Homework Ch 2

Section 1: 6, 12, 18, 24, 30, 36, 42, 48

Section 2: 6, 12, 18, 24, 30, 36, 42, 48

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CSC-7-48309
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1.6 Let s="stocks are increasing" and i="interest rates are steady."

a. Stocks are increasing but interest rates are steady.

$\boxed{s \wedge i}$

b. Neither are stocks increasing nor are interest rates steady.

$\boxed{\sim s \wedge \sim i}$

(1.12) Create a truth table for: $\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

(1.18) Are $p \vee t$ and t logically equivalent?

p	t	$p \vee t$
T	T	T
F	T	T

$p \vee t$ and t always have the same truth tables, therefore they are logically equivalent.

(1.24) Determine if $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ are logically equivalent.

p	q	r	$(p \vee q)$	$(p \wedge r)$	$(p \vee q) \wedge r$	$(p \vee q) \vee (p \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	F	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	F	F	F	F
F	F	F	F	F	F	F

$(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ do not always have the same truth values, so they are not logically equivalent.

(1.30) Use De Morgan's law to write negotiations for the statement: The dollar is at an all-time high and the stock market is at a record low.

The dollar is not at an all time high and the stock market is not at a record low.

(1.36) Assume x is a particular real number and use De Morgan's laws to write a negation for $1 > x \geq -3$

$$\boxed{1 \leq x \text{ or } x < -3}$$

(1.42) Use truth tables to establish if the following statement form is a tautology or contridiction

$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

p	q	r	$(\sim p \wedge q)$	$(q \wedge r)$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
T	T	T	F	T	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	T	T	F
F	F	T	F	F	F
F	T	F	T	F	F
F	F	F	F	F	F

* All truth values are F, therefore it is a contradiction

1.48 Supply a reason for each step.

$$(p \wedge \sim q) \vee (p \wedge q) \equiv p \wedge (\sim q \vee q) \quad \text{by } \underline{(a)}$$

$$\equiv p \wedge (q \vee \sim q) \quad \text{by } \underline{(b)}$$

$$\equiv p \wedge t \quad \text{by } \underline{(c)}$$

$$\equiv p \quad \text{by } \underline{(d)}$$

Therefore, $(p \wedge \sim q) \vee (p \wedge q) \equiv p$

a. Distributive Law

b. Communicative Law for V

c. Negation Law for V

d. Identity Law for V

(2.6) Construct a truth table for

$\sim p \vee q \rightarrow r$

p	q	$\underbrace{r}_{\text{Conclusion}}$	$\sim p$	$\underbrace{\sim p \vee q}_{\text{Hypothesis}}$	$\sim p \vee q \rightarrow r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	F

2.12

Using the logical equivalence established in: $p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$ to rewrite

If $x > 2$ or $x < -2$, then $x^2 > 4$

* If $x > 2$ then $x^2 > 4$, and if $x < -2$ then $x^2 > 4$

(2.18) Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Include truth tables and a few words of explanation.

If it walks like a duck and it talks like a duck, then it is a duck.

* $p \wedge q \rightarrow r$

Either it does not walk like a duck or it does not talk like a duck, or it is a duck.

* $(\sim p \vee \sim q) \vee r$

If it does not walk like a duck and it does not talk like a duck, then it is not a duck.

* $\sim p \wedge \sim q \rightarrow \sim r$

		Conclusion 1			Conclusion 2	Hypothesis 1		Hypothesis 2			
p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \wedge q$	$(\sim p \vee \sim q)$	$\sim p \wedge \sim q$	$p \wedge q \rightarrow r$	$(\sim p \vee \sim q) \vee r$	$\sim p \wedge \sim q \rightarrow \sim r$
T	T	T	F	F	F	T	F	F	T	T	T
T	T	F	F	F	T	T	F	F	F	F	T
T	F	T	F	T	F	F	T	F	T	T	T
T	F	F	F	T	T	F	T	F	T	T	T
F	T	T	T	F	F	F	T	F	T	T	T
F	T	F	T	F	T	F	T	F	T	T	T
F	F	T	T	T	F	F	T	T	T	T	F
F	F	F	T	T	T	F	T	T	T	T	T

$p \wedge q \rightarrow r$ and $(\sim p \vee \sim q) \vee r$ share the same truth and therefor are logically equivalent.

(2.24) Use truth tables to establish the truth of the following statement.

A conditional statement is not logically equivalent to its inverse.

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$p \to q$ and $q \to p$ do not share the same truth values, therefore they are not logically equivalent.

