Berner Fachhochschule - Technik und Informatik

Object-Oriented Programming 2

Topic 2: Collections

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Outline

The Collection Framework

Lists

Sets and Sorted Sets

Queues and Deques

Maps and Sorted Maps

Iterators

The Minimax Algorithm

Outline

The Collection Framework

Lists

Sets and Sorted Sets
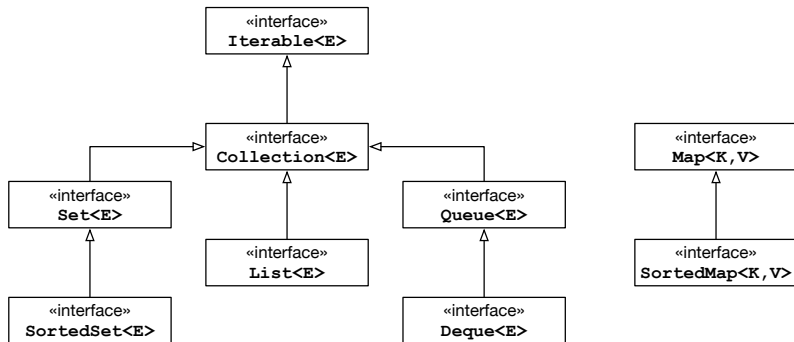
Queues and Deques

Maps and Sorted Maps

Iterators

The Minimax Algorithm

The Collection Framework

- A collection represents an iterable group of elements
- There are various types of collections, depending on how the elements are stored and on how manipulations work
  - → Some allow duplicate elements, others do not
  - → Some are ordered, others are unordered
  - → Some have a fixed (or maximal) size, others have a variable size
  - → Some can be modified, others are immutable
- The generic interface `Collection<E>` is the root of the Java Collections Framework (JCF)
- The JCF exists since Java 1.2 (generic types added in Java 5)
- The interfaces `Map<K,V>` and `SortedMap<K,V>` do not inherit from `Collection<E>`, but are still members of the JCF

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Interfaces of the Collection Framework

The Collection Interface

▶ The interface `Collection` defines a number of methods that
  perform basic operations

  → `boolean isEmpty()`: Returns `true` if the collection is empty
  → `int size()`: Returns the collection's number of elements
  → `boolean add(E e)`: Ensures that the collection contains e
  → `boolean remove(Object e)`: Removes e from the collection
  → `boolean contains(Object e)`: Returns `true` if the
    collection contains e
  → `void clear()`: Removes all elements form the collection
  → `Object[] toArray()`: Returns an array containing all of the
    elements from the collection

▶ An additional method is inherited from `Iterable<E>`

  → `Iterator<E> iterator()`: Returns an iterator over all
    elements

Classes of the Collection Framework
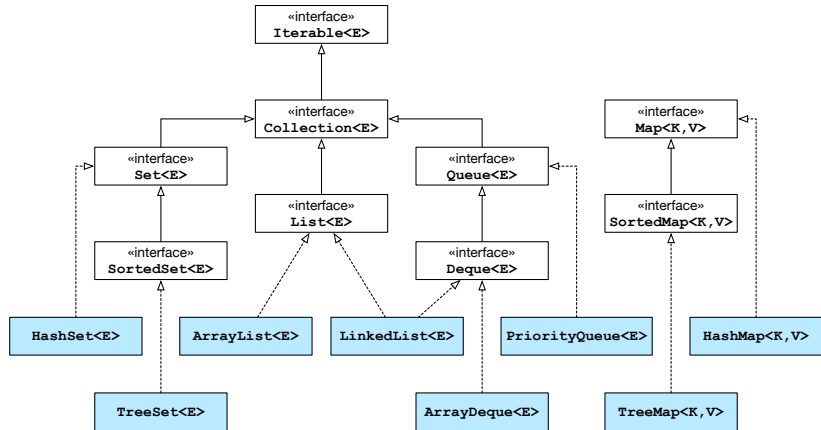
Classes of the Collection Framework

▶ A class that implements `Collection<E>` is supposed to provide at least two constuctors for constructing . . .

  → an empty collection
  → a collection containing all elements of another collection

▶ Examples:

  → `ArrayList()`, `ArrayList(Collection<? extends E> c)`
  → `LinkedList()`, `LinkedList(Collection<? extends E> c)`
  → `HashSet()`, `HashSet(Collection<? extends E> c)`
  → etc.

