Homework Assignment 4

Section 1: 4, 5, 11, 12, 15, 20, 25
Section 2: 4, 7, 10, 14, 15, 20, 24
Section 3: 4, 9, 10, 13, 15, 22, 23, 27
Section 4: 5, 8, 11, 16, 20, 22, 26
Section 6: 4, 5, 10, 12, 15, 21, 22
Section 7: 4, 5, 10, 12, 15, 21, 22, 27

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4.1.4

Prove the statement: There are integers m and n such that $m > 1$ and $n > 1$ and $\frac{1}{m} + \frac{1}{n}$ is an integer.

Let $m = 2$ and $n = 2$. $\frac{1}{m} + \frac{1}{n} = \frac{1}{2} + \frac{1}{2} = 1$. 1 is an integer.

4.1.5

Prove the statement: There are distinct integers m and n such that $\frac{1}{m} + \frac{1}{n}$ is an integer.

Let $m = -2$ and $n = 2$. $\frac{1}{m} + \frac{1}{n} = -\frac{1}{2} + \frac{1}{2} = 0$. 0 is an integer.

4.1.11

Disprove the statement by giving a counterexample. For all real numbers a and b, if $a < b$ then $a^2 < b^2$.

Let $a = -2$ and $b = -1$. Then $a < b$ because $-2 < -1$ but $a^2 \nless b^2$ because $(-2)^2 = 4$ and $(-1)^2 = 1$ and $4 \nless 1$

4.1.12

Disprove the statement by giving a counterexample. For all integers n, if n is odd then $\frac{n-1}{2}$ is odd.

> Let $n = 5$. 5 is an odd integer, but $\frac{n-1}{2}$ is not odd because $\frac{5-1}{2} = \frac{4}{2} = 2$ and 2 is not odd.

4.1.15

Determine whether the property is true for all integers, true for no integers, or true for some integers and false for other integers. Justify the answer. $-a^n = (-a)^n$

> The statement is true for some integers and false for other integers. For all integers n where n is odd, the statement is true. For all integers n where n is even, the statement is false.
>
> Let $a = 2$ and $n = 3$. $-a^n = (-a)^n$ is true because $-2^3 = -8$ and $(-2)^3 = -8$ and $-8 = -8$.
>
> Let $a = 2$ and $n = 2$. $-a^n = (-a)^n$ is false because $-2^2 = -4$ and $(-2)^2 = 4$ and $-4 \neq 4$.

4.1.20

The following statement is true. (a) Rewrite the statement with the quantification implicit as If _____, then _____, and (b) write the first sentence of a proof (the "starting point") and the last sentence of a proof (the "conclusion to be shown").

For all integers m, if $m > 1$ then $0 < \frac{1}{m} < 1$

a. If an integer is greater than 1, then its reciprocal is between 0 and 1.
b. Start of Proof: Suppose m is any integer such that $m > 1$.
   Conclusion to be shown: $0 < \frac{1}{m} < 1$.

4.1.25

Prove the following statement. Use only the definitions of the terms and the Assumptions listed on page 146, not any previously established properties of odd and even integers. Follow the directions given in this section for writing proofs of universal statements.
The difference of any even integer minus any odd integer is odd.

> Proof: Suppose a is any even integer and b is any odd integer.
> To show: $a - b$ is odd.
> Using the definition of even and odd integers, $a = 2r$ and $b = 2s + 1$ for some integers r and s. By using substitution and algebra:
> $a - b = 2r - (2s + 1) = 2r - 2s - 1 = 2(r - s - 1) + 1$. Let $t = (r - s - 1)$. t is an integer because it is the difference of integers. Therefore, $a - b = 2t + 1$, where t is an integer. By definition of odd integers, $a - b$ is odd.

4.2.4

The following number is rational. Write the number as a ratio of two integers. 0.37373737…

Let $x = 0.37373737\ldots$

Then $100x = 37.37373737\ldots$

So, $100x - x = 37.37373737\ldots - 0.37373737\ldots$

Therefore, $99x = 37$

$x = \frac{99}{37}$

4.2.7

The following number is rational. Write the number as a ratio of two integers.
52.4672167216721…

Note to self, for the larger number make it have one iteration of the repeated numbers on the left, the smaller number should have the first iteration of repeated numbers right of the decimal

Furthermore, you are trying to make a whole number with the difference.