(2.30) Use $\leftrightarrow$ to convert the following logical equivalency, then use a truth table to verify the tautology.

$$(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$$

p	q	r	$p \wedge q$	$p \wedge r$	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	F	F	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

(3.36) Taking the long view on your education, you go to the Prestige Corporation and ask what you should do in college to be hired when you graduate. The personnel director replies that you will be hired only if you major in mathematics or computer science. You become a math major, get a B+ average, and take accounting. You return and make a formal application, and are turned down. Did the personnel lie to you?

* If you major in mathematics or computer science, then you will be hired.
* If you do not major in mathematics or computer science, then you will not be hired.
* The personnel lied to you.

2.42 Use the contrapositive to rewrite the following statement:

Being divisible by 3 is a necessary condition for this number to be divisible by 9.

* Not being divisible by 3 is a necessary condition for this number to not be divisible by 9.

(2.48) (a) Use the logical equivalencies $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the given statement form without using the symbols $\to$ or $\leftrightarrow$

(b) Use the logical equivalence $p \vee q \equiv \sim(\sim p \wedge \sim q)$ to rewrite each statement from using only $\wedge$ and $\sim$.

$$p \vee \sim q \to r \vee q$$

a. $\sim(p \vee \sim q) \vee (r \vee q)$

b. $\sim[\sim(\sim(p \wedge \sim q) \wedge \sim(r \wedge q)]$

$\boxed{\sim[(p \wedge \sim q) \wedge \sim(r \wedge q)]}$

(3.6) Use truth tables to determine whether the argument form is valid.

$$p \to q$$
$$q \to p$$
$$\therefore p \vee q$$

		Premises		Conclusion
p	q	$p \to q$	$q \to p$	$p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

The last row shows it is possible for this argument form to have true premises and false conclusions, therefore this argument form is invalid.

3.12 Use truth tables to show that the following forms of argument are invalid.

a. $p \to q$
q
$\therefore p$
(converse error)

b. $p \to q$
$\sim p$
$\therefore \sim q$
(inverse error)

a.

p	q	$\underbrace{p \to q \quad q}_{\text{Premises}}$		$\underbrace{p}_{\text{Conclusion}}$
		$p \to q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

The third row shows it is possible for an argument of this form to have a false conclusion with true premises, therefore the argument form is invalid

b.

p	q	$p \to q$ (Premises)	$\sim p$ (Premises)	$\sim q$ (Conclusion)
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

The third row shows it is possible for an argument of this form to have true premises with a false conclusion, therefore the argument form is invalid.

(3.18) Use a truth table to show the following argument form is valid.

$p \vee q$

$\sim q$

$\therefore p$

		Premises		Conclusions
p	q	$p \vee q$	$\sim q$	p
T	T	T	F	
T	F	T	T	T
F	T	T	F	
F	F	F	T	

Row 2 shows the only situation where both premises are true. Since the conclusion is also true, the arguement form is valid

(3.24) Use symbols to write the logical form of the argument. If the argument is valid, identify the rule of interference that guarantees its validity. Otherwise state whether the converse or inverse error is made.

- If Jules solved this problem correctly, then Jules obtained the answer 2.
- Jules obtained the answer 2
- $\therefore$ Jules solved this problem correctly.

$*$ $p \to q$

q invalid: converse error

$\therefore p$

3.30

If this computer program is correct, then it produces the correct output when run with the test data my teacher gave me

This computer program produces the correct when run with the test data my teacher gave me

$\therefore$ This computer program is correct

* $p \to q$

q converse error

$\therefore p$

3.36 Find the mistake

a. There is an undeclared variable or there is a syntax error in the first five lines.

b. If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.

c. There is not a missing semicolon.

d. There is not a misspelled variable.

* There is an undeclared variable.
There is not a missing semicolon and there is not a misspelled variable. (c and d by $\wedge$)

3.42 Use the valid argument forms to deduce the conclusion from the premises.

a. $p \vee q$

b. $q \to r$

c. $p \wedge s \to t$

d. $\sim r$

e. $\sim q \to u \wedge s$

f. $\therefore t$

1. $q \to r$ by premise (b)
   $\sim r$ by premise (d)
   $\therefore \sim q$ by modus tollens

