



# Energy Systems Modelling

BTU Cottbus

Chair of Energy Economics

Prof. Dr. Felix Müsgens

# General Information

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- Lecture and tutorial slides will be available via **Moodle** portal
- Note: please keep in mind that registration in **Moodle** is **NOT** a registration for the examination

# Structure of ESM

- ♦ Lectures and exercises:
  - Classes on Wednesday 09:15 – 10:45 and Thursday 15:30 – 17:00
  - Next lecture November 6<sup>th</sup>, first tutorial November 7<sup>th</sup>
- ♦ Homework and assignments [30% of the final grade]
- ♦ Project work and consultations [35% of the final grade]
  - Single person projects
  - Paper (5 pages)
  - Project presentations
- ♦ Exam [35% of the final grade]

# Literature Recommendations

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## Related books:

- McCarl, B.A. and Spreen, T.H. (2011): “Applied Mathematical Programming Using Algebraic System”. Department of Agricultural Economics, Texas A&M University.

<http://agecon2.tamu.edu/people/faculty/mccarl-bruce/mccspr/thebook.pdf>

- ◆ Further related articles, home tasks, GAMS guides and all other materials will be uploaded to Moodle portal: Energy Systems Modelling | WiSe 19/20

# Homework

Homework materials are uploaded to Moodle

- A. Read the GAMS tutorial (~20 pages)
- B. Complete tasks from the “Toy electricity model” file.
- C. Send a 1-page report (tasks 1-3) and a gams code (task 4) until 4 November to  
[iegor.riepin@b-tu.de](mailto:iegor.riepin@b-tu.de)

(You will not require a GAMS license for this assignment.)

## Assignments & Homeworks



Homework 1: GAMS Tutorial



Homework 1: Toy electricity market model

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## Materials



GAMS guides [advanced]



GAMS online documentation

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## Communication



Modeller's dilemma



YouTube channel of our chair

# Toy Model: a True Story

The Springfield's energy program is one of the world's most ambitious initiatives to transform and decarbonize the entire energy system of a region. The program includes deregulation of the electricity sector. From now on, independent suppliers have to participate on the newly established electricity market to win the right to sell energy.

The Springfield's Ministry for Energy (SMfE) urgently needs tools that allow for modelling and efficient planning of electricity market operation. Homer Jay Simpson, a research scientist at the SMfE, is asked to develop a cost optimization model of electricity market.

Your task is to help Homer!



Homer reads a GAMS guide

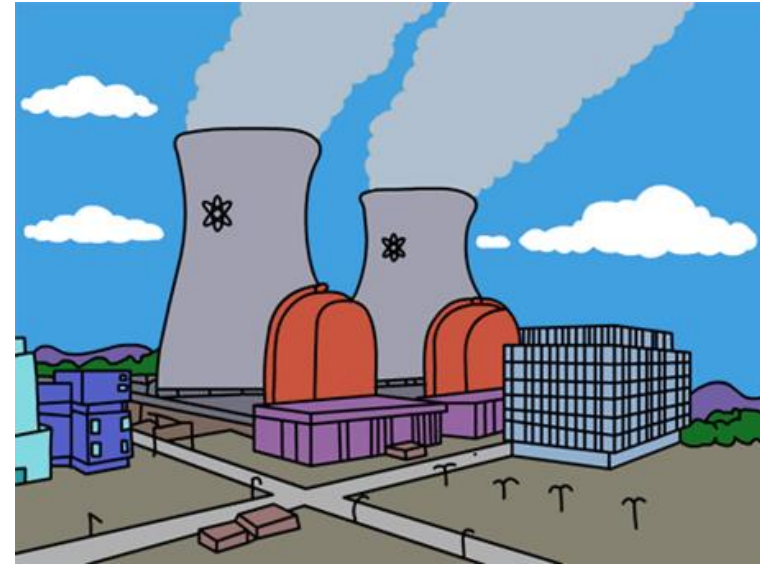
# Toy model: Market Data and Tasks

Springfield has two power plants with the following characteristics:

Variable costs:  $C_1 = 4$  and  $C_2 = 7$  [Cent/kWh].

Available capacities:  $X_1 = 800$  and  $X_2 = 500$  [MW].

The electricity demand:  $D = 1000$  [MW].



The Springfield's 800 MW power plant.

Tasks:

1. Draw the merit-order chart representing Springfield's electricity market.
2. Write the complete mathematical formulation of the cost minimization problem.
3. Specify the optimal solution to this problem.
4. Solve the problem in GAMS using linear programming.

# Energy Systems Modelling: Content

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- 1) Introduction
- 2) Basic theory
- 3) Linear problems
- 4) Linear integer problems
- 5) Non-linear problems
- 6) Modelling strategic behavior



# Energy Systems Modelling

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## **1) Introduction**

- 1) General definitions
- 2) Merit-Order
- 2) Basic theory
- 3) Linear problems
- 4) Linear integer problems
- 5) Non-linear problems
- 6) Modelling strategic behavior

# Goals of the Lecture

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## **The goal of the lecture is to understand ...**

- fundamental interactions in energy systems
- LP, MILP, NLP and MCP modelling approaches
- empirical energy models

# Planning and Optimization in Modern Society



Public transport traffic



Power supply



Telecommunication traffic



Engineering problems

Operations research encompasses a wide range of techniques applied in the pursuit of improved decision-making and efficiency.

These techniques involve the construction of mathematical models that attempt to represent real world systems.



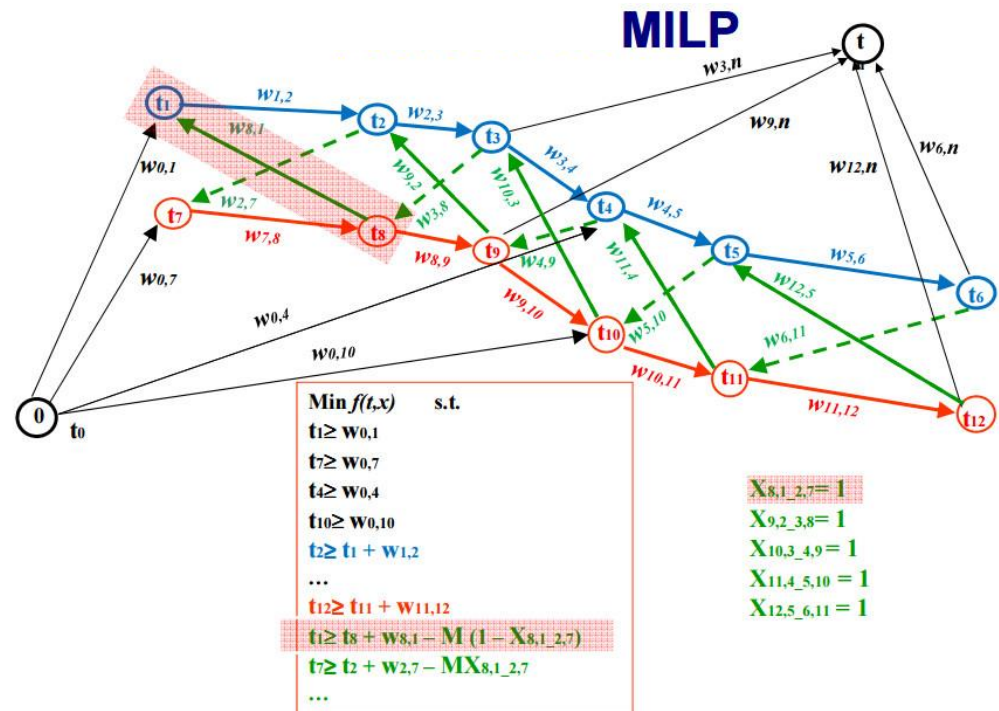
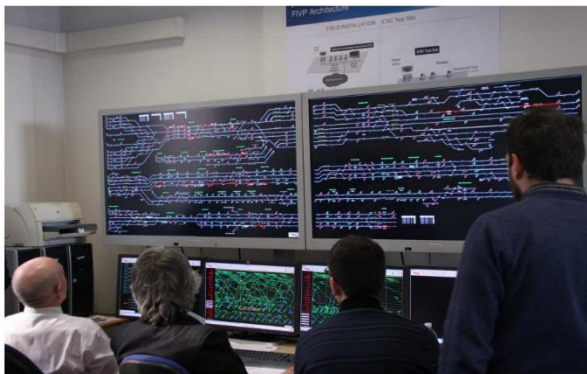
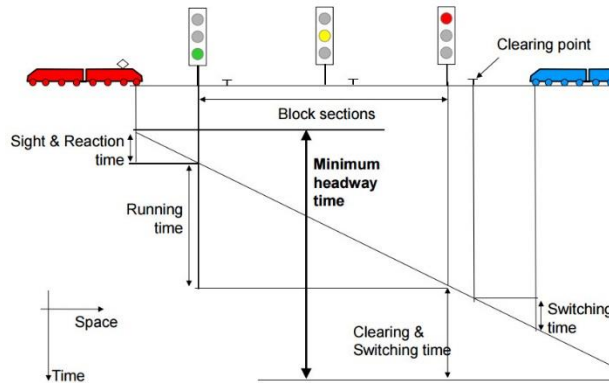
Logistics, e.g. food supply

# Examples of Operations Research Applications I

Selected slides from presentation of Prof. Dario Pacciarelli, head of the Aut.O.R.I. Lab:

## 'Scheduling of real-time railway traffic and management of congestion'

- Aim: "Development of novel railway traffic management systems for a precise and effective train traffic regulation in terms of punctuality increase"



# Examples of Operations Research Applications II

Selected figures from article by Gaetan Kenway and Joaquim R. R. A. Martin "**Aerostructural Shape Optimization of Wind Turbine Blades Considering Site-Specific Winds**", 12th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference September 10–12, 2008

- "The goal is to reduce the end unit cost of electricity, amortized over the turbine lifetime, for a particular location."

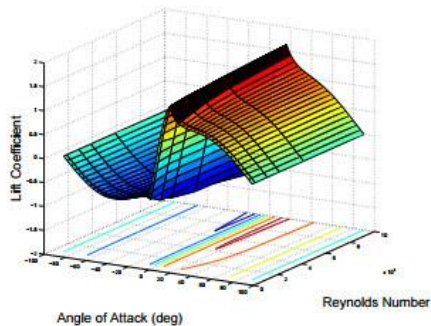


Figure 3. NACA 4415  $C_L$  map

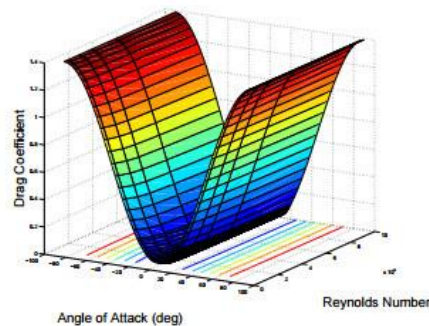


Figure 4. NACA 4415  $C_D$  map

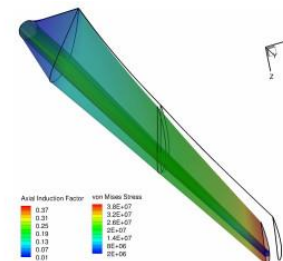


Figure 9. St. Lawrence optimized blade at 7m/s

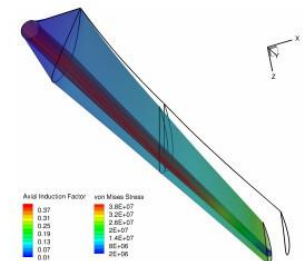


Figure 10. St. Lawrence optimized blade at 12m/s

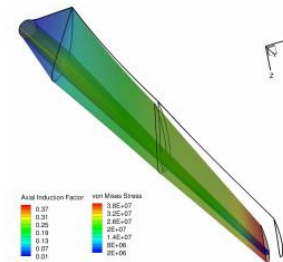


Figure 11. UTIAS optimized blade at 7m/s

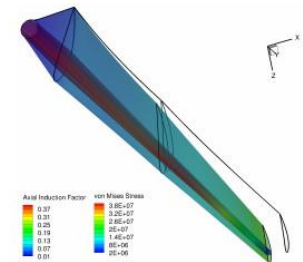


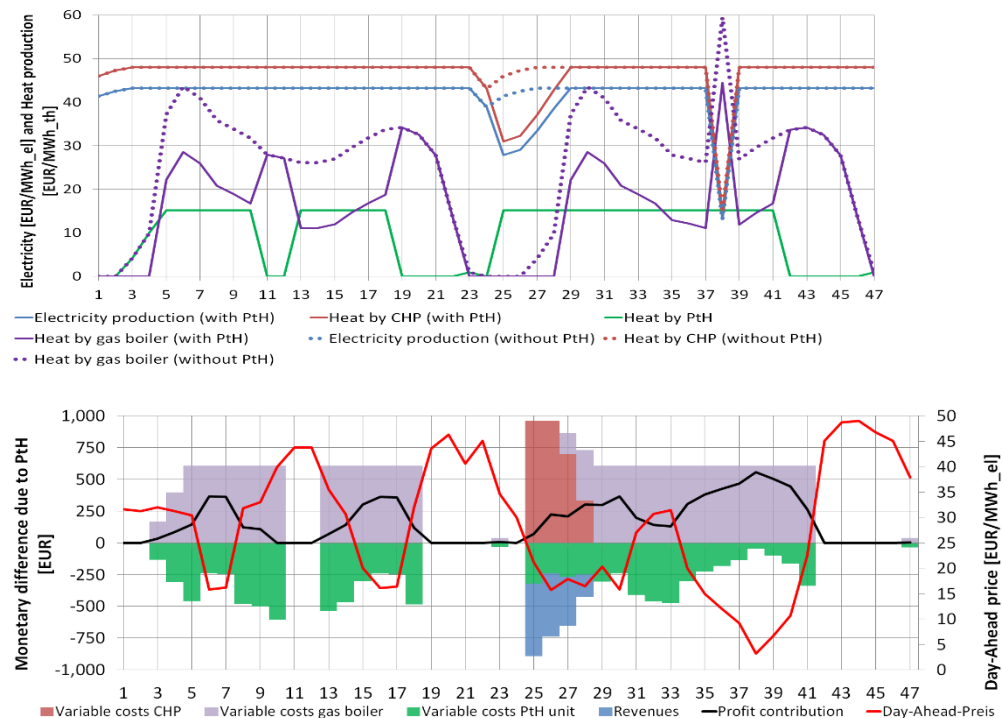
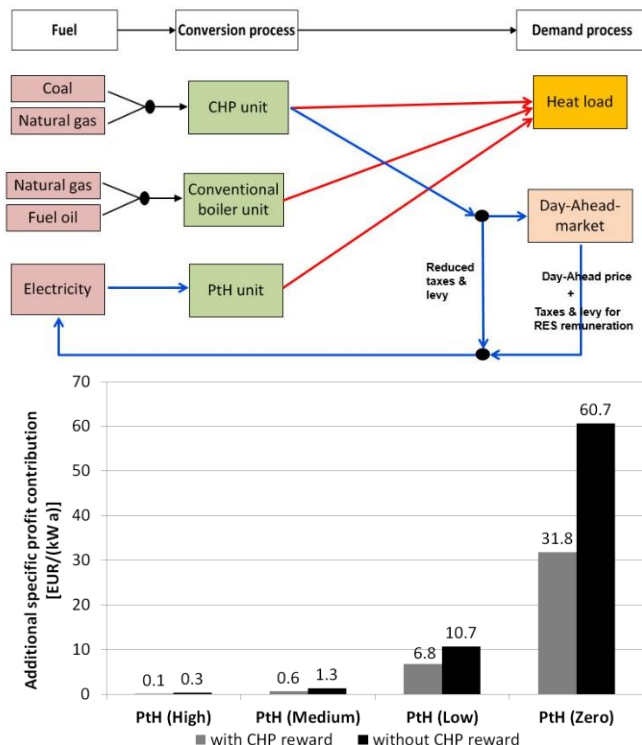
Figure 12. UTIAS optimized blade at 12m/s



# Examples of Operations Research Applications III

Selected slides from presentation of Daniel Scholz, research assistant at a Chair of energy economics, BTU C-S: **'Increasing Flexibility of Combined Heat and Power Plants with Power-to-Heat'**, 12<sup>th</sup> International Conference on the European Energy Market, Lisbon, 2015

- Aim: "...evaluate the flexibility gain by power-to-heat technology for an inflexible combined heat and power plant"