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Example of Using Collections I

```java
public class CollectionTester {

  public static void main(String[] args) {

    Collection<String> c1 = new ArrayList<>();
    c1.add("Hello");
    c1.add("World");
    c1.size(); // returns 2
    System.out.println(c1); // prints "[Hello, World]"

    Collection<String> c2 = new LinkedList<>(c1);
    c2.add("!");
    c2.add("!");
    c2.size(); // returns 4
    c2.remove("World");
    c2.size(); // returns 3
    System.out.println(c2); // prints "[Hello, !, !]"
```

Example of Using Collections II

```
    Collection<String> c3 = new HashSet(c2);
    c3.add("World");
    c3.add("World");
    c3.size(); // returns 3
    System.out.println(c3); // prints "[!, Hello, World]"
  }

}
```

Outline

The Collection Framework

Lists

Sets and Sorted Sets

Queues and Deques

Maps and Sorted Maps

Iterators

The Minimax Algorithm

Lists

▶ The interfaces List, Queue, and Deque extend the interface Collection by introducing ...

  → an ordering of the elements stored in the collection
  → additional access methods

▶ A list offers positional access, i.e., manipulations of elements are based on their numerical position (index) in the list
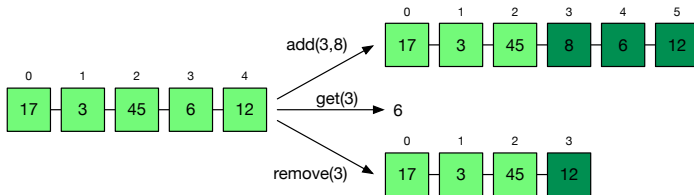
The List Interface

- `E get(int i)`: Returns the element at the specified index `i`
- `E set(int index, E element)`: Replaces the element at the specified index `i` with `e`
- `void add(int i, E e)`: Inserts `e` at the specified index `i`
- `E remove(int i)`: Removes the element at the specified index `i`
- `int indexOf(Object e)`: Returns the index of the first occurrence of `e`
- `int lastIndexOf(Object e)`: Returns the index of the last occurrence of `e`

ArrayList vs. LinkedList

▶ The main difference between the classes `ArrayList` and `LinkedList` is their internal way of storing the elements

→ `ArrayList` uses internally an array (which needs to be replaced by a bigger one if it gets full)

→ `LinkedList` uses internally a doubly linked chain of nodes, which carry the elements

▶ This has implications on the running times of some methods

→ `ArrayList` allows for fast random access, but adding or removing elements may require existing elements to be shifted

→ `LinkedList` allows for constant-time insertions or removals, but only sequential access

▶ Recommendation: use `ArrayList` except in cases where insertions or removals are the dominant operations

Example of Using Sets I

```java
public class ListTester {

  public static int ROUNDS = 500000;

  public static void main(String[] args) {

    List<String> l1 = new ArrayList<>();
    System.out.println("ArrayList: start...");
    for (int i = 1; i <= ROUNDS; i++) {
      l1.add(0, "Hello");
    }
    System.out.println("done");

    List<String> l2 = new LinkedList<>();
    System.out.println("LinkedList: start...");
    for (int i = 1; i <= ROUNDS; i++) {
      l2.add(0, "World");
```

Example of Using Sets II

```
    }
    System.out.println("done");
  }

}
```

Outline

Sets

- A set in Java is a collection that contains no duplicates
- Duplicates are elements e1 and e2 such that `e1.equals(e2)`
- Compared to a list, there are two important differences:
  - → A list may contain duplicates
  - → Elements in a list are ordered (positional access)
- Java sets correspond to sets in mathematics, except that Java sets can be modified
  - → `boolean contains(Object x)`: tests if $x \in X$
  - → `boolean containsAll(Collection<?> y)`: tests if $Y \subseteq X$
  - → `boolean addAll(Collection<? extends E> y)`: computes $X \cup Y$
  - → `boolean retainAll(Collection<?> y)`: computes $X \cap Y$
  - → `boolean removeAll(Collection<?> y)`: computes $X \setminus Y$

Sorted Sets

▶ A sorted set in Java is a set that provides a total order on its elements, which is determined by their natural order or by specifying a comparator

▶ The interface `SortedSet` provides some additional methods that exploit the existence of an order

