Homework 3

Section 1: 4, 8, 12, 16, 20, 24, 28, 32

Section 2: 6, 12, 18, 24, 30, 36, 42, 48

Section 3: 10, 20, 30, 40, 50, 60

Section 4: 4, 8, 12, 16, 20, 24, 28, 32

CSC-7-48309

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1.4 Let $Q(n)$ be the predicate "$n^2 \leq 30$"

a. Write $Q(2)$, $Q(-2)$, $Q(7)$, and $Q(-7)$, and indicate which of these statements are true and which are false.

b. Find the truth set of $Q(n)$ if the domain of n is Z, the set of all integers.

c. If the domain is the set Z^+ of all positive integers, what is the truth set of $Q(n)$.

a. $Q(2) = (2)^2 = 4$, $Q(-2) = (-2)^2 = 4$, $Q(7) = 7^2 = 49$, $Q(-7) = (-7)^2 = -49$

$Q(2)$ and $Q(-2)$ are true, $Q(7)$ and $Q(-7)$ are false.

b. $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

c. $\{1, 2, 3, 4, 5\}$

1.8 Let $B(x)$ be "$-10 < x < 10$." Find the truth set of $B(x)$ for each of the following domains.

a. Z b. Z^+ c. the set of all even integers

a. $\{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0$
$1, 2, 3, 4, 5, 6, 7, 8, 9\}$

b. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

c. $\{-8, -6, -4, -2, 0, 2, 4, 6, 8\}$

1.12 Find a counterexample to show the following statement is false.

$\forall$ real numbers x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

Let $x=1,\ y=1: \sqrt{1+1} = \sqrt{1} + \sqrt{1}$

$\sqrt{2} = 1+1$

$\sqrt{2} \neq 2$

(1.16) Rewrite each of the following statements in the form "$\forall$___ x, ____."

a. All dinosaurs are extinct

* $\forall$ dinosaurs x, x is extinct.

b. Every real number is positive, negative, or zero

* $\forall$ real numbers x, x is positive, negative, or zero

c. No irrational numbers are integers.

* $\forall$ irrational numbers x, x is not an integer

d. No logicians are lazy

* $\forall$ logicians x, x is not lazy

e. The number 2,147,581,953 is not equal to the square root of any integer.

* $\forall$ integers x, x^2 is not 2,147,581,953

f. The number -1 is not equal to the square root of real number.

* $\forall$ real numbers x, x^2 is not -1

(1.20) Rewrite the following statement informally in at least two different ways without using variables or the symbol $\forall$ or the words "for all".

1. The square root of a positive real number is positive.

2. Every positive real number has a positive square root.

$\forall$ real numbers x, if x is positive, then the square root of x positive

(1.24) Rewrite the following statements in the two forms "$\exists$ ___ x such that ___" and "$\exists x$ such that ___ and ___."

a. Some hatters are mad.

"$\exists$ a hatter x, such that x is mad"

"$\exists x$ such that x is a hatter and x is mad"

b. Some questions are easy

"$\exists$ a question x, such that x is easy"

"$\exists x$ such that x is a question and x is easy"

(1.28) Rewrite each statement without using quantifiers or variables. Indicate which are true and which are false and justify why.

Let the domain of x be the set D of objects discussed in mathematics courses, and let Real(x) be "x is a real number," Pos(x) be "x is a positive real number," Neg(x) be "x is a negative real number," and Int(x) be "x is an integer."

a. Pos(0)

* 0 is a positive real number. This statement is false as 0 is neither positive or negative

b. $\forall x$, $Real(x) \wedge Neg(x) \rightarrow Pos(-x)$

* If a real number is negative, then its opposite is positive. This is true as $-(-|x|) = |x|$

c. $\forall x$, $Int(x) \rightarrow Real(x)$

* If a number is a integer, the that number is a real number. This is true as all integers are real numbers.

d. $\exists x$ such that $Real(x) \wedge \sim Int(x)$

* There is a real number that is not an integer. This is true, for example $1/2$ is a real number but not an integer.

