Quantum Phase Estimation

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Qiskit User Meetup

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Why Quantum Phase Estimation?

Why Quantum Phase Estimation?

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Fig ref: H. Ritsch, Quantum computer factorizes more efficiently, IQOQI (2016).

Why Quantum Phase Estimation?

Qiskit

Please visit!

https://qiskit.org/textbook/

Deutsch-Jozsa Algorithm	Bernstein-Vazirani Algorithm	Simon's Algorithm	Shor's Algorithm

Quantum Phase Estimation

vs Shor’s Algorithm

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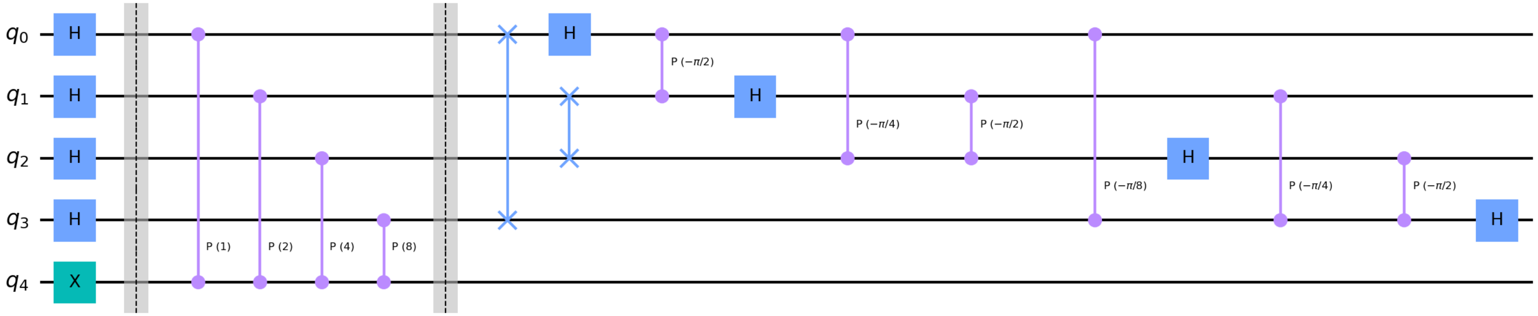
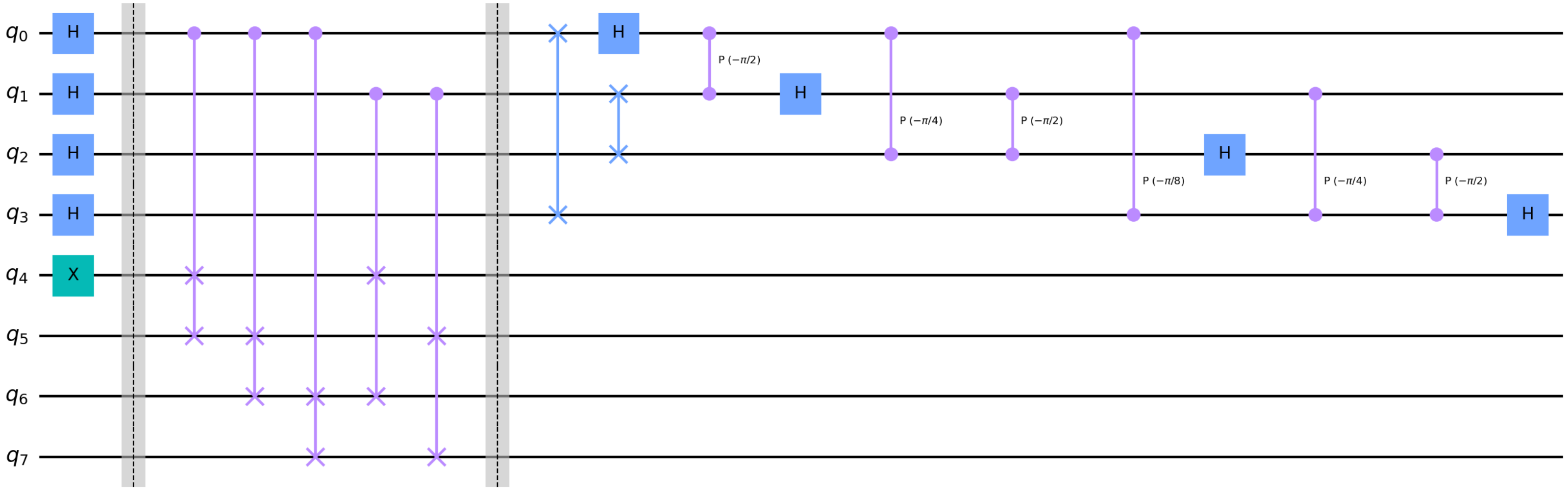
QPE



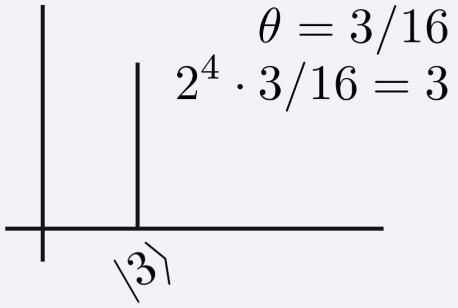
Shor’s Algorithm

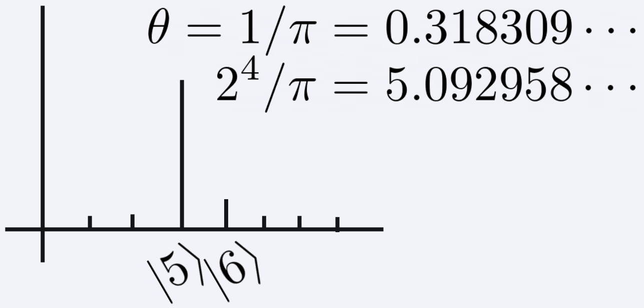


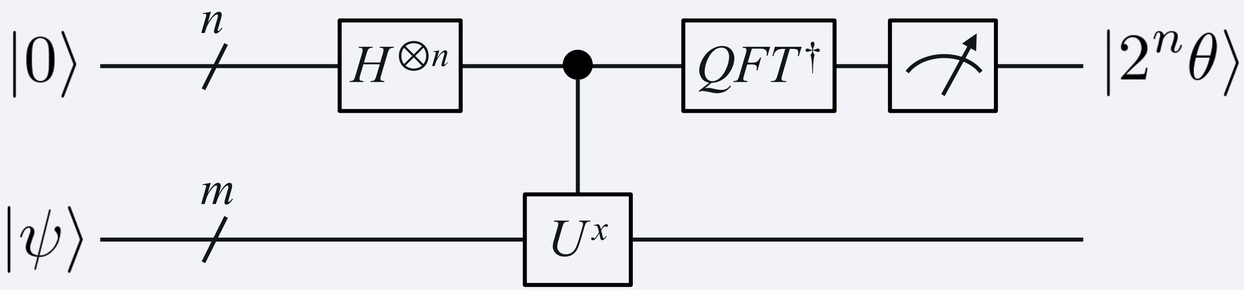
Quantum Phase Estimation

How can we measure the phase from a gate?