# General Purpose of Modelling

## Mathematical description solution of a complex 'real world' problem

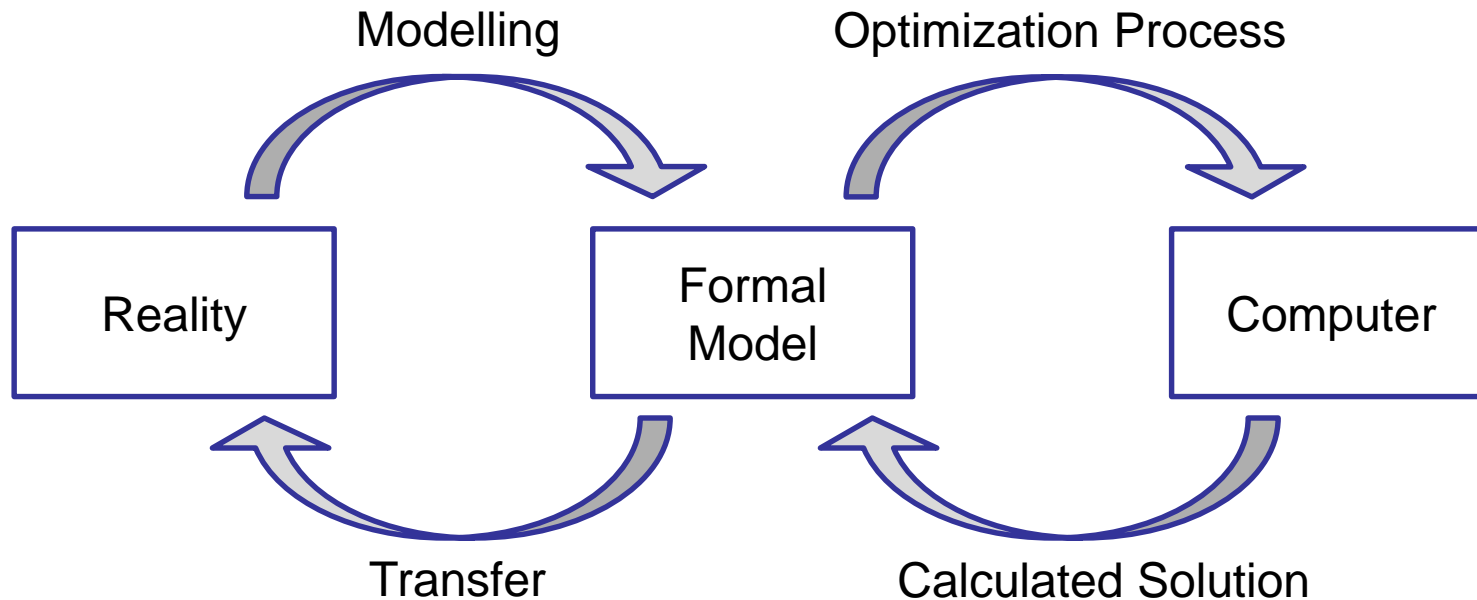


Figure: based on Suhl and Mellouli (2009)

# General Purpose of Modelling

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## **Steps to support real world planning**

- 1) Recognition and analysis of a problem
- 2) Definition of objectives and possible actions
- 3) Development of a mathematical model
- 4) Data research
- 5) Generate a solution by using an algorithm
- 6) Analysis and evaluation of the solution



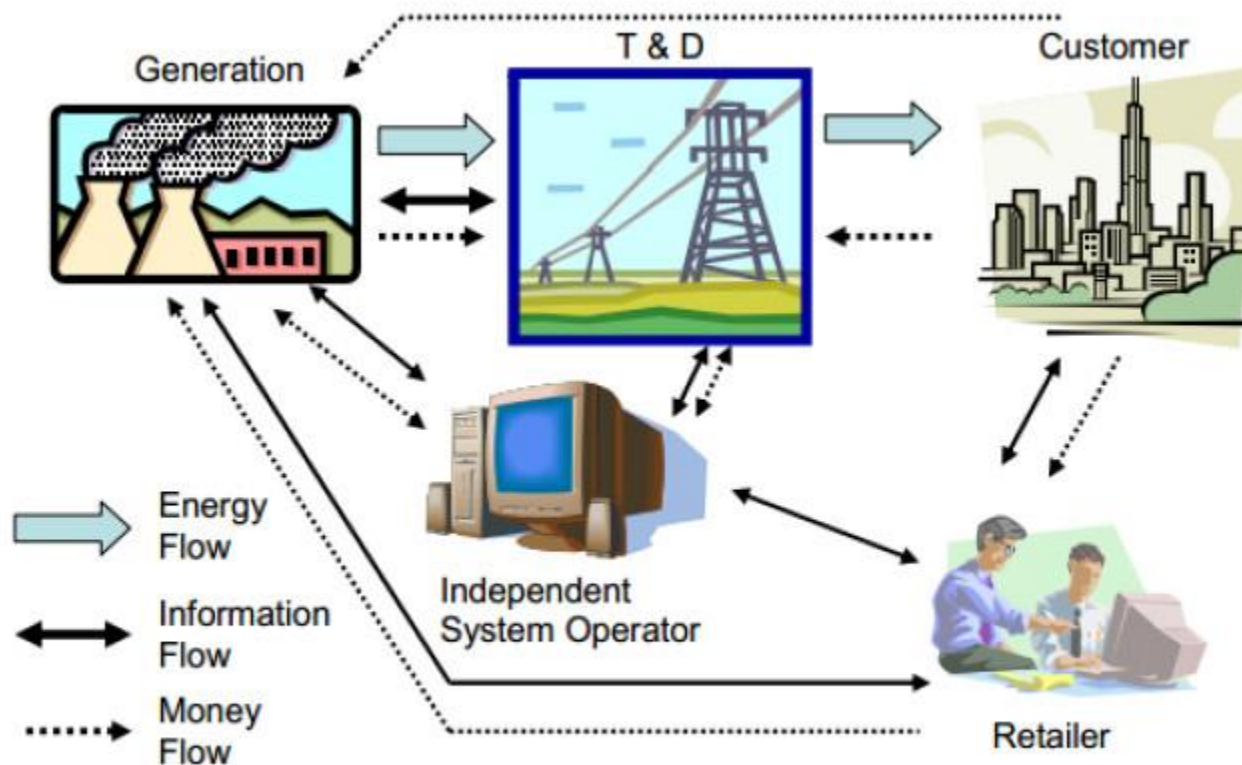
# Imagine, you are in New York!

**Disruption**

Route	Duration
03:18 bis 03:54 4 > 13 Min. alle 20 Min.	36 Min.
03:19 bis 04:07 2 > M86-SBS	48 Min.
03:11 bis 04:07 1 > M86-SBS	56 Min.
03:39 bis 04:33 2 > M79-SBS > 1	54 Min.

REISEPLANER  
Metropolitan Museum of Art erkunden

# Focusing on Energy Systems



- ✓ The energy system is a complex system.
- ✓ Empirical data as well as theories on economic and technical dependencies, causalities and correlations are available.
- ✓ For decades, this has led to efforts by the research community to develop forecasting and analysis support models in energy.
- ✓ These tools are widely used. They support business decisions as well as policy decisions.

Figure: Rishika and Nithya (2013)

# Energy Systems Modelling...

- ✓ **...has a long history**

Since the early 1970s, a wide variety of models were developed to analyse energy systems, provide of forecasts and improve decisions

- ✓ **...has multiple purposes**

Better understanding of current and future market conditions

Design of future energy supply systems in medium- and long-term

Ensuring sustainable exploitation of scarce energy resources

Better understanding of the interactions between energy markets and the rest of the economy

- ✓ **...is based on different theoretical foundations**

Engineering, economics, operations research, management science, etc.

- ✓ **...and on different techniques**

Optimization, econometrics, simulations

# Purpose of energy systems modelling

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- Energy system models are applied to predict market behavior (or the behavior of a single market participant) within a complex system
- Energy systems are influenced by
  - Design
  - Market structure
  - Regulations
  - Government policies
  - Weather
  - Unexpected events
  - etc.

# Parameters influencing the electricity price

Supply	Demand
Installed and available capacities	Weather (temperature, cloudiness)
Fuel prices	Seasonality (daily, weekly, yearly)
CO2-Prices	Special occasions (e.g. public holidays, WM-semi final, ...)
Availability of water (rain, snow melting, etc.)	Economic situation
Investment costs	Technical progress
Environmental specifications	
Life span restrictions of nuclear power plants	
Technical parameters of power plants.	
Wind velocity	
Transmission capacities	

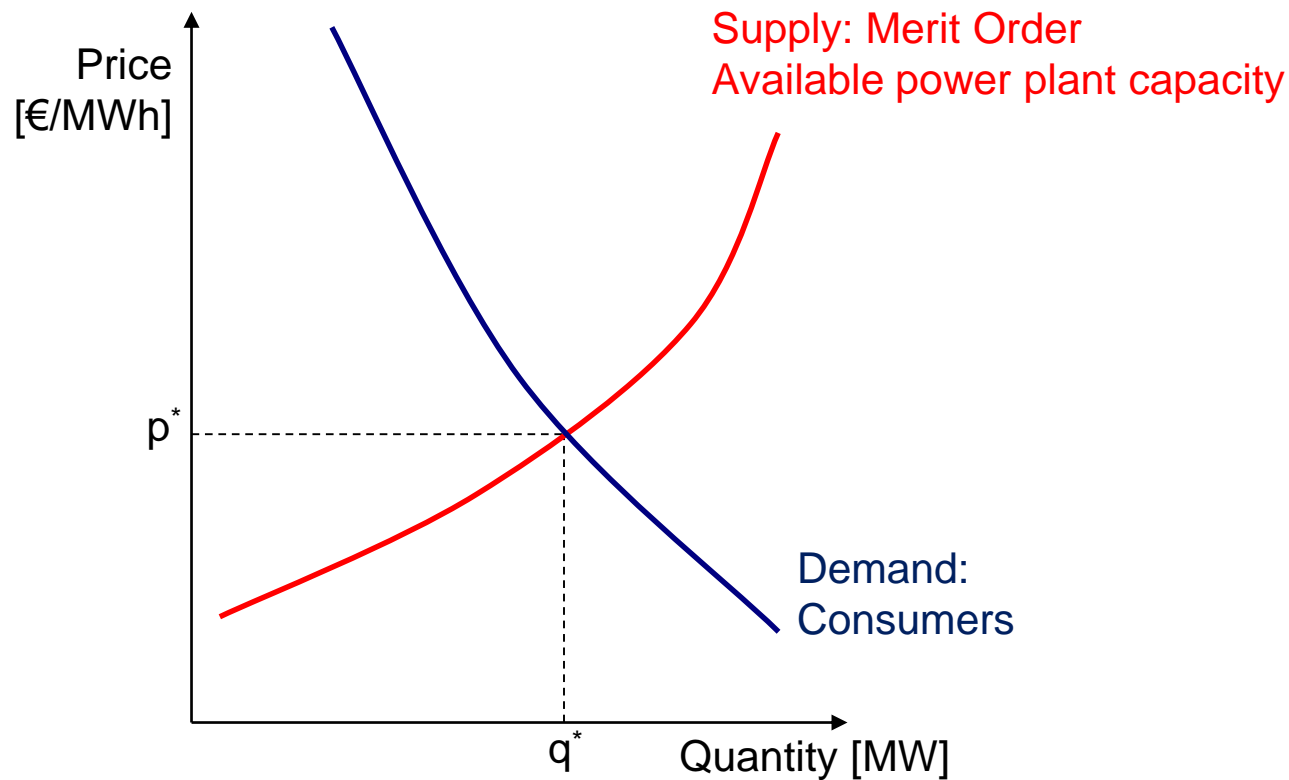
# Energy Systems Modelling

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## **1) Introduction**

- 1) General definitions
- 2) Merit-Order
- 2) Basic theory
- 3) Linear problems
- 4) Linear integer problems
- 5) Non-linear problems
- 6) Introduction to game theory
- 7) Strategic behavior in energy markets

# Price Formation





# Merit Order

## ◆ Power plant list Germany:

[http://www.bundesnetzagentur.de/DE/Sachgebiete/ElektrizitaetundGas/Unternehmen\\_Institutionen/Versorgungssicherheit/Erzeugungskapazitaeten/Kraftwerksliste/kraftwerksliste-node.html](http://www.bundesnetzagentur.de/DE/Sachgebiete/ElektrizitaetundGas/Unternehmen_Institutionen/Versorgungssicherheit/Erzeugungskapazitaeten/Kraftwerksliste/kraftwerksliste-node.html)

