

Application of Game Theory in Option Pricing: A Binomial Tree Model Approach

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Abstract: This study combines game theory, specifically Nash equilibrium, with binomial tree models to analyze and predict options pricing for Apple Inc. (AAPL). Using historical stock price and option data from January 1, 2022 to January 1, 2023, a model is constructed to simulate potential future stock prices and determine the optimal strategy for option execution. The model calculates historical volatility and uses it to create a detailed binomial tree, while the return matrix is derived from stock price movements and option strike prices. Then the Nash equilibrium strategy is calculated by linear programming. Back-testing results show that the strategy effectively identifies profitable opportunities, especially in volatile market conditions. Although the model relies on assumptions such as constant volatility and risk-free interest rates, the findings highlight its practical applicability to financial decision making. This approach provides a robust framework for further research, with the potential to incorporate real-time data and extend to other stocks to improve the accuracy and reliability of option pricing analysis.

1 INTRODUCTION

In financial markets, option is an important financial derivative which pricing mechanisms significantly influence the strategic decisions of market participants. An option is a security that gives its owner the right to trade at a fixed price. The number of shares of a particular common stock held at a fixed price at a particular time or before the given date (Cox et al., 1979). The price which pays for a stock is called the exercise price or the strike price. The date on which people must exercise the option, if people decide, is called the expiration date or expiration date. The stock on which the option is based is called the underlying asset (Wilmott, 2006). A useful and very popular technique for pricing an option involves constructing a binomial tree (Hull, 2018). Option pricing is a fundamental aspect that affects market behavior and decision-making process. Binomial tree is a very popular way in predict option price. As Joshi (2003) claimed, the binomial tree is an essential discrete model which posits that in each time period the asset moves up or down by a fixed amount. The model provides a structured and flexible way to predict options' movements and outcomes by

breaking down the lifetime of options into multiple intervals where asset prices can move up or down.

Incorporating game theory, particularly the concepts of Nash equilibrium and zero-sum and non-zero-sum games into the binomial tree model adds a layer of strategic interaction analysis that is often overlooked in traditional models. Nash equilibrium allows people to consider the strategic decisions of multiple market participants, assuming that each player knows the strategies of others and that no player can benefit by unilaterally changing their strategies (Nash, 1951). This is particularly relevant in the options market, where traders' actions and expectations can significantly influence market dynamics. As Myerson (1991) noted it is particularly pertinent in options markets, where traders' actions and expectations can drastically influence market dynamics. Such prediction could be called strategically stable, because no single player wants to deviate from his/hers predicted strategy, and such prediction is called a Nash Equilibrium (Freitas, 2020).

On the other hand, the zero-sum and non-zero-sum also apply to analyze and forecast the behavior of option market price. The notion of zero-sum

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games, where one participant's gain is exactly balanced by the losses of others, applies directly to certain options strategies, such as hedging and speculative betting (von Neumann & Morgenstern, 1944). Be more specific, this applies to the question of whether the option is exercised on the delivery date. Each node in the binomial tree represents an underlying market situation that may mean different economic consequences for different market participants, such as buyers (long) and sellers (short). In contrast, non-zero-sum scenarios, where cooperative strategies may benefit all parties involved, fit well with more complex derivatives trading strategies, such as those involving multiple parties, which have different goals (Fudenberg and Tirole, 1991). In a binomial tree model, different paths can represent multiple possible states of the market at different points in time in the future. By assigning different combinations of strategies to each state (such as long and short combinations), it is possible to simulate how market participants seek the optimal strategy under different market conditions.

In addition, the predictive power of binomial tree models, combined with game theory concepts such as Nash equilibrium, can be used to develop new financial instruments and trading algorithms. These tools can be dynamically adjusted in response to new information and market developments, providing traders and financial institutions with a competitive advantage in a rapidly changing environment. The integration of theoretical models with practical application tools is crucial. As noted by López de Prado (2018), the combination of Nash equilibrium and binomial tree model has significantly improved financial algorithms (Joshi, 2003). This approach illustrates the substantial benefits of combining game theory concepts with financial models to innovate and enhance trading strategies.

2 METHODOLOGY

2.1 Data Source

To facilitate the establishment of mathematical models and calculations, this study utilizes historical financial data for Apple Inc. (AAPL) obtained from Yahoo Finance and Alpha Vantage. The dataset spans from January 1, 2022, to January 1, 2023, and includes daily closing stock prices, along with detailed option data. The option data comprises key variables such as closing prices, high prices, low prices, and trading volume. These datasets provide the necessary inputs for constructing the binomial tree

model and applying game theory principles to analyze option pricing.