Let $x = 52.4672167216721...$

Then $100000x = 5246721.67216721...$

And $10x = 524.672167216721...$

So, $100000x - 10x = 5246721 - 524$

Thus, $99990x = 5246197$

Therefore, $x = \frac{5246197}{99990}$

4.2.10

Assume that m and n are both integers and that $n \neq 0$. Explain why $(5m + 12n)/(4n)$ must be a rational number.

> Because m and n are both integers, $5m$, $12n$, and $4n$ are also integers since the products of integers are integers. Therefore, $(5m + 12n)$ is an integer because the sum of integers is an integer. Also, $4n$ is not zero because of the zero product rule property. Thus, $(5m + 12n)/(4n)$ is a rational number because the quotient of two integers with a non zero denominator is rational.

4.2.14

Consider the statement: The square of any rational number is a rational number.

a. Write the statement formally using a quantifier and a variable.
b. Determine whether the statement is true or false and justify your answer.

a. $\forall$ rational numbers r, r^2 is a rational number.
b. Proof: Suppose r is a rational number. Show that r^2 is rational. By definition of rational, $r = \frac{a}{b}$, for some integers a and b where $b \neq 0$. Therefore $r^2 = (\frac{a}{b})^2 = \frac{a^2}{b^2} = \frac{a \cdot a}{b \cdot b}$. The product of two integers is an integer so $a \cdot a$ and $b \cdot b$ are integers and $b \cdot b \neq 0$ because of the zero product property. Therefore, r^2 is a rational number because the quotient of two integers with a non zero denominator is rational.

4.2.15

In case the statement is false, determine whether a small change would make it true. If so, make the change and prove the new statement. Follow the directions for writing proofs on page 154.

The product of any two rational numbers is a rational number.

> Proof: Suppose r and s are rational numbers. Show that rs is rational. By definition of rational, $r = \frac{a}{b}$ and $s = \frac{c}{d}$ for some integers a, b, c, and d where $b \neq 0$ and $d \neq 0$. Then, $rs = \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. ac and bd are both integers because they are products of integers and $bd \neq 0$ because of the zero product property. Thus, rs is a quotient of integers with a nonzero denominator and by definition of rational is rational.

4.2.20

In case the statement is false, determine whether a small change would make it true. If so, make the change and prove the new statement. Follow the directions for writing proofs on page 154.

Given any two rational numbers r and s with $r < s$, there is another rational number between r and s.

Proof: Suppose r, and s are rational numbers.

Show: $r < n < s$

For some value n, $n = \frac{r+s}{2}$. By definition or rational, $r = \frac{a}{b}$ and $s = \frac{c}{d}$ for some integers a, b, c, and d where $b \neq 0$ and $d \neq 0$. Then, $r + s = \frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$.

$n = (\frac{1}{2})(\frac{ad+cb}{bd}) = \frac{ad+cb}{2bd}$ So ad, cb, and $2bd$ are integers because they are products of integers and $2bd \neq 0$ because of the zero product property. Therefore, n is a quotient of integers with a nonzero denominator and by definition of rational is rational as well as between r and s.

4.2.24

Derive the statement as corollaries of Theorems 4.2.1, 4.2.2, and the results of exercises 12, 13, 14, 15, and 17

For any rational numbers r and s, $2r + 3s$ is rational.

> Proof: Suppose r and s are rational numbers. By theorem 4.2.1 2 and 3 are rational, and by exercise 15, $2r$ and $3s$ are both rational. Thus, by theorem 4.2.2, $2r + 3s$ is rational.

4.3.4

Give a reason for your answer. Assume that all variables represent integers.
Does 3 divide $(3k + 1)(3k + 2)(3k + 3)$?

Yes, $(3k + 1)(3k + 2)(3k + 3) = 3[(3k + 1)(3k + 2)(k + 1)]$ and $(3k + 1)(3k + 2)(k + 1)$ is an integer because k is an integer and the sums and products of integers are integers.

4.3.9

Give a reason for your answer. Assume that all variables represent integers.

Is 4 a factor of $2a \cdot 34b$?

> Yes, $2a \cdot 34b = 68ab = 4(17ab)$. $17ab$ is an integer because the products of integers are integers.

4.3.10

Give a reason for your answer. Assume that all variables represent integers. Does 7 | 34?

No, $\frac{37}{4} \approx 4.857$ which is not an integer.

