Statement Calculus & Predicate Calcul

* Connectivities.
* Truth tables, & Tautology & Contradiction (7M)
* Well formed formulaes (WFF), Duality law.
* Without using truth tables. } 7M
* Normal forms.

1. PDNF
2. PCNF

* Theory of Inference (7M)
    - Type-1
    - Type-2
    - Type-3
    - Type-4
    - Type-5.
* Quantifiers (7M).
    - Type-1
    - Type-2

1. Name: Negation.

Representation:- $\sim, \neg$ (not)

Truth table:-

p	$\sim p$
T	F
F	T

Ex:-

p: Ramu is a good boy.

$\sim p$: Ramu is not a good boy.

Note:-

$\sim(\sim p) \Leftrightarrow p$.

2. Name:- Conjuction.

Representation:- $\wedge$ (and, meet).

Truth table:-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex:-

p: Ramu is a good boy

q: Ravi is a bad boy.

$p \wedge q$:- Ramu is a good boy and Ravi is a bad boy.

3. Name:

Repres

Truth

p: Ra

q: Ra

$p \vee q$: Ra

bo

4. Na

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Tru

p:

q:

p_

3. Name: Disjunction.

Representation: V (or, join).

Truth table:-

p	q	pVq
T	T	T
T	F	T
F	T	T
F	F	F

p: Ramu is good boy

q: Ravi is bad boy.

pVq: Ramu is good boy or Ravi is bad boy.

4. Name:- Conditional.

Representation:- $\rightarrow$, $\Rightarrow$ (If, then).

Truth table:-

p	q	$p\rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p: Raju is studying well.

q: Raju will pass the exam.

$p\rightarrow q$: If Raju is studying well then Raju will pass the exam.

Ravi

5. Name: Bi-conditional.

Representation:- $\leftrightarrow, \Leftrightarrow$ (If and only if).

Truth Table:-

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p: Raju is studying well.

q: Raju will pass the exam.

$p \leftrightarrow q$: Raju is studying well if and only if Raju will pass the exam.

Construction of Truth tables:-

1. $\sim(p \wedge q) \leftrightarrow \sim p \vee \sim q$.
2. $\{q \wedge (p \rightarrow q)\} \rightarrow p$
3. $\sim[p \vee (q \wedge r)] \leftrightarrow (p \wedge q) \wedge (p \vee r)$.

3.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$\sim(p \vee (q \wedge r))$	$p \wedge q$	$p \vee r$	$(p \wedge q) \wedge (p \vee r)$	$A \leftrightarrow B$
T	T	T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	T	T	F
T	F	T	F	T	F	F	T	F	T
T	F	F	F	T	F	F	T	F	T
F	T	T	T	T	F	F	T	F	T
F	T	F	F	F	T	F	F	F	F
F	F	T	F	F	T	F	T	F	F
F	F	F	F	F	T	F	F	F	F

2.

p	q	$p \to q$	$q \wedge (p \to q)$	$q \wedge (p \to q) \to p$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

1.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(p \wedge q) \leftrightarrow \sim p \vee \sim q$.
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Tauto

The

state

value

Taut

Q: Show

is a

$A \leftrightarrow B$

$(P \to Q) \vee (P \to R)$

Tautology:-

The final column of the given statement formula of the truth values is always true is called Tautology.

Q: Show that $[P \to (Q \vee R)] \leftrightarrow (P \to Q) \vee (P \to R)$ is a tautology.

P	Q	R	$Q \vee R$	$P \to (Q \vee R)$ (A)	$P \to Q$	$P \to R$	$(P \to Q) \vee (P \to R)$ (B)	$A \leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

2. $[(p \to q) \to r] \to [(p \to q) \to (p \to r)]$

p	q	r	$p \to q$	$\overset{A}{(p \to q) \to r}$	$p \to q$	$p \to r$	$\overset{B}{(p \to q) \to (p \to r)}$	$A \to B$.
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F	T
T	F	T	F	T	F	T	T	T
T	F	F	F	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	F	T	T	T	T

$[(P \wedge \sim Q) \to R] \to$

P	Q	R	$\sim Q$	$P \wedge \sim Q$	$\overset{A}{(P \wedge \sim Q) \to R}$	$Q \vee R$	$\overset{B}{P \to (Q \vee R)}$	$A \to B$
T	T	T	F	F	T	T	T	T

$[(P \wedge \sim Q) \to R] \to [P \to (Q \vee R)]$

P	Q	R	$\sim Q$	$P \wedge \sim Q$	$\overset{A}{(P \wedge \sim Q) \to R}$	$Q \vee R$	$\overset{B}{P \to (Q \vee R)}$	$A \to B$
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	F	F	T
F	T	T	F	F	T	T	T	T
F	T	F	F	F	T	T	T	T
F	F	T	T	F	T	T	T	T
F	F	F	T	F	T	F	T	T

$[\{(p \vee q) \wedge (p \to r)\} \wedge [(q \to r)]] \to r\}$

p	q	r	$p \vee q$	$p \to r$	$\overset{A}{(p \vee q) \wedge (p \to r)}$	$q \to r$	$\overset{B}{A \wedge (q \to r)}$	$B \to r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

Establish that $\sim\{[((p \vee q) \to r)] \wedge (\sim p)] \to (q \to r)]$

p q r $p \vee q$ $p \vee q \to r$ $\sim p$ $\overset{A}{(p \vee q \to r) \wedge \sim p}$ $\overset{B}{q \to r}$ $\overset{C}{A \to B}$ $\sim C$

Contradiction

The final

statement

Values is a

Contradicti

Establish t

Contradiction:-

The final column of the given statement formula of the truth values is always False is called Contradiction.

