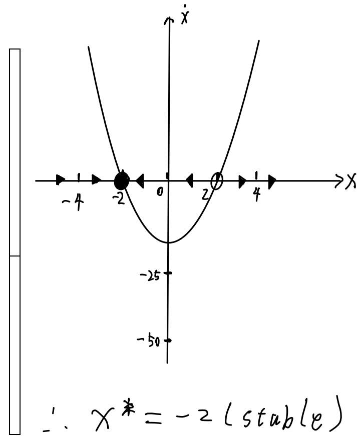
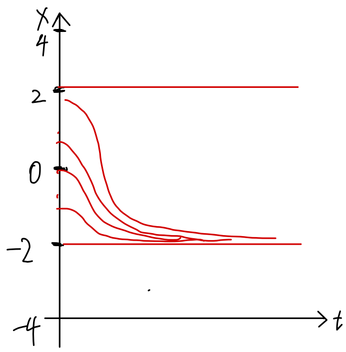
2.2.1 $\dot{x} = 4x^2 - 16$





$\therefore\ x^* = -2$ (stable)

$x^* = 2$ (unstable).

$\dot{x} = 4x^2 - 16.$

$$\frac{dx}{dt} = 4x^2 - 16$$

$$dt = \frac{1}{4x^2-16}\,dx$$

$$\int dt = \int \frac{1}{4x^2-16}\,dx$$

$$t + c_1 = \frac{1}{16}\log\left(\frac{2-x}{x+2}\right)$$

modify:

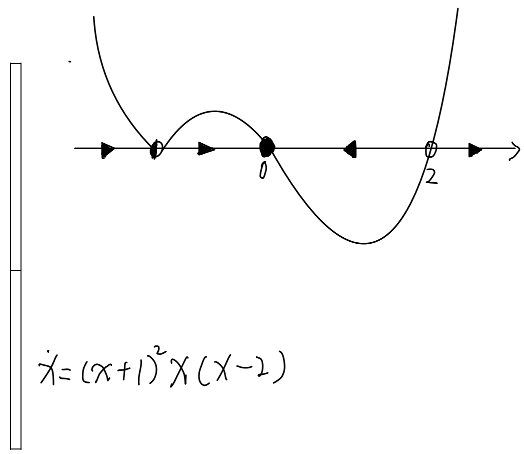
$$x(t) = \frac{-2(e^{4c_1+16t}-1)}{e^{4c_1+16t}+1}$$

