Introduction to Quantum Machine Learning
&
Parameterized Quantum Circuit

Siheon Park, M.S. Candidate
KAIST School of Electrical Engineering

Qiskit

Introduction to

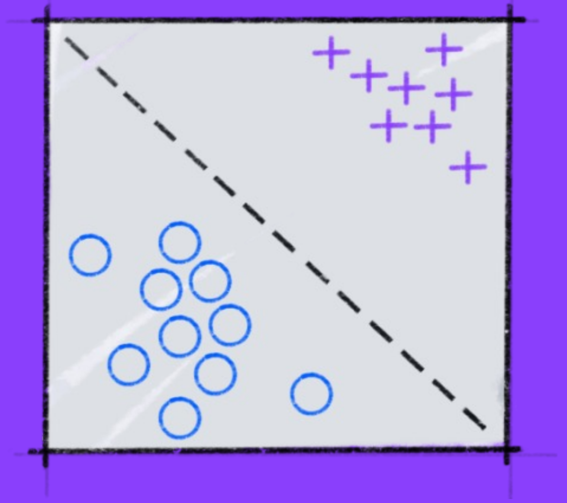
Quantum
Machine
Learning

Qiskit

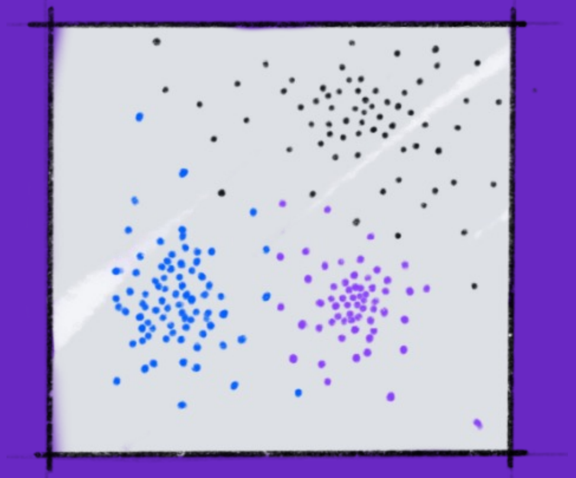
Recap: Machine Learning

- Supervised learning
  - $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^{M},\ p(\hat{y}|\hat{x}, \mathcal{D})$
  - Data classification, regression, etc.
- Unsupervised learning
  - $\mathcal{D} = \{x_i\}_{i=1}^{M},\ p(\hat{x}|\mathcal{D})$
  - Data clustering, generative models, etc.
- Reinforcement learning (RL)
  - Agent, Environment
  - Robotic control, game play, etc.