Establish that

Establish that $\sim[((p \vee q) \rightarrow r)) \overset{A}{\wedge} (\sim p)] \overset{B}{\rightarrow} (q \rightarrow r)$

p	q	r	$p \vee q$	$p \vee q \rightarrow r$	$\sim p$	$(p \vee q \rightarrow r) \wedge \sim p$	$q \rightarrow r$	$A \overset{C}{\rightarrow} B$	$\sim C$
T	T	T	T	T	F	F	T	T	F
T	T	F	T	F	F	F	F	T	F
T	F	T	T	T	F	F	T	T	F
T	F	F	T	F	F	F	T	T	F
F	T	T	T	T	T	T	T	T	F
F	T	F	T	F	T	F	F	T	F
F	F	T	F	T	T	T	T	T	F
F	F	F	F	T	T	T	T	T	F

Well Formed Formula (WFF):-

1. A statement variable standing alone is well formed formula.
2. If A is well formed formula then negation A is also well formed formula.
3. If A,B are 2 well formed formulas then $(A \wedge B)$, $(A \vee B)$, $(A \to B)$, $(A \leftrightarrow B)$ are also well formed formulas.
4. A string of symbols containing statement variables, connectivities, parenthesis is well formed formulas and the finite applications of Rules 1,2,3

Ex: $(p \wedge q)$ ✓ $\quad [(p \vee q) \to r]$ ✓

$p \wedge q$ ✗ $\quad (p \vee q) \to r$ ✗

Duality Law:-

Let X and X' are two statement formulas are said to be dual to each other if one is obtained from another by replacing $\wedge$ (and) by $\vee$ (or) T(true) by F (False)

Find the Duality of the Following.

1. $(P \vee Q) \wedge R : (P \wedge Q) \vee R$.
2. $(P \wedge Q) \vee T = (P \vee Q) \wedge F$.

Laws of Logic

1. Idempotent

$P \wedge P \Leftrightarrow P$

$P \vee P \Leftrightarrow P$

3. Associative

$P \wedge (q \wedge r) \Leftrightarrow (\ldots$

$P \vee (q \vee r) \Leftrightarrow$

5. Dominatio

$P \vee T \Leftrightarrow T$

$P \vee F \Leftrightarrow F$

7. Demorga

$\sim(P \vee q) \Leftrightarrow$

$\sim(P \wedge q) \Leftrightarrow$

8. Distribu

$P \wedge (q \vee r)$

$P \vee (q \wedge r)$

9. Double

$\sim(\sim P) \Leftrightarrow$

10. Absorpt

$P \vee (P \wedge q$

$P \wedge (P \vee$

Note:-

* $P \to$
* $P \leftrightarrow q$

Laws of Logic:

1. Idempotent Law.

$p \wedge p \Leftrightarrow p$

$p \vee p \Leftrightarrow p$

2. Commutative Law

$p \wedge q \Leftrightarrow q \wedge p$

$p \vee q \Leftrightarrow q \vee p$

3. Associative Law.

$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$

4. Identity Law:

$p \wedge T \Leftrightarrow p$

$p \vee F \Leftrightarrow p$

5. Domination Law:

$p \vee T \Leftrightarrow T$

$p \vee F \Leftrightarrow F$

6. Negation Law:

$p \wedge \sim p \Leftrightarrow F$

$p \vee \sim p \Leftrightarrow T$

7. Demorgans Law:

$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

8. Distributive Law:

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

9. Double Negation Law:

$\sim(\sim p) \Leftrightarrow p$

10. Absorptive Law:

$p \vee (p \wedge q) \Leftrightarrow p$

$p \wedge (p \vee q) \Leftrightarrow p$

Note:

* $p \rightarrow q \Leftrightarrow \sim p \vee q$

* $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

$(p \wedge q) \vee (\sim p \wedge \sim q)$.

$p \to q$

Converse: $q \to p$

Contrapositive: $\sim q \to \sim p$

Inverse: $\sim p \to \sim q$.

1. Show that $p \to (q \to r) \leftrightarrow p \to (q \vee r) \Leftrightarrow (p \wedge q) \to r$

Rules:-

* To eliminate $\to, \leftrightarrow$ by using relevant formulas.
* To eliminate $\sim$ before the statement formulas.
* By using De morgan's laws & Distributed laws repeatedly.

1. Sol. $p \to (q \to r) \Leftrightarrow p \to (\sim q \vee r)$

$$\Leftrightarrow \sim p \vee (\sim q \vee r)$$

$$(\sim p \vee \sim q) \vee r$$

$$\sim (p \wedge q) \vee r$$

$$(p \wedge q) \to r.$$

Show that $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$.

$= ((\sim p \wedge \sim q) \wedge r) \vee ((q \vee p) \wedge r)$ (associative & distributive law)

$= (\sim (p \vee q) \wedge r) \vee [(p \vee q) \wedge r]$ (Demorgans law & commutative law)

$= [\sim (p \vee q) \vee (p \vee q)] \wedge r$. (Distributive law)

$= T \wedge r$ (Negation Law)

$= r$ (Identity law).

