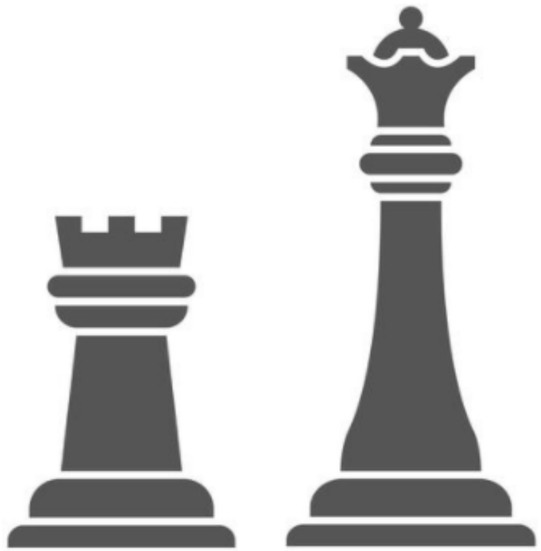
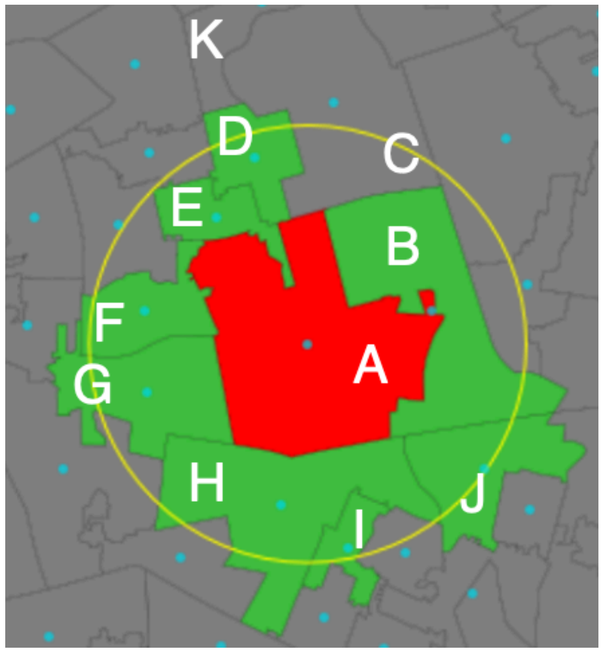
Lecture 4: Spatial weights

Instructor: Michael Szell
Feb 24, 2022

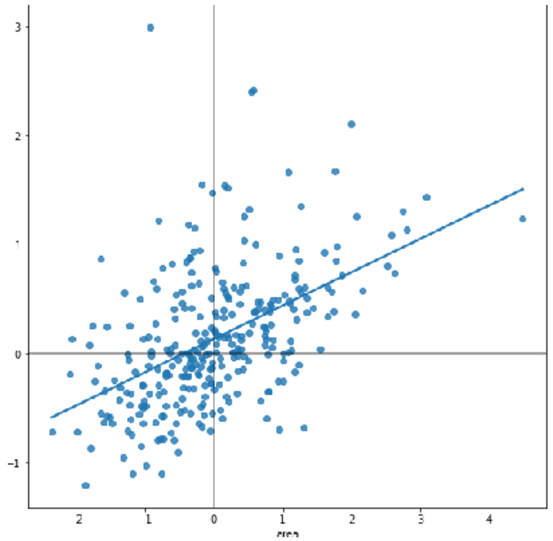


Today you will learn about spatial weight matrices

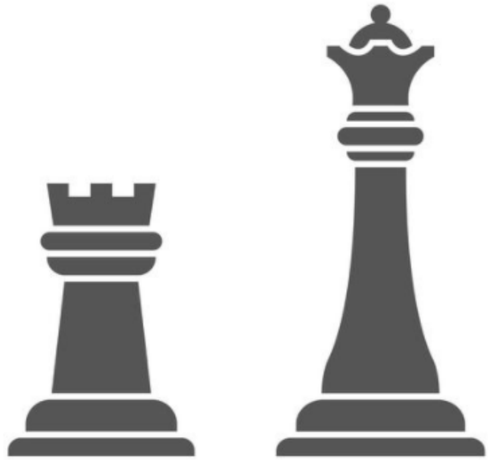
What they are,
why they are important



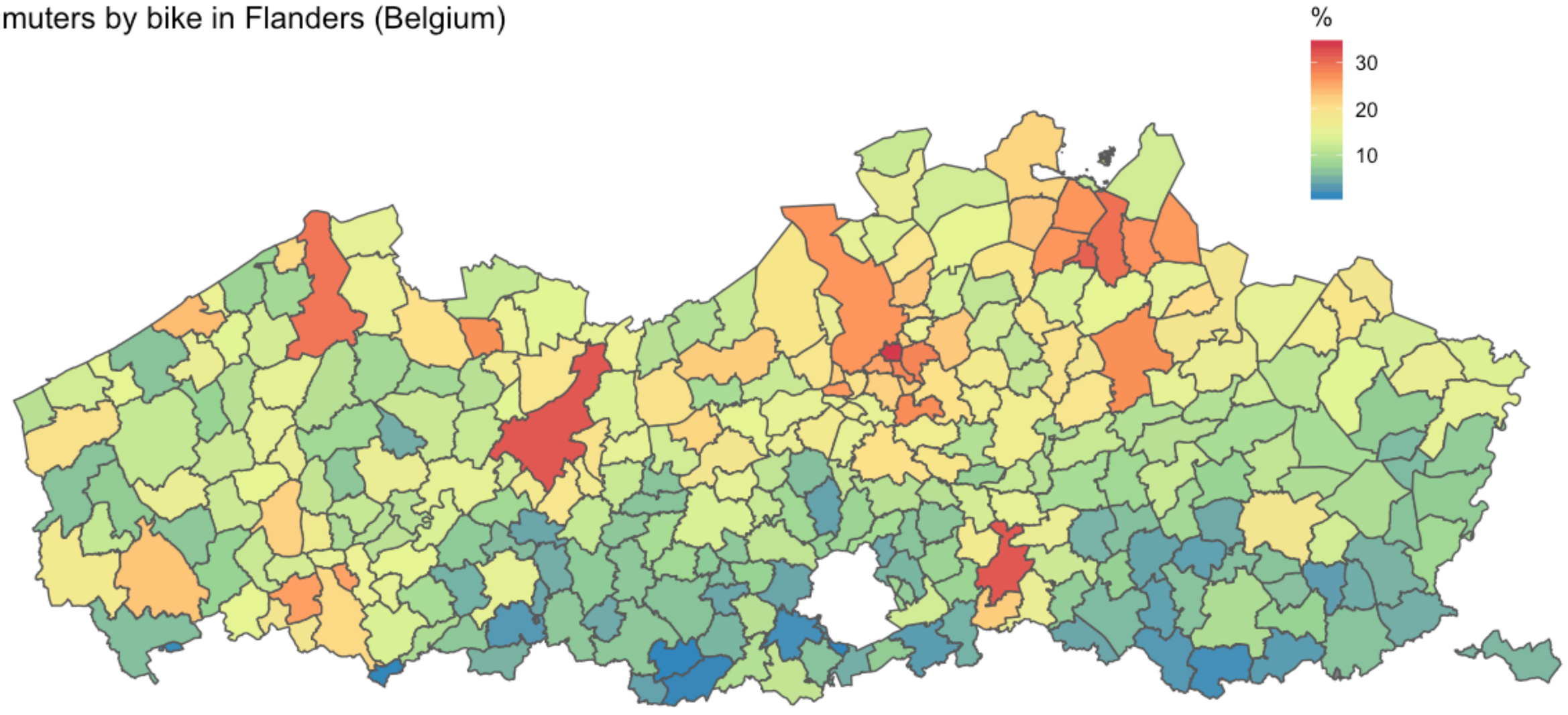
How to use them
in Python



Ways of defining them



Commuters by bike in Flanders (Belgium)



Data: Gemeentemonitor 2017

How are objects related to each other in space?

The spatial weight matrix W encodes the spatial relation between N objects

$$W = \begin{pmatrix} 0 & w_{12} & \dots & w_{1N} \\ w_{21} & \ddots & w_{ij} & \vdots \\ \vdots & w_{ji} & 0 & \vdots \\ w_{N1} & \dots & \dots & 0 \end{pmatrix}$$

N times N, positive

$w_{ii} = 0$

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N times N, positive

$w_{ii} = 0$

Generally, all non-zero elements in a row i are called the neighbors of object i. How to define neighbour?

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Contiguity Is object 2 "next to" object 1?

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N times N, positive

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Contiguity — Is object 2 "next to" object 1?

Distance — Is object 2 "close" to object 1?

How are objects related to each other in space?

The spatial weight matrix W encodes the spatial relation between N objects

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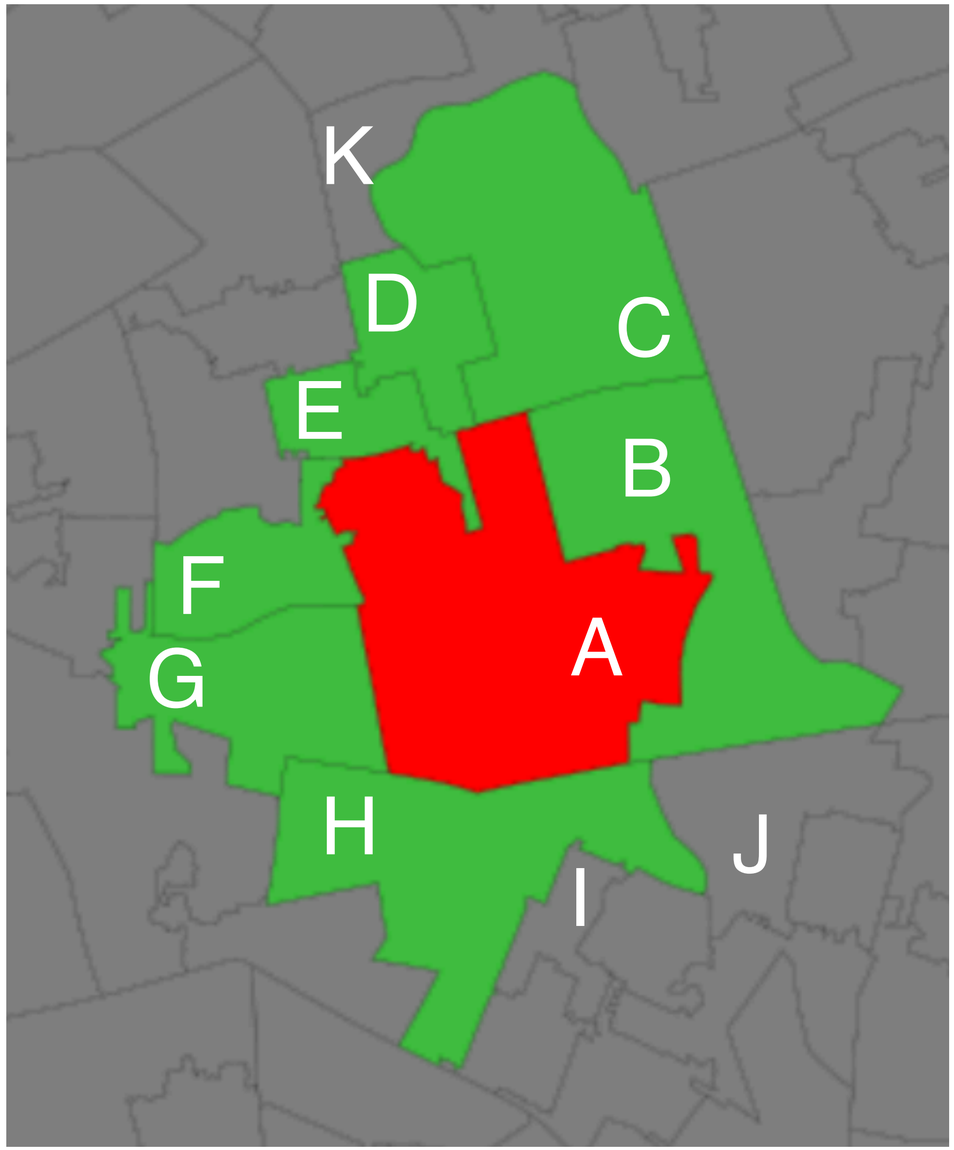
N times N, positive

