UNIT-3

Relations, Lattices, Boolean Algebra

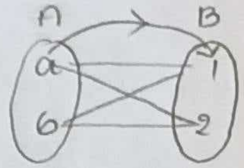
Relations:

Any set of order pairs is a relation or binary relaton

Binary relation represents the relationship b/w two elements

Example: $A = \{a, b\}$ $\quad A \times B = \{(a,1)\,(a,2),(b,1),(b,2)\}$

$B = \{1, 2\}$



Properties of relation:

i. **Reflexive relation:** If R is a relation defined on a set 'x' such that $xRx \ \forall x \in X$. In other words $xRx \Rightarrow (x, x) \in R$

Ex: $\leq$ satisifies reflexive relation and

$<$ Not satisifies reflexive relation.

ii. **Symmetric relation:** If R is a relation defined on a set 'x' is said to be symmetric then $xRy \Rightarrow yRx$. In other words ~~$xRy \Rightarrow (x,y) \in R$~~ $xRy \Rightarrow yRx$.

Ex: $=$ Satisifies symmetric relation

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iii. **Transitive relation:** let $x, y, z \in x$ and R is a relation defined on x is said to be Tranisitive.

If xRy, yRz then xRz.

Ex: Equality of sets satisifies tranisitive relation

If A, B, C are any three non-empty sets then, If $A = B$, $B = C$ then $A = C$.

iv. **Anti-symmetric relation:** Let 'x', be any non empty set and R is a relation defined on the set 'x' such that if xRy & yRx then $x = y$

Ex: The relation (subset are equal to) $\subseteq$ satisifies anti symmetric relation.

Let A, B are any two non-empty sets if $A \subseteq B$ & $B \subseteq A$ then $A = B$.

v, Irreflexive relation: Let 'X' be any non-empty set and R is a relation defined on a set 'X' such that $x \not R x \ \forall x \in X$.

Ex: The relation < satisifies irreflexive relation.

Ex: $1 < 1$ ✗ $2 < 2$ ✗

vi, Isymmetric relation: Let 'R' be the relation defined on the Non-Empty set 'X'. Such that xRy then $y \not R x$.

Ex: The relation < satisifies Isymmetric relation

Ex: [$2 < 3$; $3 \not< 2$] let $2, 3 \in X$ if $2 < 3$ then $3 \not< 2$.

1. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and R is a relation defined as $R = \{(x, y) / x + y = 10\}$. What are the properties satisifies on R.

Given that $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

& Relation $R = \{(x, y) / x + y = 10\}$

Reflexive: Let R is a relation defined on 's'.

$1 \not R 1$; $2 \not R 2$; $3 \not R 3$; $4 \not R 4$, $5 R 5$, $6 \not R 6$,

$7 \not R 7$; $8 \not R 8$; $9 \not R 9$; $10 \not R 10$

$\therefore$ R is not reflexive.

Symmetric: Let R is a relation defined on 's'.

The possible ordered pairs to give the sum is 10 are

$\{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5)\} \in R$.

If $(1, 9) \in R$ then $(9, 1) \in R$; $(4, 6) \in R$ then $(6, 4) \in R$

$(2, 8) \in R$ then $(8, 2) \in R$; $(5, 5) \in R$ then $(5, 5) \in R$.

$(3, 7) \in R$ then $(7, 3) \in R$

$\therefore$ R is Symmetric relation

Anti-Symmetric: If R is a relation defined on S. If $1,9 \in R$ and $9,1 \in R$ then $1 \neq 9$.

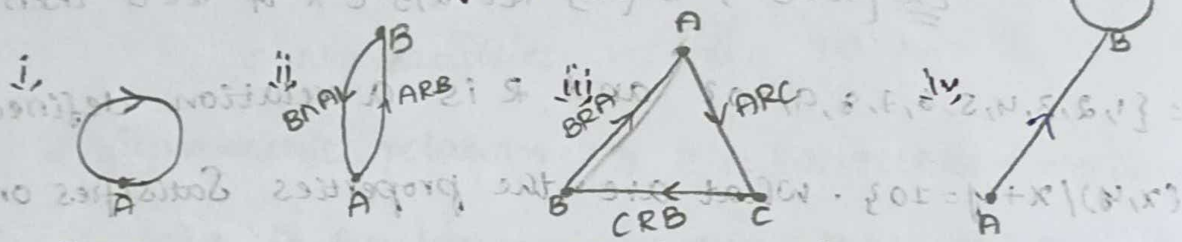
$\therefore$ R is not a Anti symmetric.

Irreflexive: Let R is a relation defined on S. If $5,5 \in R$ then R is Irreflexive. and also Assymmetric.