  → `E first()`: Returns the minimal element currently in this set
  → `E last()`: Returns the maximal element currently in this set
  → `SortedSet<E> tailSet(E e)`: Returns elements that are greater than or equal to e
  → `SortedSet<E> headSet(E e)`: Returns elements that are strictly less than e
  → `SortedSet<E> subSet(E e1, E e2)`: Returns the elements that range from e1 (inclusive) to e2 (exclusive)

HashSet vs. TreeSet

- ▶ In a `HashSet`, the order of the elements is unspecified
- ▶ In a `TreeSet`, the order is determined by their natural order or by specifying a comparator (similar to `PriorityQueue`)
    - → `TreeSet()`
    - → `TreeSet(Comparator<? super E> comparator)`
- ▶ Another difference is their internal way of storing the elements
    - → `HashSet` uses internally a hash table (array)
    - → `TreeSet` uses internally a red-black tree
- ▶ All critical operations are efficient: average $O(1)$ for `HashSet`, worst-case $O(\log n)$ for `TreeSet`

Example of Using Sets I

```java
public class SetTester {

  public static void main(String[] args) {

    Set<Integer> s1 = new HashSet<>(Arrays.asList(new Integer
        []{1,2,3,4}));
    Set<Integer> s2 = new HashSet<>(Arrays.asList(new Integer
        []{3,4,5}));
    Set<Integer> s3 = new HashSet<>(Arrays.asList(new Integer
        []{2,5,6}));

    s2.addAll(s3);
    System.out.println(s2); // s2 = s2 cup s3 = {2,3,4,5,6}

    s2.retainAll(s1);
    System.out.println(s2); // s2 = s2 cap s1 = {2,3,4}
```

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Example of Using Sets II

```
    s2.removeAll(s3);
    System.out.println(s2); // s2 = s2 minus s3 = {3,4}

    s2.add(3);
    s2.add(5);
    s2.add(5);
    System.out.println(s2); // s2 = s2 cup {3,5,5} = {3,4,5}

    SortedSet<Integer> s = new TreeSet<>(Arrays.asList(new
        Integer[]{1,3,7,2,5,4,7}));
    System.out.println(s.headSet(5)); // {1,2,3,4}
    System.out.println(s.tailSet(5)); // {5,7}
    System.out.println(s.subSet(2,5)); // {2,3,4}
  }
}
```

The Equals Method

- ▶ Duplicates in sets and sorted sets are defined differently:
    - → Sets: `e1.equals(e2)`
    - → Sorted sets:
      `e1.compareTo(e2) == 0` or `c.compareTo(e1,e2) == 0`
- ▶ The method `equals(Object obj)` as defined in `Object` only performs an identity check `e1 == e2`
- ▶ **Important**: If you need a `Set<MyClass>`, you must override `equals(Object obj)` in your class `MyClass`
    - → `e.equals(null) == false`
    - → `e.equals(e) == true` (Reflexivity)
    - → `e1.equals(e2) == e2.equals(e1)` (Symmetry)
    - → `e1.equals(e2) && e2.equals(e3) == e1.equals(e3)`
      (Transitivity)

Auto-Generated Equals Method I

```java
public class MyClass {

  private int x;
  private String y;

  @Override
  public boolean equals(Object obj) {
    if (obj == null) {
      return false;
    }
    if (this == obj) {
      return true;
    }
    if (getClass() != obj.getClass()) {
      return false;
    }
    MyClass other = (MyClass) obj;
```

Auto-Generated Equals Method II

```java
    if (this.x != other.x) {
      return false;
    }
    if (this.y == null) {
      if (other.y != null) {
        return false;
      }
    } else if (!this.y.equals(other.y)) {
      return false;
    }
    return true;
  }
}
```

The HashCode Method

- A set implemented based on hash tables (e.g. `HashSet`) requires a hash function that is compatible with `equals`
- The method `hashCode()` as defined in `Object` only transforms the memory address of the object into an integer
- **Important**: If you need a `HashSet<MyClass>`, you must override `hashCode()` in your class `MyClass`
  - → If `e1.equals(e2)`, then `e1.hashCode() == e2.hashCode()`
  - → Otherwise, `e1.hashCode() !=e2.hashCode()` with high probability
- General recommendation: Always override `hashCode()` when you override `equals(Object obj)`