4.3.13

Give a reason for your answer. Assume that all variables represent integers. If $n = 4k + 3$, does 8 divide $n^2 - 1$?

Yes, $n^2 - 1 = (4k + 3)^2 - 1 = (16k^2 + 24k + 9) - 1 = 16k^2 + 24k + 8$.
$16k^2 + 24k + 8 = 8(2k^2 + 3k + 1)$ and $2k^2 + 3k + 1$ is an integer because the sum and products of integers are integers.

4.3.15

Prove the statement directly from the definition of divisibility:
For all integers a, b, and c, if $a|b$ and $a|c$ then $a|(b + c)$.

Proof: Suppose a, b, and c are any integers such that $a|b$ and $a|c$.
Show: $a|(b + c)$.
By definition of division, $b = ar$ and $c = as$ for some integers r and s.
Then $b + c = ar + as = a(r + s)$. Let $t = r + s$. Then t is an integer because the sum of integers is an integer. So, $b + c = at$, where t is an integer. By definition of division $a|(b + c)$.

4.3.22

For the following statement, determine whether the statement is true or false. Prove the statement directly from the definitions if it is true, and give a counterexample if it is false.
A necessary condition for an integer to be divisible by 6 is that it be divisible by 2

The statement is true.
Proof: Suppose a and b are any integers such that $a = 6b$.
Then $a = 6b = (2)(3b)$. $3b$ is an integer because the product of integers is an integer.
Let $c = 3b$. So, $a = 2c$. By the distributive law of algebra an integer is divisible by 2 if it is divisible by 6.

4.3.23

For the following statement, determine whether the statement is true or false. Prove the statement directly from the definitions if it is true, and give a counterexample if it is false.
A sufficient condition for an integer to be divisible by 8 is that it be divisible by 16.

The statement is false.
Counterexample: Let a be any integer divisible by 8. Let $a = 8$. $8|a = 8/8 = 1$, but $16|a = 8/16 = 0.5$.

4.3.27

For the following statement, determine whether the statement is true or false. Prove the statement directly from the definitions if it is true, and give a counterexample if it is false.
For all integers a, b, c, if $a|(b + c)$ then $a|b$ or $a|c$

This statement is false.
Counterexample: Let $a = 3$, $b = 2$, and $c = 4$.
$a|(b + c) = (2 + 4)/3 = 6/3 = 2$, but $a|b = 2/3 \approx 0.667$ and $a|c = 4/3 \approx 1.333$

4.4.5

For the values of n and d given in the following statement, find integers q and r such that $n = dq + r$ and $0 \leq r < d$. $n =- 45$. $d = 11$

$$-45 = 11q + r, 0 \leq r < 11 \rightarrow -45 = 11(-5) + 10, 0 \leq 10 < 11$$
$$q =- 5, r = 10$$

4.4.8

Evaluate the expression.

a. $50\ div\ 7$
b. $50\ mod\ 7$

   a. 7
   b. 1

4.4.11

Check the correctness of formula (4.4.1) given in Example 4.4.3 for the following values of $DayT$ and N.

a. $DayT = 6(Saturday)$ and $N = 15$
b. $DayT = 0(Sunday)$ and $N = 7$
c. $DayT = 4(Thursday)$ and $N = 12$

a. 15 days is two weeks plus one day, so 15 days from Saturday is Sunday. So, $DayN$ should be 0. $DayN = (DayT + N)mod7 = (6 + 15)mod7 = 21mod7 = 0$
b. 7 days is one week, so 7 days from Sunday is Sunday. So, $DayN$ should be 0. $DayN = (DayT + N)mod7 = (0 + 7)mod7 = 0$
c. 12 days is one week and 5 days, so 12 days from Thursday is Tuesday. So $DayN$ should be 2. $DayN = (DayT + N)mod7 = (4 + 12)mod7 = 2$

4.4.16

Suppose d is a positive integer and n is any integer. If $d|n$, what is the remainder obtained when the quotient-remainder theorem is applied to n with divisor d?

Because $d|n$, $n = dq + 0$ for some integer q. So, the remainder is 0.

4.4.20

Suppose a is an integer. If $a \, mod \, 7 = 4$, what is $5a \, mod \, 7$? In other words, if division of a by 7 gives a remainder of 4, what is the remainder when $5a$ is divided by 7?