~~Relation Matr~~ Transitive relation: Let 'R' is a relation defined on 'S'. If (1,9) belongs to R. And remaining Ordered Pair does not exit.

$\therefore$ R is not a transitive relation.

Relation Matrix and Digraph:



ARA

ARB∧BRA

ARC∧CRB∧BRA

ARB∧BRB

Relation Matrix:

1. Let $A = \{1,2,3,4\}$ and the Relation R is defined as $R = \{(1,1),(2,1)(1,3)(4,4)\}$ Find the relation Matrix.

Given, $A = \{1,2,3,4\}$.

$R = \{(1,1),(2,1)(1,3)(4,4)\}$

$$M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

2. Let $A = \{1,2,3,4\}$ and $B = \{b_1, b_2, b_3\}$ and the relation Matrix

$$M_R \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

. Find the relation 'R'.

$$M_R = \begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 1 & 1 \\ 4 & 0 & 0 & 1 \end{array}$$

$R = \{(1,B_1),(1,B_2),(2,B_2),(2,B_3),(3,B_1)(3,B_2),(3,B_3),(4,B_3)\}$

NOTE:

	Equivalent Relation E.R	partial ordering Relation P.O.R	Compatability relation C.R
R.R	✓	✓	✓
S.R	✓		✓
T.R	✓	✓	
A.S.R		✓	

Equivalent relation: If R is a relation defined on a set 'x' such that if it satisifies Reflexive, Symmetric Relation, Transitive relation.

1. If R,S is an equivalent relation then show that R∩S is an equivalence relation.

Let 'x' = {a,b,c,d} and R,S are relations defined on x is an equivalence relations then show that R∩S is an equivalence relation.

Reflexive: Since, R is reflexive then $(a,a) \in R\ \forall\ a \in X$.

Since, 'S' is reflexive then $(a,a) \in S\ \forall\ a \in X$.

$\therefore (a,a) \in R \cap S\ \forall\ a \in X$.

$\therefore$ R∩S is reflexive.

Symmetric: Since, 'R' is symmetric

If $(a,b) \in R$ then $(b,a) \in R$

Since, 'S' is symmetric

If $(a,b) \in S$ then $(b,a) \in S$.

$\therefore$ If $(a,b) \in R \cap S \Rightarrow (b,a) \in R \cap S$.

Transitive relation: Since 'R' is transitive.

If $(a,b) \in R$

$(b,c) \in R$

$\Rightarrow (a,c) R$,

$x \in 4, y \in 1, x \in 2$

Since 's' is transitive.

If $(a,b) \in S$

$(b,c) \in S$

$\Rightarrow (a,c) S$

$\therefore (a,b) \in R \cap S$ and $(b,c) \in R \cap S$ then

$(a,c) \in R \cap S$

$\therefore R \cap S$ is Transitive.

$\therefore R \cap S$ is a equivalent relation.

2. Let 'R' be the relation defined on ordered pairs in the set of positive integers and defined as $xv = yu$ if and only if (iff) $(x,y) R (u,v)$ then show that R is an equivalence relation.

R is a relation defined on the set of positive integers.

Let 'x', 'y', 'u', 'v', 'm', 'n' are positive integers then show that R is an equivalence relation.

R is defined as $\boxed{xv = yu \text{ iff } (x,y) R (u,v)}$.

1. Reflexive: Let x, y be any positive integers

$xy = yx$

$(x,y) R (x,y)$

$\therefore$ 'R' is Reflexive.

2. Symmetric relation: Let x, y, u, v be positive integers

$(x,y) R (u,v) \Leftrightarrow xv = yu$

$\Leftrightarrow yu = xv$

$\Leftrightarrow uy = vx$

$\Leftrightarrow (u,v) R (x,y)$

$\therefore$ 'R' is Symmetric Relation.

3. Transitive Relation: Let x, y, u, v, m, n be positive Integers.

If $(x,y) R (u,v) \Leftrightarrow xv = yu$

If $(u,v) R (m,n) \Leftrightarrow un = vm$

$\Leftrightarrow xvum = yuvm$

$\Leftrightarrow xn = ym$

$\Leftrightarrow (x,y) R (m,n)$

'R' is Transitive Relation.

26/07/23.

1) Define Congruence modulo(m) and prove that it is an equivalence relation

Let N be the set of positive integers and n is any positive integer. The Congruence modulo(m) is defined as $a \equiv b (\mathrm{mod}\, m) \Leftrightarrow (a-b)$ is divisible by m. $\left(\frac{a-b}{m}\right)$.

