Topography-based modeling: TOPMODEL

Surface saturation in convergence areas, Rietholzbach

WAGENINGEN UNIVERSITY & RESEARCH

Catchment hydrology (HWM 32806)
Course schedule 2017/2018

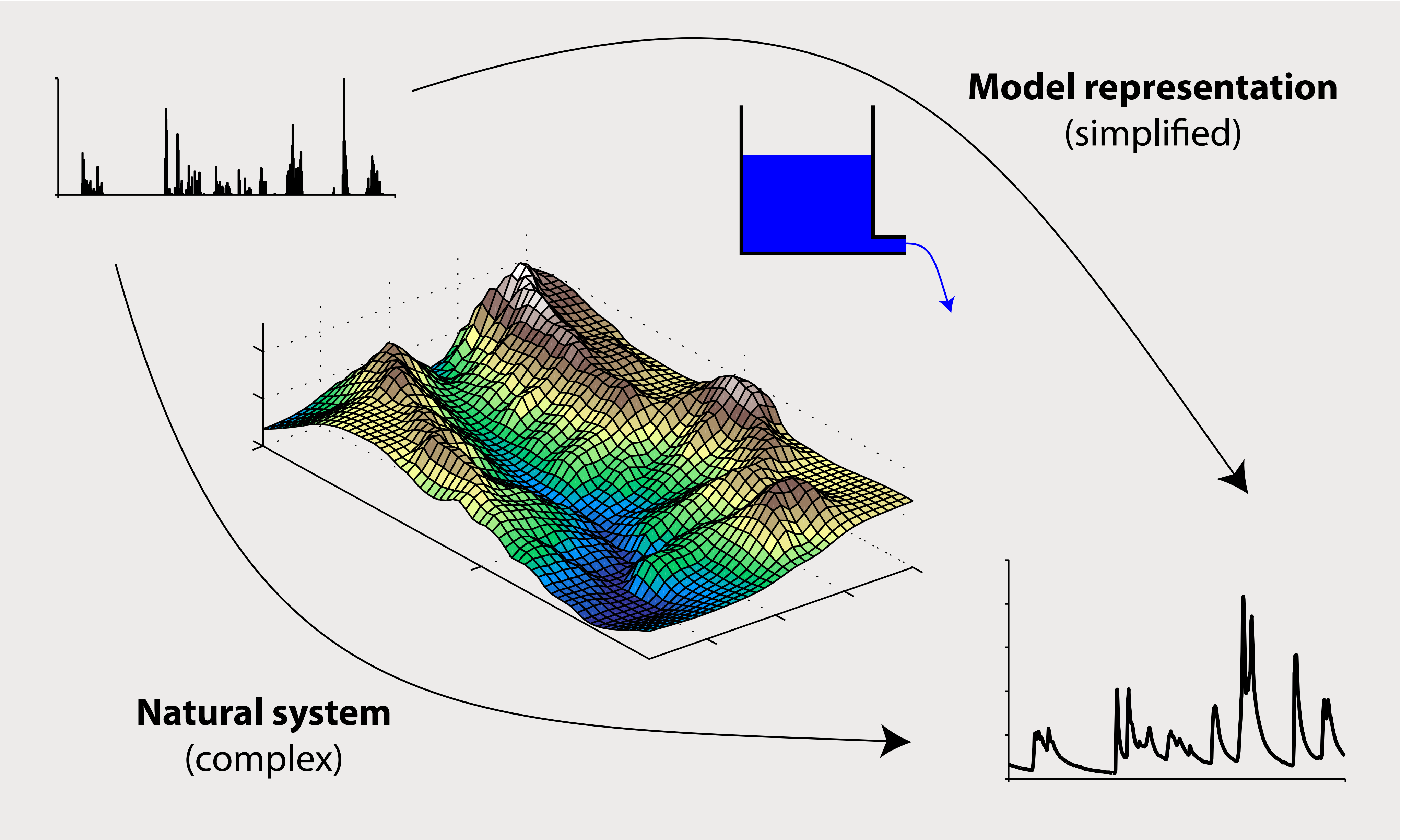
Date	Time		Room	Module: Topic	Lecturer(s)	Study material
Week 29						
Tue 20/3	8^{30}-10^{15}	L	C0092	1: Introduction to modeling concepts and the water balance	Ryan Teuling	Krause et al. (2005), AiG Schaefli & Gupta (2007), HP Gupta et al. (2009), JoH
Wed 21/3	10^{30}-12^{15}	L	P0631	2: Soil moisture	Ryan Teuling	Teuling et al. (2007), GRL
Thu 22/3	8^{30}-10^{15}	L	P0631	3: Evapotranspiration	Ryan Teuling	Zhang et al. (2001), WRR
Fri 23/3	8^{30}-12^{15}	P	PC0012 PC0015	2: Stochastic soil moisture and water balance modeling (ClimHydroTool_1.0)	Ryan Teuling, Joost Buitink	Laio et al. (2001), AWR
Week 30						
Mon 26/3	8^{30}-10^{15}	L	P0635	4: Runoff	Ryan Teuling	Kirchner (2003), HP Gevaert et al. (2014), HESS
Tue 27/3	8^{30}-10^{15}	L	C0092	5: Hydrological extremes: flash floods	Ryan Teuling, Lieke Melsen	Lobligeois et al. (2014), HESS
Wed 28/3	10^{30}-12^{15}	L	P0631	6: Recession-based modeling	Ryan Teuling	Kirchner (2009), WRR
Thu 29/3	8^{30}-12^{15}	P	PC0014	6: Recession-based modeling	Claudia Brauer, Joost Buitink	
Week 31						
Tue 3/4	8^{30}-10^{15}	L	C0092	7: Lowland hydrology	Claudia Brauer	Brauer et al. (2014), GMD
Wed 4/4	10^{30}-12^{15}	L	P0631	8: Cold region hydrology: snow & ice	Ryan Teuling	Koster et al. (2010), NGEO
Thu 5/4	8^{30}-10^{15}	L	P0631	8: Cold region hydrology: permafrost	Victor Bense	Lyon et al. (2009), HESS
Fri 6/4	8^{30}-12^{15}	P	PC0012 PC0015	7: Lowland hydrology (WALRUS)	Claudia Brauer, Mo Wannasin	
Week 32						
Tue 10/4	8^{30}-10^{15}	L	C0092	9: (Un)certainty in hydrological modeling	Lieke Melsen	Oreskes et al. (1994), SCI
Wed 11/4	10^{30}-12^{15}	L	P0631	9: (Un)certainty in hydrological modeling	Lieke Melsen	Melsen et al. (2016), HESS
Thu 12/4	8^{30}-12^{15}	P	PC0012 PC0014	8: Cold region hydrology (HBV_3.1)	Lieke Melsen, Femke Jansen	Uhlenbrook et al. (1999), HSJ

Week 33						
Tue 17/4	8^{30}-10^{15}	L	C0092	10: Topography-based modeling	Ryan Teuling	Chapter by Beven (see BB)
Wed 18/4	9^{30}-12^{15}	S	Lumen 1/2	11: Current topics in catchment hydrology (Symposium)	Valentijn Pauwels (Monash Univ.) tba (Univ.)	
Thu 19/4	8^{30}-12^{15}	P	PC0012 PC0014	10: Topography-based modeling (TOPMODEL)	Claudia Brauer, Ryan Teuling	
Fri 20/4	8^{30}-10^{15}	L	P0631	12: Flood risk & hydrological predictability	Albrecht Weerts	Yossef et al. (2013), WRR
Week 34						
Tue 24/4	8^{30}-10^{15}	L	C0092	5: Hydrological extremes: drought	Ryan Teuling	Van Loon et al. (2016), HESS
Wed 25/4	10^{30}-12^{15}	L	P0631	13: Synthesis & Assignment introduction	Ryan Teuling	Wrede et al. (2014), HP McDonnell (2003), HP
Thu 26/4	8^{30}-12^{15}	P	PC0012 PC0014	13: Synthesis assignment	Ryan Teuling	
Week 35						
Tue 1/5	8^{30}-10^{15}	L	C0092	14: Introduction to Field excursion Iceland	Ryan Teuling, Roel Dijksma	
	10^{30}-12^{15}		C0092	Question time	Ryan Teuling	
Week 36						
Mon 7/5	8^{30}-11^{30}	E	C1040	Exam (course)		
Week 49/50						
Fri 10/8-Fr 17/8		F	Iceland	14: Field excursion Iceland	Roel Dijksma, Victor Bense, Lieke Melsen, Ryan Teuling, Joost Buitink, Elise van Winden	Excursion guide, Papers for group presentation
To be announced		E		Exam (field excursion)	Roel Dijksma	

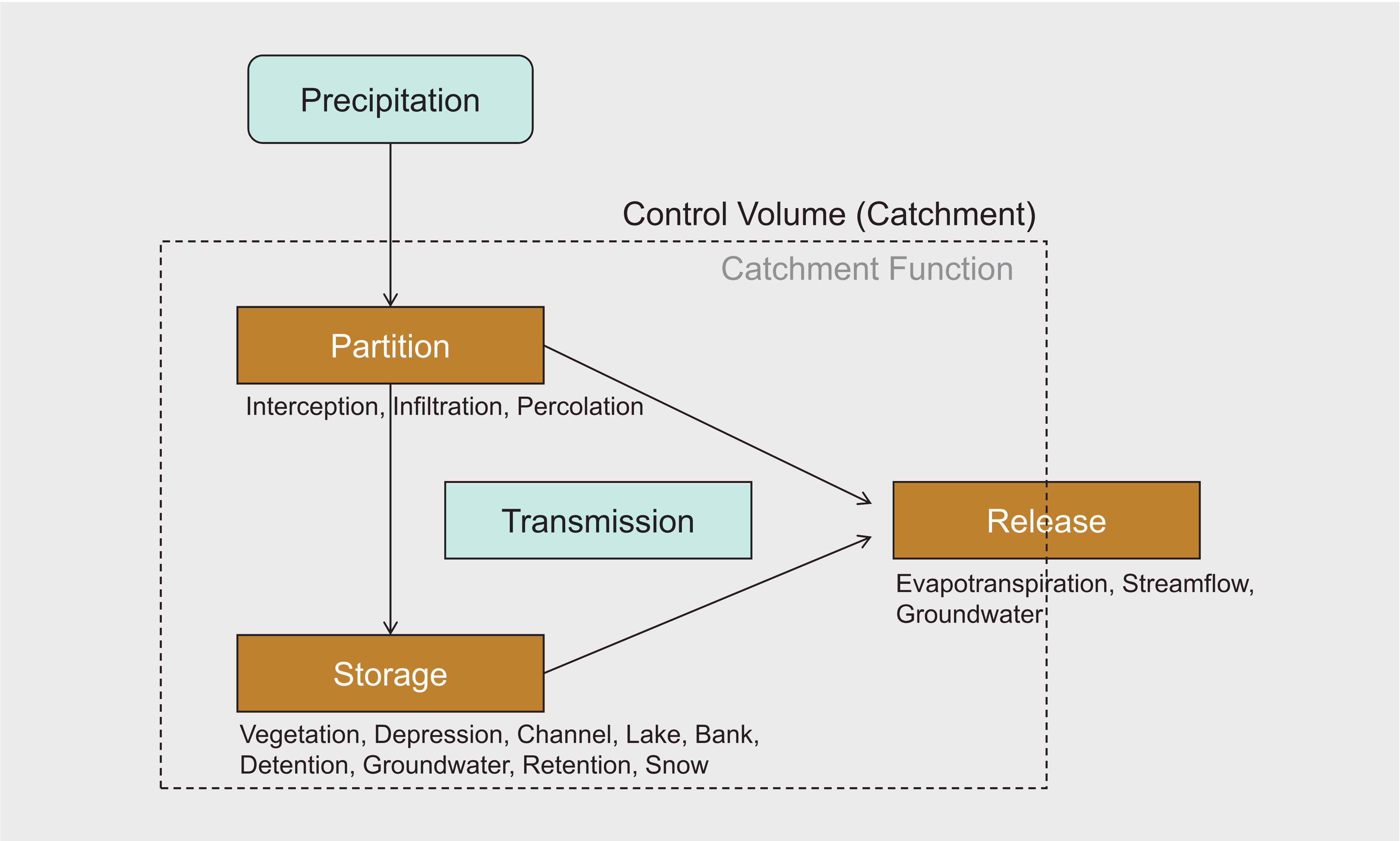
L	Lecture (modeling)
L	Lecture (processes)
P	Computer practical
S	Symposium
F	Field excursion
E	Exam

C0092: Building 107 (Radix)
P0631: Building 102 (Forum)
PC0012: Building 304 (de Valk)
PC0014: Building 304 (de Valk)
PC0015: Building 304 (de Valk)
P0635: Building 102 (Forum)
C1040: Building 102 (Orion)

Catchments as signal filters

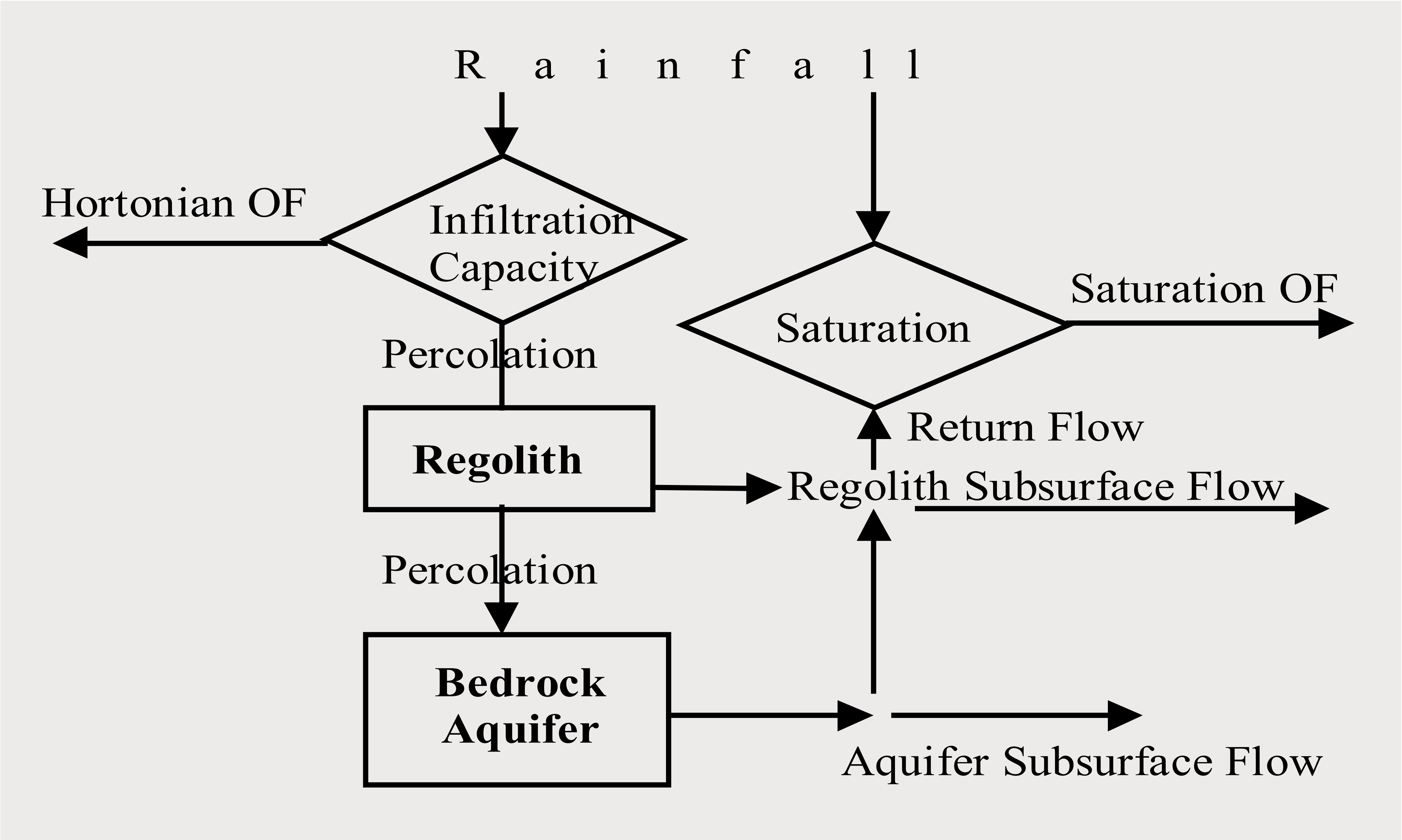


Catchment function



Wagener et al., 2007. Catchment classification and hydrologic similarity. *Geography Compass* **1/4**