Auto-Generated HashCode Method

▶ Let $h_1, \ldots, h_n$ be the hash codes of the n fields of an object x, then a good practice is to define $h_0 = 1$ and compute

$$hashCode(x) = \sum_{i=0}^{n} h_i \cdot 31^{n-i}$$

▶ Example: Let $n = 2$, then $hashCode(x) = 31^2 + 31h_1 + h_2$

```java
@Override
public int hashCode() {
  final int prime = 31;
  int result = 1;
  result = prime * result + x;
  result = prime * result + ((y == null) ? 0 : y.hashCode());
  return result;
}
```

Outline

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Queues and Deques (and Stacks)

- A queue offers first-in-first-out access (FIFO) to its elements

  - → Elements are added to the back of the queue
  - → Elements are removed from the front of the queue

- A deque (short form for "double ended queue", pronounced as "deck") offers access to both of its extremities

  - → Elements are added to the front or the back of the deque
  - → Elements are removed from the front or the back of the deque

- A deque also includes typical methods of a stack, which offers last-in-first-out access (LIFO)

  - → Elements are added to the front (push)
  - → Elements are removed from the front (pop)

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The Queue Interface

- ▶ `E element(), E peek()`: Retrieves, but does not remove, the first element of the queue
- ▶ `boolean add(E e), boolean offer(E e)`: Inserts e into the queue if it is possible
- ▶ `E remove(), E poll()`: Retrieves and removes the first element of the queue
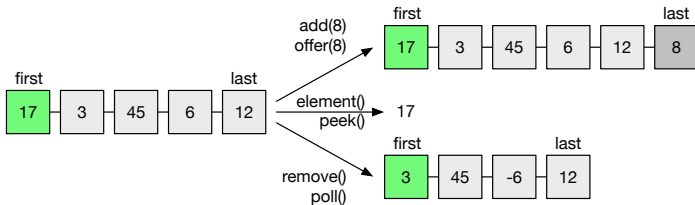
The Deque Interface

- `E getFirst()`, `E peekFirst()`: Retrieves, but does not remove, the first element of the deque
- `E getLast()`, `E peekLast()`: Retrieves, but does not remove, the last element of the deque
- `void addFirst(E e)`, `void offerFirst(E e)`, `void push(E e)`: Inserts e at the front of the deque
- `void addLast(E e)`, `void offerLast(E e)`: Inserts e at the end of the deque
- `E removeFirst()`, `E pollFirst()`, `E pop()`: Retrieves and removes the first element of the deque
- `E removeLast()`, `E pollLast()`: Retrieves and removes the last element of the deque

The Deque Interface
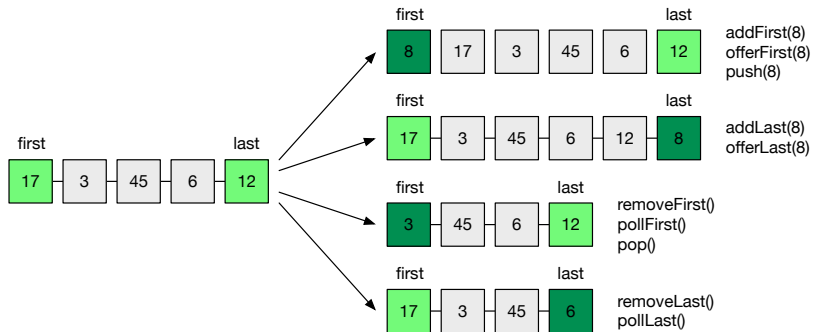
ArrayDeque vs. PriorityQueue

- In a `ArrayDeque`, the order of the elements is determined by the sequence of insertion operations

- In a `PriorityQueue`, the order of the elements is determined by their natural order or by specifying a comparator
  - → `PriorityQueue()`
  - → `PriorityQueue(Comparator<? super E> comparator)`

- Another difference is their internal way of storing the elements
  - → `ArrayDeque` uses internally an array (similar to `ArrayList`)
  - → `PriorityQueue` uses internally a heap

- All critical operations are efficient: average $O(1)$ for `ArrayDeque`, worst-case $O(\log n)$ for `PriorityQueue`

Example of Using Queues I

```java
public class QueueTester {

  public static void main(String[] args) {

    Queue<String> p1 = new ArrayDeque<>(); // Insertion order
    p1.add("Peter");
    p1.add("John");
    p1.add("Tom");
    p1.add("Andrew");
    while (!p1.isEmpty()) {
      System.out.println(p1.remove());
    } // Loop prints "Peter", "John", "Tom", "Andrew"

    Queue<String> p2 = new PriorityQueue<>(); // Natural order
    p2.add("Peter");
    p2.add("John");
    p2.add("Tom");
```