Where a, b belong to 'N' $(a, b \in N)$

To shown that Congruence is an equivalence relation.

1. Reflexive Relation:

$$\frac{0}{m} \Leftrightarrow \frac{a-a}{m}$$

$$\Leftrightarrow a \equiv a (\mathrm{modus}) \ \forall a \in N$$

$\equiv$ is a reflexive.

2. Symmetric relation:

$$\text{If } a \equiv b (\mathrm{modus}) \Leftrightarrow \frac{a-b}{m}$$

$$\Leftrightarrow \frac{-(a-b)}{m}$$

$$\Leftrightarrow \frac{b-a}{m}$$

$$\Leftrightarrow b \equiv a (\mathrm{mod}\, m)$$

$$\forall a, b \in N.$$

$\equiv$' is Symmetric

3. Transitive Relation:

If $a \equiv b \pmod m \Leftrightarrow \frac{a-b}{m}$ and $b \equiv c \pmod m \Leftrightarrow \frac{b-c}{m}$

$$\Leftrightarrow \frac{a-\cancel{b}+\cancel{b}-c}{m}$$

$$\Leftrightarrow \frac{a-c}{m} \quad \forall a,b,c \in N$$

$$\Leftrightarrow a \equiv c \pmod m$$

'$\equiv$' is an Transitive. $\therefore$ Congruence modulo is an equalance Relation.

2. Define Compactability with an example.

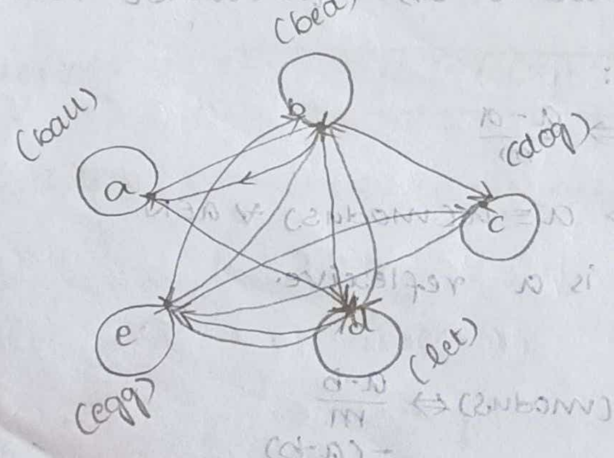
If R is a Relation defined on x is Said to be Compactability.

If R satisifies both symmetric and reflecive relation.

Ex: Let $X = \{$ball, bed, dog, let, egg$\}$ & R is a relation is defined as $R = \{(x,y) \,/\,$ If x, y contains atleast one Common letter$\}$

Given that, $X = \{$Ball, bed, dog, let, egg$\}$ and the Compactability relation is denoted by '$\approx$' $R = \{(x,y) \,/\,$ If x, y contains atleast one Common letter$\}$

1.	2.	3.	4.	5.
ball $\approx$ ball	bed $\approx$ ball	Dog $\not\approx$ ball	Let $\not\approx$ ball	Egg $\not\approx$ ball
ball $\approx$ bed	bed $\approx$ bed	Dog $\not\approx$ bed	Let $\approx$ bed	Egg $\not\approx$ bed
ball $\not\approx$ dog	bed $\not\approx$ dog	Dog $\approx$ dog	Let $\not\approx$ dog	Egg $\approx$ dog
ball $\approx$ let	bed $\approx$ let	Dog $\not\approx$ let	Let $\approx$ let	Egg $\approx$ let
ball $\not\approx$ egg	bed $\approx$ egg	Dog $\approx$ egg	Let $\approx$ egg	Egg $\approx$ egg



3. Transitive closure:

Suppose $R \subseteq A \times A$ (ordered pairs), is Said to be transitive closer of 'R' and defined as R^+ $R^+ = R \cup R^2 \cup R^3 \ldots$

1. Find the transitive closer of $R = \{(1,2)\,(2,3)\,(3,3)\}$ on the Set $A = \{1,2,3\}$

Given that,

$A = \{1, 2, 3\}$

$R = \{(1,2)(2,3)(3,3)\}$

$R^2 = R o R.$

$= \{(1,2)(2,3)(3,3)\} o \{(1,2)(2,3)(3,3)\}$

$= \{(1,3)(2,3)(3,3)\}$

$R^3 = R^2 o R$

$= \{(1,3),(2,3)(3,3)\} o \{(1,2)(2,3)(3,3)\}$

$= \{(1,3)(2,3)(3,3)\}.$

Transitive closure of R is

$R^+ = R \cup R^2 \cup R^3$

$= \{(1,2)(2,3)(3,3)(1,3)\}.$

2. Find the transitive closure of R where $R = \{(1,2)(2,3)(2,1)(3,4)\}$ where $A = \{1,2,3,4\}$

Given that,

$A = \{1,2,3,4\}$; $R = \{(1,2)(2,3)(2,1)(3,4)\}$

Wrong [$R^2 = R o R$

$= \{(1,2),(2,3)(2,1)(3,4)\} o \{(1,2)(2,3)(2,1)(3,4)\}$

$\Rightarrow \{(1,3),(2,3)(2,2)(3,$] Wrong

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 1: We can introduce C_1 & R_1