Runoff pathways



after McDonnell and Kirkby

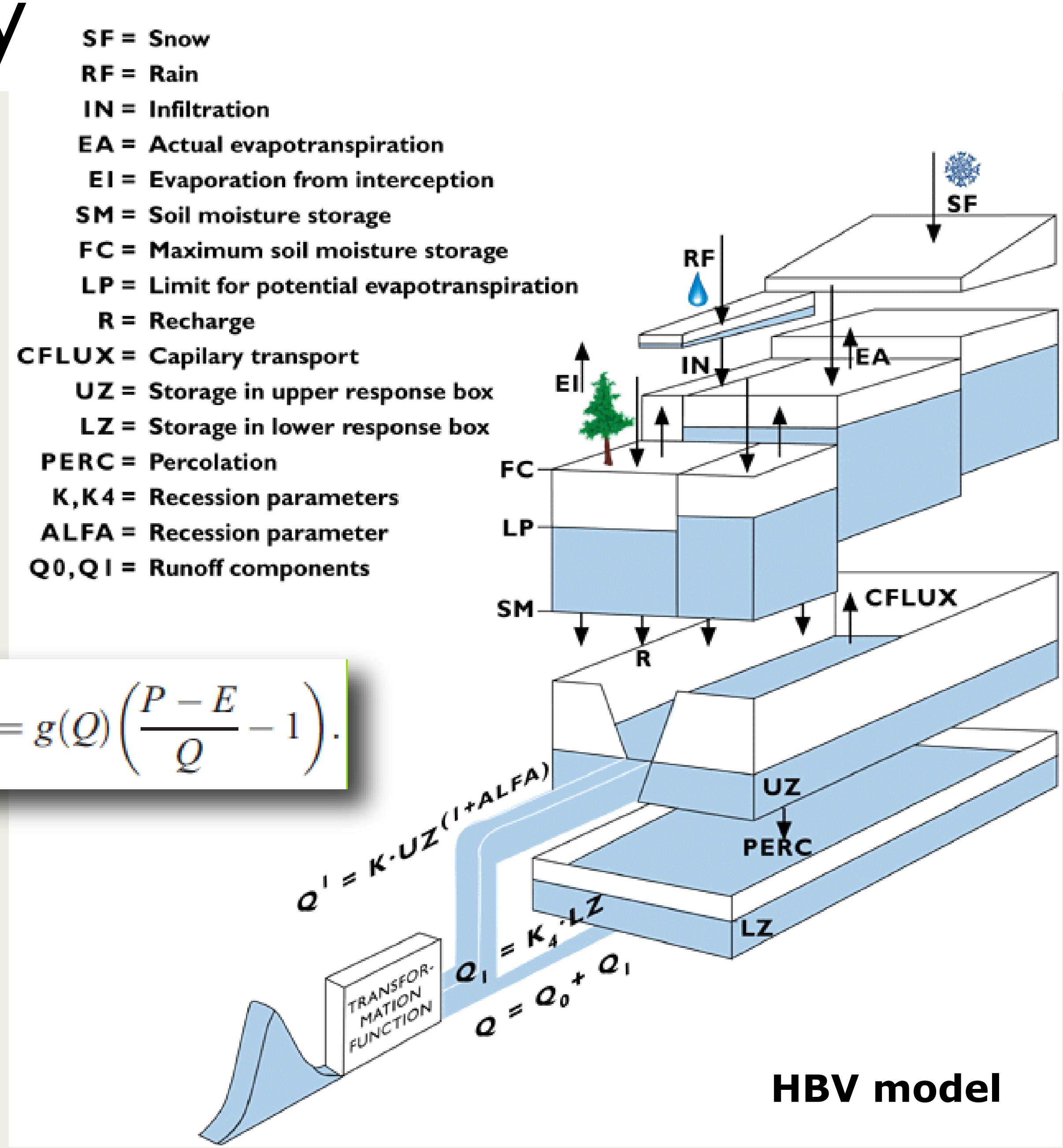
Model complexity

Models to simulate hydrographs can vary in complexity from a single differential equation to models with several interacting stores

Simple dynamical systems approach

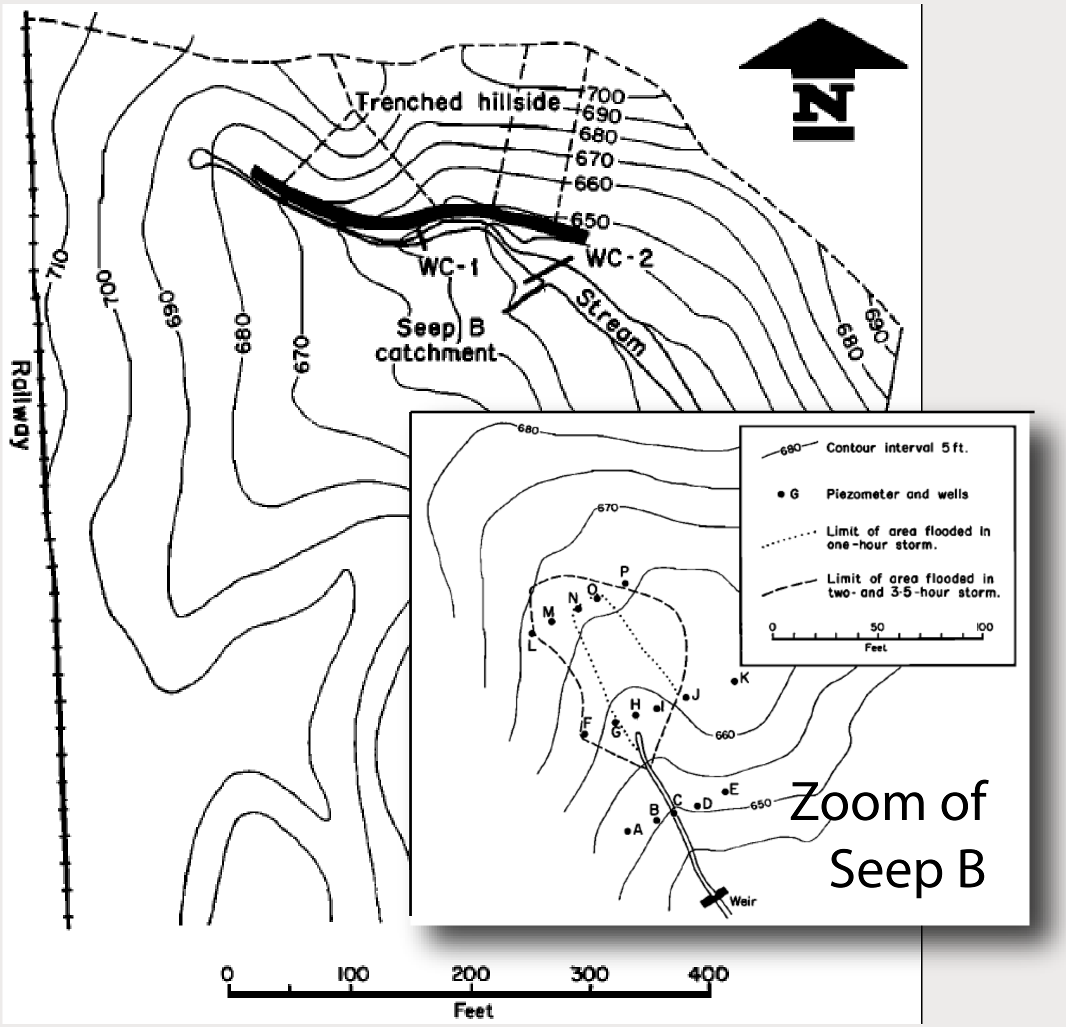
$$\frac{d(\ln(Q))}{dt} = \frac{1}{Q}\frac{dQ}{dt} = \frac{g(Q)}{Q}(P - E - Q) = g(Q)\left(\frac{P - E}{Q} - 1\right).$$

More complex models simulate more processes, but also require more parameters in order to do so.

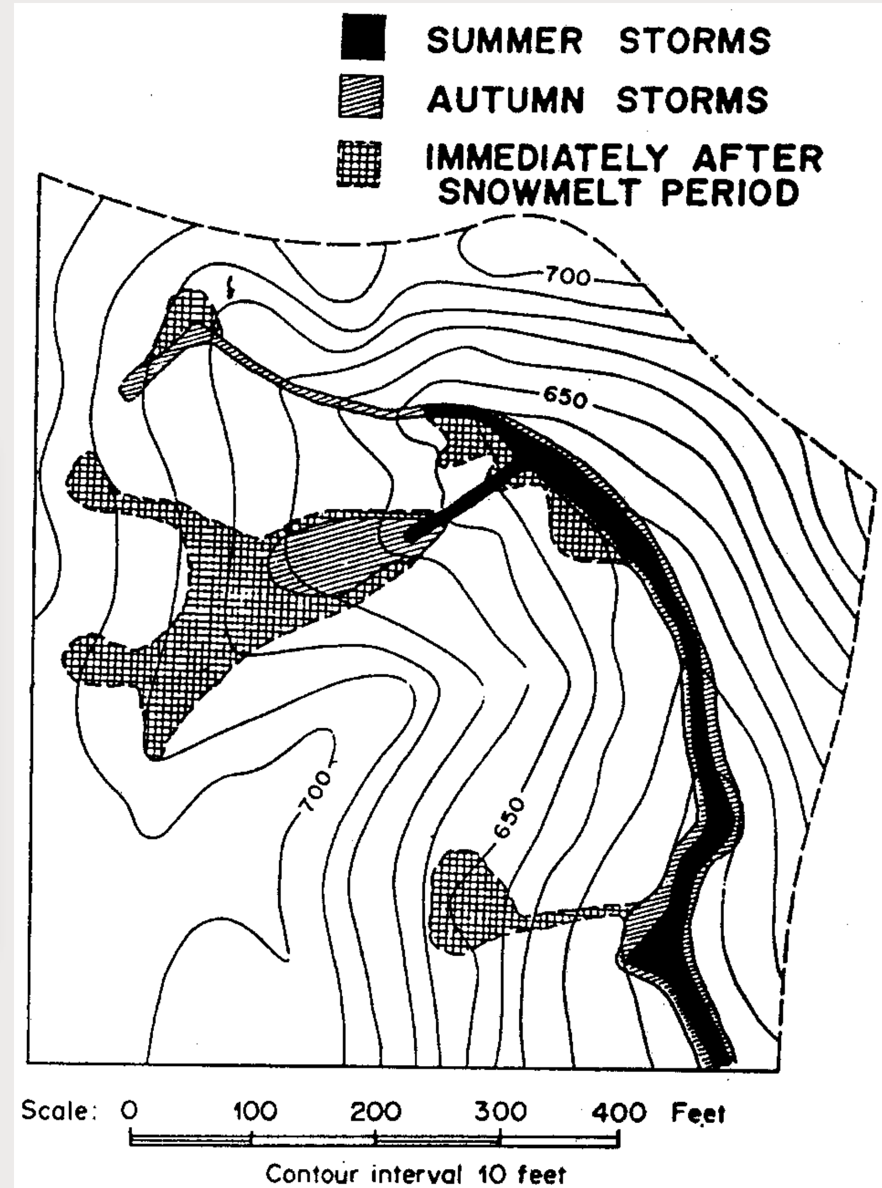


HBV model

Dunne and Black: Variable source area concept



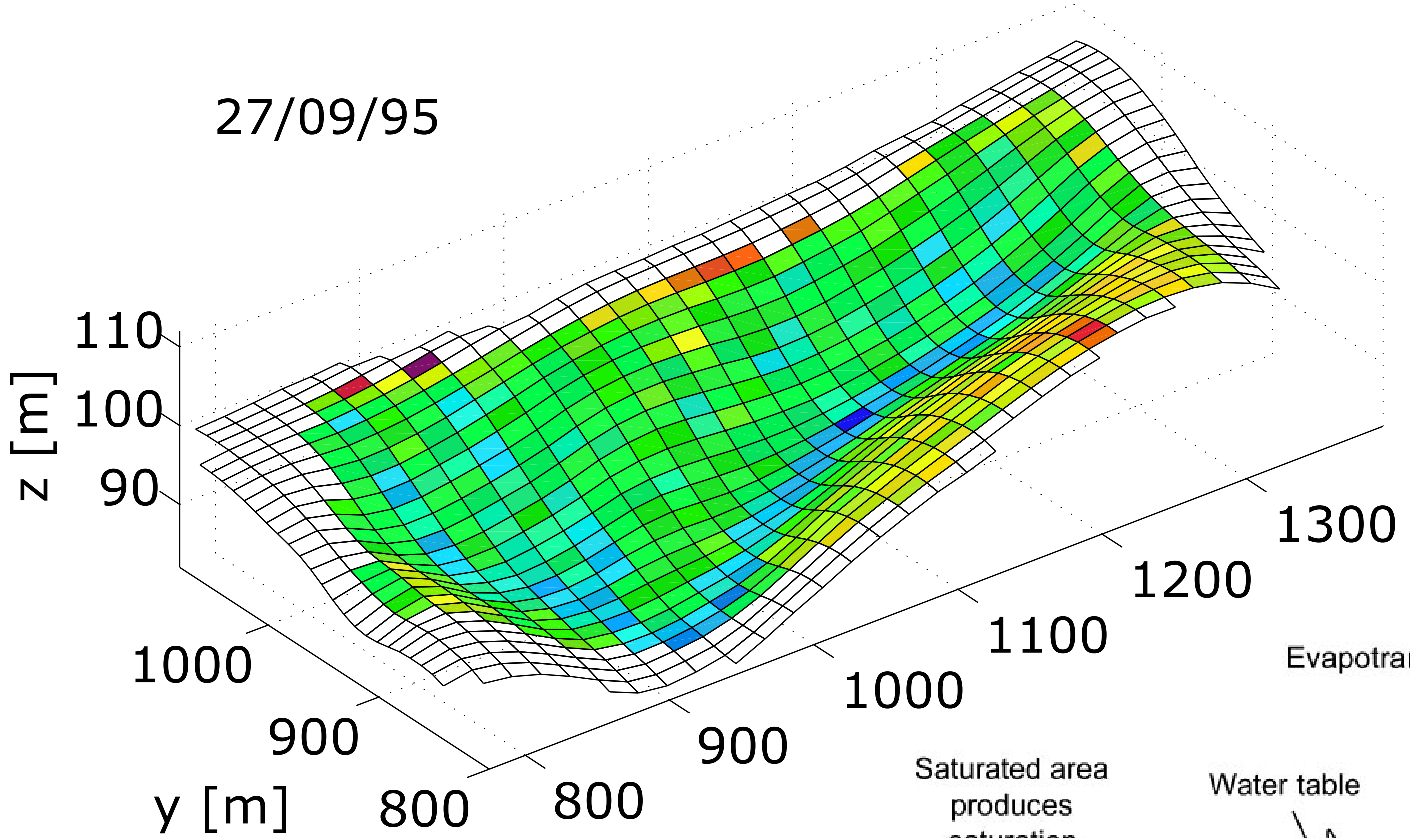
Mapping of areas contributing to direct runoff in experimental basin (Vermont, US)



Dunne & Black, 1970. Partial area contributions to storm runoff in a small New England watershed. *Water Resour. Res.* **6**

Dunne et al., 1975. Recognition and prediction of runoff-producing zones in humid regions. *Hydrol. Sci. Bull.* **3**

