

Berner Fachhochschule - Technik und Informatik

# Object-Oriented Programming 2

## Topic 2: Collections

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FS 2016

# Outline

The Collection Framework

Lists

Queues and Deques

Sets and Sorted Sets

Maps and Sorted Maps

Iterators

The Minimax Algorithm

# Outline

## The Collection Framework

Lists

Queues and Deques

Sets and Sorted Sets

Maps and Sorted Maps

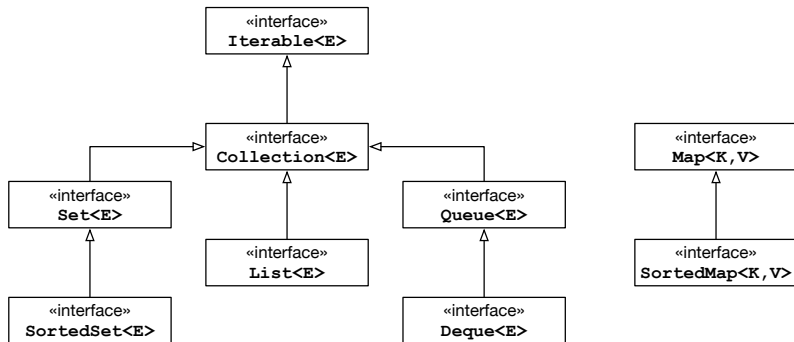
Iterators

The Minimax Algorithm

# The Collection Framework

- ▶ A **collection** represents an iterable group of elements
- ▶ There are various types of collections, depending on how the elements are stored and on how manipulations work
  - Some allow duplicate elements, others do not
  - Some are ordered, others are unordered
  - Some have a fixed (or maximal) size, others have a variable size
  - Some can be modified, others are immutable
- ▶ The generic interface `Collection<E>` is the root of the **Java Collections Framework** (JCF)
- ▶ The JCF exists since Java 1.2 (generic types added in Java 5)
- ▶ The interfaces `Map<K,V>` and `SortedMap<K,V>` do not inherit from `Collection<E>`, but are still members of the JCF

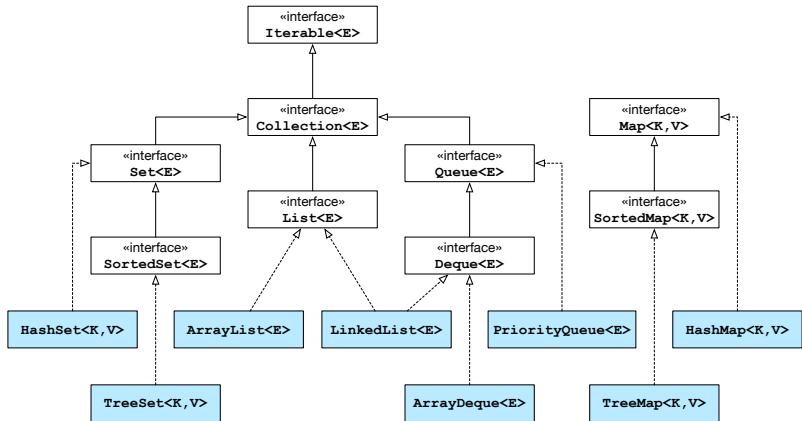
# Interfaces of the Collection Framework



# The Collection Interface

- ▶ The interface `Collection` defines a number of methods that perform basic operations
  - `boolean isEmpty()`: Returns true if the collection is empty
  - `int size()`: Returns the collection's number of elements
  - `boolean add(E e)`: Ensures that the collection contains `e`
  - `boolean remove(Object e)`: Removes `e` from the collection
  - `boolean contains(Object e)`: Returns true if the collection contains `e`
  - `void clear()`: Removes all elements from the collection
  - `Object[] toArray()`: Returns an array containing all of the elements from the collection
- ▶ An additional method is inherited from `Iterable<E>`
  - `Iterator<E> iterator()`: Returns an iterator over all elements

# Classes of the Collection Framework



# Classes of the Collection Framework

- ▶ A class that implements `Collection<E>` is supposed to provide at least two constructors for constructing ...
  - an empty collection
  - a collection containing all elements of another collection
- ▶ Examples:
  - `ArrayList()`, `ArrayList(Collection<? extends E> c)`
  - `LinkedList()`, `LinkedList(Collection<? extends E> c)`
  - `HashSet()`, `HashSet(Collection<? extends E> c)`
  - etc.