> Because $a \, mod \, 7 = 4$, $a = 7q + 4$ for some integer q.
> Thus, $5a = 35q + 20 = 35q + 14 + 6 = 7(5q + 2) + 6$. Since $5q + 2$ is an integer because the products and sums of integers are integers and since $6 < 7$, the remainder obtained when $5a$ is divided by 7 is 6. Therefore, $5a \, mod \, 7 = 6$

4.4.22

Suppose c is an integer. If $c \, mod \, 15 = 3$, what is $10c \, mod \, 15$? In other words, if division of c by 15 gives a remainder of 3, what is the remainder when $10c$ is divided by 15?

> Because $c \, mod \, 15 = 3$, $c = 15q + 3$ for some integer q.
> Thus $10c = 150q + 30 = 15(10c + 2) + 0$. Since $10c + 2$ is an integer because the product and sum of integers are integers and since $0 < 15$, the remainder obtained when $10c$ is divided by 15 is 0. Therefore, $10c \, mod \, 15 = 0$

4.4.26

Prove that a necessary and sufficient condition for a non-negative integer n to be divisible by a positive integer d is that $n \, mod \, d = 0$.

> Proof: Suppose that n and d are any such integer such that $d|n$.
> Show: $n \, mod \, d = 0$.
> Using the quotient remainder theorem, $n = dq + r$ for some integers q and r with $0 \leq r < d$. Because $d|n$, $n = dq + 0$. Therefore, $r = 0$.
> By definition of mod, $n \, mod \, d = r$. Using substitution, $n \, mod \, d = 0$.

4.6.4

Use proof by contradiction to show that for all integers m, $7m + 4$ is not divisible by 7.

> Proof: Suppose not. That is, suppose there is an integer m such that $7m + 4$ is divisible by 7. By definition of divisibility, $7m + 4 = 7k$ for some integer k. Subtracting $7m$ from both sides gives $4 = 7k - 7m = 7(k - m)$. So, by definition of divisibility $7|4$. But by Theorem 4.3.1 this implies that $7 \leq 4$, which contradicts the fact that $7 > 4$. Thus for all integers m, $7m + 4$ is not divisible by 7.

4.6.5
Carefully formulate the negation of the following statement. Then prove the statement by contradiction.
There is no greatest even integer.

Negation: There is a greatest even integer.
Proof: Suppose not. That is, suppose there is a greatest even integer; call it N. Then N is an even integer and $N \geq n$ for every even integer n. Let $M = N + 2$. M is an even integer because it is the sum of even integers, and $M > N$ since $M = N + 2$. This contradicts the supposition that $N \geq n$ for every even integer n.

4.6.10

Prove the following statement by contradiction. The square root of any irrational number is irrational.

Proof: Suppose not. Suppose there is an irrational number x such that $\sqrt{x}$ is rational. By definition of rational, $\sqrt{x} = \frac{a}{b}$ for some integers a and b with $b \neq 0$.

By substitution, $(\sqrt{x})^2 = (\frac{a}{b})^2$. So, by algebra $x = \frac{a^2}{b^2}$. However, a^2 and b^2 are both integers because they are products of integers and b^2 is nonzero by the zero product property. Thus, $\frac{a^2}{b^2}$ is rational. So x is both irrational and rational, which is a contradiction.

4.6.12

Prove the following statement by contradiction. If a and b are rational numbers, $b \neq 0$, and r is an irrational number, then $a + br$ is irrational.

> Proof: Suppose not. Suppose there are rational numbers a and b and an irrational number r such that $b \neq 0$ and $a + br$ is rational. By definition of rational, $a = \frac{c}{d}$, $b = \frac{e}{f}$, and $a + br = \frac{g}{h}$ for some integers c, d, e, f, g, and h with $d \neq 0$, $f \neq 0$, and $h \neq 0$. Using substitution, $\frac{c}{d} + \frac{e}{f}r = \frac{g}{h}$. Subtracting $\frac{c}{d}$ from both sides gives $\frac{e}{f}r = \frac{g}{h} - \frac{c}{d}$. Since $b \neq 0$, $e \neq 0$. Multiplying both sides by $\frac{f}{e}$ gives
> $r = \frac{f}{e}(\frac{g}{h} - \frac{c}{d}) = \frac{fg}{eh} - \frac{fc}{ed} = \frac{fgd - fch}{ehd}$. $fgd - fch$ and ehd are integers because the difference and products of integers are integers. ehd is nonzero because of the zero product property. Thus, $\frac{fgd-fch}{ehd}$ is rational. So r is both irrational and rational, which is a contradiction.