2. $p \vee q$ by premise (a)
   $\sim q$ by (1)
   $\therefore p$ by elimination

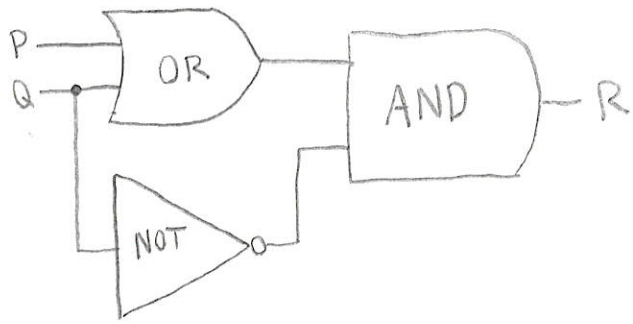
3. $\sim q \to u \wedge s$ by premise (e)
   $\sim q$ by (1)
   $\therefore u \wedge s$ by modus ponens

4. $u \wedge s$ by (3)
   $\therefore s$ by specialization

5. p by (2)
   s by (4)
   $\therefore p \wedge s$ by conjunction

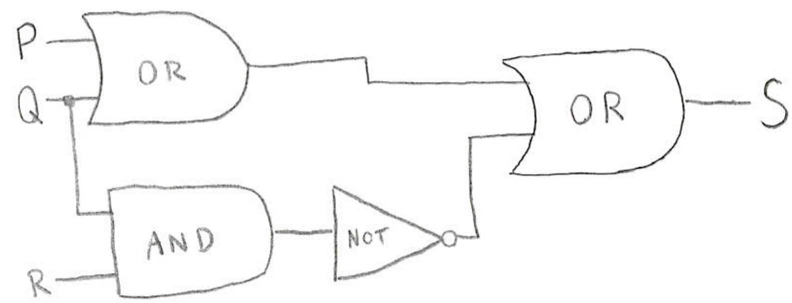
6. $p \wedge s \to t$ by premise (c)
   $p \wedge s$ by (5)
   $\therefore t$ by modus ponens

(4.6) Write an input / output table for the circuit.



Inputs		Output
P	Q	R
1	1	0
1	0	1
0	1	0
0	0	0

4.12 Find the boolean expression that corresponds to the circuit.



$\boxed{(P \vee Q) \vee \sim(Q \wedge R)}$

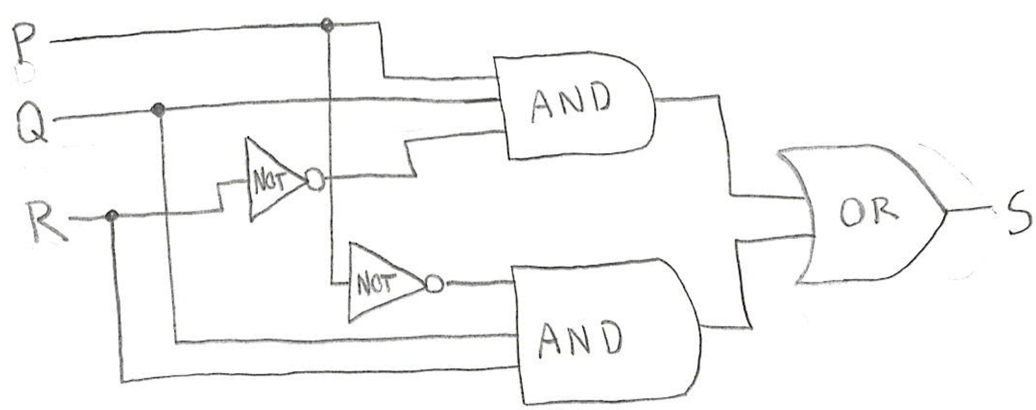
4.18

a. Construct a boolean expression having the given table as its truth table.

b. Construct a circuit having the given truth table as its input/output.

Input			Output
P	Q	R	S
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

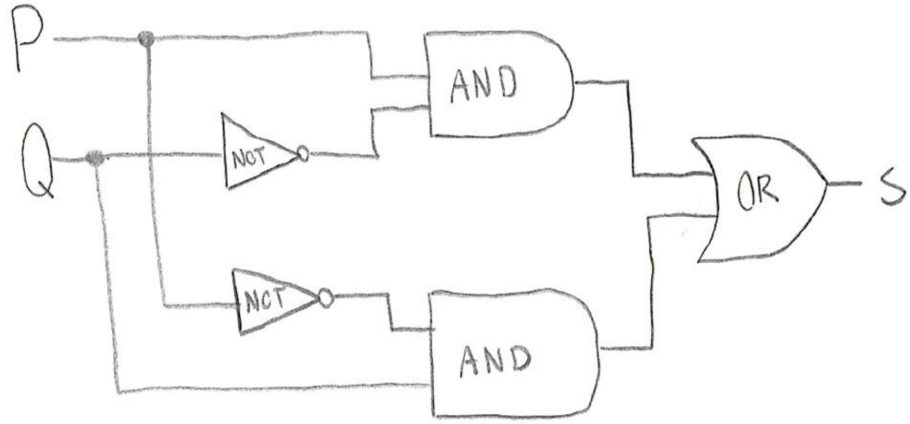
$$\boxed{(P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R)}$$



4.24 The lights are controlled by two lightswitches: one in the back and one in the front. Moving either switch to the opposite position turns of the lights if they are off or on if they are on. Assume that the lights were installed so that when both switches are down the lights are off. Design a circuit to control the switches.