$w_{ii} = 0$

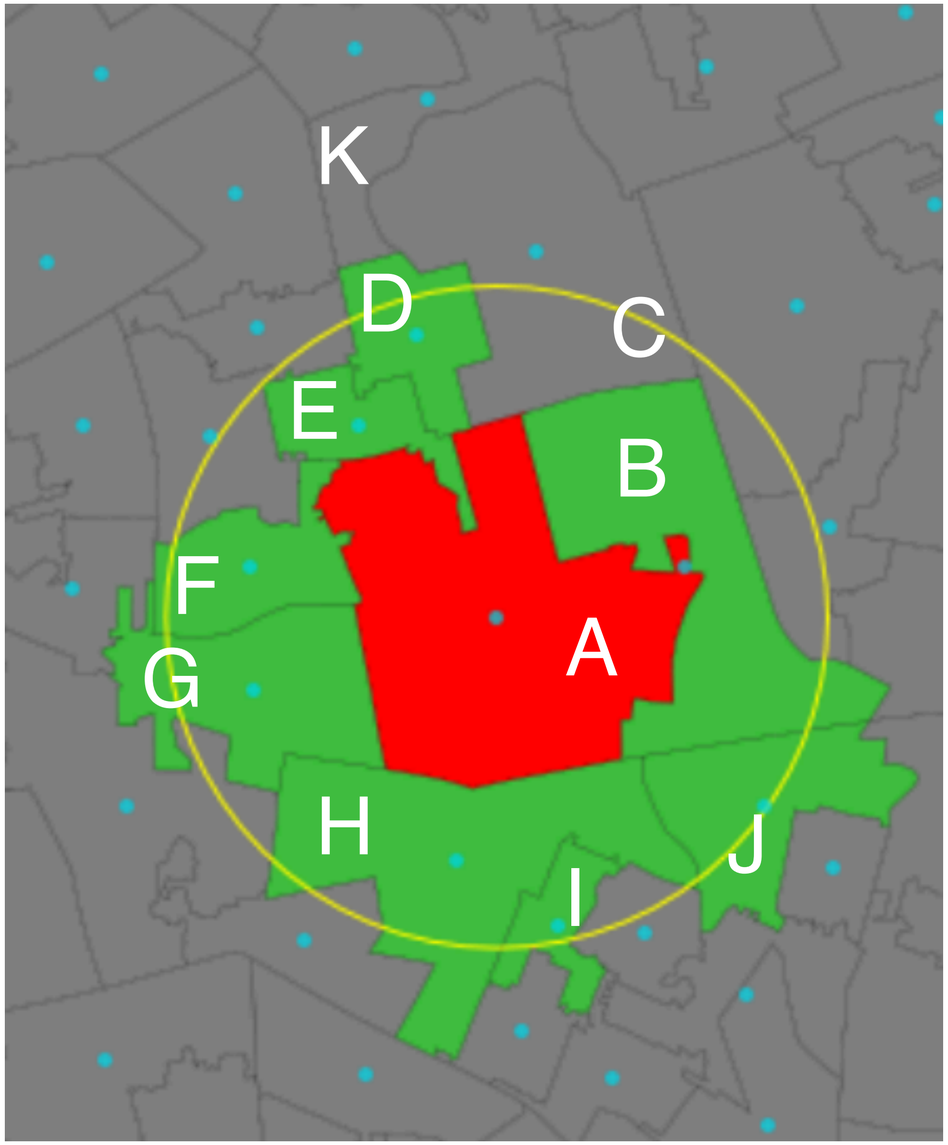
Contiguity	Is object 2 "next to" object 1?
Distance	Is object 2 "close" to object 1?
Block	Is object 2 in the same "place" as object 1?

Contiguity-based W: sharing a boundary



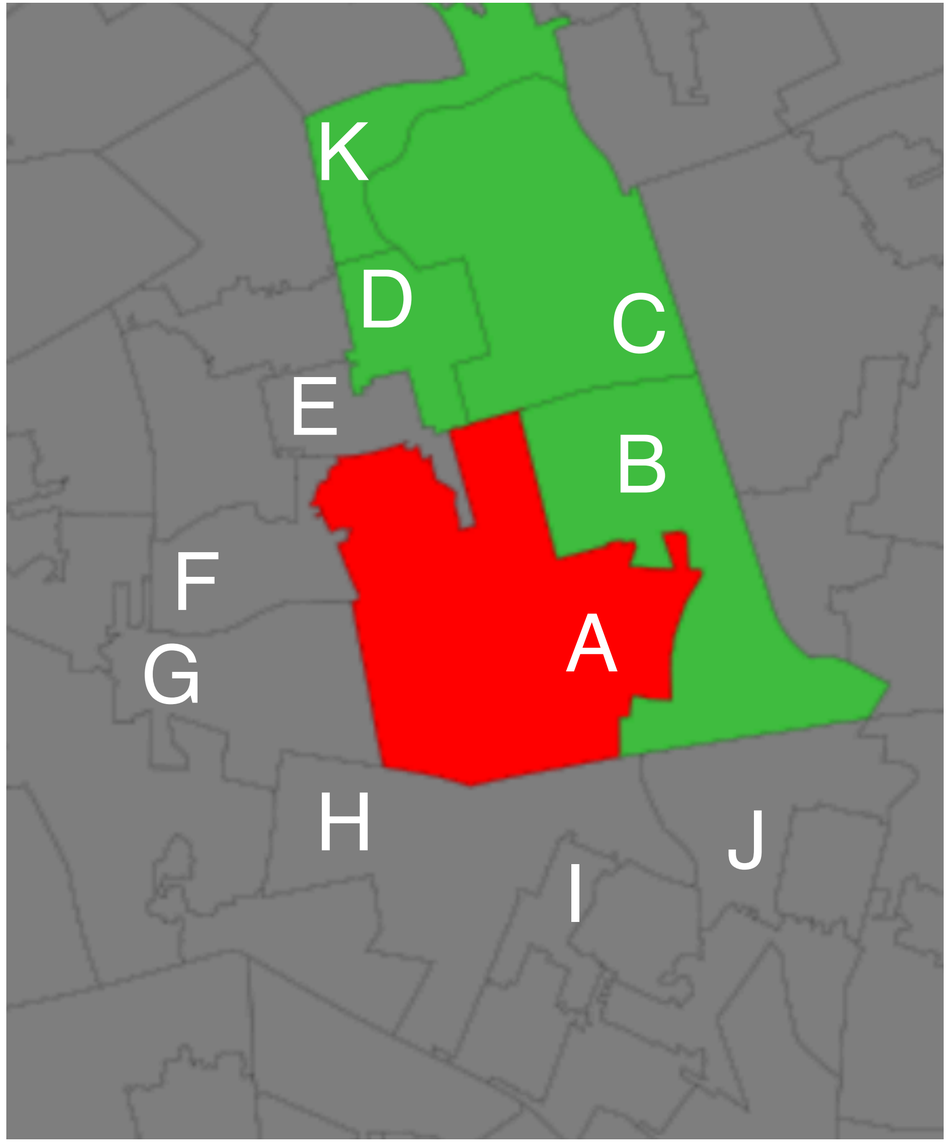
	A	B	C	D	E	F	G	H	I	J	K
...	...	...	...	...	...	...	...	...	...	...	...
A	0	1	1	1	1	1	1	1	0	0	0
...	...	...	...	...	...	...	...	...	...	...	...

Distance-based W: being closer than a threshold



	A	B	C	D	E	F	G	H	I	J	K
...	...	...	...	...	...	...	...	...	...	...	...
A	0	1	0	1	1	1	1	1	1	1	0
...	...	...	...	...	...	...	...	...	...	...	...

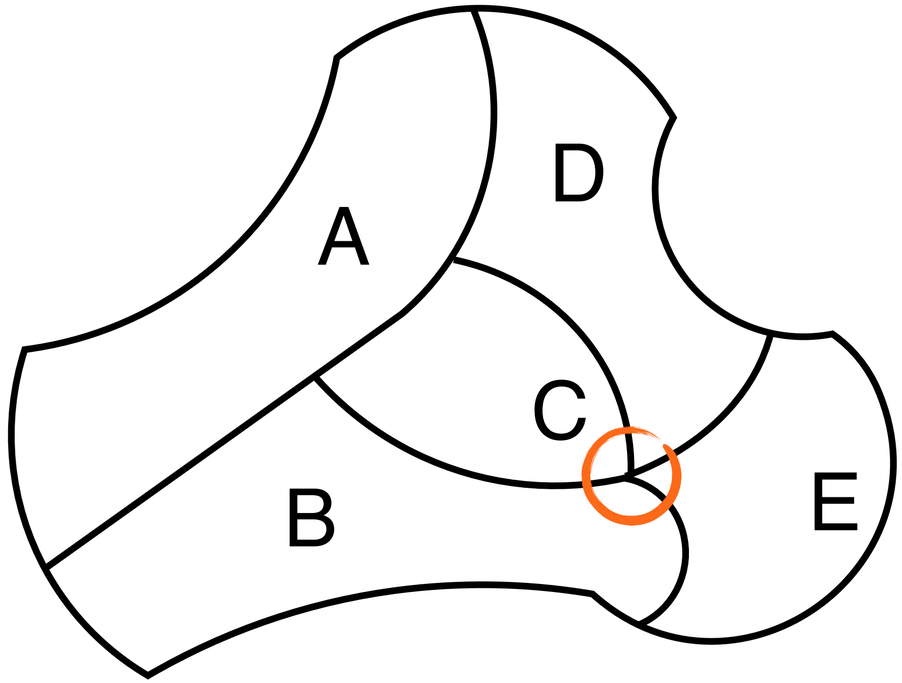
Block-based W: being in the same administrative unit



	A	B	C	D	E	F	G	H	I	J	K
...	...	...	...	...	...	...	...	...	...	...	...
A	0	1	1	1	0	0	0	0	0	0	1
...	...	...	...	...	...	...	...	...	...	...	...

