ANALYSIS OF QUEUEING SYSTEMS USING GAME THEORY

A Project Report Submitted

for the Course

MA498 Project I

by

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to the

DEPARTMENT OF MATHEMATICS

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GUWAHATI - 781039, INDIA

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CERTIFICATE

This is to certify that the work contained in this project report entitled "Analysis of Queueing Systems using Game Theory" submitted by Karan Gupta (Roll No. 180123064 and Ashish Kumar Barnawal (Roll No.: 180123006) to the Department of Mathematics, Indian Institute of Technology Guwahati towards partial requirement of Bachelor of Technology in Mathematics and Computing has been carried out by him/her under my supervision.

It is also certified that this report is a survey work based on the references in the bibliography.

OR

It is also certified that, along with literature survey, a few new results are established/computational implementations have been carried out/simulation studies have been carried out/empirical analysis has been done by the student under the project.

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Guwahati - 781 039 (Prof. N. Selvaraju)

November 2021 Project Supervisor

ABSTRACT

The main aim of the project ....

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In this paper, our main goal is to analyze the application of game theory to networks of queues in order to optimize a payoff function associated with the queueing game. We explore non-cooperative queueing games in Chapter 2 by presenting a model and stating theorems that prove/disprove the existence of Nash Equilibrium for certain sub-classes of queueing games. In Chapter 3, we have introduced an algorithmic approach to check the existence of pure strategy (mixed also?) Nash Equilibria for non-cooperative N-player games on a generalized network of queues, with a predefined strategy space for each player. We have also explored and analyzed the Best-response algorithm which shows that, for a game with a continuous strategy space, a pure-strategy Nash Equilibrium always exists. In Chapter 4, we conclude the paper with a brief overview of our findings; and provide an array of avenues which would look to explore and work on, in the future.

1.1 Queueing Theory

Some text here ...

Definition 1.1.1. *$M/M/1$ Queue* An M/M/1 queue is a single-server queue, and according to the Kendall's notation, has arrival rate(λ) following the Markovian(M) distribution, which means the inter-arrival times of customers entering the queue are exponential. The service rate(μ) of the queue is also Markovian(M) and hence is an exponential service time. The maximum number of customers in the queue at the same time is unbounded or infinite.

Theorem 1.1.2. *The expected waiting time for an M/M/1 queue with arrival rate λ and service rate μ is equal to $\frac{1}{\mu-\lambda}$.*

Proof. Let n be the number of customers at a given time, in the queue. We can make the flow-balance equations for n $\geq$ 1 and for the state with no customers as follows:

$$(\lambda + \mu)p_n = \mu p_{n+1} + \lambda p_{n-1}$$
$$\lambda p_0 = \mu p_1$$

where p_i is the long-term fraction of time with i customers in the system. The following figure shows a state diagram for the number of customers in the system at a time along with the rates of transition to the next or previous state. ¡insert figure of rate transition diagram here¿

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

Using Little's Law ($L = \lambda W$), we get,

$$W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda}$$

□

Corollary 1.1.3. *A corollary to the theorem is....*

Remark 1.1.4. Some remark.......

You may have to type many equations inside the text. The equation can be typed as below.

$$f(x) = \frac{x^2 - 5x + 2}{e^x - 2} = \frac{y^5 - 3}{e^x - 2} \tag{1.1}$$

This can be referred as (**??**) and so on.....

You may have to type a set of equations. For this you may proceed as given below.

$$\begin{aligned} f(x) &= e^{1+2(x-a)} + \dots \\ &= \log(x+a) + \sin(x+y) + \cdots \end{aligned} \tag{1.2}$$

You may have to cite the articles. You may do so as [**?**] and so on..... Note that you have already created the 'bib.bib' file and included the entry with the above name. Only then you can cite it as above.

1.2 Game Theory

Definition 1.2.1. Some definition....

Remark 1.2.2. Some remark.......

1.2.1 Subsection name

Theorem 1.2.3. *Some theorem.......*

Proof. Proof is as follows.... By Definition **??** □

[The figure will be displayed here.]

