is first divided into m -bit units. The generator then creates an extra m -bit unit called the **checksum**, which is sent with the message. At the destination, the checker creates a new checksum from the combination of the message and sent checksum. If the new checksum is all 0s, the message is accepted; otherwise, the message is discarded (Figure 10.15). Note that in the real implementation, the checksum unit is not necessarily added at the end of the message; it can be inserted in the middle of the message.

Figure 10.15 Checksum



10.4.1 Concept

The idea of the traditional checksum is simple. We show this using a simple example.

Example 10.11

Suppose the message is a list of five 4-bit numbers that we want to send to a destination. In addition to sending these numbers, we send the sum of the numbers. For example, if the set of numbers is (7, 11, 12, 0, 6), we send (7, 11, 12, 0, 6, **36**), where 36 is the sum of the original numbers. The receiver adds the five numbers and compares the result with the sum. If the two are the same, the receiver assumes no error, accepts the five numbers, and discards the sum. Otherwise, there is an error somewhere and the message is not accepted.

One's Complement Addition

The previous example has one major drawback. Each number can be written as a 4-bit word (each is less than 15) except for the sum. One solution is to use **one's complement** arithmetic. In this arithmetic, we can represent unsigned numbers between 0 and $2^m - 1$ using only m bits. If the number has more than m bits, the extra leftmost bits need to be added to the m rightmost bits (wrapping).

Example 10.12

In the previous example, the decimal number 36 in binary is $(100100)_2$. To change it to a 4-bit number we add the extra leftmost bit to the right four bits as shown below.

$$(10)_2 + (0100)_2 = (0110)_2 \rightarrow (6)_{10}$$

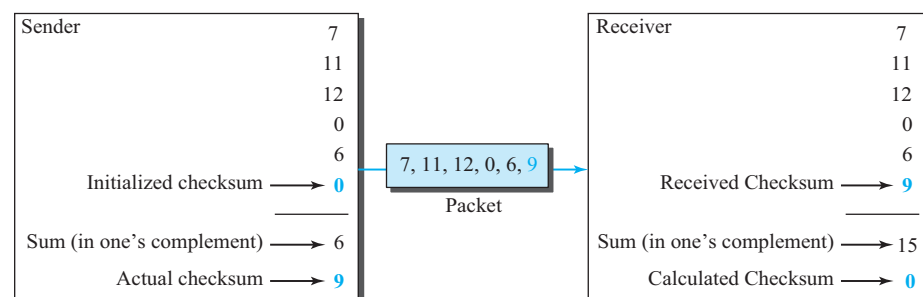
Instead of sending 36 as the sum, we can send 6 as the sum (7, 11, 12, 0, 6, 6). The receiver can add the first five numbers in one's complement arithmetic. If the result is 6, the numbers are accepted; otherwise, they are rejected.

Checksum

We can make the job of the receiver easier if we send the complement of the sum, the checksum. In one's complement arithmetic, the complement of a number is found by completing all bits (changing all 1s to 0s and all 0s to 1s). This is the same as subtracting the number from $2^m - 1$. In one's complement arithmetic, we have two 0s: one positive and one negative, which are complements of each other. The positive zero has all m bits set to 0; the negative zero has all bits set to 1 (it is $2^m - 1$). If we add a number with its complement, we get a negative zero (a number with all bits set to 1). When the receiver adds all five numbers (including the checksum), it gets a negative zero. The receiver can complement the result again to get a positive zero.

Example 10.13

Let us use the idea of the checksum in Example 10.12. The sender adds all five numbers in one's complement to get the sum = 6. The sender then complements the result to get the checksum = 9, which is $15 - 6$. Note that $6 = (0110)_2$ and $9 = (1001)_2$; they are complements of each other. The sender sends the five data numbers and the checksum (7, 11, 12, 0, 6, 9). If there is no corruption in transmission, the receiver receives (7, 11, 12, 0, 6, 9) and adds them in one's complement to get 15. The sender complements 15 to get 0. This shows that data have not been corrupted. Figure 10.16 shows the process.

Figure 10.16 Example 10.13

Internet Checksum

Traditionally, the Internet has used a 16-bit checksum. The sender and the receiver follow the steps depicted in Table 10.5. The sender or the receiver uses five steps.

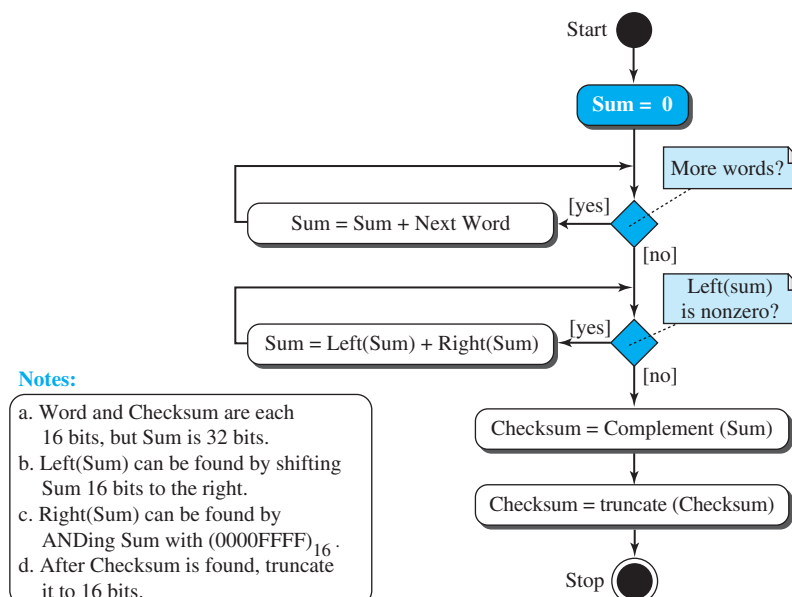
Table 10.5 Procedure to calculate the traditional checksum

Sender	Receiver
1. The message is divided into 16-bit words.	1. The message and the checksum are received.
2. The value of the checksum word is initially set to zero.	2. The message is divided into 16-bit words.
3. All words including the checksum are added using one's complement addition.	3. All words are added using one's complement addition.
4. The sum is complemented and becomes the checksum.	4. The sum is complemented and becomes the new checksum.
5. The checksum is sent with the data.	5. If the value of the checksum is 0, the message is accepted; otherwise, it is rejected.

Algorithm

We can use the flow diagram of Figure 10.17 to show the algorithm for calculation of the checksum. A program in any language can easily be written based on the algorithm. Note that the first loop just calculates the sum of the data units in two's complement; the second loop wraps the extra bits created from the two's complement calculation to simulate the calculations in one's complement. This is needed because almost all computers today do calculation in two's complement.

Figure 10.17 Algorithm to calculate a traditional checksum



Performance

The traditional checksum uses a small number of bits (16) to detect errors in a message of any size (sometimes thousands of bits). However, it is not as strong as the CRC in error-checking capability. For example, if the value of one word is incremented and the value of another word is decremented by the same amount, the two errors cannot be detected because the sum and checksum remain the same. Also, if the values of several words are incremented but the sum and the checksum do not change, the errors are not detected. Fletcher and Adler have proposed some weighted checksums that eliminate the first problem. However, the tendency in the Internet, particularly in designing new protocols, is to replace the checksum with a CRC.

10.4.2 Other Approaches to the Checksum

As mentioned before, there is one major problem with the traditional checksum calculation. If two 16-bit items are transposed in transmission, the checksum cannot catch this error. The reason is that the traditional checksum is not weighted: it treats each data item equally. In other words, the order of data items is immaterial to the calculation. Several approaches have been used to prevent this problem. We mention two of them here: Fletcher and Adler.

Fletcher Checksum

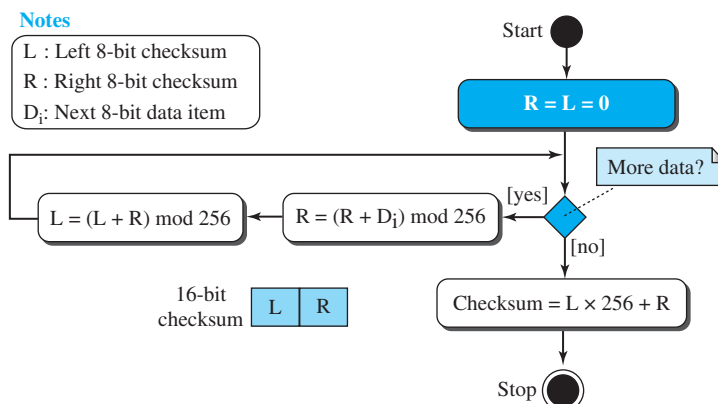
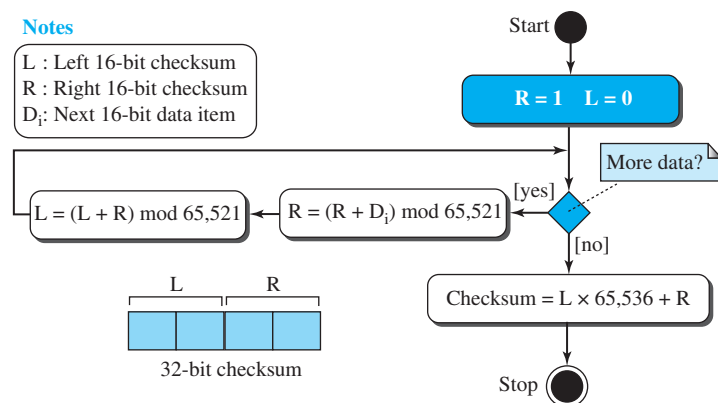
The Fletcher checksum was devised to weight each data item according to its position. Fletcher has proposed two algorithms: 8-bit and 16-bit. The first, 8-bit Fletcher, calculates on 8-bit data items and creates a 16-bit checksum. The second, 16-bit Fletcher, calculates on 16-bit data items and creates a 32-bit checksum.