	A	B	C	D	E	F	K	L	M	N	O	P	Q	R	S	T	V	X	AB	AC	
1		count	company	name	postcode	city	commissioned	retrofit	shutdown	status	fuel	technology	type	eeg	chp	capacity	BNetzA	chp_capacity	efficiency_d	voltage	network_operator
464	BNÄ0753	DE	Energieversorgung	HKW 2	46147	Oberhausen	1995			operating	gas	ST		no	yes	24.5				110	Oberhausener Net
465	BNÄ0755a	DE	Kraftwerk Obernbu	Obernburg	63784	Obernburg	1920			operating	gas	CC	IPP	no	yes	36				20	Bayernwerk AG
466	BNÄ0755b	DE	Kraftwerk Obernbu	Obernburg	63784	Obernburg	1995			operating	gas	CC	IPP	no	yes	64				20	Bayernwerk AG
467	BNÄ0756	DE	EnBW Klenk Holzenergie GmbH		74420	Oberrot	2000			operating	biomass	ST		yes		18.8			MS		Netze BW GmbH
468	BNÄ0758	DE	Energieversorgung	Heizkraftwerk Offenbach	63067	Offenbach	1990			operating	coal	ST		no	yes	54				110	Netrion GmbH
469	BNÄ0759	DE	E.ON Kraftwerke Gr	Itzehoe	25588	Oldendorf	1972			operating	oil	ST		no	no	88				110	Schleswig-Holsteir
470	BNÄ0766	DE	E.ON Kraftwerke Gr	Audorf	24783	Osterrönfeld	1973			operating	oil	ST		no	no	87				110	Schleswig-Holsteir
471	BNÄ0775	DE	EEV BioEnergie GmH	Biomasse-HKW	26871	Papenburg	2003			operating	biomass	ST		yes	no	20				20	EWE NETZ GmbH
472	BNÄ0777	AT	EnBW Energie Bade	Kopswerk I	A-6794	Partenen	1968			operating	pumped_storage	PSP		no	no	247				220	Transnet BW GmbH
473	BNÄ0778	AT	EnBW Energie Bade	Kopswerk II	A-6794	Partenen	2008			operating	pumped_storage	PSP		no	no	525				220	Transnet BW GmbH
474	BNÄ0779	AT	EnBW Energie Bade	Obervermuntwerk	A-6794	Partenen	1968			operating	reservoir	RES		no	no	29				220	Transnet BW GmbH
475	BNÄ0780	AT	EnBW Energie Bade	Vermuntwerk	A-6794	Partenen	1930			operating	reservoir	RES		no	no	158				220	Transnet BW GmbH
476	BNÄ0782	DE	Rhein-Main-Donau	Kachlet	A-94034	Passau	1927			operating	run_of_river	ROR		no	no	53.7				110	Bayernwerk AG
477	BNÄ0784	DE	Innwerk AG	Passau-Ingling	94036	Passau	1965			operating	run_of_river	ROR		no	no	43.2				110	APG/Bayernwerk AG
478	BNÄ0785	DE	Vattenfall Europe GKW	Jänschwalde	3185	Peitz	1981	1996		operating	lignite	ST		no	yes	465	76.3	0.375		380	50 Hertz Transmiss
479	BNÄ0786	DE	Vattenfall Europe GKW	Jänschwalde	3185	Peitz	1982	1996		operating	lignite	ST		no	yes	465	76.3	0.375		380	50 Hertz Transmiss
480	BNÄ0787	DE	Vattenfall Europe GKW	Jänschwalde	3185	Peitz	1984	1996		operating	lignite	ST		no	yes	465	76.3	0.375		380	50 Hertz Transmiss
481	BNÄ0788	DE	Vattenfall Europe GKW	Jänschwalde	3185	Peitz	1985	1996		operating	lignite	ST		no	yes	465	76.3	0.375		380	50 Hertz Transmiss
482	BNÄ0789	DE	Vattenfall Europe GKW	Jänschwalde	3185	Peitz	1987	1996		operating	lignite	ST		no	yes	465	76.3	0.375		380	50 Hertz Transmiss
483	BNÄ0790	DE	Vattenfall Europe GKW	Jänschwalde	3185	Peitz	1989	1996		operating	lignite	ST		no	yes	465	76.3	0.375		380	50 Hertz Transmiss
484	BNÄ0792	DE	VERBUND-Innkraftv	Perach	84567	Perach	1977			operating	run_of_river	ROR		no	no	19.4				110	Bayernwerk AG
485	BNÄ0793	DE	E.ON Kraftwerke Gr	Heyden	32469	Petershagen	1987			operating	coal	ST		no	no	875			0.42	380	Tennet TSO GmbH
486	BNÄ0795	DE	Rhein-Main-Donau	Geisling	93102	Pfaffing	1985			operating	run_of_river	ROR		no	no	25				20	Bayernwerk AG
487	BNÄ0797	DE	Heizkraftwerk Pforz	Heizkraftwerk Pforzheim GmbH	75175	Pforzheim	2004			operating	biomass	ST		yes	yes	12.3				110	Stadtwerke Pforzh
488	BNÄ0799	DE	Heizkraftwerk Pforz	Heizkraftwerk Pforzheim GmbH	75175	Pforzheim	1969			operating	gas	ST		no	yes	11.3				110	Stadtwerke Pforzh
489	BNÄ0800	DE	Heizkraftwerk Pforz	Heizkraftwerk Pforzheim GmbH	75175	Pforzheim	1980			operating	gas	CC		no	yes	41.2				110	Stadtwerke Pforzh
490	BNÄ0801	DE	Heizkraftwerk Pforz	Heizkraftwerk Pforzheim GmbH	75175	Pforzheim	1990			operating	coal	ST		no	yes	26.9				110	Stadtwerke Pforzh
491	BNÄ0802	DE	EnBW Energie Bade	Kernkraftwerk Philippsburg 2	76661	Philippsburg	1985			operating	uranium	ST		no	no	1402				380	Transnet BW GmbH
492	BNÄ0804a	DE	K+S AG	Hattorf	36269	Philippsthal	1962			operating	gas	ST		no	no	35				110	Avacon AG
493	BNÄ0804b	DE	K+S AG	Hattorf	36269	Philippsthal	2013			operating	gas	ST		no	yes	17.3				110	Avacon AG
494	BNÄ0805	DE	Daimler AG / UPM	CKraftwerk Plattling	94447	Plattling	2010			operating	gas	CC	IPP	no	yes	97.9		150		110	Bayernwerk AG
495	BNÄ0810	DE	Gemeinschaftskraft	Kraftwerk Veltheim	32457	Porta Westfalica	1974		2015	shutdown	gas	GT		no	no	65				110	Netz Veltheim Gm
496	BNÄ0811	DE	Gemeinschaftskraft	Kraftwerk Veltheim	32457	Porta Westfalica	1975		2015	shutdown	gas	ST		no	no	335				220	Netz Veltheim Gm
497	BNÄ0812	DE	Gemeinschaftskraft	Kraftwerk Veltheim	32457	Porta Westfalica	1965		2012	shutdown	coal	ST		no	no	93				110	Netz Veltheim Gm
498	BNÄ0813	DE	Gemeinschaftskraft	Kraftwerk Veltheim	32457	Porta Westfalica	1970		2015	shutdown	coal	ST		no	no	303				220	Netz Veltheim Gm
499	BNÄ0814	DE	Energie und Wasser	HKW Potsdam-Süd	14478	Potsdam	1996			operating	gas	ST		no	yes	81.8				110	Netzgesellschaft P
500	BNÄ0816	AT	TIWAG-Tiroler Was	KW Kaunertal	A-6522	Prutz	1964			operating	reservoir	RES		no	no	392				220	TIWAG-Netz AG/AI
501	BNÄ0830	DE	ETEG Energie-Service	GmbH	65383	Quilshausen	1978			operating	coal	ST		no	no	655.6		30	0.37	330	AVAG GmbH



# Bottom Up

Jänschwalde Block A		
Technology	Steam Turbine	
Fuel	Lignite	
Capacity [MW <sub>net</sub> ]	465	
Efficiency [%]	39	
Emission Factor CO <sub>2</sub> [t <sub>CO2</sub> /MWh <sub>el</sub> ]	1.01	
Minimum Load [%]	40	
Partial Load Efficiency [%]	34.04	
Ramping Load Gradient [%/min]	1.75	
Startup Time [h]	Hot Startup	1
	Cold Startup	8
Minimum Overtime [h]	8	
Technical Availability [%]	85	
Fuel Costs [€/MWh <sub>el</sub> ]	11.66	
CO <sub>2</sub> -Costs [€/MWh <sub>el</sub> ]	4.99	
Variable O&M Costs [€/MWh <sub>el</sub> ]	7.42	

# Bottom Up – Creating a Merit-Order

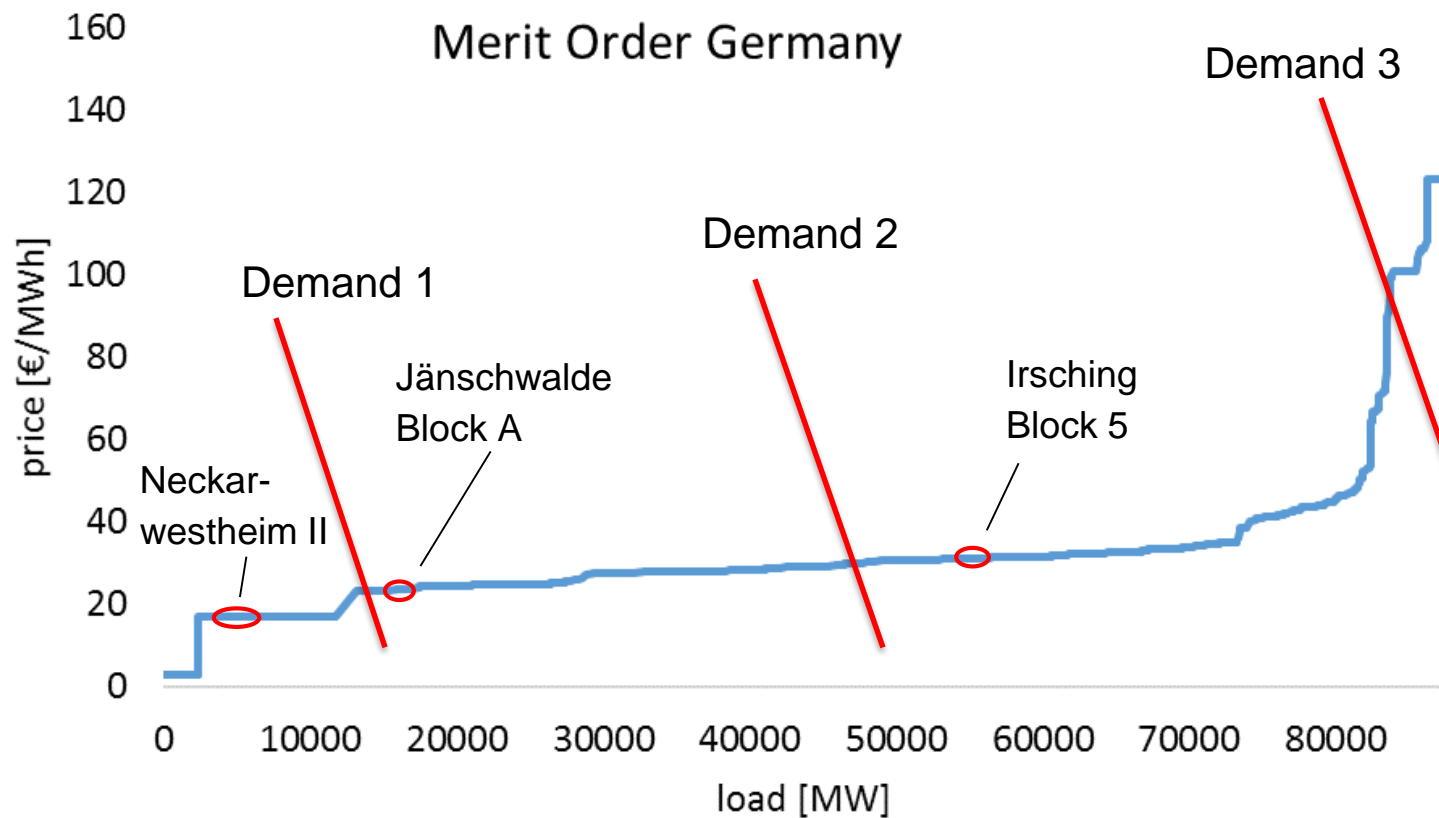
	<b>Jänschwalde Block A</b>	<b>Neckarwestheim II</b>	<b>Irsching Block 5</b>
<b>Capacity [MW<sub>net</sub>]</b>	465	1310	845
<b>Fuel price [€/MWh<sub>el</sub>]</b>	11,66	9,1	24,44
<b>CO<sub>2</sub> price [€/MWh<sub>el</sub>]</b>	4,99	0	2,01
<b>Variable O&amp;M costs [€/MWh<sub>el</sub>]</b>	7,42	7,94	4,24
<b>Variable generation costs [€/MWh<sub>el</sub>]</b>	<b>24,07</b>	<b>17,04</b>	<b>30,69</b>



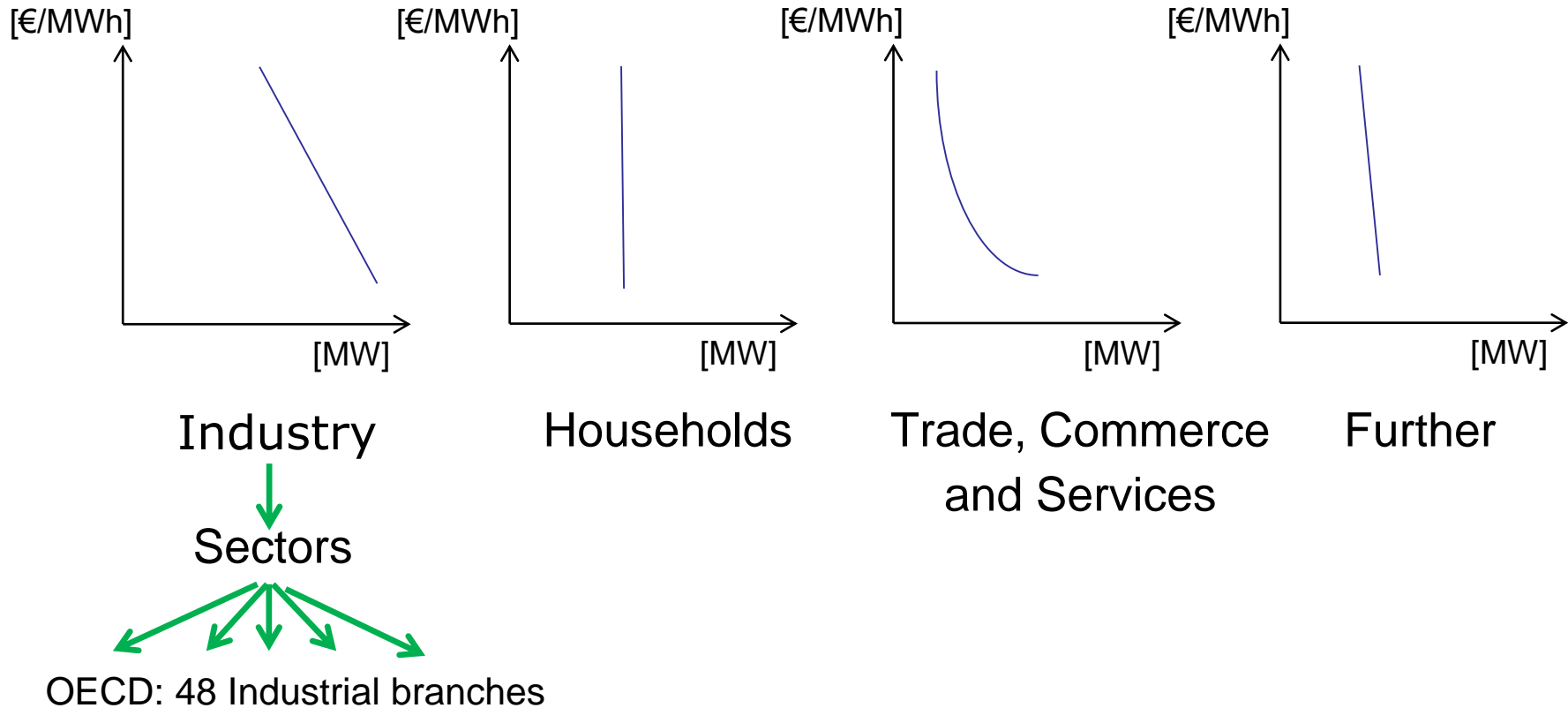
Sort with increasing generation costs

	<b>Neckarwestheim II</b>	<b>Jänschwalde Block A</b>	<b>Irsching Block 5</b>
<b>Capacity [MW<sub>net</sub>]</b>	1310	465	845
<b>Fuel price [€/MWh<sub>el</sub>]</b>	9,1	11,66	24,44
<b>CO<sub>2</sub> price [€/MWh<sub>el</sub>]</b>	0	4,99	2,01
<b>Variable O&amp;M costs [€/MWh<sub>el</sub>]</b>	7,94	7,42	4,24
<b>Variable generation costs [€/MWh<sub>el</sub>]</b>	<b>17,04</b>	<b>24,07</b>	<b>30,69</b>

# Merit-Order



# Demand and Price Elasticity



# Energy System Modelling

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- 1) Introduction
- 2) Basic theory**
- 3) Linear problems
- 4) Linear integer problems
- 5) Non-linear problems
- 6) Introduction to game theory
- 7) Strategic behavior in energy markets

# Energy System Modelling

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1) Introduction

**2) Basic theory**

1) What models exist?

2) Merit order model

3) The idea of an optimization model

4) Example of power market optimization model: EUDIS

3) Linear problems

4) Linear integer problems

5) Non-linear problems

6) Introduction to game theory

7) Strategic behavior in energy markets

# Energy System Model Classification: Purposes

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- ♦ To predict or forecast the future
- ♦ To explore the future (scenario analysis)
  - A limited number of “intervention” scenarios may be compared with a “reference” (BaU) scenario
  - The alternative scenarios are only relevant in the context of the reference scenario and rely on assumptions rather than parameters extracted from the data
- ♦ To look back from the future to present (“backcasting”)
  - Constructing visions about desired future and subsequently looking back at what is to be changed to accomplish it.

sources: [1], [2]

# Energy System Model Classification: Independent Dimensions 1

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- ♦ Geographical coverage
  - Energy system models may analyse different levels of geographical coverage. The world economy is investigated in “global” models at a large scale. “National” models focus in detail on energy sector(s) inside one country, i.e. model them endogenously, while neighbouring countries are considered exogenously.
- ♦ Sectoral coverage
  - i.e. electricity consumption may be divided into certain sectors
  - electricity, gas, heat, transportation?
- ♦ Time horizon
  - i.e. short-term (hours, days), medium-term (usually up to 10 years), long-term (beyond 10 years)

sources: [1], [2], [3]



# Energy System Model Classification: Independent Dimensions 2

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- ♦ Development of Capacities
  - conventional capacity
  - renewable capacity
- ♦ Data type
  - Aggregated or disaggregated, quantitative or qualitative
- ♦ The extent of the description of the non-energy sector
- ♦ The extent of the description of energy end-users
- ♦ The extent of the description of energy supply technology

sources: [1], [2], [3]

# Model classification: major methodologies 1

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## 1) Econometric models

- Apply statistical methods to extrapolate past market behaviour into the future (same assumptions as time series).
- Rely on aggregated data that have been measured in the past to calculate correlations.
- Econometric methods use historical information and forecasts for independent variables to predict the future

# Model classification: major methodologies 2

## 2) Macro-economic models

- The macro-economic methodologies consider the entire economy of a society and the interaction between sectors.
- Often do not concentrate on energy specifically but on the economy as a whole, of which energy is only a part

## 3) Economic equilibrium models

- Consider energy sector as a part of the overall economy and focus on interrelations between the energy sector and the rest of the economy
- Partial equilibrium models only focus on equilibria in parts of the economy, such as the equilibrium between energy demand and supply.
- General equilibrium models are particularly concerned with the conditions which allow for simultaneous equilibrium in all markets, as well as the determinants and properties of such an economy-wide set of equilibria.

sources: [1], [3]

# Model classification: major methodologies 3

## 4) Optimization models

- An optimization problem implies finding a good choice out of a set of alternatives by minimizing or maximizing one or some real functions. Input values are selected from an allowed set and must satisfy some constraints.
- Underlying assumption of optimization methodologies is that all acting agents behave rationally under given constraints

## 5) Simulation models

- Descriptive models based on a logical representation of an energy system, aimed at reproducing a simplified operation of this system (World Energy Conference (1986)).