# Example of Using Collections I

```
public class CollectionTester {  
  
    public static void main(String[] args) {  
  
        Collection<String> c1 = new ArrayList<>();  
        c1.add("Hello");  
        c1.add("World");  
        c1.size(); // returns 2  
        System.out.println(c1); // prints "[Hello, World]"  
  
        Collection<String> c2 = new LinkedList<>(c1);  
        c2.add("!");  
        c2.add("!");  
        c2.size(); // returns 4  
        c2.remove("World");  
        c2.size(); // returns 3  
        System.out.println(c2); // prints "[Hello, !, !]"  
    }  
}
```

## Example of Using Collections II

```
Collection<String> c3 = new HashSet(c2);  
c3.add("World");  
c3.add("World");  
c3.size(); // returns 3  
System.out.println(c3); // prints "[!, Hello, World]"  
}  
  
}
```

# Outline

The Collection Framework

**Lists**

Queues and Deques

Sets and Sorted Sets

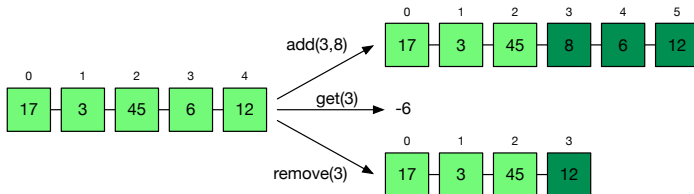
Maps and Sorted Maps

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# Lists

- ▶ The interfaces `List`, `Queue`, and `Deque` extend the interface `Collection` by introducing ...
  - an ordering of the elements stored in the collection
  - additional access methods
- ▶ A `list` offers **positional access**, i.e., manipulations of elements are based on their numerical position (index) in the list



# The List Interface

- ▶ `E get(int i)`: Returns the element at the specified index `i`
- ▶ `E set(int index, E element)`: Replaces the element at the specified index `i` with `e`
- ▶ `void add(int i, E e)`: Inserts `e` at the specified index `i`
- ▶ `E remove(int i)`: Removes the element at the specified index `i`
- ▶ `int indexOf(Object e)`: Returns the index of the first occurrence of `e`
- ▶ `int lastIndexOf(Object e)`: Returns the index of the last occurrence of `e`

# ArrayList vs. LinkedList

- ▶ The main difference between the classes `ArrayList` and `LinkedList` is their internal way of storing the elements
  - `ArrayList` uses internally an array (which needs to be replaced by a bigger one if it gets full)
  - `LinkedList` uses internally a doubly linked chain of nodes, which carry the elements
- ▶ This has implications on the running times of some methods
  - `ArrayList` allows for fast random access, but adding or removing elements may require existing elements to be shifted
  - `LinkedList` allows for constant-time insertions or removals, but only sequential access
- ▶ Recommendation: use `ArrayList` except in cases where insertions or removals are the dominant operations

## Example of Using Sets I

```
public class ListTester {  
  
    public static int ROUNDS = 500000;  
  
    public static void main(String[] args) {  
  
        List<String> l1 = new ArrayList<>();  
        System.out.println("ArrayList: start...");  
        for (int i = 1; i <= ROUNDS; i++) {  
            l1.add(0, "Hello");  
        }  
        System.out.println("done");  
  
        List<String> l2 = new LinkedList<>();  
        System.out.println("LinkedList: start...");  
        for (int i = 1; i <= ROUNDS; i++) {  
            l2.add(0, "World");  
        }  
    }  
}
```

## Example of Using Sets II

```
    }  
    System.out.println("done");  
}  
  
}
```



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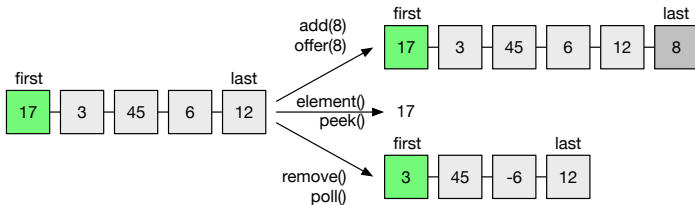
The Minimax Algorithm

# Queues and Deques (and Stacks)

- ▶ A **queue** offers **first-in-first-out access** (FIFO) to its elements
  - Elements are added to the back of the queue
  - Elements are removed from the front of the queue
- ▶ A **deque** (short form for “double ended queue”, pronounced as “deck”) offers access to both of its extremities
  - Elements are added to the front or the back of the deque
  - Elements are removed from the front or the back of the deque
- ▶ A deque also includes typical methods of a **stack**, which offers **last-in-first-out access** (LIFO)
  - Elements are added to the front (push)
  - Elements are removed from the front (pop)