Show that $\sim (p \vee (\sim p \wedge q))$ and $(\sim p \wedge \sim q)$ are logically equivalent:-

$\sim (p \vee (\sim p \wedge q)) = \sim p \wedge \sim (\sim p \wedge q)$ [Demorgans)

$= \sim p \wedge (p \vee q)$ (Demorgans & Double negation)

$= (\sim p \wedge q) \vee (\sim p \wedge q)$ (Distributive)

$= F \vee (\sim p \wedge \sim q)$ (Negation law)

$= (\sim p \wedge \sim q)$ (Identity law)

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

$(p \wedge q) \rightarrow (p \vee q) =$

$= \sim (p \wedge q) \vee (p \vee q)$

$= (\sim p \vee \sim q) \vee (p \vee q)$ (Demorgans Law).

$= (\sim p \vee p) \vee (q \vee q)$ (Negation law).

$= T \vee T$ (Idempotent)

$= T$.

Show that $(p\to q)\to q \Leftrightarrow p\vee q$

$(p\to q)\to q = \sim(p\to q)\vee q$.

$= \sim(\sim p\vee q)\vee q$.

$= (p\wedge \sim q)\vee q$ (Demorgans)

$= \cancel{p\wedge q\,(p\vee q)\wedge}$

$= (p\vee q)\wedge(\sim q\vee q)$ (Distributiv

$= (p\vee q)\wedge T$ (Identity law)

$= (p\vee q)$.

Show that $[(\overset{A}{p\vee q})\wedge\sim(\sim p\wedge(\overset{B}{\sim q\vee\sim r}))]\vee[(\underset{C}{\sim p\wedge\sim q})\vee(\underset{D}{\sim p\wedge\sim r})]$ is a tautology.

$\sim(\sim p\wedge(\sim q\vee\sim r)) = p\vee(q\wedge r)$ (Demorgans)

$= (p\vee q)\wedge(p\vee r)$ (Distributiv

$(\sim p\wedge\sim q)\vee(\sim p\wedge\sim r) = \sim[(p\vee q)\wedge(p\vee r)]$

$(A\wedge B)\vee(C\vee D) = [(p\vee q)\wedge(p\vee q)\wedge(p\vee r)]\vee$

$\sim[(p\vee q)\wedge(p\vee r)]$

$= [(p\vee q)\wedge(p\vee r)]\vee\sim[(p\vee q)\wedge(p\vee r)]$

$= p\vee\sim p$

$= T$

(H.W) Show

$(q\vee p)$.

Norma

Ele

p, q

(H.W) Show that $(p \to q) \wedge (\sim q \wedge (r \vee \sim q)) \Leftrightarrow (q \vee p)$.

Normal Forms:-

$$\text{Normal Forms} \begin{cases} \text{DNF} \to \text{PDNF (T)} \\ \text{CNF} \to \text{PCNF (F)} \end{cases}$$

Elementary Products ($\wedge$)   Min terms.

Elementary Products: p, q	Elementary Products: p, q, r	Min terms: p, q	Min terms: p, q, r
$p \wedge q$	$p \wedge q \wedge r$	$p \wedge q$	$p \wedge q \wedge r$
$p \wedge \sim q$	$\sim p \wedge q \wedge r$	$p \wedge \sim q$	$\sim p \wedge q \wedge r$
$\sim p \wedge q$	$p \wedge \sim q \wedge r$	$\sim p \wedge q$	$p \wedge \sim q \wedge r$
$\sim p \wedge \sim q$	$p \wedge q \wedge \sim r$	$\sim p \wedge \sim q$	$p \wedge q \wedge \sim r$
$p \wedge \sim p$	$\sim p \wedge \sim q \wedge r$		$\sim p \wedge \sim q \wedge r$
$q \wedge \sim q$	$p \wedge \sim q \wedge \sim r$		$p \wedge \sim q \wedge \sim r$
	$\sim p \wedge q \wedge \sim r$		$\sim p \wedge q \wedge \sim r$
	$\sim p \wedge \sim q \wedge \sim r$		$\sim p \wedge \sim q \wedge \sim r$

PDNF:- (Principle Disjunctive Normal form) :- or (Sum of Product Canonical form):-

A formula which is equivalent to given formula consists of Disjunction of minterms only is called PDNF.

* Obtain the PDNF of $\sim p \vee q$

$\sim p \vee q \Leftrightarrow (\sim p \wedge T) \vee (q \wedge T)$ [Identity Law]

$\Leftrightarrow (\sim p \wedge (q \vee \sim q)) \vee (q \wedge (p \vee \sim p))$ [Negation Law]

$\Leftrightarrow (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q) \vee (\sim p \wedge q)$ [Distributive]

$\Leftrightarrow (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ [commutative, Idempotent]

Which is required PDNF.

Truth Table Method:-

$\sim p \vee q \Leftrightarrow$

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

The PDNF of given statement formula is $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$.

Find the PDNF of $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$

$\Leftrightarrow [(p \wedge q) \wedge T] \vee [(\sim p \wedge r) \wedge T] \vee [(q \wedge r) \wedge (T)]$

$= [(p \wedge q) \wedge (r \vee \sim r)] \vee [(\sim p \wedge r) \wedge (q \vee \sim q)] \vee [(q \wedge r) \wedge [p \vee \sim p)]$

$= [(p\wedge q\wedge r)\vee(p\wedge q\wedge \sim r)\vee(\sim p\wedge r\wedge q)\vee[\sim p\wedge r\wedge \sim q$
$\vee(q\wedge r\wedge p)\vee(q\wedge r\wedge \sim p)]$

$= (p\wedge q\wedge r)\vee(p\wedge q\wedge \sim r)\vee(\sim p\wedge r\wedge q)\vee(\sim p\wedge r\wedge \sim q)$