Tarrawarra synthesis



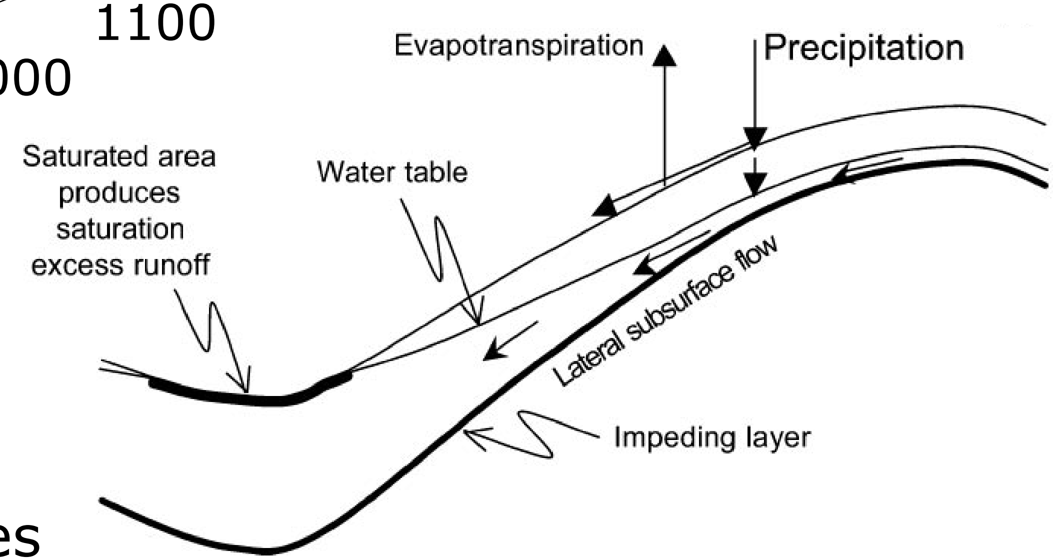
Wet state

Result of nonlocal controls

Reflects terrain/topography

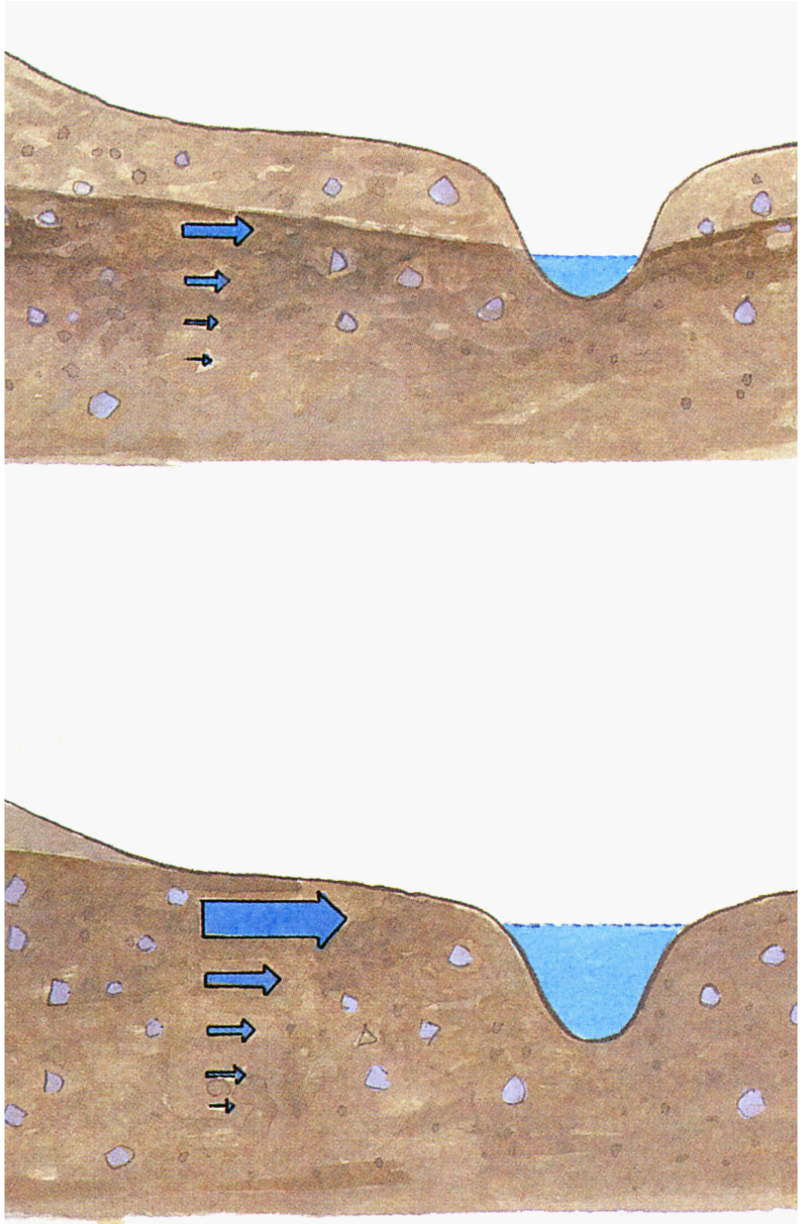
Lateral fluxes dominate

Organization along drainage lines

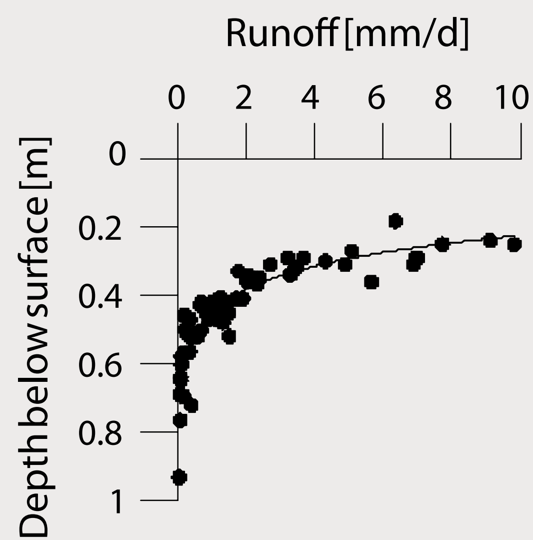


Grayson et al. (1997), *Water Resour. Res.*, **33**

Transmissivity feedback



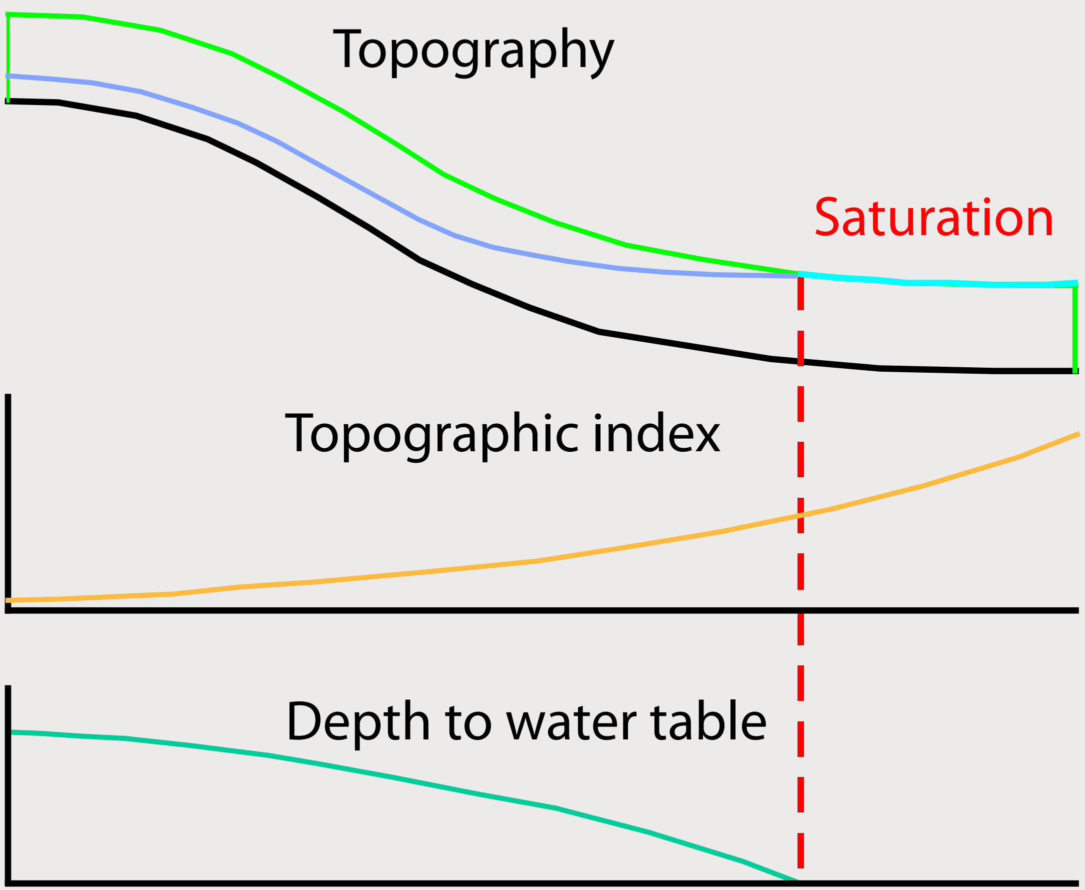
Rapid response of the groundwater outflow to infiltration may be explained by a large increase in the hydraulic conductivity of the soil towards the ground surface. This is called the **transmissivity feedback**.

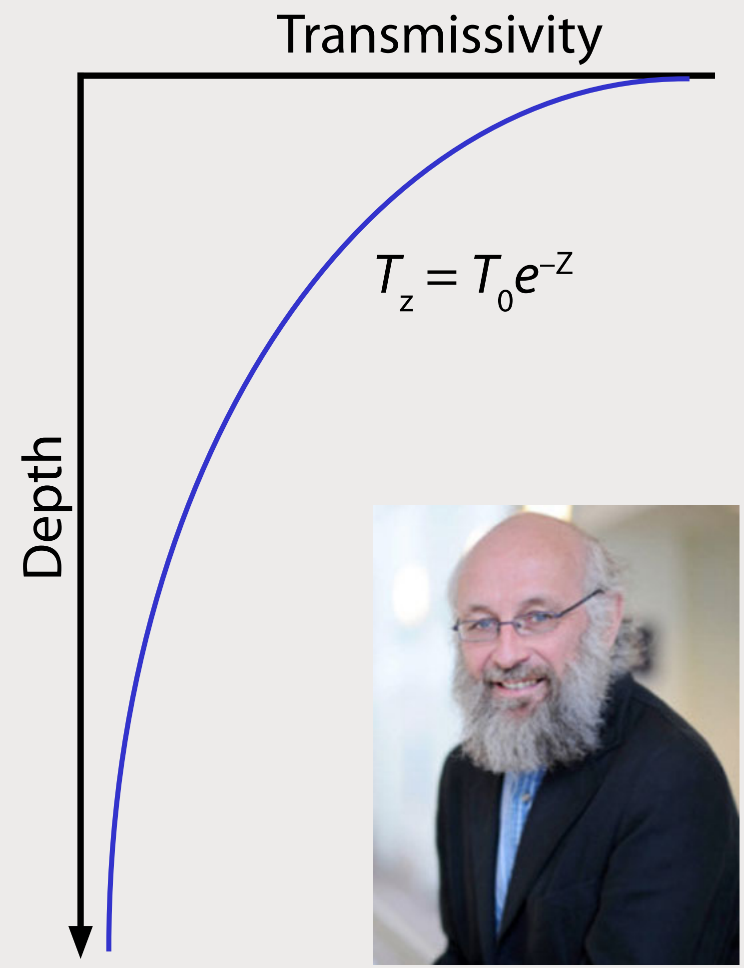


Rodhe, 1989. On the Generation of Stream Runoff in Till Soils. *Nordic Hydrol.* **20**
Bishop et al., 2004. Resolving the Double Paradox of rapidly mobilized old water with highly variable responses in runoff chemistry. *Hydrol. Process*. **18**

Current dogma: TOPMODEL

TOPMODEL predicts saturated subsurface flow based on terrain features (slope and upslope area) that can be derived from a DEM. It captures the transmissivity feed-back and the variable source area concept.





Beven & Kirkby, 1979. A physically based, variable contributing area model of basin hydrology. *Hydrol. Sci. Bull.* **24**
Adapted from: McDonnell

Principle of groundwater flow: Darcy's law

In the mid 1800s, Henry Darcy experimented with flow in tubes and filters of sand. He found that flow velocity is proportional to the hydraulic gradient:

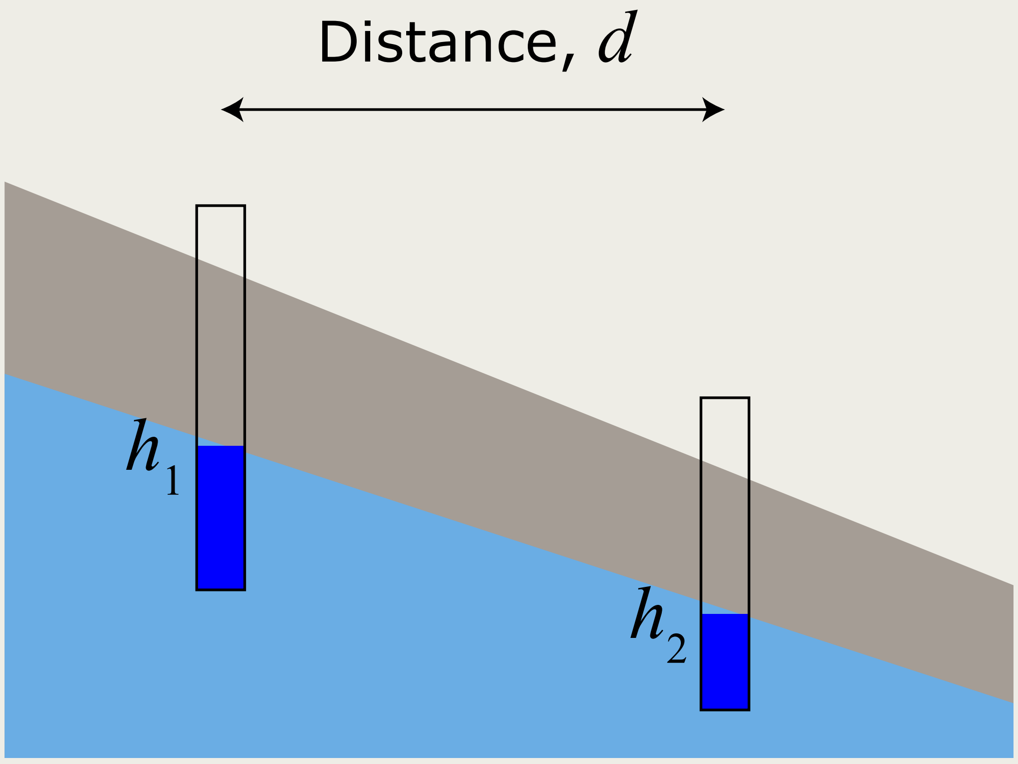
$$v \propto (h_1 - h_2)/d$$

Water also flows faster through coarser material:

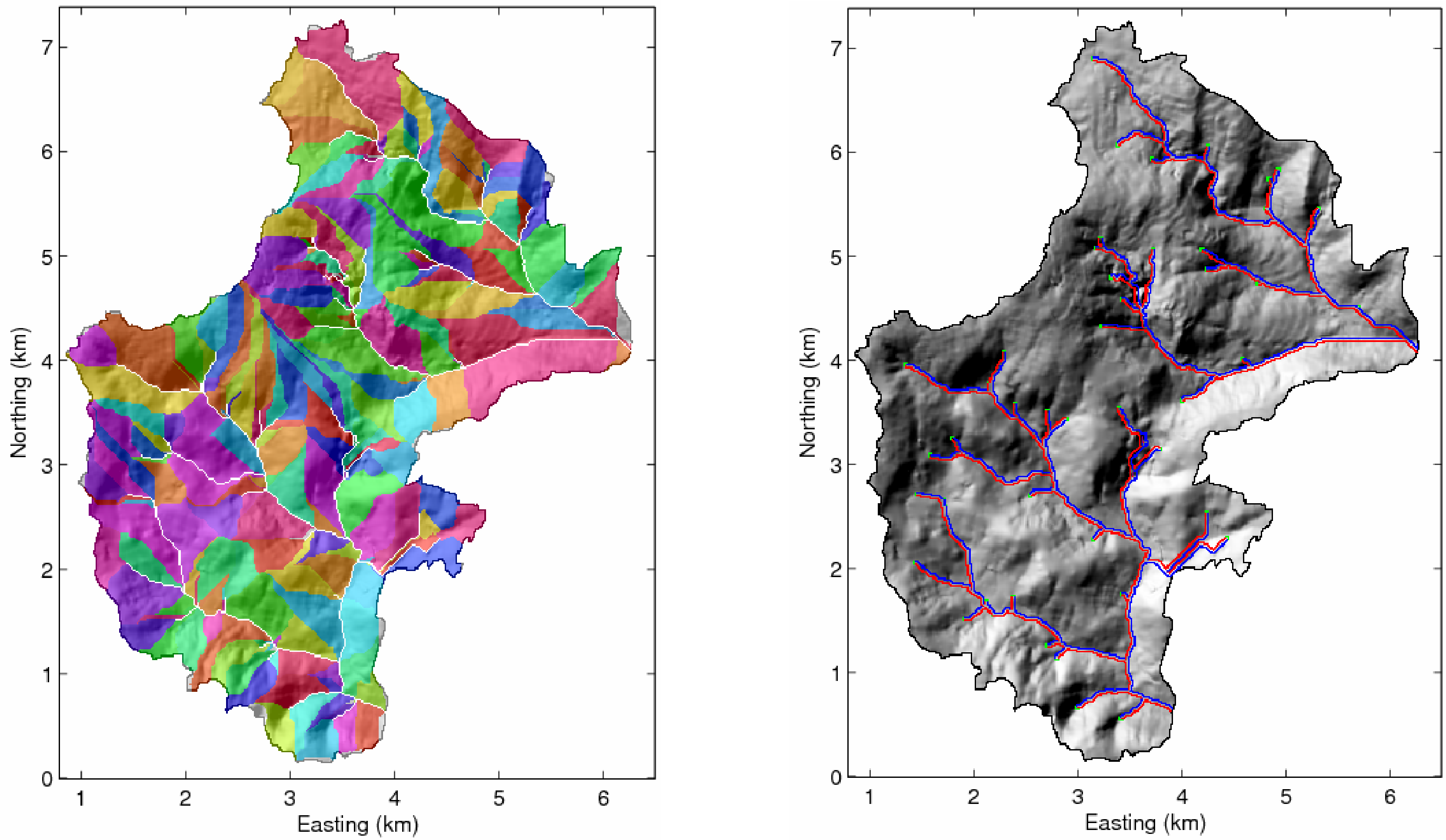
$$v \propto K$$

Combining these gives Darcy's law

$$v = K \times (h_1 - h_2)/d$$



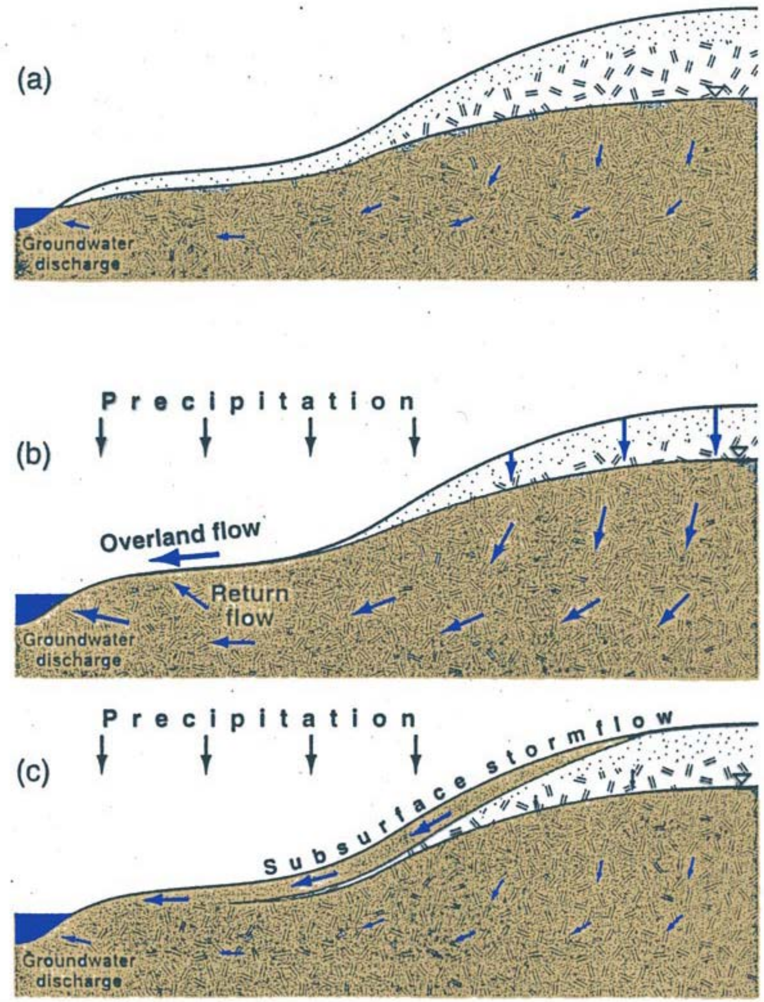
Catchment = hillslopes + channel network



(Bogaart and Troch, 2006)

Runoff processes on hillslopes

- Groundwater discharge
- Overland flow (Horton, Dunne)
- Return flow
- Subsurface storm flow
- Modelling approach: use equations of overland and groundwater flow in hillslopes → TOPMODEL



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Beven & Kirkby (1979), *Hydr.Sci.Bull.*, 24, 43–69

Hydrological Sciences–Bulletin–des Sciences Hydrologiques, **24**, 1, 3/1979

A physically based, variable contributing area model of basin hydrology

K. J. BEVEN Institute of Hydrology, Wallingford, Oxfordshire
M. J. KIRKBY School of Geography, University of Leeds, Leeds, Yorkshire

Received 14 April 1978, revised 17 August 1978

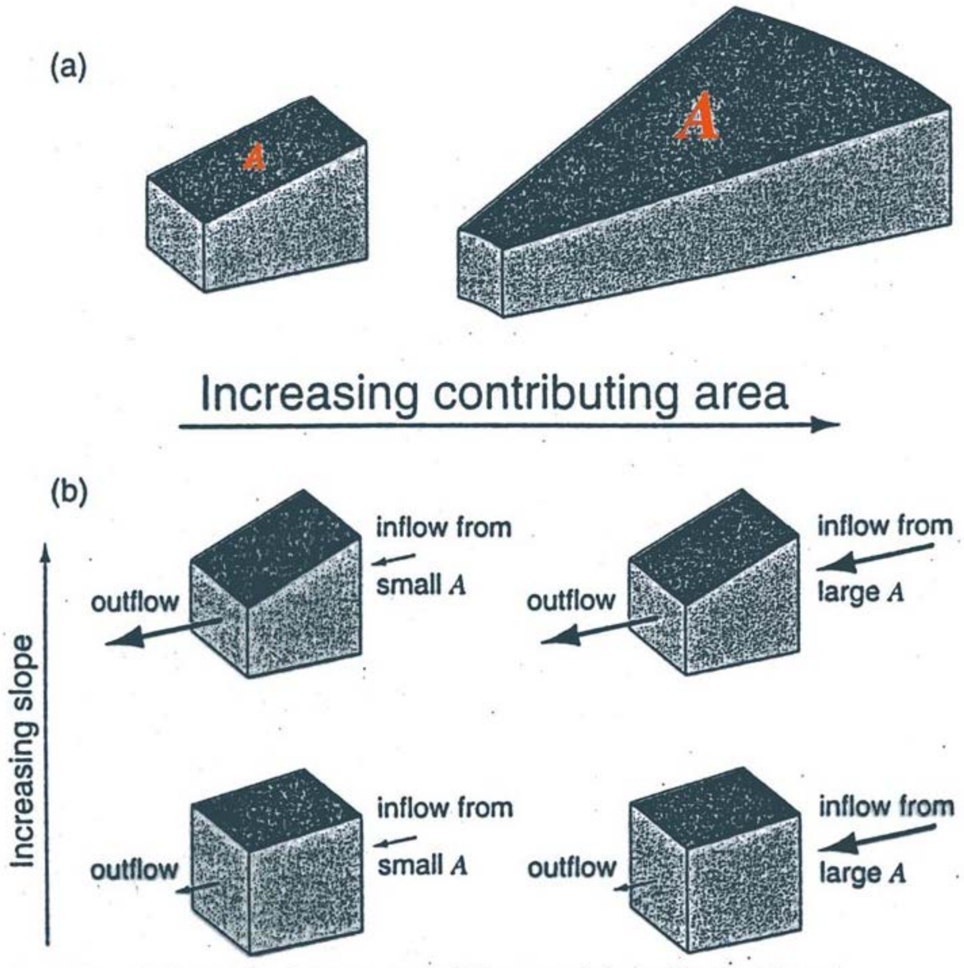
Abstract. A hydrological forecasting model is presented that attempts to combine the important distributed effects of channel network topology and dynamic contributing areas with the advantages of simple lumped parameter basin models. Quick response flow is predicted from a storage/contributing area relationship derived analytically from the topographic structure of a unit within a basin. Average soil water response is represented by a constant leakage infiltration store and an

TOPMODEL (Beven & Kirkby, 1979)

- Basic assumption: topography exerts large influence on subsurface flow in basin
- Topography is most important landscape feature controlling flow
- Extend idea of reservoirs to landscape elements as “building blocks”: Route water through blocks along hillslopes, considering inflow, outflow and local slope
- Use equation of groundwater flow to quantify slow flow and assess saturated areas to quantify overland flow (variable source area) components

Landscape elements in TOPMODEL

- Local slope and contributing area control water balance for catchment block



TOPMODEL – basic ideas

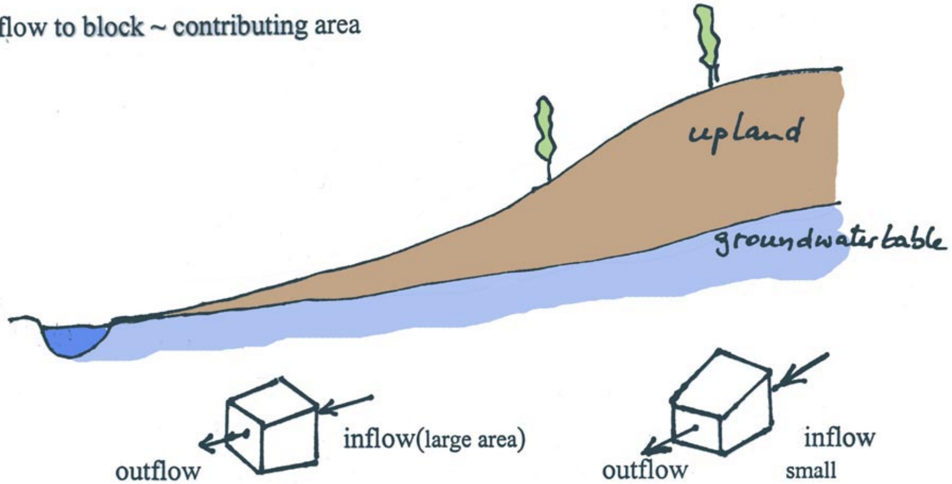
TOPMODEL

-Extend "idea of reservoir " to elements of landscape

-Break up catchments in blocks of given size

-Assign inflow ,outflow and gradient of watertable

-Inflow to block ~ contributing area



inflow> outflow

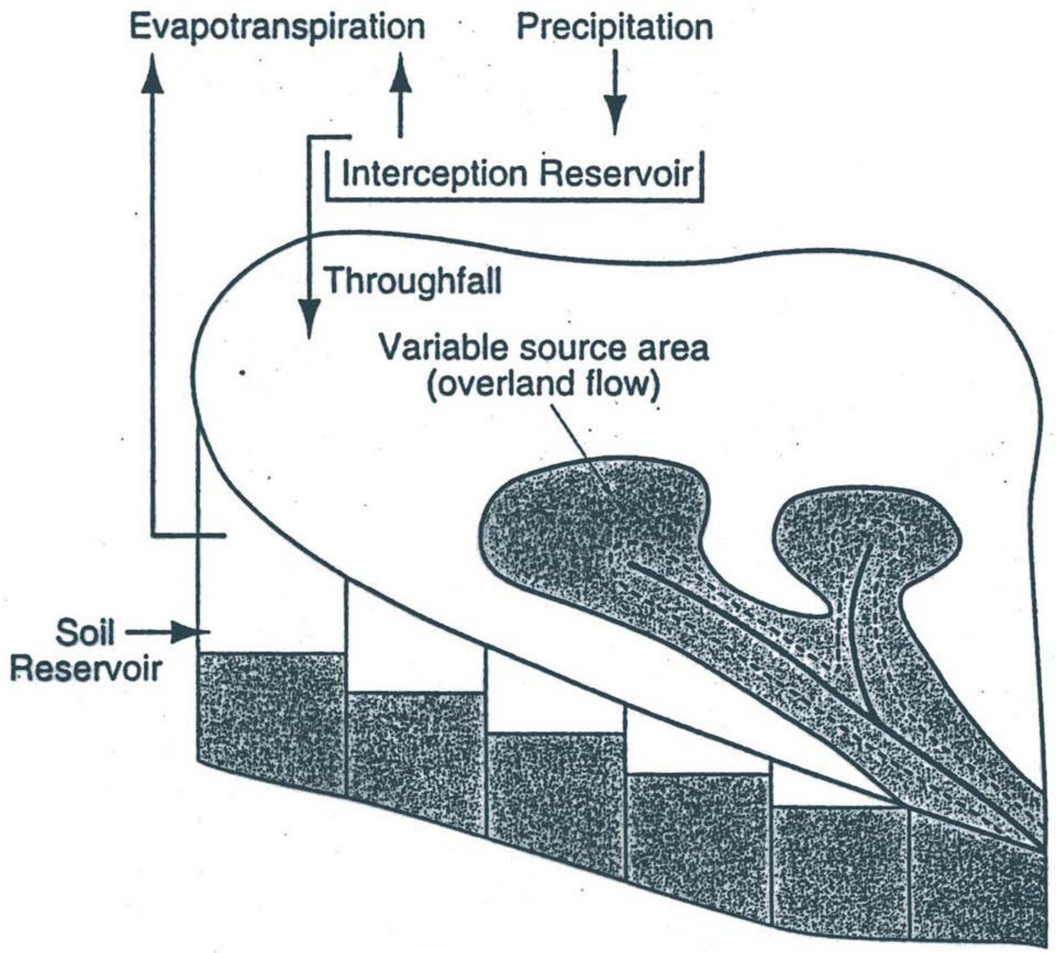
watertable rises
saturation of soil

inflow<outflow

watertable declines

Topography exerts dominant role on flow in upland basins. This is basis Topmodel

TOPMODEL concept



Schematic diagram of the TOPMODEL concept.

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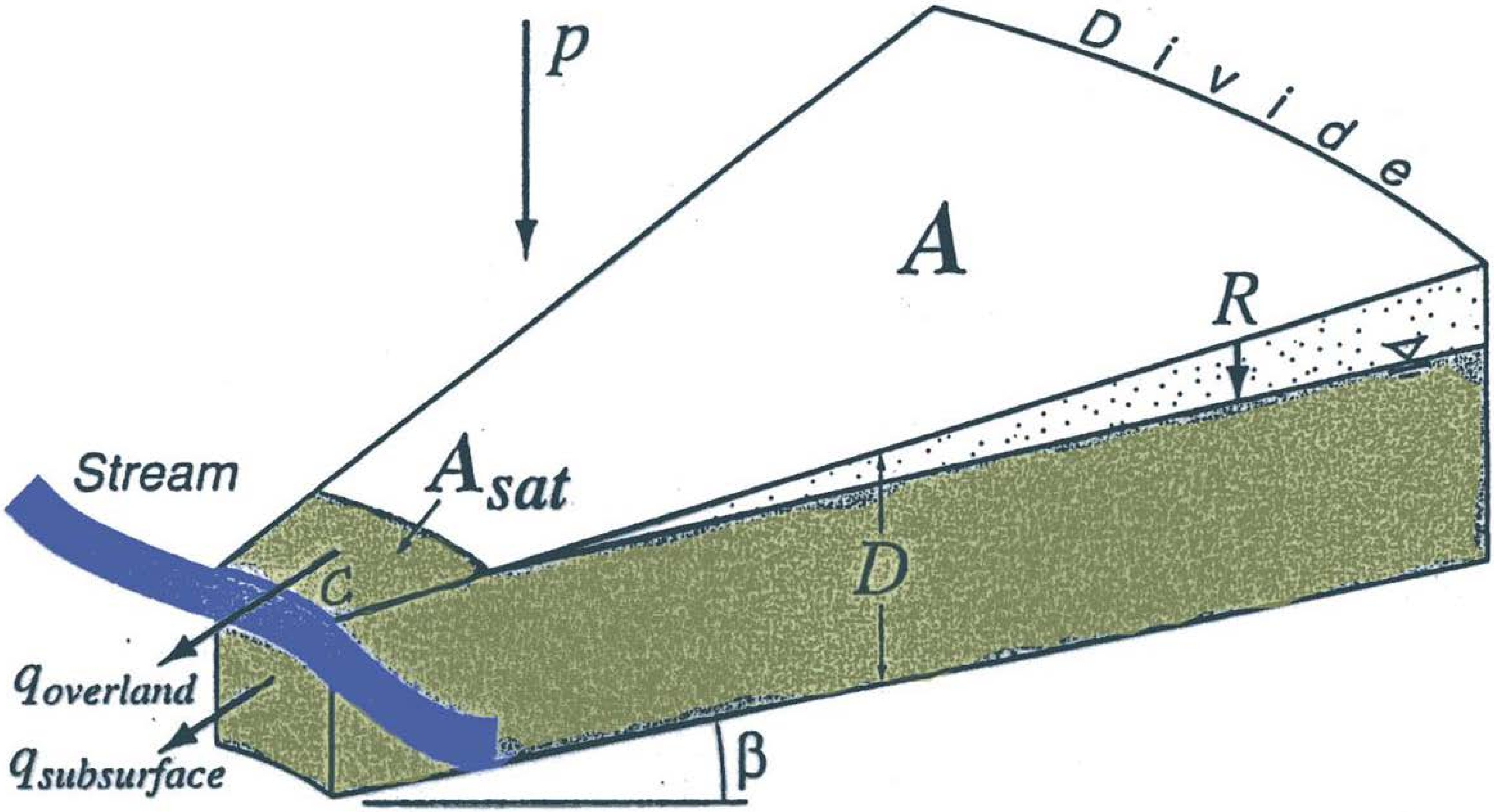
Variable source areas