1.32

Let R be the domain of the predictable variable x. Which of the following are true and which are false? Give counter examples for the statements that are false.

a. $x > 2 \Rightarrow x > 1$

* True

b. $x > 2 \Rightarrow x^2 > 4$

* True

c. $x^2 > 4 \Rightarrow x > 2$

* false; let $x = -3$: $(-3)^2 > 4$

$9 > 4$ but $-3 \not> 2$

d. $x^2 > 4 \Leftrightarrow |x| > 2$

* true

(2.6) Write a negation for each of the following statements.

a. Sets A and B do not have any points in common.

* Sets A and B have at least one point in common.

b. Towns P and Q are not connected by any road on the map.

* Towns P and Q have at least one road connecting them on the map.

2.12 Determine whether the proposed negation is correct. If not, rewrite a correct negation.

Statement: The product of any irrational number and any rational number is irrational.

Proposed negation: The product of any irrational number and any rational number is rational.

* Correct negation: There is a irrational number and rational number whose product is rational.

2.18 Write a negation for the statement

$\forall x \in R$, if $x(x+1)>0$ then $x>0$ or $x<-1$

$\exists$ a real number x such that $x(x+1)>0$ and both $x \leq 0$ and $x \geq -1$

(2.24) Rewrite the statements in each pair in if-then form and indicate the logical relationship between them.

a. All the children in Tom's family are female.
All the females in Tom's family are children

* If a person is a child in Tom's family, then the person is a female.

If a person is a female in Tom's family, then the person is a child.

The second statement is the converse of the first.

b. All the integers that are greater then 5 and end in 1, 3, 7, or 9 are prime.
All the integers that are greater then 5 and are prime end in 1, 3, 7, or 9.

* If an integer is greater than 5 and ends in 1, 3, 7, or 9, then the integer is prime.

If an integer is greater than 5 and is prime, then the integer ends in 1, 3, 7, or 9.

The first statement is the converse of the first.

2.30 Write the converse, inverse, and contrapositive for the given statement. Indicate which are true and which are false. Provide counter examples for each that is false.

statement: $\forall$ integers a, b, and c, if $a-b$ is even and $b-c$ is even, then $a-c$ is even.

* the statement is true

* converse: $\forall$ integers a, b, and c, if $a-c$ is even, then $a-b$ is even and $b-c$ is even.

false; let $a=3$, $b=2$, $c=1$: $3-1=2$, $3-2=1$, $2-1=1$

in this example $a-c$ is even, but $a-b$ and $b-c$ are not

* inverse: $\forall$ integers a, b, and c, if $a-b$ is not even and $b-c$ is not even, then $a-c$ is not even.

false; let $a=3$, $b=2$, $c=1$: $3-2=1$, $2-1=1$, $3-1=2$

in this example $a-b$ and $b-c$ are not even, but $a-c$ is even

* Contrapositive: $\forall$ integers a, b, and c, if $a-c$ is not even, then $a-b$ is not even and $b-c$ is not even.

this statement is true

(2.36) If $P(x)$ is a predicate and the domain of x is the set of all real numbers, let R be "$\forall x \in \mathbb{Z}, P(x)$," let S be "$\forall x \in \mathbb{Q}, P(x)$" and let T be "$\forall x \in \mathbb{R}, P(x)$"

a. Find a definition for $P(x)$ (but do not use $x \in \mathbb{Z}$) so that R is true and both S and T are false.

* Let $P(x)$ be $2x \neq 1$

b. Find a definition for $P(x)$ (but do not use $x \in \mathbb{Q}$) so that both R and S are true and T is false

* Let $P(x)$ be $x \neq \sqrt{2}$

2.42 Rewrite the statement in if-then form.

Passing a comprhensive exam is a necessary condition for obtaining a master's degree.

* If a person does not pass a comprehensive exam then they will not obtain a master's degree

(2.48) A frequent-flyer club brochure states, "You may select among carriers only if they offer the same lowest fare" Assuming that "only if" has its formal, logical meaning, does this statement guarantee that if two carriers offer the same lowest fare, the customer will be able to choose between them?

