

STATS 3001 / STATS 4101 / STATS 7054
Statistical Modelling III
Tutorial 6
2022

QUESTIONS:

1. 0.1 Singular value decomposition

The SVD of the $N \times p$ matrix X has the form

$$X = UDV^T,$$

where the columns of U and V are orthogonal, *i.e.*

$$U^T U = I \text{ and } V^T V = I,$$

and D is a diagonal matrix with diagonal entries $d_1 \geq d_2 \geq \dots \geq d_p \geq 0$.

(a) Show that for linear regression

$$X\hat{\beta} = X(X^T X)^{-1} X^T \mathbf{y} = U U^T \mathbf{y}$$

(b) Show that for ridge regression:

$$X\hat{\beta}_\lambda = U D (D^2 + \lambda I)^{-1} D U^T \mathbf{y}$$

(c) Hence, show that

$$X\hat{\beta}_\lambda = \sum_{j=1}^p \mathbf{u}_j \frac{d_j^2}{d_j^2 + \lambda} \mathbf{u}_j^T \mathbf{y},$$

where \mathbf{u}_j are the columns of U .