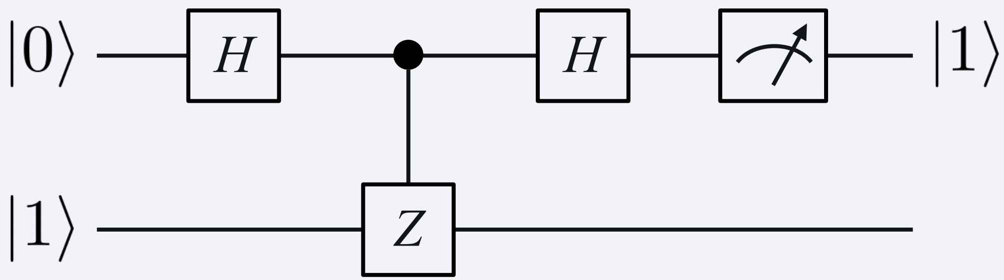
$$U\,|\psi\rangle = e^{2\pi i\theta}\,|\psi\rangle$$







Phase Kickback

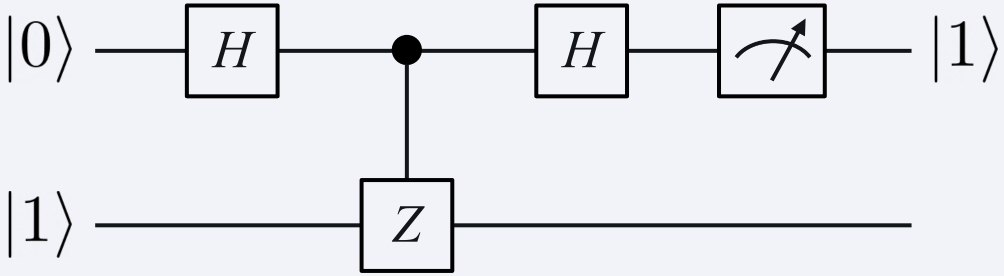


Gates in the Phase Kickback Circuit

$\boxed{H}$: Hadamard gate

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$



Gates in the Phase Kickback Circuit

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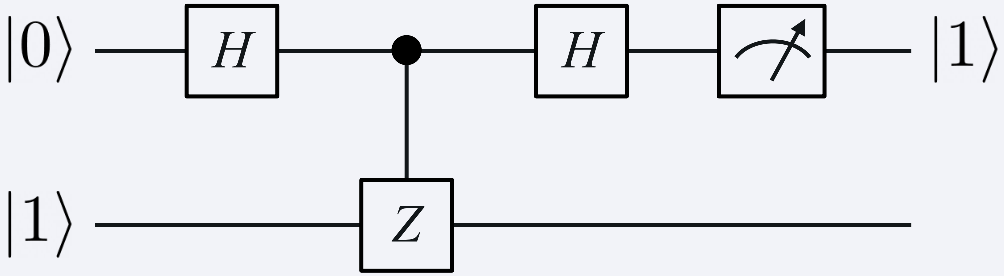
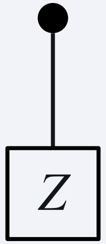
$\boxed{Z}$: Z gate

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$



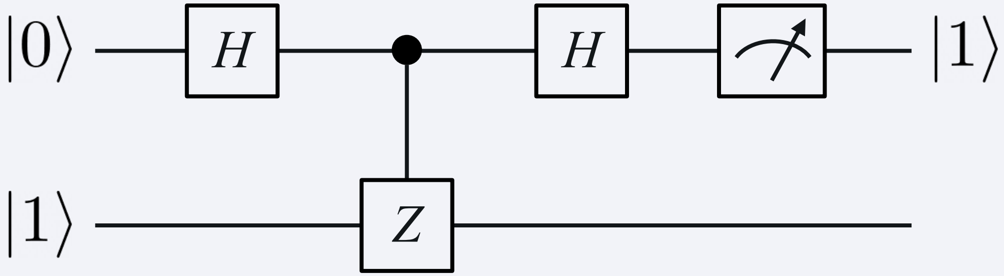
Gates in the Phase Kickback Circuit



: CZ gate

$$CZ\,|1\rangle\,|0\rangle = |1\rangle\,(Z\,|0\rangle) = |1\rangle\,|0\rangle$$

$$\begin{aligned} CZ\,|1\rangle\,|1\rangle &= |1\rangle\,(Z\,|1\rangle) \\ &= |1\rangle\,(-\,|1\rangle) = -\,|1\rangle\,|1\rangle \end{aligned}$$



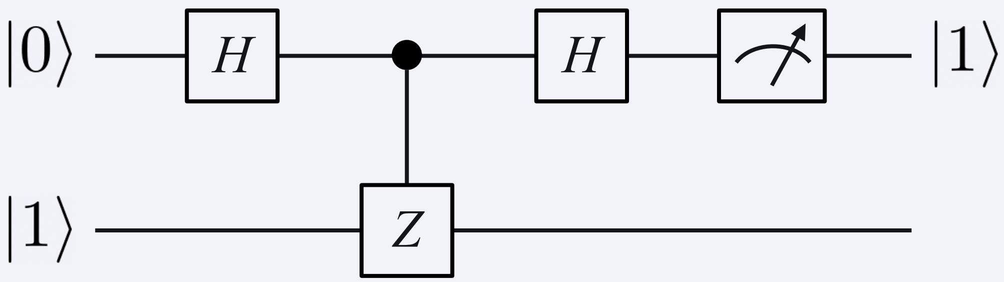
Phase Kickback

$$|0\rangle\,|1\rangle$$

$$\xrightarrow{H\otimes I} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|1\rangle = \frac{|0\rangle\,|1\rangle + |1\rangle\,|1\rangle}{\sqrt{2}}$$

