Homework Chapter 1

Section 1 - 1, 4, 7, 10

Section 2 - 1, 4, 7, 10

Section 3 - 1, 4, 7, 10, 13, 16, 19

CSC-7-48309
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1.1 Is there a real number whose square is -1?

a. Is there a real number x such that $x^2 = -1$?

b. Does there exist a real number x such that $x^2 = -1$?

1.4 Given any real number, there is a real number that is greater.

a. Given any real number r, there is a real number s such that s is greater than r.

b. For any real number r, there is a real number s such that $s > r$.

1.7 Rewrite the following statements less formally, without using variables. Determine, as best you can, whether the statements are true or false.

a. There are real numbers u and v with the property that $u+v < u-v$.

* There are real numbers whose sum is less than their difference.

* True

b. There is a real number x such that $x^2 < x$.

* There is a real number whose square is less than it.

* True.

c. For all positive integers n, $n^2 \geq n$.

* All positive integers have a larger square.

* True

d. For all real numbers a and b, $|a+b| \leq |a| + |b|$

* For all real numbers, the absolute value of their sum is less than or equal to the sum of their absolute values.

* True

1.10 Every nonzero real number has a reciprocal.

a. All nonzero real numbers have a reciprocal.

b. For all nonzero real numbers r, there is a reciprocal for r.

c. For all nonzero real numbers r, there is a real number s such that s is a reciprocal for r.

2.1 Which of the following sets are equal?

$A = \{a, b, c, d\}$ $\quad$ $B = \{d, e, a, c\}$

$C = \{d, b, a, c\}$ $\quad$ $D = \{a, a, d, e, c, e\}$

* $A = C$, $B = D$

2.4

a. Is $2 \in \{2\}$?

* yes

b. How many elements are in the set $\{2,2,2,2\}$?

* 1

c. How many elements are in the set $\{0, \{0\}\}$?

* 2

d. Is $\{0\} \in \{\{0\}, \{1\}\}$?

* yes

e. Is $0 \in \{\{0\}, \{1\}\}$?

* no

2.7 Use the set-roster notation to indicate the elements in each of the following sets.

a. $S = \{n \in Z \mid n = (-1)^k$, for some integer $k\}$.

* $\{1, -1\}$

b. $T = \{m \in Z \mid m = 1 + (-1)^i$, for some integer $k\}$.

* $\{2, 0\}$

c. $U = \{r \in Z \mid 2 \leq r \leq -2\}$.

* no elements

d. $V = \{s \in Z \mid s > 2$ or $s < 3\}$.

* Z

e. $W = \{t \in Z \mid 1 < t < -3\}$

* no elements

f. $X = \{u \in Z \mid u \leq 4$ or $u \geq 1\}$

* Z

2.10

a. Is $((-2)^2, -2^2) = (-2^2, (-2)^2)$?

* no

b. Is $(5, -5) = (-5, 5)$?

* no

c. Is $(8-9, \sqrt[3]{-1}) = (-1, -1)$?

* yes

d. Is $(\frac{-2}{-4}, (-2)^3) = (\frac{3}{6}, -8)$?

* yes

3.1 Let $A=\{2,3,4\}$ and $B=\{6,8,10\}$ and define a relation R from A to B as follows. For all $(x,y) \in A \times B$,

$$(x,y) \in R \text{ means that } \frac{y}{x} \text{ is an integer.}$$

a. Is $4 R 6$? Is $4 R 8$? Is $(3,8) \in R$? Is $(2,10) \in R$?

* no, yes, no, yes

b. Write R as a set of ordered pairs.

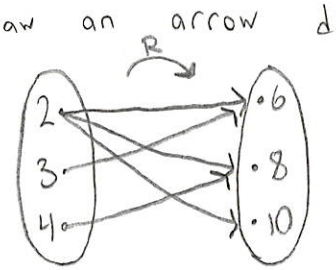
* $R=\{(2,6),(2,8),(2,10),(3,6),(4,8)\}$

c. Write the domain and co-domain of R.