The 8-bit Fletcher is calculated over data octets (bytes) and creates a 16-bit checksum. The calculation is done modulo 256 (2^8), which means the intermediate results are divided by 256 and the remainder is kept. The algorithm uses two accumulators, L and R. The first simply adds data items together; the second adds a weight to the calculation. There are many variations of the 8-bit Fletcher algorithm; we show a simple one in Figure 10.18.

The 16-bit Fletcher checksum is similar to the 8-bit Fletcher checksum, but it is calculated over 16-bit data items and creates a 32-bit checksum. The calculation is done modulo 65,536.

Adler Checksum

The Adler checksum is a 32-bit checksum. Figure 10.19 shows a simple algorithm in flowchart form. It is similar to the 16-bit Fletcher with three differences. First, calculation is done on single bytes instead of 2 bytes at a time. Second, the modulus is a prime number (65,521) instead of 65,536. Third, L is initialized to 1 instead of 0. It has been proved that a prime modulo has a better detecting capability in some combinations of data.

Figure 10.18 Algorithm to calculate an 8-bit Fletcher checksum**Figure 10.19** Algorithm to calculate an Adler checksum

To see the behavior of the different checksum algorithms, check some of the applets for this chapter at the book website.

10.5 FORWARD ERROR CORRECTION

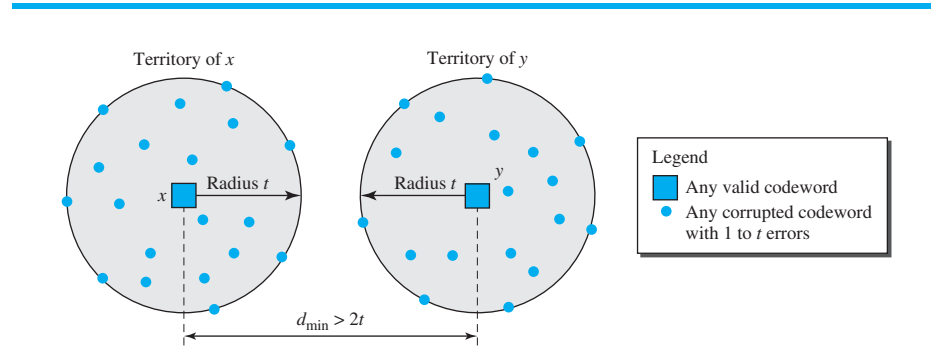
We discussed error detection and retransmission in the previous sections. However, retransmission of corrupted and lost packets is not useful for real-time multimedia transmission because it creates an unacceptable delay in reproducing: we need to wait until the lost or corrupted packet is resent. We need to correct the error or reproduce the

packet immediately. Several schemes have been designed and used in this case that are collectively referred to as **forward error correction (FEC)** techniques. We briefly discuss some of the common techniques here.

10.5.1 Using Hamming Distance

We earlier discussed the Hamming distance for error detection. We said that to detect s errors, the minimum Hamming distance should be $d_{\min} = s + 1$. For error detection, we definitely need more distance. It can be shown that to detect t errors, we need to have $d_{\min} = 2t + 1$. In other words, if we want to correct 10 bits in a packet, we need to make the minimum hamming distance 21 bits, which means a lot of redundant bits need to be sent with the data. To give an example, consider the famous BCH code. In this code, if data is 99 bits, we need to send 255 bits (extra 156 bits) to correct just 23 possible bit errors. Most of the time we cannot afford such a redundancy. We give some examples of how to calculate the required bits in the practice set. Figure 10.20 shows the geometrical representation of this concept.

Figure 10.20 Hamming distance for error correction



10.5.2 Using XOR

Another recommendation is to use the property of the exclusive OR operation as shown below.

$$R = P_1 \oplus P_2 \oplus \dots \oplus P_i \oplus \dots \oplus P_N \rightarrow P_i = P_1 \oplus P_2 \oplus \dots \oplus R \oplus \dots \oplus P_N$$

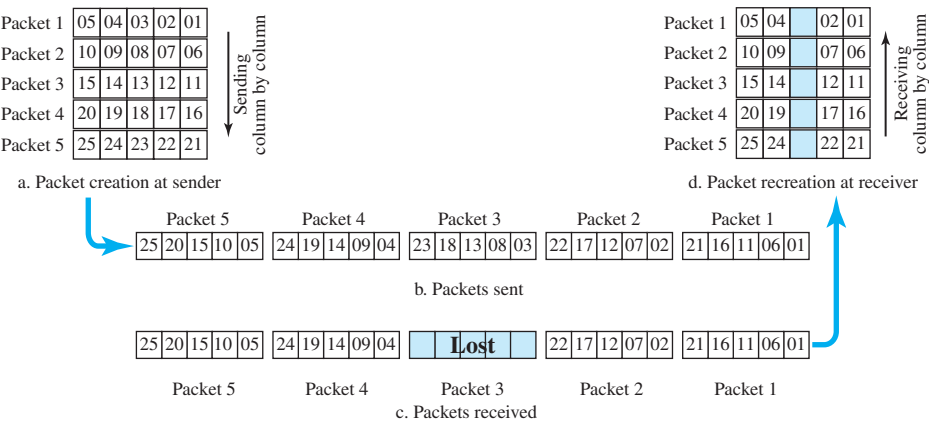
In other words, if we apply the exclusive OR operation on N data items (P_1 to P_N), we can recreate any of the data items by exclusive-ORing all of the items, replacing the one to be created by the result of the previous operation (R). This means that we can divide a packet into N chunks, create the exclusive OR of all the chunks and send $N + 1$ chunks. If any chunk is lost or corrupted, it can be created at the receiver site. Now the question is what should the value of N be. If $N = 4$, it means that we need to send 25 percent extra data and be able to correct the data if only one out of four chunks is lost.

10.5.3 Chunk Interleaving

Another way to achieve FEC in multimedia is to allow some small chunks to be missing at the receiver. We cannot afford to let all the chunks belonging to the same

packet be missing; however, we can afford to let one chunk be missing in each packet. Figure 10.21 shows that we can divide each packet into 5 chunks (normally the number is much larger). We can then create data chunk by chunk (horizontally), but combine the chunks into packets vertically. In this case, each packet sent carries a chunk from several original packets. If the packet is lost, we miss only one chunk in each packet, which is normally acceptable in multimedia communication.

Figure 10.21 Interleaving

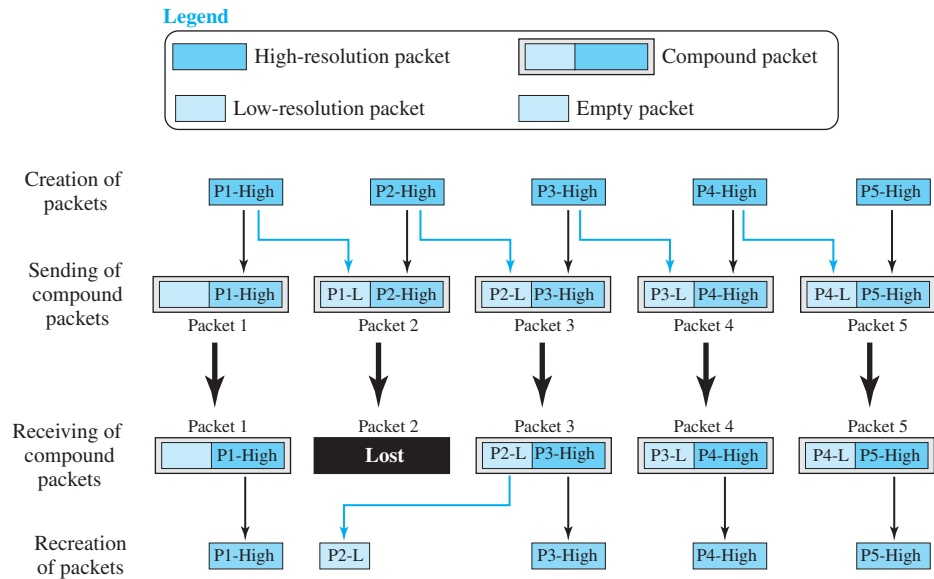


10.5.4 Combining Hamming Distance and Interleaving

Hamming distance and interleaving can be combined. We can first create n -bit packets that can correct t -bit errors. Then we interleave m rows and send the bits column by column. In this way, we can automatically correct burst errors up to $m \times t$ -bit errors.

10.5.5 Compounding High- and Low-Resolution Packets

Still another solution is to create a duplicate of each packet with a low-resolution redundancy and combine the redundant version with the next packet. For example, we can create four low-resolution packets out of five high-resolution packets and send them as shown in Figure 10.22. If a packet is lost, we can use the low-resolution version from the next packet. Note that the low-resolution section in the first packet is empty. In this method, if the last packet is lost, it cannot be recovered, but we use the low-resolution version of a packet if the lost packet is not the last one. The audio and video reproduction does not have the same quality, but the lack of quality is not recognized most of the time.

Figure 10.22 Compounding high- and low-resolution packets

10.6 END-CHAPTER MATERIALS

10.6.1 Recommended Reading

For more details about subjects discussed in this chapter, we recommend the following books and RFCs. The items in brackets [...] refer to the reference list at the end of the text.

Books

Several excellent books discuss link-layer issues. Among them we recommend [Ham 80], [Zar 02], [Ror 96], [Tan 03], [GW 04], [For 03], [KMK 04], [Sta 04], [Kes 02], [PD 03], [Kei 02], [Spu 00], [KCK 98], [Sau 98], [Izz 00], [Per 00], and [WV 00].

RFCs

A discussion of the use of the checksum in the Internet can be found in RFC 1141.