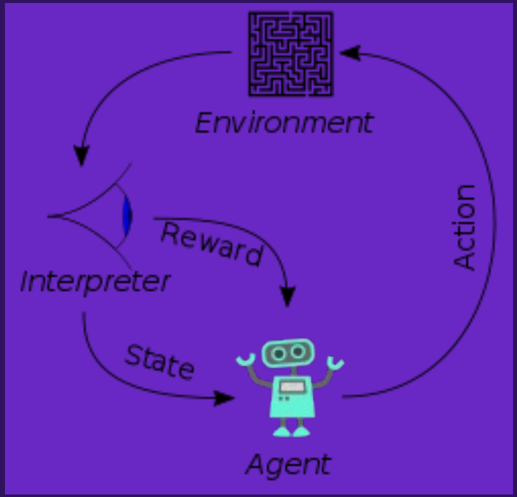
Supervised learning

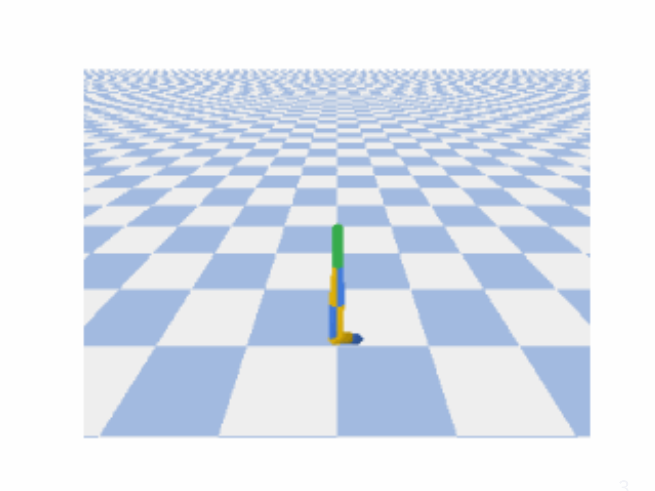


Unsupervised learning



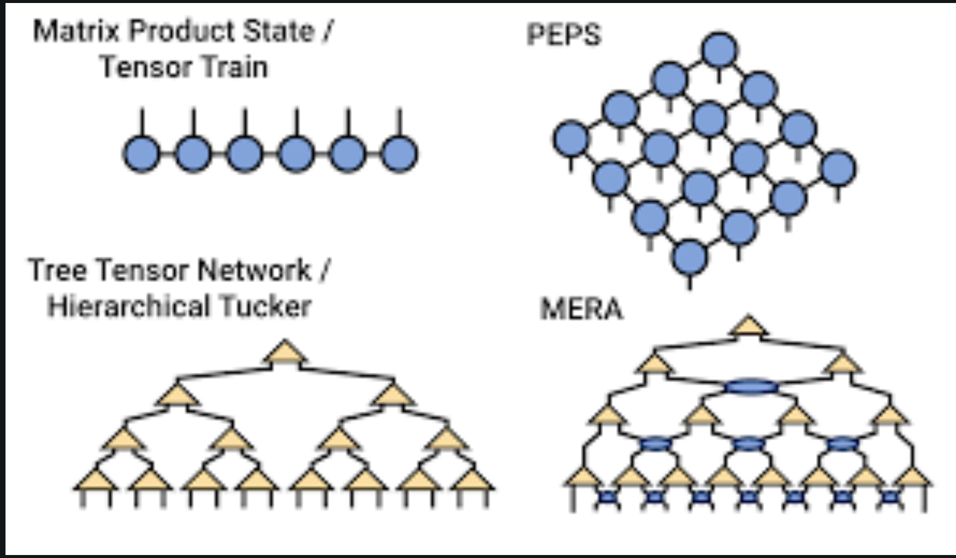
Reinforcement learning

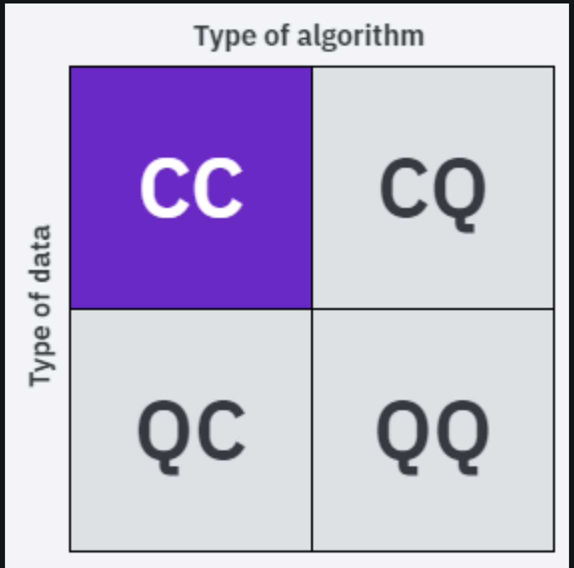




Categories of QML

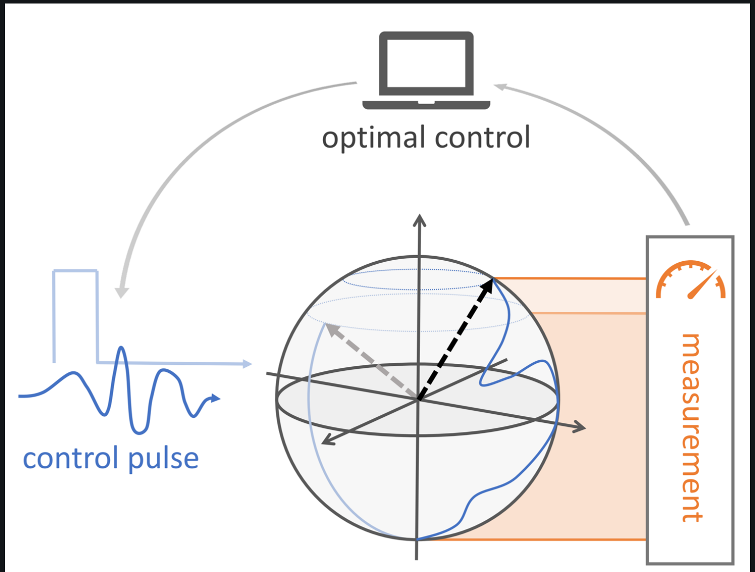
- Process classical data with classical computers
- “Inspired” by quantum systems
- Simulated Annealing, Quantum Monte Carlo, Tensor networks, etc.