Figure 1.1: The correlation coefficient as a function of ρ

Chapter 2

Introduction

In this paper, our main goal is to analyze the application of game theory to networks of queues in order to optimize a payoff function associated with the queueing game. We explore non-cooperative queueing games in Chapter 2 by presenting a model and stating theorems that prove/disprove the existence of Nash Equilibrium for certain sub-classes of queueing games. In Chapter 3, we have introduced an algorithmic approach to check the existence of pure strategy (mixed also?) Nash Equilibria for non-cooperative N-player games on a generalized network of queues, with a predefined strategy space for each player. We have also explored and analyzed the Best-response algorithm which shows that, for a game with a continuous strategy space, a pure-strategy Nash Equilibrium always exists. In Chapter 4, we conclude the paper with a brief overview of our findings; and provide an array of avenues which would look to explore and work on, in the future.

2.1 Queueing Theory

Some text here ...

Definition 2.1.1. $M/M/1$ Queue An M/M/1 queue is a single-server queue, and according to the Kendall's notation, has arrival rate(λ) following the Markovian(M) distribution, which means the inter-arrival times of customers entering the queue are exponential. The service rate(μ) of the queue is also Markovian(M) and hence is an exponential service time. The maximum number of customers in the queue at the same time is unbounded or infinite.

Theorem 2.1.2. *The expected waiting time for an M/M/1 queue with arrival rate λ and service rate μ is equal to $\frac{1}{\mu-\lambda}$.*

Proof. Let n be the number of customers at a given time, in the queue. We can make the flow-balance equations for n $\geq$ 1 and for the state with no customers as follows:

$$(\lambda + \mu)p_n = \mu p_{n+1} + \lambda p_{n-1}$$
$$\lambda p_0 = \mu p_1$$

where p_i is the long-term fraction of time with i customers in the system. The following figure shows a state diagram for the number of customers in the system at a time along with the rates of transition to the next or previous state. ¡insert figure of rate transition diagram here¿

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

Using Little's Law ($L = \lambda W$), we get,

$$W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda}$$

□

Corollary 2.1.3. *A corollary to the theorem is....*

Remark 2.1.4. Some remark.......

You may have to type many equations inside the text. The equation can be typed as below.

$$f(x) = \frac{x^2 - 5x + 2}{e^x - 2} = \frac{y^5 - 3}{e^x - 2} \tag{2.1}$$

This can be referred as (**??**) and so on.....

You may have to type a set of equations. For this you may proceed as given below.

$$\begin{aligned} f(x) &= e^{1+2(x-a)} + \ldots \\ &= \log(x+a) + \sin(x+y) + \cdots \end{aligned} \tag{2.2}$$

You may have to cite the articles. You may do so as [**?**] and so on..... Note that you have already created the 'bib.bib' file and included the entry with the above name. Only then you can cite it as above.

2.2 Game Theory

Definition 2.2.1. Some definition....

Remark 2.2.2. Some remark.......

2.2.1 Subsection name

Theorem 2.2.3. *Some theorem.......*

Proof. Proof is as follows.... By Definition **??** $\square$

[The figure will be displayed here.]

Figure 2.1: The correlation coefficient as a function of ρ

Chapter 3

Algorithms for Nash Equilibria of Queueing games

Introductory lines...

3.1 Section-1 Name

Definition 3.1.1. Some definition....

Remark 3.1.2. Some remark.......

Theorem 3.1.3. *Some theorem.......*

Proof. Proof is as follows.... □

3.2 Section-2 Name

Definition 3.2.1. Some definition....

Remark 3.2.2. Some remark.......

3.2.1 Subsection name

Theorem 3.2.3. *Some theorem.......*

Proof. Proof is as follows.... □

Chapter 4

Future Work

Introductory lines...

4.1 Section-1 Name

Definition 4.1.1. Some definition....

Remark 4.1.2. Some remark.......

Theorem 4.1.3. *Some theorem.......*

Proof. Proof is as follows.... □

4.2 Section-2 Name

Definition 4.2.1. Some definition....

Remark 4.2.2. Some remark.......

4.2.1 Subsection name

Theorem 4.2.3. *Some theorem.......*

Proof. Proof is as follows.... □