$x \in [-2, 2]$

$$t = \frac{1}{16}\ln\left(\frac{2-x}{x+2}\right).$$

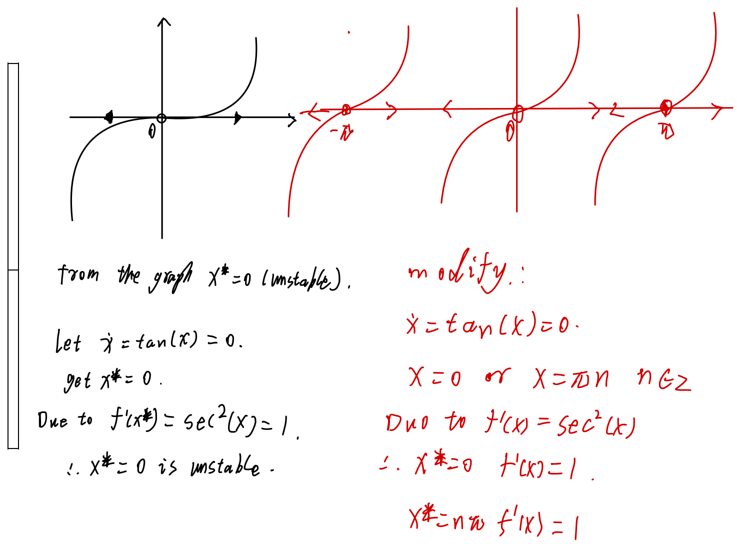
(a) 2.2.1

2.2.8



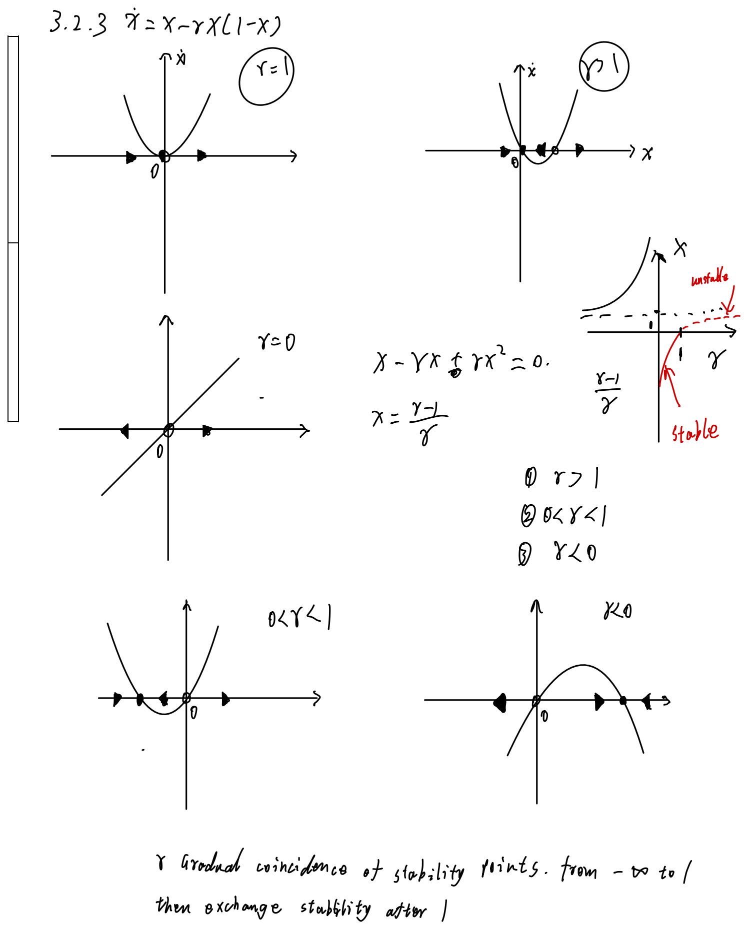
(b) 2.2.8

2.4.3


0
-π
0
π
from the graph x* = 0 (unstable).
Let ẋ = tan(x) = 0.
get x* = 0.
Due to f'(x*) = sec²(x) = 1.
∴ x* = 0 is unstable.
modify.:
ẋ = tan(x) = 0.
x = 0 or x = πn n∈Z
Duo to f'(x) = sec²(x)
∴ x* = 0 f'(x) = 1.
x* = nπ f'(x) = 1