C_1 R_1

2 2

$C_1 \times R_1 = \{(2,2)\}$

Step 2: We can introduce C_2 & R_2

C_2 R_2

1 $\{1,3\}$

$C_2 \times R_2 = \{(1,1)(1,3)\}$

Step 3: We can introduce C_3 & R_3

C_3 R_3

2 4

$C_3 \times R_3 = \{(2,4)\}$

Step 4: We can introduce C_4 and R_4

C_4 R_4

3 ϕ

The transitive closure of R is

$R^+ = \{(2,2)(2,4)(1,1)(1,3)\}$

⇒ Warshal's algorithm: To find the transitive closer of the adjacent matrix by the given relation

1. Let R is a relation defined on X where $R = \{(1,2)(1,3)(2,3)(3,1)\}$

Here $X = \{1,2,3\}$.

Given that $X = \{1,2,3\}$

$R = \{(1,2)(1,3)(2,3)(3,1)\}$

$$M_R = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}_{3\times 3} \end{array}$$

S-1 We Can Introduce C_1 and R_1

C_1	R_1
$\{3\}$	$\{2,3\}$

$C_1 \times R_1 = \{(3,2)(3,3)\}$

S-2 We Can Introduce C_2 and R_2

C_2	R_2
$\{1\}$	$\{3\}$

$C_2 \times R_2 = \{(1,3)\}$

S-3 We Can Introduce C_3 and R_3

C_3	R_3
$\{1,2\}$	$\{1\}$

$C_3 \times R_3 = \{(1,1)(2,1)\}$

∴ Transitive closure of R is

$R^+ = \{(3,2)(3,3)(1,3)(1,1)(2,1)\}$.

2. Find the transitive closure of R using Warshal's Algorithm where $R = \{(2,1)(2,3)(3,1)(3,4)(4,1)(4,3)\}$ and $A = \{1,2,3,4\}$.

⇒ Hasse Diagram:

The pictorial representation of partial ordered set is Called Hasse Diagram.

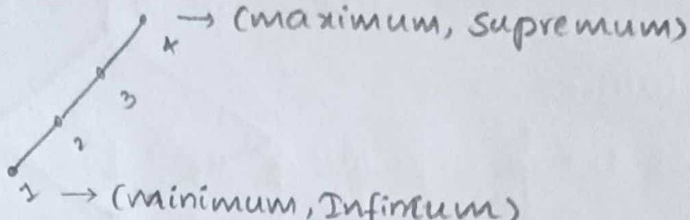
1. Draw the Hasse Diagram for the relation '≤' and the Set $A = \{1,2,3,4\}$

Given that,

$A = \{1,2,3,4\}$ and the Relation is '$\leq$'.

$[A, \leq] = \{(1,2)(1,3)(1,4)(2,3)(2,4)(3,4)\}$

Hasse Diagram:



2. Draw the Hasse Diagram for the poset ~~(X, /)~~ (X, /) where
$x = \{2,3,6,12,24\}$ → Divisibility

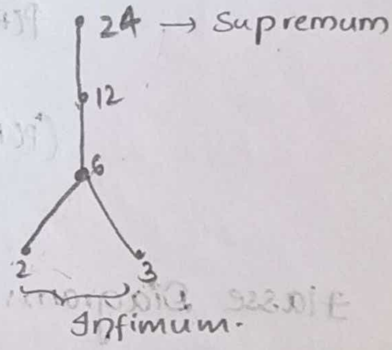
Given that,

$x = \{2,3,6,12,24\}$

and the relation is divisibility. ~~(X,X)~~

$(X, /) = \{(2,6)(2,12)(2,24)(3,6)(3,12)(3,24)(6,12), (6,24)(12,24)\}$

Hasse Diagram:



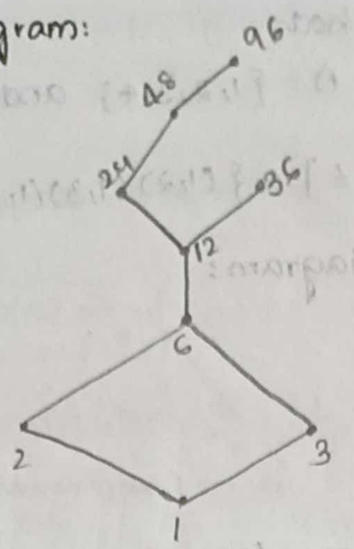
3. Draw the Hasse Diagram for the divisibility on the set $\{1,2,3,6,12,24,48,36,96\}$. ~~Let X = {~~

Given that,

Let $X = \{1,2,3,6,12,24,36,48,96\}$ and the Relation is divisibility.