Truth table Method:-

p q r p∧q ~p∧r

p	q	r	~p	~q	~r	p∧q	~p∧r	q∧r
T	T	T	F	F	F	T	F	T
T	T	F	F	F	T	T	F	F
T	F	T	F	T	F	F	F	F
T	F	F	F	T	T	F	F	F
F	T	T	T	F	F	F	T	T
F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	T	F
F	F	F	T	T	T	F	F	F

$(p\wedge q)\overset{A}{\vee}(\sim p\wedge q)$	$A\vee(q\wedge r)$
T	T
T	T
F	F
F	F
T	T
F	F
T	T
F	F

$(p\wedge q\wedge r)\vee(p\wedge q\wedge \sim r)\vee(\sim p\wedge q\wedge r)\vee(\sim p\wedge \sim q\wedge r)$

Find the Principle Disjunctive Normal form of $P \to [(P \to Q) \wedge \sim(\sim Q \vee \sim P)]$ By constructing truth tables.

$P \to [(\sim P \vee Q) \wedge \sim(\sim Q \vee \sim P)]$

$P \to [(\sim P \vee Q) \wedge (Q \wedge P)]$

$\sim P \vee [(\sim P \vee Q) \wedge (Q \wedge P)]$

$(\sim P \wedge T) \vee [(\sim P \vee Q) \wedge (Q \wedge P)]$

$(\sim P \wedge (Q \vee \sim Q)) \vee [(\sim P \vee Q) \wedge (Q \wedge P)]$

$[(\sim P \wedge Q) \vee (\sim P \wedge \sim Q)] \vee [(\sim P \vee Q) \wedge (Q \wedge P)]$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee [(\sim P \vee Q \wedge P) \wedge (Q \wedge P)]$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee [(F \wedge Q) \wedge (Q \wedge P)]$

$(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$.

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P	Q	$\sim P$	$\sim Q$	$P \to Q$	$\sim Q \vee \sim P$	$\sim(Q \vee \sim P)$ $\cancel{(P \to Q)}$	B $(P \to Q) \wedge \sim(\sim Q \vee \sim P)$	$P \to B$.
T	T	F	F	T	F	T	T	T
T	F	F	T	F	T	F	F	F
F	T	T	F	T	T	F	F	T
F	F	T	T	T	T	F	F	T

$(P \wedge Q) \vee (\sim P \wedge Q) \vee (\sim P \wedge \sim Q)$.

PCNF:- Principle Conjuctive Normal Form & Product of sum Canonical form:-

A Formula which is equivalent to given formula and consists of Product of Max terms is called PCNF.

$(\vee)\wedge(\vee)$.

Find the PCNF of $\sim(p \overset{\vee}{\wedge} q)$

$= \sim p \wedge \sim q = (\sim p \vee F) \wedge (\sim q \vee F)$

$= (\sim p \vee (q \wedge \sim q)) \wedge (\sim q \vee [p \wedge \sim p])$

$= [(\sim p \vee q) \wedge (\sim p \vee \sim q)] \wedge [(\sim q \vee p) \wedge (\sim q \vee \sim p)]$

$= (\sim p \vee q) \wedge (\sim p \vee \sim q) \wedge (p \vee \sim q)$.

Which is required PCNF.

Truth Table Method:-

p	q	$p \vee q$	$\sim(p \vee q)$	
T	T	T	F ✓	$\sim(p \wedge q) = \sim p \vee \sim q$
T	F	T	F ✓	$\sim(p \wedge \sim q) = \sim p \vee q$
F	T	T	F ✓	$\sim(\sim p \wedge q) = p \vee \sim q$
F	F	F	T	

$(\sim p \vee \sim q) \wedge (\sim p \vee q) \wedge (p \vee \sim q)$.

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Theory of Inference:-

The word Inference means we can derive new statement from old statement.

The inference is of the form

Conjuction of premises $\longrightarrow$ conclusion.

The Inference is said to be a valid then it is tautology. Otherwise it is invalid inference or faulty inference.

$$\frac{\text{premisis}}{\therefore \text{Conclusion.}}$$

Rules of Inference:-

Rule of Inference	Tatological form	Name

Rule Of Inference	Tautological form	Name.
1. $\dfrac{p}{p \vee q}$	$p \rightarrow p \vee q$	Addition
2. $\dfrac{p \wedge q}{p}$	$p \wedge q \rightarrow p$	Simplification
3. $\dfrac{\begin{matrix} p \\ p \rightarrow q \end{matrix}}{q}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus Poners
4. $\dfrac{\begin{matrix} \sim q \\ p \rightarrow q \end{matrix}}{\sim p}$	$[\sim q \wedge (p \rightarrow q)] \rightarrow \sim p$	Modus Tollens.
5. $\dfrac{\begin{matrix} p \rightarrow q \\ q \rightarrow r \end{matrix}}{p \rightarrow r}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow p \rightarrow r$	Hypothetical sylloism or Transistive Rule
6. $\dfrac{\begin{matrix} p \\ q \end{matrix}}{p \wedge q}$	$(p \wedge q) \rightarrow (p \wedge q)$	Conjunction.
7. $\dfrac{\begin{matrix} p \vee q \\ \sim p \end{matrix}}{q}$	$(p \vee q) \wedge \sim p \rightarrow q$	Disjunctive Sylloism.

Type - 1:-

1. Demonstrate that r is a valid inference from the premisis $p \rightarrow q$, $q \rightarrow r$ and p

Name.
Addition

Simplifica-tion

Modus Ponens

Modus Tollens.