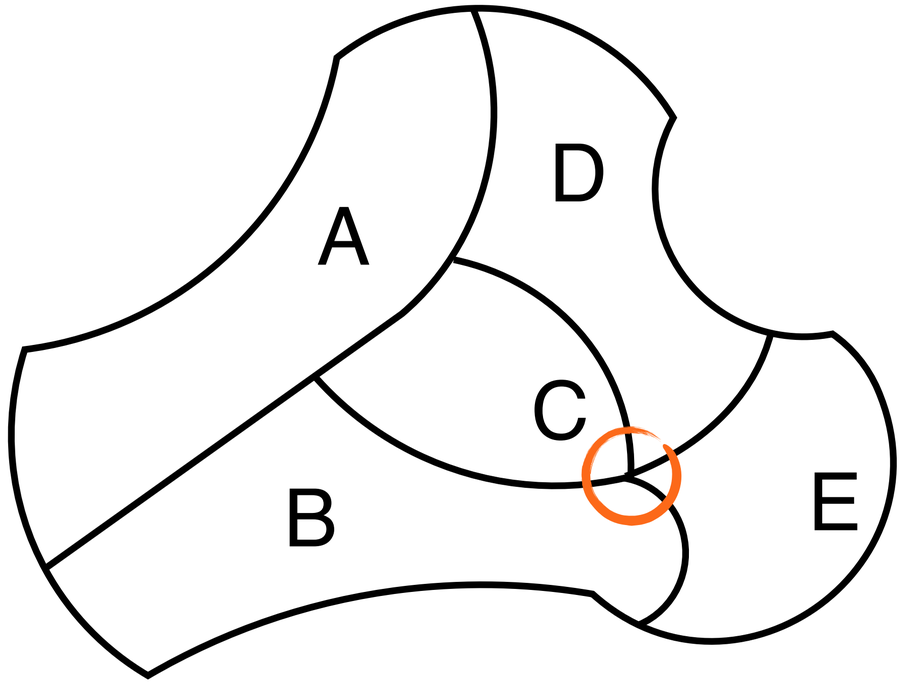
Special cases and variations

Contiguity-based W: What is a boundary?



	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	?	1
C	1	1	0	1	?
D	1	?	1	0	1
E	0	1	?	1	0

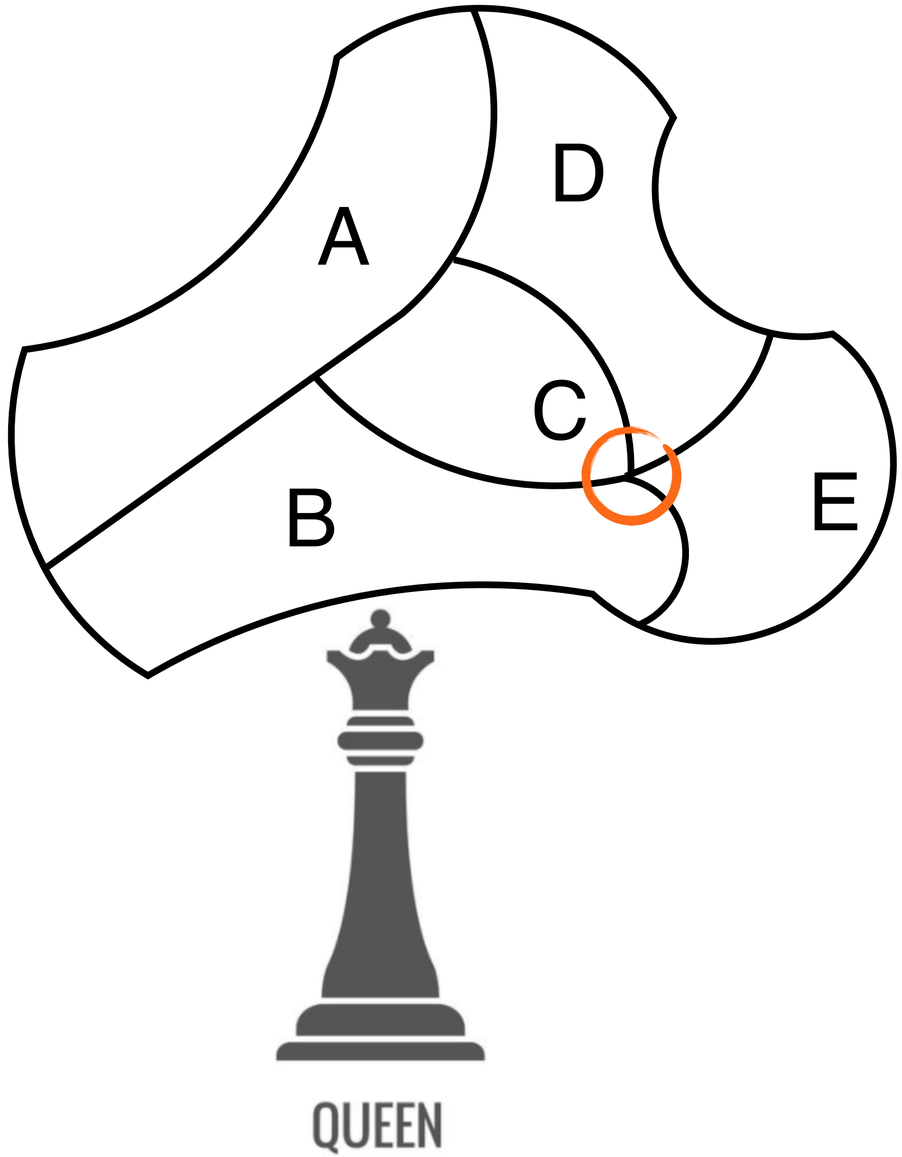
Contiguity-based W: What is a boundary?



ROOK

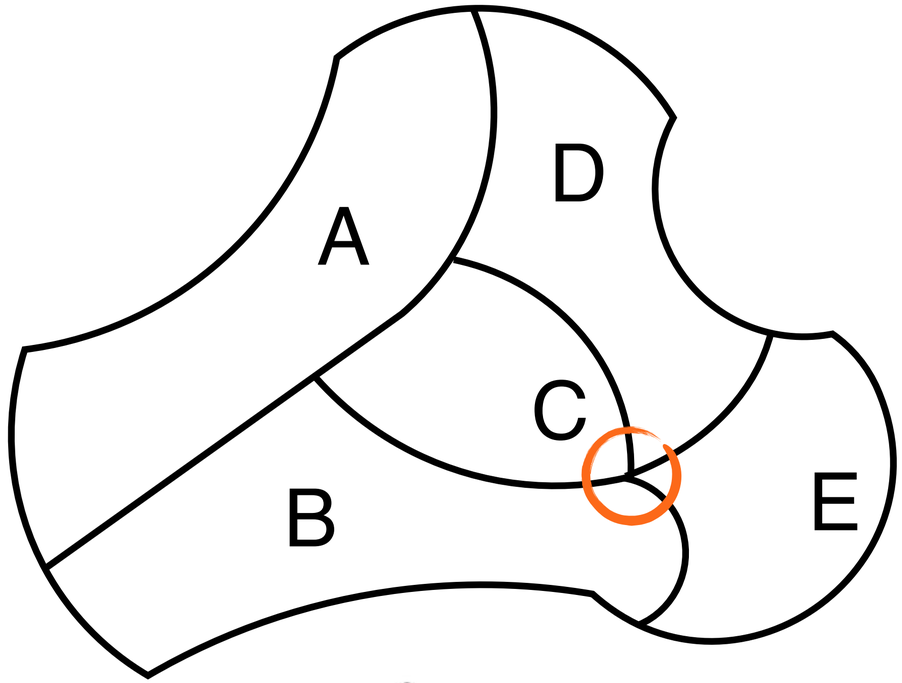
	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	1
C	1	1	0	1	0
D	1	0	1	0	1
E	0	1	0	1	0

Contiguity-based W: What is a boundary?



	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	1	1
C	1	1	0	1	1
D	1	1	1	0	1
E	0	1	1	1	0

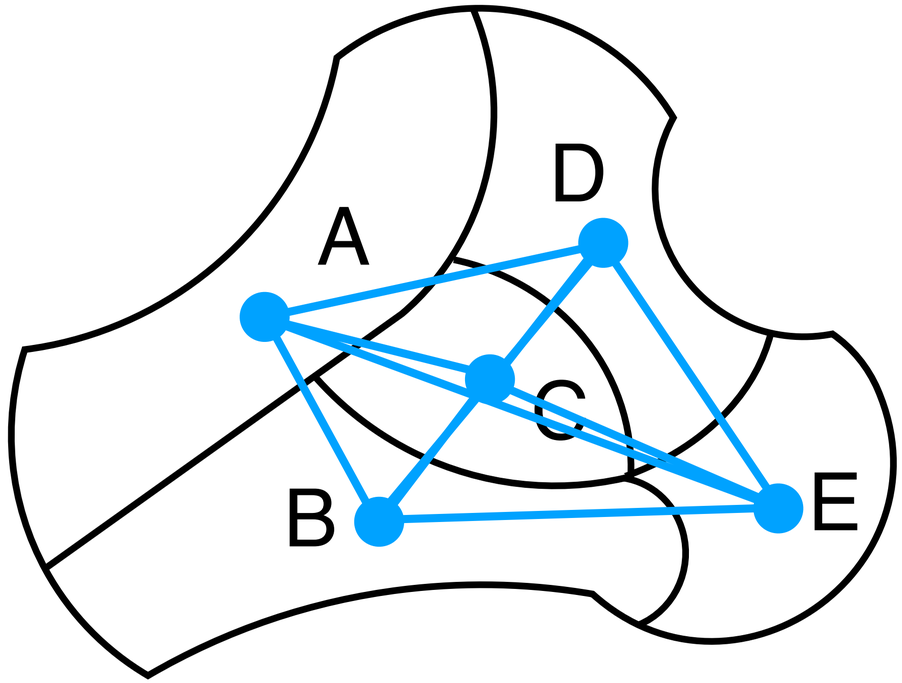
Contiguity-based W: What is a boundary?



BISHOP

	A	B	C	D	E
A	0	0	0	0	0
B	0	0	0	1	0
C	0	0	0	0	1
D	0	1	0	0	0
E	0	0	1	0	0

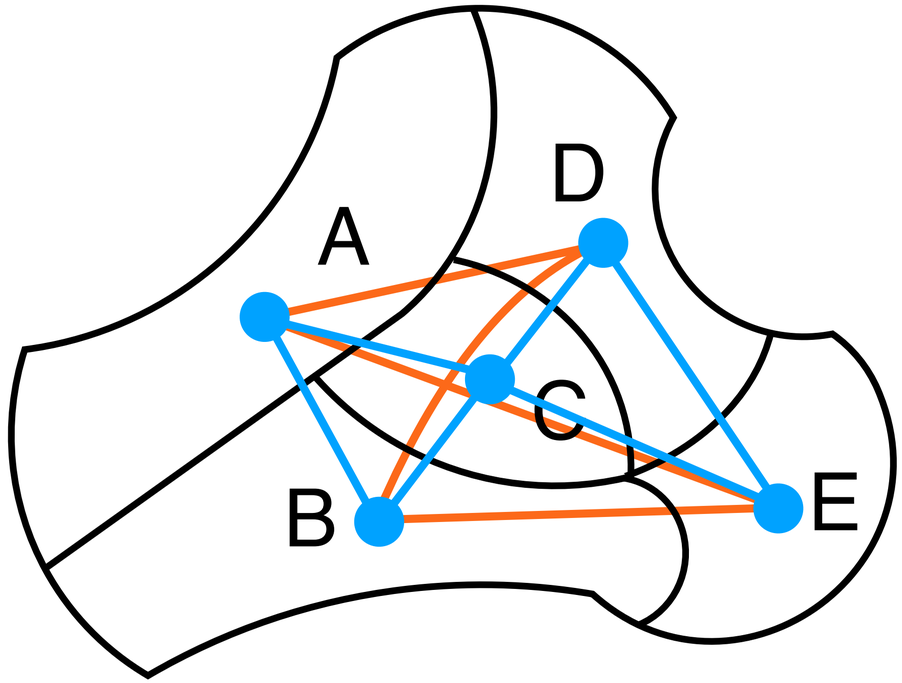
Distance-based W: Values can be continuous



$$w_{ij} = \frac{1}{d_{ij}} \quad \text{or} \quad w_{ij} = \frac{1}{d_{ij}^2}$$

	A	B	C	D	E
A	0	0.58	0.57	0.39	0.25
B	0.58	0	0.76	0.38	0.33
C	0.57	0.76	0	0.76	0.43
D	0.39	0.38	0.76	0	0.42
E	0.25	0.33	0.43	0.42	0