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Example of Using Queues II

```java
p2.add("Andrew");
while (!p2.isEmpty()) {
  System.out.println(p2.remove());
} // Loop prints "Andrew", "John", "Peter", "Tom"

// String length comparator
Comparator<String> c = new Comparator<String>() {
  @Override
  public int compare(String s1, String s2) {
    if (s1.length() < s2.length()) {
      return -1;
    }
    if (s1.length() > s2.length()) {
      return 1;
    }
    return 0;
  }
};
```

Example of Using Queues III

```
Queue<String> p3 = new PriorityQueue<>(c);
p3.add("Peter");
p3.add("John");
p3.add("Tom");
p3.add("Andrew");
while (!p3.isEmpty()) {
  System.out.println(p3.remove());
} // Loop prints "Tom", "John", "Peter", "Andrew"
  }
}
```

Outline

Maps and Sorted Maps

- A map in Java is an object that maps keys to values (similar to a function $f : \{k_1, \ldots, k_n\} \to V$ in mathematics)

$$k_1 \mapsto v_1 = f(k_1)$$
$$\vdots$$
$$k_n \mapsto v_n = f(k_n)$$

- A map cannot contain duplicate keys (each key maps to at most one value), but different keys can map to the same value

- The goal of a map is to provide a key-based access to the values stored in the map

- A sorted map is a map that provides a total ordering on its keys (similar to `SortedSet`)

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The Map Interface

- ▶ Since `Map<K,V>` does not inherit from `Collection<E>`, it defines its own methods `isEmpty()`, `size()`, and `clear()`

  - → `boolean containsKey(Object key))`: Returns `true` if the map contains a mapping with the given value
  - → `boolean containsValue(Object key))`: Returns `true` if the map contains a mapping for the given key
  - → `V get(Object key)`: Returns the value of the mapping for the given key (or `null` if the key does not exist)
  - → `V put(K key, V value)`: Adds a new mapping or replaces the value of an existing mapping
  - → `V remove(Object key)`: Removes the mapping if the given key exists
  - → `V replace(K key, V value)`: Replaces the value in the mapping if the given key exists
  - → `Collection<V> values()`: Returns a collection of all values
  - → `Set<K> keySet()`: Returns a set of all keys

HashMap vs. TreeMap

- In a `HashMap`, the order of the keys is unspecified
- In a `TreeMap`, the order is determined by their natural order or by specifying a comparator (similar to `TreeSet`)
    - → `TreeMap()`
    - → `TreeMap(Comparator<? super E> comparator)`
- Another difference is their internal way of storing the elements
    - → `HashMap` uses internally a hash table (array)
    - → `TreeMap` uses internally a red-black tree
- All critical operations are efficient: average $O(1)$ for `HashMap`, worst-case $O(\log n)$ for `TreeMap`

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Example of Using Maps I

```java
public class MapTester {

  public static void main(String[] args) {

    Map<Integer,String> map = new HashMap<>();
    map.put(1, "Hello");
    map.put(5, "World");
    map.put(7, "Hello");
    map.put(10, "Hello World");
    map.put(10, "Hello World!"); // replaces previous value

    System.out.println(map.keySet()); // prints [1, 5, 7, 10]
    System.out.println(map.values()); // prints [Hello, World,
          Hello, Hello World!]

    String str1 = map.get(1);
    if (str1 != null) {
```

Example of Using Maps II

```
      System.out.println(str1); // prints "Hello"
    }
    String str2 = map.get(2);
    if (str2 != null) {
      System.out.println(str2); // nothing is printed
    }

    // the following code is less efficient
    int key = 1;
    if (map.containsKey(key)) {
      System.out.println(map.get(key)); // prints "Hello"
    }
  }
}
```

Outline

Iterators

▶ An iterator in Java is an object that helps iterating through all elements in a collection

▶ The interface `Iterator<E>` defines the following methods:
   → `boolean hasNext()`: Returns `true` if the iteration has more elements
   → `E next()`: Returns the next element in the iteration

▶ Note that an iterator only allows a single iteration

▶ The following is a typical example of usage:

```
Iterator<String> iterator = ...;
while(iterator.hasNext()) {
  System.out.println(iterator.next());
}
```