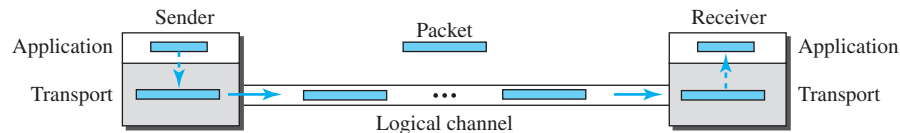
23.2 TRANSPORT-LAYER PROTOCOLS

We can create a transport-layer protocol by combining a set of services described in the previous sections. To better understand the behavior of these protocols, we start with the simplest one and gradually add more complexity. The TCP/IP protocol uses a transport-layer protocol that is either a modification or a combination of some of these protocols. We discuss these general protocols in this section to pave the way for understanding more complex ones in the rest of the chapter. To make our discussion simpler, we first discuss all of these protocols as a unidirectional protocol (i.e., simplex) in which the data packets move in one direction. At the end of the chapter, we briefly discuss how they can be changed to bidirectional protocols where data can be moved in two directions (i.e., full duplex).

23.2.1 Simple Protocol

Our first protocol is a simple connectionless protocol with neither flow nor error control. We assume that the receiver can immediately handle any packet it receives. In other words, the receiver can never be overwhelmed with incoming packets. Figure 23.17 shows the layout for this protocol.

Figure 23.17 Simple protocol



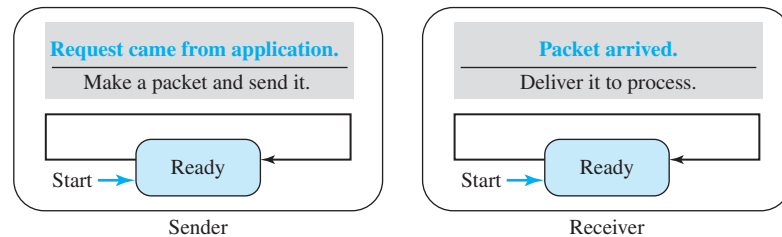
The transport layer at the sender gets a message from its application layer, makes a packet out of it, and sends the packet. The transport layer at the receiver receives a packet from its network layer, extracts the message from the packet, and delivers the message to its application layer. The transport layers of the sender and receiver provide transmission services for their application layers.

FSMs

The sender site should not send a packet until its application layer has a message to send. The receiver site cannot deliver a message to its application layer until a packet arrives. We can show these requirements using two FSMs. Each FSM has only one state, the *ready state*. The sending machine remains in the ready state until a request comes from the process in the application layer. When this event occurs, the sending machine encapsulates the message in a packet and sends it to the receiving machine. The receiving machine remains in the ready state until a packet arrives from the sending machine. When this event occurs, the receiving machine decapsulates the message out of the packet and delivers it to the process at the application layer. Figure 23.18

shows the FSMs for the simple protocol. We see later that the UDP protocol is a slight modification of this protocol.

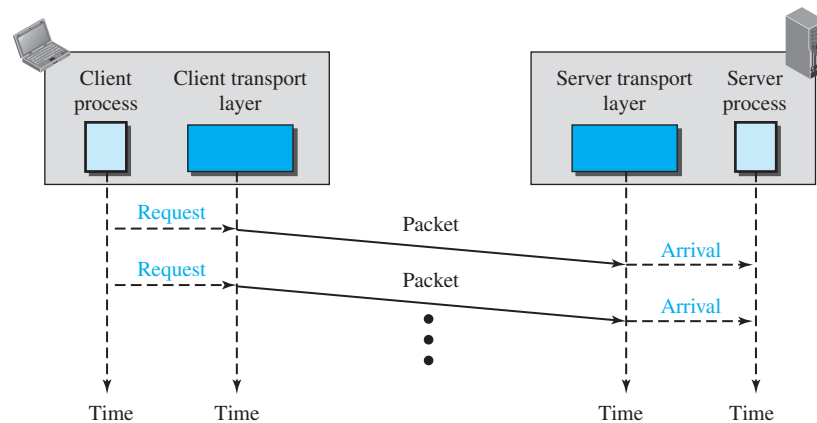
Figure 23.18 *FSMs for the simple protocol*



Example 23.3

Figure 23.19 shows an example of communication using this protocol. It is very simple. The sender sends packets one after another without even thinking about the receiver.

Figure 23.19 *Flow diagram for Example 23.3*

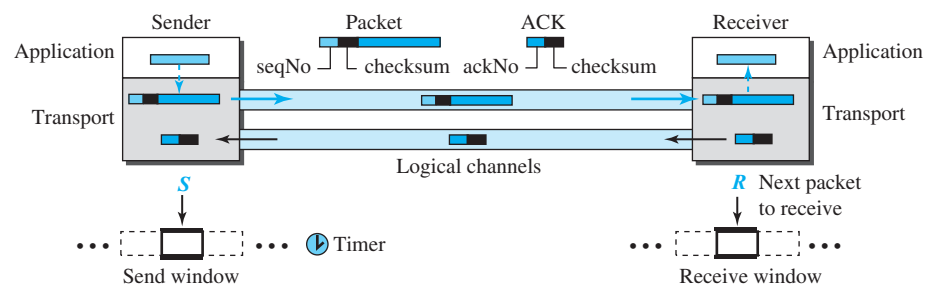


23.2.2 Stop-and-Wait Protocol

Our second protocol is a connection-oriented protocol called the **Stop-and-Wait protocol**, which uses both flow and error control. Both the sender and the receiver use a sliding window of size 1. The sender sends one packet at a time and waits for an acknowledgment before sending the next one. To detect corrupted packets, we need to add a checksum to each data packet. When a packet arrives at the receiver site, it is checked. If its checksum is incorrect, the packet is corrupted and silently discarded.

The silence of the receiver is a signal for the sender that a packet was either corrupted or lost. Every time the sender sends a packet, it starts a timer. If an acknowledgment arrives before the timer expires, the timer is stopped and the sender sends the next packet (if it has one to send). If the timer expires, the sender resends the previous packet, assuming that the packet was either lost or corrupted. This means that the sender needs to keep a copy of the packet until its acknowledgment arrives. Figure 23.20 shows the outline for the Stop-and-Wait protocol. Note that only one packet and one acknowledgment can be in the channels at any time.

Figure 23.20 Stop-and-Wait protocol



The Stop-and-Wait protocol is a connection-oriented protocol that provides flow and error control.

Sequence Numbers

To prevent duplicate packets, the protocol uses sequence numbers and acknowledgment numbers. A field is added to the packet header to hold the sequence number of that packet. One important consideration is the range of the sequence numbers. Since we want to minimize the packet size, we look for the smallest range that provides unambiguous communication. Let us discuss the range of sequence numbers we need. Assume we have used x as a sequence number; we only need to use $x + 1$ after that. There is no need for $x + 2$. To show this, assume that the sender has sent the packet with sequence number x . Three things can happen.

1. The packet arrives safe and sound at the receiver site; the receiver sends an acknowledgment. The acknowledgment arrives at the sender site, causing the sender to send the next packet numbered $x + 1$.
2. The packet is corrupted or never arrives at the receiver site; the sender resends the packet (numbered x) after the time-out. The receiver returns an acknowledgment.
3. The packet arrives safe and sound at the receiver site; the receiver sends an acknowledgment, but the acknowledgment is corrupted or lost. The sender resends the packet (numbered x) after the time-out. Note that the packet here is a duplicate. The receiver can recognize this fact because it expects packet $x + 1$ but packet x was received.

We can see that there is a need for sequence numbers x and $x + 1$ because the receiver needs to distinguish between case 1 and case 3. But there is no need for a packet to be numbered $x + 2$. In case 1, the packet can be numbered x again because packets x and $x + 1$ are acknowledged and there is no ambiguity at either site. In cases 2 and 3, the new packet is $x + 1$, not $x + 2$. If only x and $x + 1$ are needed, we can let $x = 0$ and $x + 1 = 1$. This means that the sequence is 0, 1, 0, 1, 0, and so on. This is referred to as modulo 2 arithmetic.

Acknowledgment Numbers

Since the sequence numbers must be suitable for both data packets and acknowledgments, we use this convention: The acknowledgment numbers always announce the sequence number of the *next packet expected* by the receiver. For example, if packet 0 has arrived safe and sound, the receiver sends an ACK with acknowledgment 1 (meaning packet 1 is expected next). If packet 1 has arrived safe and sound, the receiver sends an ACK with acknowledgment 0 (meaning packet 0 is expected).

In the Stop-and-Wait protocol, the acknowledgment number always announces, in modulo-2 arithmetic, the sequence number of the next packet expected.

The sender has a control variable, which we call S (sender), that points to the only slot in the send window. The receiver has a control variable, which we call R (receiver), that points to the only slot in the receive window.