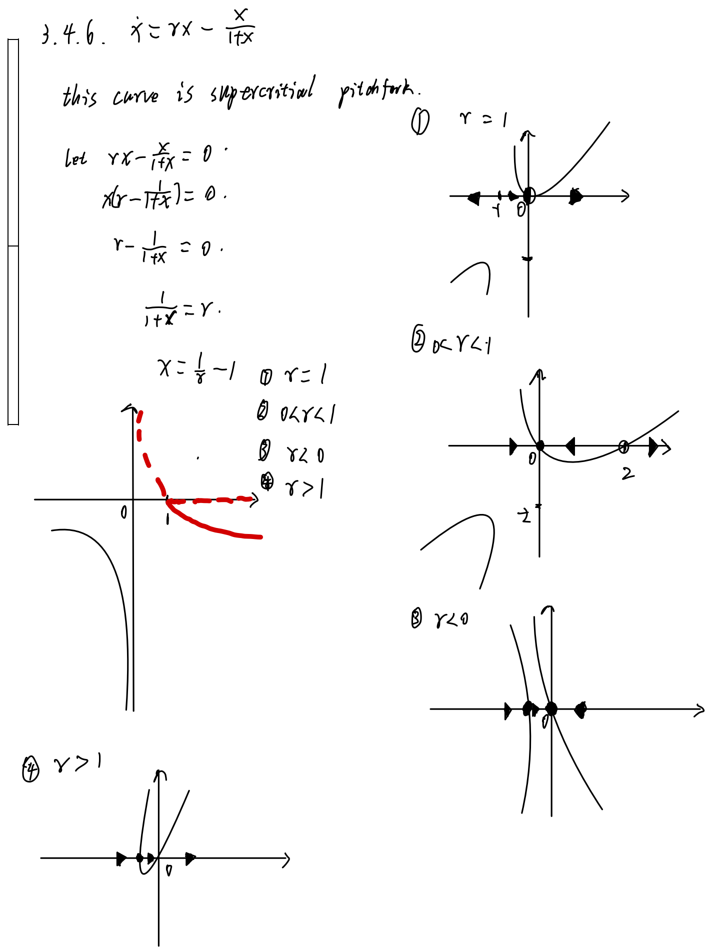
(c) 2.4.3

3.2.3


3.2.3 ẋ = x − rx(1−x)
r=1
r>1
unstable
stable
r=0
x − rx + rx² = 0.
x = (r−1)/r
① r>1
② 0<r<1
③ r<0
0<r<1
r<0
r Gradual coincidence of stability points. from −∞ to 1
then exchange stability after 1

(d) 3.2.3

3.4.6


3.4.6. $\dot{x} = rx - \frac{x}{1+x}$
this curve is supercritical pitchfork.
Let $rx - \frac{x}{1+x} = 0$.
$x(r - \frac{1}{1+x}) = 0$.
$r - \frac{1}{1+x} = 0$.
$\frac{1}{1+x} = r$.
$x = \frac{1}{r} - 1$
① $r = 1$
② $0 < r < 1$
③ $r < 0$
④ $r > 1$
① $r = 1$
② $0 < r < 1$
③ $r < 0$
④ $r > 1$

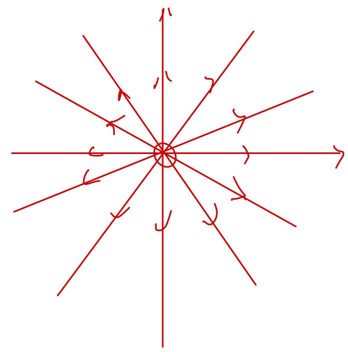
(e) 3.4.6

5.1.7

5.1.7

$\dot{x} = x$

$\dot{y} = x + y$



$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$\tau = a + d = 2.$

$\Delta = 1 - 0 = 1$

$\tau^2 - 4\Delta = 0.$

$\lambda_{1,2} = \frac{2}{2} = 1.$

(f) 5.1.7

5.2.3

5.2.3. $\dot{x} = y$

$\dot{y} = -2x - 3y$.

$A_x = \begin{pmatrix} 0, & 1 \\ -2 & -3 \end{pmatrix}$

$\tau = -3$.

$\Delta = 2$

$\lambda_{1,2} = \dfrac{-3 \pm \sqrt{(-3)^2 - (4 \times 2)}}{2}$

$= \dfrac{-3 \pm 1}{2}$

$\lambda_2 = -2 \quad \lambda_1 = -1$

$\therefore \lambda_1 = -1 \quad \lambda_2 = -2 \quad \lambda_2 < \lambda_1 < 0$.

when $\lambda_1 = -1$

$\begin{pmatrix} 1, & 1 \\ -2, & -2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

$v_1 = 1$

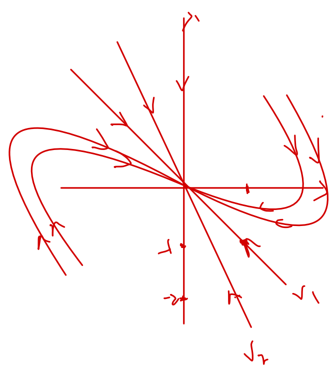
$v_2 = -1 \quad \vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

when $\lambda_2 = -2$.

$\begin{pmatrix} 2, & 1 \\ -2, & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$V_1 = 1 \quad \vec{v} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

$V_2 = -2$



(g) 5.2.3

5.2.5

5.2.5.