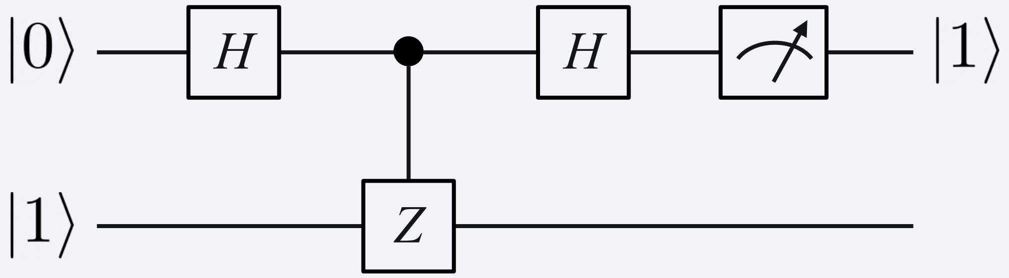
$$\xrightarrow{CZ} \frac{|0\rangle\,|1\rangle - |1\rangle\,|1\rangle}{\sqrt{2}} = \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)|1\rangle$$

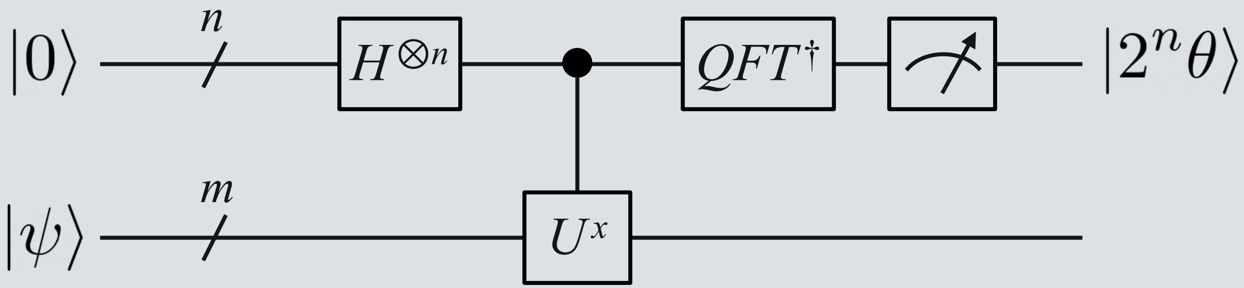
$$\xrightarrow{H\otimes I} |1\rangle\,|1\rangle$$



Phase Kickback

vs Quantum Phase Estimation

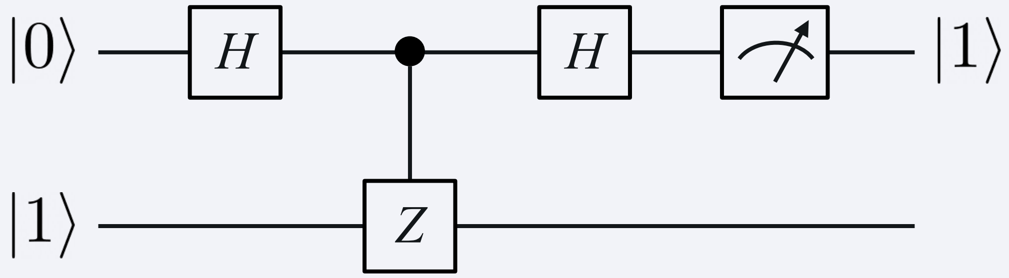




Gates in the QPE Circuit

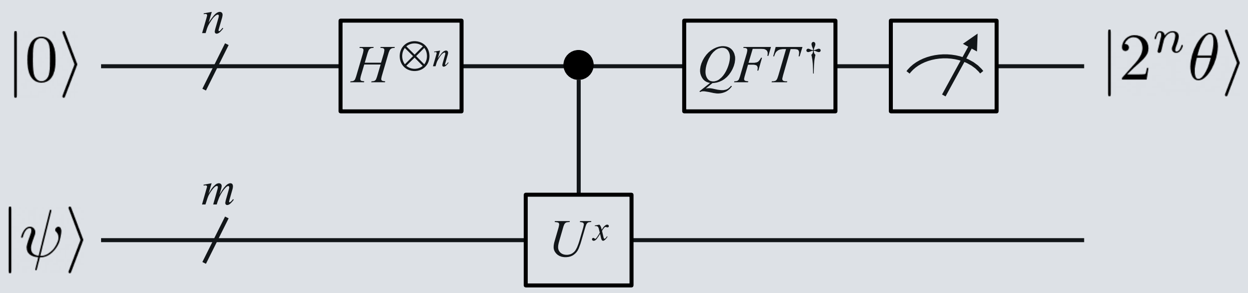
$\boxed{Z}$: Z gate

$$Z|1\rangle = -|1\rangle = e^{2\pi i \frac{1}{2}}|1\rangle$$



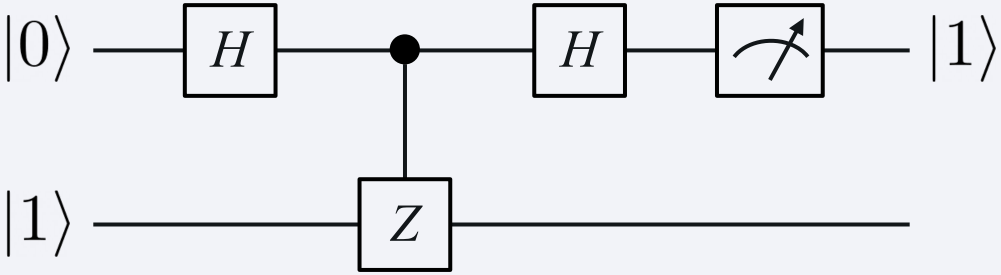
$\boxed{U}$: Some unitary gate giving phase

