BLOC-516: Principles of Money, Banking, and Finance

Portfolio theory and Pricing Models

Dr Ifigenia Georgiou

Ph.D. in Finance
Assistant Professor
Department of Accounting, Economics, and Finance
School of Business
University of Nicosia

Learning objectives

- Understand the concept of return and risk of a financial asset
- Understand how return and risk affect the investment decision
- Understand the concept of pricing assets

Outline

- Expected Return and Risk
- Portfolio Risk and Return
- Portfolio Diversification
- Two-Security Portfolio
- Portfolio selection
- Other valuation models

Expected Return and Risk

What do we mean by return?

$$R = \frac{V_f - V_i}{V_i}$$

Holding Period Return

Period: Day, week, month, year, hour…

where:

V_f = final value, including dividends and interest

V_i = initial value

Or if we want to use the logarithmic or continuously compounted form this is:

$$R_{\log} = \ln\left(\frac{V_f}{V_i}\right)$$

Expected Return and Risk

- **Expected Return**: weighted average all the possible returns of an investment, where every possible return is weighted by the probability of it happening.

$$E(r) = \sum_{i=1}^{n} P_i r_i$$

We have a share of stock that: In case the economy is booming (this scenario has a probability of 25%) the return of the stock will be 31%. If things stay as they are in the economy (probability 45%), the return of this stock will be 14%), in case that the growth of the economy declines a bit (probability 25%), the return will be − 6.75%, and if there is a recession (probability 5%), then the return will be -5.2%. What is the Expected return of the stock?

Scenario for Economy	Prob.	Ret.
Booming	25%	31%
Same	45%	14%
Growth declines	25%	-6.75%
Recession	5%	-5.2%

$$E(r) = (.25 \times .31) + (.45 \times .14) + [.25 \times (-.0675)] + [.05 \times (-.52)] = .0976$$

- **Risk**: Standard deviation

$$\sigma = \{\Sigma P_i [r_i - E(r_i)]^2\}^{1/2}$$

$$\sigma^2 = .25(.31 - .0976)^2 + .45(.14 - .0976)^2 + .25(-.0675 - .0976)^2 + .05(-.52 - .0976)^2 = .0380$$

$$\sigma = \sqrt{.0380} = .1949 = 19.49\%$$

Relative Risk Measure: Coefficient of Covariation

- If I need to compare investments:

$$\boxed{CV = \sigma_i / E(r_i)}$$

- For example: If two investments have the same risk, say σ=0,00346, but investment A has a return of 35%, while investment B has a return of 25%. Which one do you choose?
- The other one (the one you won't choose) we say that it is not an efficient investment.

Portfolio Risk and Return and Portfolio Diversification

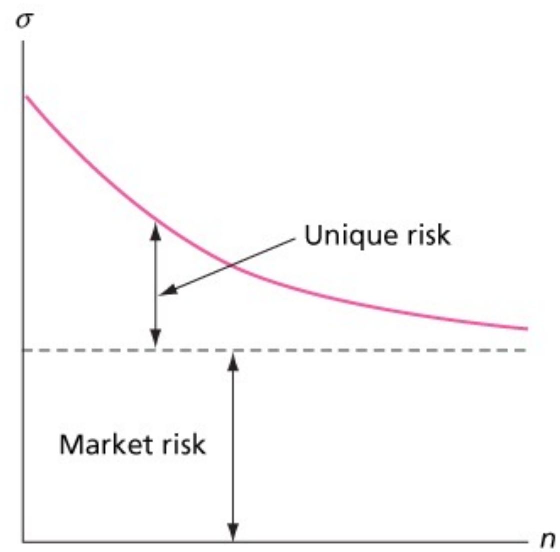
Sources of Risk

- Interest Rate Risk
  - The evolution of interest rates may have an effect on the price of a share.
- Inflation Risk
  - The risk of losing the real value of the capital due to a higher than expected increase in inflation.
- Market Risk
  - Changes in the market due to e.g. interest rate risk, exchange rate risk, etc.
- Business Risk (also Operational Risk, Strategic Risk, Legal Risk…)
  - Risk of a decline in the efficiency and productivity of a company due to bad management decisions that result in losses.
- Financial Risk
  - Comes from the company's use of borrowed funds. The higher the debt to equity ratio, the more the company is exposed to financial risk.
- Liquidity Risk
  - Liquidity risk is a financial risk and is caused by any lack of liquidity in the market, when there is no supply or demand for the security.
- Exchange Risk
  - Investments made in foreign capital markets may be accompanied by the risk of loss of capital returns from a decrease in the exchange rate or a devaluation of the currency.
- Political Risk
  - Due to political events, government election and more specifically, government decisions pertaining to critical areas of social and financial life of a country

Diversification and Portfolio Risk

- Total Risk = Systematic Risk + Non-Systematic Risk
- Systematic risk (or Market Risk)
  - Systematic or nondiversifiable
  - Interest Risk, inflation, political etc.
  - Measures with beta (β)

- Non-Systematic risk (or Firm-specific risk)
  - Diversifiable or nonsystematic
  - Financial risk, Liquidity risk, Operational risk etc.