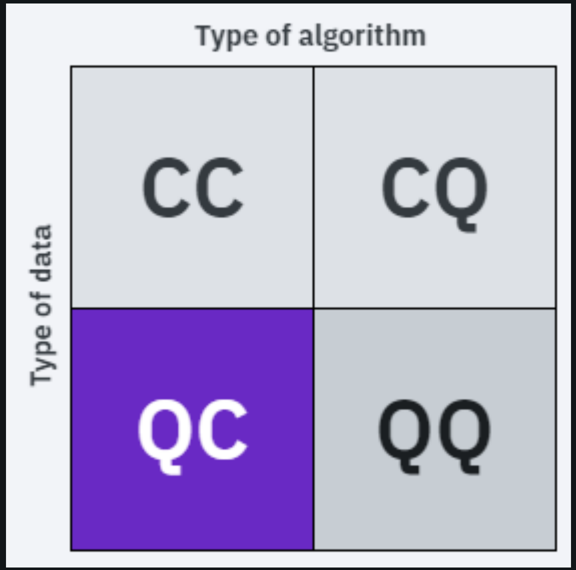




Categories of QML

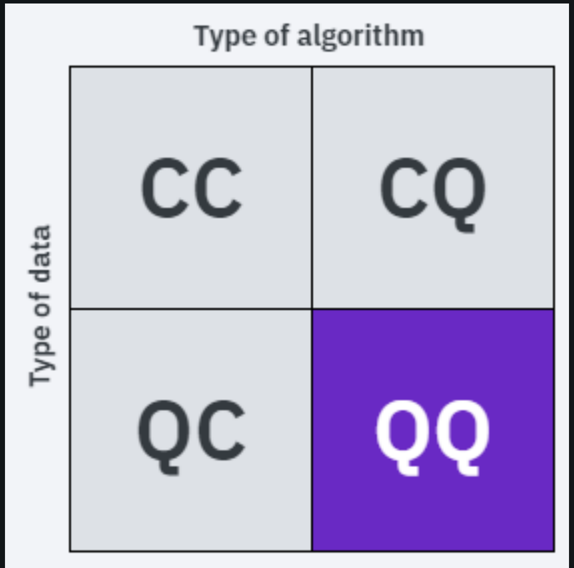
- Process quantum data with classical computers
- Inspect quantum systems with ML
- Qubit characterization, quantum optimal control, readout error mitigation, etc.

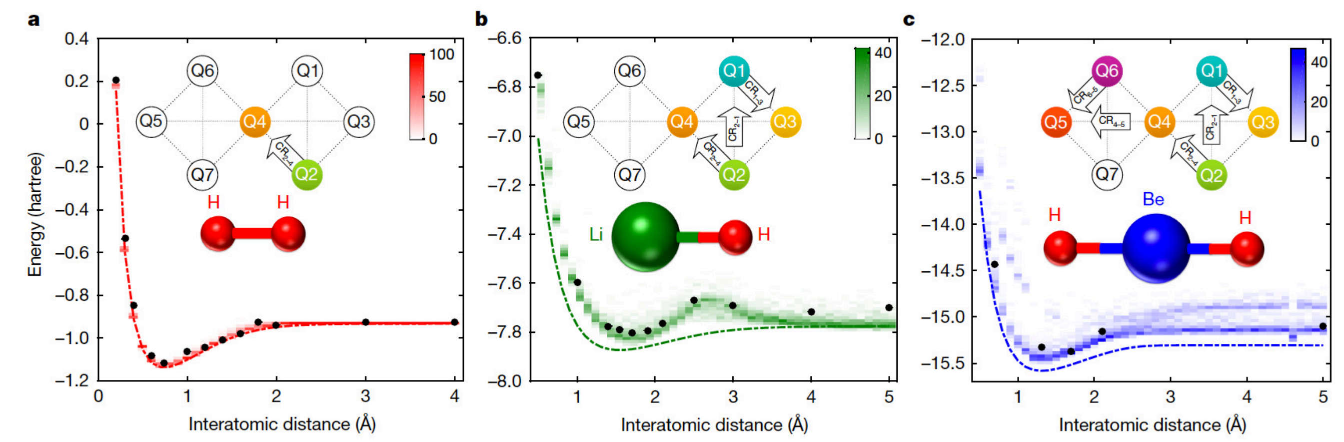




Categories of QML

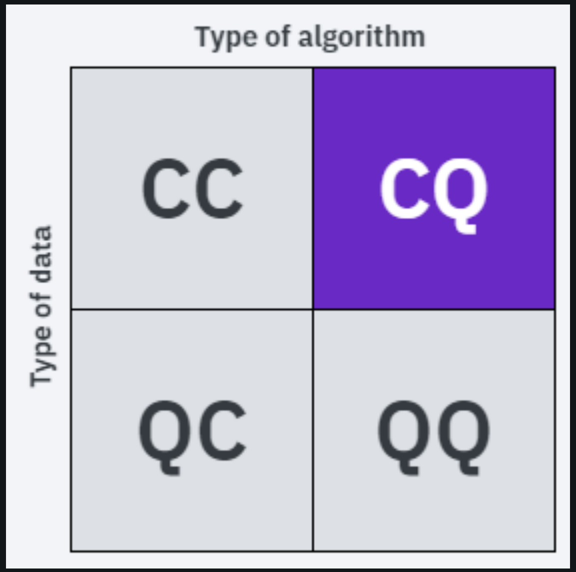
- Process quantum data with quantum computers
- Measure quantum system → quantum ML
- Or full quantum state as input
- Quantum many-body systems, Quantum Simulation, Quantum Chemistry





Categories of QML

- Process classical data with quantum computers
- Main focus!!
- 99% of all quantum-related ML (maybe?)