$\dot{x} = 3x - 4y$

$\dot{y} = x - y$

$A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

$\tau = 2.$

$\Delta = -3 - (-4) = 1$

$\lambda_{1,2} = \dfrac{2 \pm \sqrt{4 - 1 \times 4}}{2}$

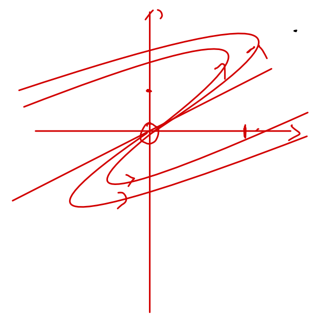
$= 1$

$\lambda_1 = \lambda_2 = 1$

$\begin{pmatrix} 3-\lambda & b \\ c & d-\lambda \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

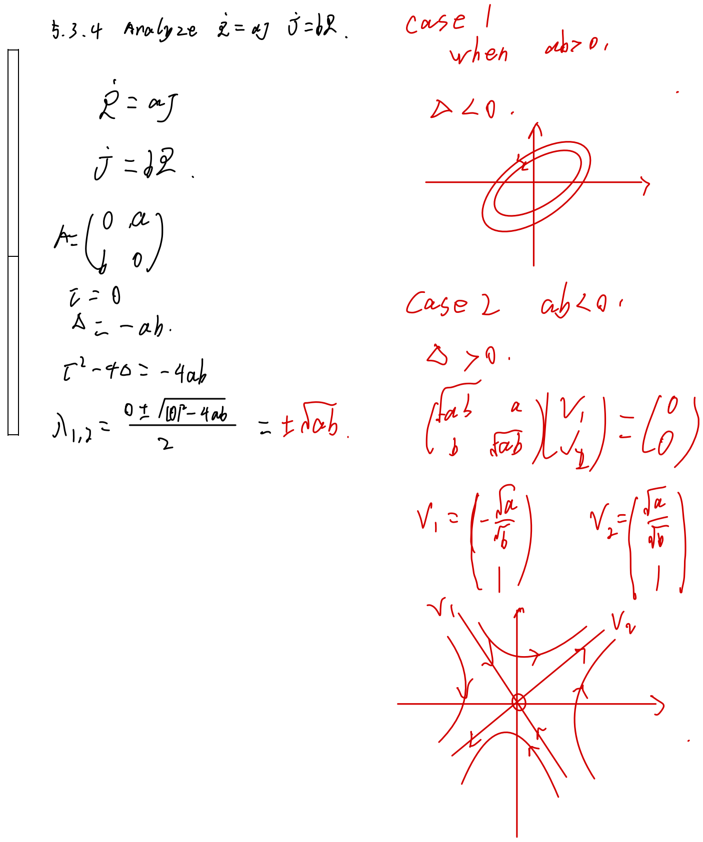
eigenvector $v = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}$



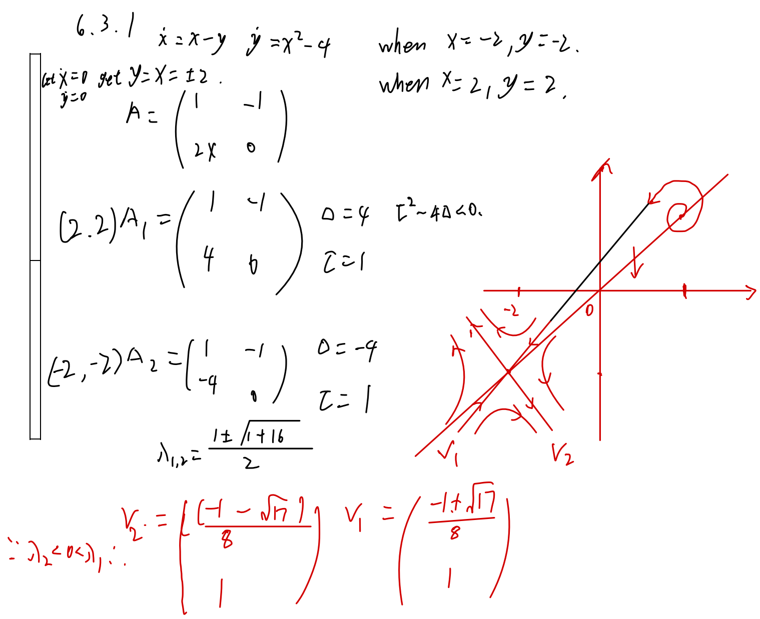
(h) 5.2.5

5.3.4


5.3.4 Analyze $\dot{x} = ay$ $\dot{y} = bx$.
$\dot{x} = ay$
$\dot{y} = bx$.
$A = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$
$\tau = 0$
$\Delta = -ab$.
$\tau^2 - 4\Delta = -4ab$
$\lambda_{1,2} = \frac{0 \pm \sqrt{(0)^2 - 4ab}}{2} = \pm\sqrt{ab}$.
Case 1
when $ab > 0$,
$\Delta < 0$.
Case 2 $ab < 0$,
$\Delta > 0$.
V_1
V_2