B: Market and unique risk

Portfolio Risk as a Function of the Number of Stocks in the Portfolio

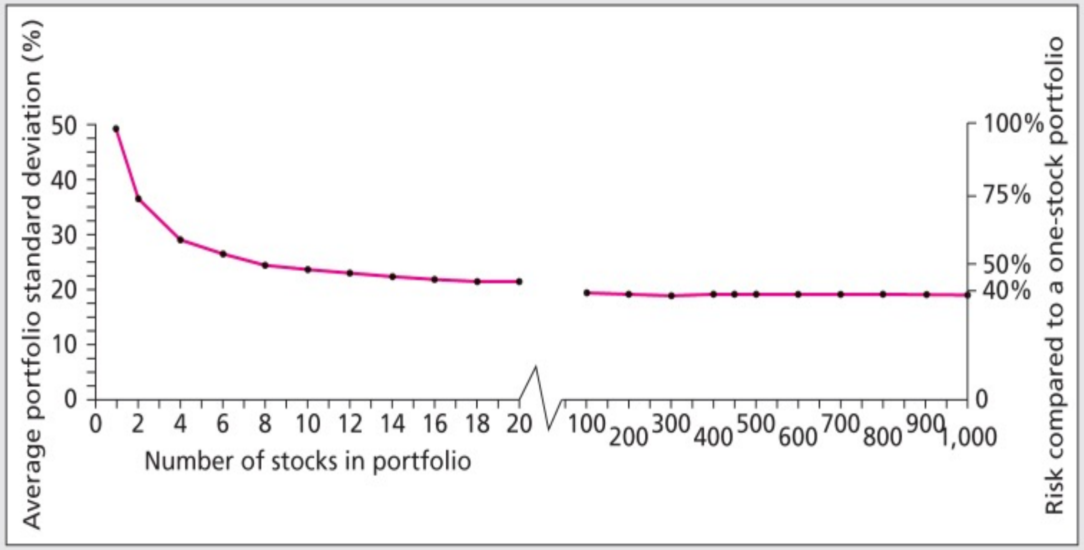
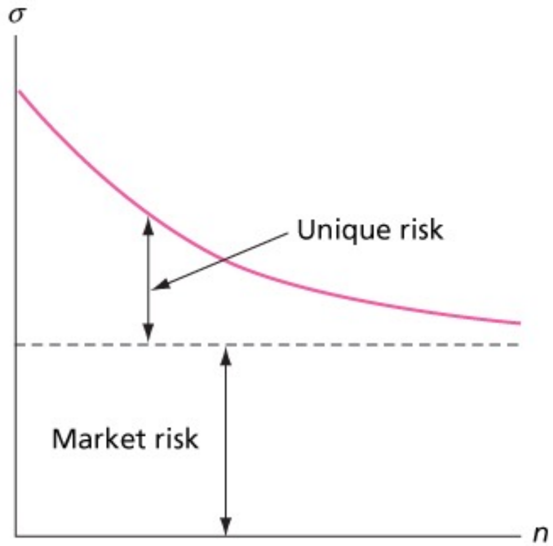


FIGURE 6.2

Portfolio risk decreases as diversification increases

Source: Meir Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22, September 1987.



B: Market and unique risk

"Don't put all your eggs in one basket!"

Covariance and Correlation

- Portfolio risk depends on the correlation between the returns of the assets in the portfolio

- Covariance and the correlation coefficient provide a measure of the way returns of two assets vary

Two-Security Portfolio

Two-Security Portfolio: Expected Return

$$r_p = w_D r_D + w_E r_E$$

r_P = Portfolio Return

w_D = Bond Weight

r_D = Bond Return

w_E = Equity Weight

r_E = Equity Return

$$\boxed{E(r_p) = w_D E(r_D) + w_E E(r_E)}$$

Example: Expected Return of a two-securities portfolio

- An investor has $100 to invest. They invest $80 in security A that has an expected return of 10% and the rest in security A that has an expected return of 20%.
- What is the expected return of this portfolio?

Answer:

- Portfolio is invested 80% (0.8) in A and 20% (Why?) in B.
- Expected Return = 0.8 *10% + 0.2*20% = 12%

Two-Security Portfolio: Risk (expressed with ρ)

$$\sigma_p^2 = w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E \sigma_D \sigma_E \rho_{DE}$$

(Pronounced "rho")

$\rho_{D,E}$ = Correlation coefficient of returns = $\mathbf{Cov(r_{D},r_{E}) / \sigma_D\sigma_E}$

Where σ_D and σ_E are the standard deviation of returns for Securities D and E respectively