4.6.15

Prove the following statement by contradiction. If a, b, and c are integers and $a^2 + b^2 = c^2$, then at least one of a and b is even.

Proof: Suppose not. That is, suppose there are some integers a, b, and c such that $a^2 + b^2 = c^2$ and a and b are odd.
By definition of odd integers, $a = 2r + 1$ and $b = 2s + 1$. Using substitution, $(2r + 1)^2 + (2s + 1)^2 = c^2$. By algebra, $c^2 = 4r^2 + 4r + 1 + 4s^2 + 4s + 1 = 4(r^2 + r + s^2 + s) + 2$. Let $t = r^2 + r + s^2 + s$, then t is an integer because the products and sums of integers are integers. So, $c^2 = 4t + 2$ or equivalently $c^2 - 2 = 4t$. By definition of divisibility, $c^2 - 2$ is divisible by 4. Therefore, $c^2 = 4m + 2 = 2(2m + 1)$ for some integer m. By definition of even integers, c^2 and n are even. Then $n = 2x$ for some integer x. Substituting into equation $n^2 - 2 = 4m$ gives $4r^2 = 4m + 2 \rightarrow 2r^2 = 2m + 1$. Since r^2 is an integer, this implies that $2m + 1$ is even, but since m is an integer, $2m + 1$ is odd. This is a contradiction because no integer is both even and odd.

4.6.21

Consider the statement "For all integers n, if n^2 is odd then n is odd."

a. Write what you would suppose and what you would need to show to prove this statement by contradiction.
b. Write what you would suppose and what you would need to show to prove this statement by contraposition.

a. Proof by contradiction: Suppose not. Suppose there is an integer n such that n^2 is odd and n is even. Show that this supposition logically leads to a contradiction.
b. Proof by contraposition: Suppose n is an integer such that n is not odd. Show that n^2 is not odd.

4.6.22

Consider the statement "For all real numbers r, if r^2 is irrational then r is irrational."

a. Write what you would suppose and what you would need to show to prove this statement by contradiction.
b. Write what you would suppose and what you would need to show to prove this statement by contraposition.

   a. Proof by contradiction: Suppose not. Suppose there is a real number r such that r^2 is irrational and r is rational.
   b. Proof by contraposition: Suppose r is a real number such that r is not irrational. Show that r^2 is not irrational.

4.7.4

Determine if the following statement is true or false. Prove if it is true and disprove if it is false.
$3\sqrt{2} - 7$ is irrational.

Suppose not. Suppose $3\sqrt{2} - 7$ is rational. By definition of rational, $3\sqrt{2} - 7 = \frac{a}{b}$ for some integers a and b with $b \neq 0$. Then, $\sqrt{2} = \frac{1}{3}(\frac{a}{b} + 7) = \frac{a+7b}{3b}$. But $a + 7b$ and $3b$ are integers because the sum and products of integers are integers and $3b \neq 0$ because of the zero product property. Therefore, $\sqrt{2}$ is rational by definition of rational. This contradicts the fact that $\sqrt{2}$ is irrational, so the supposition is false and $3\sqrt{2} - 7$ is irrational.

4.7.5

Determine if the following statement is true or false. Prove if it is true and disprove if it is false.

$\sqrt{4}$ is irrational.

The statement is false, $\sqrt{4} = 2 = \frac{2}{1}$, which is rational.

4.7.10

Determine if the following statement is true or false. Prove if it is true and disprove if it is false. If r is any rational number and s is any irrational number, then r/s is irrational.

Counterexample: Let $r = 0$ and $s = \sqrt{2}$. Then $\frac{r}{s} = \frac{0}{\sqrt{2}} = 0 = \frac{0}{1}$ which is rational. Thus the statement is false

4.7.12

Determine if the following statement is true or false. Prove if it is true and disprove if it is false. The product of any two irrational numbers is irrational.

Counterexample: Suppose x and y are two irrational numbers. Let $x = \sqrt{2}$ and $y = \sqrt{8}$. $\sqrt{x} \cdot \sqrt{y} = \sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8} = \sqrt{16} = 4$, which is rational. Thus the statement is false.