(i) 5.3.4

6.3.1



6.3.1 $\dot{x} = x - y$ $\dot{y} = x^2 - 4$ when $x = -2, y = -2$.
when $x = 2, y = 2$.

let $\dot{x} = 0$, $\dot{y} = 0$ get $y = x = \pm 2$.

$A = \begin{pmatrix} 1 & -1 \\ 2x & 0 \end{pmatrix}$

$(2,2)$ $A_1 = \begin{pmatrix} 1 & -1 \\ 4 & 0 \end{pmatrix}$ $\Delta = 4$ $\tau^2 - 4\Delta < 0$.
$\tau = 1$

$(-2,-2)$ $A_2 = \begin{pmatrix} 1 & -1 \\ -4 & 0 \end{pmatrix}$ $\Delta = -4$
$\tau = 1$

$\lambda_{1,2} = \frac{1 \pm \sqrt{1+16}}{2}$

$\because \lambda_2 < 0 < \lambda_1 \therefore$ $V_2 = \begin{pmatrix} \frac{-1-\sqrt{17}}{8} \\ 1 \end{pmatrix}$ $V_1 = \begin{pmatrix} \frac{-1+\sqrt{17}}{8} \\ 1 \end{pmatrix}$

(j) 6.3.1

6.3.3

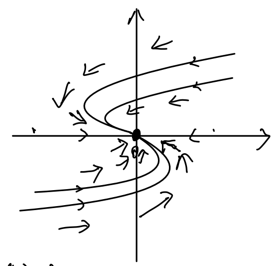
6.3.3. $\dot{x} = 1 + y - e^{-x}$, $\dot{y} = x^3 - y$

$\dot{x} = 1 + y - e^{-x} = 0$.

$\dot{y} = x^3 - y = 0$

$\therefore y = x = 0$.

$$A = \begin{pmatrix} e^{x} & 1 \\ 0 & 1 \end{pmatrix}$$



$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \tau = 2 \quad \Delta = 1 \quad \tau^2 - 4\Delta = 0.$$

(k) 6.3.3

6.4.2

6.4.2. $\dot{x} = x(3-2x-y) = 0.$
$\dot{y} = y(2-x-y) = 0,$ get $(0,2), (1,1)\ (\frac{3}{2},0)\ (0,0),$

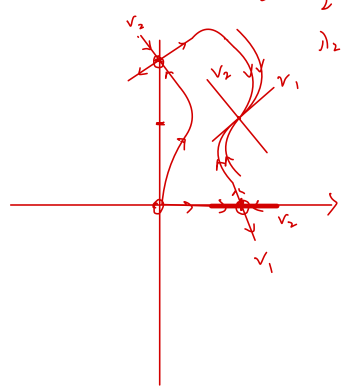
$$A = \begin{pmatrix} -4x-y+3 & -x \\ -y & -x-2y+2 \end{pmatrix} \qquad (0,0)\ A = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$

$$(0,2)\ A = \begin{pmatrix} 1 & 0 \\ -2 & -2 \end{pmatrix} \quad \lambda = 1, -2$$

$$(1,1)\ A = \begin{pmatrix} -2 & -1 \\ -1 & -1 \end{pmatrix} \quad \tau = -3,\ \Delta = 1 \quad \lambda = \frac{-3 \pm \sqrt{5}}{2}$$

$$(\tfrac{3}{2},0)\ A = \begin{pmatrix} -3 & -\frac{3}{2} \\ 0 & \frac{1}{2} \end{pmatrix} \quad \tau = -\frac{5}{2},\ \Delta = -\frac{3}{2} \quad \lambda = \frac{-\frac{5}{2} \pm \sqrt{(\frac{5}{2})^2 - \cancel{4\times}\frac{3}{2}}}{2}$$

$\lambda_2 < \lambda_1 < 0.$



(l) 6.4.2