Range of values for $\rho_{1,2}$

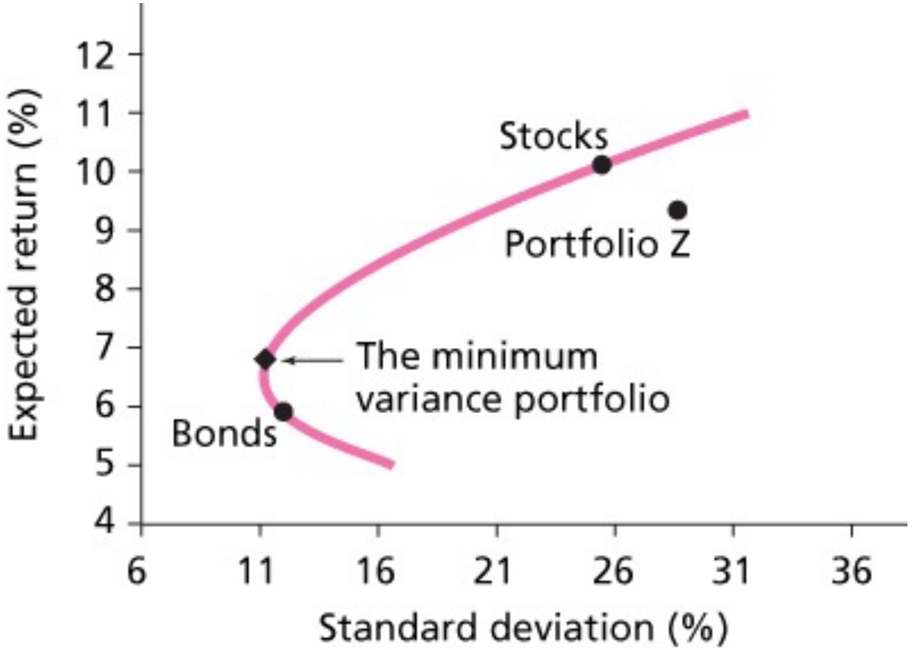
$$\boxed{+1.0 \geq \rho \geq -1.0}$$

If ρ = 1.0, the securities are perfectly positively correlated

If ρ = - 1.0, the securities are perfectly negatively correlated

Investment Opportunity Set for Stocks and Bonds

We vary the % invested in Stocks (WE) and the % invested in Bonds (WD) to draw the graph below.
The graph draws the different combinations.

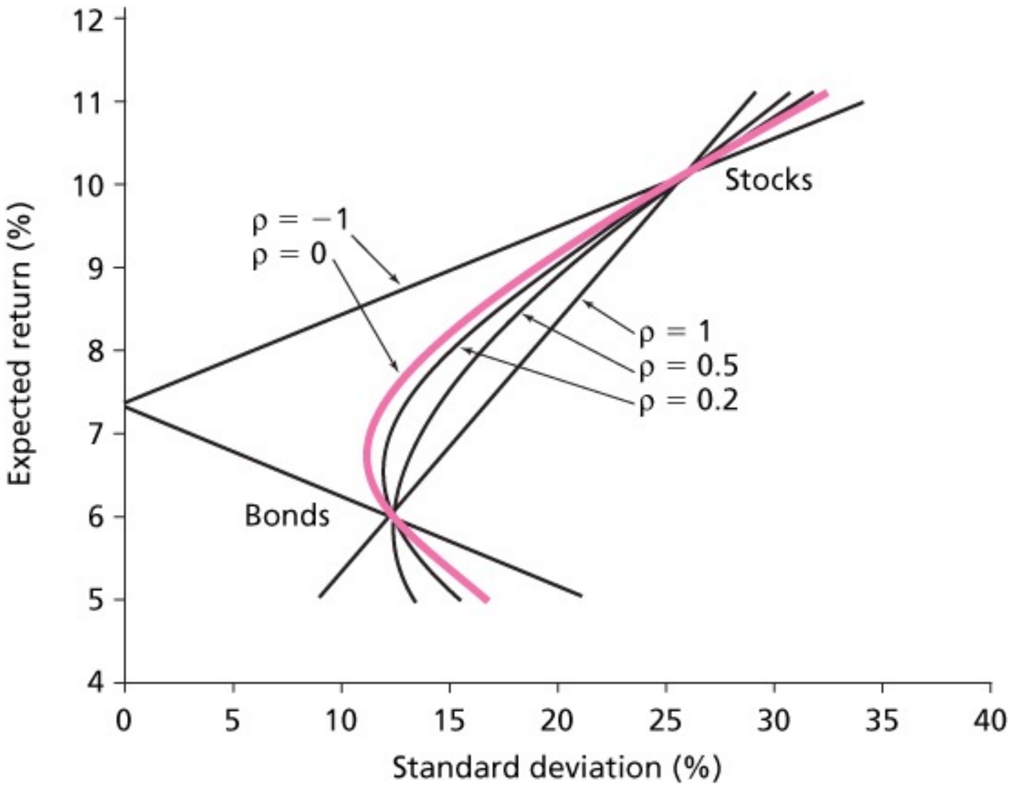


$$E(r_p) = w_D E(r_D) + w_E E(r_E)$$

$$\sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_D w_E \sigma_D \sigma_E \rho_{DE}$$

Investment Opportunity Set for Stocks and Bonds with Various Correlations (ρ)

Same as the previous graph, but for different values of ρ.