* No, interpretted formally the statement only guarantees customers cannot choose between carriers if the lowest fares are not the same.

3.10 Determine whether each of the following statements is true or false.

a. ∀ students S, ∃ a dessert D such that S chose D.

* true, all students chose a dessert

b. ∀ students S, ∃ a salad T such that S chose T

* false, there is a student that did not choose a salad

c. ∃ a dessert D, such that ∀ students S, S chose D

* true, every student chose pie

d. ∃ a beverage B such that ∀ students D, D chose B

* false, no beverage was chosen by all students

e. ∃ a item I such that ∀ students S, S did not choose I

* false, there is not an item no student chose

f. ∃ as station Z such that ∀ students S, ∃ an item I such that S chose I from Z.

* true, all students chose pie from the dessert station

(3.20) Recall that reversing the order of the quantifiers in a statement with two different quantifiers may change the truth value of the statement – but it does not necessarily do so. All the statements in the pair refer to the Tarski World of figure 3.3.1 In each pair, the order of the quantifiers is reversed but everything else is the same. For each pair, determine whether the statements have the same or opposite truth values. Justify your answers.

a. (1) For all squares y there is a triangle x such that x and y have a different color.

(2) There is a triangle x such that for all squares y, x and y have different colors.

* The statements have opposite truth values. statement (1) is true in all cases, but statement (2) is false in all cases.

b. (1) For all circles y there is a square x such that x and y have different colors.

(2) There is a square x such that for all circles y, x and y have the same color.

* The statements have the same truth values. In both statements, every case is false.

(3.30) a. Write a new statement by interchanging the symbols $\forall$ and $\exists$

b. State which is true: the given statement, the version with interchanged quantities, neither, or both.

$\exists x \in R$ such that $\forall y \in R^-$, $x > y$

a. $\forall x \in R$, $\exists y \in R^-$ such that $x > y$

b. Both statements are true

(3.40) In informal speech most sentences of the form "There is ___ every ___" are intended to be understood as meaning "$\forall$ ___ $\exists$ ___," even though the existenial quantifier there is comes before the universal quantifier every. Rewrite using quantifiers and variables.

a. There is a sucker born every minute.

* $\forall$ minute m, $\exists$ a sucker s such that s was born in minute m.

b. There is a time for every purpose under heaven.

* $\forall$ purpose p, $\exists$ a time t such that every t has a p

(3.50) For every object x, there is an object y such that $x \neq y$ and x and y have different colors.

a. indicate whether the statement is true or false and justify your answer.

b. write the given statement using the formal logical notation

c. write the negation of the given statement using the formal logical notation.

a. true, every object has one other object thats a diffirent color.

b. $\forall x(\exists x(x \neq y \rightarrow \sim SameColor(x,y)))$

c. $\exists x(\forall x(x \neq y \wedge SameColor(x,y)))$

3.60 Find the answers Prolog would give if added to the program given in example 3.3.11

a. ? isabove (w_1, g)

b. ? color $(w_2, blue)$

c. ? isabove (X, b_1)

a. no

b. no

c. $X = g$

4.4 If the first two premises of universal transitivity are written as "Any x that makes $P(x)$ true makes $Q(x)$ true" and "Any x that makes $Q(x)$ true makes $R(x)$ true," then the conclusion can be written as "____."

$\forall x$ if $P(x)$ then $Q(x)$

$\forall x$ if $Q(x)$ then $R(x)$

$\therefore \forall x$ if $P(x)$ then $R(x)$

4.8 State if the following statement is valid or invalid.

All freshmen must take writing.