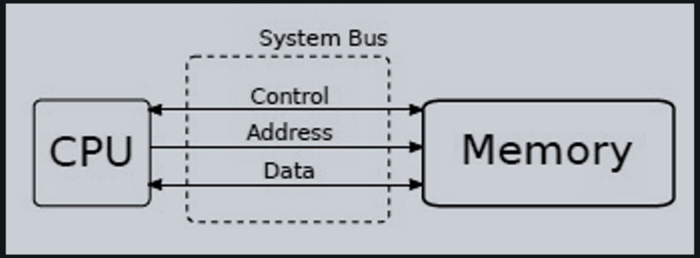


A. With Power of **QRAM** (Skeptical)
B. Without **QRAM** (Near-term)

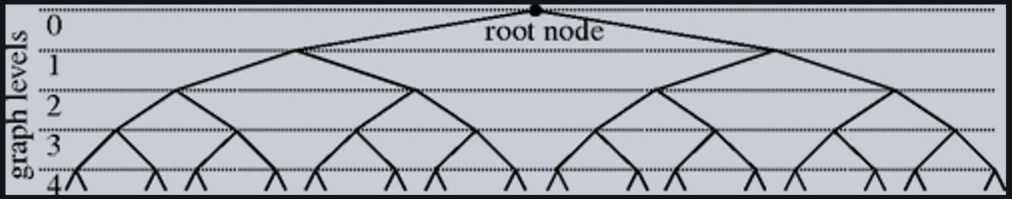
Quantum RAM

Random Access Memory

- Bus: [(ctrl)][address bits=i][data bits=0]
- → [(ctrl)][address bits=i][data bits=$data(i)$]



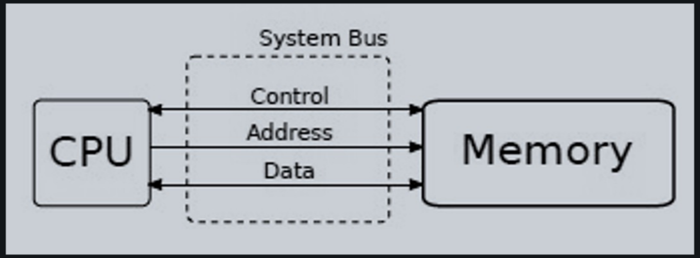
- Very fast $\mathcal{O}(\log M)$, access on at a time



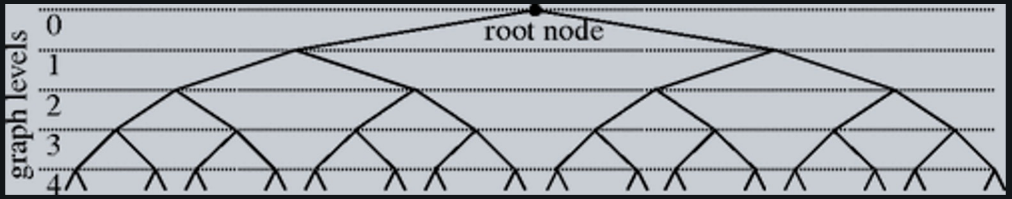
Quantum RAM

Random Access Memory

- Bus: [(ctrl)][address bits=i][data bits=0]
- → [(ctrl)][address bits=i][data bits=$data(i)$]

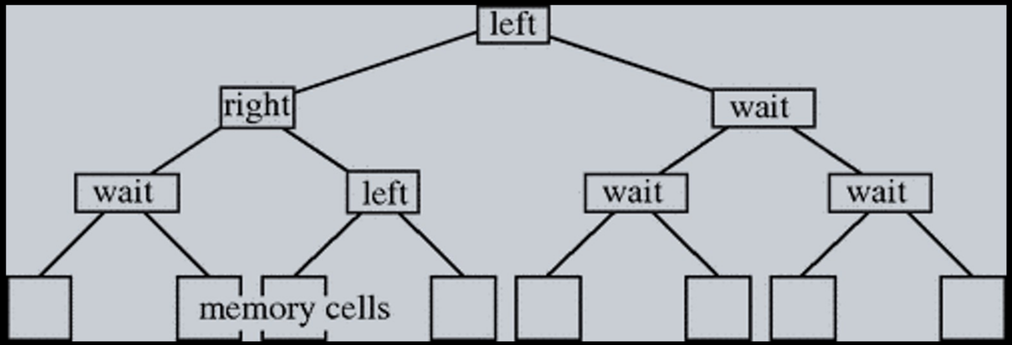


- Very fast $\mathcal{O}(\log M)$, access on at a time



Quantum Random Access Memory

- Load classical data to quantum state:
$$QRAM|i\rangle|0\rangle = |i\rangle|x_i\rangle$$
- Also, load all data simultaneously;
$$QRAM\left(\frac{1}{\sqrt{M}}\sum_{i=1}^{M}|i\rangle\right)|0\rangle = \frac{1}{\sqrt{M}}\sum_{i=1}^{M}|i\rangle|x_i\rangle$$
- Very fast $\mathcal{O}(\log M)$ ➔ Exponential Speed-up



$$|\psi_{QRAM}\rangle = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} |i\rangle|x_i\rangle$$

Index register $|i\rangle$:
$\log M$ qubits,

Data register
$|x_i\rangle = \frac{1}{\sqrt{\|x_i\|}} \sum_{j=1}^{N} x_j^{(i)} |j\rangle$:
$\log N$ qubits

System size
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$
Really?

Access time
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$

$$|\psi_{QRAM}\rangle = \frac{1}{\sqrt{M}}\sum_{i=1}^{M}|i\rangle|x_i\rangle$$

Index register $|i\rangle$:
$\log M$ qubits,

Data register
$|x_i\rangle = \frac{1}{\sqrt{\|x_i\|}}\sum_{j=1}^{N} x_j^{(i)}|j\rangle$:
$\log N$ qubits

System size
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$
Really?