$(X, /) = \{(1,2)(1,3)(1,6)(1,12)(1,24)(1,36)(1,48)(1,96), (2,6), (2,12)$
$(2,24)(2,36)(2,48)(2,96)(3,6)(3,12)(3,24)(3,36)(3,48)$
$(3,96)(6,12)(6,24)(6,36)(6,48)(6,96)(12,24)(12,36)$
$(12,48)(12,96)(24,48)(24,96)$ ~~(36,9~~ $(48,96)\}$

Hasse Diagram:



maximum element = 96; 36

minimum element = 1.

4. Draw the Hasse Diagram for the poset where $(P(A), \subseteq)$ where

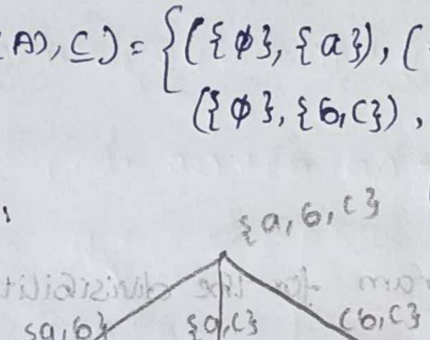
$A = \{a, b, c\}$

Given that, $A = \{a, b, c\}$

$P(A) = \{ \{\phi\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\} \}$

$(P(A), \subseteq) = \{ (\{\phi\}, \{a\}), (\{\phi\}, \{b\}), (\{\phi\}, \{c\}), (\{\phi\}, \{a,b\}),$
$(\{\phi\}, \{b,c\}), (\{\phi\}, \{a,c\}), (\{\phi\} \{a,b,c\}),$
$(\{a\}, \{a,b\}) (\{a\} \{b,c\}) (\{a\} \{a,c\}),$
$(\{a\}, \{a,b,c\}), (\{b\} \{a,b\})$
$(\{b\} \{b,c\}), (\{b\}, \{a,c\}),$
$(\{b\}, \{a,b,c\}), (\{c\}, \{a,b\}),$
$(\{c\}, \{b,c\}), (\{c\} \{a,c\}), (\{c\}, \{a,b,c\})$

Hasse Diagram:



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Lattice Theory:

* Let $(L, *, \oplus)$ is Said to be Lattice for any $(a, b) \in L$ then thelre exist a greatest lower bound $a*b \in L$ and least upper bound $a \oplus b \in L$.

$* \longrightarrow$ Product , $\oplus \longrightarrow$ Sum.

Properties of lattice:

* $a * a = a$ [$\because$ Idempontent law]
* $a*b = b*a$ [$\because$ Commutative law]
* $a*(b*c) = (a*b)*c$ [$\because$ Associative law]
* $a*(b \oplus c) = a$ [$\because$ Absorption law]
* $a \oplus a = a$ [$\because$ Idempontent law]
* $a \oplus b = b \oplus a$ [$\because$ Commutative law]
* $a \oplus (b \oplus c) = (a \oplus b) \oplus c$ [$\because$ Associative law]
* $a \oplus (b*c) = a$ [$\because$ Absorption law]

Theorem:

Let $(L, *, \oplus)$ be a lattice in which $*, \oplus$ denotes $\wedge, \vee$ respectively then show that $a \leq b \Leftrightarrow a*b = a$

$a \oplus b = b$

Proof: Given that,

$(L, *, \oplus$ be a lattice.

We have to show that,

$a \leq b \Leftrightarrow a*b = a$

$a \oplus b = b$

$a*b$ = greatest lower bound of $\{a, b\}$

$= a$

$a \oplus b$ = least upper bound of $\{a, b\}$

$= b$

$\therefore$ Infimum of $\{a, b\} = a$

Supermum of $\{a, b\} = b$

Sub lattice:

- Let $L, *, \oplus$ be a lattice and S is any non empty subset of L. then, it said to be sub lattice of L is also $(S, *, \oplus)$.