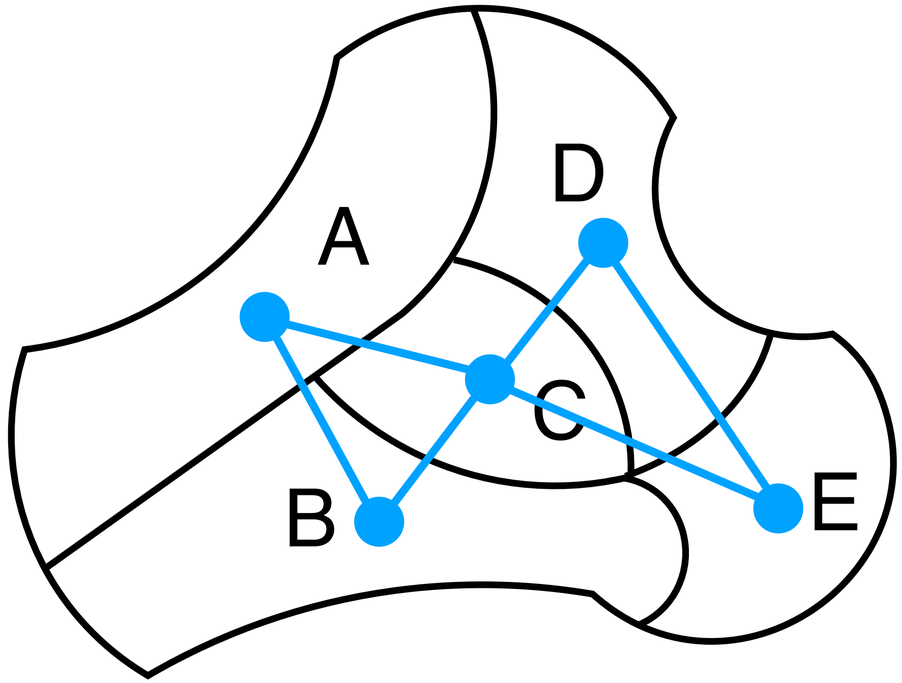
Distance-based W: Values can be continuous



We want a threshold
to keep W sparse!

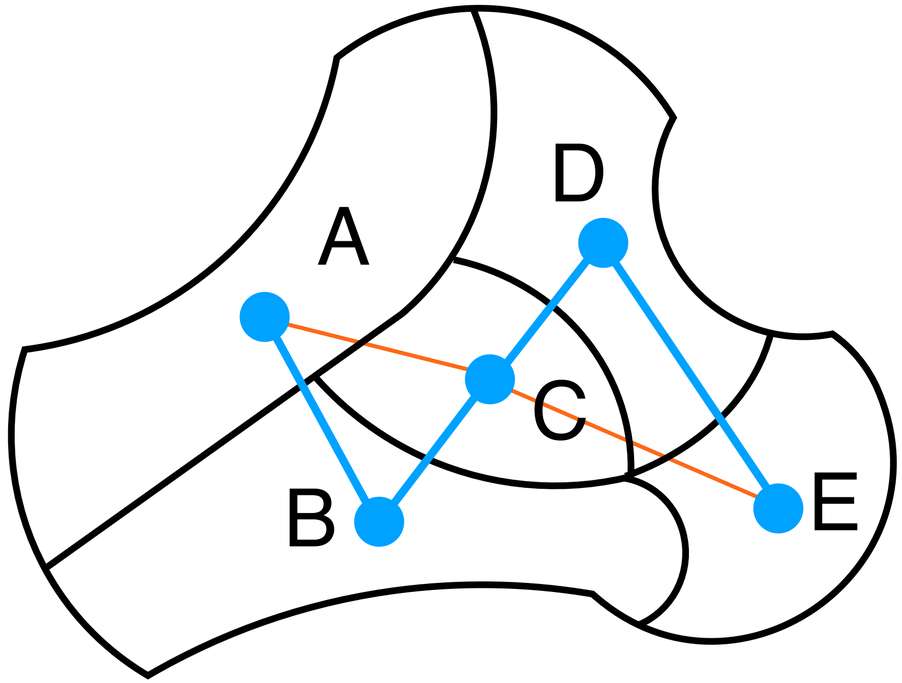
t=0.4	A	B	C	D	E
A	0	0.58	0.57	0	0
B	0.58	0	0.76	0	0
C	0.57	0.76	0	0.76	0.43
D	0	0	0.76	0	0.42
E	0	0	0.43	0.42	0

Distance-based W: KNN (k closest neighbors)



k=2	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	0	0
C	0	1	0	1	0
D	0	0	1	0	1
E	0	0	1	1	0

Distance-based W: KNN (k closest neighbors)

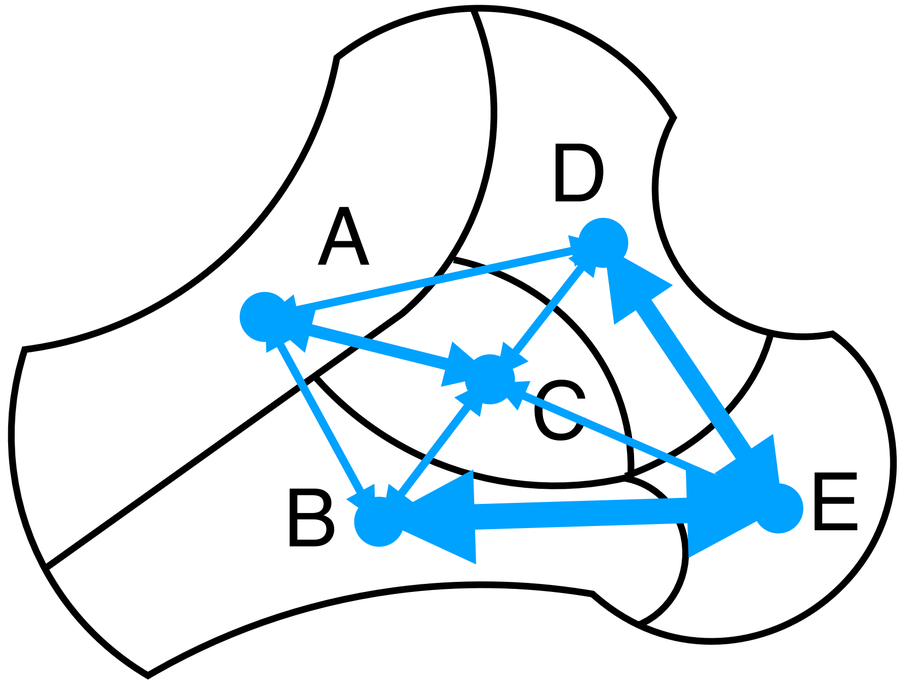


k=2	A	B	C	D	E
A	0	1	0.5	0	0
B	1	0	1	0	0
C	0.5	1	0	1	0.5
D	0	0	1	0	1
E	0	0	0.5	1	0

To make it symmetric, you could use:

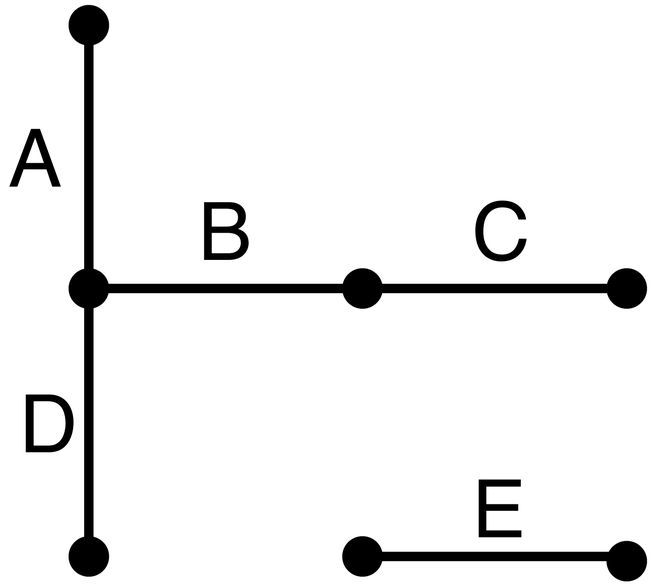
$$(W + W^T)/2$$

Interaction-based W: Flows



	A	B	C	D	E
A	0	1	2	1	0
B	1	0	1	0	5
C	2	1	0	1	1
D	1	0	1	0	2
E	0	4	1	3	0

The structure can be a network

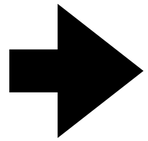


$$w_{ij} = \frac{1}{l_{ij}}$$

	A	B	C	D	E
A	0	1	0.5	0.5	0
B	1	0	1	1	0
C	0.5	1	0	0.5	0
D	0.5	1	0.5	0	0
E	0	0	0	0	0

It is common to standardize W: divide all by sum of row

	A	B	C	D	E
A	0	1	2	1	0
B	1	0	1	0	5
C	2	1	0	1	1
D	1	0	1	0	2
E	0	4	1	3	0

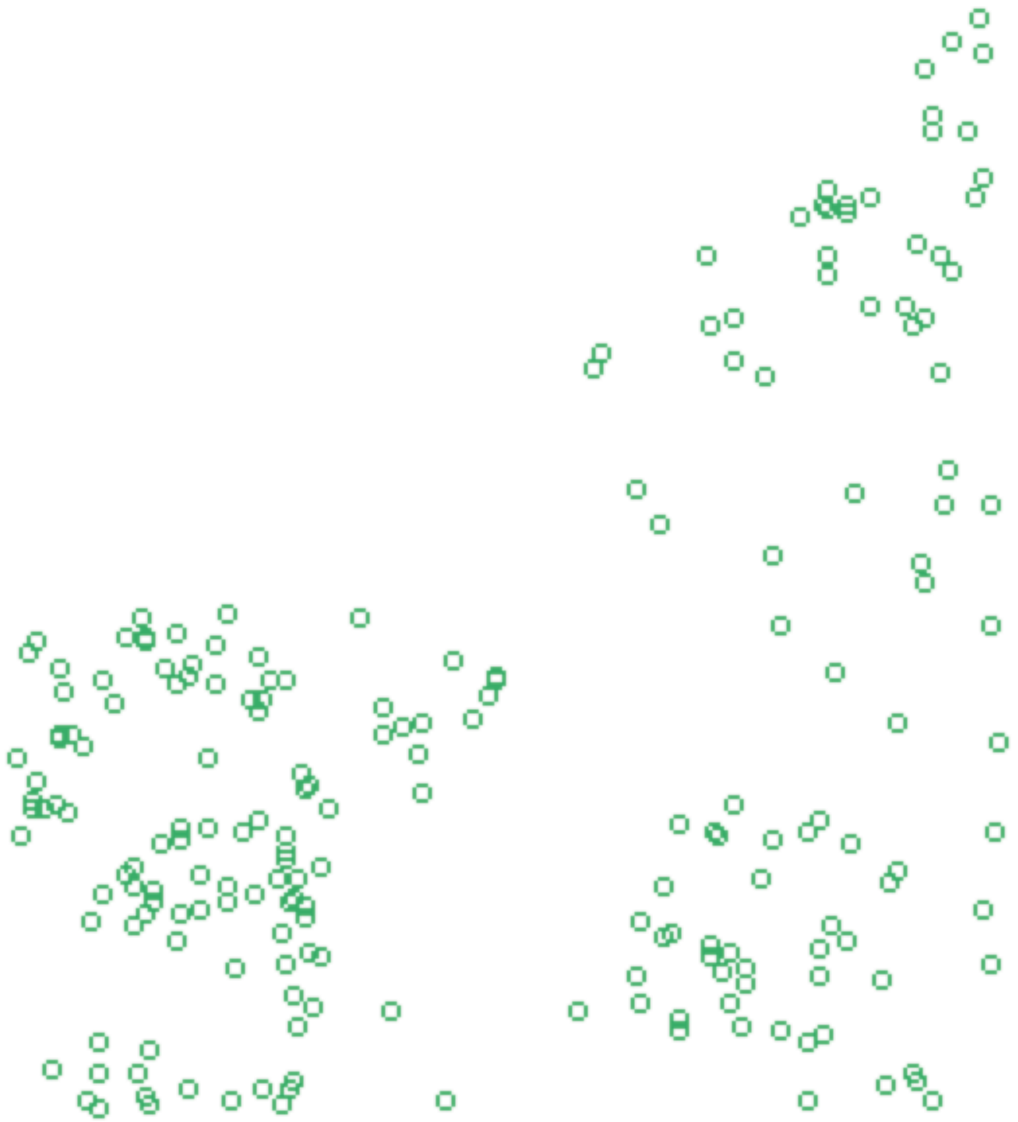


	A	B	C	D	E	Σ
A	0	1/4	1/2	1/4	0	1
B	1/7	0	1/7	0	5/7	1
C	2/5	1/5	0	1/5	1/5	1
D	1/4	0	1/4	0	1/2	1
E	0	1/2	1/8	3/8	0	1

The choice of W should reflect the studied interactions

Spreading processes like COVID	Contiguity, Flow
Accessibility	Distance
Effects of laws on counties	Block

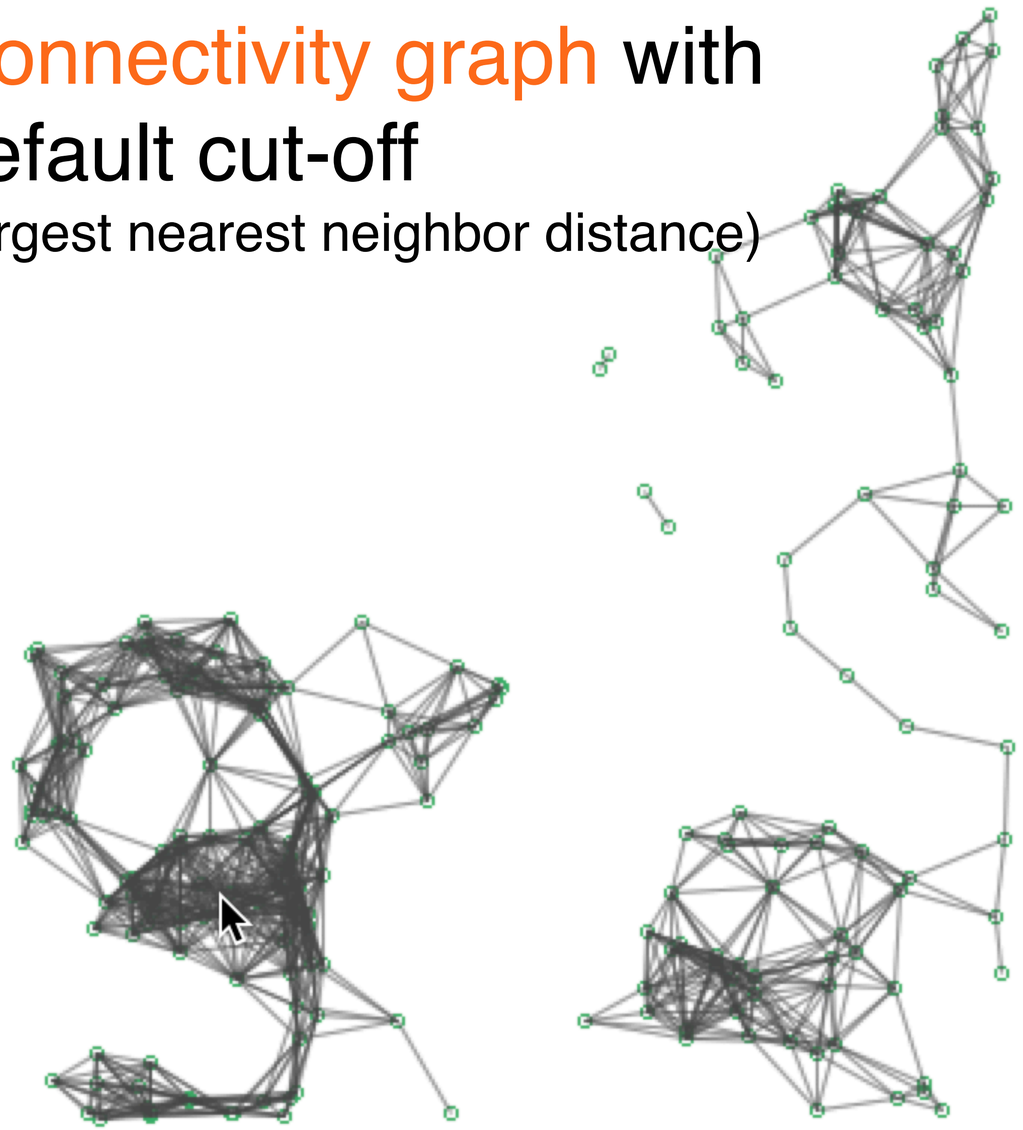
The problem with distance-bands



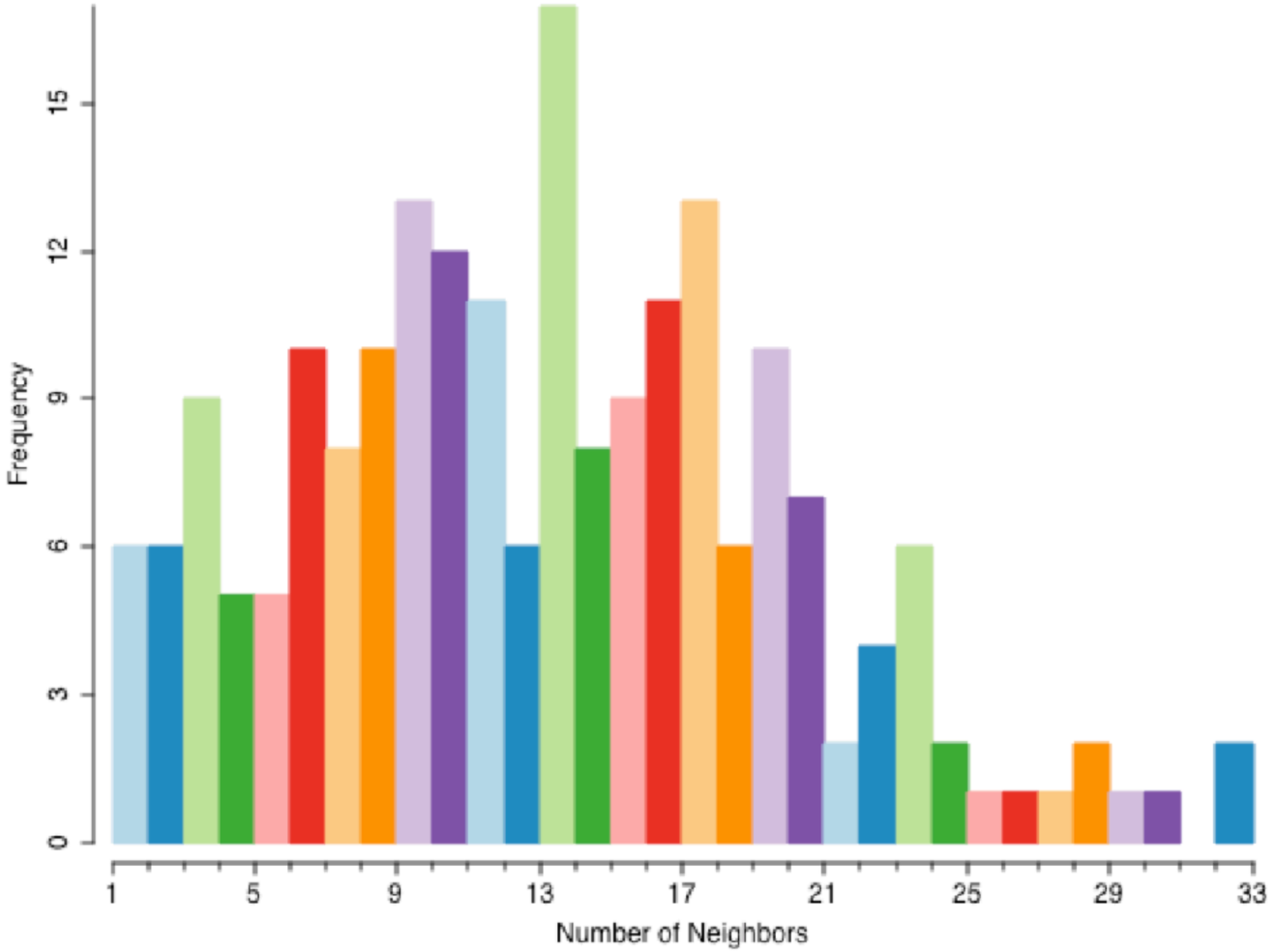
https://geoda.gitee.io/workbook/4b_dist_weights/lab4b.html

The problem with distance-bands

Connectivity graph with default cut-off
(largest nearest neighbor distance)