Basic TOPMODEL equations

- Consider segment from river to divide
- Flow driven by topography
- Variable source areas produce overland flow where saturation occurs
- $q_{total} = q_{overland} + q_{subsurface}$ (per unit area)[L/T]
- $q_{overland} = (A_{sat} / A) \cdot p$, where
  p = precipitation intensity [L/T]
- $Q_{subsurface} = T \cdot c \cdot \tan\beta$ [L^3/T]
- $T = kD$ (transmissivity) [L^2/T], c = contour length [L], $\tan\beta$ = slope water table ≈ slope land surface [–]

Water balance for catchment hillslope segment



Saturated conductivity profile in TOPMODEL

$$Q_{\text{subsurface}} = T \cdot c \cdot \tan\beta, \text{ Darcy's law with } T = kD$$

hydraulic conductivity decreases exponentially with z

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$$k(z) = k_0 e^{-fz} \Rightarrow T(z) = \int_z^D k(\zeta) d\zeta = k_0 \int_z^D e^{-f\zeta} d\zeta =$$

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$$-\frac{k_0}{f}\left[e^{-f\zeta}\right]_z^D = \frac{k_0}{f}\left(e^{-fz} - e^{-fD}\right) \Rightarrow \text{if } D >> z \text{ then}$$

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$$T(z) \approx \frac{k_0}{f} e^{-fz} \Rightarrow \boxed{Q_{\text{subsurface}} =} \frac{k_0}{f} e^{-fz} \cdot c \cdot \tan\beta$$

$$= \boxed{T_{\max} e^{-s/m} \cdot c \cdot \tan\beta} (\text{if } s = z \cdot \varphi \text{ and } m = \varphi / f)$$

Soil water balance in TOPMODEL

- TOPMODEL accounts for soil water balance by computing saturation deficit s, being the volume (mm) of water to be percolated to the ground water to rise the water table to soil surface
- Those grid cells with s = 0 produce (saturation excess) overland flow
- Since saturated areas determine saturation excess overland flow, it makes sense to compute saturation deficit s instead of water table depth z

Great 'leap' of TOPMODEL – steady state situation

$$\boxed{R \cdot A = Q_{\text{subsurface}} = T_{\max} e^{-s/m} \cdot c \cdot \tan\beta,} \text{ where:}$$

$$R = \text{recharge of or percolation to groundwater [L/T]}$$

$$A = \text{upstream basin (hillslope slice) area [L}^2\text{]}$$

$$Q_{\text{subsurface}} = \text{subsurface flow [L}^3\text{/T]}$$

$$T_{\max} = \text{maximum transmissivity [L}^2\text{/T]}$$

$$s = \text{saturation deficit (to be calculated) [L]}$$

$$m = e\text{-folding parameter of transmissivity profile [L]}$$

$$c = \text{contour length [L]}$$

$$\tan\beta = \text{surface slope [–]}$$

Saturation deficit and resulting subsurface flux

$$Q_{\text{subsurface}} = R \cdot A = T_{\max} e^{-s/m} \cdot c \cdot \tan \beta \Leftrightarrow$$

Saturation deficit and resulting subsurface flux

$$Q_{\text{subsurface}} = R \cdot A = T_{\max} e^{-s/m} \cdot c \cdot \tan\beta \Leftrightarrow$$

$$s = -m \ln\left(\frac{R}{T_{\max}}\right) - m \ln\left(\frac{a}{\tan\beta}\right), \text{ with } a = \frac{A}{c} \Rightarrow \text{average:}$$

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$$\bar{s} = -m \ln\left(\frac{R}{T_{\max}}\right) - m\bar{\lambda} \qquad (\bar{\lambda} = \text{mean topographic index})$$

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$$\Rightarrow \boxed{s = \bar{s} + m\left[\bar{\lambda} - \ln\left(\frac{a}{\tan\beta}\right)\right]} \qquad \text{(saturation deficit)}$$

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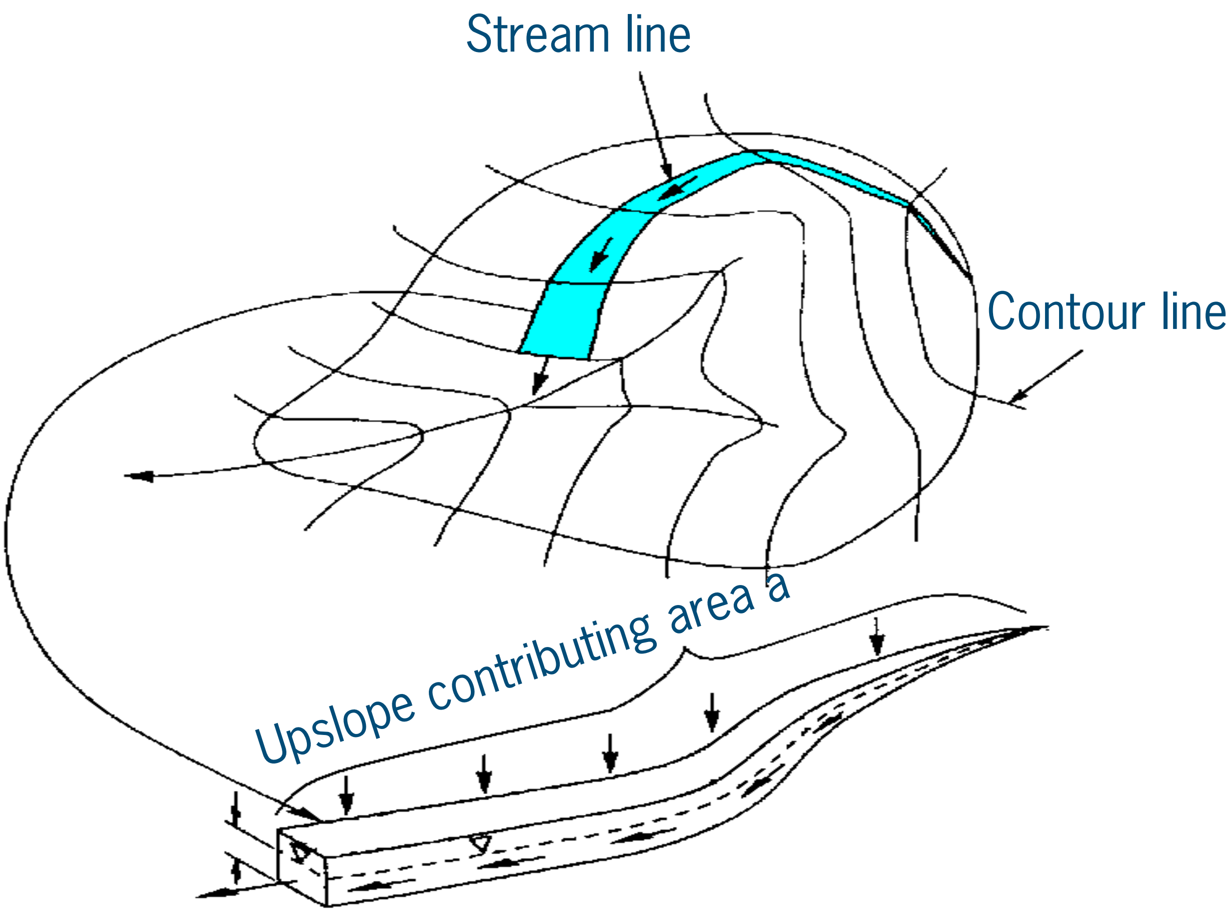
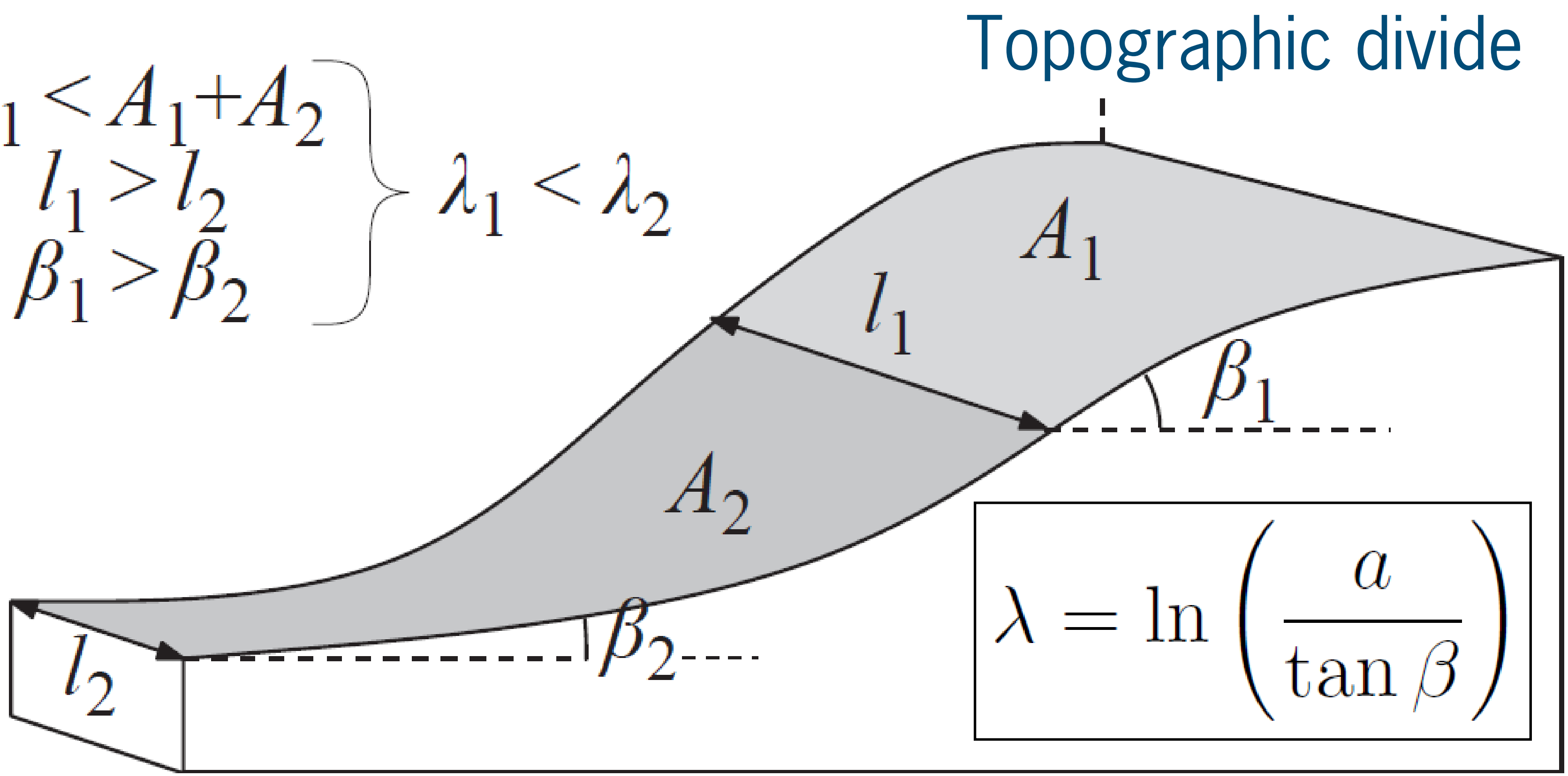
$$\boxed{q_{\text{subsurface}} = T_{\max} e^{-s/m} e^{-\lambda}} \qquad \text{(subsurface flux per unit area)}$$

Contributing area per unit contour length



Saturation excess overland flow

$$\left.\begin{array}{c} A_1 < A_1 + A_2 \\ l_1 > l_2 \\ \beta_1 > \beta_2 \end{array}\right\} \lambda_1 < \lambda_2$$



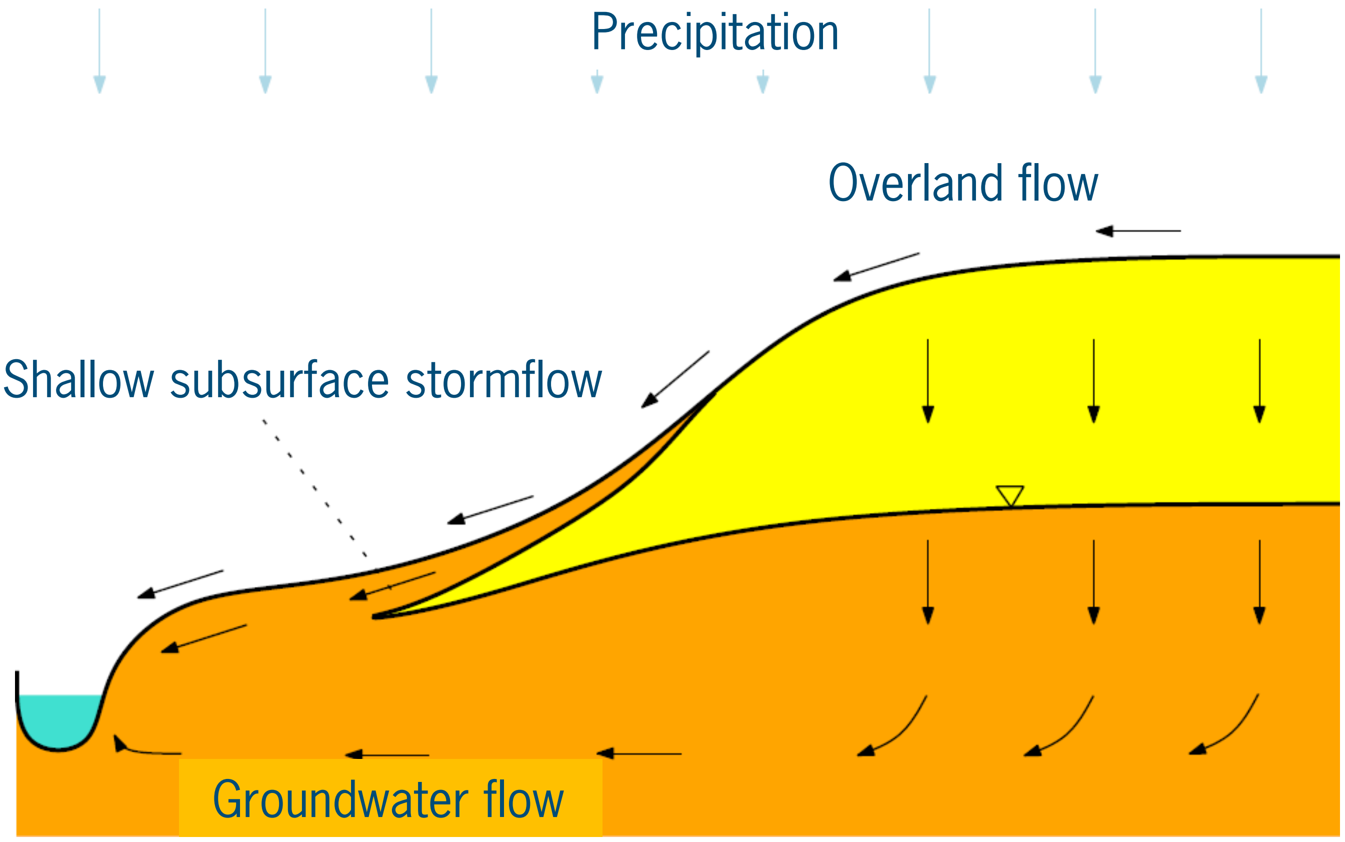
$$\lambda = \ln\left(\frac{a}{\tan\beta}\right)$$