Direct product of lattice:

- Let $L, *, \oplus$ be a lattice and another lattice $L', \wedge, \vee$ then the direct product of both lattices. Then the algebraic system is

$$(L, *, \oplus) \cdot (L', \wedge, \vee)$$

$$(L_1 \times L_2, \cdot, +)$$

$\cdot \rightarrow *, \wedge$

$+ \rightarrow \oplus, \vee$

$a_1, a_2 \in L$

$$((a_1, a_2) \cdot (b_1, b_2)) \Leftrightarrow ((a_1 * a_2), (b_1 \wedge b_2))$$

$$((a_1, a_2) + (b_1, b_2)) \Leftrightarrow ((a_1 \oplus a_2), (b_1 \vee b_2))$$

Complete Lattice:

- A Lattice $(L, *, \oplus)$ is said to complete if every non-empty subset of L as least upper bound and greater lower bound.

$L(1, 2, 3, 4, 5, 6)$

$\downarrow$ greater lower bound

$\downarrow$ Least upper bound.

Bounded

~~Boundary~~ Lattice: $(L, *, \oplus, 0, 1)$

- A lattice $(L, *, \oplus)$ is said to be a bounded lattice '0' is the least element and '1' is the greatest element exist and satisfy the following conditions.

$a * 0 \Rightarrow 0$ *-Smaller, ⊕ - greater

$a \oplus 0 = a$

$a * 1 = a$

$a \oplus 1 = 1$

Complementary lattice:

- A boundary lattice $(L, *, \oplus, 0, 1)$ then the complement of the lattice is defined as for any $b \in L$ is said to be complement of A.
- If $(a * b) = 0$ and $a \oplus b = 1$.

Properties:

- $0' = 1$
- $1' = 0$
- $(a * b)' = a' \oplus b'$
- $(a \oplus b)' = a' * b'$
- $(a * a') = 0$
- $(a \oplus a') = 1$

Distributive lattice:

- Let $(L, *, \oplus)$ be a Lattice is said to be distributive for any $(a, b, c) \in L$ then

$$a * (b \oplus c) = (a * b) \oplus (a * c)$$

$$a \oplus (b * c) = (a \oplus b) * (a \oplus b)$$

Properties:

- If $a * b = a * c$
  $a \oplus b = a \oplus c$ then $b = c$.

- If $(a \oplus b) * (b \oplus c) * (a \oplus c) = (a * b) \oplus (b * c) \oplus (a * c)$.

**

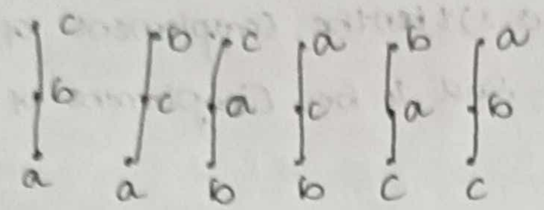
Theorem:

- Show that every chain is a distributive lattice.

Let $(L, *, \oplus)$ be a lattice is distributive then $a * (b \oplus c) =$ $a * (b \oplus c) = (a * b) \oplus (a * c)$ & $a \oplus (b * c) = (a \oplus b) * (a \oplus c)$.

- Every pair of elements in the lattice L is comparable under the relation is called a chain or total ordered set.
- Let $(a, b, c) \in L$ then the possible chains are

Cases:



$$a * (b \oplus c) = (a * b) \oplus (a * c)$$

Case 1: $a \le b \le c$

$LHS = a * (b \oplus c)$.
$= a * c$
$= a$

$RHS = (a * b) \oplus (a * c)$
$= a \oplus a$
$= a$

$LHS = RHS \Rightarrow a * (b \oplus c) = (a * b) \oplus (a * c)$

Case 2: $a \le c < b$

$LHS = a * (b \oplus c)$
$= a * b$
$= a$

$RHS = (a * b) \oplus (a * c)$
$= a \oplus a$
$= a$

$LHS = RHS \Rightarrow a * (b \oplus c) = (a * b) \oplus (a * c)$

Case 3: $b \le a \le c$.

