STATS 3001 / STATS 4101 / STATS 7054
Statistical Modelling III
Tutorial 4
2022

QUESTIONS:

1. Show for $\pi_1,\ \pi_2$ both small that

$$\log\left(\frac{\pi_1/(1-\pi_1)}{\pi_2/(1-\pi_2)}\right) \approx \log\left(\frac{\pi_1}{\pi_2}\right).$$

   Does this make the interpretation of the log-odds ratio easier?

2. If

$$\pi = \frac{e^\eta}{(1+e^\eta)}$$

   then show that

$$\frac{d\pi}{d\eta} = \pi(1-\pi).$$

3. Consider the log-likelihood function

$$\ell(\boldsymbol{\beta};\boldsymbol{y}) = \sum_{i=1}^m y_i \log \pi_i + \sum_{i=1}^m (n_i - y_i)\log(1-\pi_i) + \log \prod_{i=1}^m \binom{n_i}{y_i}$$

   where

$$\pi_i = \frac{\exp(\boldsymbol{x}_i^T\boldsymbol{\beta})}{1+\exp(\boldsymbol{x}_i^T\boldsymbol{\beta})}.$$

   Evaluate

$$\frac{\partial^2 \ell}{\partial\beta_j\partial\beta_k}$$

   and hence derive the Fisher information matrix directly.