# The Queue Interface

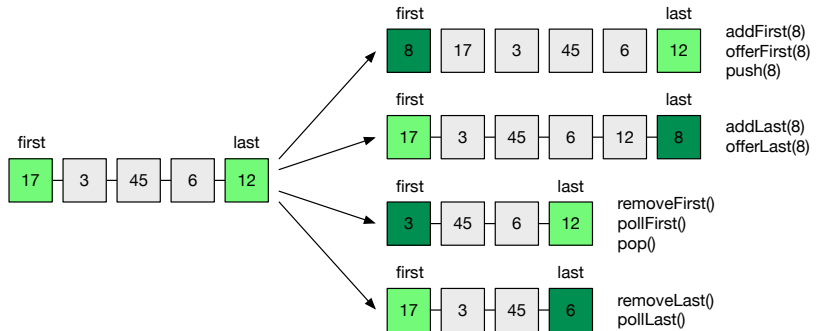
- ▶ `E element()`, `E peek()`: Retrieves, but does not remove, the first element of the queue
- ▶ `boolean add(E e)`, `boolean offer(E e)`: Inserts `e` into the queue if it is possible
- ▶ `E remove()`, `E poll()`: Retrieves and removes the first element of the queue



# The Deque Interface

- ▶ `E getFirst()`, `E peekFirst()`: Retrieves, but does not remove, the first element of the deque
- ▶ `E getLast()`, `E peekLast()`: Retrieves, but does not remove, the last element of the deque
- ▶ `void addFirst(E e)`, `void offerFirst(E e)`, `void push(E e)`: Inserts `e` at the front of the deque
- ▶ `void addLast(E e)`, `void offerLast(E e)`: Inserts `e` at the end of the deque
- ▶ `E removeFirst()`, `E pollFirst()`, `E pop()`: Retrieves and removes the first element of the deque
- ▶ `E removeLast()`, `E pollLast()`: Retrieves and removes the last element of the deque

# The Deque Interface



# ArrayDeque vs. PriorityQueue

- ▶ In a `ArrayDeque`, the order of the elements is determined by the sequence of insertion operations
- ▶ In a `PriorityQueue`, the order of the elements is determined by their natural order or by specifying a comparator
  - `PriorityQueue()`
  - `PriorityQueue(Comparator<? super E> comparator)`
- ▶ Another difference is their internal way of storing the elements
  - `ArrayDeque` uses internally an array (similar to `ArrayList`)
  - `PriorityQueue` uses internally a **heap**
- ▶ All critical operations are efficient: average  $O(1)$  for `ArrayDeque`, worst-case  $O(\log n)$  for `PriorityQueue`

## Example of Using Queues I

```
public class QueueTester {  
  
    public static void main(String[] args) {  
  
        Queue<String> p1 = new ArrayDeque<>(); // Insertion order  
        p1.add("Peter");  
        p1.add("John");  
        p1.add("Tom");  
        p1.add("Andrew");  
        while (!p1.isEmpty()) {  
            System.out.println(p1.remove());  
        } // Loop prints "Peter", "John", "Tom", "Andrew"  
  
        Queue<String> p2 = new PriorityQueue<>(); // Natural order  
        p2.add("Peter");  
        p2.add("John");  
        p2.add("Tom");  
    }  
}
```

## Example of Using Queues II

```
p2.add("Andrew");  
while (!p2.isEmpty()) {  
    System.out.println(p2.remove());  
} // Loop prints "Andrew", "John", "Peter", "Tom"  
  
// String length comparator  
Comparator<String> c = new Comparator<String>() {  
    @Override  
    public int compare(String s1, String s2) {  
        if (s1.length() < s2.length()) {  
            return -1;  
        }  
        if (s1.length() > s2.length()) {  
            return 1;  
        }  
        return 0;  
    }  
};
```



## Example of Using Queues III

```
Queue<String> p3 = new PriorityQueue<>(c);
p3.add("Peter");
p3.add("John");
p3.add("Tom");
p3.add("Andrew");
while (!p3.isEmpty()) {
    System.out.println(p3.remove());
} // Loop prints "Tom", "John", "Peter", "Andrew"
}
```

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# Sets

- ▶ A `set` in Java is a collection that contains no duplicates
- ▶ Duplicates are elements `e1` and `e2` such that `e1.equals(e2)`
- ▶ Compared to a list, there are two important differences:
  - A list may contain duplicates
  - Elements in a list are ordered (positional access)
- ▶ Java sets correspond to sets in mathematics, except that Java sets can be modified
  - `boolean contains(Object x)`: tests if  $x \in X$
  - `boolean containsAll(Collection<?> y)`: tests if  $Y \subseteq X$
  - `boolean addAll(Collection<? extends E> y)`: computes  $X \cup Y$
  - `boolean retainAll(Collection<?> y)`: computes  $X \cap Y$
  - `boolean removeAll(Collection<?> y)`: computes  $X \setminus Y$

