STATS 3001 / STATS 4101 / STATS 7054
Statistical Modelling III
Tutorial 5
2022

QUESTIONS:

1. Suppose $y_1, y_2, \dots, y_n$ are independent Poisson observations, $y_i \sim Po(\mu_i)$ and consider the **linear** regression model

$$\boldsymbol{\mu} = X\boldsymbol{\beta}$$

where X is an $n \times p$ matrix with linearly independent columns and $\boldsymbol{\beta} \in \mathbb{R}^p$ is the vector of unknown parameters.

   (a) Write down the log-likelihood

$$\ell(\boldsymbol{\mu}; \boldsymbol{y})$$

   (b) Calculate $\dfrac{\partial \ell}{\partial \mu_i}$.

   (c) Find the matrix of partial derivatives

$$\left[\frac{\partial \mu_i}{\partial \beta_j}\right].$$

   (d) Show that the score vector can be expressed as

$$\mathcal{S}(\boldsymbol{\beta}) = \left[\frac{\partial \ell}{\partial \beta_j}\right] = X^T D_\mu^{-1}(\boldsymbol{y} - \boldsymbol{\mu})$$

   where $D_\mu = diag(\mu_1, \mu_2, \dots, \mu_n)$.

   (e) Evaluate the Fisher information matrix.

   (f) Describe the Fisher scoring algorithm.

      i. State the iterative step of the algorithm to obtain $\hat{\boldsymbol{\beta}}^{(t+1)}$ from $\hat{\boldsymbol{\beta}}^{(t)}$.

      ii. Suggest a starting value, $\hat{\boldsymbol{\beta}}^{(0)}$.