- Topographic index

Saturation excess overland flow

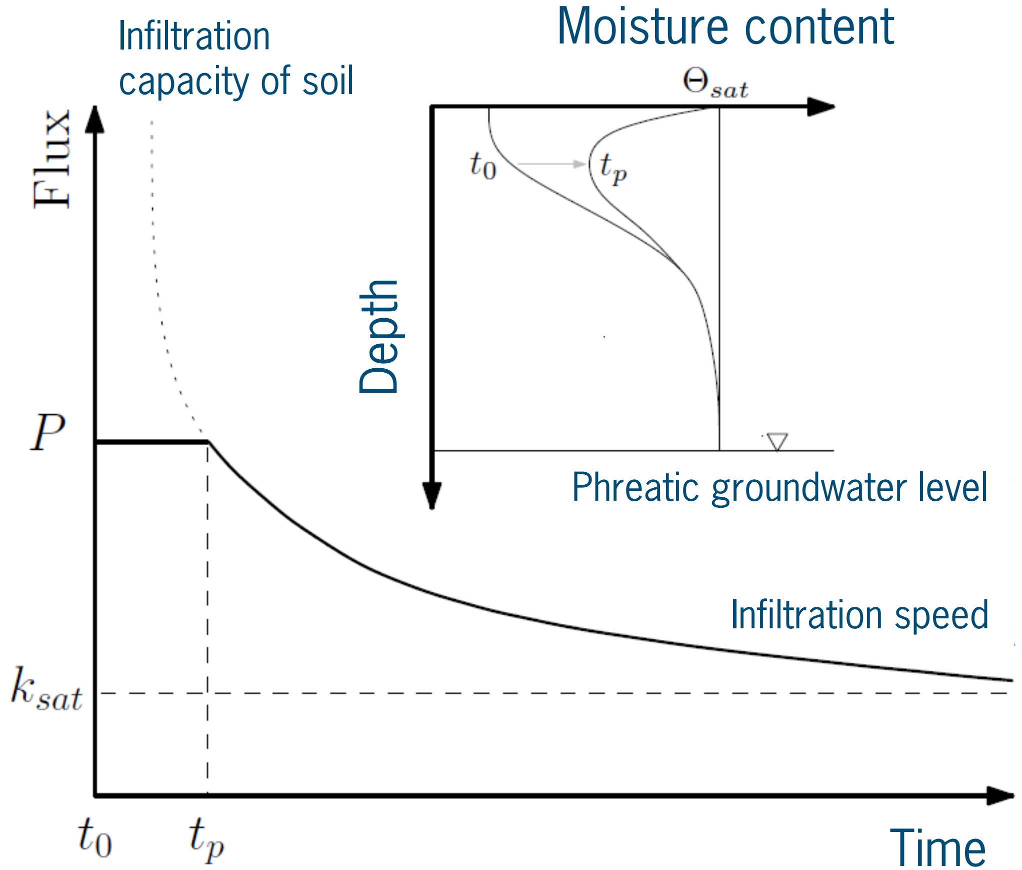
- Obtain digital elevation model (DEM) of basin
- Compute topographic index λ of each grid cell
- Where λ is large, saturation and associated overland flow can be expected
- Saturation deficit s is computed, which yields saturated area A_s
- $q_{overland} = (A_s / A) \cdot p$ [L/T]

Overland flow – infiltration excess (1)



- Horton's principle

Overland flow – infiltration excess (2)

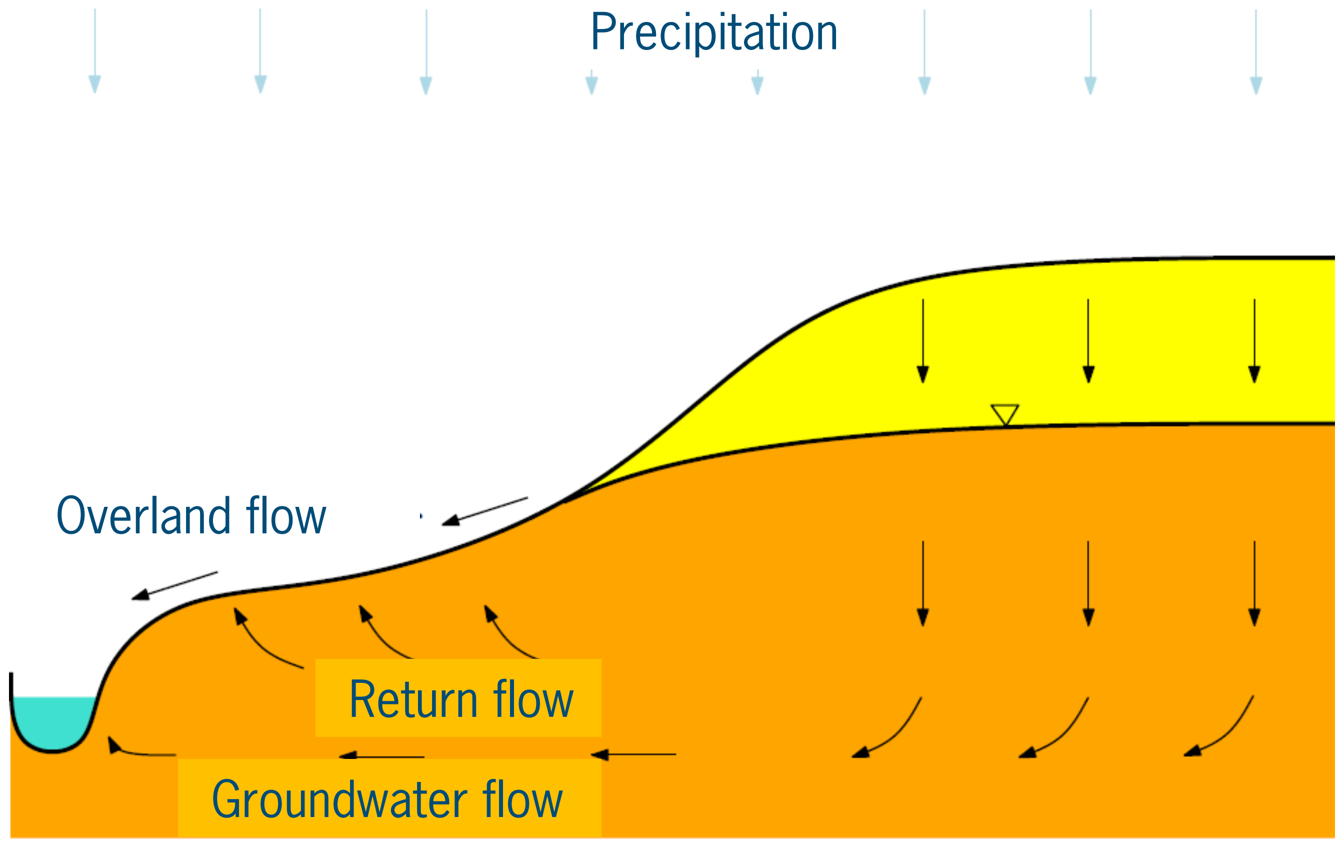


- Horton's principle

Conditions for Hortonian overland flow

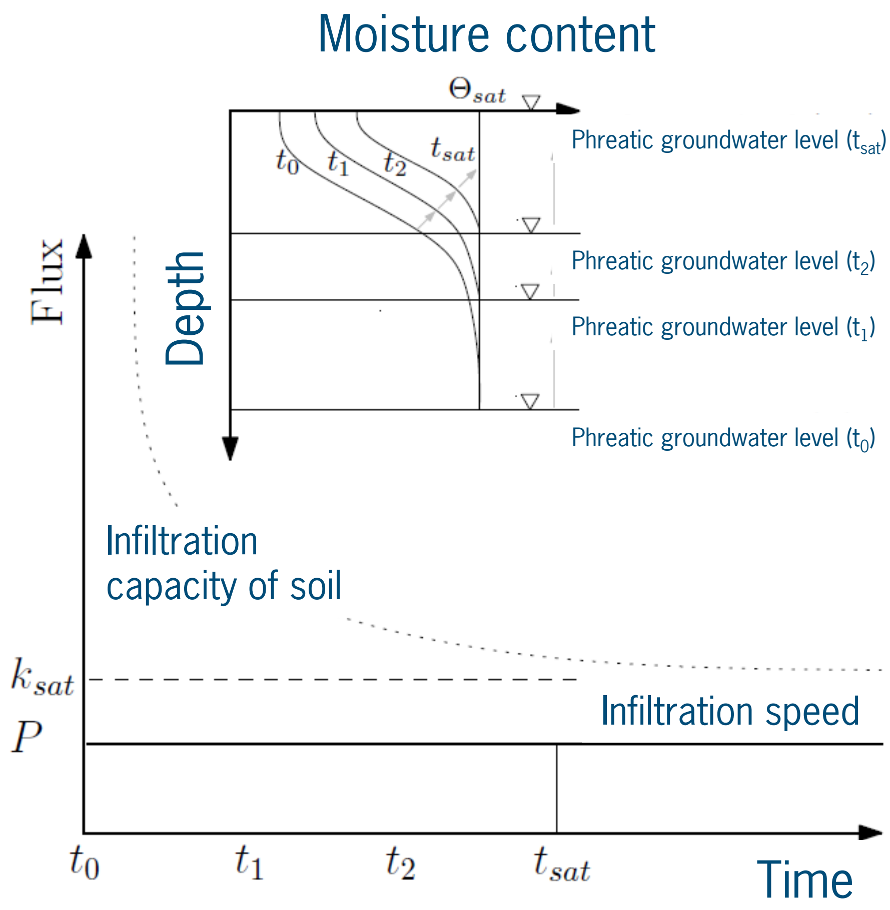
- Rainfall intensity > saturated hydraulic conductivity
- Duration of rainfall event > time to ponding
- Deep phreatic groundwater level (deep soil)

Overland flow – saturation excess (1)



- Dunne's principle

Overland flow – saturation excess (2)

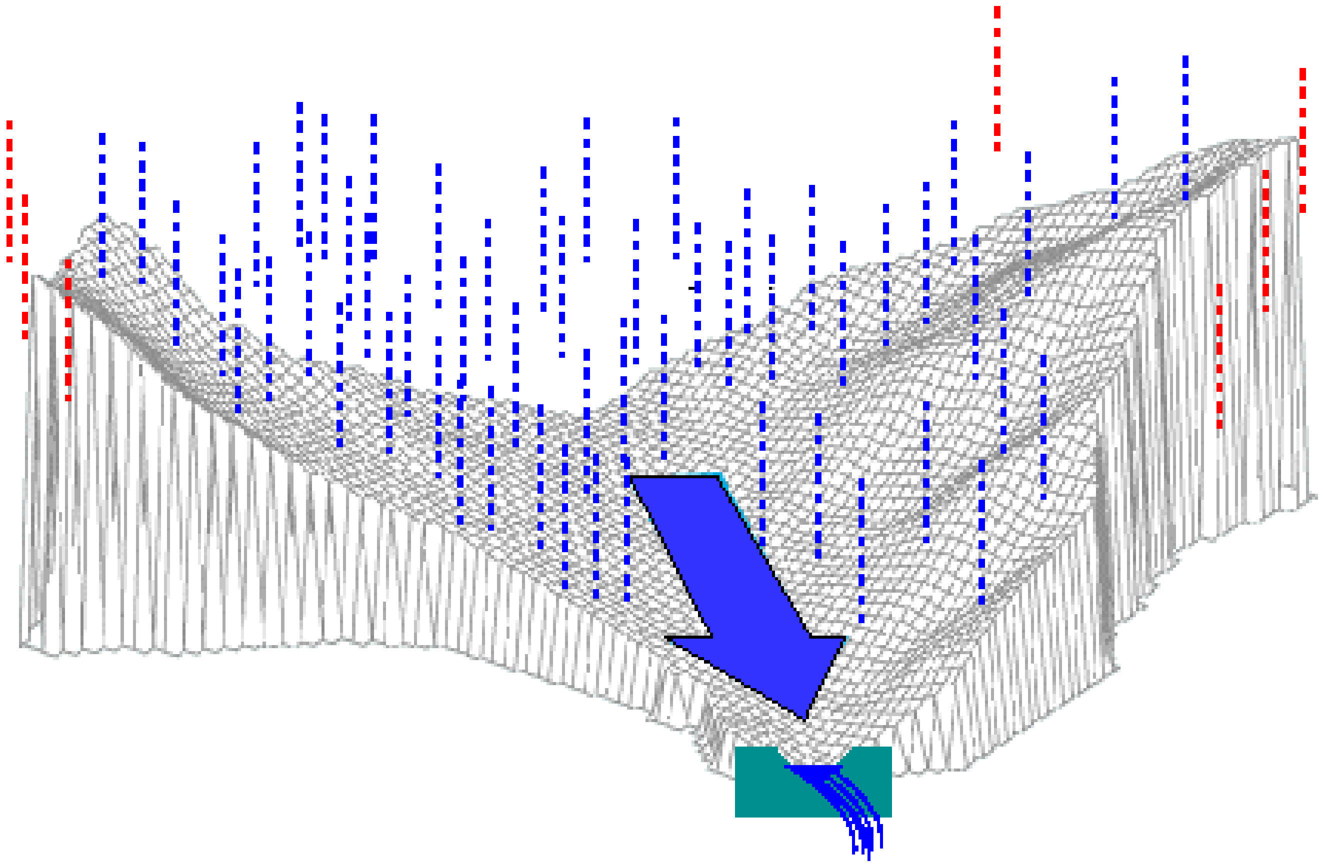


- Dunne's principle

Conditions for Dunnean overland flow

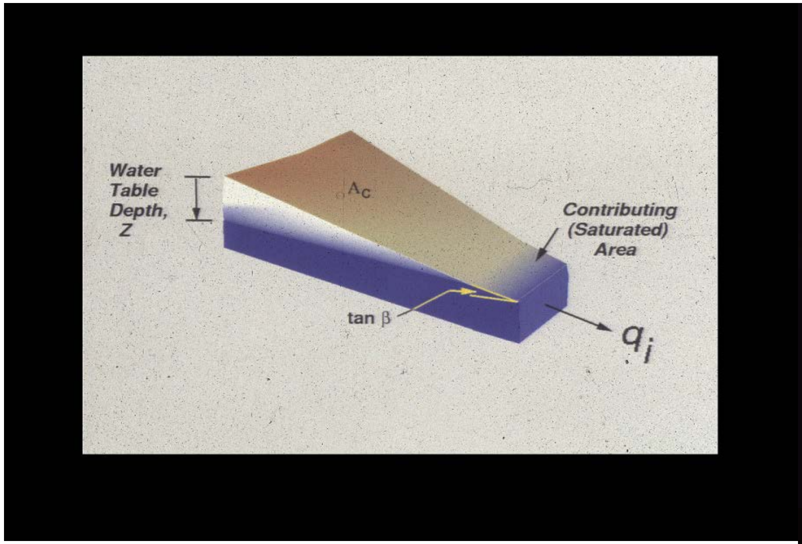
- Saturated hydraulic conductivity > rainfall intensity
- Duration of rainfall event > time to saturation
- Shallow phreatic groundwater level (shallow soil)

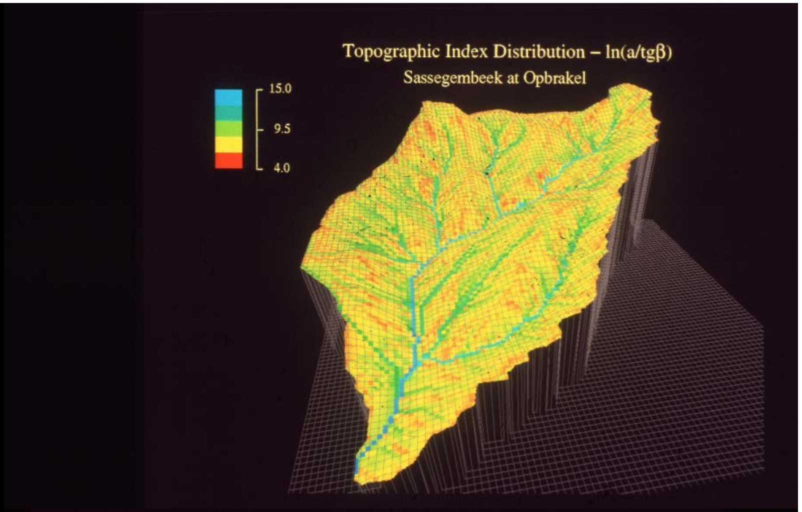
Digital elevation models (DEMs)