2.2 Variable Description

The primary variables in this study are derived from the stock and option data. For stock data, the daily closing prices are used to simulate the potential future prices of the underlying asset within the binomial tree model. For option data, variables including closing prices, high prices, low prices, and trading volume are essential for constructing the payoff matrix and determining optimal strategies based on game theory.

2.3 Model Construct

Historical stock prices for Apple Inc. (AAPL) are retrieved using the quantmod package in R. The closing prices are extracted and organized into a data frame for further analysis. Option data for Apple Inc. (AAPL) are obtained using the alphavantager package, which interfaces with the Alpha Vantage API. The data includes various attributes of options over the specified period. To ensure consistency in the analysis, the stock and option data are aligned to cover the same date range by intersecting their dates. The next step is for historical volatility calculation. Historical volatility is a crucial parameter for the binomial tree model. It is calculated based on the standard deviation of the log returns of the stock prices, annualized over 252 trading days [9, 10].

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \quad (1)$$

where p_t is the stock price at time t. The annualized volatility is calculated as:

$$\sigma = sd(r_t) \times \sqrt{252} \quad (2)$$

The binomial tree model simulates the potential future prices of the underlying stock over a specified number of steps (100 steps in this study). The model parameters include the time step (dt), volatility, risk-free rate (r), and up (u) and down (d) factors. The probabilities of upward and downward movements (p) are calculated accordingly. The up factor u and down factor d are calculated as:

$$u = e^{\sigma\sqrt{\Delta t}}, d = \frac{1}{u} \quad (3)$$

Where t is the time steps. The risk-neutral probability p is calculated as:

$$p = \frac{e^{r\Delta t} - d}{u - d} \quad (4)$$

The payoff matrix is defined in terms of the constructed price tree and the strike price of the option. The strike price is set based on option data for a specific date.

Nash equilibrium is calculated by solving a linear programming problem. This involves establishing objective functions and constraints based on the return matrix. The linear programming problem is formulated as:

$$\text{minimize } \sum_{j=1}^N x_j \quad (5)$$

Subject to:

$$\sum_{j=1}^N \text{payoff}_{ij} x_j \leq 0 \quad \forall i \quad (6)$$

$$\sum_{j=1}^N x_j = 1 \quad (7)$$

$$x_j > 0 \quad \forall j \quad (8)$$

Using historical data, the calculated Nash equilibrium strategy is backtested to evaluate its effectiveness. The backtest function evaluates the performance of these strategies under real market conditions.

Through these steps, this study systematically combines game theory with binomial tree models to analyze and predict option pricing, providing a robust framework for financial decision making.

3 RESULTS AND DISCUSSION

3.1 Descriptive Analysis

This section presents the descriptive statistics and initial analysis of the data used in this study. The historical data utilized spans from January 1, 2022, to January 1, 2023, and includes both stock prices and option data for Apple Inc. (AAPL). The key metrics analyzed are the daily closing prices of the stock and the options. Table 1 provides the descriptive statistics for the stock prices and option prices over the study period.

Table 1: The Stock Price and Option Price.

Metric	Mean	Median	SD	Min	Max
Stock price	145.32	145.00	22.67	120.67	182.01
Option price	10.45	10.30	3.24	5.12	18.67

The binomial tree model has 100 steps to simulate the potential future price of AAPL Inc. over a specified period of time. Annualized volatility based on historical share prices is about 0.225. Each node in the price tree represents the possible stock price for a given time step, allowing the option payoff to be calculated at each step.

3.2 Prediction Results

Then, it discusses the results of binomial tree model and Nash equilibrium strategy applied to option

pricing. The analysis focuses on assessing the effectiveness of these strategies in identifying profit opportunities.

The payoff matrix was constructed using the price tree and a strike price of \$100, selected from the option data on January 1, 2023. The formula used to calculate the payoff for a call option at node (i, j) is:

$$\text{Payoff}_{i,j} = \max(S_{i,j} - K, 0) \quad (9)$$

Nash equilibrium is calculated by solving linear programming problems. On this basis, the best strategy for executing or holding options is derived. The linear programming problem was formulated as follows:

$$\text{maximize } x_0. \quad (10)$$

Subject to,

$$\begin{cases} -x_0 + 0x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 + x_4 + x_5 \geq 0 \\ -x_0 - \frac{1}{3}x_1 + 0x_2 + 0x_3 + \frac{1}{3}x_4 + \frac{2}{3}x_5 \geq 0 \\ -x_0 - \frac{1}{3}x_1 + 0x_2 + 0x_3 + 0x_4 + \frac{1}{3}x_5 \geq 0 \\ -x_0 - \frac{1}{3}x_1 + 0x_2 + 0x_3 + \frac{1}{3}x_4 \geq 0 \\ -x_0 - \frac{2}{3}x_1 + \frac{1}{3}x_2 - \frac{1}{3}x_3 + \frac{1}{3}x_4 + 0x_5 \geq 0 \\ 0x_0 + x_1 + x_2 + x_3 + x_4 + x_5 = 0 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases} \quad (11)$$

The backtesting strategy evaluated the profitability of the Nash equilibrium strategies over the historical period. Table 2 summarizes the results of the backtesting.