Access time
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$

$$\rho_{QRAM} = \frac{1}{M}\sum_{i,j=1}^{M}|i\rangle\langle j| \otimes |x_i\rangle\langle x_j|$$

Let's trace out index register

$$\rho_{data} = \frac{1}{M}\sum_{i=1}^{M}|x_i\rangle\langle x_i|$$

Let's trace out data register

$$\boldsymbol{\rho_{index} = \frac{1}{M}\sum_{i,j=1}^{M}\langle x_j|x_i\rangle|i\rangle\langle j| = \frac{K}{tr(K)}}$$

$$|\psi_{QRAM}\rangle = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} |i\rangle |x_i\rangle$$

Index register $|i\rangle$:
$\log M$ qubits,

Data register
$|x_i\rangle = \frac{1}{\sqrt{\|x_i\|}} \sum_{j=1}^{N} x_j^{(i)} |j\rangle$:
$\log N$ qubits

System size
$\mathcal{O}(MN) \to \mathcal{O}(\log MN)$
Really?

Access time
$\mathcal{O}(MN) \to \mathcal{O}(\log MN)$

ρ_{QRAM}

$$= \frac{1}{M} \sum_{i,j=1}^{M} |i\rangle\langle j| \otimes |x_i\rangle\langle x_j|$$

Let's trace out index register

$$\rho_{data} = \frac{1}{M} \sum_{i=1}^{M} |x_i\rangle\langle x_i|$$

Let's trace out data register

$\boldsymbol{\rho_{index}}$

$$= \frac{1}{M} \sum_{i,j=1}^{M} \langle x_j | x_i \rangle |i\rangle\langle j|$$

$$= \frac{K}{tr(K)}$$

Playing with Density matrix

- Matrix Exponentiation
  $\rho \to e^{-i\rho\Delta t}$
- Matrix Inversion
  $|x\rangle = \rho^{-1} |b\rangle$

This techniques achieves exponential quantum speed-up
$\mathcal{O}(M^2 N) \to \mathcal{O}(\log MN)$

&

Very useful

$$|\psi_{QRAM}\rangle = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} |i\rangle |x_i\rangle$$

Index register $|i\rangle$:
$\log M$ qubits,

Data register
$|x_i\rangle = \frac{1}{\sqrt{\|x_i\|}} \sum_{j=1}^{N} x_j^{(i)} |j\rangle$:
$\log N$ qubits

System size
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$
Really?

Access time
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$

$$\rho_{QRAM} = \frac{1}{M} \sum_{i,j=1}^{M} |i\rangle\langle j| \otimes |x_i\rangle\langle x_j|$$

Let's trace out index register

$$\rho_{data} = \frac{1}{M} \sum_{i=1}^{M} |x_i\rangle\langle x_i|$$

Let's trace out data register

$$\boldsymbol{\rho_{index}} = \frac{1}{M} \sum_{i,j=1}^{M} \langle x_j | x_i \rangle |i\rangle\langle j| = \frac{K}{tr(K)}$$

Playing with Density matrix

- Matrix Exponentiation
  $\rho \rightarrow e^{-i\rho\Delta t}$
- Matrix Inversion
  $|x\rangle = \rho^{-1}|b\rangle$

This techniques achieves exponential quantum speed-up
$\mathcal{O}(M^2N) \rightarrow \mathcal{O}(\log MN)$

&

Very useful

Applications

- qPCA
$\mathcal{O}(\min\{M^2N, N^2M\})$
$\rightarrow \mathcal{O}(\epsilon^{-2} R \log \min\{M, N\})$
- qSVM
$\mathcal{O}(M^2N)$
$\rightarrow \mathcal{O}(\kappa_{eff}^3 \epsilon^{-3} \log MN)$
- qClustering
$\mathcal{O}(M^2N)$
$\rightarrow \mathcal{O}(\epsilon^{-1} k \log kMN)$

$$|\psi_{QRAM}\rangle = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} |i\rangle|x_i\rangle$$

Index register $|i\rangle$:
$\log M$ qubits,

Data register
$|x_i\rangle = \frac{1}{\sqrt{\|x_i\|}} \sum_{j=1}^{N} x_j^{(i)} |j\rangle$:
$\log N$ qubits

System size
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$
Really?

Access time
$\mathcal{O}(MN) \rightarrow \mathcal{O}(\log MN)$

$$\rho_{QRAM} = \frac{1}{M} \sum_{i,j=1}^{M} |i\rangle\langle j| \otimes |x_i\rangle\langle x_j|$$

Let's trace out index register

$$\rho_{data} = \frac{1}{M} \sum_{i=1}^{M} |x_i\rangle\langle x_i|$$

Let's trace out data register

$$\boldsymbol{\rho_{index}} = \frac{1}{M} \sum_{i,j=1}^{M} \langle x_j | x_i \rangle |i\rangle\langle j| = \frac{K}{tr(K)}$$

Playing with Density matrix

- Matrix Exponentiation
  $\rho \rightarrow e^{-i\rho\Delta t}$
- Matrix Inversion
  $|x\rangle = \rho^{-1}|b\rangle$