$$U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$$

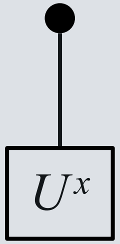


Gates in the QPE Circuit

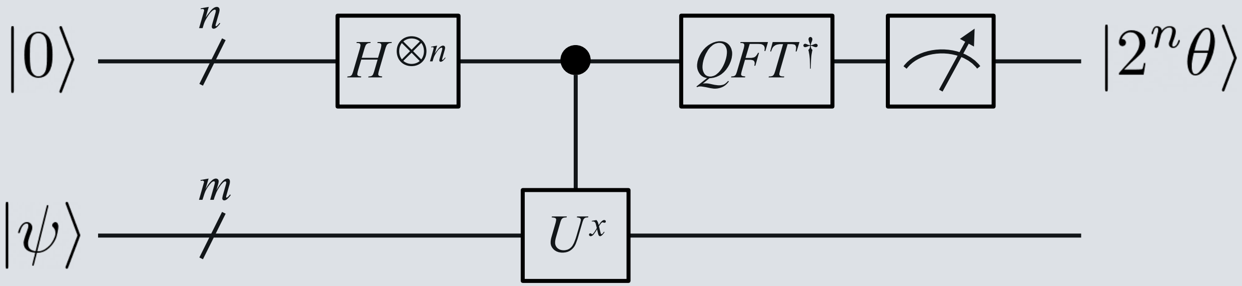
: Z gate



$$Z|1\rangle = -|1\rangle = e^{2\pi i \frac{1}{2}}|1\rangle$$



: Controlled unitary gate with input x

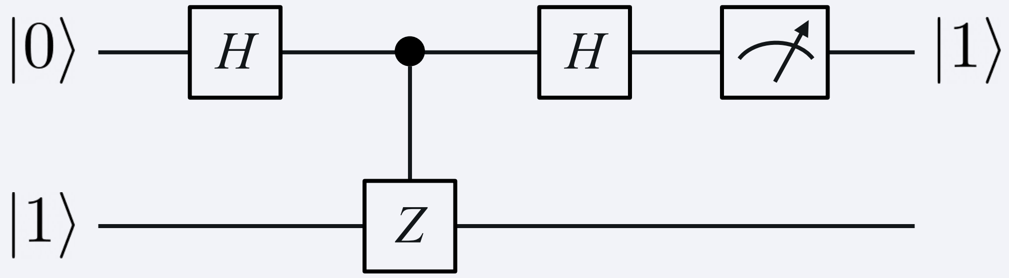


$$\begin{aligned} CU|x\rangle|\psi\rangle &= |x\rangle(U^x|\psi\rangle) \\ &= e^{2\pi i\theta x}|x\rangle|\psi\rangle \end{aligned}$$

Gates in the QPE Circuit

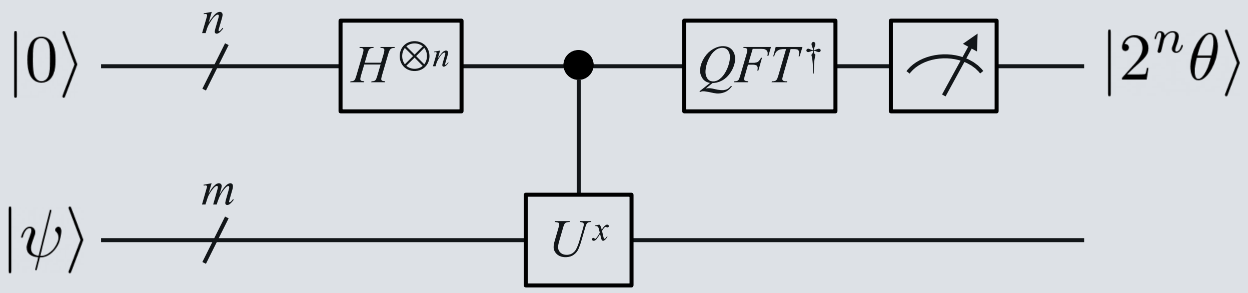
$\boxed{H}$: Hadamard gate

$$H\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = |1\rangle$$



$\boxed{QFT^\dagger}$: Quantum Fourier Transform (inversed)

$$QFT^\dagger\left(\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i \frac{2^n\theta}{2^n}k}\,|k\rangle\right) = |2^n\theta\rangle$$



Gates in the QPE Circuit

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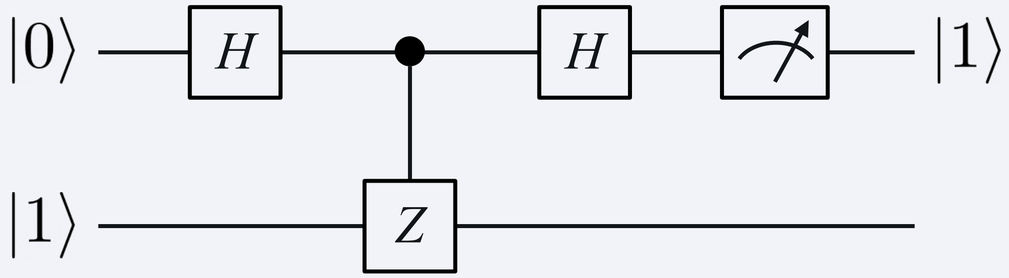
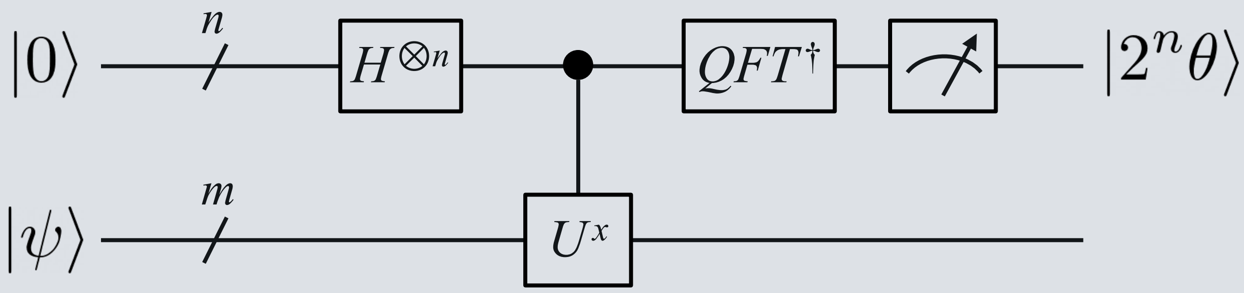
$\boxed{H}$: Hadamard gate

$$H\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = H\left(\frac{|0\rangle + e^{2\pi i \frac{1}{2}1}|1\rangle}{\sqrt{2}}\right) = \left|2^1 \cdot \frac{1}{2}\right\rangle = |1\rangle$$