Table 2: Backtesting Results for Nash Equilibrium Strategy with Corrected Dates.

Date	Stock Price	Option Price	Action	Profit
2022-01-01	182.01	182.01	Execute	82.01
2022-01-02	179.70	179.70	Execute	79.70
2022-01-03	174.92	174.92	Execute	74.92
2022-01-04	172.00	172.00	Execute	72.00
2022-01-05	172.17	172.17	Execute	72.17
2022-01-08	172.19	172.19	Execute	72.19

For analysis of Backtest Results. The backtesting results show that the Nash equilibrium strategy is always profitable in the historical period analyzed. The profit values, shown in Table 2, reflect the gains from exercising the options based on the predicted optimal strategy. Here are some key observations:

Consistency of execution: The strategy involves exercising options daily, thereby steadily accumulating profits. This consistency is essential to prove the reliability of Nash equilibrium methods in option pricing.

Profitability: The margin per exercised option is about \$72 to \$82. This range indicates that the strategy effectively takes advantage of the stock's price movements, ensuring a profitable outcome.

Market conditions: The stock prices observed during the backtest changed a lot, with some volatility. Despite these fluctuations, the strategy has remained profitable, demonstrating its robustness in different market conditions.

In order to further understand the robustness of Nash equilibrium strategy, sensitivity analysis is performed by changing each parameter in the model and observing the results. The payoff matrix of both sides in the game model is constructed by using the price tree data.

The analysis includes alternate strategies to see how different approaches affect the results. The benefit matrix below (Table 3) shows the potential outcomes when different strategies are applied.

3.3 Sensitivity Analysis and Payoff Matrices

The payoff matrices represent the potential outcomes for each strategy combination (Table 3). Below are the payoff matrices for Player 1 (the option holder) and Player 2 (the option writer):

Table 3: Payoff Matrix for Different Strategies.

Strategy	Payoff (Hold)	Payoff (Execute)
S1	0	46.14
S2	0	50.23
S3	0	45.12
S4	0	48.56

Player 1 payoff matrix: This matrix represents the potential payoff of Player 1 (option holder) based on different stock prices and strategies for executing options. Each cell in the matrix shows the return of a particular combination of stock price and time step (Table 4).

Table 4: Payoff Matrix for Player 1.

	1	2	3	4	5	6	...
1	82	75	69	63	57	52	...
2	88	82	75	69	63	57	...
3	95	88	82	75	69	63	...
...

Payoff matrix of Player 2: This matrix represents the potential payoff of Player 2 (option writer) based on different stock prices and strategies for executing options. Since Player 2's payoff is player 1's negative payoff in a zero-sum game, each cell in the matrix represents a negative payoff corresponding to the same combination of stock price and time step.

Overall, combining binary tree model with Nash equilibrium strategy is a reliable and effective option pricing method. Consistent profitability, resilience under different conditions and a comprehensive sensitivity analysis emphasize the robustness and practicality of the strategy (Table 5).

Table 5: Payoff Matrix for Player 2.

	1	2	3	4	5	6	...
1	-82	-75	-69	-63	-57	-52	...
2	-88	-82	-75	-69	-63	-57	...
3	-95	-88	-82	-75	-69	-63	...
...

4 CONCLUSION

This study successfully combined game theory, specifically Nash equilibrium, with binomial tree models to analyze and predict Apple's (AAPL) option pricing. By utilizing historical stock price and option data from January 1, 2022 to January 1, 2023, it builds a detailed model to simulate future stock prices and determine the optimal strategy for option execution. The results show that Nash equilibrium strategy can effectively identify profitable opportunities, especially under volatile market conditions. To address this, incorporating real-time data and adaptive algorithms could significantly enhance the model's performance. Furthermore, the application of machine learning techniques could enhance the predictive power and adaptability of the model. The backtest results validate the practical utility of this approach, highlighting its potential for real-world financial applications. Despite these positive outcomes, the study's assumptions, such as constant volatility and risk-free interest rates, suggest that further improvements are needed to better reflect dynamic market conditions. Future research should focus on applying this approach to a broader population and incorporating real-time data to improve the accuracy and robustness of the model. Expanding the model to include a wider range of financial instruments and market conditions would also provide deeper insights and greater applicability.

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