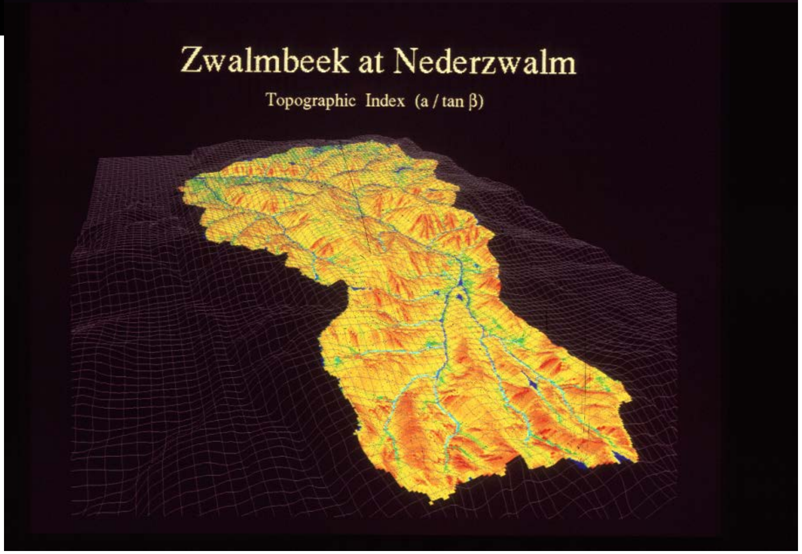


- DEM shows λ-values and associated saturated areas

Topographic index extracted from DEM

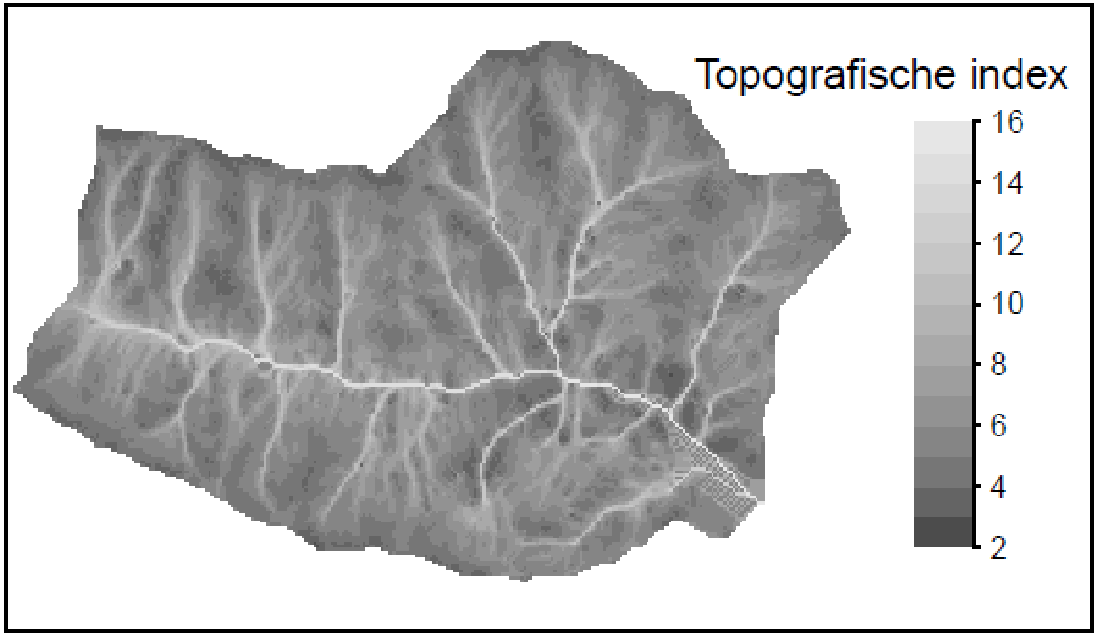


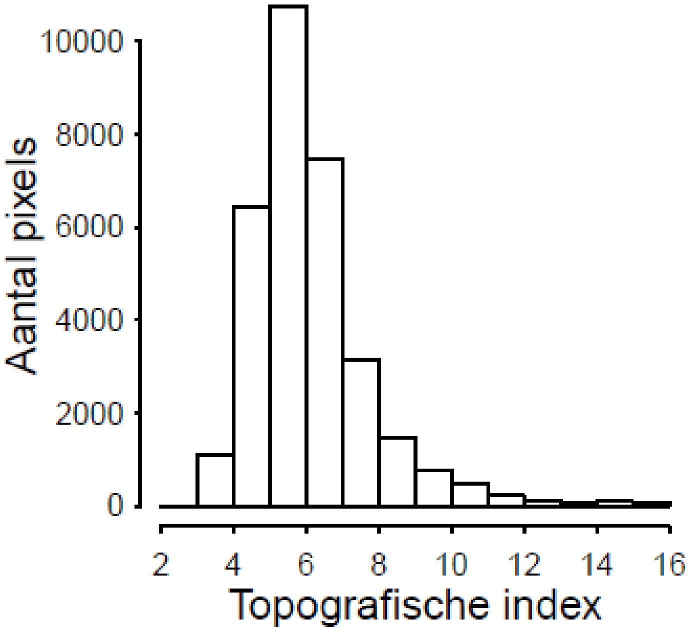




$$\lambda = \ln\left(\frac{a}{\tan\beta}\right)$$

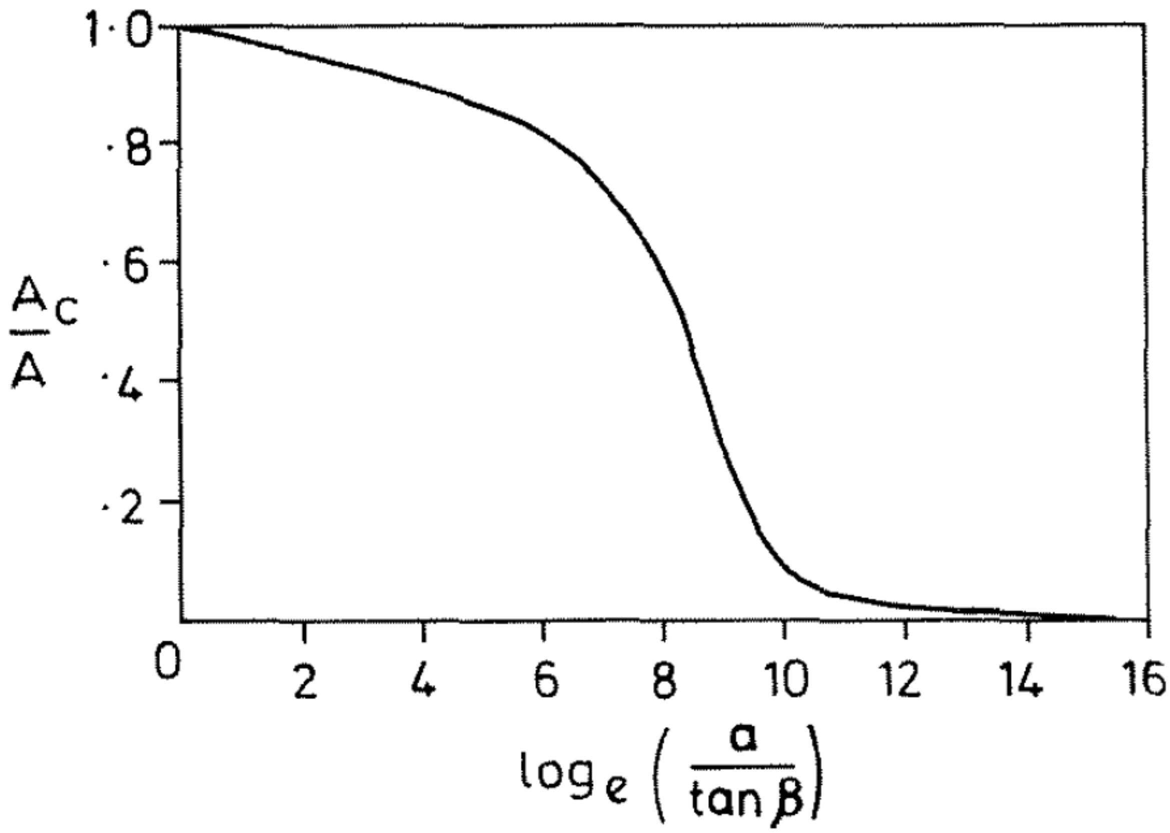
Topographic index distribution





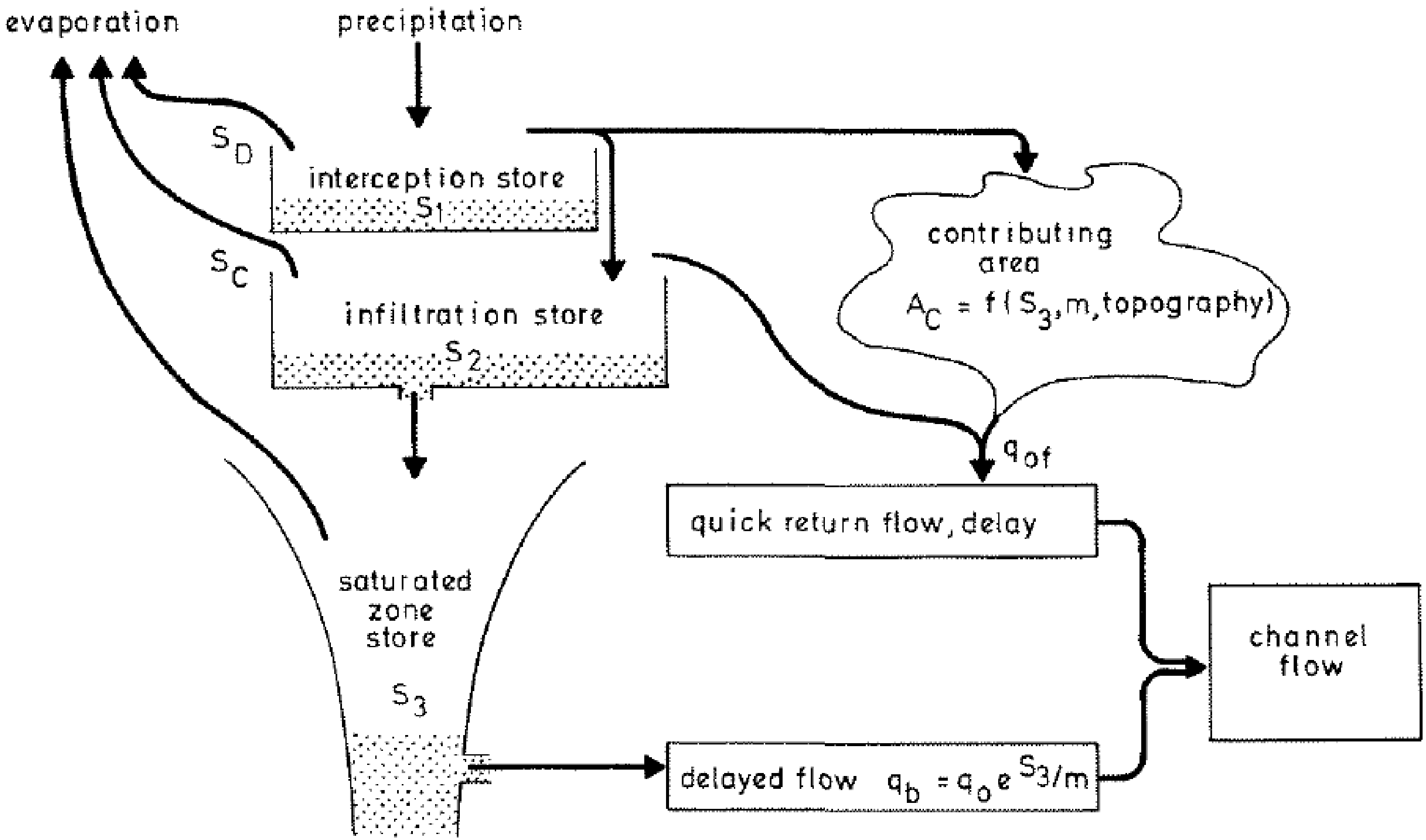
- Rietholzbach catchment, Switzerland (3.4 km^2)

Topographic index distribution

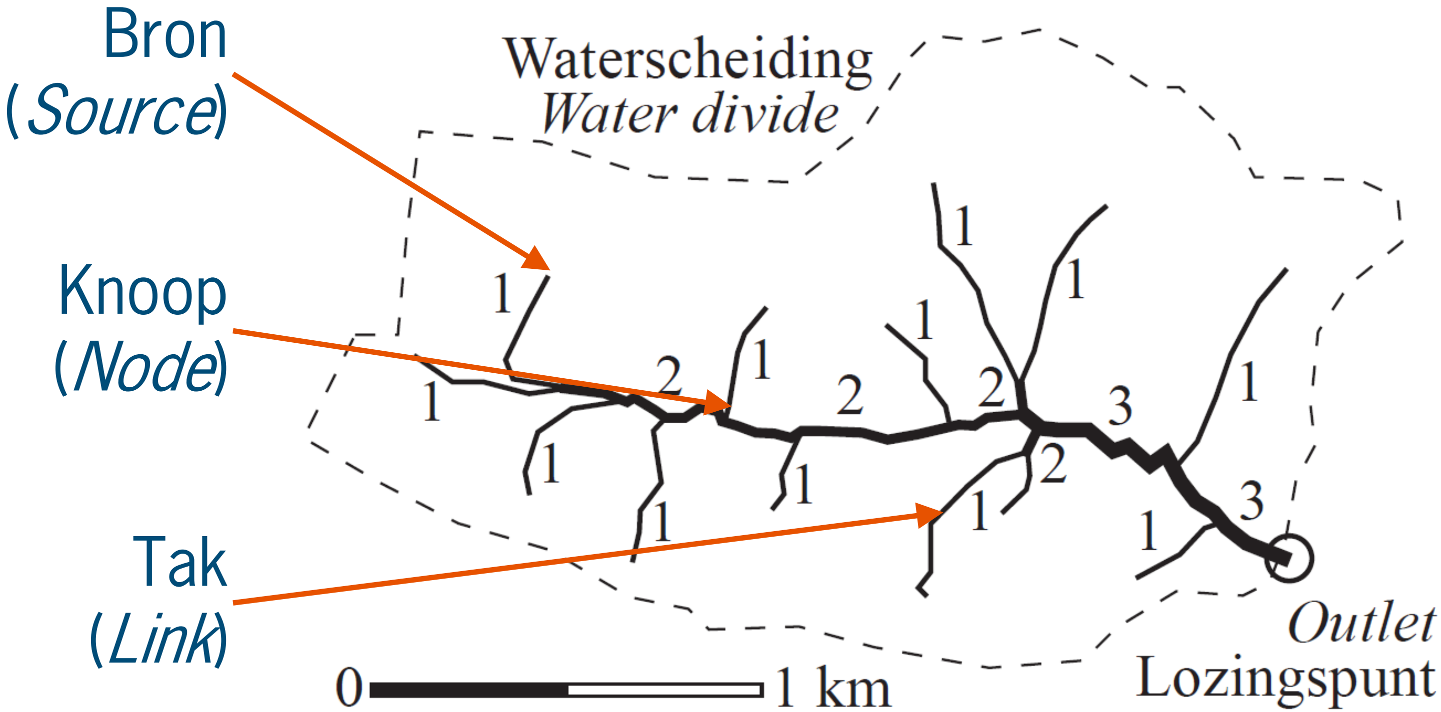


- Beven & Kirkby (1979)

TOPMODEL (Beven & Kirkby, 1979)



Order numbering of channel network



- Rietholzbach catchment, Switzerland (3.4 km^2)

Contribution of channel network to response

- Impulse-response function of advection-diffusion equation for unsteady open channel flow:

$$\boxed{u(0,t) = \frac{x}{2\sqrt{\pi D t^3}} \exp\left[-\frac{(x-ct)^2}{4Dt} \right]}, \text{ where:}$$

$$x = \text{distance from upstream end of section [L]}$$

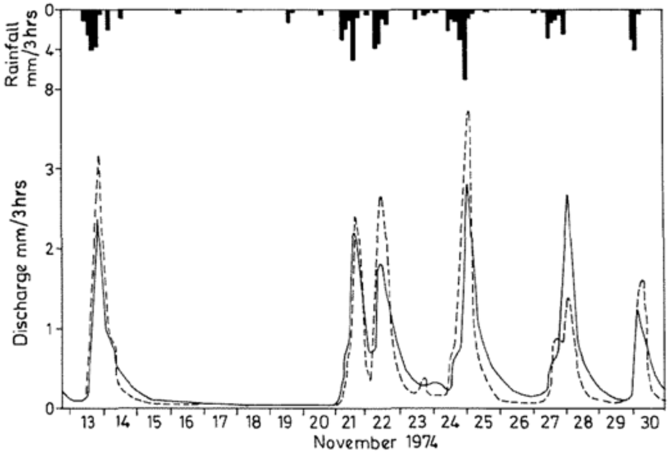
$$t = \text{time [T]}$$

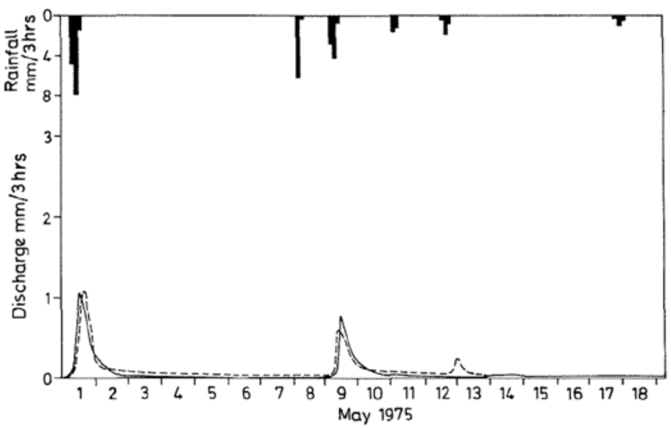
$$c = \text{advection constant [L/T]}$$

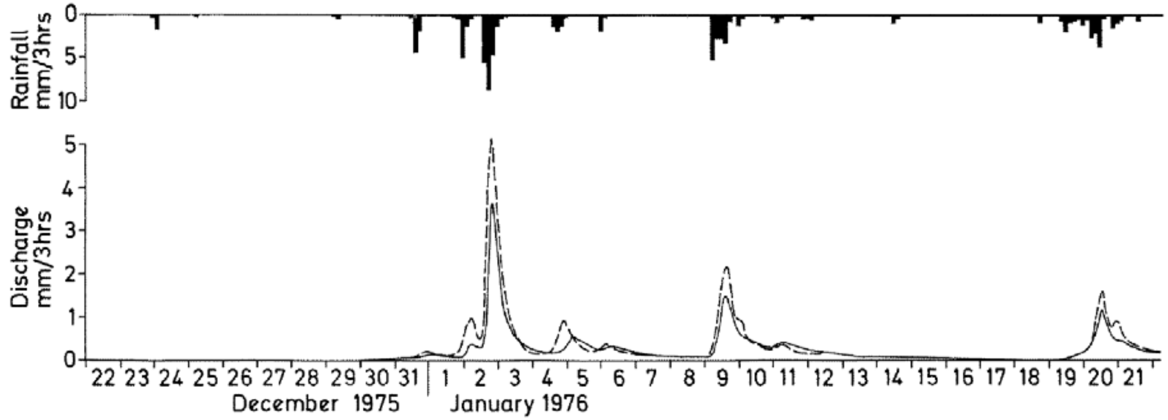
$$D = \text{diffusion constant } [\mathrm{L}^2 / T]$$

$$\mu = x / c \text{ (mean)}; \quad \sigma^2 = 2Dx / c^3 \text{ (variance)}$$

Simulation results (Beven & Kirkby, 1979)

