Unit-II permutations and Combinations

1) principle of Inclusion (or) Exclusion

2)

1) principle of Inclusion (or) Exclusion :-

i) Let A, B are any two non empty sets then $|A \cup B| = |A| + |B| - |A \cap B|$

ii) Let A, B, C are any three sets then

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

Proof :-

$L.H.S = |A \cup B \cup C|$

$= |A| + |B \cup C| - |A \cap (B \cup C)|$

$= |A| + |B| + |C| - |B \cap C| - |(A \cap B) \cup (A \cap C)|$

$= |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| +$

$|A \cap B \cap A \cap C|$

$= |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= R.H.S.$

iii) Let A_1, A_2, A_3, A_4 are any four nonempty sets then $|A_1 \cup A_2 \cup A_3 \cup A_4| = |A_1| + |A_2| + |A_3| + |A_4| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_3 \cap A_4| - |A_4 \cap A_1| - |A_1 \cap A_3| - |A_2 \cap A_4| + |A_1 \cap A_2 \cap A_3| + |A_2 \cap A_3 \cap A_4| + |A_3 \cap A_4 \cap A_1| + |A_4 \cap A_1 \cap A_2| - |A_1 \cap A_2 \cap A_3 \cap A_4|$

Note :-

$|(A_1 \cup A_2 \cup A_3 \cup A_4)| = |U| - |A_1 \cup A_2 \cup A_3 \cup A_4|$

1. How many positive integers not exceeding 2000 which are divisible by 7 or 11.

Sol Let U denotes set of positive integers not exceeding 2000

$\therefore |U| = 2000$

Let A denotes set of positive integers which are divisible by 7.

$$|A| = \left| \frac{2000}{7} \right|$$

$$= 286$$

Let B denotes set of positive integers which are divisible by 11.

$$|B| = \left\lfloor \frac{2000}{11} \right\rfloor$$

$$= 182$$

Let $A \cap B$ denotes the set of positive integers which are divisible by 7 and 11. $[7 \times 11 = 77]$

$$|A \cap B| = \left\lfloor \frac{2000}{77} \right\rfloor$$

$$= 26$$

$\therefore$ The total no of positive integers which are divisible by 7 or 11

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= 286 + 182 - 26$$

$$= 442$$

2. Find the no of integers from 1 to 250 that are divisible by any of the integers 2, 3 and 6.

Sol Let 'U' denotes set of positive integers not exceeding 250.

$|U|$

Let 'A' denotes set of positive integers which are divisible by 2

$|A| = \left| \frac{250}{2} \right|$

$= 125$

Let 'B' denotes set of positive integers which are divisible by 3

$|B| = \left| \frac{250}{3} \right|$

$= 84$

Let 'c' denotes set of positive integers which are divisible by 6

$|C| = \left| \frac{250}{6} \right|$

$= 42$

Let $A \cap B$ denotes set of positive integers which are divisible by 2, 3. $[2 \times 3 = 6]$

$|A \cap B| = \left| \frac{250}{6} \right|$

$= 42$

Let $B \cap C$ denotes set of positive integers which are divisible by 3, 6. $[3 \times 6 = 18]$

$|B \cap C| = \left| \frac{250}{18} \right|$

$= 14$

Let $C \cap A$ denotes set of positive integers which are divisible by 6, 2. $[6 \times 2 = 12]$

$|C \cap A| = \left| \frac{250}{12} \right|$

$= 21$

Let $A \cap B \cap C$ denotes set of positive integers which are divisible by 2, 3 and 6.

$[2 \times 3 \times 6 = 36]$

$$|A \cap B \cap C| = \left| \frac{250}{36} \right|$$

$$= 7$$

$\therefore$ The total no of positive integers which are divisible by 2, 3 and 6

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

$$= 125 + 84 + 42 - 42 - 14 - 21 + 7$$

$$= 181$$

3. Determine the no of positive integers n, where ~~1 ≤ n ≤ 2000~~ $1 \leq n \leq 2000$ and n is not divisible by 2, 3, 5 but it is divisible by 7.