Degree distribution
Connectivity histogram



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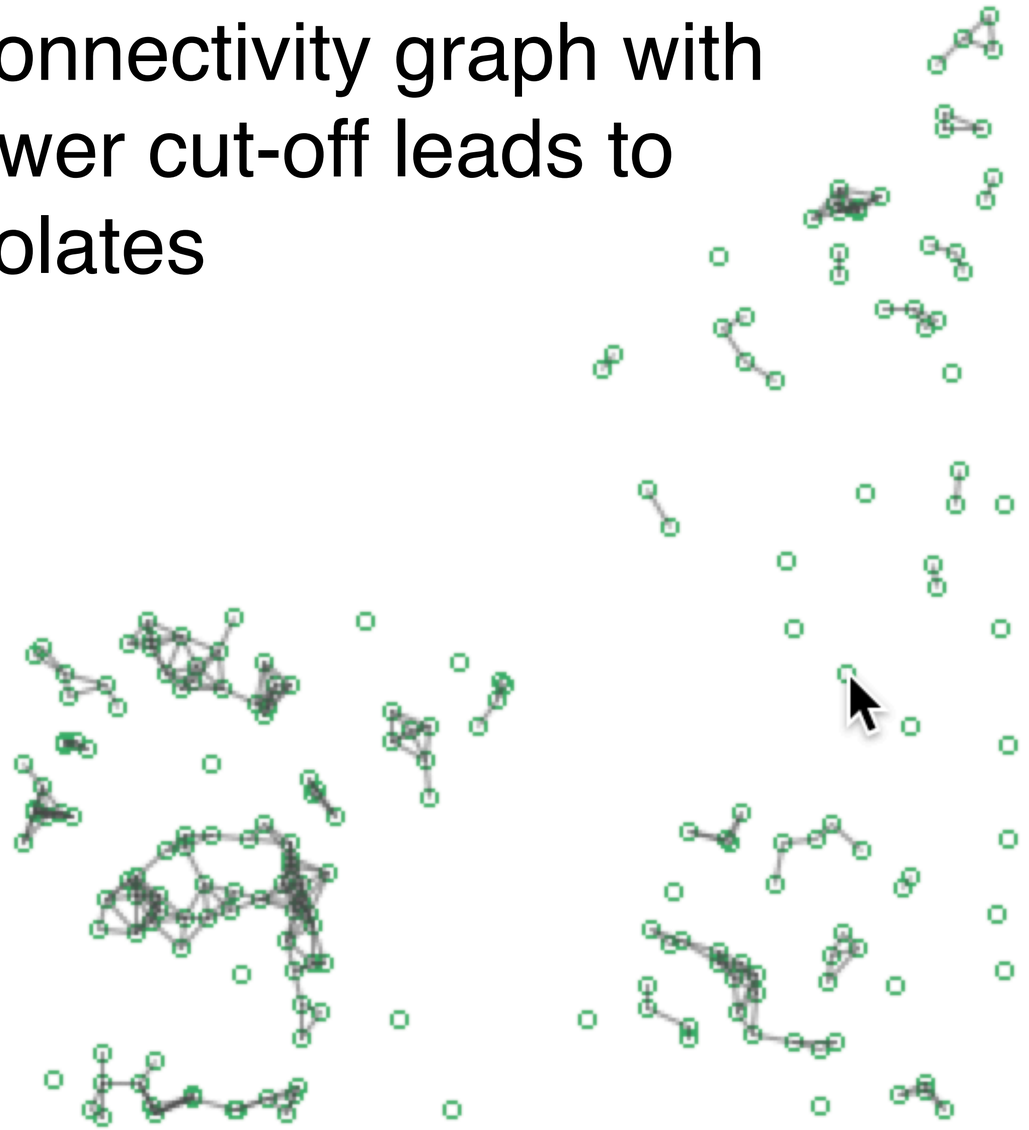
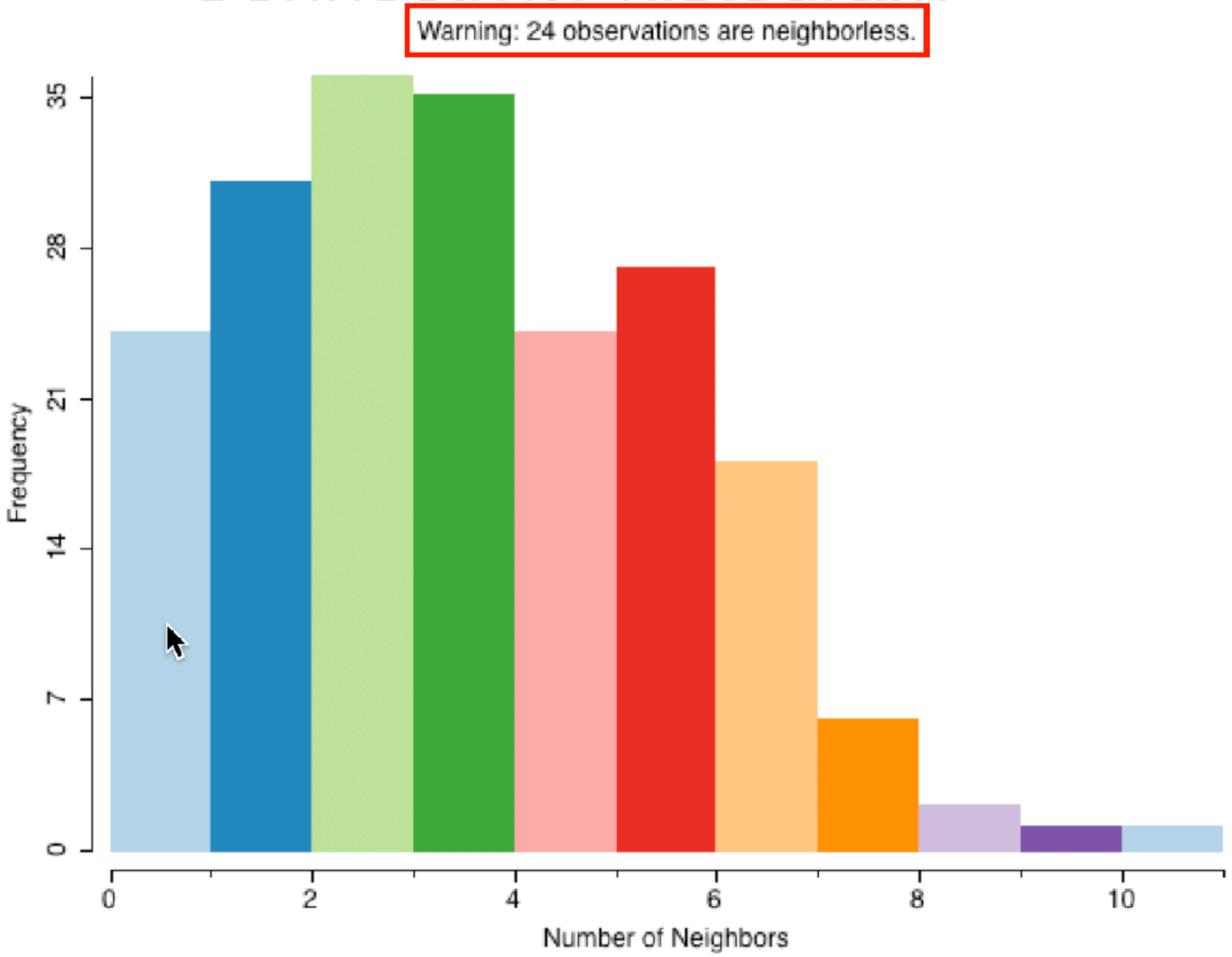
The problem with distance-bands

Connectivity graph with lower cut-off leads to isolates



Connectivity histogram



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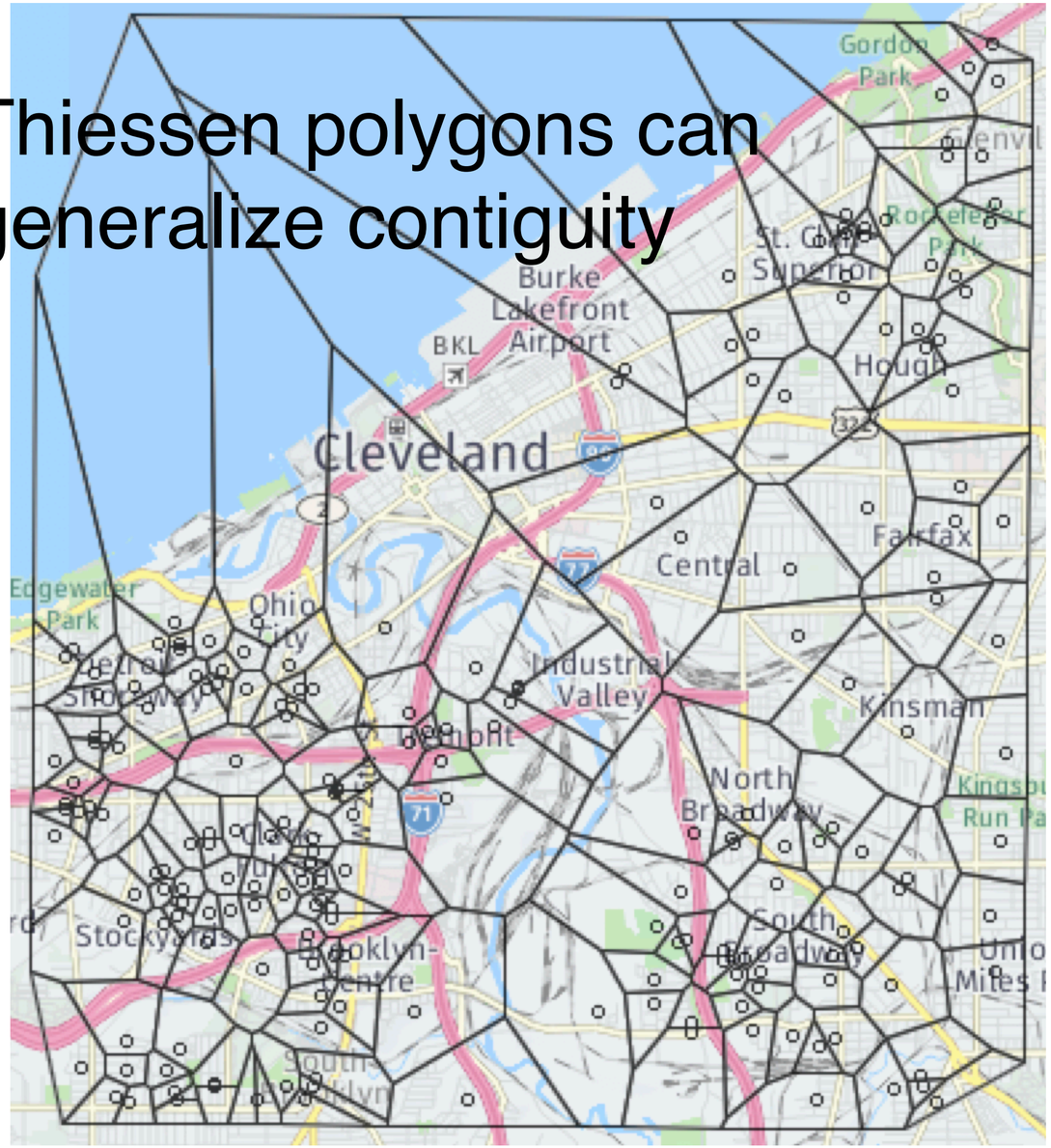
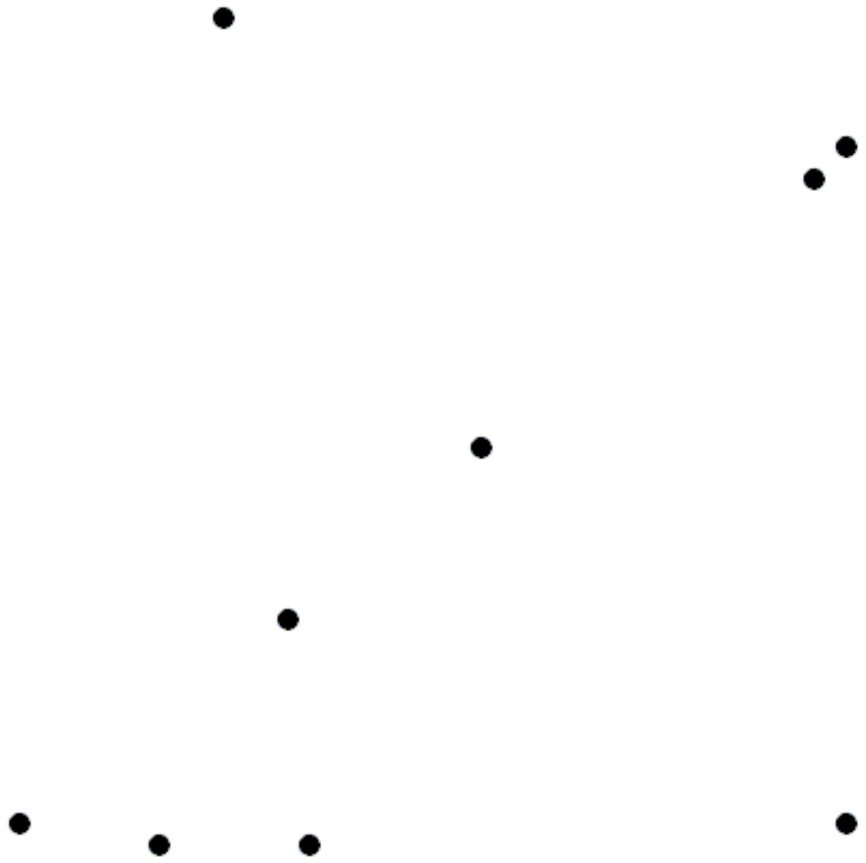
The problem with distance-bands