## Sorted Sets

- ▶ A **sorted set** in Java is a set that provides a total order on its elements, which is determined by their natural order or by specifying a comparator
- ▶ The interface `SortedSet` provides some additional methods that exploit the existence of an order
  - `E first()`: Returns the minimal element currently in this set
  - `E last()`: Returns the maximal element currently in this set
  - `SortedSet<E> tailSet(E e)`: Returns elements that are greater than or equal to `e`
  - `SortedSet<E> headSet(E e)`: Returns elements that are strictly less than `e`
  - `SortedSet<E> subSet(E e1, E e2)`: Returns the elements that range from `e1` (inclusive) to `e2` (exclusive)

# HashSet vs. TreeSet

- ▶ In a HashSet, the order of the elements is unspecified
- ▶ In a TreeSet, the order is determined by their natural order or by specifying a comparator (similar to PriorityQueue)
  - `TreeSet()`
  - `TreeSet(Comparator<? super E> comparator)`
- ▶ Another difference is their internal way of storing the elements
  - HashSet uses internally a **hash table** (array)
  - TreeSet uses internally a **red-black tree**
- ▶ All critical operations are efficient: average  $O(1)$  for HashSet, worst-case  $O(\log n)$  for TreeSet

## Example of Using Sets I

```
public class SetTester {  
  
    public static void main(String[] args) {  
  
        Set<Integer> s1 = new HashSet<>(Arrays.asList(new Integer  
            []{1,2,3,4}));  
        Set<Integer> s2 = new HashSet<>(Arrays.asList(new Integer  
            []{3,4,5}));  
        Set<Integer> s3 = new HashSet<>(Arrays.asList(new Integer  
            []{2,5,6}));  
  
        s2.addAll(s3);  
        System.out.println(s2); // s2 = s2 cup s3 = {2,3,4,5,6}  
  
        s2.retainAll(s1); // s2 = s2 cap s1 = {2,3,4}  
        System.out.println(s2); // s2 = s2 cap s1 = {2,3,4}
```

## Example of Using Sets II

```
s2.removeAll(s3);  
System.out.println(s2); // s2 = s2 minus s3 = {3,4}  
  
s2.add(3);  
s2.add(5);  
s2.add(5);  
System.out.println(s2); // s2 = s2 cup {3,5,5} = {3,4,5}  
  
SortedSet<Integer> s = new TreeSet<>(Arrays.asList(new  
    Integer[]{1,3,7,2,5,4,7}));  
System.out.println(s.headSet(5)); // {1,2,3,4}  
System.out.println(s.tailSet(5)); // {5,7}  
System.out.println(s.subSet(2,5)); // {2,3,4}  
}  
}
```

# The Equals Method

- ▶ Duplicates in sets and sorted sets are defined differently:
  - Sets: `e1.equals(e2)`
  - Sorted sets:  
`e1.compareTo(e2) == 0` or `c.compareTo(e1,e2) == 0`
- ▶ The method `equals(Object obj)` as defined in `Object` only performs an identity check `e1 == e2`
- ▶ **Important:** If you need a `Set<MyClass>`, you must override `equals(Object obj)` in your class `MyClass`
  - `e.equals(null) == false`
  - `e.equals(e) == true` (Reflexivity)
  - `e1.equals(e2) == e2.equals(e1)` (Symmetry)
  - `e1.equals(e2) && e2.equals(e3) == e1.equals(e3)` (Transitivity)



# Auto-Generated Equals Method I

```
public class MyClass {  
  
    private int x;  
    private String y;  
  
    @Override  
    public boolean equals(Object obj) {  
        if (obj == null) {  
            return false;  
        }  
        if (this == obj) {  
            return true;  
        }  
        if (getClass() != obj.getClass()) {  
            return false;  
        }  
        MyClass other = (MyClass) obj;
```

# Auto-Generated Equals Method II

```
if (this.x != other.x) {  
    return false;  
}  
if (this.y == null) {  
    if (other.y != null) {  
        return false;  
    }  
} else if (!this.y.equals(other.y)) {  
    return false;  
}  
return true;  
}
```