Hypothetical syllogism or transistive Rule

Conjunction.

Disjunctive syllogism.

a valid premisis

{1} (1) $p \to q$ Rule P

{2} (2) $q \to r$ Rule P

{1,2} (3) $p \to r$ T.R

{4} (4) p Rule P

{1,2,4} (5) r Modus P

2. Show that $\sim q, p \to q \Rightarrow \sim p$

The given premesis are $\sim q, p \to q$

Conclusion is $\sim p$.

{1} (1) $\sim q$ Rule P

{2} (2) $p \to q$ Rule P

{1,2} (3) $\sim p$ Modus Tollens

3. Show that $r \vee s$ follows logically from the premises $(c \vee d), (c \vee d) \to \sim h$, $\sim h \to (a \wedge \sim b)$ and $(a \wedge \sim b) \to (r \vee s)$

{1} (1) $c \vee d$ Rule P

{2} (2) $(c \vee d) \to \sim h$ Rule P

{1,2} (3) $\sim h$ Modus Ponens.

{4} (4) $\sim h \to (a \wedge \sim b)$ Rule P

{1,2,4} (5) $(a \wedge \sim b)$ Modus Ponens

{6} (6) $(a \wedge \sim b) \to (r \vee s)$ Rule P

{1,2,4,6} (7) $q \vee s$ Modern Ponens

show that $s \vee r$ is tautologically implied by $(p \vee q) \wedge (p \to r) \wedge (q \to s)$

$p \vee q \Leftrightarrow \sim p \to q$.

{1} (1) $p \vee q$ Rule P.

{1} (2) $\sim p \to q$ Rule T

{3} (3) $q \to s$ Rule P

{1,3} (4) $\sim p \to s$ Transistive Rule

{1,3} (5) $\sim s \to p$ Contrapositive

{6} (6) $p \to r$ Rule P

{1,3,6} (7) $\sim s \to r$ Transistive Rule

{1,3,6} (8) $s \vee r$ Rule T

Show that $R \wedge (P \vee Q)$ is a valid conclusion for the premisis $P \vee Q$, $Q \to R$, $P \to M$, $\sim M$

{1} (1) $P \vee Q$ Rule P

{1} (2) $\sim P \to Q$ Rule T

{3} (3) $Q \to R$ Rule P.

{1,3} (4) $\sim P \to R$ Transitive Rule

{1,3} (5) $\sim R \to P$ Contrapositive

{6} (6) $P \to M$ Rule P

{1,3,6} (7) $\sim R \to M$ Transitive Rule

{1,3,6} (8) $\sim M$

{9} (9) $\sim$

{1,3,6,9} (10)

{1,3,6,9} (11) $R \wedge$

Type-2:-

Test the

argument:

If sonia i

is not stu

studying t

buy a sc

father wi

Let us

as follow

P: Sonia

Q: sonia

R: Sonia

The prem

The con

{1,3,6}	(8)	$\sim M \rightarrow R$	Contrapositive
{9}	(9)	$\sim M$	Rule P.
{1,3,6,9}	(10)	R	Modus ponens
{1,3,6,9}	(11)	$R \wedge (P \vee Q)$	conjuction.

Type-2:-

Test the validity of following argument: "Sonia is watching TV. If sonia is watching TV then she is not studying. If she is not studying then her father will not buy a scooty. Therefore, sonia's father will not buy a scooty.

Let us indicate the statements as follows.

P: Sonia is watching TV.

Q: sonia is ~~no~~ studying.

R: Sonia's father will buy a scooty

The premises are

$$P$$

$$P \rightarrow \sim Q$$

$$\sim Q \rightarrow \sim R.$$

The conclusion is

$$\sim R$$

{1}	(1)	P	Rule P
{2}	(2)	$P \to \sim Q$	Rule P
{1,2}	(3)	$\sim Q$	Modus ponens
{4}	(4)	$\sim Q \to \sim R$	Rule P.
{1,2,4}	(5)	$\sim P$	Modus ponens

"If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not diffi-cult. They arrived on time. Therefore, There was no ball game." show that these statements constitute a valid argument.

Let us indicate the statement as follows.

P: there was a ball game

Q: travelling was difficult.

R: they arrived on time.

The premises are

$$P \to Q$$
$$R \to \sim Q$$
$$R$$

The Conclusion is $\sim P$

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ument.
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{1}	(1)	$P \to Q$	Rule P.
{2}	(2)	$R \to \sim Q$	Rule P
{2}	(3)	$Q \to \sim R$	contrapositive
{1,2}	(4)	$P \to \sim R$	Mo Transsistive Rule
{1,2}	(5)	$R \to \sim P$	Contrapositive
{6}	(6)	R	Rule P
{1,2,6}	(7)	$\sim P$	Modus ponens.

Type-3:-

Rule CP (conditional proof).

* You can derive S from R set of premises then the conclusion is of the form $R \to S$

* If $P \to Q$ is a conclusion then we take P as additional premisi

Show that $r \to s$ can be derived from the premesis $p \to (q \to s)$, $\sim r \vee p$ and q.