# Model classification: mathematical approaches

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- ♦ Linear programming (LP)
- ♦ Mixed integer programming (MIP)
- ♦ Nonlinear programming (NLP)
- ♦ Mixed integer nonlinear programming (MINLP)
- ♦ Mixed complementarity programming (MCP)
- ♦ Dynamic programming
- ♦ Stochastic Programming (SP)

# Mathematical approaches: an overview

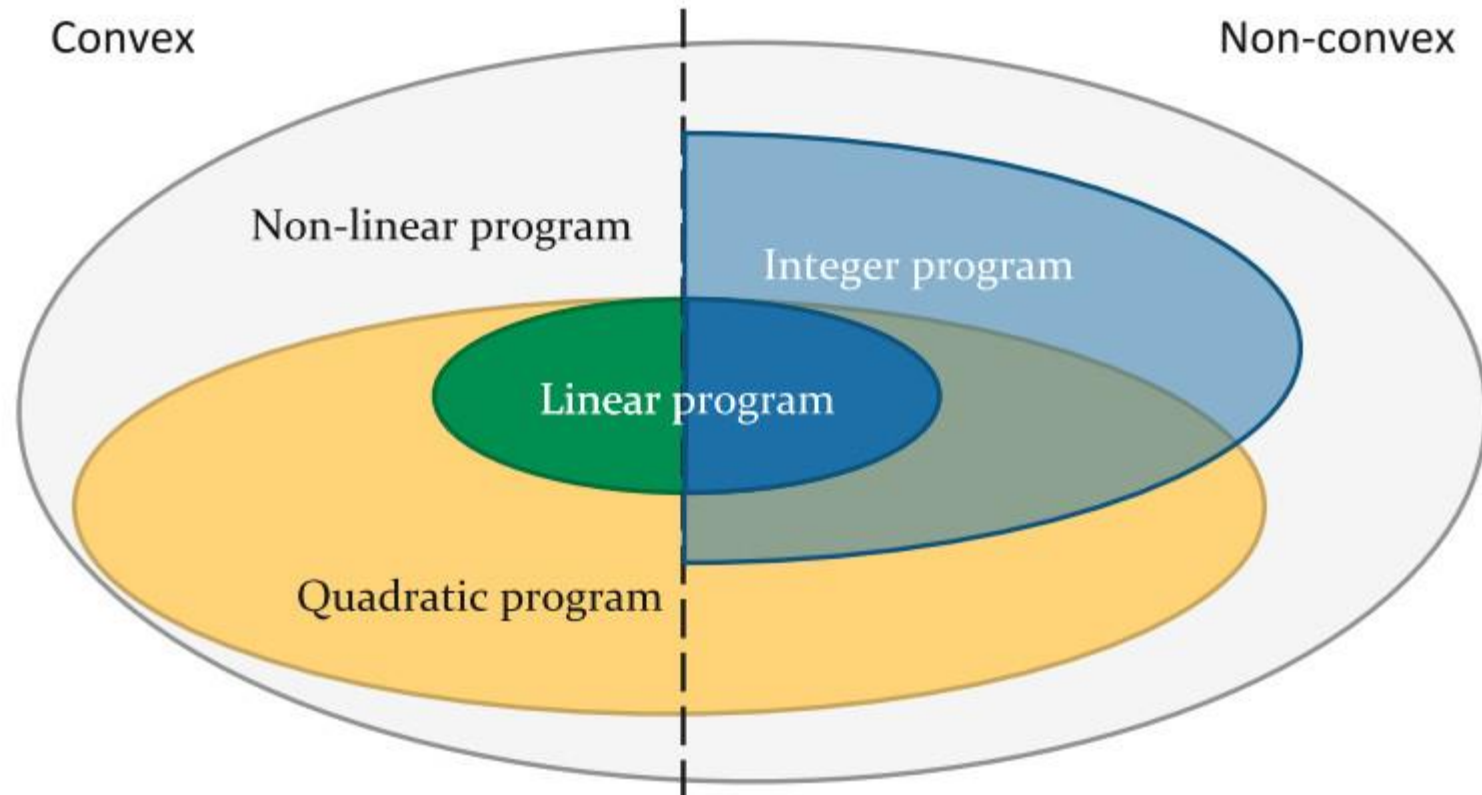


Figure from "One and Two-Level Energy Market Equilibrium Modeling", Dr. Daniel Huppmann

# Energy System Modelling

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1) Introduction

**2) Basic theory**

1) What models exist?

2) Merit order model

3) The idea of an optimization model

4) Example of power market optimization model: EUDIS

3) Linear problems

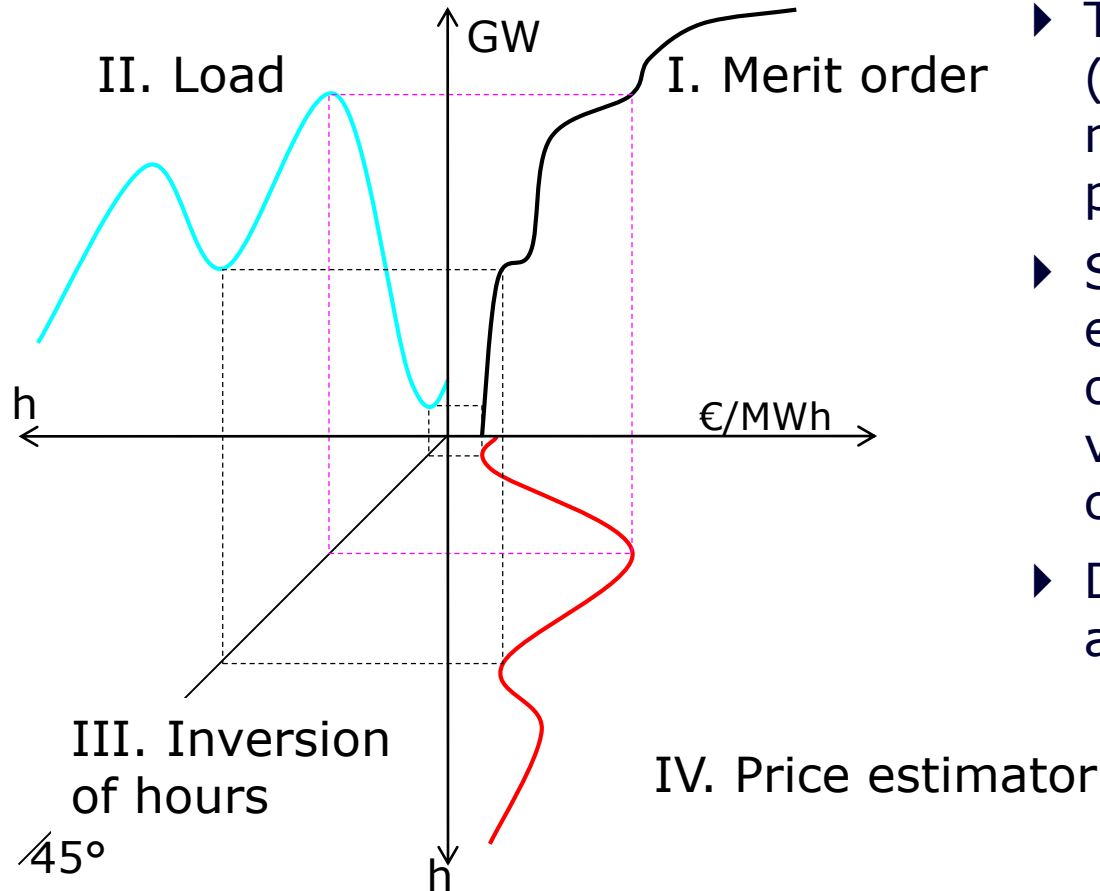
4) Linear integer problems

5) Non-linear problems

6) Introduction to game theory

7) Strategic behavior in energy markets

# Estimating prices based on Merit order models



- ▶ These merit order models (also known as „stack models“) are widely used in practice.
- ▶ Strengths/advantages are evident especially in times of high fundamental volatility and structural changes.
- ▶ Disadvantage: Static approach.



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# Modeling prices – the concept of marginal costs

- ♦ How pricing works when dynamic effects are considered:

→ The concept of marginal costs of demand

- Marginal costs are the costs that occur due to a marginal increase in demand.
- It is the 'shadow price' of the demand constraint.
- This is an application of Operations Research theory to economic questions.
- The shadow price can be used as price estimator in a competitive market.

# The idea of an optimization model (I)

Assume there are two generating technologies (1 and 2).

The variable generation costs are:  $C_1 = 1, C_2 = 2$

The capacity available is:  $x_1 = 100, x_2 = 180$

The demand is:  $D = 150$

Mathematical formulation of the optimization problem:

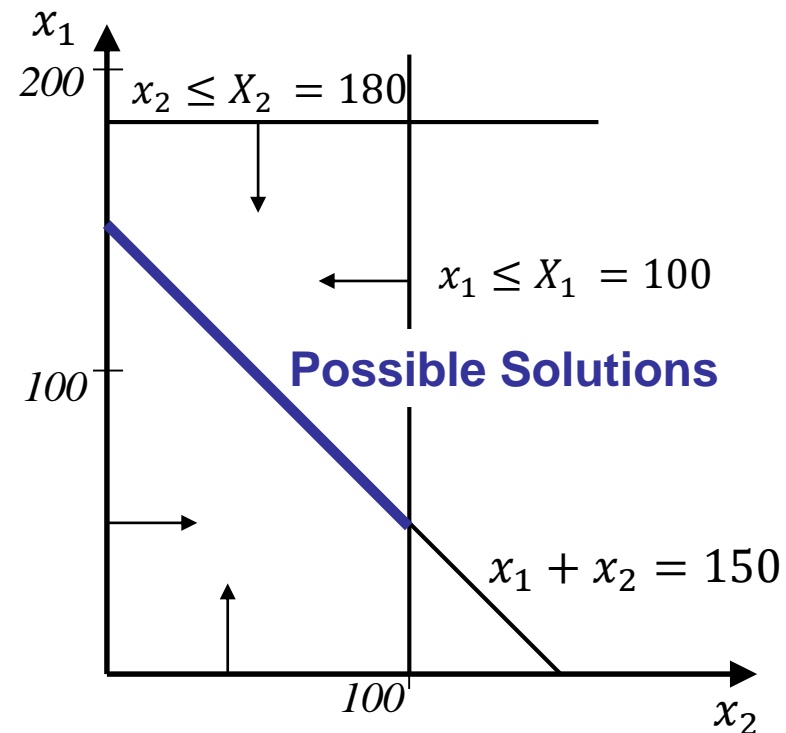
$$\min_{x_1, x_2} \text{cost} = C_1 x_1 + C_2 x_2$$

s.t.

$$x_1 + x_2 = D$$

$$x_i \leq X_i \quad \forall i$$

$$x_i \geq 0 \quad \forall i$$

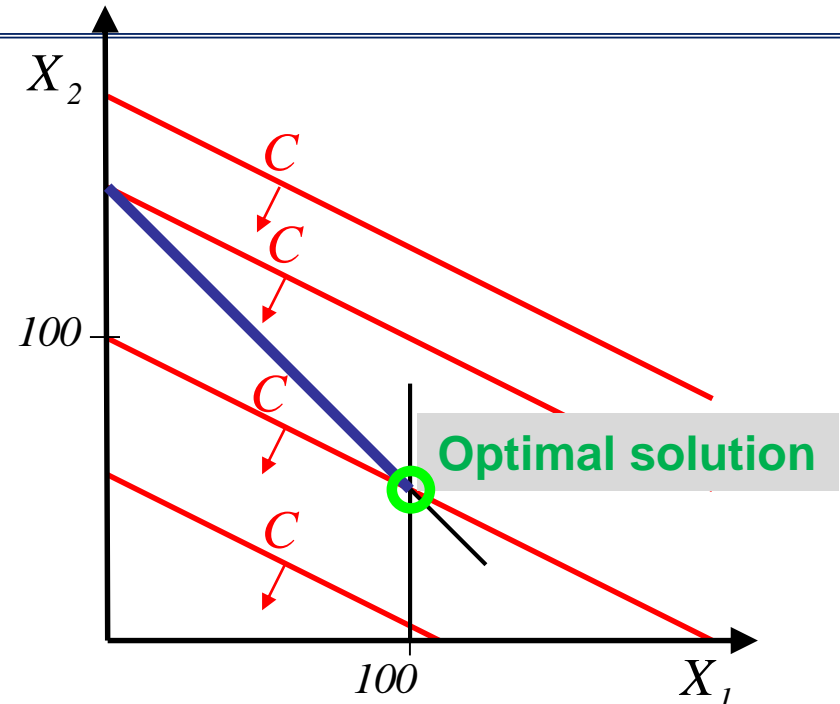


# The idea of an optimization model (II)

**Optimal solution:**

$$x_1 = 100, x_2 = 150$$

$$\text{cost} = 200$$



- ♦ **Price estimators** are determined by the equation of demand. They correspond to the shadow price of this constraint.
- ♦ They answer the question of how much the cost will rise if demand increases by one additional unit (simplified!).
- ♦ In our example: A marginal increase in demand would be covered by additional production of technology 2 (technology 1 does not have any free capacity).  
 → The marginal costs (price estimators) are 2 (production costs of technology 2).