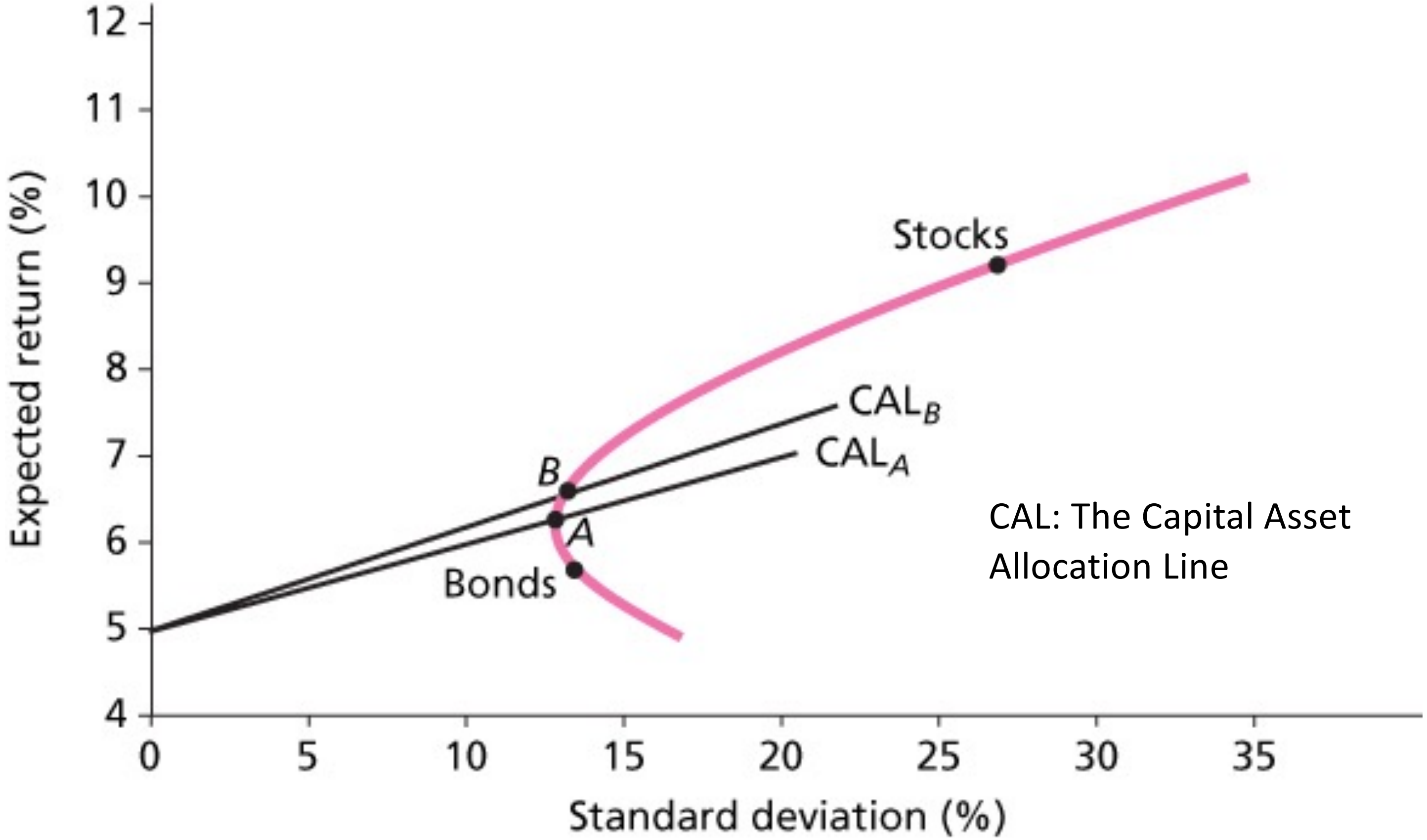


$$E(r_p) = w_D E(r_D) + w_E E(r_E)$$

$$\sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_D w_E \sigma_D \sigma_E \rho_{DE}$$