Array Iterator I

```java
public class ArrayIterator<E> implements Iterator<E> {

  private E[] array;
  private int currentIndex;

  public ArrayIterator(E[] array) {
    this.array = array;
    this.currentIndex = 0;
  }

  @Override
  public boolean hasNext() {
    return this.currentIndex < this.array.length;
  }

  @Override
  public E next() {
```

Array Iterator II

```java
    if (!this.hasNext()){
      throw new NoSuchElementException();
    }
    return this.array[this.currentIndex++];
  }

}
```

Reverse Array Iterator I

```java
public class ReverseArrayIterator<E> implements Iterator<E> {

  private E[] array;
  private int currentIndex;

  public ReverseArrayIterator(E[] array) {
    this.array = array;
    this.currentIndex = array.length - 1;
  }

  @Override
  public boolean hasNext() {
    return this.currentIndex >= 0;
  }

  @Override
  public E next() {
```

Reverse Array Iterator II

```java
    if (!this.hasNext()) {
      throw new NoSuchElementException();
    }
    return this.array[this.currentIndex--];
  }

}
```

Iterator Tester I

```java
public class IteratorTester {

  public static void main(String[] args) {

    Integer[] values = new Integer[] { 1, 3, 3, 5, 6, 9 };

    Iterator<Integer> i1 = new ArrayIterator<>(values);
    while (i1.hasNext()) {
      System.out.print(i1.next() + " ");
    }
    System.out.println();
    // prints 1 3 3 5 6 9

    Iterator<Integer> i2 = new ReverseArrayIterator<>(values);
    while (i2.hasNext()) {
      System.out.print(i2.next() + " ");
    }
```

Iterator Tester II

```
    System.out.println();
    // prints 9 6 5 3 3 1
  }

}
```

List Iterators

- ▶ The `ListIterator<E>` interface extends `Iterator<E>` with methods for traversing a list backwards

    - → `boolean hasPrevious()`: Returns `true` if a backward iteration has more elements
    - → `E previous()`: Returns the previous element in a backward iteration

- ▶ It also provides methods for adding, removing, or replacing element in the list

    - → `void add(E e)`: Inserts an element into the list
    - → `void remove()`: Removes the current element from the list
    - → `void set(E e)`: Replaces the current element in the list

Obtaining Iterators from Collections

▶ An iterator can be obtained from a collection using a method
   inherited from Iterable<E>
   → Iterator<E> iterator(): Returns an iterator over all
      elements in the collection (the order is unspecified)

▶ Similarly, a list iterator can be obtained from a list
   → ListIterator<E> listIterator(): Returns a list iterator
      over all elements in the list

▶ Note that the Java for-each loop work for any class that
   implements the Iterable<E> interface

```
Iterable<Integer> values =Arrays.asList(new Integer
    []{1,3,3,5,7,9});
int sum = 0;
for (Integer value: values) {
  sum = sum + value;
}
```

Outline

The Minimax-Algorithm

▶ The minimax algorithm is an optimal game playing algorithm for two player games such as tic-tac-toe, chess, go, uril, ...

▶ Let A and B be the two players and S the finite set of possible game states, where ...

   → $s_0 \in S$ is the initial state
   → $S^* \subseteq S$ are final states, in which the game ends

▶ For all final states $s^* \in S^*$, the winner of the game is defined by a function $E : S^* \to [-1, 1]$, where

$$E(s^*) = \begin{cases} 1, & \text{if } A \text{ wins} \\ 0, & \text{if the game ends as draw} \\ -1, & \text{if } B \text{ wins} \end{cases}$$

Example: Subtraction Game

- In a (k, n)-subtraction game, A and B take turns in removing up to k objects from a pile with initially n objects
    - → A begins
    - → Whoever removes the last object from the pile wins

- Example: $(2, 5)$-subtraction game
    - → States: $S = \{(5, A), (3, A), \ldots, (0, A), (4, B), \ldots, (0, B)\}$
    - → Initial state: $s_0 = (5, A)$
    - → Final states: $S^* = \{(0, A), (0, B)\}$
    - → $E(s^*) = \begin{cases} 1, & \text{for } s^* = (0, B) \\ -1, & \text{for } s^* = (0, A) \end{cases}$