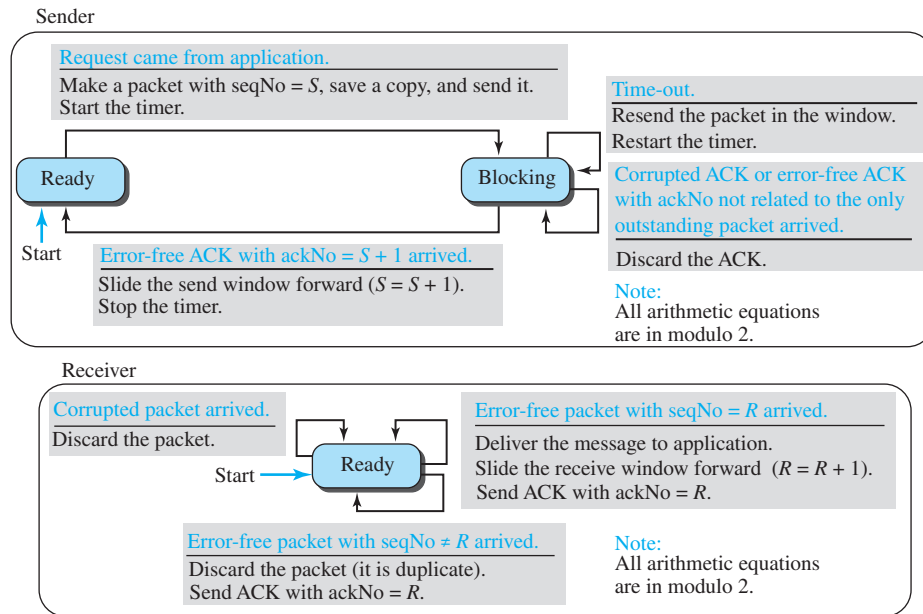
FSMs

Figure 23.21 shows the FSMs for the Stop-and-Wait protocol. Since the protocol is a connection-oriented protocol, both ends should be in the *established* state before exchanging data packets. The states are actually nested in the *established* state.

Sender

The sender is initially in the ready state, but it can move between the ready and blocking state. The variable S is initialized to 0.

- **Ready state.** When the sender is in this state, it is only waiting for one event to occur. If a request comes from the application layer, the sender creates a packet with the sequence number set to S . A copy of the packet is stored, and the packet is sent. The sender then starts the only timer. The sender then moves to the blocking state.
- **Blocking state.** When the sender is in this state, three events can occur:
 - a. If an error-free ACK arrives with the `ackNo` related to the next packet to be sent, which means $\text{ackNo} = (S + 1) \text{ modulo } 2$, then the timer is stopped. The window slides, $S = (S + 1) \text{ modulo } 2$. Finally, the sender moves to the ready state.
 - b. If a corrupted ACK or an error-free ACK with the $\text{ackNo} \neq (S + 1) \text{ modulo } 2$ arrives, the ACK is discarded.
 - c. If a time-out occurs, the sender resends the only outstanding packet and restarts the timer.

Figure 23.21 FSMs for the Stop-and-Wait protocol**Receiver**

The receiver is always in the *ready* state. Three events may occur:

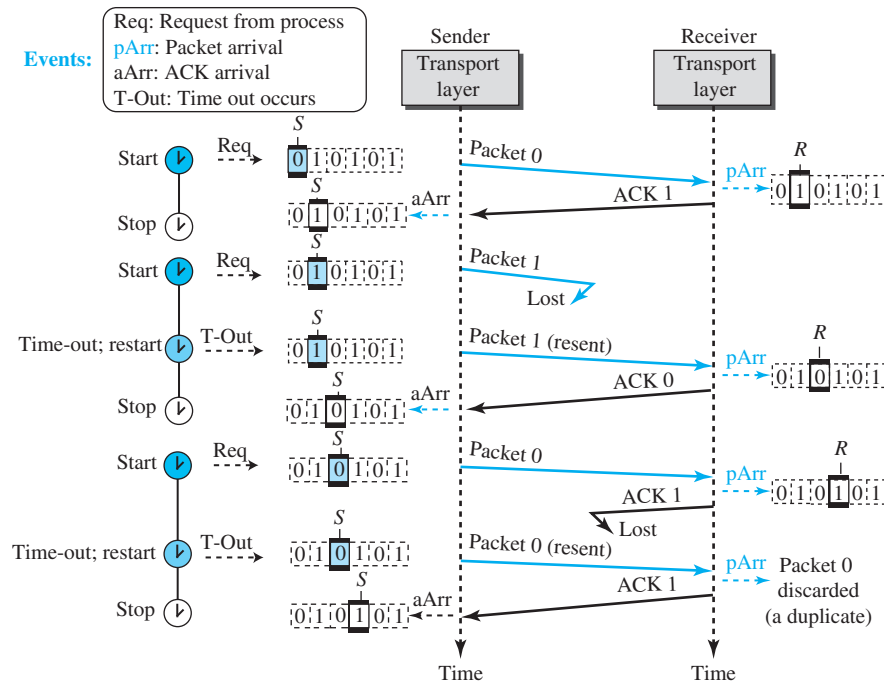
- If an error-free packet with seqNo = R arrives, the message in the packet is delivered to the application layer. The window then slides, $R = (R + 1)$ modulo 2. Finally an ACK with ackNo = R is sent.
- If an error-free packet with seqNo $\neq R$ arrives, the packet is discarded, but an ACK with ackNo = R is sent.
- If a corrupted packet arrives, the packet is discarded.

Example 23.4

Figure 23.22 shows an example of the Stop-and-Wait protocol. Packet 0 is sent and acknowledged. Packet 1 is lost and resent after the time-out. The resent packet 1 is acknowledged and the timer stops. Packet 0 is sent and acknowledged, but the acknowledgment is lost. The sender has no idea if the packet or the acknowledgment is lost, so after the time-out, it resends packet 0, which is acknowledged.

Efficiency

The Stop-and-Wait protocol is very inefficient if our channel is *thick* and *long*. By *thick*, we mean that our channel has a large bandwidth (high data rate); by *long*, we mean the round-trip delay is long. The product of these two is called the **bandwidth-delay product**. We can think of the channel as a pipe. The bandwidth-delay product then is the volume of the pipe in bits. The pipe is always there. It is not efficient if it

Figure 23.22 Flow diagram for Example 23.4

is not used. The bandwidth-delay product is a measure of the number of bits a sender can transmit through the system while waiting for an acknowledgment from the receiver.

Example 23.5

Assume that, in a Stop-and-Wait system, the bandwidth of the line is 1 Mbps, and 1 bit takes 20 milliseconds to make a round trip. What is the bandwidth-delay product? If the system data packets are 1,000 bits in length, what is the utilization percentage of the link?

Solution

The bandwidth-delay product is $(1 \times 10^6) \times (20 \times 10^{-3}) = 20,000$ bits. The system can send 20,000 bits during the time it takes for the data to go from the sender to the receiver and the acknowledgment to come back. However, the system sends only 1,000 bits. We can say that the link utilization is only $1,000/20,000$, or 5 percent. For this reason, in a link with a high bandwidth or long delay, the use of Stop-and-Wait wastes the capacity of the link.

Example 23.6

What is the utilization percentage of the link in Example 23.5 if we have a protocol that can send up to 15 packets before stopping and worrying about the acknowledgments?

Solution

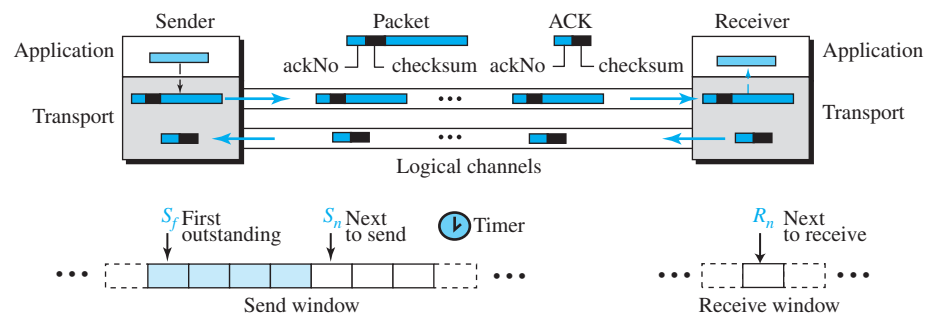
The bandwidth-delay product is still 20,000 bits. The system can send up to 15 packets or 15,000 bits during a round trip. This means the utilization is 15,000/20,000, or 75 percent. Of course, if there are damaged packets, the utilization percentage is much less because packets have to be resent.

Pipelining

In networking and in other areas, a task is often begun before the previous task has ended. This is known as **pipelining**. There is no pipelining in the Stop-and-Wait protocol because a sender must wait for a packet to reach the destination and be acknowledged before the next packet can be sent. However, pipelining does apply to our next two protocols because several packets can be sent before a sender receives feedback about the previous packets. Pipelining improves the efficiency of the transmission if the number of bits in transition is large with respect to the bandwidth-delay product.

23.2.3 Go-Back-N Protocol (GBN)

To improve the efficiency of transmission (to fill the pipe), multiple packets must be in transition while the sender is waiting for acknowledgment. In other words, we need to let more than one packet be outstanding to keep the channel busy while the sender is waiting for acknowledgment. In this section, we discuss one protocol that can achieve this goal; in the next section, we discuss a second. The first is called **Go-Back-N (GBN)** (the rationale for the name will become clear later). The key to Go-back-N is that we can send several packets before receiving acknowledgments, but the receiver can only buffer one packet. We keep a copy of the sent packets until the acknowledgments arrive. Figure 23.23 shows the outline of the protocol. Note that several data packets and acknowledgments can be in the channel at the same time.

Figure 23.23 Go-Back-N protocol**Sequence Numbers**

As we mentioned before, the sequence numbers are modulo 2^m , where m is the size of the sequence number field in bits.

Acknowledgment Numbers

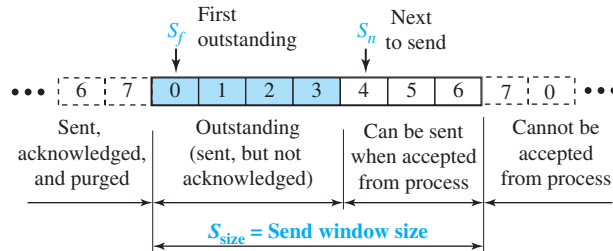
An acknowledgment number in this protocol is cumulative and defines the sequence number of the next packet expected. For example, if the acknowledgment number (ackNo) is 7, it means all packets with sequence number up to 6 have arrived, safe and sound, and the receiver is expecting the packet with sequence number 7.