# The hashCode Method

- ▶ A set implemented based on hash tables (e.g. HashSet) requires a **hash function** that is compatible with equals
- ▶ The method `hashCode()` as defined in `Object` only transforms the memory address of the object into an integer
- ▶ **Important:** If you need a `HashSet<MyClass>`, you must override `hashCode()` in your class `MyClass`
  - If `e1.equals(e2)`, then `e1.hashCode() == e2.hashCode()`
  - Otherwise, `e1.hashCode() != e2.hashCode()` with high probability
- ▶ General recommendation: Always override `hashCode()` when you override `equals(Object obj)`

# Auto-Generated hashCode Method

- ▶ Let  $h_1, \dots, h_n$  be the hash codes of the  $n$  fields of an object  $x$ , then a good practice is to define  $h_0 = 1$  and compute

$$\text{hashCode}(x) = \sum_{i=0}^n h_i \cdot 31^{n-i}$$

- ▶ Example: Let  $n = 2$ , then  $\text{hashCode}(x) = 31^2 + 31h_1 + h_2$

```
@Override
public int hashCode() {
    final int prime = 31;
    int result = 1;
    result = prime * result + x;
    result = prime * result + ((y == null) ? 0 : y.hashCode());
    return result;
}
```

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# Maps and Sorted Maps

- ▶ A **map** in Java is an object that maps **keys** to **values** (similar to a function  $f : \{k_1, \dots, k_n\} \rightarrow V$  in mathematics)

$$k_1 \mapsto v_1 = f(k_1)$$

$$\vdots$$

$$k_n \mapsto v_n = f(k_n)$$

- ▶ A map cannot contain duplicate keys (each key maps to at most one value), but different keys can map to the same value
- ▶ The goal of a map is to provide a **key-based access** to the values stored in the map
- ▶ A **sorted map** is a map that provides a total ordering on its keys (similar to SortedSet)

# The Map Interface

- ▶ Since `Map<K,V>` does not inherit from `Collection<E>`, it defines its own methods `isEmpty()`, `size()`, and `clear()`
  - `boolean containsKey(Object key)`: Returns true if the map contains a mapping with the given value
  - `boolean containsValue(Object key)`: Returns true if the map contains a mapping for the given key
  - `V get(Object key)`: Returns the value of the mapping for the given key (or null if the key does not exist)
  - `V put(K key, V value)`: Adds a new mapping or replaces the value of an existing mapping
  - `V remove(Object key)`: Removes the mapping if the given key exists
  - `V replace(K key, V value)`: Replaces the value in the mapping if the given key exists
  - `Collection<V> values()`: Returns a collection of all values
  - `Set<K> keySet()`: Returns a set of all keys

# HashMap vs. TreeMap

- ▶ In a HashMap, the order of the keys is unspecified
- ▶ In a TreeMap, the order is determined by their natural order or by specifying a comparator (similar to TreeSet)
  - `TreeMap()`
  - `TreeMap(Comparator<? super E> comparator)`
- ▶ Another difference is their internal way of storing the elements
  - HashMap uses internally a **hash table** (array)
  - TreeMap uses internally a **red-black tree**
- ▶ All critical operations are efficient: average  $O(1)$  for HashMap, worst-case  $O(\log n)$  for TreeMap



## Example of Using Maps I

```
public class MapTester {  
  
    public static void main(String[] args) {  
  
        Map<Integer,String> map = new HashMap<>();  
        map.put(1, "Hello");  
        map.put(5, "World");  
        map.put(7, "Hello");  
        map.put(10, "Hello World");  
        map.put(10, "Hello World!"); // replaces previous value  
  
        System.out.println(map.keySet()); // prints [1, 5, 7, 10]  
        System.out.println(map.values()); // prints [Hello, World,  
            Hello, Hello World!]  
  
        String str1 = map.get(1);  
        if (str1 != null) {
```

## Example of Using Maps II

```
    System.out.println(str1); // prints "Hello"
}
String str2 = map.get(2);
if (str2 != null) {
    System.out.println(str2); // nothing is printed
}

// the following code is less efficient
int key = 1;
if (map.containsKey(key)) {
    System.out.println(map.get(key)); // prints "Hello"
}
}
```

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# Iterators

- ▶ An **iterator** in Java is an object that helps iterating through all elements in a collection
- ▶ The interface `Iterator<E>` defines the following methods:
  - `boolean hasNext()`: Returns true if the iteration has more elements
  - `E next()`: Returns the next element in the iteration
- ▶ Note that an iterator only allows a single iteration
- ▶ The following is a typical example of usage:

```
Iterator<String> iterator = ...;  
while(iterator.hasNext()) {  
    System.out.println(iterator.next());  
}
```