Thiessen polygons can generalize contiguity





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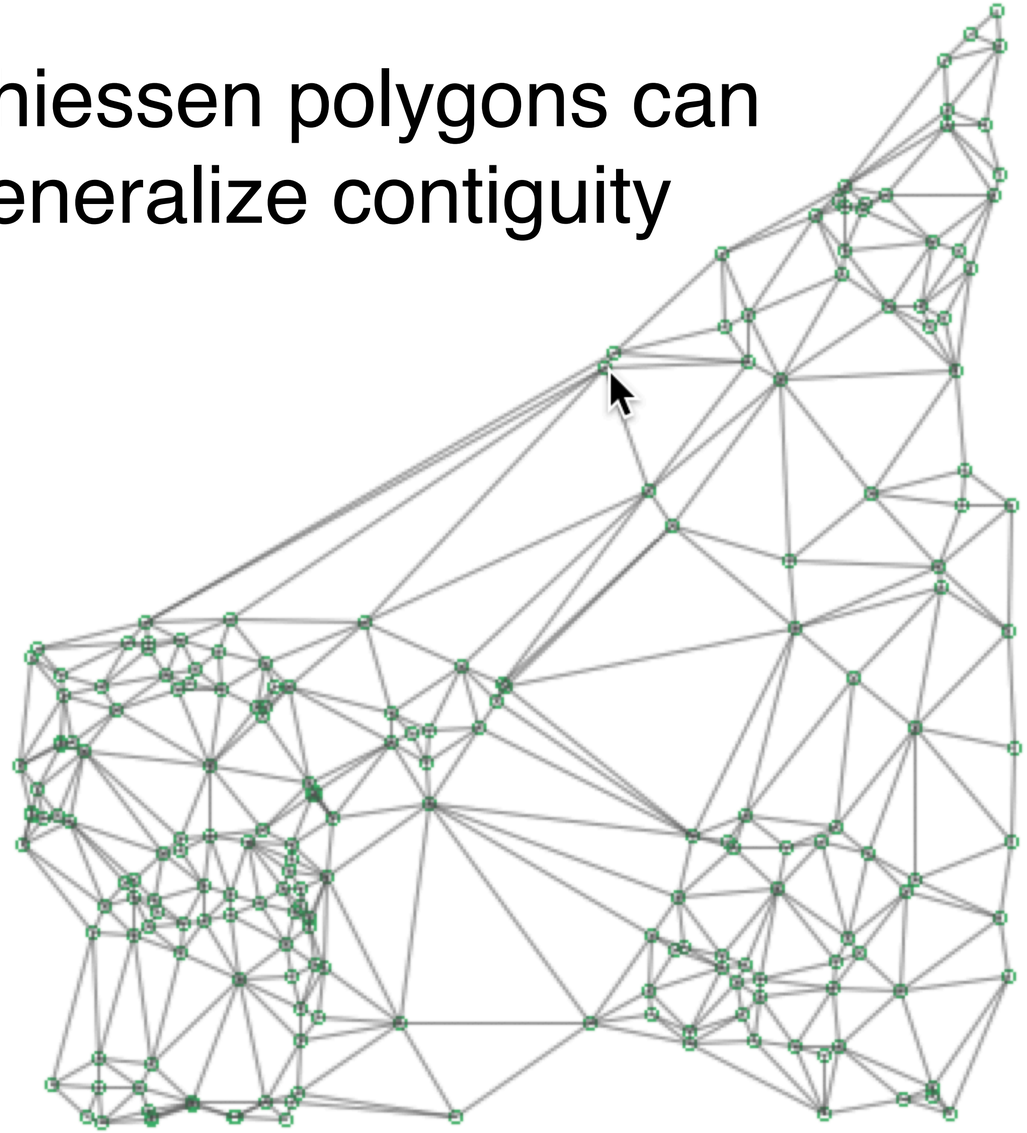
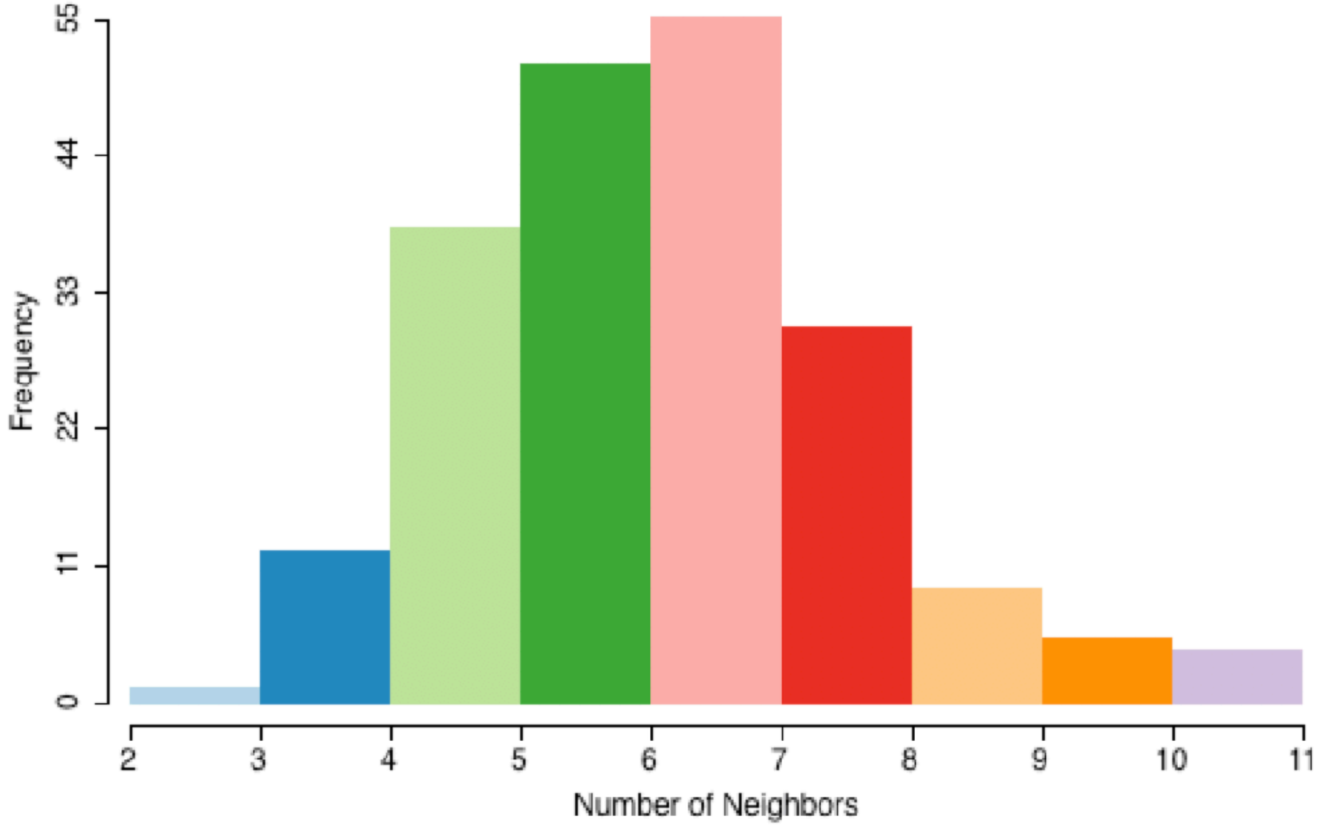


Connectivity histogram



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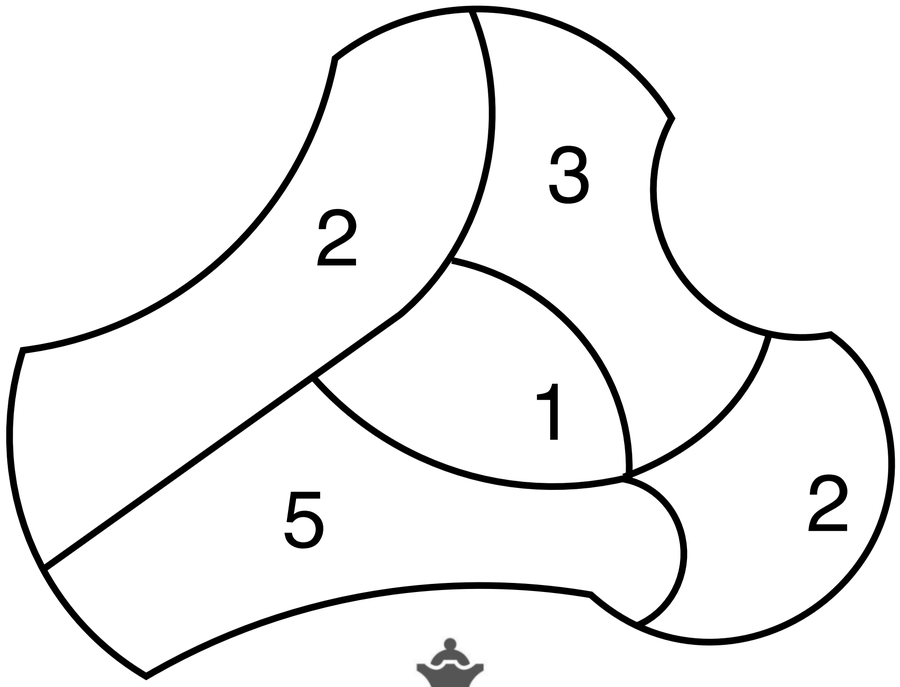
Spatial lag