In the Go-Back- N protocol, the acknowledgment number is cumulative and defines the sequence number of the next packet expected to arrive.

Send Window

The send window is an imaginary box covering the sequence numbers of the data packets that can be in transit or can be sent. In each window position, some of these sequence numbers define the packets that have been sent; others define those that can be sent. The maximum size of the window is $2^m - 1$, for reasons that we discuss later. In this chapter, we let the size be fixed and set to the maximum value, but we will see later that some protocols may have a variable window size. Figure 23.24 shows a sliding window of size 7 ($m = 3$) for the Go-Back- N protocol.

Figure 23.24 Send window for Go-Back- N



The send window at any time divides the possible sequence numbers into four regions. The first region, left of the window, defines the sequence numbers belonging to packets that are already acknowledged. The sender does not worry about these packets and keeps no copies of them. The second region, colored, defines the range of sequence numbers belonging to the packets that have been sent, but have an unknown status. The sender needs to wait to find out if these packets have been received or were lost. We call these *outstanding* packets. The third range, white in the figure, defines the range of sequence numbers for packets that can be sent; however, the corresponding data have not yet been received from the application layer. Finally, the fourth region, right of the window, defines sequence numbers that cannot be used until the window slides.

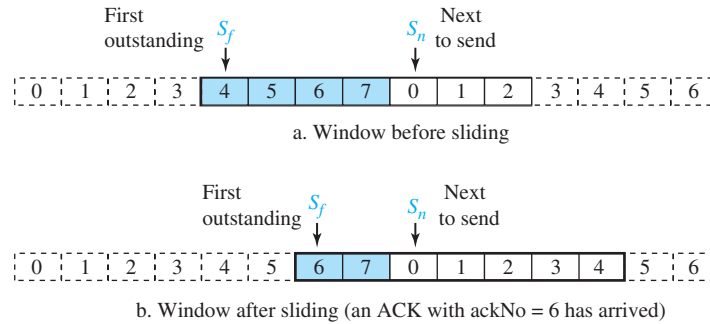
The window itself is an abstraction; three variables define its size and location at any time. We call these variables S_f (send window, the first outstanding packet), S_n (send window, the next packet to be sent), and S_{size} (send window, size). The variable S_f defines the sequence number of the first (oldest) outstanding packet. The variable

S_n holds the sequence number that will be assigned to the next packet to be sent. Finally, the variable S_{size} defines the size of the window, which is fixed in our protocol.

The send window is an abstract concept defining an imaginary box of maximum size = $2^m - 1$ with three variables: S_f , S_n , and S_{size} .

Figure 23.25 shows how a send window can slide one or more slots to the right when an acknowledgment arrives from the other end. In the figure, an acknowledgment with $\text{ackNo} = 6$ has arrived. This means that the receiver is waiting for packets with sequence number 6.

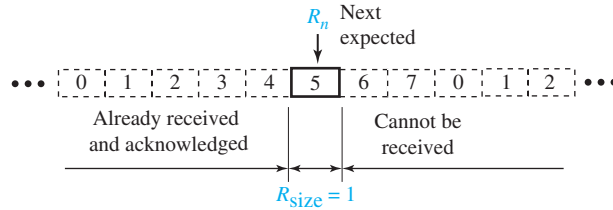
Figure 23.25 Sliding the send window



The send window can slide one or more slots when an error-free ACK with ackNo greater than or equal to S_f and less than S_n (in modular arithmetic) arrives.

Receive Window

The receive window makes sure that the correct data packets are received and that the correct acknowledgments are sent. In Go-Back- N , the size of the receive window is always 1. The receiver is always looking for the arrival of a specific packet. Any packet arriving out of order is discarded and needs to be resent. Figure 23.26 shows the receive window. Note that we need only one variable, R_n (receive window, next packet expected), to define this abstraction. The sequence numbers to the left of the window belong to the packets already received and acknowledged; the sequence numbers to the right of this window define the packets that cannot be received. Any received packet with a sequence number in these two regions is discarded. Only a packet with a sequence number matching the value of R_n is accepted and acknowledged. The receive window also slides, but only one slot at a time. When a correct packet is received, the window slides, $R_n = (R_n + 1) \text{ modulo } 2^m$.

Figure 23.26 Receive window for Go-Back-N

The receive window is an abstract concept defining an imaginary box of size 1 with a single variable R_n . The window slides when a correct packet has arrived; sliding occurs one slot at a time.

Timers

Although there can be a timer for each packet that is sent, in our protocol we use only one. The reason is that the timer for the first outstanding packet always expires first. We resend all outstanding packets when this timer expires.

Resending packets

When the timer expires, the sender resends all outstanding packets. For example, suppose the sender has already sent packet 6 ($S_n = 7$), but the only timer expires. If $S_f = 3$, this means that packets 3, 4, 5, and 6 have not been acknowledged; the sender goes back and resends packets 3, 4, 5, and 6. That is why the protocol is called *Go-Back-N*. On a time-out, the machine goes back N locations and resends all packets.

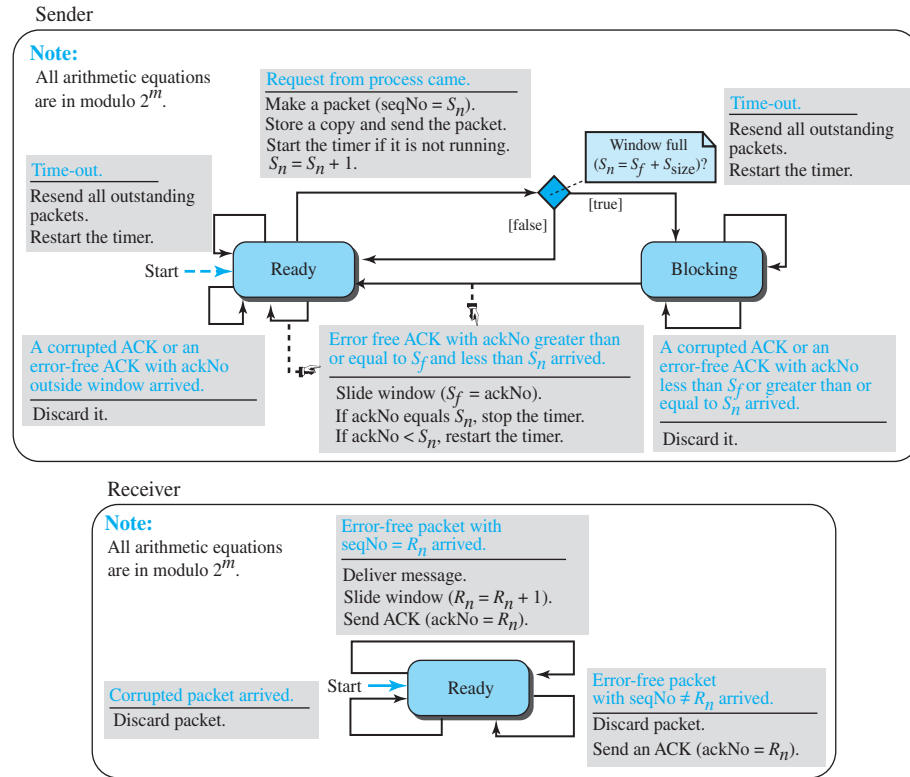
FSMs

Figure 23.27 shows the FSMs for the GBN protocol.

Sender

The sender starts in the ready state, but thereafter it can be in one of the two states: *ready* or *blocking*. The two variables are normally initialized to 0 ($S_f = S_n = 0$).

- **Ready state.** Four events may occur when the sender is in ready state.
 - a. If a request comes from the application layer, the sender creates a packet with the sequence number set to S_n . A copy of the packet is stored, and the packet is sent. The sender also starts the only timer if it is not running. The value of S_n is now incremented, $(S_n = S_n + 1) \text{ modulo } 2^m$. If the window is full, $S_n = (S_f + S_{\text{size}}) \text{ modulo } 2^m$, the sender goes to the blocking state.
 - b. If an error-free ACK arrives with ackNo related to one of the outstanding packets, the sender slides the window (set $S_f = \text{ackNo}$), and if all outstanding packets are acknowledged ($\text{ackNo} = S_n$), then the timer is stopped. If all outstanding packets are not acknowledged, the timer is restarted.

Figure 23.27 FSMs for the Go-Back-N protocol

- c. If a corrupted ACK or an error-free ACK with ackNo not related to the outstanding packet arrives, it is discarded.
 - d. If a time-out occurs, the sender resends all outstanding packets and restarts the timer.
- **Blocking state.** Three events may occur in this case:
- a. If an error-free ACK arrives with ackNo related to one of the outstanding packets, the sender slides the window (set $S_f = \text{ackNo}$) and if all outstanding packets are acknowledged (ackNo = S_n), then the timer is stopped. If all outstanding packets are not acknowledged, the timer is restarted. The sender then moves to the ready state.
 - b. If a corrupted ACK or an error-free ACK with the ackNo not related to the outstanding packets arrives, the ACK is discarded.
 - c. If a time-out occurs, the sender sends all outstanding packets and restarts the timer.