Sol. Let U denotes set of positive integers

$$|U| = 2000$$

Let A denotes set of positive integers which are divisible by 2

$$|A| = \left| \frac{2000}{2} \right|$$

$$= 1000$$

Let B denotes set
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$|B| = |$

$= 6$

Let C denotes s
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Let $A \cap B$
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Let B denotes set of positive integers which are divisible by 3

$$|B| = \left| \frac{2000}{3} \right|$$
$$= 667$$

Let C denotes set of positive integers which are divisible by 5

$$|C| = \left| \frac{2000}{5} \right|$$
$$= 400$$

Let $A \cap B$ denotes set of positive integers which are divisible by 2, 3. $[2 \times 3 = 6]$

$$|A \cap B| = \left| \frac{2000}{6} \right|$$
$$= 334$$

Let $B \cap C$ denotes set of positive integers which are divisible by 3, 5. $[3 \times 5 = 15]$

$$|B \cap C| = \left| \frac{2000}{15} \right|$$
$$= 134$$

Let $C \cap A$ denotes set of positive integers which are divisible by 5, 2. $[5 \times 2 = 10]$

$$|C \cap A| = \left| \frac{2000}{10} \right|$$
$$= 200$$

Let $A \cap B \cap C$ denotes set of positive integers which are divisible by 2, 3, 5.
$[2 \times 3 \times 5 = 30]$

$$|A \cap B \cap C| = \left\lfloor \frac{2000}{30} \right\rfloor$$

$$= 67$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

$$= 1000 + 667 + 400 - 334 - 134 - 200 + 67$$

$$= 1466$$

$$|(A \cup B \cup C)'| = |U| - |A \cup B \cup C|$$

$$= 2000 - 1466$$

$$= 534$$

The no of integers which are not divisible by 2, 3, 5. But divisible by 7 :

=.

4. How many integers from 1 to 1000 are not divisible by 2, 3, 5, 7.

5. Determine no of primes not exceeding 100 and not divisible 2, 3, 5, 7.

permutations :

1) Product rule :-

If 1st task can be done in 'm' ways and 2nd task can be done in 'n' ways then the both task can be done with together is ('mn') ways.

2) Sum rule :-

If 1st task can be done in 'm' ways and 2nd task can be done in 'n' ways then the both task can be done with at a time one task after another task is :- (m+n) ways

permutation :-

① $^{n}P_{r} = \frac{n!}{(n-r)!}$

② $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

③ $0! = 1$

④ $^{n}P_{1} = n$

⑤ $^{n}P_{0} = 1$

permutations without reputations :-

① How many possibilities are there to select a 1st prize winner, a 2nd prize winner and a 3rd prize winner from 50 different people you have entered a contest.

Sol Here $n = 50$

The total no of ways we can select 1^{st} prize winner, 2^{nd} prize winner, 3^{rd} prize winner out of 50 people $= {}^{50}P_1 \times {}^{49}P_1 \times {}^{48}P_1$

$= 50 \times 49 \times 48$

$= 2450 \times 48$

$= 1,17,600$ ways

② Consider 6 digits 1, 2, 3, 5, 6, 7. Assuming that reputations are not permitted answer the following :-

(i) How many four digit numbers can be formed from 6 digits?

(ii) How many of these numbers are less than 4000?

(iii) How many of the numbers in (i) are even?

(iv) How many of the numbers in (i) are odd?

(v) How many of the numbers in (i) are multiples of 5?

(vi) How many of the numbers in (i) contains both the digits 5 and 7?

Sol (i) The total no of 4 digit numbers can be formed using 6 digits 1, 2, 3, 5, 6, 7 =

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Select 1st prize

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$^6P_1 \times {^5P_1} \times {^4P_1} \times {^3P_1}$

$= 6 \times 5 \times 4 \times 3$

$= 360$ ways

[or]

$^6P_4$ ways

$= 360$ ways

(ii) The total no of 4 digit numbers which are less than 4000

$= {^3P_1} \times {^5P_1} \times {^4P_1} \times {^3P_1}$

$= 3 \times 5 \times 4 \times 3$

$= 180$ ways

(iii) The total no of even numbers :-

$= {^2P_1} \times {^5P_1} \times {^4P_1} \times {^3P_1}$

$= 2 \times 5 \times 4 \times 3$

$= 120$ ways

(iv) The total no of odd numbers :-

$=$ total $-$ even

$= 360 - 120$

$= 240$ ways

(v) The total numbers which are multiple by 5

$= {^1P_1} \times {^5P_1} \times {^4P_1} \times {^3P_1}$

$= 1 \times 5 \times 4 \times 3$

$= 60$ ways

(vi) The total no of numbers which contain both the digits 5 and 7

$$= {}^4P_2 \times {}^4P_2$$

$$= \frac{4!}{(4-2)!} \times \frac{4!}{(4-2)!}$$

$$= \frac{4!}{2!} \times \frac{4!}{2!}$$

$$= \frac{4\times3\times\cancel{2\times1}}{\cancel{2\times1}} \times \frac{4\times3\times\cancel{2\times1}}{\cancel{2\times1}}$$

$$= 12\times12$$

$$= 144 \text{ ways}$$

(3) Reputations are not permitted consider the digits 2, 3, 5, 6, 7, 9 :

i) How many 3 digit numbers can be formed from 6 digits ~~2, 3, 5~~ ?

ii) How many of these numbers are less than 400 ?

iii) How many of these numbers are even?

iv) How many of these numbers are odd?