4.7.15

a. Prove that for all integers a, if a^3 is even then a is even.
b. Prove that $\sqrt[3]{2}$ is irrational.

a. Suppose a is not even. Show a^3 is not even. By definition of odd integers, $a = 2k + 1$. So, $a^3 = (2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1$. Using algebra, $8k^3 + 12k^2 + 6k + 1 = 2(4k^3 + 6k^2 + 3k) + 1$. Let $t = 4k^3 + 6k^2 + 3k$. t is an integer because the sum and product of integers is an integer. Thus, $a^3 = 2t + 1$. Therefore, a^3 is odd according to the definition of odd integers.
b. Suppose not. Suppose $\sqrt[3]{2}$ is rational. By definition of rational $\sqrt[3]{2} = \frac{a}{b}$ for some integers a and b with $b \neq 0$. Without loss of generality, assume that a and b have no common factor. Thus, $2 = (\frac{a}{b})^3 = \frac{a^3}{b^3}$. So, $2b^3 = a^3$. Thus a^3 is divisible by 2, which means a is also divisible by 2. By definition of divisibility, $a = 2k$ for some integer k, thus $a^2 = 8k^3$. Using substitution, $b^3 = 4k^3$. So, b^3 is divisible by 4 which means b is divisible by 4. Consequently, both a and b are divisible by 2 which contradicts the assumption that a and b have no common factor. Therefore the supposition is false, so $\sqrt[3]{2}$ is irrational.

4.7.21 An alternative proof of the irrationality of $\sqrt{2}$ counts the number of 2's on the two sides of the equation $2n^2 = m^2$ and uses the unique factorization of integers theorem to deduce a contradiction. Write a proof that uses this approach.

Proof by contradiction: Suppose not. Suppose $\sqrt{2}$ is rational. By definition of rational, $\sqrt{2} = \frac{a}{b}$ for some integers a and b with $b \neq 0$. Then $2 = \frac{a^2}{b^2}$ so $a^2 = 2b^2$. By the unique factorization of integers theorem, the factorizations of a^2 and $2b^2$ are unique except for the order in which the factors are written. Because every prime factor for a occurs twice in the same prime factorization of a^2, the prime factorization of a^2 contains an even number of 2's. Because every prime factor of b occurs twice in the prime factorization of b^2, the prime factorization of $2b^2$ contains an odd number of 2's. Therefore, the equation $a^2 = 2b^2$ cannot be true. So the supposition is false, and $\sqrt{2}$ is irrational.

4.7.22 Use the proof technique illustrated in exercise 21 to prove that if n is any integer that is not a perfect square, then $\sqrt{n}$ is irrational.

Proof by contradiction: Suppose not. Suppose an integer n exists that is not a perfect square such that $\sqrt{n}$ is rational. By definition of rational, $\sqrt{n} = \frac{a}{b}$ for some integers a and b with $b \neq 0$. Then $n = \frac{a^2}{b^2}$ so $a^2 = nb^2$. By the unique factorization of integers theorem, the factorizations of a^2 and nb^2 are unique except for the order in which the factors are written. Because every prime factor for a occurs twice in the same prime factorization of a^2, the prime factorization of a^2 contains an even number of n's. Because every prime factor of b occurs twice in the prime factorization of b^2, the prime factorization of nb^2 contains an odd number of n's. Therefore, the equation $a^2 = nb^2$ cannot be true. So the supposition is false, and $\sqrt{n}$ is irrational.

4.7.27 Let $p_1, p_2, p_3 \dots$ be a list of all prime numbers in ascending order. Here is a table for the first six:

p_1	p_2	p_3	p_4	p_5	p_6
2	3	5	7	11	13

a. For each $i = 1, 2, 3, 4, 5, 6$, let $N_i = p_1, p_2 \dots p_i + 1$. Calculate $N_1, N_2, N_3, N_4, N_5, N_6$.
b. For each $i = 1, 2, 3, 4, 5, 6$, find the smallest prime number q_i such that q_i divides N_i.

a. $N_1 = 2 + 1 = 3$, $N_2 = 2 \cdot 3 + 1 = 7$, $N_3 = 2 \cdot 3 \cdot 5 + 1 = 31$,
$N_4 = 2 \cdot 3 \cdot 5 \cdot 7 + 1 = 211$, $N_5 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 + 1 = 2311$,
$N_6 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 + 1 = 30031$
b. $q_1 = 3$, $q_2 = 7$, $q_3 = 31$, $q_4 = 211$, $q_5 = 2311$, $q_6 = 59$