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 Random Sample $(X_1, X_2, X_3, \dots, X_n)$ from a normal population with parameters θ_1 (mean) and θ_2 (variance)

Maximum Likelihood Estimates for θ_1 & θ_2 are as follows:-

Likelihood function for normal distribution -

$$L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

log-likelihood function -

$$l(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = \sum_{i=1}^n \left(-\frac{1}{2} \log(2\pi\theta_2) - \frac{(x_i - \theta_1)^2}{2\theta_2} \right)$$

→ We have to maximize this log likelihood function. So we differentiate it w.r.t θ_1 & θ_2 and equate it to 0, then solve for θ_1 & θ_2 .

w.r.t θ_1

$$\frac{\partial l}{\partial \theta_1} = \sum_{i=1}^n \frac{x_i - \theta_1}{\theta_2} = 0$$

$$\Rightarrow \frac{\partial l}{\partial \theta_1} = \sum_{i=1}^n (x_i - \theta_1) = 0$$

$$\Rightarrow \sum_{i=1}^n x_i - n\theta_1 = 0$$

$$\Rightarrow \left[\frac{n\theta_1}{n} = \frac{1}{n} \sum_{i=1}^n x_i \right]$$

w.r.t θ_2

$$\frac{\partial l}{\partial \theta_2} = \sum_{i=1}^n \left(-\frac{1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right) = 0$$

$$\Rightarrow -\frac{n}{2\theta_2} + \sum_{i=1}^n \frac{(x_i - \theta_1)^2}{2\theta_2^2} = 0$$

$$\Rightarrow \frac{\sum_{i=1}^n (x_i - \theta_1)^2}{\theta_2^2} = n$$

$$\Rightarrow \theta_2 = \frac{\sum_{i=1}^n (x_i - \theta_1)^2}{n}$$

Random sample (x_1, x_2, \dots, x_n) from a binomial distribution with parameters n and θ , θ is unknown.

log-likelihood function for binomial distribution

$$l(\theta | x_1, x_2, \dots, x_n) = \sum_{i=1}^n \left(\log \binom{n}{x_i} + x_i \log \theta + (n - x_i) \log (1 - \theta) \right)$$

differentiation w.r.t. θ

$$\frac{\partial l}{\partial \theta} = \sum_{i=1}^n \left(\frac{x_i}{\theta} - \frac{n - x_i}{1 - \theta} \right) = 0$$

$$\Rightarrow \sum_{i=1}^n (x_i^0 (1-\theta) - (n \cdot x_i^0) \theta) = 0$$

$$\Rightarrow \sum_{i=1}^n (x_i^0 - n\theta) = 0$$

$$\Rightarrow \sum_{i=1}^n x_i^0 - n n \theta = 0$$

~~0~~

$$\Rightarrow \theta = \frac{1}{nn} \sum_{i=1}^n x_i^0$$