$\boxed{QFT^\dagger}$: Quantum Fourier Transform (inversed)

$$QFT^\dagger\left(\frac{1}{\sqrt{2^n}}\sum_{k=0}^{2^n-1} e^{2\pi i \frac{2^n\theta}{2^n}k}|k\rangle\right) = |2^n\theta\rangle$$



Gates in the QPE Circuit

$\boxed{H}$:

$H\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right.$

$\boxed{QFT^\dagger}$

$QFT^\dagger\left(\frac{1}{\sqrt{2^n}}\sum_{k=0}\right)$

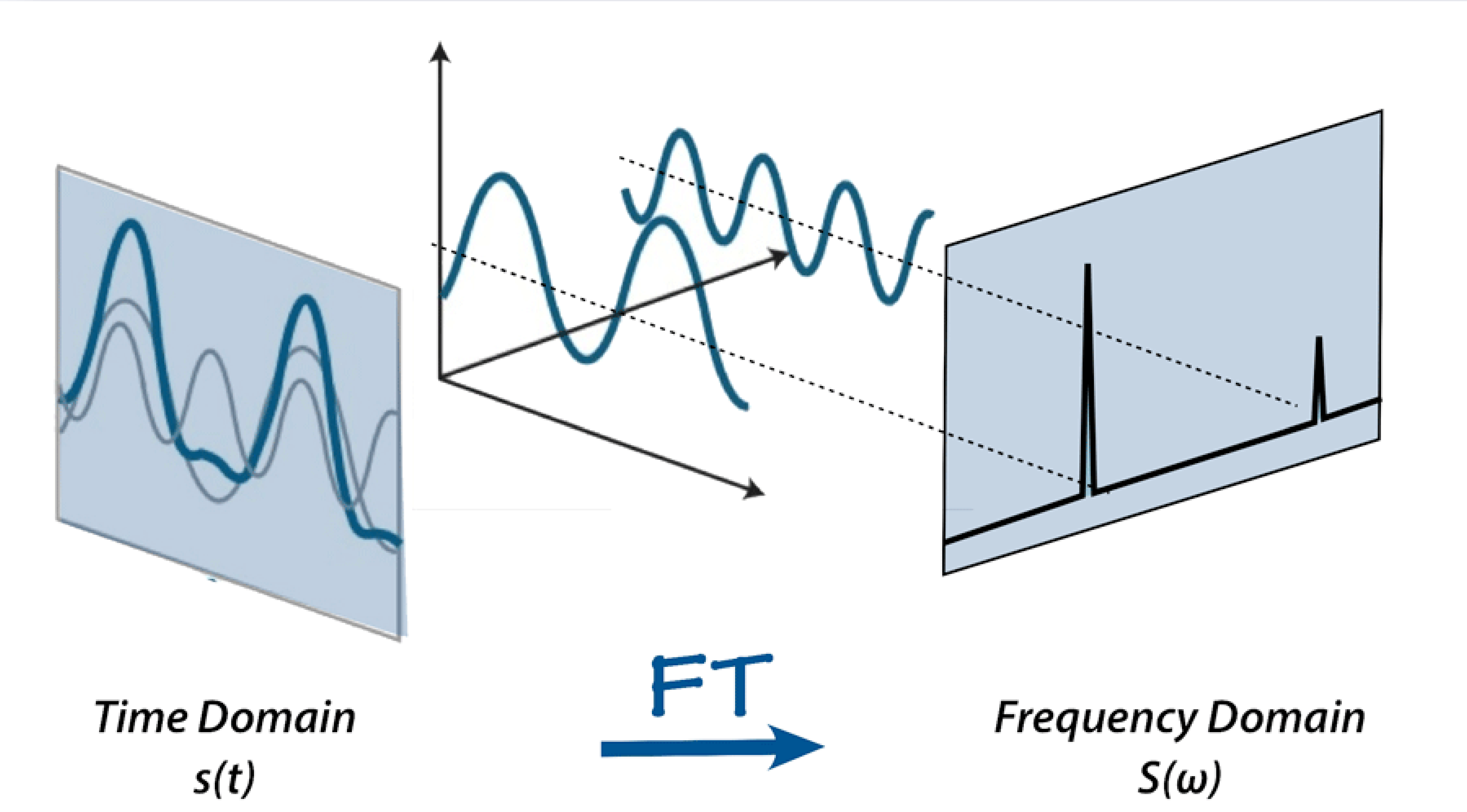


Figure ref: https://mriquestions.com/fourier-transform-ft.html

$|1\rangle$

$|2^n\theta\rangle$

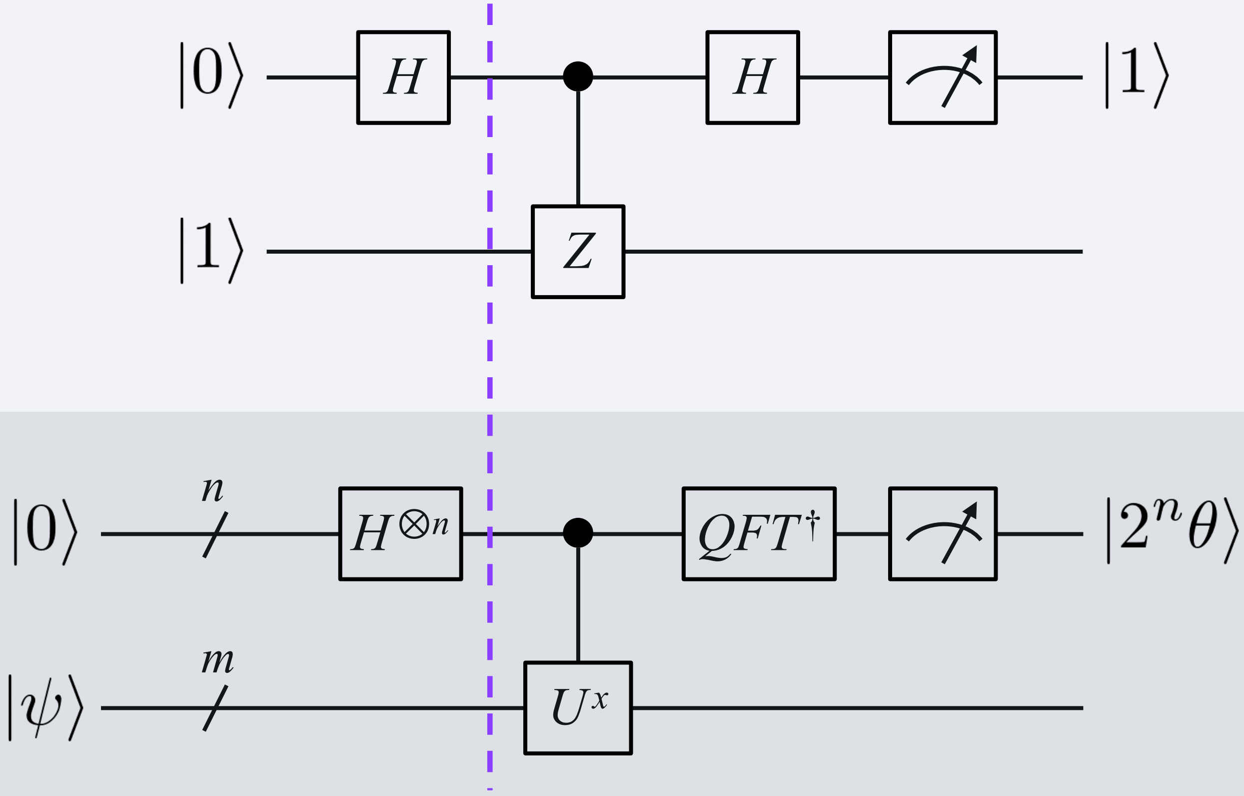
Superposition

$$|0\rangle\,|1\rangle$$