Example: Subtraction Game
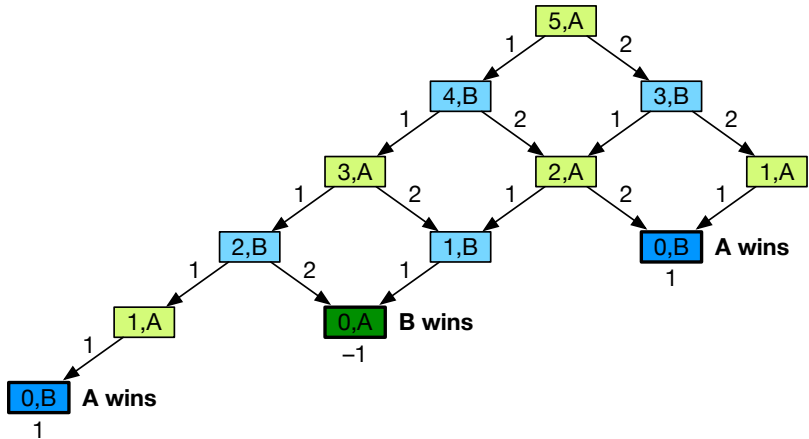
Minimax Algorithm: General Idea

- Let $next(s) \subseteq S$ be the reachable states from $s \in S$
- Example: $(2, 5)$-subtraction game
  - $\rightarrow$ $next((5, A)) = \{(4, B), (3, B)\}$
  - $\rightarrow$ $next((4, B)) = \{(3, A), (2, A)\}$
    - $\vdots$
  - $\rightarrow$ $next((0, A)) = next(0, B) = \{\}$
- The minimax algorithm extends E from $E : S^* \rightarrow [-1, 1]$ to $F : S \rightarrow [-1, 1]$ by computing $F(s)$ recursively by for all $s \in S$

$$F(s) = \begin{cases} E(s), & \text{if } s \in S^* \\ \max\{F(s') : s' \in next(s)\}, & \text{if it is } A\text{'s turn} \\ \min\{F(s') : s' \in next(s)\}, & \text{if it is } B\text{'s turn} \end{cases}$$

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Minimax Algorithm: Pseudocode

Algorithm: Minimax(s, d)

if $s \in S^*$ **then**   // game ends
    | **return** $E(s)$

if $d \bmod 2 = 0$ **then**   // A's turn
    | $m \leftarrow -1$
    | **for** $s' \in next(s)$ **do**
    | | $m \leftarrow \max(m, \text{Minimax}(s', d + 1))$

else   // B's turn
    | $m \leftarrow 1$
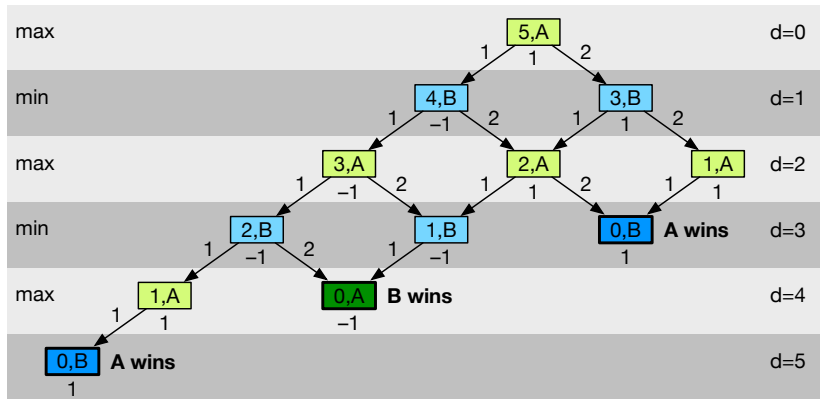 | **for** $s' \in next(s)$ **do**
    | | $m \leftarrow \min(m, \text{Minimax}(s', d + 1))$

return m

Example: Subtraction Game

Initial call: Minimax$((5, A), 0)$
Return value: $F((5, A)) = 1$, i.e. A wins by removing 2 objects

Minimax Algorithm: Performance

▶ The branching factor b of a game is the average number legal moves (children in the game tree)

▶ Examples:
  → (k, n)-subtraction game: $b \leq k$, $h \leq n$
  → Chess: $b \approx 35$, $h \approx 80$
  → Go: $b \approx 250$

▶ The minimax algorithms runs in $O(b^h)$ time, where h denotes the height (maximal depth) of the game tree

▶ In other words, exploring the full game tree is impossible for most non-trivial games

Pruned Minimax Algorithm

▶ To apply the minimax algorithm to non-trivial games, the game tree exploration must be pruned