This techniques achieves exponential quantum speed-up
$\mathcal{O}(M^2 N) \rightarrow \mathcal{O}(\log MN)$

&

Very useful

Applications

- qPCA
  $\mathcal{O}(\min\{M^2 N, N^2 M\})$
  $\rightarrow \mathcal{O}(\epsilon^{-2} R \log \min\{M, N\})$
- qSVM
  $\mathcal{O}(M^2 N)$
  $\rightarrow \mathcal{O}\left(\kappa_{eff}^3 \epsilon^{-3} \log MN\right)$
- qClustering
  $\mathcal{O}(M^2 N)$
  $\rightarrow \mathcal{O}(\epsilon^{-1} k \log kMN)$

Issue with QRAM-based algorithms

Qiskit

- Complex quantum subroutines → Requires Fault-tolerant QC
- Low-rank Approximation (Read the fine print!!)
- Not good for non-linear algebra
- No viable hardware candidate for realizing qRAM (not that I know of…)

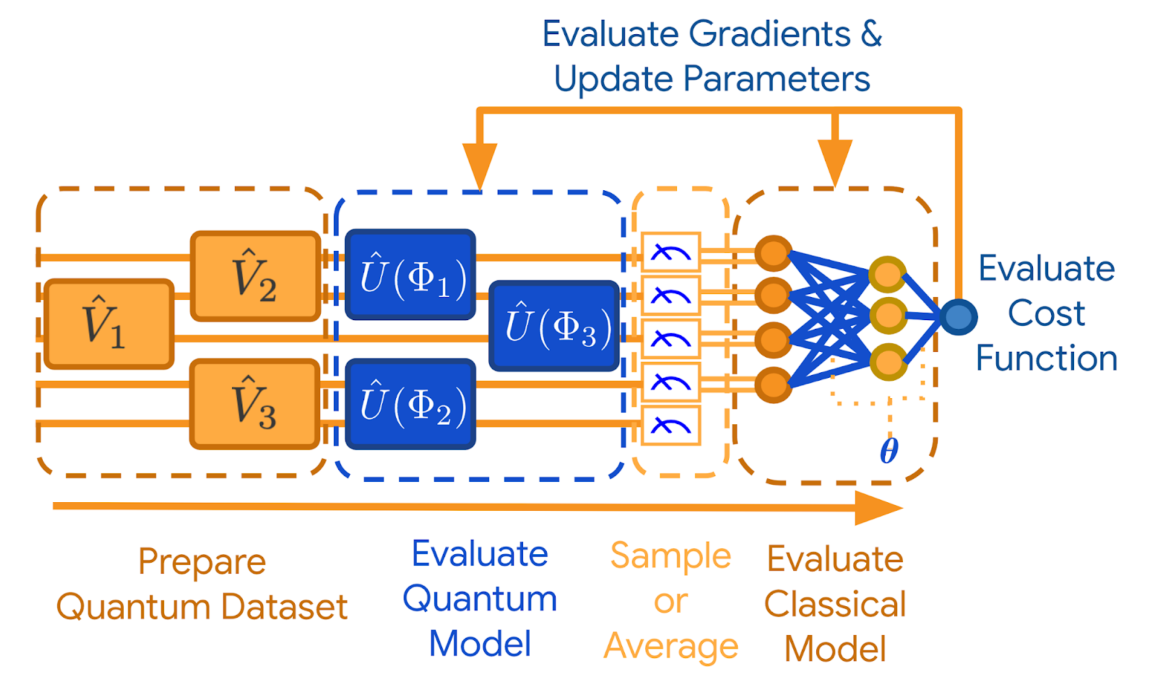
- QML without efficient data loading assumption!!

