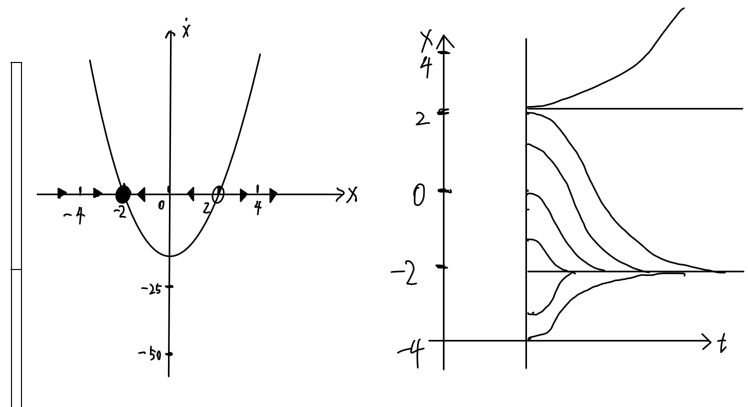
Task 1: pages 36-40 ex. 2.2.1, 2.2.3, 2.2.8, 2.4.1, 2.4.3;
Task 2: pages 79-83 ex. 3.1.1, 3.2.3, 3.4.1, 3.4.5, 3.4.6;
Task 3: pages 141-144 ex. 5.1.7, 5.2.3, 5.2.5, 5.3.4;
Task 4: pages 182-184 ex. 6.3.1, 6.3.3, 6.4.2.
2.2.1 $\dot{x} = 4x^2 - 16$



∴ $x^* = -2$ (stable)

$x^* = 2$ (unstable).

$\dot{x} = 4x^2 - 16.$

$\frac{dx}{dt} = 4x^2 - 16$

$dt = \frac{1}{4x^2-16} dx$

$\int dt = \int \frac{1}{4x^2-16} dx$

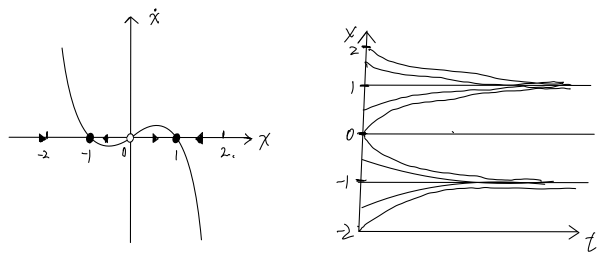
$4t + C = \frac{1}{4} \ln\left(\frac{x-2}{x+2}\right)$

∴ $x(t) = \frac{-2(1+Ce^{16t})}{1-Ce^{16t}}$

$C = \frac{x-2}{x+2}.$

(a) 2.2.1

2.2.3 $\dot{x} = x - x^3$



$x^* = +1$
$x^* = -1$ (stable) $\quad x^* = 0$ (unable.)

$\dot{x} = x - x^3$

$\int dt = \int \frac{1}{x-x^3} dx$

$\int dt = \int \frac{1}{x(1-x^2)} dx$

$t + C_1 = \int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{x+1}dx - \frac{1}{2}\int\frac{1}{x-1}dx$

$t + C_1 = -\frac{1}{2}\ln(x^2-1) + \ln(x)$

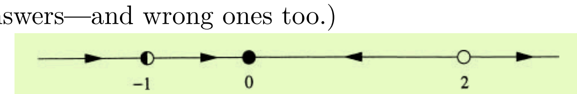
$2t + 2C_1 = \ln\left(\frac{x^2}{1-x^2}\right)$

$e^{2t+2C_1} = \frac{x^2}{1-x^2}$

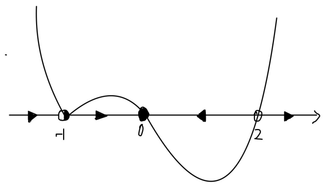
$\therefore x(t) = \pm\frac{C_1 e^t}{\sqrt{C_1^2 e^{2t}+1}}$

(b) 2.2.3

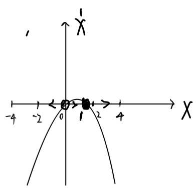
2.2.8 (Working backwards, from flows to equations) Given an equation , we know how to sketch the corresponding flow on the real line. Here you are asked to solve the opposite problem: For the phase portrait shown in Figure 1, find an equation that is consistent with it. (There are an infinite number of correct answers—and wrong ones too.)



$$(x+1)^2 x(x-2)$$



2.4.1 $\dot{x} = x(1-x)$



from the graph get $x_1^* = 0$ (unstable) $x_2^* = 1$ (stable)

Let $\dot{x} = x(1-x) = 0$

$x_1^* = 0 \quad x_2^* = 1$

Due to $f(x) = 1 - 2x$

$\therefore f'(x_1^*) = 1 - 2x_1^*$

$= 1$

$\therefore f'(x_2^*) = 1 - 2x_2^*$

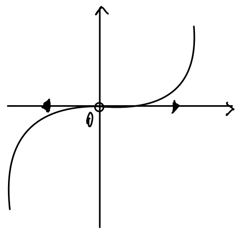
$= -1$

$\therefore x_1^* = 0$ (unstable)

$x_2^* = 1$ (stable)

(c) 2.4.1

2.4.3 $\dot{x} = tan(x)$



from the graph $x^* = 0$ (unstable).