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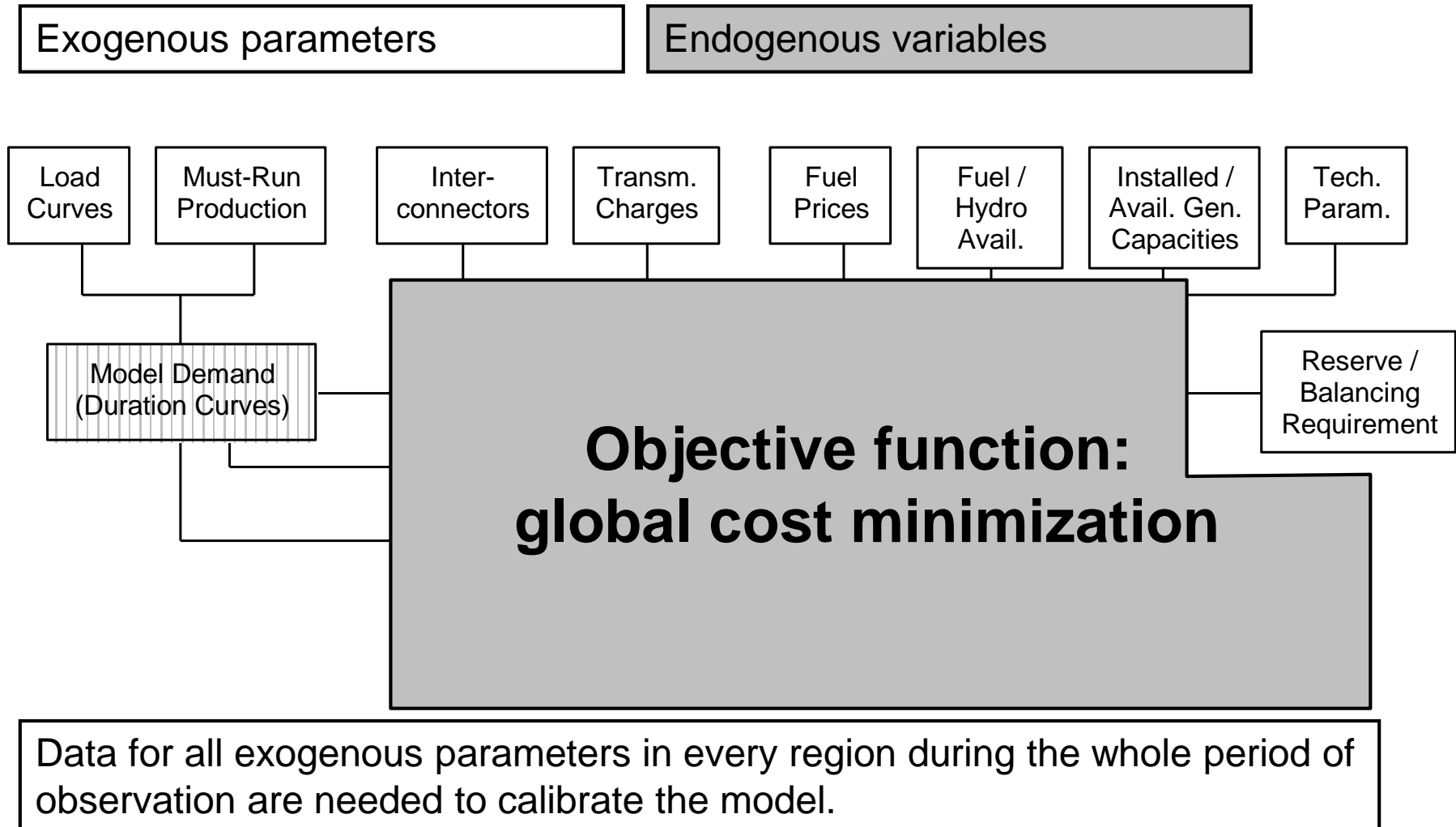
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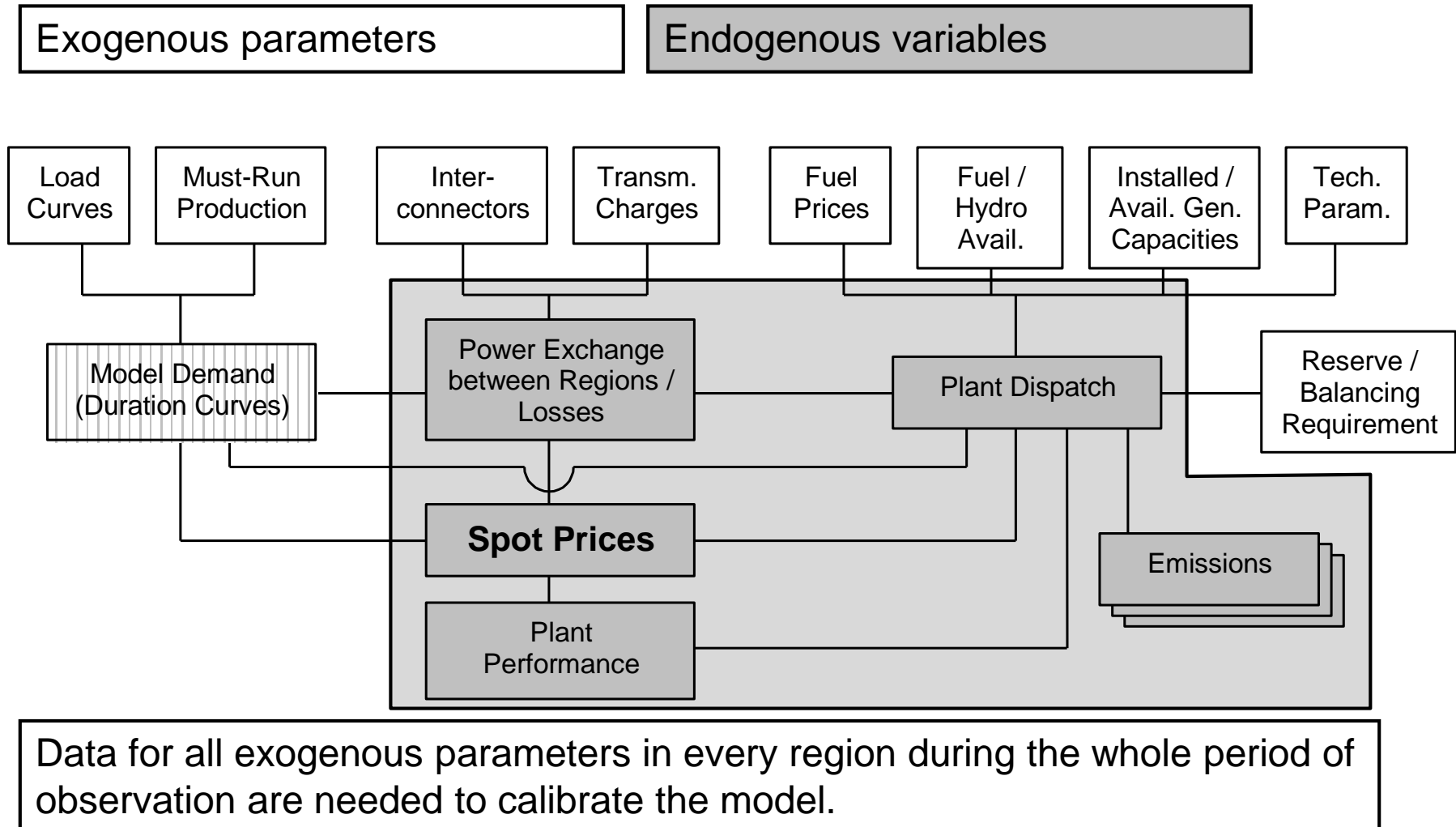
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# EUDIS – Model Structure and Data Input



# EUDIS – Model Structure and Data Input



# Sources

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- [1] Beeck, N.M.J.P. van, 1999. Classification of Energy Models. *FEW Research Memorandum; Vol. 777. Tilburg: Operations research;*
- [2] Hourcade, J.C. et. al. (1996). "Estimating the Cost of Mitigating Greenhouse Gases." in: J.P. Bruce, H. Lee, and E.F. Haites (eds.), *Climate Change 1995: Economic and Social Dimensions of Climate Change. Contribution of Working Group III to the Second Assessment Report of the IPCC.* University Press, Cambridge, 263-296.
- [3] Neshat, N., Amin-Naseri, M.R. & Danesh, F., 2014. Energy models: Methods and characteristics. *Journal of Energy in Southern Africa*, 25, pp.101–111. Available at [http://www.scielo.org.za/scielo.php?script=sci\\_arttext&pid=S1021-447X2014000400010&nrm=iso](http://www.scielo.org.za/scielo.php?script=sci_arttext&pid=S1021-447X2014000400010&nrm=iso).



# Energy System Modelling

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- 1) Introduction
- 2) Basic theory
- 3) Linear problems**
- 4) Linear integer problems
- 5) Non-linear problems
- 6) Modelling strategic behavior

# Energy System Modelling

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1) Introduction

2) Basic theory

## **3) Linear problems**

1) Introduction to linear programming

2) Application of LP for electricity market modelling

3) Implementing dynamics

4) Linear integer problems

5) Non-linear problems

6) Modelling strategic behavior

# Introduction to linear programming

## Key points of lectures 1 and 2:

Mathematical programming is used to find the **optimal solution** to a problem that requires a decision or set of decisions about how best to use a set of limited resources to achieve a state goal of objectives.

## Steps involved in mathematical programming:

- Conversion of stated problem into a mathematical model that abstracts all the essential elements of the problem.
  - Exploration of different solutions of the problem.
  - Finding out the most suitable or optimum solution.
- ✓ **Linear programming** requires that all the mathematical functions in the model are **linear functions**.

# Introduction to linear programming

Define:  $x_1, x_2, x_3, \dots, x_n$  -> **Decision variables**  
 $z$  -> **Objective function**

Objective: Maximization of the function  $z$

$$z = C_1x_1 + C_2x_2 + C_3x_3 + \dots + C_nx_n$$

Subject to the constraints (s.t.):

$$A_{11}x_1 + A_{12}x_2 + \dots + A_{1n}x_n \leq B_1$$

$$A_{21}x_1 + A_{22}x_2 + \dots + A_{2n}x_n \leq B_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$A_{m1}x_1 + A_{m2}x_2 + \dots + A_{mn}x_n \leq B_n$$

$$x_j \geq 0 \quad \forall j$$

... where  $A_{ij}$ ,  $B_i$  and  $C_j$  are given parameters

# Introduction to linear programming

The linear programming model can be written in a general notation as:

Objective:  $\max_{x_j} z = \sum_{j=1}^n c_j^T \cdot x_j$  — Objective Function

s.t.:  $\sum_{j=1}^n A_{ij} \cdot x_j \leq B_i$  — “regular” constraints

$x_j \geq 0$  — Non-negativity constraints

where:  $i = 1, 2 \dots m$   
 $j = 1, 2 \dots n$

The decision variables,  $x_1, x_2, \dots, x_n$ , represent levels of  $n$  competing activities

# Energy System Modelling

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# Application to an Electricity Market

- ♦ The target is to determine a cost minimal dispatch for an electricity market
- ♦ There are various technologies, which are characterized by constant unit costs (**VC**, €/MWh) and their maximum capacity (**CAP**, MW)
- ♦ Generation (**g**, MWh/h) has to equal demand (**DEMAND**, MWh/h) in each hour
- ♦ Generation can not be greater than the available capacity (**CAP**, MW)
- ♦ Generation can not have a negative value

# Application to an Electricity Market

## Sets

$i$	set of technologies
$t$	time

## Variables

$tc$	objective value: total cost
$g_{i,t}$	generation by conventional power plants

## Parameters

$VC_i$	variable production costs [€/MWh]
$CAP_i$	installed capacity
$AF_i$	availability factor of power plants
$DEMAND_t$	reference demand [MW]



# Application to an Electricity Market

## Objective function

$$\min_{g_{i,t}} tc = \sum_{i,t} VC_i * g_{i,t}$$

## Constraints

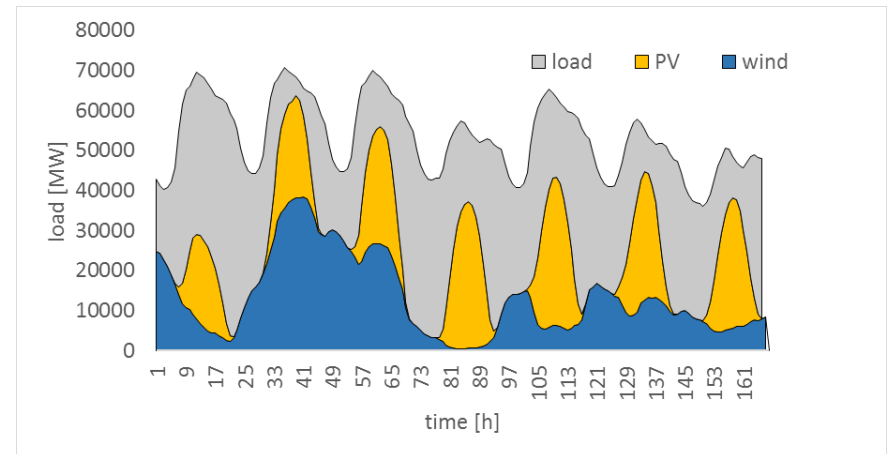
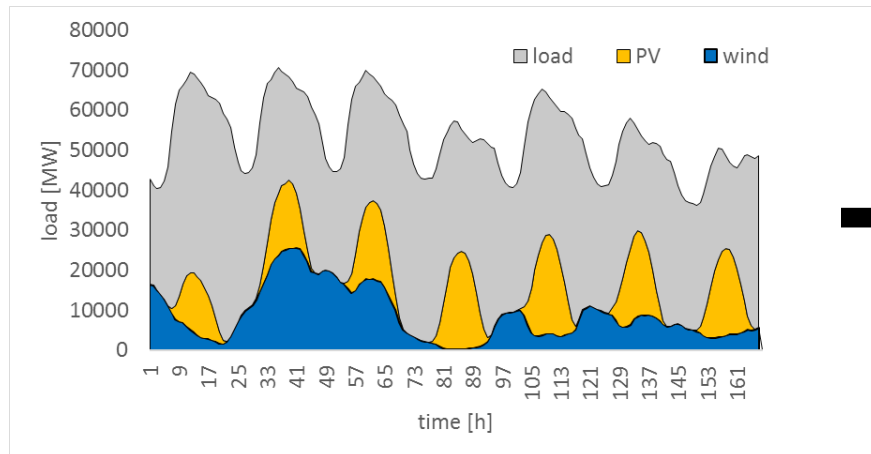
- ♦ Capacity constraint  $g_{i,t} \leq CAP_i * AF_i \quad \forall i, t$
- ♦ Energy Balance  $\sum_i g_{i,t} = DEMAND_t \quad \forall t$
- ♦ Non-negativity constraints  $0 \leq g_{i,t} \quad \forall i, t$

# Modelling RES feed-in

- ♦ Dispatchable RES
  - E.g. Biomass
  - Treated as conventional technologies → constant availability factor for each hour
- ♦ Non-dispatchable RES
  - E.g. Wind, PV
  - Varying weather situation → varying availability factor for each hour
  - But: curtailment is still possible

# Fluctuating RES

- The following pictures show the German load profile first week of June 2015
- On the left: the situation with the original RES feed-in
  - On the right: the situation with a RES feed-in increased by 50%



# Modelling RES feed-in

Using subsets for the set of technologies to distinguish between different technology classifications in order to describe the electricity production by RES

## set

$i$	set of technologies
$c(i)$	set of conventional (or dispatchable) technologies
$r(i)$	set of non-dispatchable technologies (renewables)

## Parameters

$AF_{i,t}$	availability factor of power plants depending on technology <u>and</u> time
	➤ particularly the availability of intermittent RES depends on time

# Modelling RES feed-in

## Implementing constraints which control the electricity production by RES

- ♦ Upper limit  $g_{r,t} \leq AF_{r,t} * CAP_r \quad \forall r, t$
- ♦ Lower limit  $0 \leq g_{r,t} \quad \forall r, t$

- RES curtailment occurs if  $g_{r,t}$  deviates from the upper limit
- How to determine  $AF_{r,t}$ ?



# Energy System Modelling

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2) Basic theory

**3) Linear problems**

- a. Introduction to linear programming
- b. Application of LP for electricity market modelling
- c. Implementing dynamics

