

# Homework 4

Due Friday, September 26

**Problem 1** (15 pts). This problem is an exercise to practice set operations. No formal proof is required.

- (a) Determine the sets  $A, B$  where  $A - B = \{1, 3, 7, 11\}$ ,  $B - A = \{2, 6, 8\}$ , and  $A \cap B = \{4, 9\}$ .
- (b) Let  $A = \{x \mid \exists n : x = 2n\}$  and  $B = \{x \mid \exists n : x = 3n\}$  in the universe of integers. Find  $A \cup B$ ,  $A \cap B$ , and  $A^c \cap B$ . You can answer either in English or in mathematical notation.
- (c) Let  $A = \{x \in \mathbb{R} : x^2 < 4\}$ , and  $B = \{x \in \mathbb{R} : x > 1\}$ . Find  $A \cup B$ ,  $A \cap B$ , and  $A - B$ .

**Problem 2** (15 pts). In this problem, you will prove several identities about sets. Each part is worth 5 points.

- (a) Prove that  $(A \cap B)^C = A^C \cup B^C$  is true for all sets  $A$  and  $B$ .
- (b) Prove that the symmetric difference is associative. That is, prove that for all sets  $A$ ,  $B$ , and  $C$ , it must be true that

$$(A \triangle B) \triangle C = A \triangle (B \triangle C).$$

- (c) Prove that if  $A$  and  $B$  are sets, then  $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$ . (Recall from class that the power set  $\mathcal{P}(A)$  is the set of all subsets of  $A$ .)

**Bonus 1** (1 pt). Approximately how long did you spend on this homework assignment?