P	Q	S
1	1	0
1	0	1
0	1	1
0	0	0

$(P \wedge \sim Q) \vee (\sim P \wedge Q)$



4.30 For the circuit corresponding to the boolean expression there is an equivalent circuit with at most two logic gates. Find such a circuit.

$$(P\wedge Q)\vee(\sim P\wedge Q)\vee(\sim P\wedge\sim Q)$$

$$
\begin{aligned}
(P\wedge Q)\vee(\sim P\wedge Q)\vee(\sim P\wedge Q) &\equiv (P\wedge Q)\vee((\sim P\wedge Q)\vee(\sim P\wedge\sim Q)) && \text{by Associative Law}\\
&\equiv (P\wedge Q)\vee(\sim P\wedge(Q\vee\sim Q)) && \text{by Distributive Law}\\
&\equiv (P\wedge Q)\vee(\sim P\wedge t) && \text{by Negation Law}\\
&\equiv (P\wedge Q)\vee\sim P && \text{by Identity Law}\\
&\equiv \sim P\vee(P\wedge Q) && \text{by Commutative Law}\\
&\equiv (\sim P\vee P)\wedge(\sim P\vee Q) && \text{by Distributive Law}\\
&\equiv t\wedge(\sim P\vee Q) && \text{by Negation Law}\\
&\equiv (\sim P\vee Q)\wedge t && \text{by Commutative Law}\\
&\equiv \sim P\vee Q && \text{by Identity Law}
\end{aligned}
$$

(5.6) Represent the decimal integer 1424 in binary notation.

$1424_{10} \rightarrow \_\_\_\__{2}$

1424/2	712	0
712/2	356	0
356/2	178	0
178/2	89	0
89/2	44	1
44/2	22	0
22/2	11	0
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

$1424_{10} = \boxed{10110010000_{2}}$

5.12 Represent the integer 1011011_2 in decimal notation.

$1011011_2 \rightarrow \_\_\_\__{10}$

$1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^1 + 1 \cdot 2^0 = 91$

$1011011_2 \boxed{= 91_{10}}$

5.18 Perform the arithmatic using binary notation

$$\begin{array}{r} \cancel{1}\cancel{1}\cancel{0}\cancel{1}\cancel{0}_2 \\ -\ 1101_2 \\ \hline 1101 \end{array}$$

$$\boxed{1101_2}$$

(5.24) Find the 8-bit two's complements for the integer 67.

$67_{10} \to \_\_\__2$

67/2	33	1
33/2	16	1
16/2	8	0
8/2	4	0
4/2	2	0
2/2	1	0
1/2	0	1

$$\begin{array}{r} 01000011 \\ \hline 10111100 \\ + \quad 1 \\ \hline 10111101 \end{array}$$

$\boxed{10111101_2}$

5.30 Find the decimal representations for the integer with the 8-bit representation

Because the leading bit is one, it is a 8-bit representation of a negative number.

$$\begin{array}{l} 10111010 \\ \hline 01000101 \\ \hline 01000110_2 \end{array}$$

$$-(1\cdot 2^6+1\cdot 2^2+1\cdot 2^1)=\boxed{-70_{10}}$$

5.36 Use 8-bit representations to compute the sum

$123 + (-94)$

123/2	61	1
61/2	30	1
30/2	15	0
15/2	7	1
7/2	3	1
3/2	1	1
1/2	0	1

01111011

94	47	0
47	23	1
23	11	1
11	5	1
5	2	1
2	1	0
1	0	1

01011110

10100001

10100010

$$\begin{array}{r} 01111011 \\ +10100010 \\ \hline 1|00011101 \end{array}$$

11101

$1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^0 = 29_{10}$

$\boxed{29_{10}}$

(5.42) Convert the hexadecimal from hexadecimal to binary notation.

$B53DF8_{16}$

1011	0101	0011	1101	1111	1000
B	5	3	D	F	8

$B53DF8_{16} = \boxed{101101010011110111111000_2}$