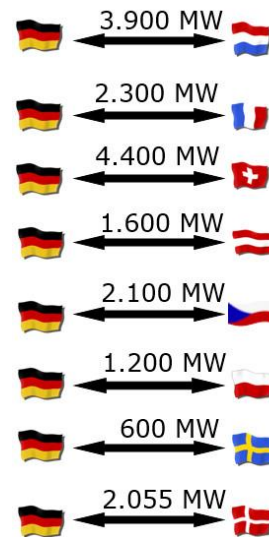
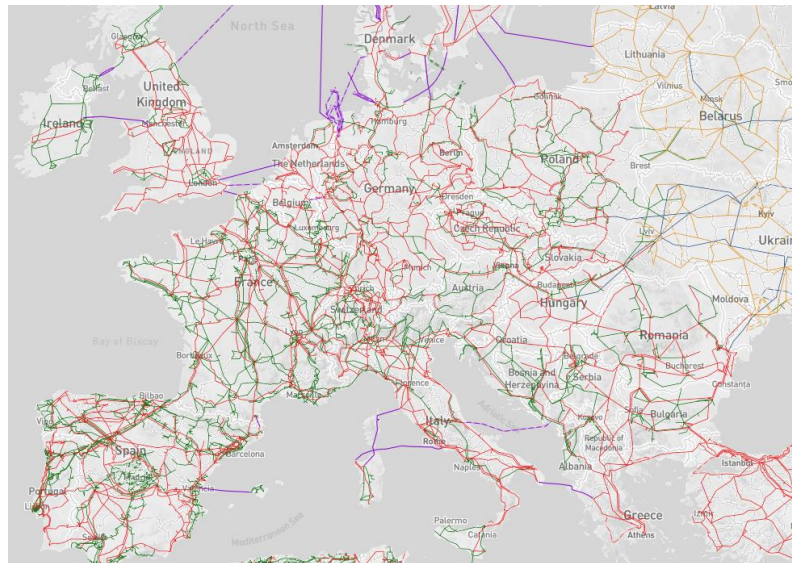
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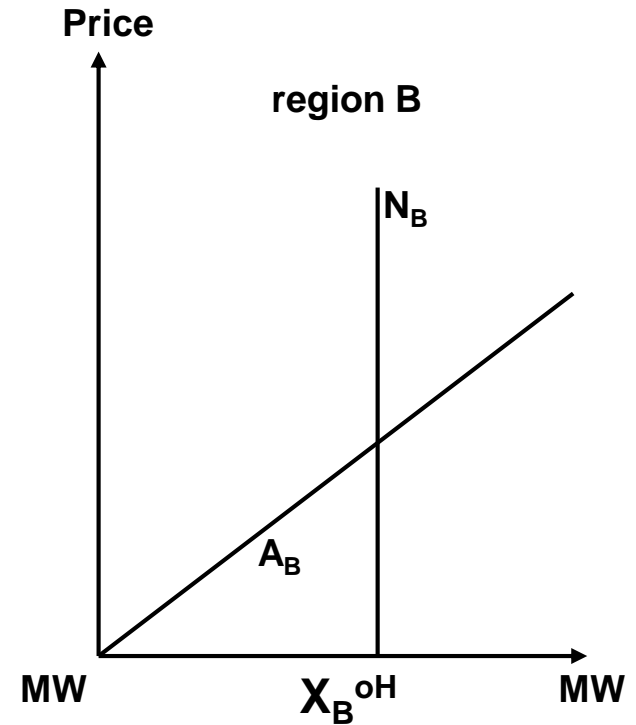
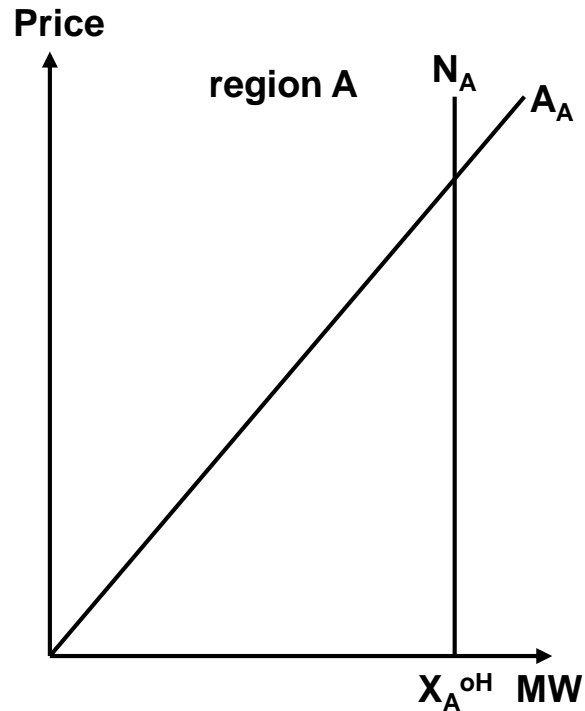
# Interspatial effects

- ♦ European electricity markets are highly interconnected



NTC values Summer 2010 in MW  
 Source: entsoe.com

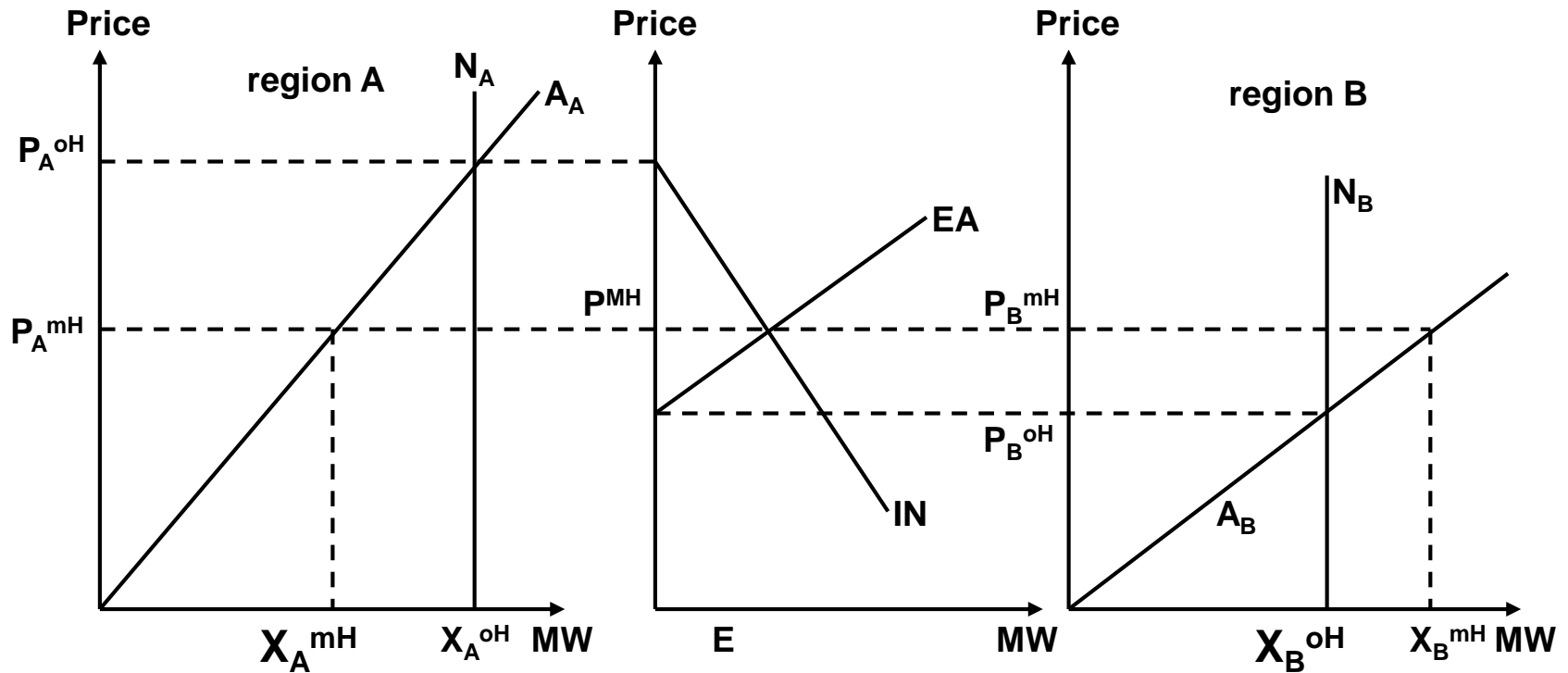
# Prices With and Without Congestions



Source: Schwarz and Lang (2006): Europäische Stromerzeugungsmärkte am Beispiel Zentraleuropas: Stand der Integration und Handlungsbedarf

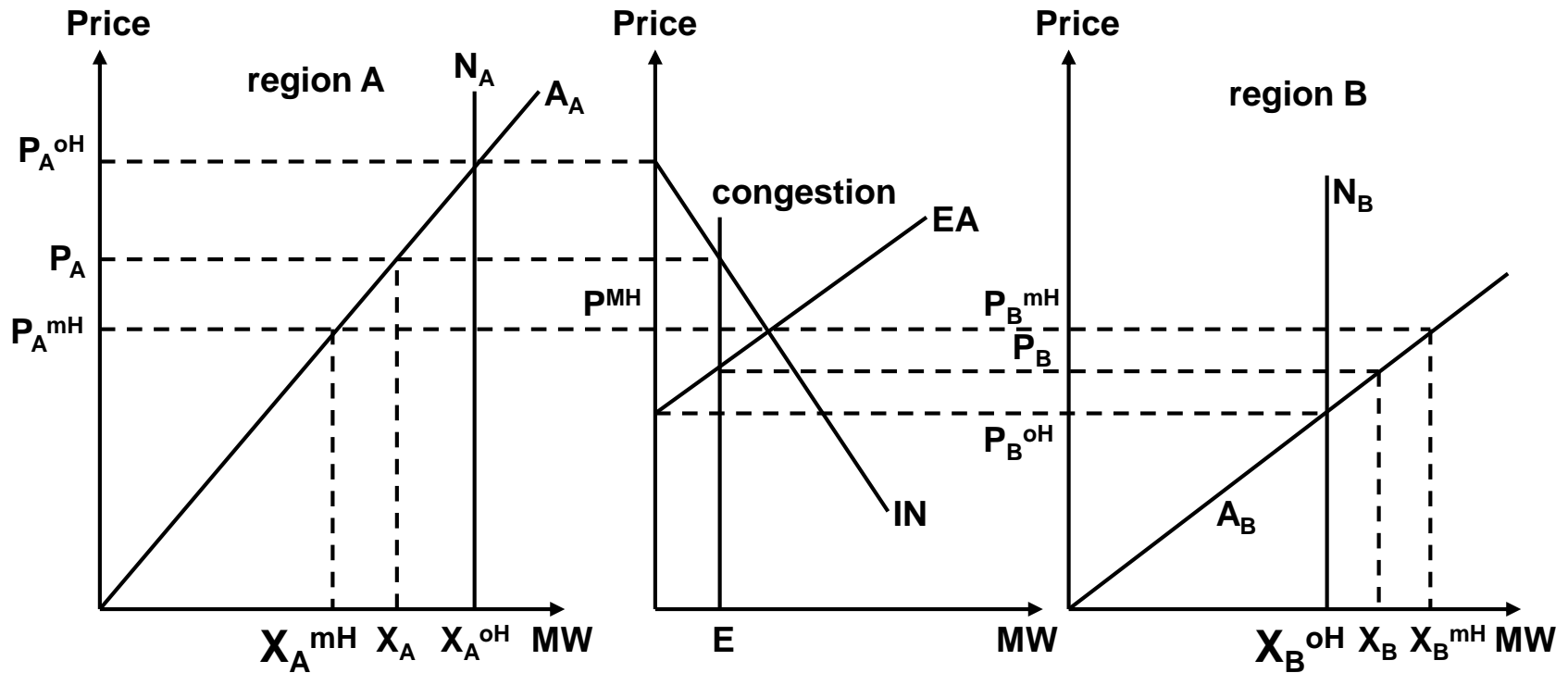


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# Prices With and Without Congestions



Source: Schwarz and Lang (2006): Europäische Stromerzeugungsmärkte am Beispiel Zentraleuropas: Stand der Integration und Handlungsbedarf

# Interspatial effects

## Energy trade – mathematical description

- ♦ Markets (countries) are declared by a set  $N$
- ♦ The transmission capacities can be defined by Net Transfer Capacities (NTC)
- ♦ Every pair of nodes is linked by one NTC ( $NTC_{n,nn}$ )
- ♦ The NTC's restrict the energy flow  $flow_{n,nn,t}$

$n \in N$	markets (countries)
$NTC_{n,nn}$	net transfer capacity from node n to node nn [MW]
$flow_{n,nn,t}$	Transmission flow from node n to node nn at time t [MW]

# Interspatial effects

## Energy trade – mathematical description

- ♦ Energy flow constraint:

$$flow_{n,nn,t} \leq NTC_{n,nn} \quad \forall n, nn, t$$

$$flow_{nn,n,t} \leq NTC_{nn,n} \quad \forall n, nn, t$$

- ♦ Imports and exports affect the energy balance:

$$\begin{aligned}
 DEMAND_{n,t} = & \sum_p g_{p,n,t} \\
 & + \sum_{nn} flow_{n,nn,t} \\
 & - \sum_{nn} flow_{nn,n,t}
 \end{aligned}$$

If  $g_{p,n,t}$  is a generation of power plant **p** in a country **n** at time **t**  
 Import from country nn  
 Export to country nn

# Implementing DSM

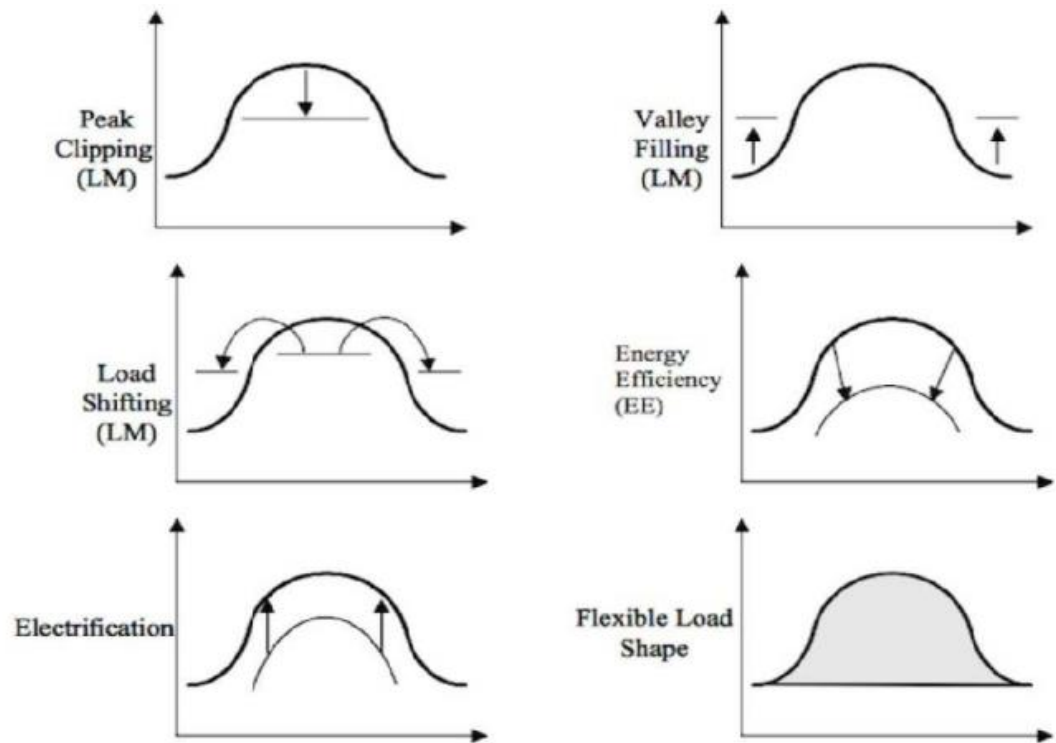
## Demand Side Management (DSM)

- ♦ DSM generally can be understood as the 'modifications in the demand side energy consumption pattern to foster better efficiency and operations in electrical energy systems.' (Behrangrad, 2015)
  
- ♦ DSM examples:
  - Demand Response
  - Energy efficiency

# Implementing DSM

DSM can be implemented in different ways:

The most classic form of demand response is the peak load clipping that can be modelled in a linear formulation by using the value of lost load (VOLL) parameter of the consumers

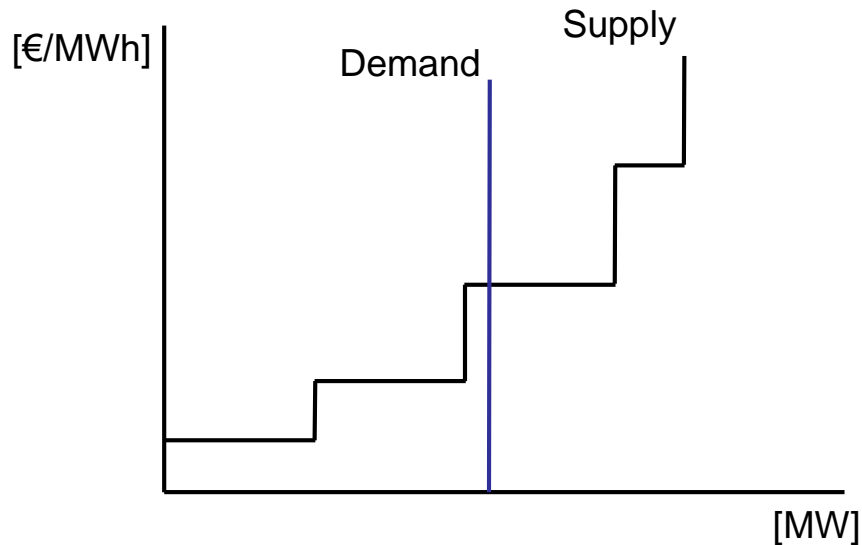


Source: Gellings and Chamberlin (1993)