Adding the Risk Free Asset

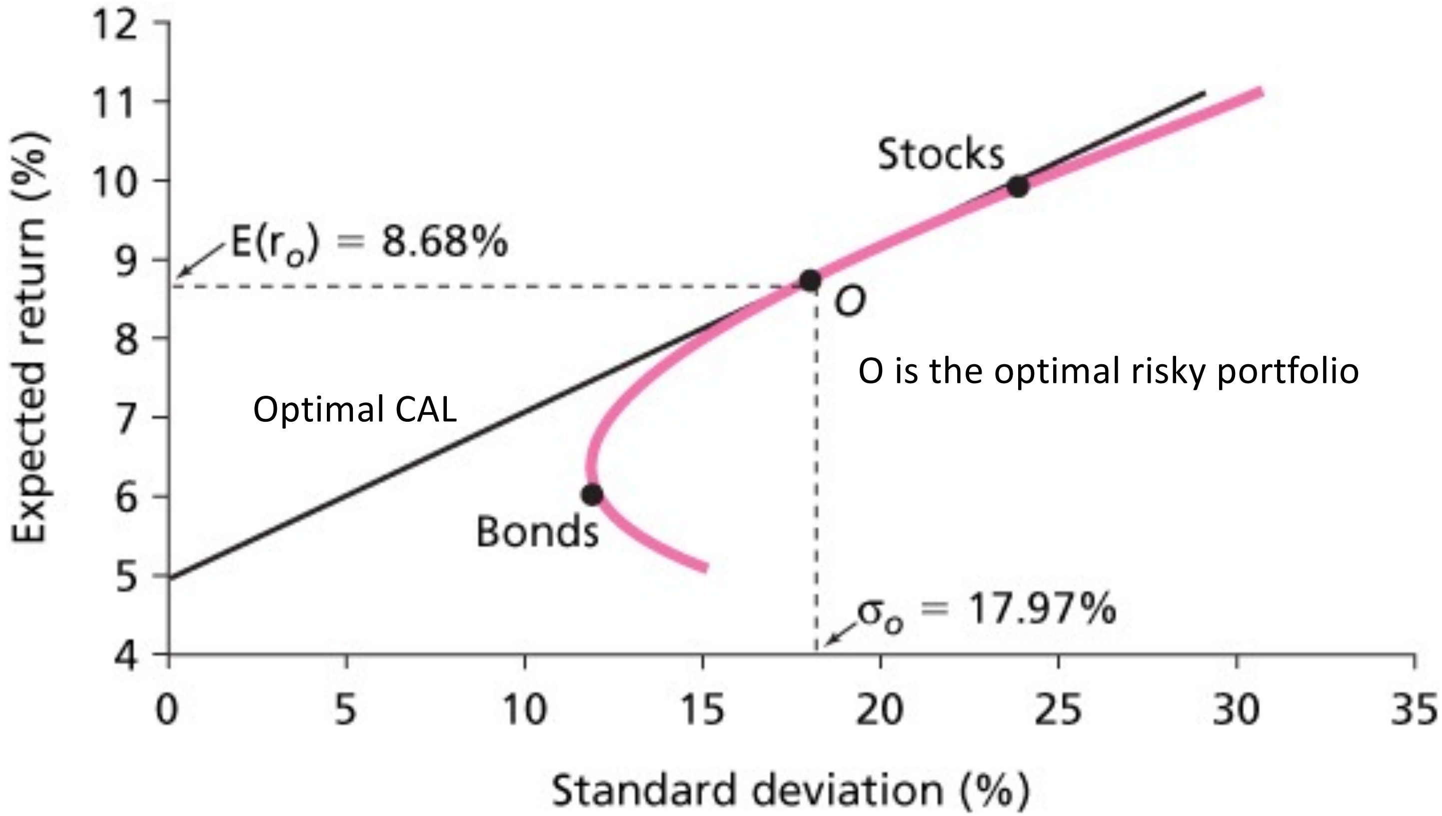
Figure 6.5 Opportunity Set Using Stocks and Bonds and Two Capital Allocation Lines



Rf = 5%
Risk = 0

CAL: The Capital Asset Allocation Line

Optimal Capital Allocation Line for Bonds, Stocks and T-Bills (Rf)



O is the optimal risky portfolio

If the Risky Portfolio is the Market Portfolio, then CAL is called CML (Capital Market Line).

Portfolio Selection

Portfolio Selection

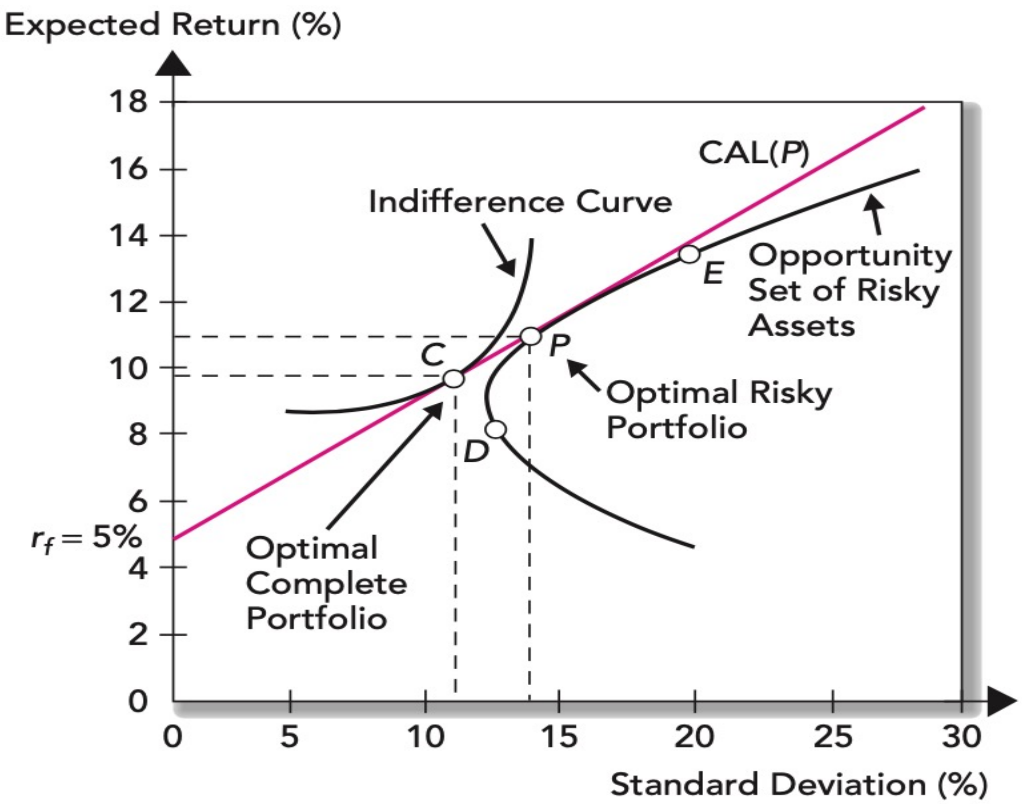
- **The separation principle** (Tobin, 1958)*
- It says that the portfolio choice problem may be separated into two independent tasks:

1. First: Determination of the optimal risky portfolio – a purely technical task. Given the available investments as input, the best risky portfolio is the same for all clients, regardless of risk aversion.
    - See the Markowitz Model (Markowitz, 1952)**, a portfolio optimization model, how to chose the optimal portfolio
2. Second: Allocation of the complete portfolio to T-bills versus the risky portfolio. This task depends on personal preference. Here the client is the decision maker.