$LHS = a * (b \oplus c)$
$= \cancel{a * c} = a * c$
$= \cancel{a} = a$

$RHS = (a * b) \oplus (a * c)$
$= b \oplus a$
$= a$

$LHS = RHS \Rightarrow a * (b \oplus c) = (a * b) \oplus (a * c)$

Case 4:

$b \leq c \leq a$

$LHS = a * (b \oplus c)$	$RHS = (a * b) \oplus (a * c)$
$= a * c$	$= b \oplus c$
$= c$	$= c$

$LHS = RHS \Rightarrow a * (b \oplus c) = (a * b) \oplus (a * c).$

Case 5:

$c \leq a \leq b$

$LHS = a * (b \oplus c)$	$RHS = (a * b) \oplus (a * c)$
$= a * b$	$= a \oplus c$
$= a$	$= a$

$LHS = RHS \Rightarrow a * (b \oplus c) = (a * b) \oplus (a * c)$

Case 6:

$c \leq b \leq a$

$LHS = a * (b \oplus c)$	$RHS = (a * b) \oplus (a * c)$
$= a * b$	$= b \oplus c$
$= b$	$= b$

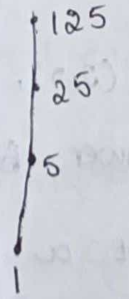
$LHS = RHS \Rightarrow a * (b \oplus c) = (a * b) \oplus (a * c).$

30/05/23:

Determine the poset $\{(1, 5, 25, 125), /\}$ is a lattice

The given poset is $\{(1, 5, 25, 125), /\}$

The corresponding Hassie diagram is



Least upper bound of $(1, 5) = 5$

greater lower bound of $(1, 5) = 1$

L.U.B of (1,25) = 25

G.U.B of (1,25) = 1

L.U.B. of (1,125) = 125

G.U.B of (1,125) = 1

L.U.B. of (5,25) = 25

G.U.B of (5,25) = 5

L.U.B of (5,125) = 125

G.U.B of (5,125) = 5

L.U.B of (25,125) = 125

G.U.B of (25,125) = 25

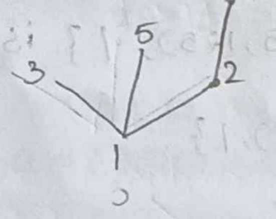
Every pair of poset as least upper bound and greater lower bound.

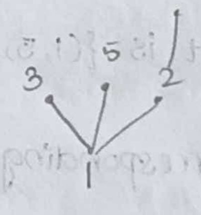
∴ The poset {(1,5, 25,125)/} is a lattice.

Determine the poset {(1,2,3,4,5)/} is a lattice.

Given poset {(1,2,3,4,5)/}

The Corresponding hassie diagram is





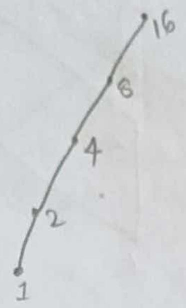
The order pairs (2, 3,) (3, 5), (2,5), (5,4) are not a least upper bound and greates lower bound does not exit.

So, the poset {1,2,3,4,5} is not a lattice.

Determine the poset {(1,2,4,8, 16),/}

The given poset {(1,2,4,8,16)/}

The corresponding hassie diagram is



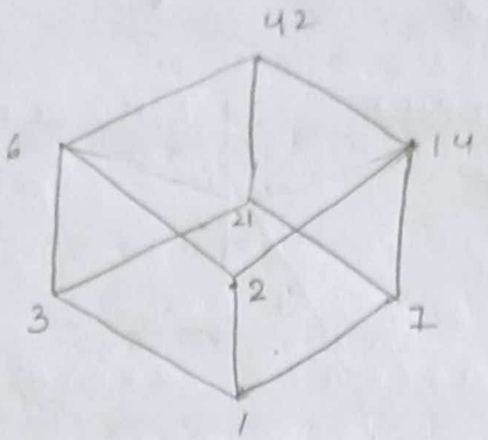
L.U.B of (1,2) = 2 ;	L.U.B of (2,4) = 4
G.U.B of (1,2) = 1 ;	G.U.B of (2,4) = 2
L.U.B of (4,8) = 8 ;	L.U.B of (8,16) = 16
G.U.B of (4,8) = 4 ;	G.U.B of (8,16) = 8
L.U.B of (1,4) = 4 ;	L.U.B of (1,8) = 8
L.U.B of (1,4) = 1 ;	G.U.B of (1,8) = 1
L.U.B of (1,16) = 16 ;	L.U.B of (2,8) = 8
G.U.B of (1,16) = 1 ;	G.U.B of (2,8) = 2
L.U.B of (2,16) = 16 ;	L.U.B of (4,16) = 16
G.U.B of (2,16) = 2 ;	G.U.B of (4,16) = 4

Every pair of poset as least upper bound and greater lower bound

∴ The poset { (1,2,4,8,16), | }.

Prove that D_{42} is a Complementary lattice.