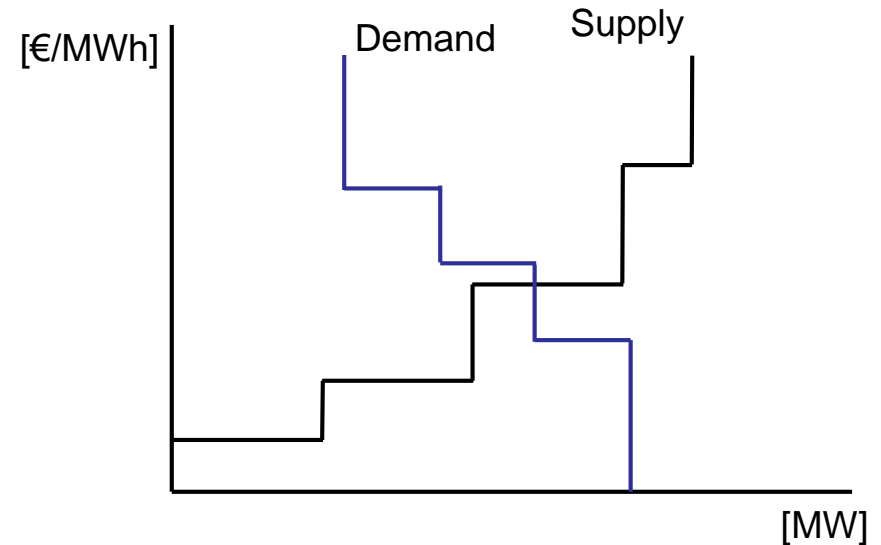
# Implementing DSM

Load shedding at different price levels (e.g. for different consumers)  
converts the constant demand curve to a step function

**Constant demand**



**Demand curve including DSM**



# Intertemporal dynamics

---

Intertemporal dynamics - a decision made at one time step has an effect on the optimal decisions in other time steps

- ♦ Q: What are the causes of intertemporal dynamics in electricity markets?
  - Energy storages
  - Investment decisions
  - Start-up constraints
    - Partial-load costs
    - Start-up costs



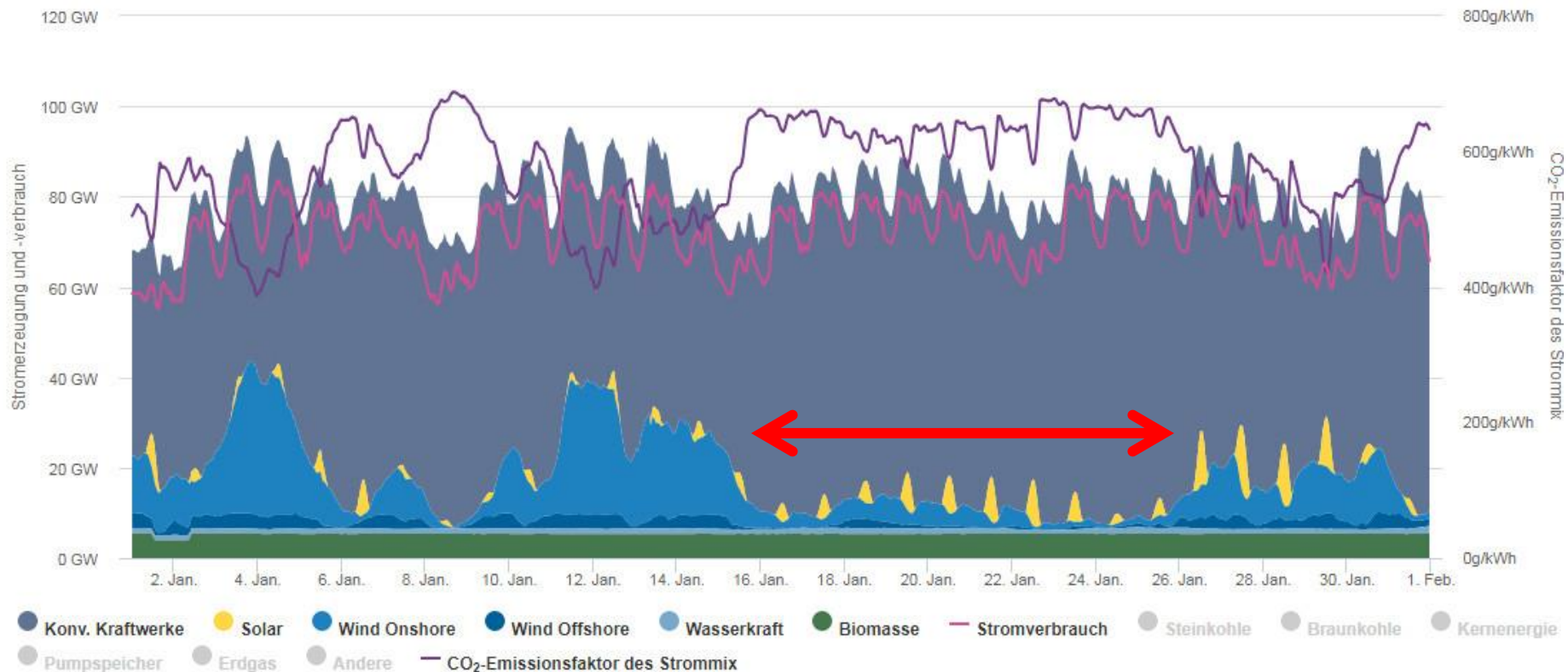
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“Dunkelflaute”: German term for the co-occurrence of ‘Dunkelheit’ (darkness) and ‘Windflaute’ (windlessness)



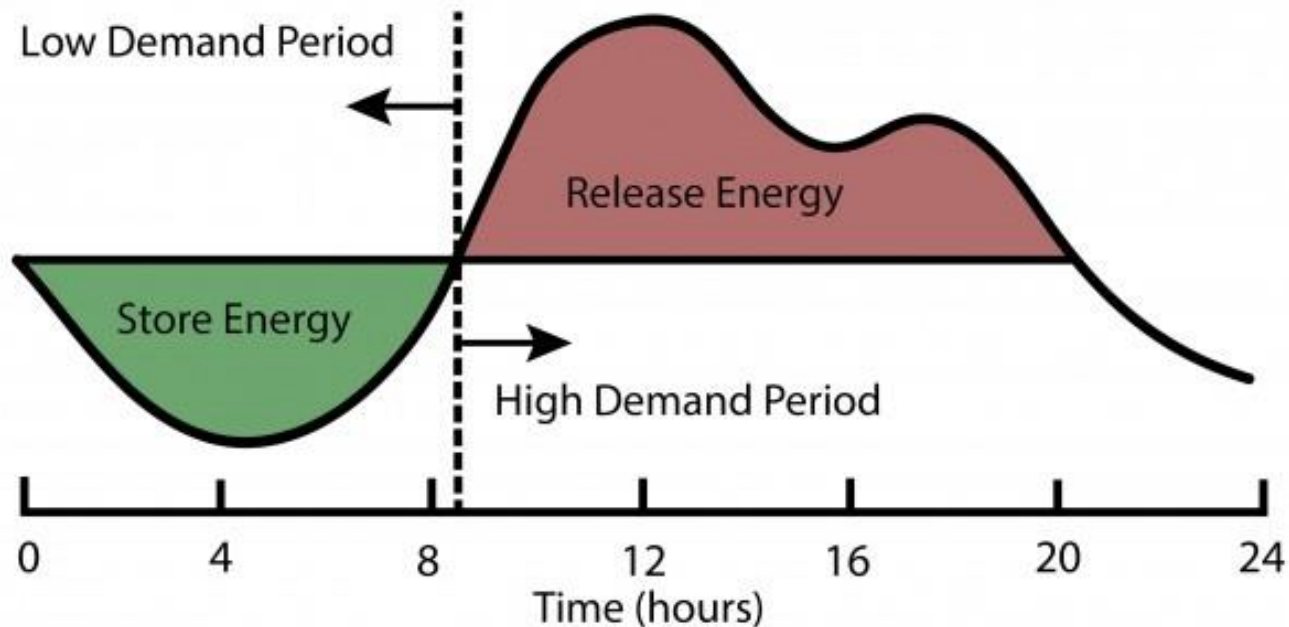
January 2017, 10 days of “Dunkelflaute”

source: Agora Energiewende

[https://www.agora-energiewende.de/service/agorameter/chart/power\\_generation/01.01.2017/31.01.2017/](https://www.agora-energiewende.de/service/agorameter/chart/power_generation/01.01.2017/31.01.2017/)

# Illustration of storage activity

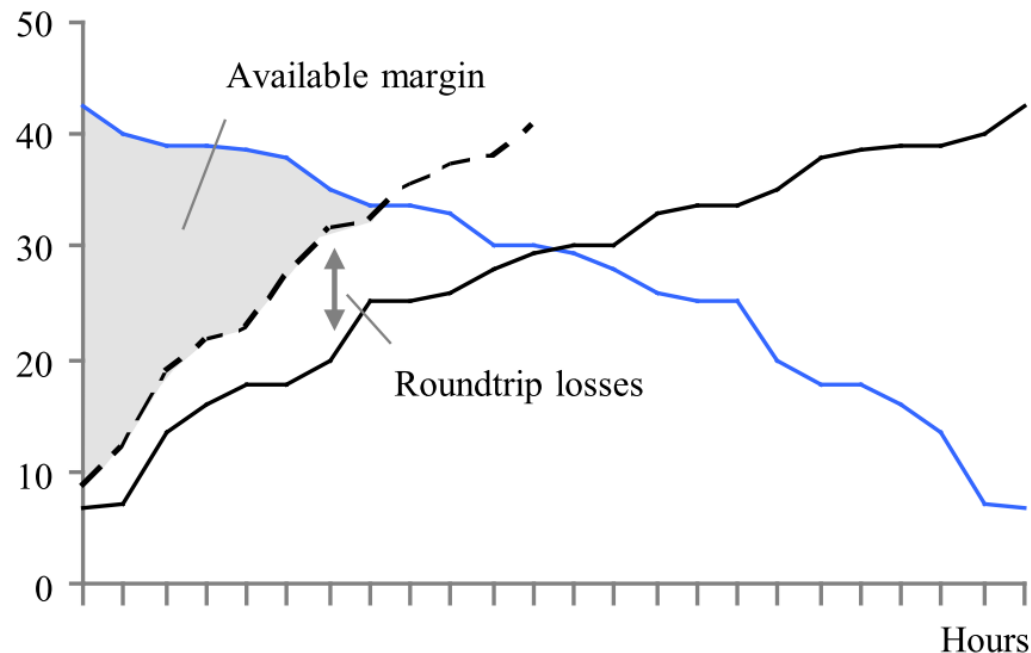
- Storing energy during low demand (off-peak periods) and using that energy during high demand (on-peak periods)



Source: U.S. Grid energy storage factsheet  
<http://css.umich.edu/factsheets/us-grid-energy-storage-factsheet>

# Illustration of time spread margin

Price duration curves EPEX Spot, 01/08/2010  
€/MWh



- (a) Hourly prices in decreasing order = **generation revenues**
- (b) Hourly prices in increasing order
- - (c) Prices (b) after 20% roundtrip losses = **pumping costs**

Discussion: What are the other revenue sources (in addition to energy arbitrage at a spot market) for energy storage?

Source: Steffen, Bjarne, Prospects for Pumped-Hydro Storage in Germany (December 8, 2011). EWL Working Paper No. 07/2011.

<http://dx.doi.org/10.2139/ssrn.1969767>

# Implementing energy storage

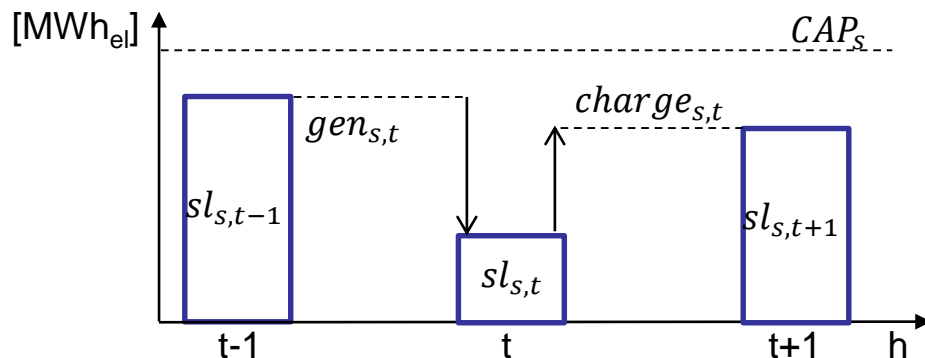
## Storages – mathematical description on the electricity market

Defining a storage level of a storage  $s$  at time  $t$ :

$$sl_{s,t} \leq sl_{s,t-1} + charge_{s,t} - gen_{s,t} \quad \forall s, t$$

Defining maximum storage capacity:

$$sl_{s,t} \leq CAP_s \quad \forall s, t$$



*New index:*

$s$  index for storage technologies

*New parameters:*

$CAP_s$  storage capacity [MWh]

$POWER\_X_s$  generation/charging power [MW]

*New variables:*

$sl_{s,t}$  storage level [MWh]

$charge_{s,t}$  charging storage [MW]

$gen_{s,t}$  generation by storage [MW]

# Implementing energy storage

## Storages – mathematical description on the electricity market

Constraint for the charging/generation power:

$$charge_{s,t} \leq POWER\_PUMP_s \quad \forall s, t$$

$$gen_{s,t} \leq POWER\_TURB_s \quad \forall s, t$$

Constraint for the generation quantity:

$$gen_{s,t} \leq sl_{s,t} \quad \forall s, t$$

Non-negativity constraints:

$$0 \leq sl_{s,t}, gen_{s,t}, charge_{s,t} \quad \forall t$$

*New index:*

$s$  index for storage technologies

*New parameters:*

$CAP_s$  storage capacity [MWh]

$POWER\_X_s$  generation/charging power [MW]

*New variables:*

$sl_{s,t}$  storage level [MWh]

$charge_{s,t}$  charging storage [MW]

$gen_{s,t}$  generation by storage [MW]

# Implementing energy storage

## Storages – mathematical description on the electricity market

### 2) Effects on the energy balance

$$\sum_c g_{c,t} + \sum_r g_{r,t} = DEMAND_t + \sum_s (charge_{s,t} - gen_{s,t} * (1 - LOSS_s)) \quad \forall t$$

Charging the storage increases demand for electricity

Losses in storage cycle  
 (here: LOSS parameter represents the losses of the entire storage cycle)

# Intertemporal dynamics

---

Intertemporal dynamics - a decision made at one time step has an effect on the optimal decisions in other time steps

- ♦ Q: What are the causes of intertemporal dynamics in electricity markets?
  - Energy storages
  - **Investment decisions**
  - Start-up constraints
    - Partial-load costs
    - Start-up costs

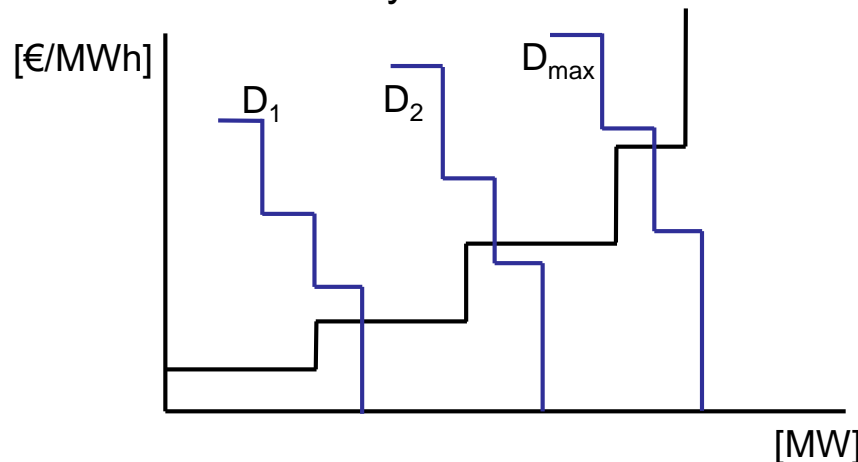


# Implementing investments

## Investments in the electricity market – finding an equilibrium

### 1) Excess of generation capacities

- ♦ The total installed capacity is never fully used, hence, enough capacity to cover demand in each time is available → electricity prices are set by the variable costs of the most expensive unit which is “in the money”



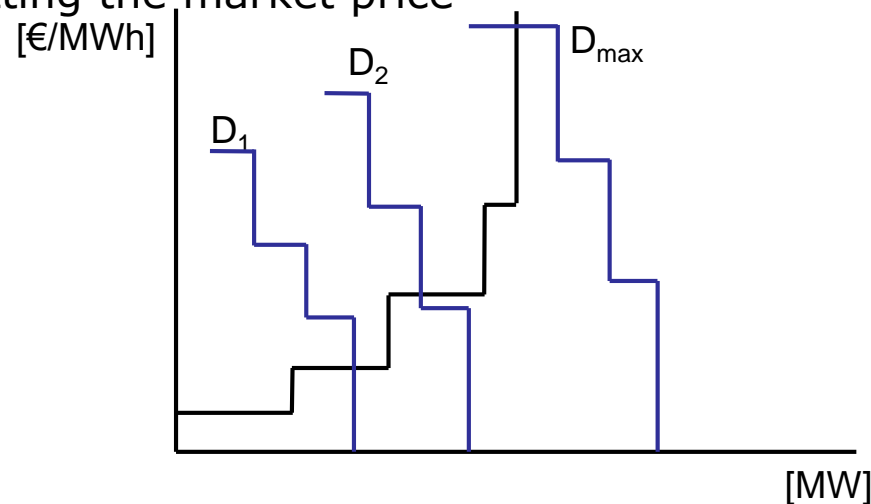
- ♦ Old capacity leaves the market but no incentives to replace it or to invest in further generation capacity (“missing money” problem?)