Introduction to

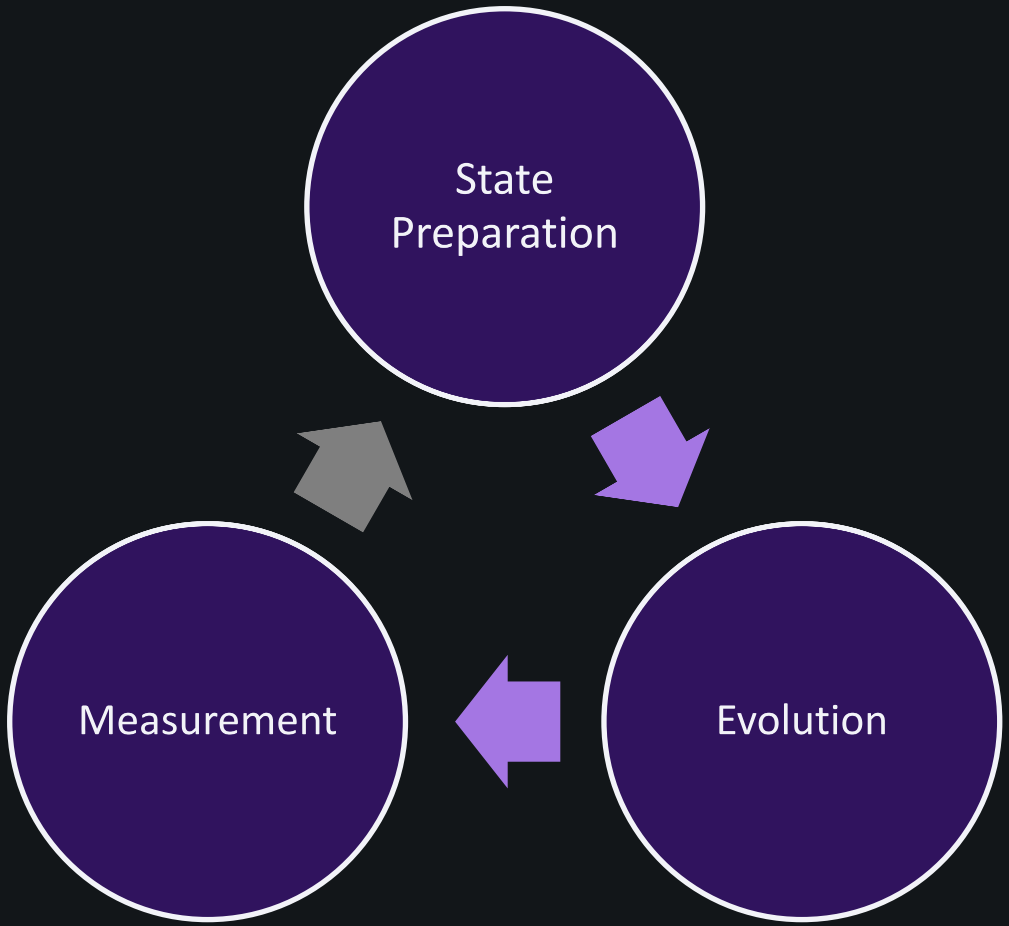
Parameterized Quantum Circuit

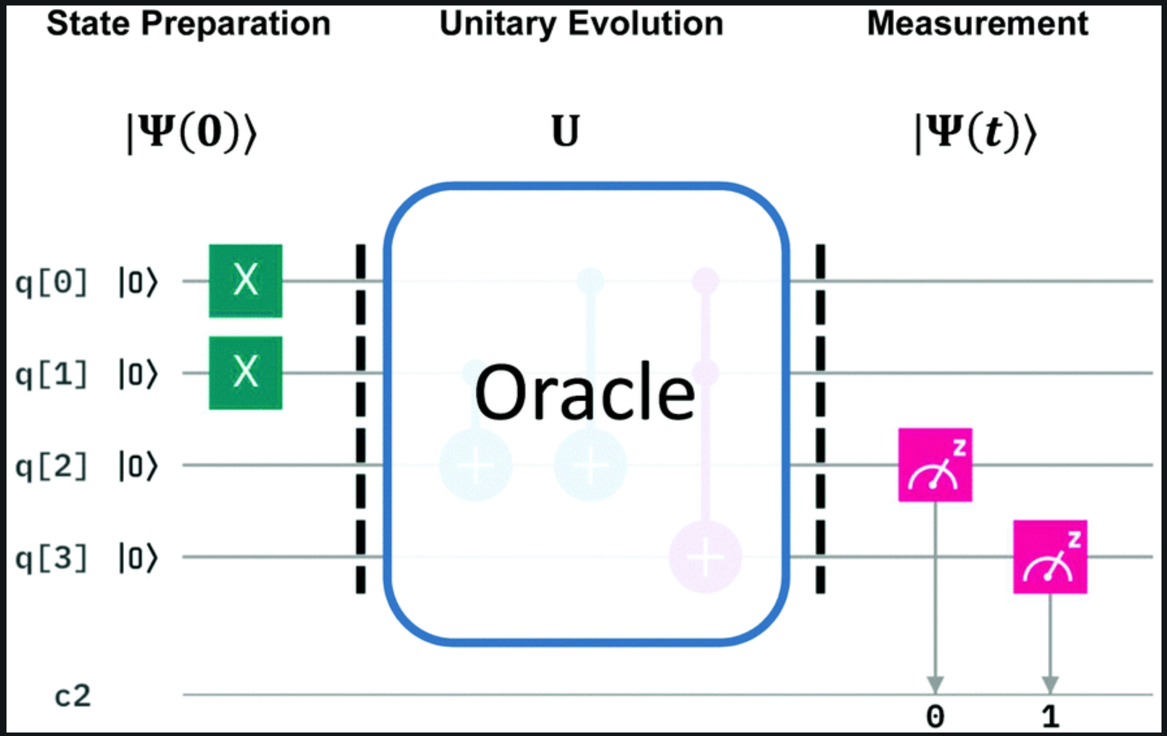
Hybrid Quantum-Classical Algorithm

- Do not assume loading data is not trivial
- Do assume NISQ
- Q: How to perform QML in near-future?
  - How to encode classical data to QC?
  - How to tune circuit?
  - How to tune measurement basis
- A: Parameterize Quantum Circuit + Variational Quantum Eigensolver/Algorithm
  - $E_0 = \min_{\psi} \langle \psi | H | \psi \rangle$