Receiver

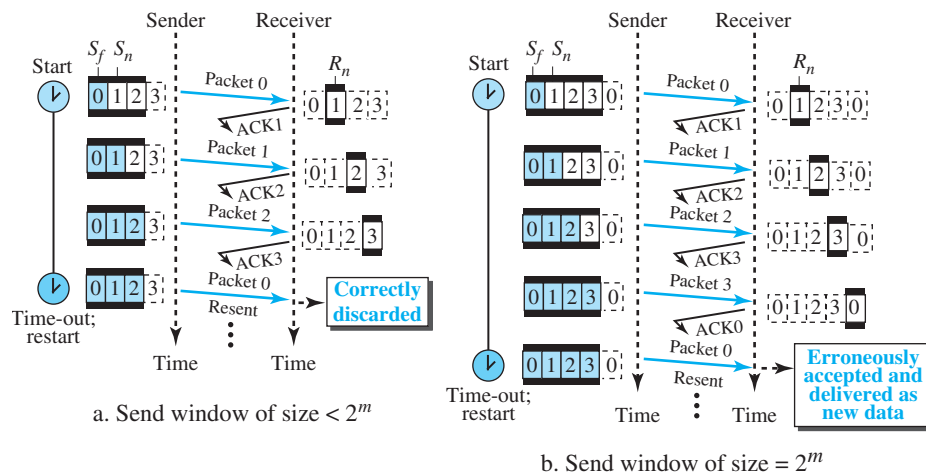
The receiver is always in the *ready* state. The only variable, R_n , is initialized to 0. Three events may occur:

- If an error-free packet with seqNo = R_n arrives, the message in the packet is delivered to the application layer. The window then slides, $R_n = (R_n + 1)$ modulo 2^m . Finally an ACK is sent with ackNo = R_n .
- If an error-free packet with seqNo outside the window arrives, the packet is discarded, but an ACK with ackNo = R_n is sent.
- If a corrupted packet arrives, it is discarded.

Send Window Size

We can now show why the size of the send window must be less than 2^m . As an example, we choose $m = 2$, which means the size of the window can be $2^m - 1$, or 3. Figure 23.28 compares a window size of 3 against a window size of 4. If the size of the window is 3 (less than 2^m) and all three acknowledgments are lost, the only timer expires and all three packets are resent. The receiver is now expecting packet 3, not packet 0, so the duplicate packet is correctly discarded. On the other hand, if the size of the window is 4 (equal to 2^2) and all acknowledgments are lost, the sender will send a duplicate of packet 0. However, this time the window of the receiver expects to receive packet 0 (in the next cycle), so it accepts packet 0, not as a duplicate, but as the first packet in the next cycle. This is an error. This shows that the size of the send window must be less than 2^m .

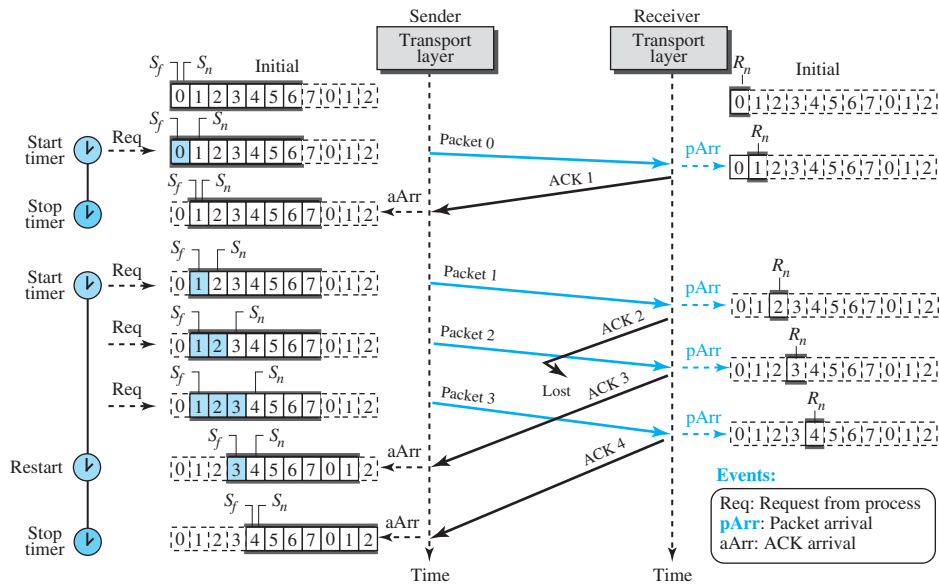
Figure 23.28 Send window size for Go-Back-N



In the Go-Back-N protocol, the size of the send window must be less than 2^m ; the size of the receive window is always 1.

Example 23.7

Figure 23.29 shows an example of Go-Back- N . This is an example of a case where the forward channel is reliable, but the reverse is not. No data packets are lost, but some ACKs are delayed and one is lost. The example also shows how cumulative acknowledgments can help if acknowledgments are delayed or lost.

Figure 23.29 Flow diagram for Example 23.7

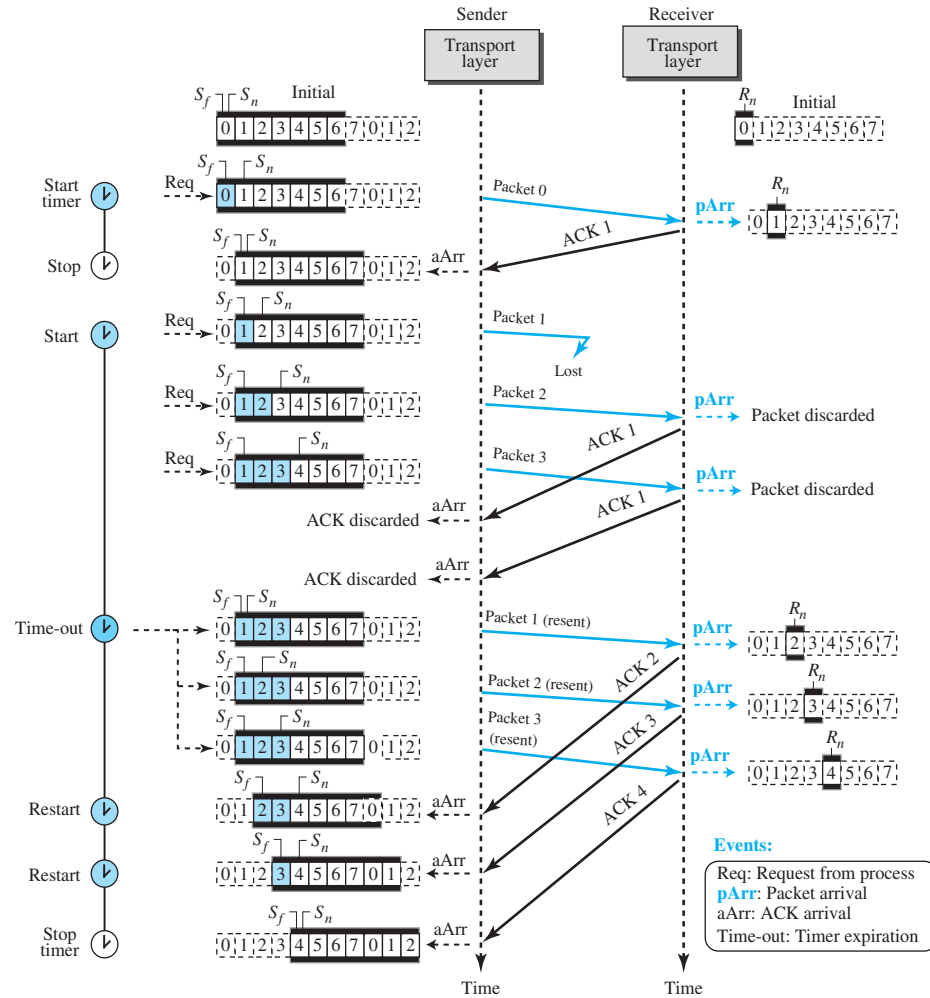
After initialization, there are some sender events. Request events are triggered by message chunks from the application layer; arrival events are triggered by ACKs received from the network layer. There is no time-out event here because all outstanding packets are acknowledged before the timer expires. Note that although ACK 2 is lost, ACK 3 is cumulative and serves as both ACK 2 and ACK 3. There are four events at the receiver site.

Example 23.8

Figure 23.30 shows what happens when a packet is lost. Packets 0, 1, 2, and 3 are sent. However, packet 1 is lost. The receiver receives packets 2 and 3, but they are discarded because they are received out of order (packet 1 is expected). When the receiver receives packets 2 and 3, it sends ACK1 to show that it expects to receive packet 1. However, these ACKs are not useful for the sender because the ackNo is equal to S_f , not greater than S_f . So the sender discards them. When the time-out occurs, the sender resends packets 1, 2, and 3, which are acknowledged.

Go-Back- N versus Stop-and-Wait

The reader may find that there is a similarity between the Go-Back- N protocol and the Stop-and-Wait protocol. The Stop-and-Wait protocol is actually a Go-Back- N protocol in

Figure 23.30 Flow diagram for Example 23.8

which there are only two sequence numbers and the send window size is 1. In other words, $m = 1$ and $2^m - 1 = 1$. In Go-Back- N , we said that the arithmetic is modulo 2^m ; in Stop-and-Wait it is modulo 2, which is the same as 2^m when $m = 1$.

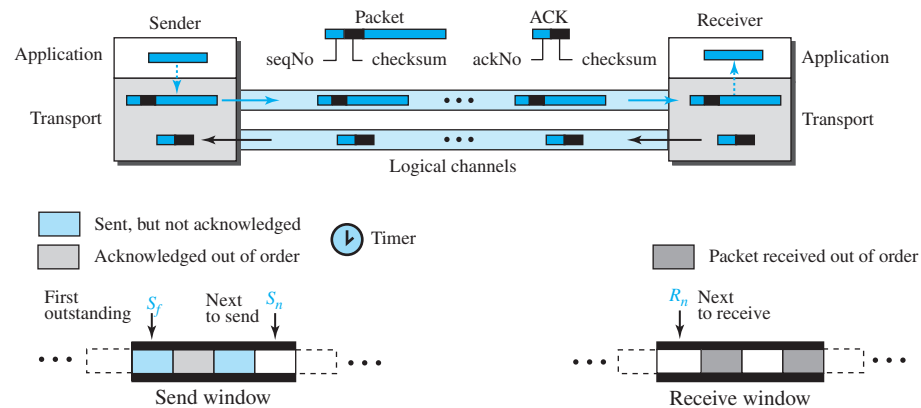
23.2.4 Selective-Repeat Protocol

The Go-Back- N protocol simplifies the process at the receiver. The receiver keeps track of only one variable, and there is no need to buffer out-of-order packets; they are simply discarded. However, this protocol is inefficient if the underlying network protocol loses a lot of packets. Each time a single packet is lost or corrupted, the sender

resends all outstanding packets, even though some of these packets may have been received safe and sound but out of order. If the network layer is losing many packets because of congestion in the network, the resending of all of these outstanding packets makes the congestion worse, and eventually more packets are lost. This has an avalanche effect that may result in the total collapse of the network.