▶ The simplest pruning method is to stop the recursion when a maximal depth d_{max} is reached

▶ When the recursion stops at state $s \in S$, then a evaluation function $\tilde{E} : S \to [-1, 1]$ is applied to s

   → For $s \in S^*$, let $\tilde{E}(s) = E(s)$
   → Otherwise, let $\tilde{E}(s)$ be an estimate of the advantage of state s relative to A and B, such that $\tilde{E}(s) = 1$ means maximal advantage for A and $\tilde{E}(s) = -1$ maximal advantage for B

▶ The quality of the estimate $\tilde{E}(s)$ and d_{max} determine the quality and accuracy of the final minimax return value

Pruned Minimax Algorithm: Pseudocode

Algorithm: $\mathrm{Minimax}(s, d, d_{\max})$

if $s \in S^*$ **or** $d = d_{\max}$ **then**   // game ends
$\quad$ **return** $\tilde{E}(s)$

if $d \bmod 2 = 0$ **then**   // A's turn
$\quad$ $m \leftarrow -1$
$\quad$ **for** $s' \in next(s)$ **do**
$\quad\quad$ $m \leftarrow \max(m, \mathrm{Minimax}(s', d + 1, d_{\max}))$

else   // B's turn
$\quad$ $m \leftarrow 1$
$\quad$ **for** $s' \in next(s)$ **do**
$\quad\quad$ $m \leftarrow \min(m, \mathrm{Minimax}(s', d + 1, , d_{\max}))$

return m

Defining the Evaluation Function

▶ One popular strategy for constructing an evaluation function
$\tilde{E} : S \rightarrow [-1, 1]$ is as a weighted sum

$$\tilde{E}(s) = \frac{1}{W} \sum_{i=1}^{k} w_i \cdot \tilde{E}_i(s)$$

of k individual evaluation criteria

▶ The value $w_i \in [0, 1]$ denotes the weight of criterion i and
$W = \sum_{i=1}^{k} w_i$ denotes the total weight of all criteria

▶ $\tilde{E}_i : S \rightarrow [-1, 1]$ defines the evaluation function of criterion i

▶ Example from chess:
$$\tilde{E}(s) = 9 \cdot (Q - Q') + 5 \cdot \frac{R - R'}{2} + 3 \cdot \frac{B - B'}{2} + 3 \cdot \frac{N - N'}{2} + \frac{P - P'}{8} + \dots$$

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Alpha-Beta Pruning

- There are many ways of optimizing the minimax algorithm
- The general idea is to prune branches of the game tree that will not influence the final minimax return value
- The simplest optimization is known as alpha-beta pruning
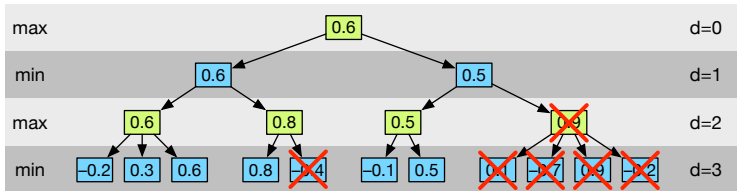
- The following version of the minimax algorithm is initially called with $Minimax(s_0, 0, d_{\max}, -1, 1)$

Alpha-Beta Pruning: Pseudocode I

Algorithm: $\text{Minimax}(s, d, d_{\max}, \alpha, \beta)$

if $s \in S^*$ **or** $d = d_{\max}$ **then**   // game ends
  └ **return** $\tilde{E}(s)$

if $d \bmod 2 = 0$ **then**   // A's turn
  │ $m \leftarrow -1$
  │ **for** $s' \in next(s)$ **do**
  │   │ $m \leftarrow \max(m, \text{Minimax}(s', d + 1, d_{\max}, \alpha, \beta))$
  │   │ $\alpha \leftarrow \max(\alpha, m)$
  │   │ **if** $\alpha \geq \beta$ **then**
  │   │   └ **return** m // β cutoff

Alpha-Beta Pruning: Pseudocode II

else $//$ B's turn
    $m \leftarrow 1$
    for $s' \in next(s)$ **do**
        $m \leftarrow \min(m, \text{Minimax}(s', d + 1, , d_{\max}, \alpha, \beta))$
        $\beta \leftarrow \min(\beta, m)$
        if $\alpha \geq \beta$ **then**
            return m $//$ α cutoff
return m