# Array Iterator I

```
public class ArrayIterator<E> implements Iterator<E> {  
  
    private E[] array;  
    private int currentIndex;  
  
    public ArrayIterator(E[] array) {  
        this.array = array;  
        this.currentIndex = 0;  
    }  
  
    @Override  
    public boolean hasNext() {  
        return this.currentIndex < this.array.length;  
    }  
  
    @Override  
    public E next() {
```

# Array Iterator II

```
if (!this.hasNext()){  
    throw new NoSuchElementException();  
}  
return this.array[this.currentIndex++];  
}  
  
}
```

# Reverse Array Iterator I

```
public class ReverseArrayIterator<E> implements Iterator<E> {  
  
    private E[] array;  
    private int currentIndex;  
  
    public ReverseArrayIterator(E[] array) {  
        this.array = array;  
        this.currentIndex = array.length - 1;  
    }  
  
    @Override  
    public boolean hasNext() {  
        return this.currentIndex >= 0;  
    }  
  
    @Override  
    public E next() {
```

# Reverse Array Iterator II

```
if (!this.hasNext()) {  
    throw new NoSuchElementException();  
}  
return this.array[this.currentIndex--];  
}  
  
}
```



# Iterator Tester I

```
public class IteratorTester {  
  
    public static void main(String[] args) {  
  
        Integer[] values = new Integer[] { 1, 3, 3, 5, 6, 9 };  
  
        Iterator<Integer> i1 = new ArrayIterator<>(values);  
        while (i1.hasNext()) {  
            System.out.print(i1.next() + " ");  
        }  
        System.out.println();  
        // prints 1 3 3 5 6 9  
  
        Iterator<Integer> i2 = new ReverseArrayIterator<>(values);  
        while (i2.hasNext()) {  
            System.out.print(i2.next() + " ");  
        }  
    }  
}
```

# Iterator Tester II

```
System.out.println();  
// prints 9 6 5 3 3 1  
}  
  
}
```

# List Iterators

- ▶ The `ListIterator<E>` interface extends `Iterator<E>` with methods for traversing a list backwards
  - `boolean hasPrevious()`: Returns true if a backward iteration has more elements
  - `E previous()`: Returns the previous element in a backward iteration
- ▶ It also provides methods for adding, removing, or replacing element in the list
  - `void add(E e)`: Inserts an element into the list
  - `void remove()`: Removes the current element from the list
  - `void set(E e)`: Replaces the current element in the list

## Obtaining Iterators from Collections

- ▶ An iterator can be obtained from a collection using a method inherited from `Iterable<E>`
  - `Iterator<E> iterator()`: Returns an iterator over all elements in the collection (the order is unspecified)
- ▶ Similarly, a list iterator can be obtained from a list
  - `ListIterator<E> listIterator()`: Returns a list iterator over all elements in the list
- ▶ Note that the Java for-each loop work for any class that implements the `Iterable<E>` interface

```
Iterable<Integer> values =Arrays.asList(new Integer
    []{1,3,3,5,7,9});
int sum = 0;
for (Integer value: values) {
    sum = sum + value;
}
```

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# The Minimax-Algorithm

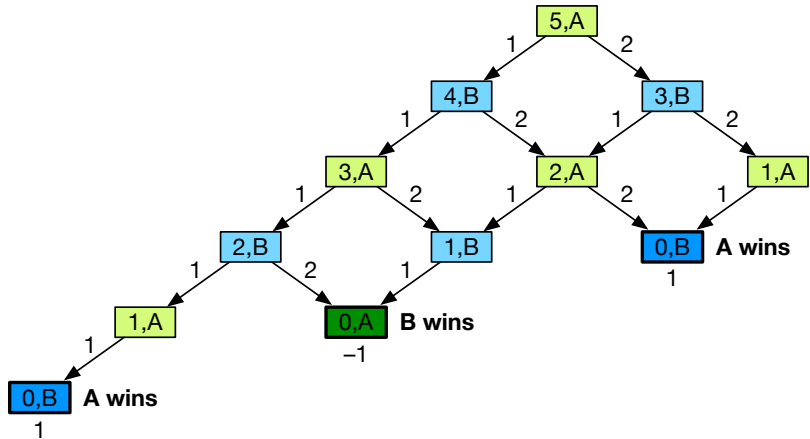
- ▶ The **minimax algorithm** is an optimal game playing algorithm for two player games such as tic-tac-toe, chess, go, uril, ...
- ▶ Let  $A$  and  $B$  be the two players and  $S$  the finite set of possible **game states**, where ...
  - $s_0 \in S$  is the **initial state**
  - $S^* \subseteq S$  are **final states**, in which the game ends
- ▶ For all final states  $s^* \in S^*$ , the winner of the game is defined by a function  $E : S^* \rightarrow [-1, 1]$ , where

$$E(s^*) = \begin{cases} 1, & \text{if } A \text{ wins} \\ 0, & \text{if the game ends as draw} \\ -1, & \text{if } B \text{ wins} \end{cases}$$