Another protocol, called the **Selective-Repeat (SR) protocol**, has been devised, which, as the name implies, resends only selective packets, those that are actually lost. The outline of this protocol is shown in Figure 23.31.

Figure 23.31 Outline of Selective-Repeat

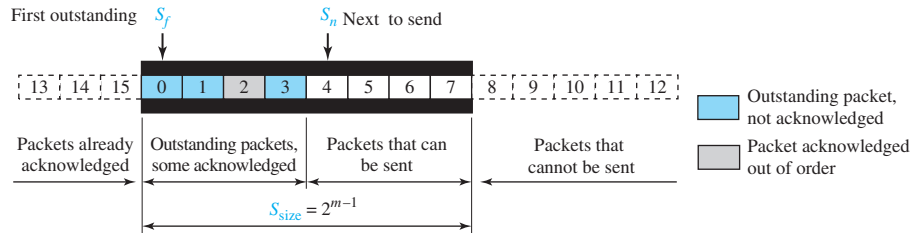


Windows

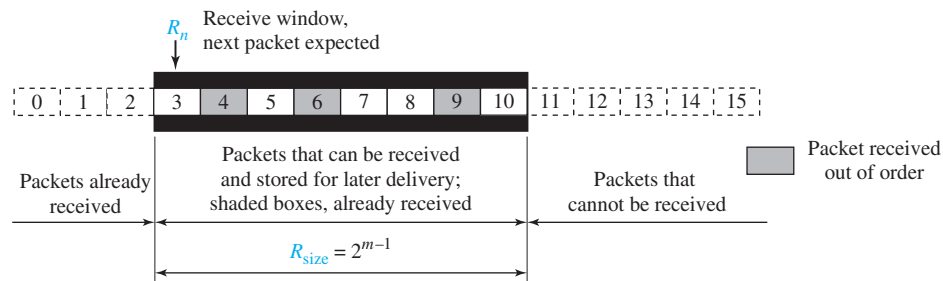
The Selective-Repeat protocol also uses two windows: a send window and a receive window. However, there are differences between the windows in this protocol and the ones in Go-Back- N . First, the maximum size of the send window is much smaller; it is 2^{m-1} . The reason for this will be discussed later. Second, the receive window is the same size as the send window.

The send window maximum size can be 2^{m-1} . For example, if $m = 4$, the sequence numbers go from 0 to 15, but the maximum size of the window is just 8 (it is 15 in the Go-Back- N Protocol). We show the Selective-Repeat send window in Figure 23.32 to emphasize the size.

The receive window in Selective-Repeat is totally different from the one in Go-Back- N . The size of the receive window is the same as the size of the send window (maximum 2^{m-1}). The Selective-Repeat protocol allows as many packets as the size of the receive window to arrive out of order and be kept until there is a set of consecutive packets to be delivered to the application layer. Because the sizes of the send window and receive window are the same, all the packets in the send packet can arrive out of order and be stored until they can be delivered. We need, however, to emphasize that in a reliable protocol the receiver *never* delivers packets out of order to the application layer. Figure 23.33 shows the receive window in Selective-Repeat. Those slots inside the

Figure 23.32 Send window for Selective-Repeat protocol

window that are shaded define packets that have arrived out of order and are waiting for the earlier transmitted packet to arrive before delivery to the application layer.

Figure 23.33 Receive window for Selective-Repeat protocol

Timer

Theoretically, Selective-Repeat uses one timer for each outstanding packet. When a timer expires, only the corresponding packet is resent. In other words, GBN treats outstanding packets as a group; SR treats them individually. However, most transport-layer protocols that implement SR use only a single timer. For this reason, we use only one timer.

Acknowledgments

There is yet another difference between the two protocols. In GBN an ackNo is cumulative; it defines the sequence number of the next packet expected, confirming that all previous packets have been received safe and sound. The semantics of acknowledgment is different in SR. In SR, an ackNo defines the sequence number of a single packet that is received safe and sound; there is no feedback for any other.

In the Selective-Repeat protocol, an acknowledgment number defines the sequence number of the error-free packet received.

Example 23.9

Assume a sender sends 6 packets: packets 0, 1, 2, 3, 4, and 5. The sender receives an ACK with $\text{ackNo} = 3$. What is the interpretation if the system is using GBN or SR?

Solution

If the system is using GBN, it means that packets 0, 1, and 2 have been received uncorrupted and the receiver is expecting packet 3. If the system is using SR, it means that packet 3 has been received uncorrupted; the ACK does not say anything about other packets.

FSMs

Figure 23.34 shows the FSMs for the Selective-Repeat protocol. It is similar to the ones for the GBN, but there are some differences.

Sender

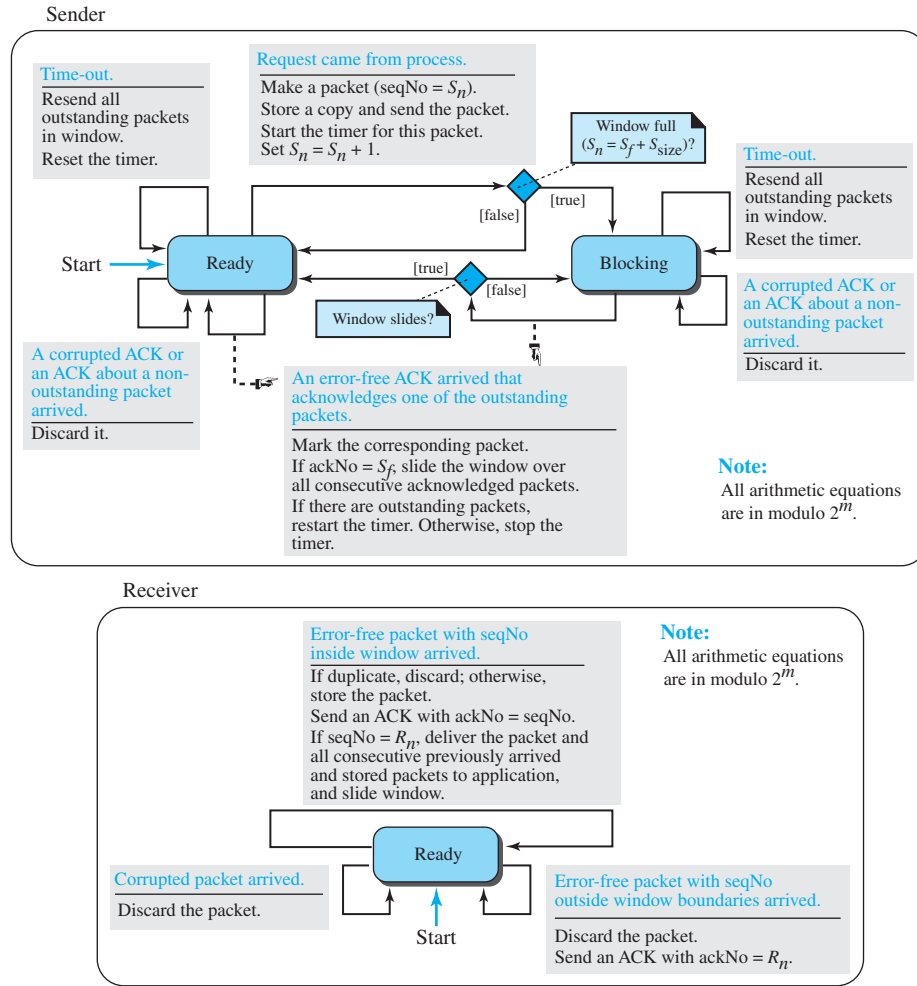
The sender starts in the *ready* state, but later it can be in one of the two states: *ready* or *blocking*. The following shows the events and the corresponding actions in each state.

- **Ready state.** Four events may occur in this case:
 - a. If a request comes from the application layer, the sender creates a packet with the sequence number set to S_n . A copy of the packet is stored, and the packet is sent. If the timer is not running, the sender starts the timer. The value of S_n is now incremented, $S_n = (S_n + 1) \text{ modulo } 2^m$. If the window is full, $S_n = (S_f + S_{\text{size}}) \text{ modulo } 2^m$, the sender goes to the blocking state.
 - b. If an error-free ACK arrives with ackNo related to one of the outstanding packets, that packet is marked as acknowledged. If the $\text{ackNo} = S_f$, the window slides to the right until the S_f points to the first unacknowledged packet (all consecutive acknowledged packets are now outside the window). If there are outstanding packets, the timer is restarted; otherwise, the timer is stopped.
 - c. If a corrupted ACK or an error-free ACK with ackNo not related to an outstanding packet arrives, it is discarded.
 - d. If a time-out occurs, the sender resends all unacknowledged packets in the window and restarts the timer.
- **Blocking state.** Three events may occur in this case:
 - a. If an error-free ACK arrives with ackNo related to one of the outstanding packets, that packet is marked as acknowledged. In addition, if the $\text{ackNo} = S_f$, the window is slid to the right until the S_f points to the first unacknowledged packet (all consecutive acknowledged packets are now outside the window). If the window has slid, the sender moves to the ready state.
 - b. If a corrupted ACK or an error-free ACK with the ackNo not related to outstanding packets arrives, the ACK is discarded.
 - c. If a time-out occurs, the sender resends all unacknowledged packets in the window and restarts the timer.