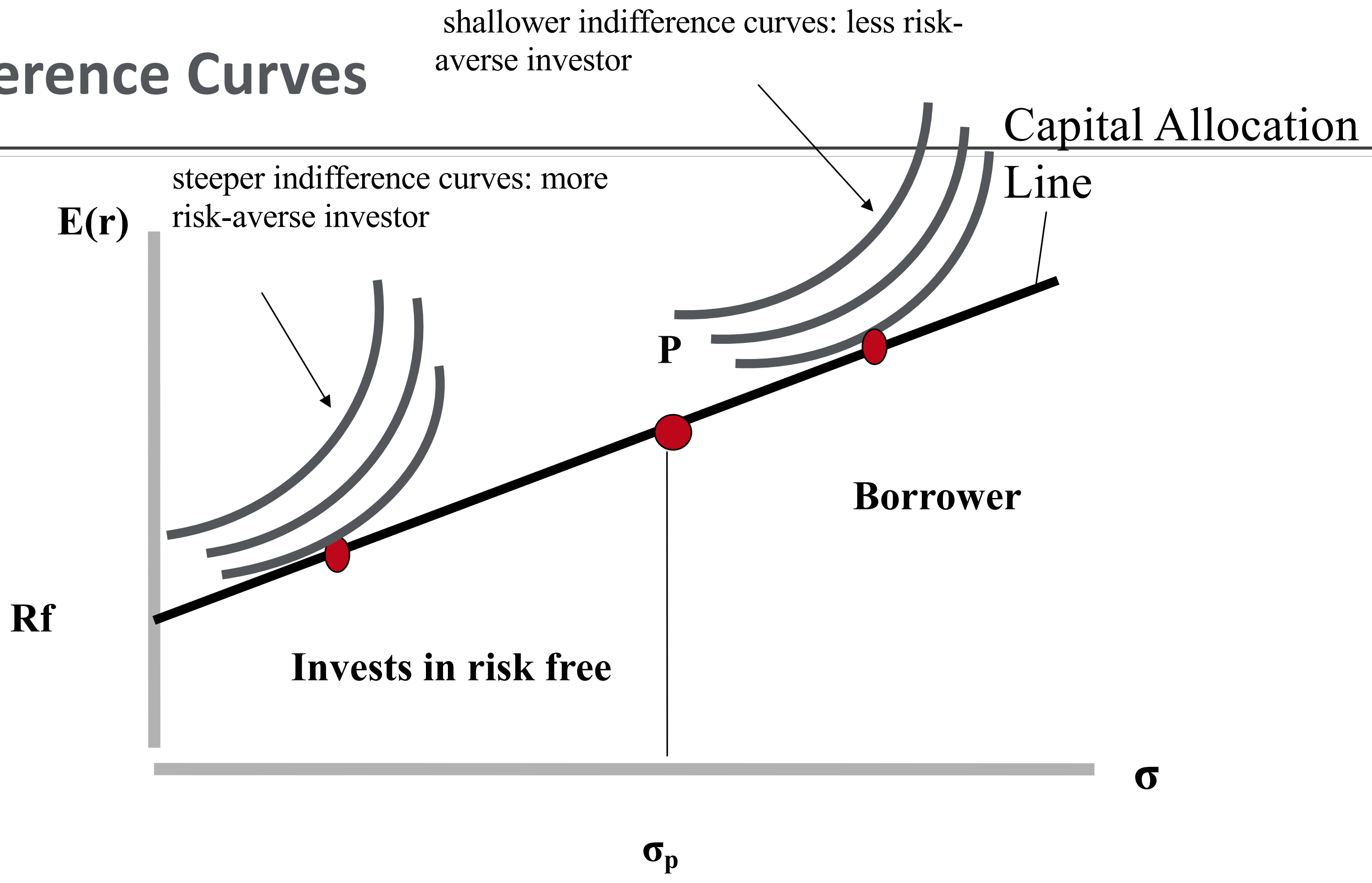
*Tobin, J., 1958. Liquidity preference as behavior towards risk. The review of economic studies, 25(2), pp.65-86.