$$\xrightarrow{H\otimes I} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|1\rangle = \frac{|0\rangle\,|1\rangle + |1\rangle\,|1\rangle}{\sqrt{2}}$$

$$|0\rangle^{\otimes n}\,|\psi\rangle$$

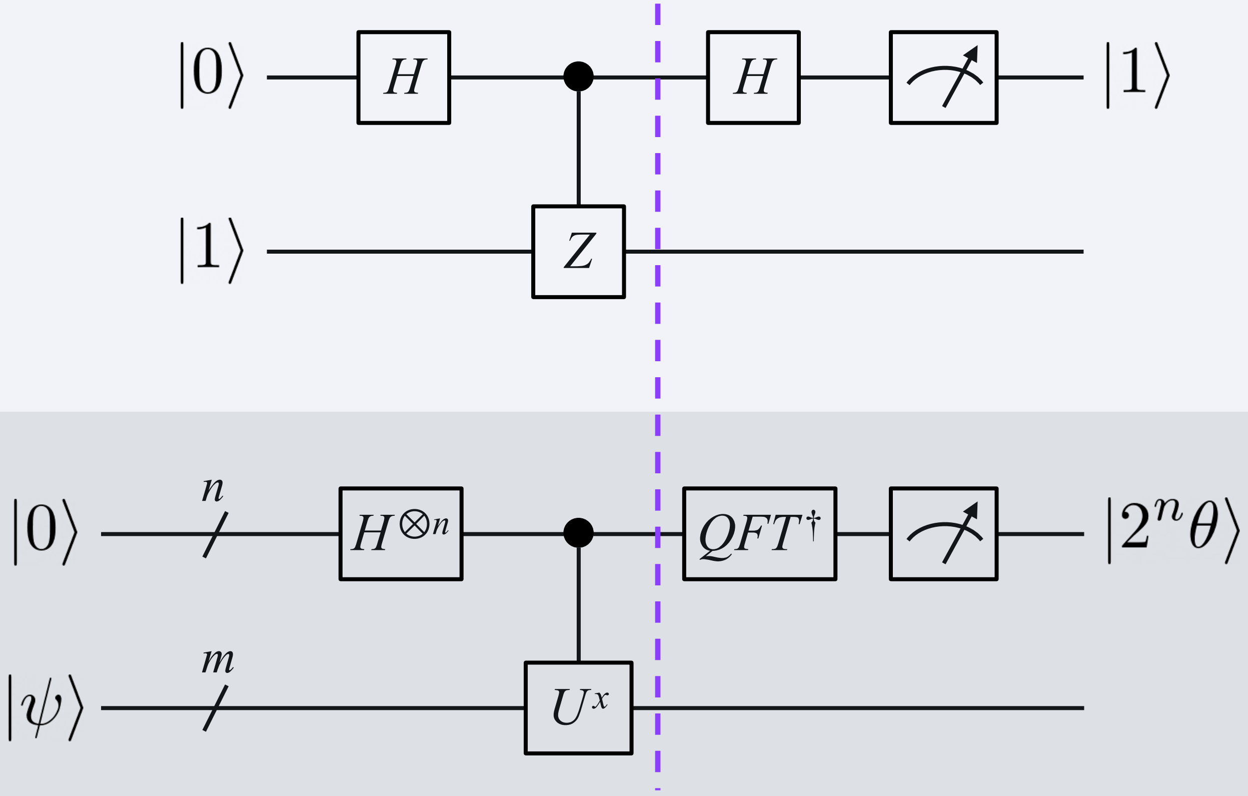
$$\xrightarrow{H^{\otimes n}\otimes I} \frac{1}{\sqrt{2^n}}\left(\sum_{k=0}^{2^n-1}|k\rangle\right)|\psi\rangle$$



Phase Kickback

$$\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|1\rangle$$

$$\xrightarrow{CZ}\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|1\rangle$$



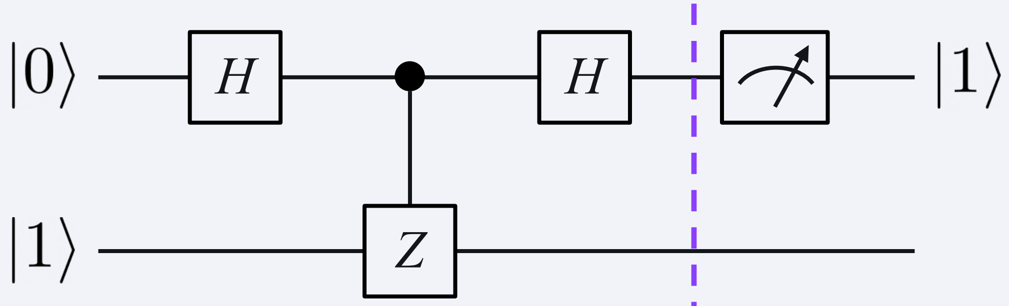
$$\frac{1}{\sqrt{2^n}}\left(\sum_{k=0}^{2^n-1}|k\rangle\right)|\psi\rangle$$

$$\xrightarrow{CU}\frac{1}{\sqrt{2^n}}\left(\sum_{k=0}^{2^n-1}e^{2\pi i\theta k}|k\rangle\right)|\psi\rangle$$