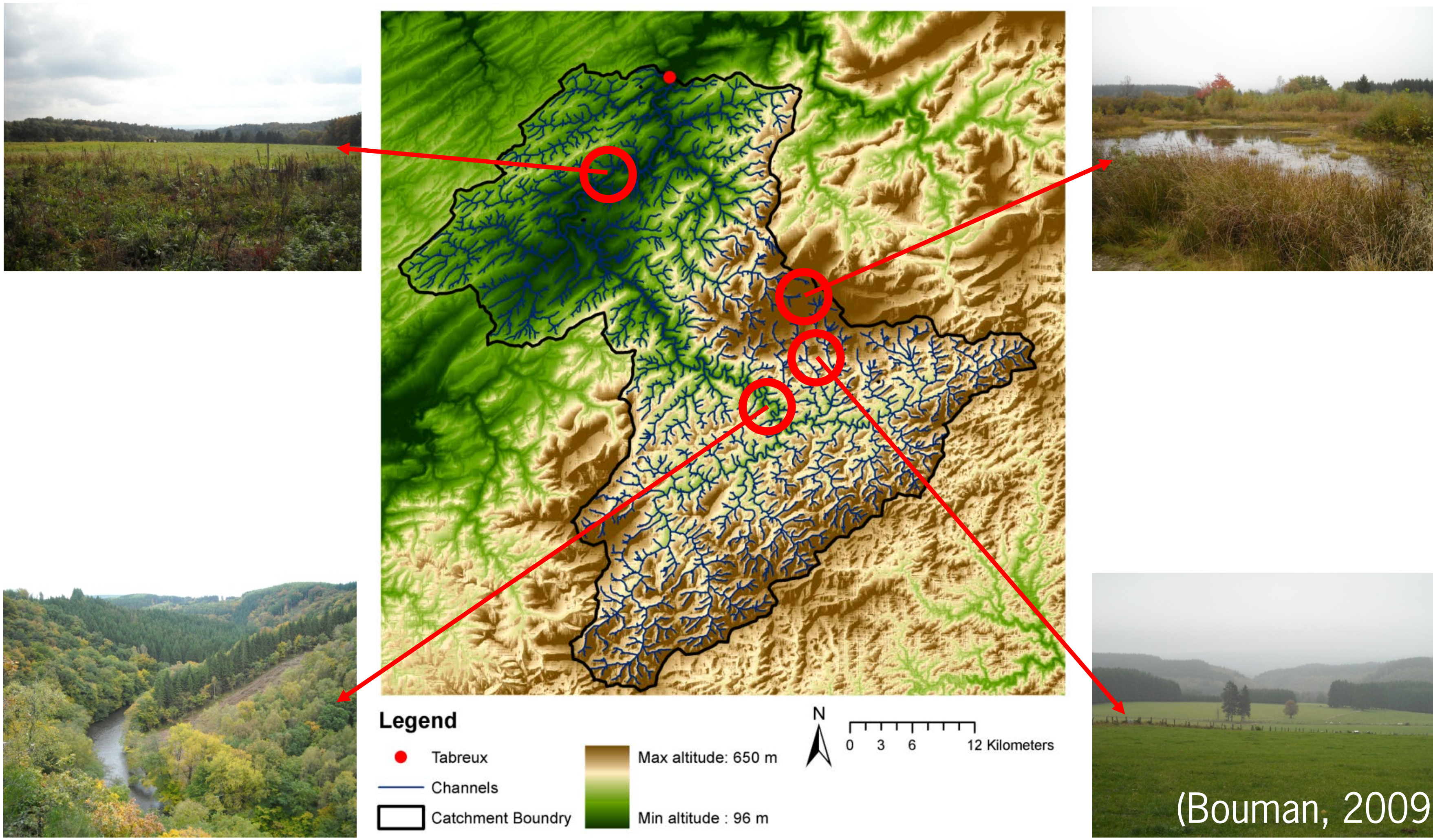




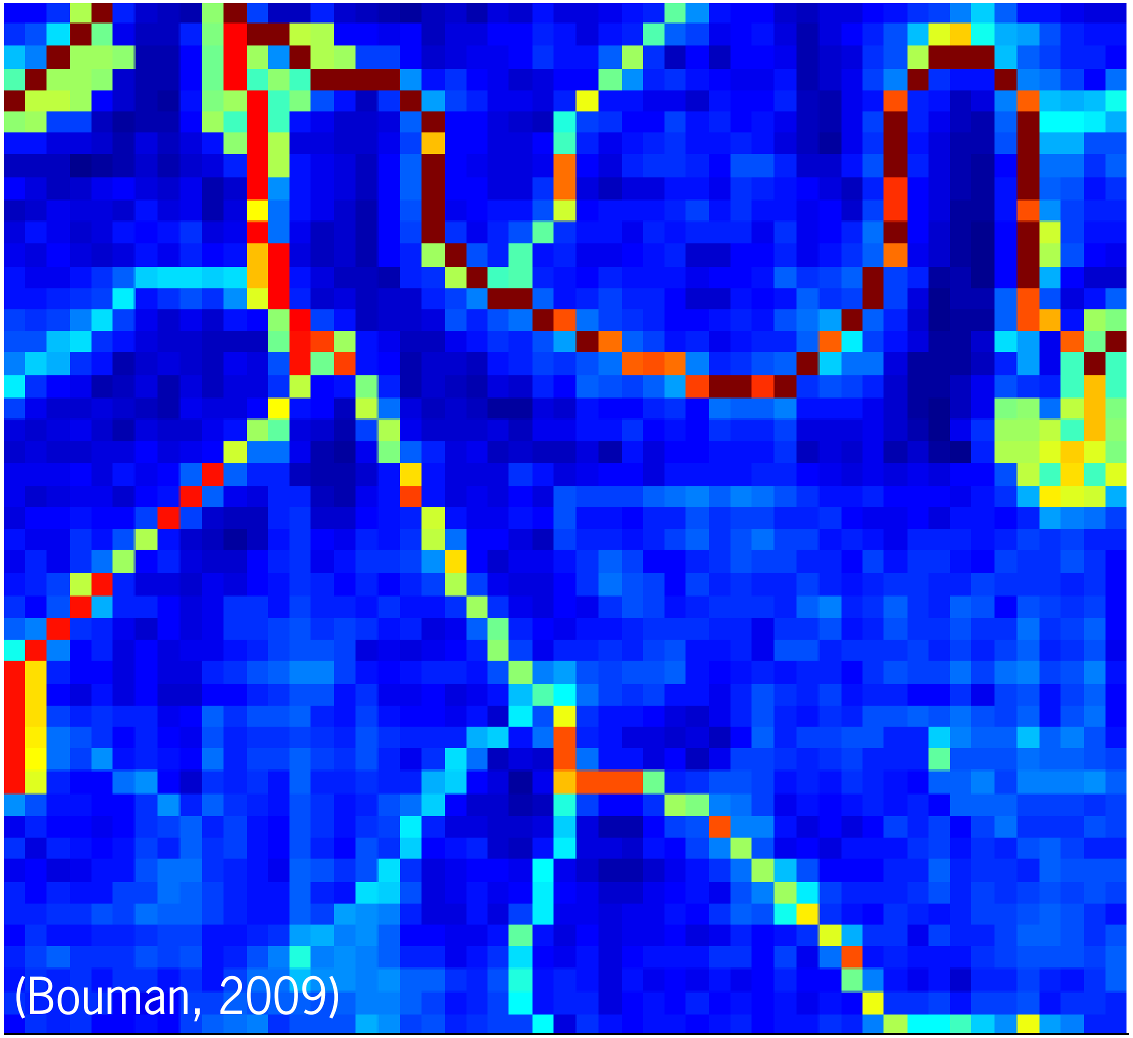


- Left: Lanshaw sub-basin (<1 km^2); Right: Crimple Beck basin (~8 km^2)

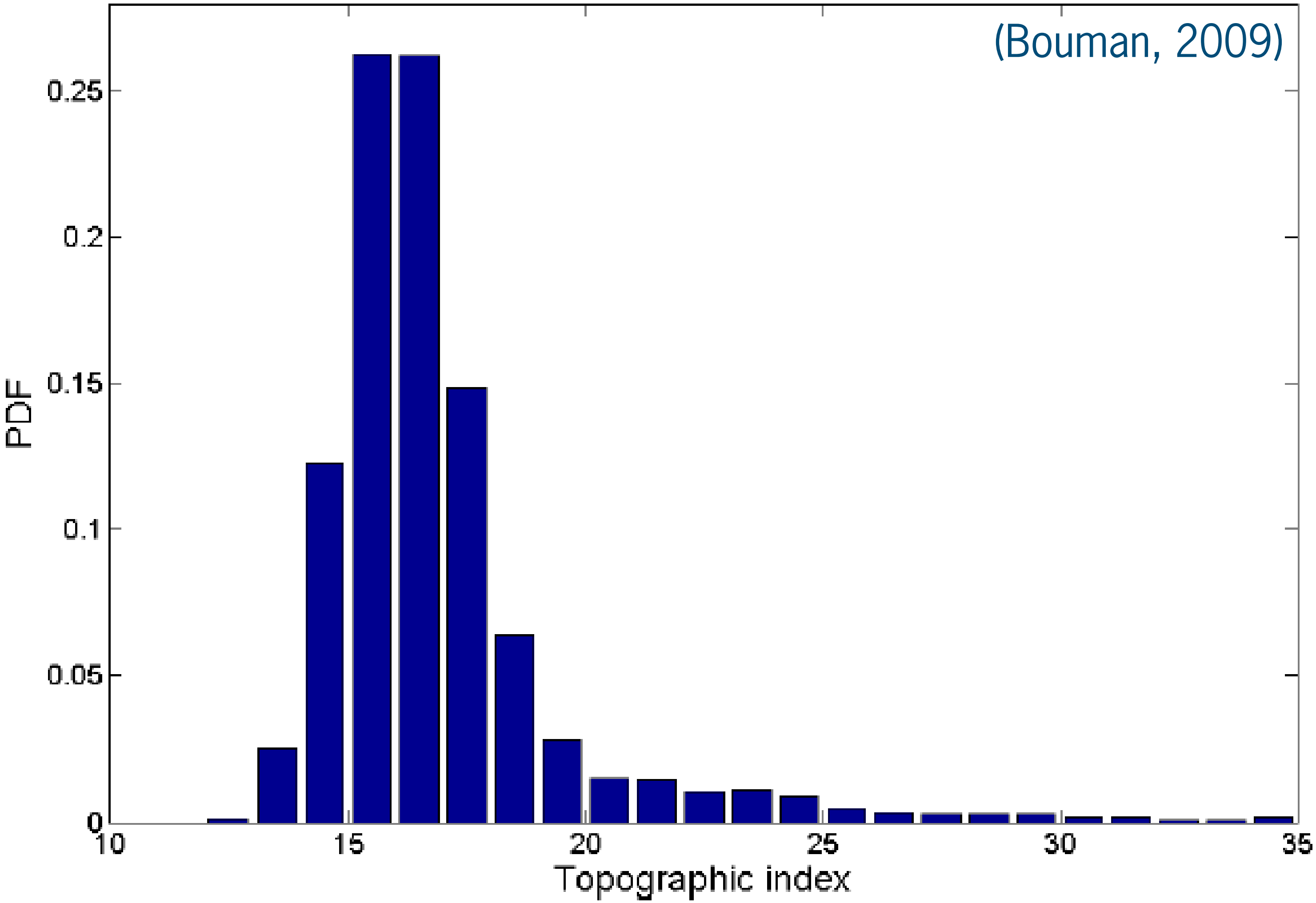
Ourthe catchment, Belgium (~1600 km^2)



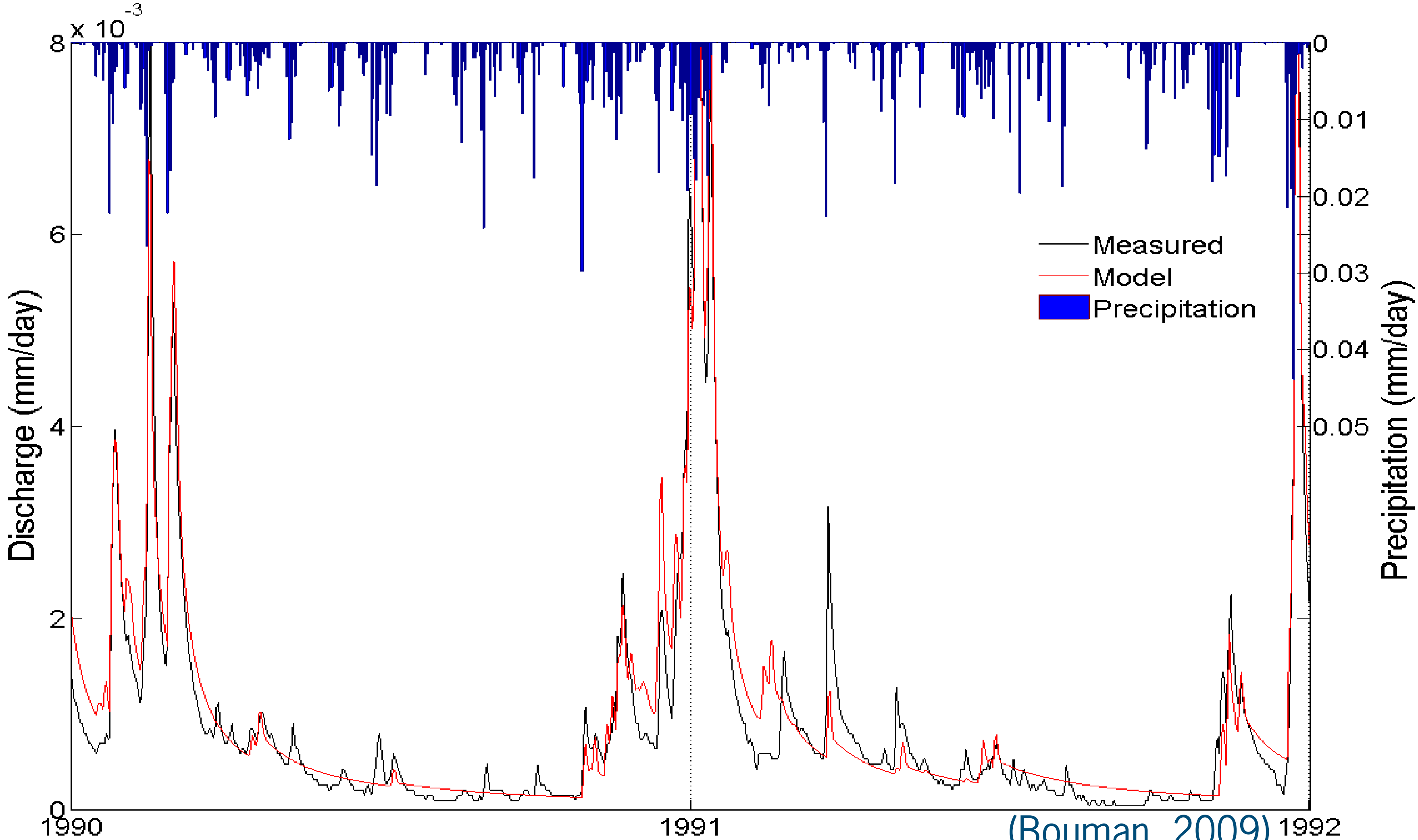
Topographic index (60 x 60 m pixels)



Probability distribution of topographic index



Simulated versus measured hydrograph



(Bouman, 2009)

WAGENINGEN UNIVERSITY
WAGENINGEN UR

Remko.Uijlenhoet@wur.nl

WAGENINGEN UNIVERSITY
WAGENINGEN UR