Receiver

The receiver is always in the *ready* state. Three events may occur:

Figure 23.34 FSMs for SR protocol

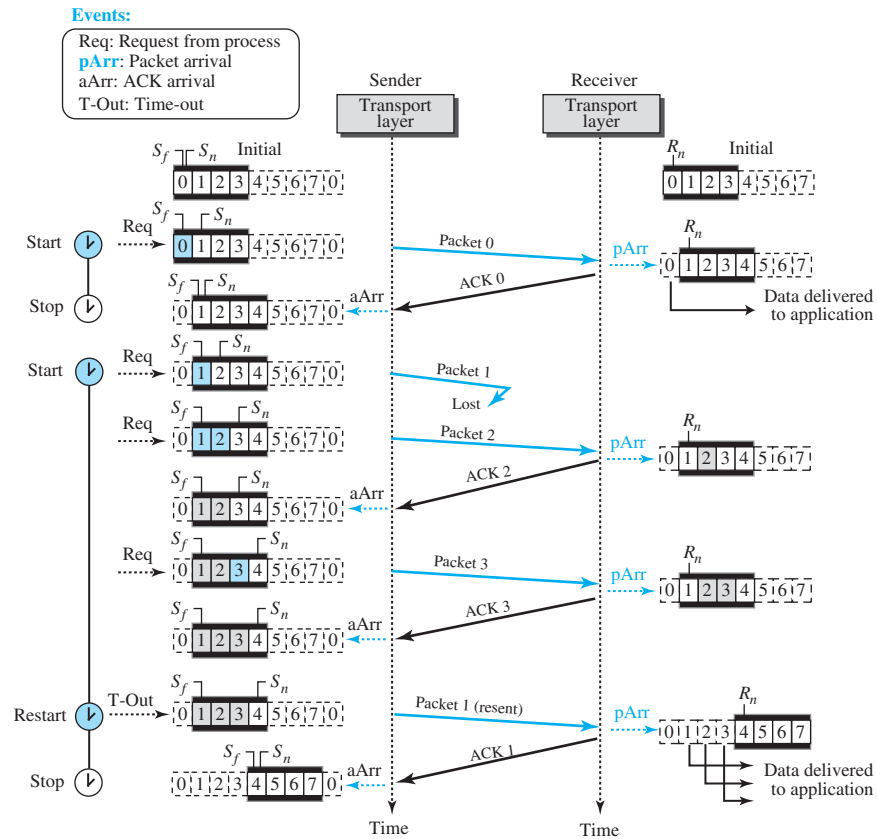


- If an error-free packet with seqNo in the window arrives, the packet is stored and an ACK with ackNo = seqNo is sent. In addition, if the seqNo = R_n , then the packet and all previously arrived consecutive packets are delivered to the application layer and the window slides so that the R_n points to the first empty slot.
- If an error-free packet with seqNo outside the window arrives, the packet is discarded, but an ACK with ackNo = R_n is returned to the sender. This is needed to let the sender slide its window if some ACKs related to packets with seqNo < R_n were lost.
- If a corrupted packet arrives, the packet is discarded.

Example 23.10

This example is similar to Example 23.8 (Figure 23.30) in which packet 1 is lost. We show how Selective-Repeat behaves in this case. Figure 23.35 shows the situation.

Figure 23.35 Flow diagram for Example 23.10



At the sender, packet 0 is transmitted and acknowledged. Packet 1 is lost. Packets 2 and 3 arrive out of order and are acknowledged. When the timer times out, packet 1 (the only unacknowledged packet) is resent and is acknowledged. The send window then slides.

At the receiver site we need to distinguish between the acceptance of a packet and its delivery to the application layer. At the second arrival, packet 2 arrives and is stored and marked (shaded slot), but it cannot be delivered because packet 1 is missing. At the next arrival, packet 3 arrives and is marked and stored, but still none of the packets can be delivered. Only at the last arrival, when finally a copy of packet 1 arrives, can packets 1, 2, and 3 be delivered to the application layer. There are two conditions for the delivery of packets to the application layer: First, a set of consecutive packets must have arrived. Second, the set starts from the beginning of the window. After the first arrival, there was only one packet and it started from the beginning of the window. After the last arrival, there are three packets and the first one starts from the beginning of the window. The key is that a reliable transport layer promises to deliver packets in order.

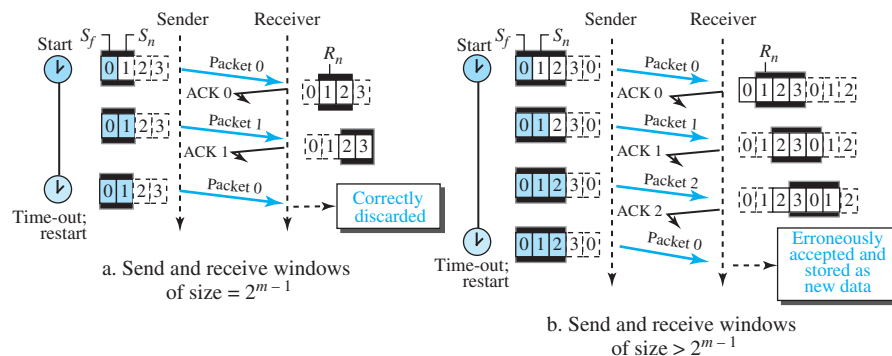
Window Sizes

We can now show why the size of the sender and receiver windows can be at most one-half of 2^m . For an example, we choose $m = 2$, which means the size of the window is $2^m/2$ or $2^{(m-1)} = 2$. Figure 23.36 compares a window size of 2 with a window size of 3.

If the size of the window is 2 and all acknowledgments are lost, the timer for packet 0 expires and packet 0 is resent. However, the window of the receiver is now expecting packet 2, not packet 0, so this duplicate packet is correctly discarded (the sequence number 0 is not in the window). When the size of the window is 3 and all acknowledgments are lost, the sender sends a duplicate of packet 0. However, this time, the window of the receiver expects to receive packet 0 (0 is part of the window), so it accepts packet 0, not as a duplicate, but as a packet in the next cycle. This is clearly an error.

In Selective-Repeat, the size of the sender and receiver window can be at most one-half of 2^m .

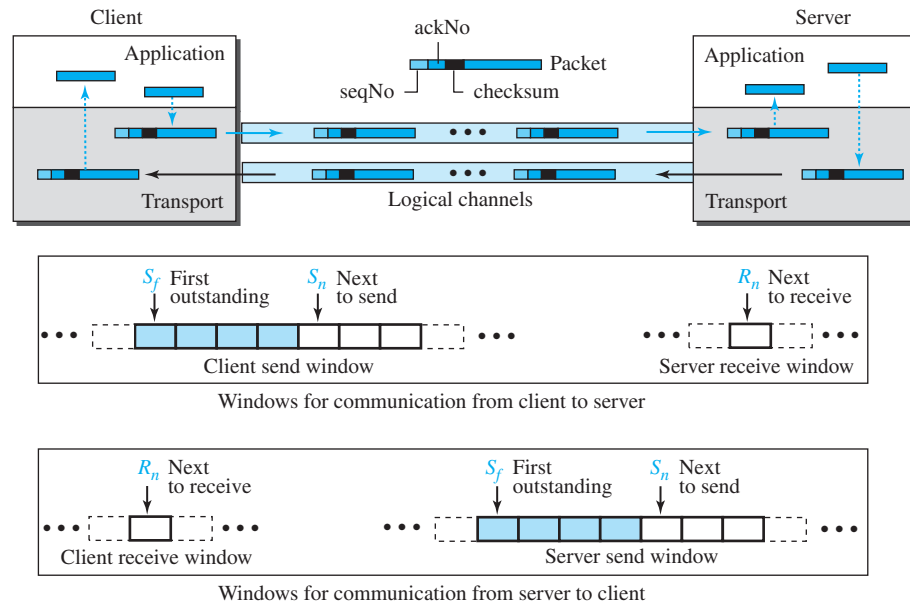
Figure 23.36 Selective-Repeat, window size



23.2.5 Bidirectional Protocols: Piggybacking

The four protocols we discussed earlier in this section are all unidirectional: data packets flow in only one direction and acknowledgments travel in the other direction. In real life, data packets are normally flowing in both directions: from client to server and from server to client. This means that acknowledgments also need to flow in both directions. A technique called **piggybacking** is used to improve the efficiency of the bidirectional protocols. When a packet is carrying data from A to B, it can also carry acknowledgment feedback about arrived packets from B; when a packet is carrying data from B to A, it can also carry acknowledgment feedback about the arrived packets from A.

Figure 23.37 shows the layout for the GBN protocol implemented bidirectionally using piggybacking. The client and server each use two independent windows: send and receive.

Figure 23.37 Design of piggybacking in Go-Back-N

23.3 END-CHAPTER MATERIALS

23.3.1 Recommended Reading

For more details about subjects discussed in this chapter, we recommend the following books.

Books

Several books give information about transport-layer protocols. The items enclosed in brackets refer to the reference list at the end of the book: In particular, we recommend [Com 06], [PD 03], [GW 04], [Far 04], [Tan 03], and [Sta 04].

23.3.2 Key Terms

bandwidth-delay product
client-server paradigm
congestion
congestion control
demultiplexing
ephemeral port number
finite state machine (FSM)
Go-Back-N protocol (GBN)
multiplexing
piggybacking

pipelining
port number
process-to-process communication
Selective-Repeat (SR) protocol
sequence number
sliding window
socket address
Stop-and-Wait protocol
well-known port number