Sol i) The total no of 3 digit number =

$${}^6P_3$$

$$= \frac{6!}{(6-3)!}$$

$$= \frac{6!}{3!}$$

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number :

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$

$= 120$ ways

ii) The total no of 3 digit numbers which are less than 400 :-

$= {}^2P_1 \times {}^5P_1 \times {}^4P_1$

$= 40$ ways

iii) The total no of even numbers are :-

$= {}^2P_1 \times {}^5P_1 \times {}^4P_1$

$= 40$ ways

iv) The total no of odd numbers are :-

$=$ total $-$ even

$= 120 - 40$

$= 80$ ways

(9) Suppose that a salesman has to visit 9 different cities. He must begin with his trip in a specific city, but he can visit the other 8 cities in any other he wishes. How many possible orders can the salesman used when visiting the cities.

Sol Specific city needs the first you can select in ${}^1P_1$ ways

The total no of ways he can visit

remaining 8 cities $= {}^{8}P_{8}$ ways

The total no of ways you can select one specific city and can visit other 8 cities

$= {}^{1}P_{1} \times {}^{8}P_{8}$

$= 1 \times 8!$

$= 8!$ ways [or] $= 40{,}320$ ways

(5) How many permutations of the letters A, B, C, D, E, F, G, H contains :

(i) The string CDE

(ii) The string AB and FG

(iii) The strings ABC and CDE

(iv) The strings ABC and BED

Sol (i) The total no of permutation of string CDE:

A B (C D E) F G H

${}^{6}P_{6}$

$= 720$ ways

(ii) The total no of permutations of string AB and FG :-

(A B) C D E (F G) H

${}^{6}P_{6}$

$= 720$ ways

(iii) The total n

ABC a

(A B C D

(iv) The total

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(7) Find

(i) P(n

(ii) P(

(iii)

(iii) The total no of permutations of string ABC and CDE : [ABCDE as a string]

$\text{(ABCDE)}\ F\ G\ H$

$^4P_4$

$= 24$ ways

(iv) The total no of permutations of string ABC and BED

$= 0$ ways

(6) In how many ways can 4 mathematics books, 3 dictionary books and 2 social books on a self, such that all books of same subject are together.

Sol) The total no of books are arranged of same subjects are together $= 3! \times 4! \times 3! \times 2!$

$= \overset{6}{\cancel{10}} \times 24 \times 60 \times 2$

$= 1728$ ways

(7) Find the following

(i) $P(n,2) = 72$

(ii) $P(n,4) = 42P(n,2)$

(iii) $2P(n,2) + 50 = P(2n,2)$

iii) Sol

$$2\frac{n!}{(n-2)!} + 50 = \frac{2n!}{(2n-2)!}$$

$$2\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} + 50 = \frac{(2n)(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}}$$

$$2(n^2-n)+50 = 4n^2-2n$$

$$2n^2-\cancel{2n}+50 = 4n^2-\cancel{2n}$$

$$2n^2 = 50$$

$$n^2 = 25$$

$$n = \pm 5$$

$$n = 5, -5 \times$$

$$\therefore \boxed{n=5}$$

i) $P(n,2) = 72$

Sol

$$\frac{n!}{(n-2)!} = 72$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 72$$

$$n(n-1) = 72$$

$$n^2-n = 72$$

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$(2n-1)(2n-3)$

$(2n-2)!$

(8) In how many ways, 6 men and 4 women

i) sit in a row

ii) In how many ways, can they sit in a row if all the men sit together and all the women sit together.

iii) In how many ways can they sit in a row if just the women sit together.

iv) In how many ways can they sit in a row if just the men sit together.