The given premesis are

$$p \to (q \to s)$$
$$\sim r \vee p$$
$$q$$

The conclusion is $r \to s$

{1}	(1)	r	Additional premise Rule P
{2}	(2)	$\sim r \vee p$	Rule T
{2}	(3)	$r \to p$	Modus ponens
{1,2}	(4)	p	Rule P
{5}	(5)	$p \to (q \to s)$	Modus pones
{1,2,5}	(6)	$q \to s$	rule P
{7}	(7)	q	Modus ponens
{1,2,5,7}	(8)	s	rule CP.
{1,2,5,7}	(9)	$r \to s$	

By using the method of derivation show that the following statements consistute a valid argument:

"If A works hard then either B or C will enjoy. If B enjoys, A will not work hard. If D enjoys, then C will not enjoy. therefore, If A works hard then D will not enjoy".

Let us indicate the statements as follows.

P: A works hard

Q: B enjoy

R: C will enjoy

S: D will enjoy

Premises are

$P \to (Q$

~~$Q \to \sim$~~

$Q \to \sim$

$S \to \sim$

The conclusi

{1}	(1)	P
{2}	(2)	$P \to ($
{1,2}	(3)	$Q \vee$
{1,2,4}	(4)	$\sim Q$
{5}	(5)	S
{5}	(6)	
{1,2,5}	(7)	$\sim$
{1,2,3}	(8)	
{9}	(9)	
{1,2,3,9}	(10)	
{1,2,3,9}	(11	

Premises are

$P \to (Q \vee R)$

$\cancel{Q \to \sim RS}$

$Q \to \sim P$

$S \to \sim R$.

The conclusion is :- $P \to \sim S$.

$\{1\}$	(1)	P	Additional premesi
$\{2\}$	(2)	$P \to (Q \vee R)$	Rule P
$\{1,2\}$	(3)	$Q \vee R$	Modes pomenes.
$\{1,2\}$ $\cancel{\{1,2,4\}}$	(4)	$\sim Q \to R$	Rule T
$\{5\}$	(5)	$S \to \sim R$	Rule P
$\{5\}$	(6)	$R \to \sim S$	Contrapositive.
$\{1,2,5\}$	(7)	$\sim Q \to \sim S$	T.R ~~Modus pomemus~~
$\{1,2,3\}$	(8)	$S \to Q$	contrapositive
$\{9\}$	(9)	$Q \to \sim P$	Rule P ~~T.R contrapostive~~
$\{1,2,3,9\}$	(10)	$S \to \sim P$	T.R
$\{1,2,3,9\}$	(11)	$P \to \sim S$	contrapositive

Type-4:- (Indirect Method of Proof).

Show that $\sim(P\wedge q)$ follows from $\sim P\wedge\sim q$

{1}	(1)	$\sim[\sim(P\wedge q)]$	Rule P.
{1}	(2)	$P\wedge q$	Double Negation
{1}	(3)	P	simplification
{1}	(4)	q	"
{5}	(5)	$\sim P\wedge\sim q$	Rule P
{5}	(6)	$\sim P$	simplification
{5}	(7)	$\sim q$	"
{1,5}	(8)	$P\wedge\sim P$	Conjunction
	(9)	F	Negation law

* Our assumption is wrong.
* $\sim(P\wedge q)$ follows from $\sim P\wedge\sim q$.

Using the Indirect Method show that $P\rightarrow q,\ q\rightarrow r,\ \sim(P\wedge r),\ (P\vee r)\Leftrightarrow r$

{1}	(1)	$\sim r$	Rule P (assumed)
{2}	(2)	$q\rightarrow r$	Rule P
{1,2}	(3)	$\sim q$	Modus Tollens
{4}	(4)	$P\rightarrow q$	Rule P
{1,2,4}	(5)	$\sim P$	Modus Tollens.
{6}	(6)	$\sim(P\wedge r)$	Rule P
{6}	(7)	$\sim P\vee\sim r$	Demorgans law

{6} (8) $P\rightarrow$

{6} (9) $\sim P$

{1,2,4,6} (10)

{1,2,4,6} (11) $\sim r$

*Our assu

Type-5:- (co

*Show that

$d\rightarrow(b\wedge\sim c)$,

{1} (1)

{1} (2)

{1} (3)

{4} (4)

{1,4} (5)

{1,4} (6)

{7} (7)

{7} (8)

{7} (9)

{1,4,7} (1

{1,4,7} (

$d\wedge\sim$

wrong

* So c

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ō Proof).
from $\sim p \wedge q$
e P.
ble Negation
nplification
e P
nplification
junction
ation Law
rong..
$\wedge \sim q$.
hod show
$(p \vee r) \Leftrightarrow r$
(assumed)
Tollens
Tollens.
gams Law

{6} (8) $p \rightarrow \sim r$ Rule T

{6} (9) $\sim p \rightarrow r$ Inverse law

{1,3,4,6} (10) r Modus ponens.

{1,2,4,6} (11) $\sim r \wedge r$ conjunction.
F

* Our assumption is wrong.

Type - 5:- (consistency of Premesis):-

* Show that the premesis $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge \sim c)$, $a \wedge d$

{1} (1) $a \wedge d$ Rule P

{1} (2) a Simplification

{1} (3) d "

{4} (4) $a \rightarrow (b \rightarrow c)$ Rule P

{1,4} (5) $b \rightarrow c$ Modus ponens

{1,4} (6) $\sim b \vee c$ Rule T

{7} (7) $d \rightarrow (b \wedge \sim c)$ Rule P

{7} (8) $\sim(b \wedge \sim c) \rightarrow \sim d$ contrapositive

{7} (9) $\sim b \vee c \rightarrow \sim d$ Demorgams

{1,4,7} (10) $\sim d$ Modus ponens

{1,4,7} (11) $d \wedge \sim d$ conjunction

∴ $d \wedge \sim d \Rightarrow F$. Our assumption is wrong.