## Example: Subtraction Game

- ▶ In a  $(k, n)$ -subtraction game,  $A$  and  $B$  take turns in removing up to  $k$  objects from a pile with initially  $n$  objects
  - $A$  begins
  - Whoever removes the last object from the pile wins
- ▶ Example:  $(2, 5)$ -subtraction game
  - States:  $S = \{(5, A), (4, A), \dots, (0, A), (4, B), \dots, (0, B)\}$
  - Initial state:  $s_0 = (5, A)$
  - Final states:  $S^* = \{(0, A), (0, B)\}$
  - $$E(s^*) = \begin{cases} 1, & \text{for } s^* = (0, B) \\ -1, & \text{for } s^* = (0, A) \end{cases}$$

# Example: Subtraction Game





# Minimax Algorithm: General Idea

- ▶ Let  $next(s) \subseteq S$  be the reachable states from  $s \in S$
- ▶ Example: (2,5)-subtraction game
  - $next((5, A)) = \{(4, B), (3, B)\}$
  - $next((4, B)) = \{(3, A), (2, A)\}$
  - $\vdots$
  - $next((0, A)) = next(0, B) = \{\}$
- ▶ The minimax algorithm extends  $E$  from  $E : S^* \rightarrow [-1, 1]$  to  $F : S \rightarrow [-1, 1]$  by computing  $F(s)$  recursively by for all  $s \in S$

$$F(s) = \begin{cases} E(s), & \text{if } s \in S^* \\ \max\{F(s') : s' \in next(s)\}, & \text{if it is } A\text{'s turn} \\ \min\{F(s') : s' \in next(s)\}, & \text{if it is } B\text{'s turn} \end{cases}$$

# Minimax Algorithm: Pseudocode

**Algorithm:** Minimax( $s, d$ )

**if**  $s \in S^*$  **then** // game ends  
     $\text{return } E(s)$

**if**  $d \bmod 2 = 0$  **then** // A's turn

$m \leftarrow -1$

**for**  $s' \in \text{next}(s)$  **do**

$m \leftarrow \max(m, \text{Minimax}(s', d + 1))$

**else** // B's turn

$m \leftarrow 1$

**for**  $s' \in \text{next}(s)$  **do**

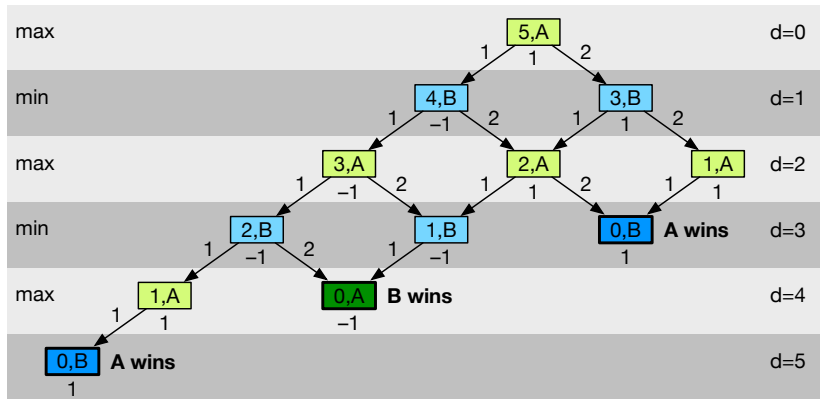
$m \leftarrow \min(m, \text{Minimax}(s', d + 1))$

**return**  $m$

## Example: Subtraction Game

Initial call:  $\text{Minimax}((5, A), 0)$

Return value:  $F((5, A)) = 1$ , i.e. A wins by removing 2 objects



# Minimax Algorithm: Performance

- ▶ The **branching factor**  $b$  of a game is the average number legal moves (children in the game tree)
- ▶ Examples:
  - $(k, n)$ -subtraction game:  $b \leq k, h \leq n$
  - Chess:  $b \approx 35$
  - Go:  $b \approx 250$
- ▶ The minimax algorithm runs in  $O(b^h)$  time, where  $h$  denotes the height (maximal depth) of the game tree
- ▶ In other words, exploring the full game tree is impossible for most non-trivial games