Sol. Total men + women = 10

i) The total no of ways the 10 members can be sit in a row $= {}^{10}P_{10} = 10!$

ii) The total no of ways if all the mens sit together and all the womens sit together $= 2! \times 6! \times 4!$

iii) The total no of ways can they sit in a row if just the women sit together

$= 7! \times 4!$

iv) The total no of ways can they sit in a row if just the men sit together $= 5! \times 6!$

(9) In how many different orders can 3 men and 3 women in a row of 6 sits if

i) Any one ~~[illegible]~~ sit in any of these sits.

ii) The first and last sits must be filled by men

iii) men occupied first 3 sits and women occup[illegible] last 3. sits.

Sol The total men + women = 6

i) The total no of way that anyone sit in any of these sits = $6P_6 = 6!$

ii) The total no of ways the first and last sits must be filled by men $= 3P_2 \times 4!$

$= 3! \times 4!$

iii) The total no of ways men occupied first 3 sits and women occupied last 3 sits $= 3! \times 3!$

permutations

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permutations with repitation :-

1) How many different ~~strings~~ strings can be made from the letters of the word "SUCCESS" using all the letters.

2) How many different strings can be made from the letters in "ABRACADABRA" using all the letters.

1) Sol

The total no of strings can be from the letters of the "SUCCESS".

$$= \frac{7!}{3! \times 1! \times 2! \times 1!} \text{ ways}$$

2) The total no of strings can be from the letters of the "ABRACADABRA"

$$= \frac{11!}{5! \times 2! \times 2! \times 1! \times 1!} \text{ ways}$$

Combinations :-

1) $C(n, r)$ or $^{n}C_{r} = \frac{n!}{r!(n-r)!}$

2) $^{n}C_{0} = 1$

3) $^{n}C_{1} = 1$

4) $^{n}C_{n} = 1$

5) $^{n}C_{r} = {^{n}C_{s}}$

$\boxed{n = r + s}$

6) $^nC_r = {}^nC_{n-r}$

Combinations without repetation :-

1) A club has 25 members how many ways are there to choose 4 members of the club to serve on an executive Committee

Sol The total no of ways we can select 4 members out of 25 members = $^{25}C_4$ ways

2) Suppose a department consists of 10 men and 15 women. How many ways are there to form a committee with 6 members if it must have 3 men and 3 women.

Sol The total no of ways we can select 3 mens out of 10 men and 3 womens out of 15 womens = $^{10}C_3 \times {}^{15}C_3$ ways

3) Suppose a department consists of 8 men and 9 women in how many ways can we select a committee of

i) 3 men and 4 women

ii) 4 persons that has atleast 1 women

iii) 4 persons that has atmost 1 man

iv) 4 persons that has persons of both

men and women.

i) The total no of ways of 3 men out of 8 men and 4 women out of 9 women

$= {}^8C_3 \times {}^9C_4$ ways

ii) The total no of ways that 4 persons has atleast 1 women

m	w
3	1
2	2
1	3
0	4

$= {}^8C_3 \times {}^9C_1 + {}^8C_2 \times {}^9C_2 + {}^8C_1 \times {}^9C_3 + {}^8C_0 \times {}^9C_4$ ways

iii) The total no of ways that 4 persons has atmost 1 man

m	w
1	3
0	4

$= {}^8C_1 \times {}^9C_3 + {}^8C_0 \times {}^9C_4$ ways

iv) The total no of ways that 4 persons has persons of both men and women

m	w
3	1
2	2
1	3

$= {}^8C_3 \times {}^9C_1 + {}^8C_2 \times {}^9C_2 + {}^8C_1 \times {}^9C_3$

(4) A certain question paper contains two parts 'A' and 'B'. Each containing '6' questions. How many different ways a student can 7 questions. atleast 3 questions from each set.

Sol Total no. of questions in part A = 6

Total no. of questions in part B = 6

The no of ways we can select 4 questions from part A and 3 questions from part B

$= 6C_4 \times 6C_3$ ways

The no of ways we can select 3 questions from part A and 4 questions from part B

$= 6C_3 \times 6C_4$ ways

The total no of ways we can select atleast 3 questions from part A and 3 questions from part B.

$= (6C_4 \times 6C_3) + (6C_3 \times 6C_4)$ ways

Circular

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Circular permutations :-

The total no of circular permutations of n objects = $(n-1)!$

① How many ways are their to sit 10 boys and 10 girs arround a circular table if boys and girls sit alternatively.

Sol The total no of boys = 10

The total no of girls = 10

The total no of ways we can sit 10 boys and 10 girls around a circular table If boys and girls sit alternatively =

$10! \times 9!$ ways

② (i) If 8 people P, Q, R, S, T, U, V, W are sitted around a table

(ii) How many different circular arrangements are possible if array Elements are considered the same then one obtain the other by rotation.

