QUESTIONS:

1. The purpose of this tutorial is to derive the maximum likelihood estimates for the multiple regression model. It is intended partly to help with understanding the derivation of the Box-Cox profile likelihood.

   Consider the multiple regression model

   $$\boldsymbol{Y} = X\boldsymbol{\beta} + \boldsymbol{\mathcal{E}}$$

   where $E_1, E_2, \ldots, E_n$ are independent with

   $$E_i \sim N(0, \sigma^2)$$

   (a) Show that the model can be expressed equivalently as

   $$Y_i \sim N(\boldsymbol{x}_i^T \boldsymbol{\beta}, \sigma^2)$$

   independently for $i = 1, 2, \ldots, n$ where $\boldsymbol{x}_i^T$ is the ith row of X.

   (b) Write down the log-likelihood function, $\ell(\boldsymbol{\beta}, \sigma^2; \boldsymbol{y})$.

   (c) Show that for any value of $\sigma^2 > 0$ the log-likelihood is maximized with respect to $\boldsymbol{\beta}$ by the ordinary least squares estimate $\hat{\boldsymbol{\beta}}$.

   (d) Hence show that the maximum likelihood estimate for σ^2 is

   $$\hat{\sigma}^2 = \frac{RSS(\boldsymbol{y})}{n}$$

   where the residual sum of squares (RSS) is defined by

   $$RSS(\boldsymbol{y}) = \sum_{i=1}^{n}(y_i - \boldsymbol{x}_i^T \hat{\boldsymbol{\beta}})^2.$$

   (e) If c is a scalar then show that

   $$RSS(c\boldsymbol{y}) = c^2 RSS(\boldsymbol{y}).$$