Quantum Fourier Transform

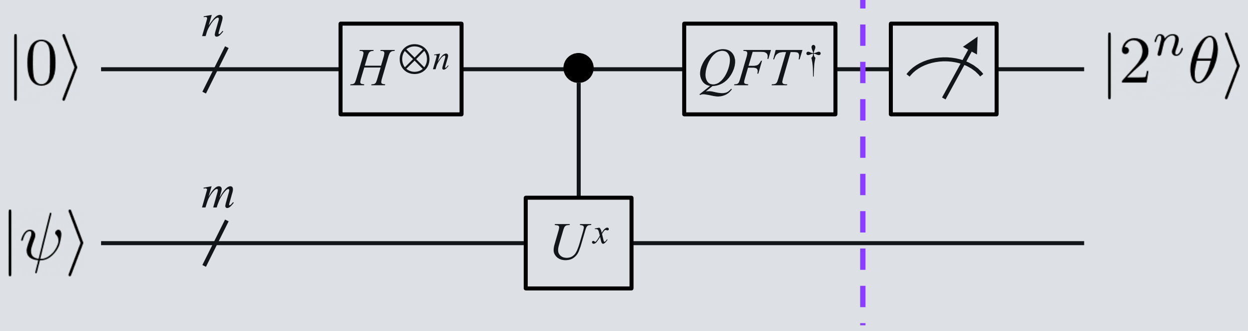
$$\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)|1\rangle$$

$$\xrightarrow{H\otimes I} |1\rangle\,|1\rangle$$



$$\frac{1}{\sqrt{2^n}}\left(\sum_{k=0}^{2^n-1} e^{2\pi i\theta k}\,|k\rangle\right)|\psi\rangle$$

$$\xrightarrow{QFT^\dagger} \frac{1}{2^n}\left(\sum_{x,k=0}^{2^n-1} e^{-\frac{2\pi i k}{2^n}(x-2^n\theta)}\,|x\rangle\right)|\psi\rangle = |2^n\theta\rangle\,|\psi\rangle$$



Quantum Phase Estimation

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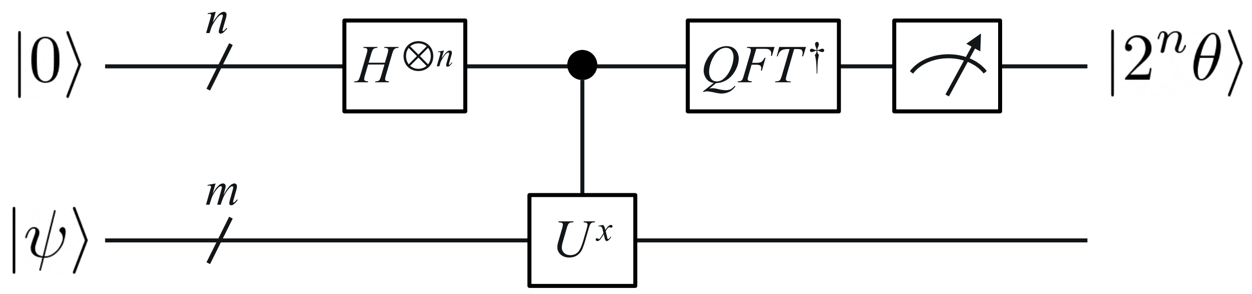
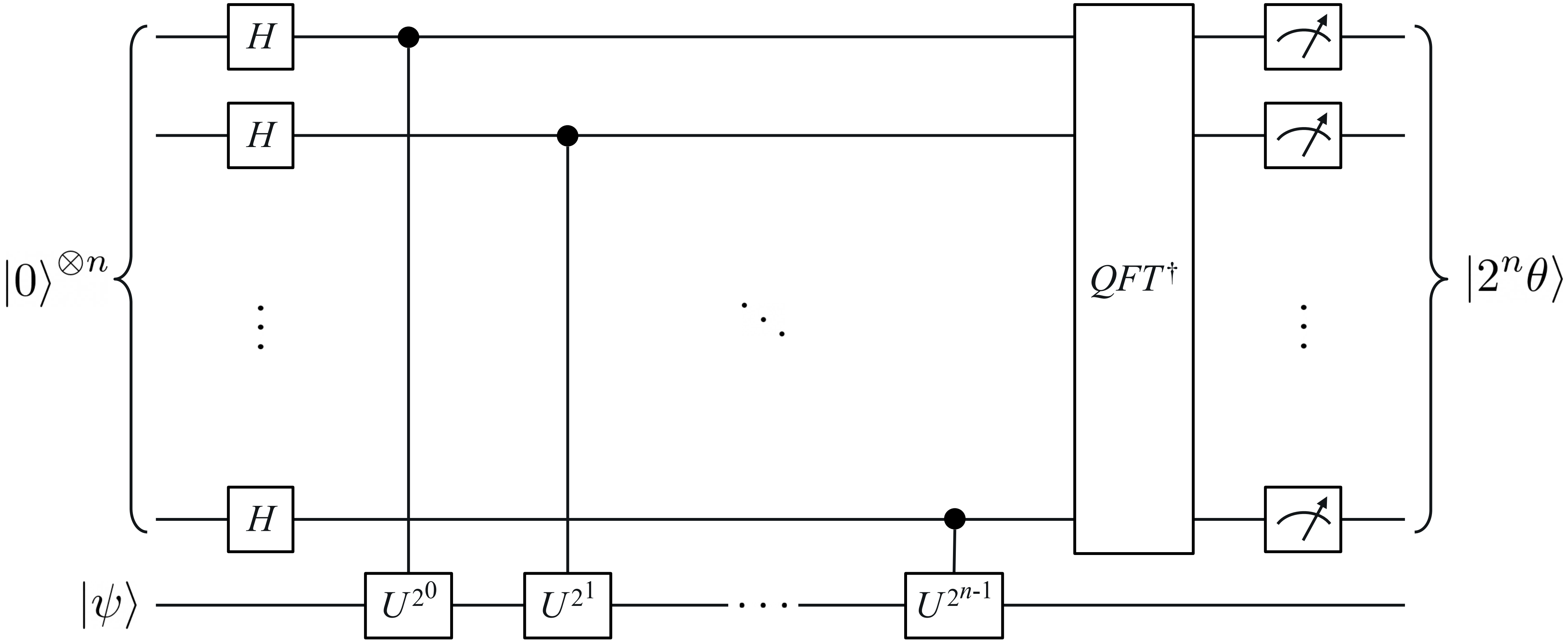