# Implementing investments

## Investments in the electricity market – finding an equilibrium

### 2) Lack of generation capacity

- ♦ The total installed capacity is not able to cover demand in all time periods → a scarcity situation and scarcity prices appears where the VOLL of the consumers is setting the market price



- ♦ Incentives to invest is given by the scarcity price

# Implementing investments

## Adjustment to the objective function of the electricity market model

$$\min tc = \sum_{i,t} VC_i * g_{i,t} + \boxed{cap\_inv_i} * \boxed{INV\_C_i}$$

The installed capacity of technologies (i) are now decision variables

Investment costs (usually annualized) for technology i

# Intertemporal dynamics

Intertemporal dynamics - a decision made at one time step has an effect on the optimal decisions in other time steps

- ♦ Q: What are the causes of intertemporal dynamics in electricity markets?
  - Energy storages
  - Investment decisions
  - **Start-up constraints**
    - **Partial-load costs**
    - **Start-up costs**

# Implementing start-up constraints

- ♦ Generation capacity has to be started up to produce electricity
- ♦ Starting up capacity causes costs
  - fuel is consumed to heat power plant without electricity input
  - increased attrition due to temperature changes
- ♦ Running capacities have to produce between minimum and maximum load level
- ♦ Furthermore, operating power plants below optimal load levels (usually around max generation) causes efficiency losses (i.e. operating at higher variable costs)

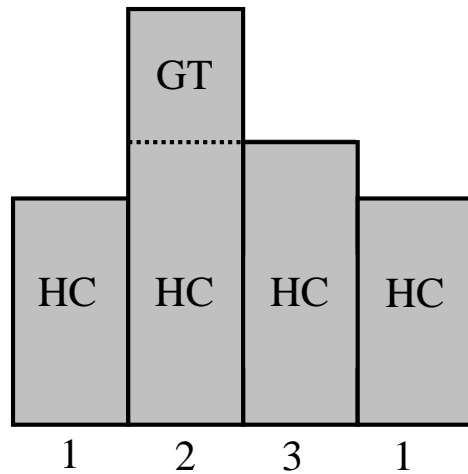
# Implementing start-up constraints: theoretical background [PSE2 lecture]

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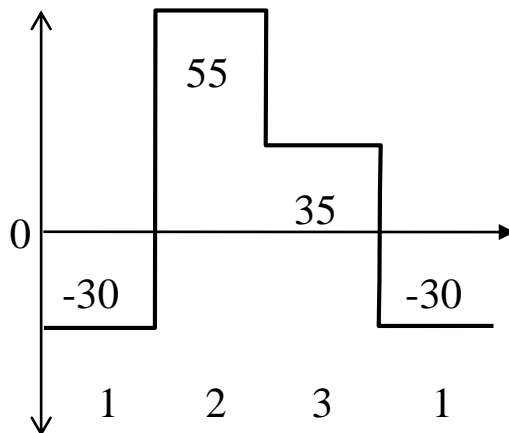
- ♦ Let's look at a market with three load-structures which repeat periodically
- ♦ We ,linearize' the problem and assume a very small (infinitesimal) capacity of the power plant.  
I.e. each capacity that has to be started up entails start-up costs which have to be covered

## Load and dispatch of p.p.

4)

Marginal costs of the system  
[€/MW]

5)

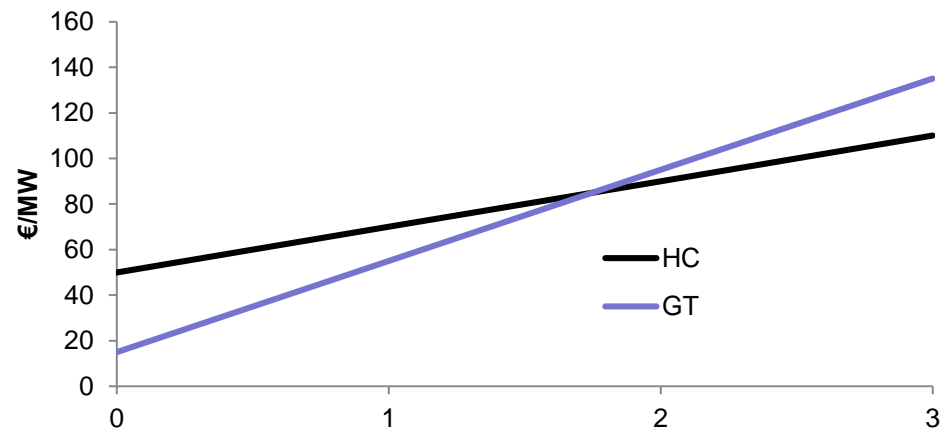


## Start-up- and generation costs

2)

	Start-up costs [€/MW]	Variable generation costs [€/MWh]
Hard coal	50	20
Gas turbine	15	40

3)



	additional costs	savings	
55	= $SC_{GT} + VC_{GT}$		= 15 + 40
35	= $SC_{HC} + 2 \cdot VC_{HC}$	- ( $SC_{GT} + VC_{GT}$ )	= 90 - 55
-30	= $VC_{HC}$	- $SC_{HC}$	= 20 - 50

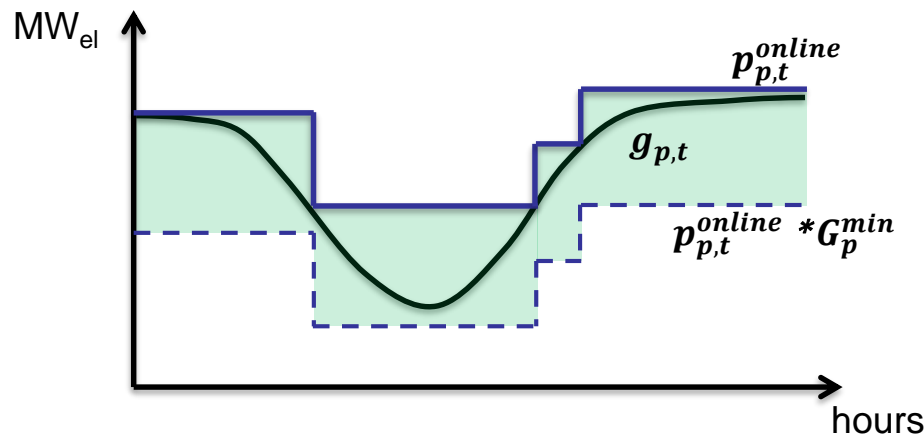
# Implementing start-up constraints: mathematical formulation

- Capacity 'online' is restricted by the installed capacity and its technical availability
- An upper limit for electricity generation
- A lower limit for electricity generation

$$p_{p,t}^{online} \leq CAP_p * AF_{p,t} \quad \forall p, t$$

$$g_{p,t} \leq p_{p,t}^{online} \quad \forall p, t$$

$$p_{p,t}^{online} * G_p^{min} \leq g_{p,t} \quad \forall p, t$$





# Implementing start-up constraints: mathematical formulation

- A start-up activity increases 'online' capacity

$$p_{p,t}^{online} - p_{p,t-1}^{online} \leq su_{p,t} \quad \forall p, t$$

Adjustments in the objective function:

$$\min tc = \sum_{p,t} VC_p^{full\ load} * g_{p,t} + \sum_{p,t} SC_p * su_{p,t} + \sum_{p,t} (p_{p,t}^{online} - g_{p,t}) * (VC_p^{min\ load} - VC_p^{full\ load}) * \frac{g_p^{min}}{(1 - g_p^{min})}$$

Starting up capacity causes start-up costs

Efficiency losses of power plants running at partial load cause higher production costs.

# Energy System Modelling

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- 1) Introduction
- 2) Basic theory
- 3) Linear problems
- 4) Linear integer problems**
  - 1) Introduction to integer programming
  - 2) Application of ILP
- 5) Non-linear problems
- 6) Modelling strategic behavior

# Introduction to integer programming

- ✓ An **integer programming** problem involves some (or all) of the variables that are restricted to be **integers**.

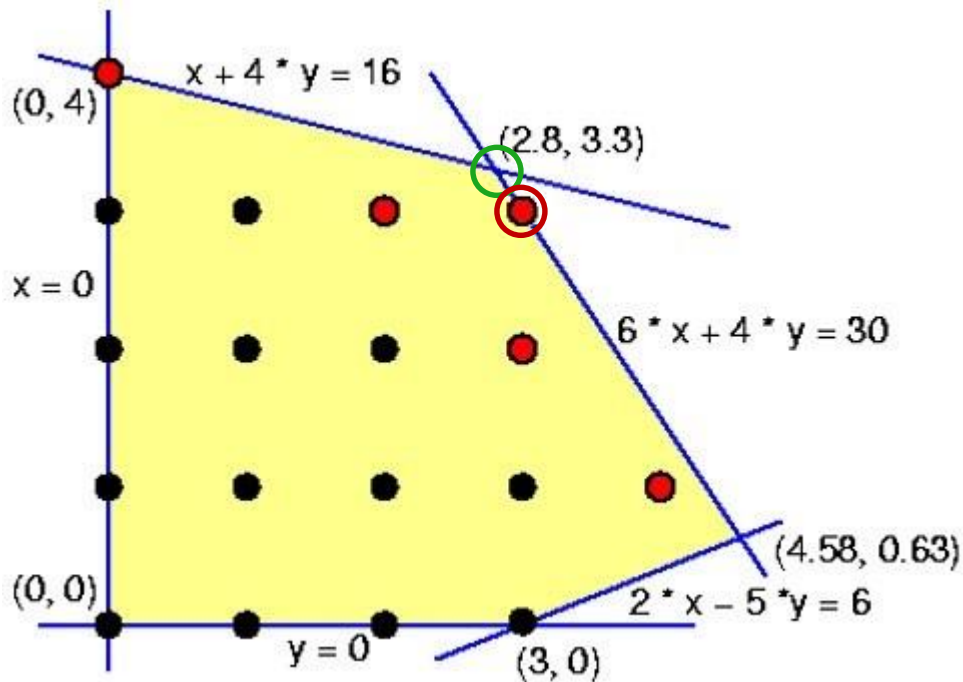
## What are the reasons for using integer variables?

- The integers represent objects/processes that can only be integer (it is not possible to start 2.3 blocks of coal power plant).
- The integer variables may represent decisions (investment decision: "1" for "yes" and "0" for "no").



# Integer programming

Solve the following integer programming example in GAMS (using IP solver) and find solution points that are shown below:



Objective: Maximization of the function  $Z$

$$f(x,y): Z = 6x + 5y$$

s.t.

$$x + 4y \leq 16$$

$$6x + 4y \leq 30$$

$$2x - 5y \leq 6$$

Optimum LP solution:

$$(x, y) = (2.80, 3.30)$$

$$obj = 33.30$$

Optimum ILP solution:

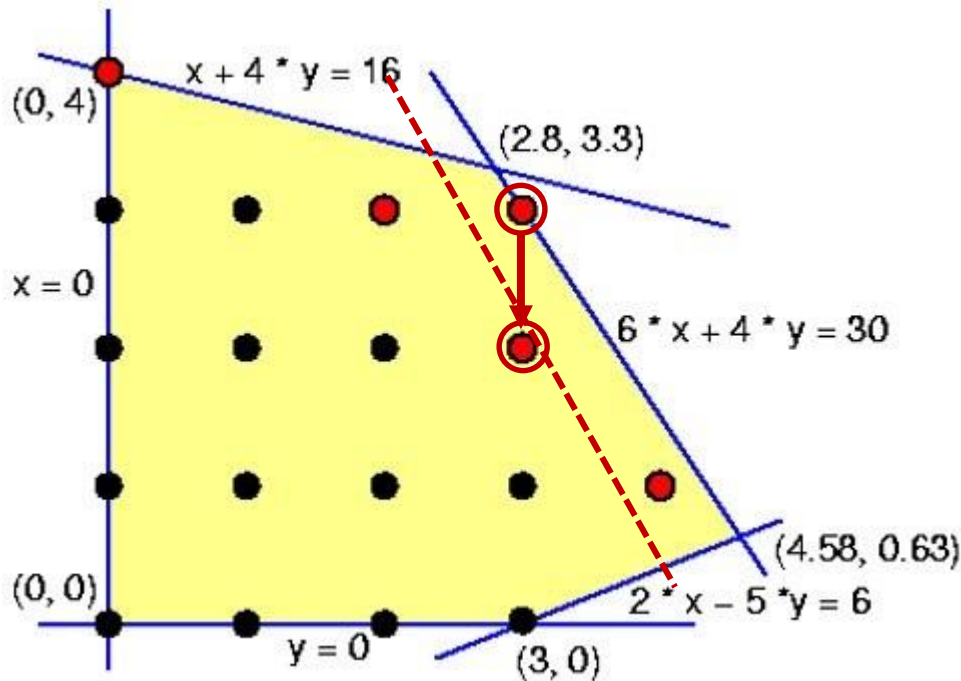
$$(x, y) = (3, 3)$$

$$obj = 33$$

Image adapted from [http://users.informatik.uni-halle.de/~jopsi/drand04/linear\\_programming.gif](http://users.informatik.uni-halle.de/~jopsi/drand04/linear_programming.gif)  
GAMS code for this example will be uploaded to Moodle

# Integer programming

Solve the following integer programming example in GAMS (using IP solver) and find solution points that are shown below:



Objective: Maximization of the function  $Z$

$$f(x,y): Z = 6x + 5y$$

s.t.

$$\begin{aligned} x + 4y &\leq 16 \\ 6x + 4y &\leq 30 \\ 2x - 5y &\leq 6 \end{aligned}$$

Optimum LP solution:

$$\begin{aligned} (x, y) &= (2.80, 3.30) \\ \text{obj} &= 33.30 \end{aligned}$$

Optimum ILP solution:

$$\begin{aligned} (x, y) &= (3, 3) \\ \text{obj} &= 33 \end{aligned}$$

s.t.

$$\begin{aligned} x + 4y &\leq 16 \\ 7x + 4y &\leq 30 \\ 2x - 5y &\leq 6 \end{aligned}$$

Optimum LP solution:

$$\begin{aligned} (x, y) &= (2.33, 3.42) \\ \text{obj} &= 31.08 \end{aligned}$$

Optimum ILP solution:

$$\begin{aligned} (x, y) &= (3, 2) \\ \text{obj} &= 28 \end{aligned}$$



# Implementing start-up constraints with integer variables

- ♦ The single MW of capacity is physically not independent from other units
- ♦ Start-up activities can be seen as a binary decisions (0 or 1)
  - Starting up a single power plant unit (block): YES  $\rightarrow 1$
  - Starting up a single power plant unit (block): NO  $\rightarrow 0$

Hence, start-up decisions  $su_{i,t}$  are expressed as a binary variables:

$$su_{p,t} \in \{0, 1\}$$

- ♦ Furthermore, it is necessary to implement a variable  $down_{i,t}$  that defines the shutdown decision of a power plant block

Shutdown decisions:

$$down_{p,t} \in \{0, 1\}$$

# Implementing start-up constraints with integer variables

- Adjustments in the objective function

$$\min tc = \sum_{p,t} VC_p * g_{p,t} + \sum_{p,t} SC_p * su_{p,t} * CAP_p * G_i^{min}$$

$su_{p,t}$  is the start-up decision for a power plant block which has the capacity  $CAP_p$ .

- Adjustments of the constraint defining the running capacity

$$p_{p,t}^{online} - p_{p,t-1}^{online} = su_{p,t} * CAP_p - down_{p,t} * CAP_p \quad \forall p, t$$

Generation unit is started up

In case of a shutdown  $DOWN_{p,t}$  the full capacity  $cap_p$  has to shut down