* So our premesis are inconsis-tent

show that the following are set of premesis are incosistent:
"If the contract is valid", then john is liable for penalty. If john is liable for penalty, He will go bankrupt. If the bank will loan him money, He will not go bankrupt. As a matter of fact, the contract is valid and the bank will loan him money.

Let us indicate the statement as follows:-

p: The contract is valid.

q: John liable for penaltly

r: John will go bankrupt.

s: The bank will loan him mon

The premesis are

$p \to q, q \to r, s \to \sim r, p \land s$

{1}	(1)	$p \to q$	Rule p
{2}	(2)	$q \to r$	Rule p
{1,2}	(3)	$p \to r$	T.R
{4}	(4)	$s \to \sim r$	Rule p
{4}	(5)	$r \to \sim s$	contrapositive
{1,2,4}	(6)	$p \to \sim s$	T.R

{1,2,4) (
{1,2,4} (
{9} (
{1,2,4,9} (
$(p \land s) \land \sim($
The prem
show tha
are inc
i. If Jack
because
High sch
ii. If Jac
unedu
iii. If Jo
then
iv. Jack
because
lot of
Let us
as foll
p: Jack
ause
q: He
r: Jack
s: Jack

$\{1,2,4\}$	(7)	$\sim P \vee \sim S$	Rule T
$\{1,2,4\}$	(8)	$\sim(P \wedge S)$	Demorgans
$\{9\}$	(9)	$P \wedge S$	Rule P
$\{1,2,4,9\}$	(10)	$(P \wedge S) \wedge \sim(P \wedge S)$	conjunction

$(P \wedge S) \wedge \sim(P \wedge S) \Leftrightarrow F$.

The premesis are inconsistent.

Show that the following premes are inconsistent:

i. If Jack misses many classes because of illness, then he ~~face~~ fails High school.

ii. If Jack fails H.S, then he is uneducated.

iii. If Jack reads a lot of book then he is not uneducated

iv. Jack misses many classes because of illness and reads a lot of books.

Let us indicate the statement as follows:-

P: Jack misses many classes bec ause of illness.

q: He fails High school.

r: Jack is uneducated

s: Jack reads a lot of books

The premesis are:

$p \to q$, $q \to r$, $s \to \sim r$, $p \wedge s$.

{1}	(1)	$p \to q$	Rule P
{2}	(2)	$q \to r$	Rule P
{1,2}	(3)	$p \to r$	T.R
{4}	(4)	$s \to \sim r$	Rule P
{4}	(5)	$r \to \sim s$	Contrapositive
{1,2,4}	(6)	$p \to \sim s$	T.R
{1,2,4}	(7)	$\sim p \vee \sim s$	Rule T
{1,2,4}	(8)	$\sim(p \wedge s)$	Demorgans
{9}	(9)	$p \wedge s$	Rule P
{1,2,4,9}	(10)	$(p \wedge s) \wedge \sim(p \wedge s)$	conjunction

* $(p \wedge s) \wedge \sim(p \wedge s) \Leftrightarrow F$. Our premesis is inconsistent.

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Rules

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2) Ru

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Quantifiers:-

Quantifier:-

* It means to measure the quantity of the statement.
* these are 2 types:-
  1. Universal Quantifiers.
  2. Existential Quantifiers.

Universal Quantifiers:-

* ALL
* Everyone
* Everything
* atleast
* These are denoted by (x).

Existential Quantifiers:-

* something
* someone
* forsome, atmost.
* These are denoted by $(\exists x)$

Rules of Quantifiers:-

1) Rule universal Specification:-

$$(x)A(x) \Rightarrow A(y)$$

2) Rule Existential Specification:-

$$(\exists x)A(x) \Rightarrow A(y)$$

Rule Universal Genalisation:-

$$A(y) \Rightarrow (x)A(x)$$

4) Rule Existential Generalization:-

$A(y) \Rightarrow (\exists x)A(x)$.

Note:-

* $(\exists x)(P(x) \vee Q(x)) \Leftrightarrow (\exists x)P(x) \vee (\exists x)Q(x)$
* $(x)(P(x) \wedge Q(x)) \Leftrightarrow (x)P(x) \wedge (x)Q(x)$
* $\sim(\exists x)P(x) \Leftrightarrow (x)\cdot P(x)$.
* $\sim(x)P(x) \Leftrightarrow (\exists x)P(x)$.
*

Type-1:- show that $(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$

{1}	(1)	$(x)(P(x) \rightarrow Q(x))$	Rule P
{1}	(2)	$P(y) \rightarrow Q(y)$	Rule US.
{3}	(3)	$(x)(Q(x) \rightarrow R(x))$	Rule P
{3}	(4)	$Q(y) \rightarrow R(y)$	Rule US.
{1,3}	(5)	$P(y) \rightarrow R(y)$	T.R
{1,3}	(6)	$(x)(P(x) \rightarrow R(x))$	Rule UG.

Show that $(\exists x)M(x)$ follows logically from the premesis $(x)(H(x) \rightarrow M(x))$ and $(\exists x)H(x)$.