Quantum Phase Estimation

Qiskit



Thank you

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Shor's Algorithm

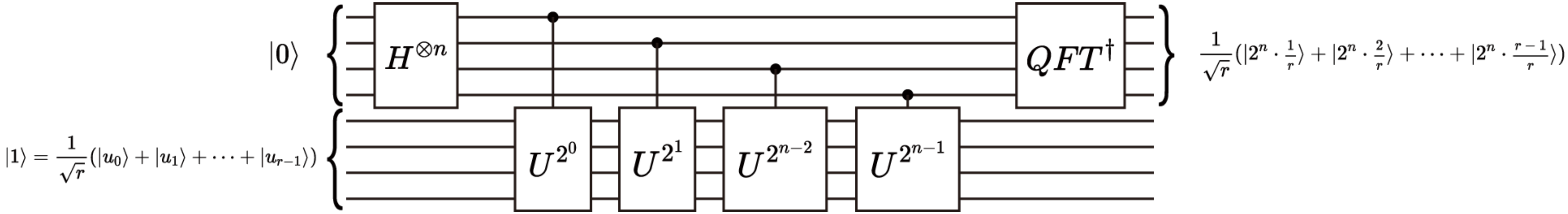

Figure ref: https://qiskit.org/textbook/ch-algorithms/shor.html

Answering question vs Building algorithm

Qiskit

Hard

```
   13546843495787 5665
+  76234821497465 1348
   -------------------
                     ?
```

Easy

```
                  _1_1_
   135468434957875665
+  762348214974651928
   ------------------
               ......7593
```

Quantum Fourier Transform

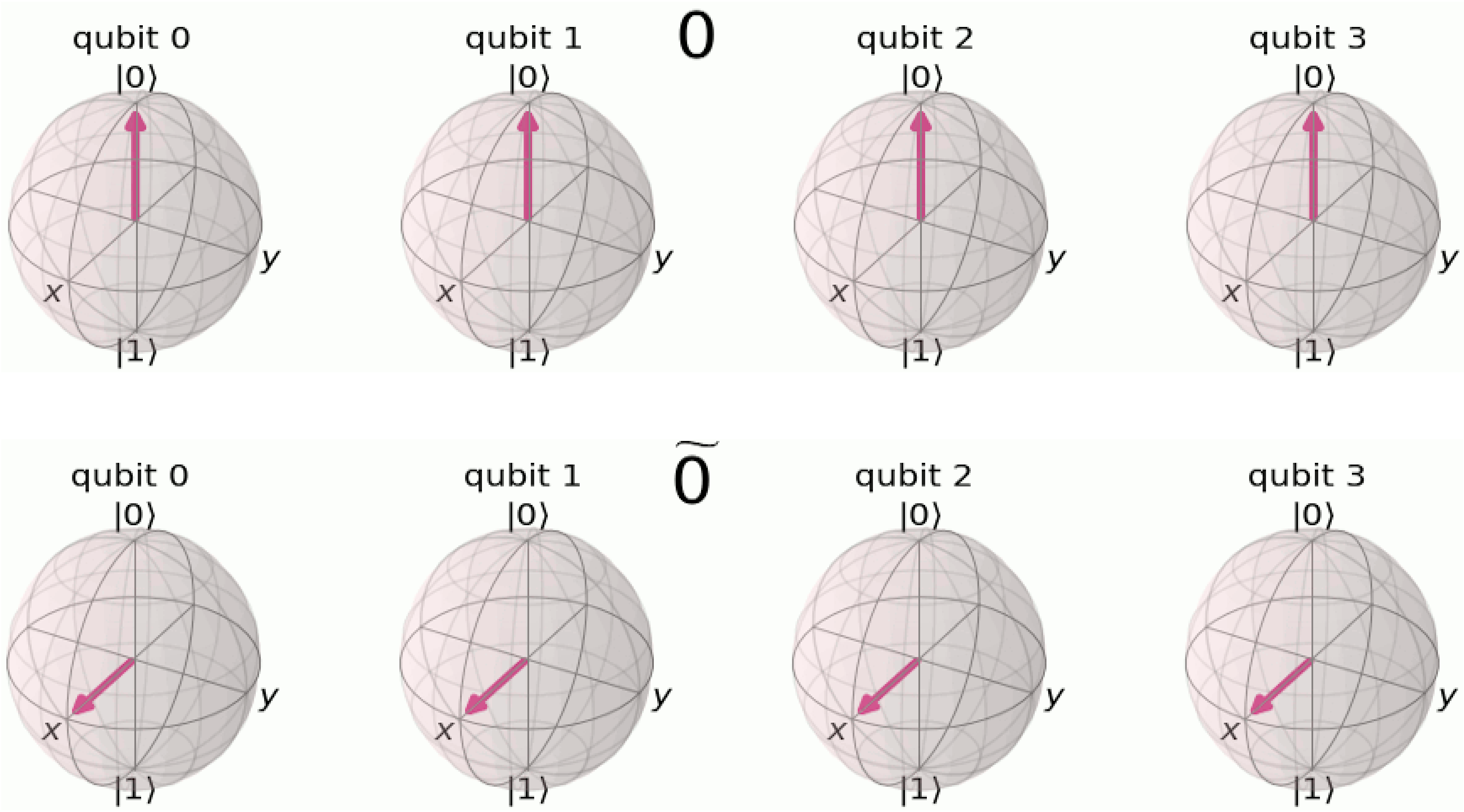

Figure ref: https://qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html