Principle of QC



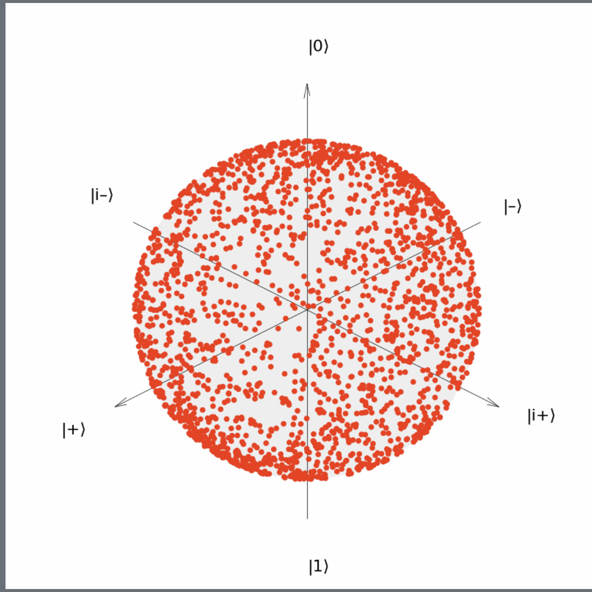


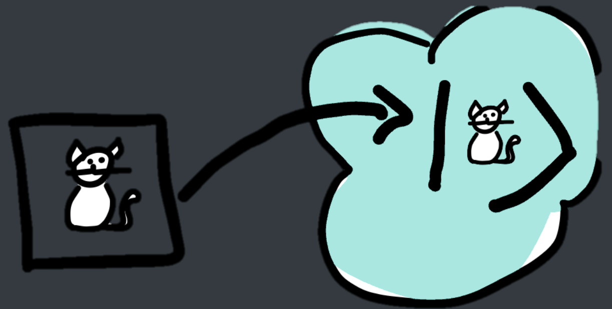
Learning With Qiskit

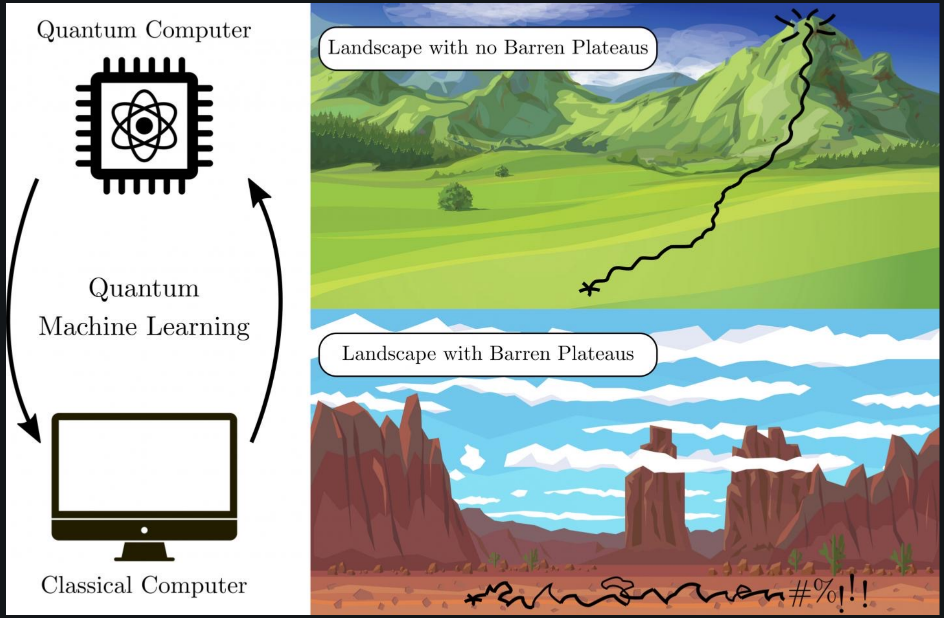
Qiskit

Next Lecture

- PQC Properties
  - Expressibility
  - Entanglement Capacity
  - Hardware Efficiency
- Data encoding
  - Amplitude encoding, etc.
- Trainability
  - Gradients of PQC
  - Barren plateaus







Thank you

Park, Siheon

Qiskit Machine Learning https://learn.qiskit.org/course/machine-learning/introduction

Lloyd, Seth, Masoud Mohseni, and Patrick Rebentrost. "Quantum principal component analysis." *Nature Physics* 10.9 (2014): 631-633. https://www.nature.com/articles/nphys3029

Lloyd, Seth, Masoud Mohseni, and Patrick Rebentrost. "Quantum algorithms for supervised and unsupervised machine learning." *arXiv preprint arXiv:1307.0411* (2013). https://arxiv.org/abs/1307.0411

Giovannetti, Vittorio, Seth Lloyd, and Lorenzo Maccone. "Quantum random access memory." *Physical review letters* 100.16 (2008): 160501. https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.100.160501

Rebentrost, Patrick, Masoud Mohseni, and Seth Lloyd. "Quantum support vector machine for big data classification." *Physical review letters* 113.13 (2014): 130503. https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.113.130503

Alchieri, Leonardo, et al. "An introduction to quantum machine learning: from quantum logic to quantum deep learning." *Quantum Machine Intelligence* 3 (2021): 1-30. https://link.springer.com/article/10.1007/s42484-021-00056-8#Sec15