{1}	(1)	$(x)(H(x) \rightarrow M(x))$	Rule P
{1}	(2)	$H(y) \rightarrow M(y)$	Rule US
{3}	(3)	$(\exists x)(H(x))$	Rule P
{3}	(4)	$H(y)$	Rule ES

{1,3} (5)

{1,3} (6)

Show that

$\wedge (\exists x)Q(x)$

{1} (1)

{1} (2)

{1} (3)

{1} (4)

{1} (5

{1} (6

{1}

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conclusio

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{4}

{4}

{1,3}	(5)	$M(y)$	Modes pones.
{1,3}	(6)	$(\exists x)M(x)$	Rule EG

Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)(P(x) \wedge (\exists x)Q(x)$

{1}	(1)	$(\exists x)(P(x) \wedge Q(x))$	Rule P
{1}	(2)	$P(y) \wedge Q(y)$	Rule ES
{1}	(3)	$P(y)$	simplifica
~~{1}~~	~~(4)~~	~~$Q(y)$~~	~~simplificat~~
{1}	(4)	$(\exists x)P(x)$	Rule EG
{1}	(5)	$Q(y)$	simplifica
{1}	(6)	$(\exists x)Q(x)$	Rule EG.
{1}	(7)	$(\exists x)P(x) \wedge (\exists x)Q(x)$	conjuncti

S.T from $(\exists x)[F(x) \wedge S(x)] \rightarrow (y)[M(y) \rightarrow W(y)]$ and $(\exists y)[M(y) \wedge \sim(W)(y))$ the conclusion $(x)[F(x) \rightarrow \sim(S)(x)]$ follows.

{1}	(1)	$(\exists y)[M(y) \wedge \sim W(y)]$	Rule P
{1}	(2)	$M(z) \wedge \sim W(z)$	Rule ES
{1}	(3)	$\sim [M(z) \rightarrow W(z)]$	Rule T
{4}	(4)	$(\exists x)[F(x) \wedge S(x)] \rightarrow (y)[M(y) \rightarrow W(y)]$	Rule P.
{4}	(5)	$F(z) \wedge S(z) \rightarrow [M(z) \rightarrow W(z)]$	Rule US & ES

{1,4}	(6)	$\sim[F(z) \wedge S(z)]$	Modus Tollens
{1,4}	(7)	$\sim F(z) \vee \sim S(z)$	Demorgans Law
{1,4}	(8)	$[F(z) \rightarrow \sim S(z)]$	
	(9)	$(x)[F(x)]$.	

Type-2:- Verify the validity for following arguments: "All men are mortal. Socraties is a man. Therefore, Socraties is mortal."

Let us indicate the statement as follows:

$A(x)$: x is a man.

$M(x)$: x is a mortal

s : socraties

The premesis are

$(x)[A(x) \rightarrow M(x)], A(s)$

The conclusion is $M(s)$

{1}	(1)	$(x)[A(x) \longrightarrow M(x)]$	Rule P
{1}	(2)	$A(s) \rightarrow M(s)$	Rule US
{3}	(3)	$A(s)$	Rule P
{1,3}	(4)	$M(s)$	Modus ponens

Verify the
argument
is a plan
Goldfish
a plant.
Therefore
heart
Let us
as follo
$P(x)$: x
$A(x)$: x
$H(x)$: x
$f(x)$:
The Gi
$(x)[P($
The co
{1} (1)
{1} (2)
{1} (3)
{4} (4)
{4}
{1,4}
{7}
{1,4,7}

Verify the Validity of the following arguments; "Every living thing is a plant or an animal. Joe's Goldfish is alive and it is not a plant. All animals has hearts. Therefore Joe's Gold fish has a heart

Let us indicate the statements as follows:

$P(x)$: x is a plant

$A(x)$: x is an animal

$H(x)$: x has a heart.

$f(x)$: x is Joe's Goldfish.

The Given premesis

$(x)[P(x) \vee A(x)]$, $\sim P(f)$, $(x)[A(x) \rightarrow H(x)]$

The conclusion: $H(f)$.

$\{1\}$	(1)	$(x)(P(x) \vee A(x))$	Rule P
$\{1\}$	(2)	$P(f) \vee A(f)$	Rule US
$\{1\}$	(3)	$\sim P(f) \rightarrow A(f)$	Rule T
$\{4\}$	(4)	$(x)[A(x) \rightarrow H(x)]$	Rule P
$\{4\}$	(5)	$A(f) \rightarrow H(f)$	Rule US
$\{1,4\}$	(6)	$\sim P(f) \rightarrow H(f)$	T.R
$\{7\}$	(7)	$\sim P(f)$	Rule P
$\{1,4,7\}$	(8)	$H(f)$	Modus polens

③ Establish the validity of the following argument : "All integers are rational numbers. Some integers are powers of 2. Therefore some rational numbers are powers of 2.

Sol Let us indicate the statement as follows

$I(x)$: x is an integer

$R(x)$; x is a rational numbers

$P(x)$: x is a powers of 2

The given premises are $(x)(I(x) \rightarrow R(x))$, $(\exists x)(I(x) \wedge P(x))$

The conclusion is $(\exists x)(R(x) \wedge P(x))$

{1}	(1)	$(x)(I(x) \rightarrow R(x))$	Rule P
{1}	(2)	$I(y) \rightarrow R(y)$	Rule US
{3}	(3)	$(\exists x)(I(x) \wedge P(x))$	Rule P
{3}	(4)	$I(y) \wedge P(y)$	Rule ES
{3}	(5)	$I(y)$	Simplification
{3}	(6)	$P(y)$	Simplification
{1,3}	(7)	$R(y)$	modus ponens
{1,3}	(8)	$R(y) \wedge P(y)$	Conjunction
{1,3}	(9)	$(\exists x)(R(x) \wedge P(x))$	Rule EG