5.2.3. $\dot{x} = y$

$\dot{y} = -2x - 3y.$

$Ax = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$

$\tau = -3.$

$\Delta = 2$

$\lambda_{1,2} = \dfrac{-3 \pm \sqrt{(-3)^2 - (4 \times 2)}}{2}$

$= \dfrac{-3 \pm 1}{2}$

$\lambda_2 = -2 \quad \lambda_1 = -1$

$\therefore \lambda_1 = -1 \quad \lambda_2 = -2 \quad \lambda_2 < \lambda_1 < 0.$

when $\lambda_1 = -1$

$\begin{pmatrix} 1 & 1 \\ -2 & -2 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$

$V_1 = 1$

$V_2 = -1 \qquad \vec{V} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

when $\lambda_2 = -2.$

$\begin{pmatrix} 2 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$V_1 = 1$

$V_2 = -2 \qquad \vec{V} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$