* Domain $R=A=\{2,3,4\}$, co-domain $R=B=\{6,8,10\}$

d. Draw an arrow diagram for R

*



(3.4) Let $G = \{-2, 0, 2\}$ and $H = \{4, 6, 8\}$ and define a relation V from G to H as follows:
For all $(x, y) \in G \times H$,

$(x, y) \in V$ means that $\frac{x-y}{4}$ is an integer

a. Is $2 V 6$? Is $(-2) V (-6)$? Is $(0, 6) \in V$? Is $(2, 4) \in V$?

* yes, yes, no, no

b. Write V as a set of ordered pairs.

* $V = \{(-2, 6), (0, 4), (0, 8), (2, 6)\}$

c. Write the domain and co-domain of V.

* Domain $V = G = \{-2, 0, 2\}$, co-domain $V = H = \{4, 6, 8\}$

d. Draw an arrow diagram for V.

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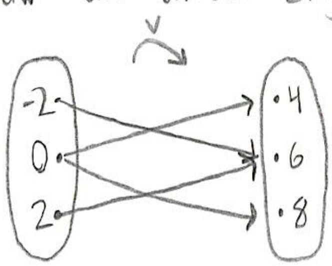

3.7 Let $A = \{4, 5, 6\}$ and $B = \{5, 6, 7\}$ and define relations R, S, T from A to B as follows:

$(x, y) \in R$ means that $x \geq y$.

$(x, y) \in S$ means that $\frac{x-y}{2}$ is an integer.

$T = \{(4,7), (6,5), (6,7)\}$.

a. Draw arrow diagrams for R, S, and T.

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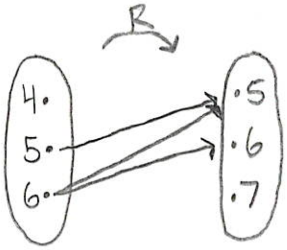
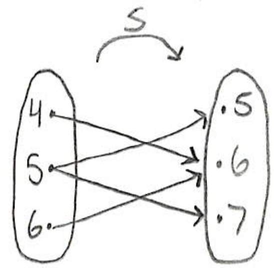
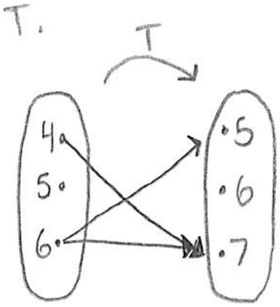






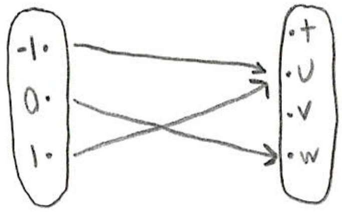
b. Indicate whether any of the relations R, S, T are functions.

* none are functions.

(3.10) Find four relations from $\{a,b\}$ to $\{x,y\}$ that are not functions from $\{a,b\}$ to $\{x,y\}$.

* $R_1 = \{(a,x)\}$
  $R_2 = \{(b,y)\}$
  $R_3 = \{(a,x),(a,y)\}$
  $R_4 = \{(b,x),(b,y)\}$

(3.13) Let $A = \{-1, 0, 1\}$ and $B = \{t, u, v, w\}$. Define a function $F: A \to B$ by the following arrow diagram.



a. Write the domain and co-domain of F.

* domain $F = A = \{-1, 0, 1\}$, co-domain $F = B = \{t, u, v, w\}$

b. Find $F(-1)$, $F(0)$, $F(1)$.

* $F(-1) = u$, $F(0) = w$, $F(1) = u$

3.16 Let f be the squaring function defined in example 1.3.6

Find $f(-1)$, $f(0)$, and $f(\frac{1}{2})$.

* $f(-1) = (-1)^2 = 1$, $f(0) = (0)^2 = 0$, $f(\frac{1}{2}) = (\frac{1}{2})^2 = \frac{1}{4}$

(3.19) Define functions f and g from R to R by the following formulas: For all $x \in R$,

$$f(x) = 2x \quad \text{and} \quad g(x) = \frac{2x^3 + 2x}{x^2 + 1}$$

Does $f = g$? Explain.

* $g(x) = \frac{2x^3 + 2x}{x^2 + 1} = \frac{2x(x^2+1)}{x^2+1} = 2x = f(x)$, therefore $f = g$.