Caroline is a freshman

∴ Caroline must take writing

* Valid via universal modus ponens

4.12

All honest people pay their taxes

Darth is not honest

$\therefore$ Darth does not pay taxes

* Invalid due to converse error

(4.16)

If a number is even, then twice that number is even

The number $2n$ is even, for a particular number n

$\therefore$ The particular number n is even

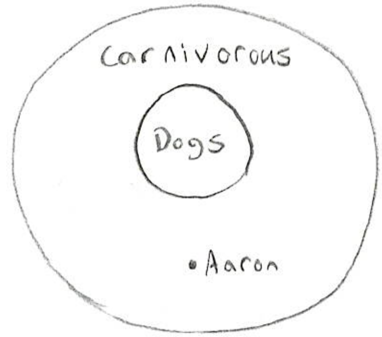
* Invalid due to converse error

4.20 a. Use a diagram to show that the following arguement can have true premises and a false conclusion.

All dogs are carnivorus

Aaron is not a dog

$\therefore$ Aaron is not carnivorous



b. What can you conclude about the validity or invalidity of the following arguement form? Explain how the result from (a) leads to this conclusion.

$\forall x$, if $P(x)$ then $Q(x)$

$\sim P(a)$ for a particular a

$\therefore \sim Q(a)$

*The argument is invalid due to the converse error.

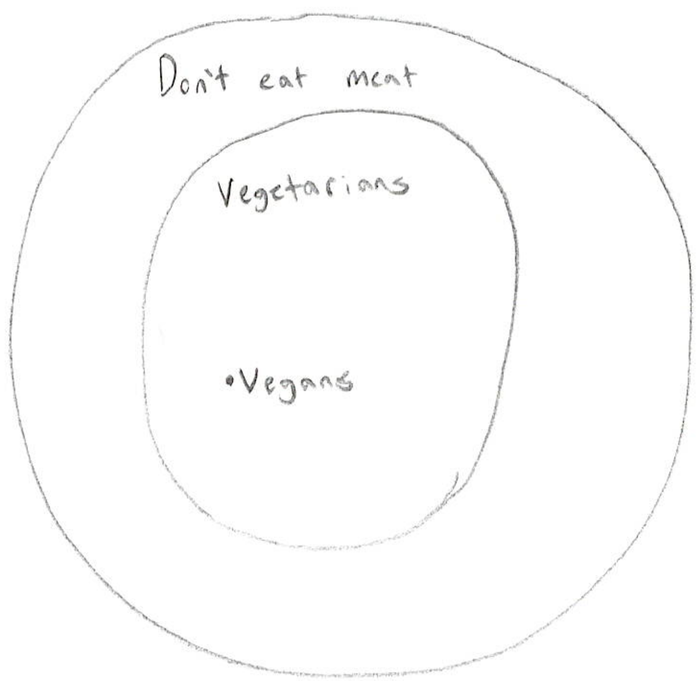
(a) is invalid because of this error

4.24 Indicate if valid or invalid.

No vegetarians eat meat

All vegans are vegetarians

∴ No vegans eat meat.



* Valid

4.28 Reorder the premises to show that the conclusion follows as a valid consequence from the premises.

1. Every object that is to the right of all the blue objects is above all the triangles
2. If an object is a circle, then it is to the right of all the blue objects.
3. If an object is not a circle, then it is not gray

∴ all gray objects are above all the triangles

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3. Contrapositive; if object is gray, then object is a circle
2. If an object is a circle, then it is to the right of all blue objects.
1. Every object right of all blue objects is above all triangles.

∴ All gray objects are above all the triangles

4.32 Reorder the following premises to show that the conclusion follows a valid consequence from the premises.

1. When I work a logic example without grumbling, you may be sure it is one I understand.

2. The arguments in these examples are not arranged in regular like the ones I'm used to.

3. No easy examples make my head ache

4. I can't understand examples if the arguments are not arranged like I'm used too

5. I never grumble at an example unless it gives me a headache

∴ These questions are not easy

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1. grumble → ~understand
2. ~ordered
3. ~easy → headache
4. ~ordered → ~understand
5. headache → grumble

→

2. ~ordered
4. ~ordered → ~understand
1. ~understand → grumble
5. grumble → headache
3. headache → ~easy
∴ ~easy

$\boxed{(2) \to (4) \to (1) \to (5) \to (3) \to \therefore \text{not easy}}$