# Pruned Minimax Algorithm

- ▶ To apply the minimax algorithm to non-trivial games, the game tree exploration must be pruned
- ▶ The simplest pruning method is to stop the recursion when a **maximal depth**  $d_{\max}$  is reached
- ▶ When the recursion stops at state  $s \in S$ , then a **evaluation function**  $\tilde{E} : S \rightarrow [-1, 1]$  is applied to  $s$ 
  - For  $s \in S^*$ , let  $\tilde{E}(s) = E(s)$
  - Otherwise, let  $\tilde{E}(s)$  be an estimate of the **advantage** of state  $s$  relative to  $A$  and  $B$ , such that  $\tilde{E}(s) = 1$  means maximal advantage for  $A$  and  $\tilde{E}(s) = -1$  maximal advantage for  $B$
- ▶ The quality of the estimate  $\tilde{E}(s)$  and  $d_{\max}$  determine the quality and accuracy of the final minimax return value

# Pruned Minimax Algorithm: Pseudocode

**Algorithm:** Minimax( $s, d, d_{\max}$ )

**if**  $s \in S^*$  **or**  $d = d_{\max}$  **then** // game ends

**return**  $\tilde{E}(s)$

**if**  $d \bmod 2 = 0$  **then** // A's turn

$m \leftarrow -1$

**for**  $s' \in \text{next}(s)$  **do**

$m \leftarrow \max(m, \text{Minimax}(s', d + 1, d_{\max}))$

**else** // B's turn

$m \leftarrow 1$

**for**  $s' \in \text{next}(s)$  **do**

$m \leftarrow \min(m, \text{Minimax}(s', d + 1, d_{\max}))$

**return**  $m$

## Defining the Evaluation Function

- ▶ One popular strategy for constructing an evaluation function  $\tilde{E} : S \rightarrow [-1, 1]$  is as a **weighted sum**

$$\tilde{E}(s) = \frac{1}{W} \sum_{i=1}^k w_i \cdot \tilde{E}_i(s)$$

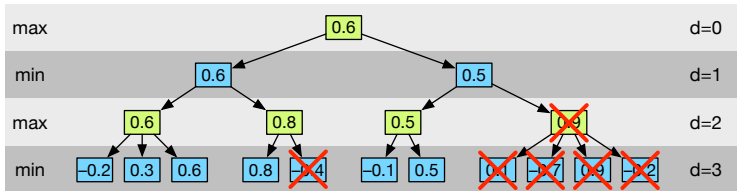
of  $k$  individual evaluation criteria

- ▶ The value  $w_i \in [0, 1]$  denotes the **weight** of criterion  $i$  and  $W = \sum_{i=1}^k w_i$  denotes the total weight of all criteria
- ▶  $\tilde{E}_i : S \rightarrow [-1, 1]$  defines the evaluation function of criterion  $i$
- ▶ Example from chess:

$$\tilde{E}(s) = 9 \cdot (Q - Q') + 5 \cdot \frac{R - R'}{2} + 3 \cdot \frac{B - B'}{2} + 3 \cdot \frac{N - N'}{2} + \frac{P - P'}{8} + \dots$$

# Alpha-Beta Pruning

- ▶ There are many ways of optimizing the minimax algorithm
- ▶ The general idea is to prune branches of the game tree that will not influence the final minimax return value
- ▶ The simplest optimization is known as **alpha-beta pruning**



- ▶ The following version of the minimax algorithm is initially called with  $\text{Minimax}(s_0, 0, d_{\max}, -1, 1)$



# Alpha-Beta Pruning: Pseudocode I

```

Algorithm: Minimax( $s, d, d_{\max}, \alpha, \beta$ )
if  $s \in S^*$  or  $d = d_{\max}$  then                                // game ends
  | return  $\tilde{E}(s)$ 
if  $d \bmod 2 = 0$  then                                           // A's turn
  |  $m \leftarrow -1$ 
  | for  $s' \in \text{next}(s)$  do
  | |  $m \leftarrow \max(m, \text{Minimax}(s', d + 1, d_{\max}, \alpha, \beta))$ 
  | |  $\alpha \leftarrow \max(\alpha, m)$ 
  | | if  $\alpha \geq \beta$  then
  | | | return  $m$                                               //  $\beta$  cutoff
  |

```

# Alpha-Beta Pruning: Pseudocode II

```
else // B's turn
   $m \leftarrow 1$ 
  for  $s' \in next(s)$  do
     $m \leftarrow \min(m, \text{Minimax}(s', d + 1, , d_{\max}, \alpha, \beta))$ 
     $\beta \leftarrow \min(\beta, m)$ 
    if  $\alpha \geq \beta$  then
      return  $m$  //  $\alpha$  cutoff
  return  $m$ 
```