**Markowitz, Harry, 1952, Portfolio selection, Journal of Finance 7, 77-91.

Which portfolio will be chosen by a certain investor?
Enter Indifference Curves (and Utility)



Indifference Curves



Markowitz (1952)

- Aim to find the minimum variance portfolio, subject to certain constraints.
- The model will give back the optimal weights, w, to invest in each asset.

$$\sigma_p^2 = \begin{pmatrix} w_1 & w_2 & w_3 & w_4 \end{pmatrix} \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix}$$

- The problem with the Markowitz model is that it requires a lot of inputs, i.e. Estimates, calculations.
- For a portfolio containing (n) securities, we need to calculate n expected returns, (n) variances or standard deviations (σ), and [n (n-1)] / 2 covariances.
- That is, a total of [n (n + 3)] / 2 estimates will be needed.
- For example, if n = 100 securities, then 5,150 estimates will be required.

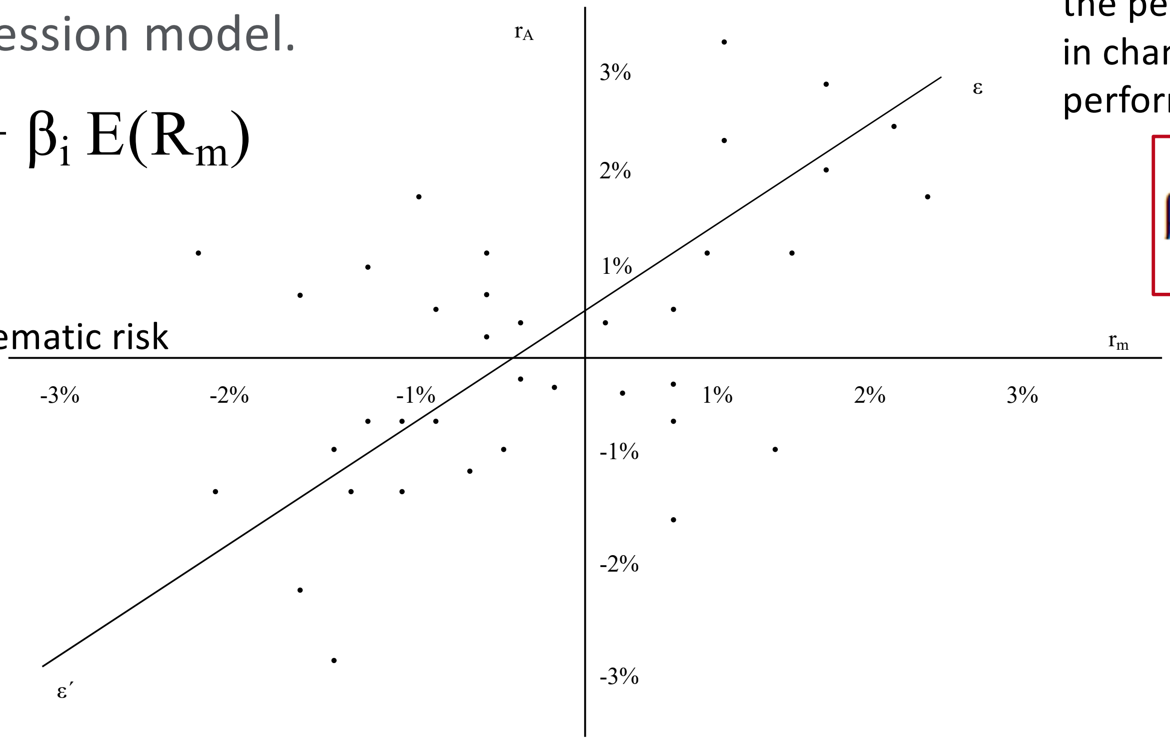
Single Index Model (Sharpe, 1963)

- Can be estimated with a simple regression model.

$$E(R_i) = \alpha_i + \beta_i\, E(R_m)$$

$$\sigma_i^2 = \boxed{\beta_i^2\, \sigma_m^2} + \boxed{\sigma_{\varepsilon i}^2}$$

Systematic risk + nonsystematic risk



Slope is βA, i.e., the sensitivity of the performance of the security A in changes of the market performance, Rm