Sol (i) The total no of circular arrangements of 8 objects = $(8-1)! = 7!$ ways

(ii) The total no of boys = 4 [P, Q, R, S]

The total no of girls = 4 [T, U, V, W]

The total no of circular arrangements of 8 people if sit alternatively = $3! \times 4!$ ways

Integral equations :- $c[n-1+r, r]$

① How many integral solutions are their to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$. where each $x_i \geq 2$

Sol we put 2 objects in each box. Then the total no of objects to be filled is 10. The remaining objects can be filled with other words.

The total no of integral solutions =

$y_1 = x_1 - 2$; $y_2 = x_2 - 2$; $y_3 = x_3 - 2$;

$y_4 = x_4 - 2$; $y_5 = x_5 - 2$

$y_1 + y_2 + y_3 + y_4 + y_5 = x_1 + x_2 + x_3 + x_4 + x_5 - 10$

$= 20 - 10$

$y_1 + y_2 + y_3 + y_4 + y_5 = 10$

Here $n = 10$

$r = 5$

is of the form :- $c(n-1+r, r)$

$= c(10-1+5, 5)$

$= c(14, 5)$

(2) How many integral Solutions are their to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$. where $x_1 \geq 3$, $x_2 \geq 2$, $x_3 \geq 4$, $x_4 \geq 6$, $x_5 \geq 0$.

Sol: we put 3 objects in First box, 2 objects in second box, 4 objects in third box, 6 objects in Fourth box. The remainining objects can be Filled in other words.

The total no of integral Solutions =

$y_1 = x_1 - 3$; $y_2 = x_2 - 2$; $y_3 = x_3 - 4$;

$y_4 = x_4 - 6$; $y_5 = x_5 - 0$

$y_1 + y_2 + y_3 + y_4 + y_5 = x_1 + x_2 + x_3 + x_4 + x_5 - 15$

$= 20 - 15$

$y_1 + y_2 + y_3 + y_4 + y_5 = 5$

$\therefore$ Here $n = 5$

$r = 5$

is of the form : $c(n-1+r, r)$

$= c(5-1+5, 5)$

$= c(9, 5)$

3 How many integral solutions are their $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ wher $x_1 \geq -3$, $x_2 \geq 0$, $x_3 \geq 4$, $x_4 \geq 2$, $x_5 \geq 2$

Sol Given $x_1 \geq -3$ means to add 3 in the total no of boxes.

Now, the total no of boxes is 23.

we put 4 objects in 3^{rd} box, 2 objects in 4^{th} box and 2 objects in 5^{th} box.

The remaining objects can be filled with other words.

The total no of integral solutions :

$y_1 = x_1$; $y_2 = x_2 - 0$; $y_3 = x_3 - 4$; $y_4 = x_4 - 2$

$y_5 = x_5 - 2$.

$y_1 + y_2 + y_3 + y_4 + y_5 = x_1 + x_2 + x_3 + x_4 + x_5 - 8$

$= 23 - 8$

$= 15$

$\therefore$ Here $n = 5$

$r = 15$

is of the form $C(n-1+r, r)$

$= C(5-1+15, 15)$

$= C(19, 15)$

Binomial Coef

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Binomial coefficients :-

① Find the Binomial coefficient of x^{101}, y^{99} in the expansion of $(2x-3y)^{200}$

Sol Here $n = 200$

$q_1 = 101$

$q_2 = 99$

The coefficient of $x^{101} y^{99}$ in the Expansion of $(2x-3y)^{200} = \frac{200!}{101!\,99!} (2^{101} (-3)^{99})$

② Find the coefficient of $x^9 y^3$ in the expansion of $(x+3y)^{12}$

Sol Here $n = 12$

$q_1 = 9$

$q_2 = 3$

The coefficient of $x^9 y^3$ in the Expansion of $(x+3y)^{12}$

$= \frac{12!}{9!\,3!} (1^9 3^3)$

③ what is the coefficient of $u^2 w^3 x^4 y^2$ in the expansion of $(u+v+2w+x+y+z)^{11}$

Sol Here $n = 11$

$q_1 = 2$

$q_2 = 3$

$q_3 = 4$

$q_4 = 2$

The coefficient of $u^2 w^3 x^4 y^2$ in the Expansion of $(u + v + 2w + x + y + z)^{11}$

$$= \frac{11!}{2!\,3!\,4!\,2!}\left(1^2\, 2^3\, 1^4\, 1^2\right)$$