The spatial lag is the weighted average value of neighbors

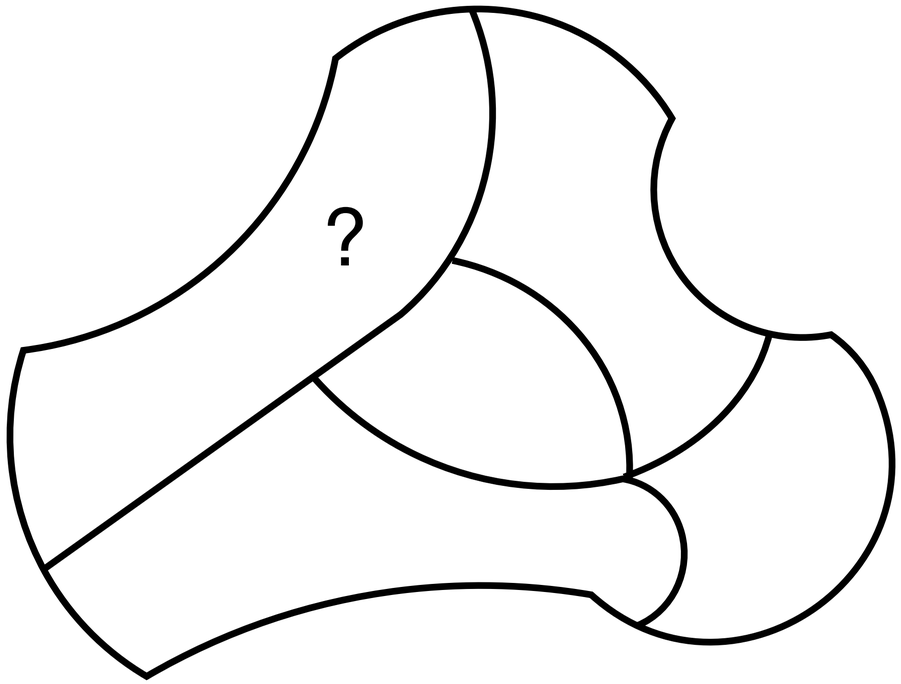
if W is standardized

Values $\mathbf{y}$



QUEEN

Spatial lag $\mathbf{y}_{\text{lag}}$



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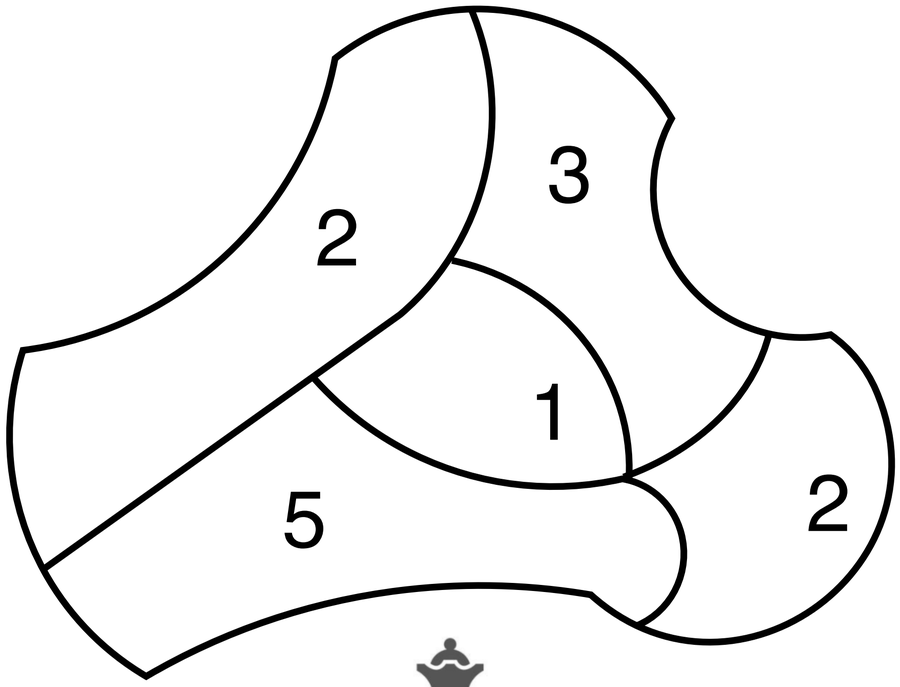
Values $\mathbf{y}$



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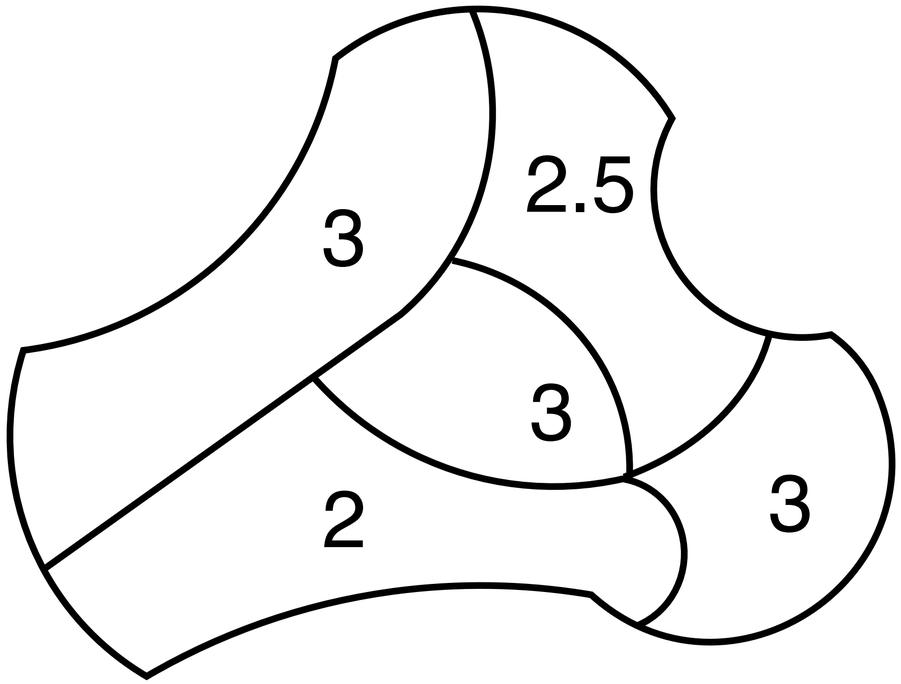

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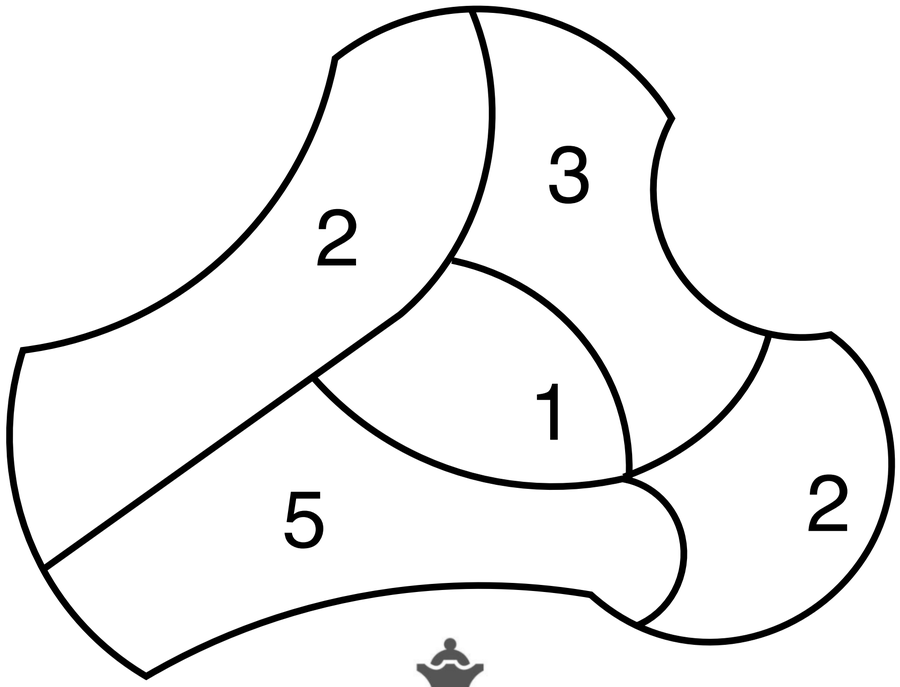
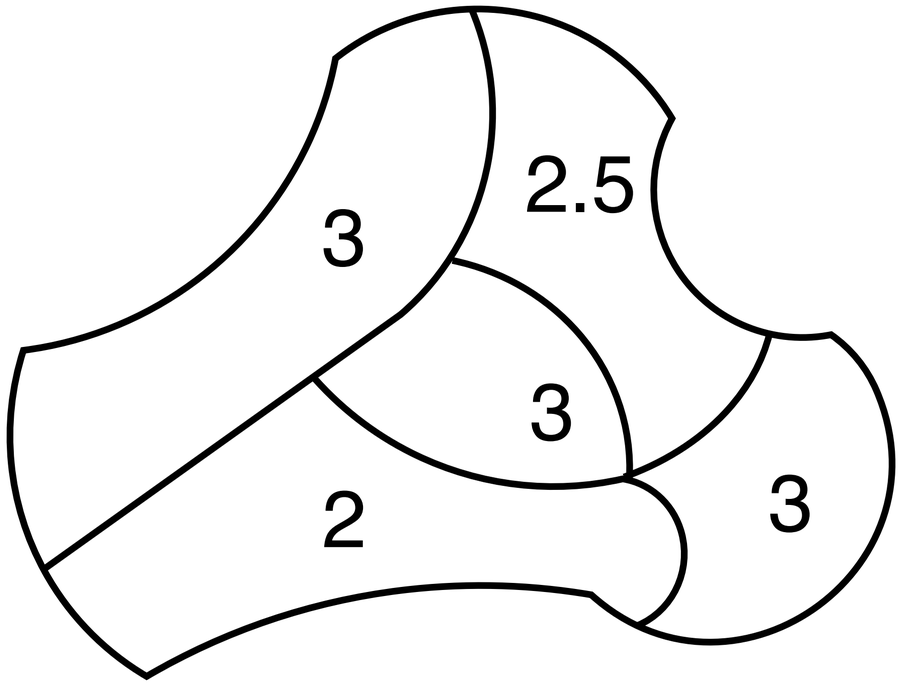
Values $\mathbf{y}$



Spatial lag $\mathbf{y}_{\text{lag}}$



It is a smoother: It brings all values closer to the average

The spatial lag is the sum of products of weights and values

$$y_{\text{lag},i} = w_{i1}y_i + w_{i2}y_2 + \cdots + w_{in}y_n = \sum_{j=1}^{n} w_{ij}y_j$$

The spatial lag is the sum of products of weights and values

$$y_{\text{lag},i} = w_{i1}y_i + w_{i2}y_2 + \cdots + w_{in}y_n = \sum_{j=1}^{n} w_{ij}y_j$$

$$\mathbf{y}_{\text{lag}} = \left(\sum_{j=1}^{n} w_{ij}y_j \right)_i = W\mathbf{y}$$

The spatial lag is the sum of products of weights and values

$$y_{\text{lag},i} = w_{i1}y_i + w_{i2}y_2 + \cdots + w_{in}y_n = \sum_{j=1}^{n} w_{ij}y_j$$

$$\mathbf{y}_{\text{lag}} = \left(\sum_{j=1}^{n} w_{ij}y_j \right)_i = W\mathbf{y}$$

The spatial lag appears in
many tools and models

Early evaluation

mentimeter link

Jupyter

Sources and further materials for today's class

Geographic Data Science with Python

https://geographicdata.science/book/notebooks/05_choropleth.html

https://darribas.org/gds_course/content/bE/concepts_E.html

https://geoda.gitee.io/workbook/4b_dist_weights/lab4b.html