$$\boxed{\beta_i = \frac{COV(R_m, R_i)}{\sigma_m^2}}$$

Expected return and risk of a portfolio using the Single Index Model

$$E(R_p) = \alpha_p + \beta_p \, E(R_m)$$

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i$$

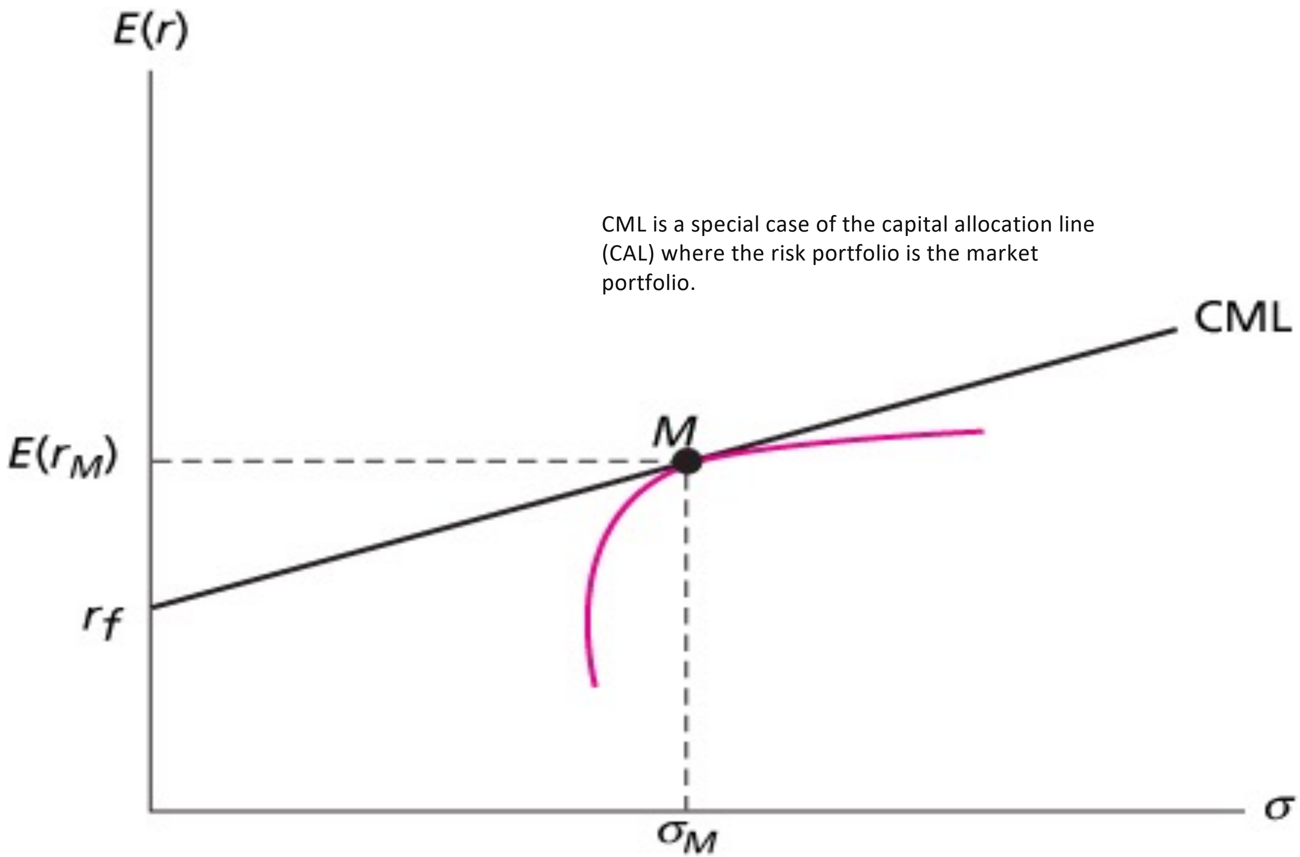
$$\alpha_p = \sum_{i=1}^{n} w_i \alpha_i$$

$$\sigma_p^{2} = \beta_p^{2} \, \sigma_m^{2}$$

The Capital Asset Pricing Model

- Treynor (1961, 1962), Sharpe (1964), Lintner (1965) and Mossin (1966) independently
- Based on Markowitz's work on diversification and modern portfolio theory.
- Nobel Prize in Economics (1990) jointly to Sharpe, Markowitz and Merton Miller.

The Efficient Frontier and the Capital Market Line



CML is a special case of the capital allocation line (CAL) where the risk portfolio is the market portfolio.

CAPM EQUATION

$$E(ri) = R_f + \beta_i (E(rm) - R_f)$$

- E(ri) = return required on financial asset i
- Rf = risk-free rate of return
- βi = beta value for financial asset i
- E(rm) = average return on the capital market

$[E(r_M) - r_f]/\sigma m$ =Market price of risk =
=Slope of the CAPM

CAPM Example

Suppose you are considering investing in a stock with a beta of 1.2. The risk-free rate of return is 3%, and the market risk premium is 7%. What is the expected return on the stock according to the CAPM?

Solution:

- Expected return = Risk-free rate + Beta * Market risk premium

Plugging in the values given in the problem, we get:

- Expected return = 3% + 1.2 * 7% = 11.4%

Therefore, according to the CAPM, we can expect to earn an 11.4% return on this stock given its level of risk (as measured by its beta) relative to the overall market.

Other valuation models

Multifactor models

- Fama and French (1993) three factor model: Performance of the **Market**, **Size**, Book value relative to market value (**Book-to-Market**) – Nobel 2013
- Carhart (1997) added the factor of **momentum**
- Fama and French (2015) proposed a five-factor model by adding **profitability** and **investment** pattern factors

Also:

- Arbitrage Pricing Theory (APT)

Readings

From Bodie, Kane, and Marcus

From the Bodie, Kane, and Marcus (2018) book:

(read very selectively!)

- Capital Allocation to Risky Assets (BKM 6)
- Optimal Risky Portfolios (BKM 7)
- Index Models (BKM 8)
- The Capital Asset Pricing Model (BKM 9)
- Arbitrage Pricing Theory and Multifactor Models of Risk and Return (BKM 10)
- We only covered "the peak of the iceberg" – the above are the relevant chapters, that is, you may find what we discussed about there, but also much much much more. If you are interested in the subject, you can read more - for the exams: we don't ask you about things we did not cover in class.
- You may also find the chapter: Risk, Return, and the Historical Record (BKM 5) interesting.

UNIVERSITY *of*
NICOSIA