Let $\dot{x} = \tan(x) = 0$.

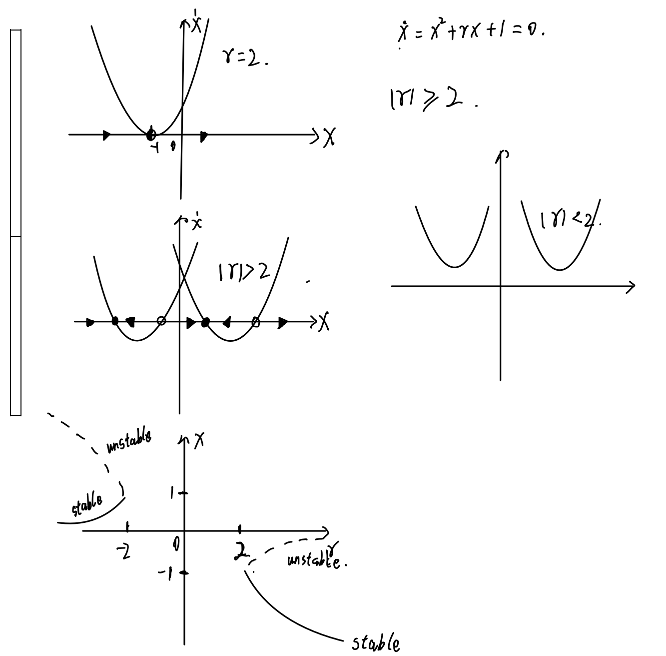
get $x^* = 0$.

Due to $f'(x^*) = \sec^2(x) = 1$.

$\therefore x^* = 0$ is unstable.

(d) 2.4.3

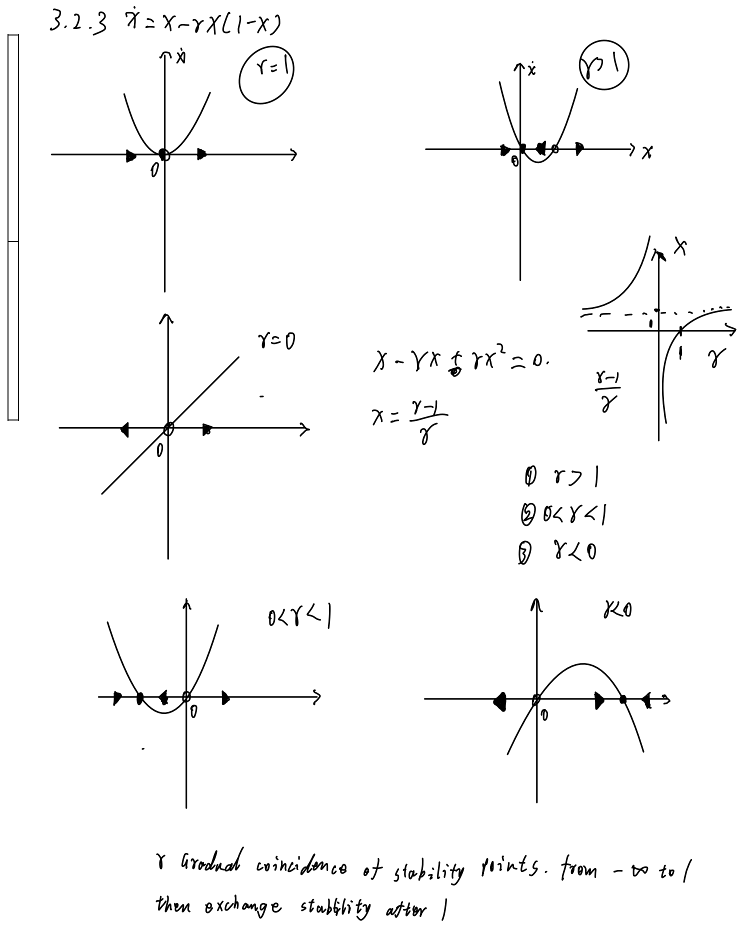
3.1.1 $\dot{x} = 1 + rx + x^2$



(e) 3.1.1

3.2 Transcritical Bifurcation For each of the following exercises, sketch all the qualitatively different vector fields that occur as r is varied. Show that a transcritical bifurcation occurs at a critical value of r, to be determined. Finally, sketch the bifurcation diagram of fixed points x∗ vs. r.

3.2.3 $\dot{x} = xrx(1x)$


3.2.3 ẋ = x − rx(1−x)
r=1
r>1
r=0
x − rx + rx² = 0.
x = (r−1)/r
① r>1
② 0<r<1
③ r<0
0<r<1
r<0
r Gradual coincidence of stability points. from −∞ to 1
then exchange stablity after 1

(f) 3.2.3