Given, $D_{42} = \{1, 2, 3, 6, 7, 14, 21, 42\}$



(1, 42)

(2, 21)

(3, 14)

(6, 7)

$1 \oplus 42 = \text{l.c.m of } (1, 42)$

$= 42$

$1 * 42 = \text{g.c.d of } (1, 42)$

$= \cancel{42}\ 1$

The Complement of 1 is 42

$2 \oplus 21 = \text{l.c.m of } (2, 21)$

$= 42$

$2 * 21 = \text{g.c.d of } (2, 21)$

$= \cancel{42}\ 1$

The Complement of 2 is 21

$3 \oplus 14 = \text{l.c.m of } (3, 14)$

$= 42$

$3 * 14 = \text{G.c.d of } (3, 14)$

$= 1$

The Complement of ~~(3,14)~~ 3 is 14.

$6 \oplus 7 = \text{L.c.m of } (6, 7)$

$= 42$

$6 * 7 = \text{G.C.D of } (6, 7)$

$= 1$

The Complement of 6 is 7.

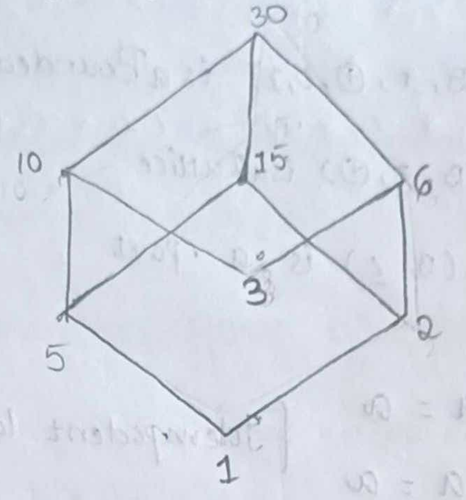
Every element of D_{42} is a Complement.

$\therefore$ D_{42} is a Complementary lattice.

Prove that D_{30} is a Complementary lattice.

Given,

$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$



(1,30)
(2,15)
(3,10)
(5,6)
(6,5)
(10,3)
(15,2)
(30,1)

$1 \oplus 30$ = L.C.M of (1,30)

= 30

$1 * 30$ = g.c.d of (1,30)

= 1

The Complement of 1 is 30.

$2 \oplus 15$ = L.C.M of (2,15)

= 30

$2 * 15$ = g.c.d of (2,15)

The Complement of 2 is 15.

$3 \oplus 10$ = L.C.M of (3,10)

= 30

$3 * 10$ = G.C.d of (3,10)

= 1

The Complement of 3 is 10

$5 \oplus 6$ = L.C.M of (5,6)

= 30

$5 * 6$ = G.C.D of (5,6)

= 1

The Complement of 5 is 6.

Every element of D_{30} is a Complement:

$\therefore$ D_{30} is a Complementary lattice.

02/06/23

Boolean Algebra:

A Boolean Algebra is a Complemented distributive lattice and it is denoted by $(B, *, \oplus, ', 0, 1)$ where,

where, $(B, *, \oplus, 0, 1)$ is a Bounded lattice.

where, $(B, *, \oplus)$ is a Lattice

where, $(B, \leq)$ is a Poset

Properties:

1. $a * a = a$
   $a \oplus a = a$ [Idempodent law]

2. $a * b = b * a$
   $a \oplus b = b \oplus a$ [Commutative Law]

3. $a * (b * c) = (a * b) * c$
   $a \oplus (b \oplus c) = (a \oplus b) \oplus c$ [Associative Law]

4. $a * (b \oplus c) = (a * b) \oplus (a * c)$
   $a \oplus (b * c) = (a \oplus b) * (a \oplus c)$ [Distributive Law]

5. $a * (a \oplus b) = a$
   $a \oplus (a * b) = a$ [Absorption Law]

6. $a * 0 = 0$
   $a \oplus 0 = a$

7. $a * 1 = a$
   $a \oplus 1 = 1$

8. $a' * a = 0$
   $a' \oplus a = 1$

9. $0' = 1$
   $1' = 0$

10. $(a * b)' = a' \oplus b'$

$(a \oplus b)' = a' * b'$ [Demorgon's law]

PDNF & PCNF :

* PDNF : A Boolean expression which is equalant to Some of min terms is Called, Sum of Product canonical form (or) Principal disjentive normal form. $(*) \oplus (*)$

* PCNF : The Complement of PDNF is Called PCNF

Express the following expression into PDNF which involves the variables x_1, x_2, x_3

$x_1 * x_2$

PDNF: $(x_1 * x_2) * 1.$

$(x_1 * x_2) * (x_3 \oplus x_3')$

$(x_1 * x_2 * x_3) \oplus (x_1 * x_2 * x_3')$

$\therefore$ The required PDNF is $(x_1 * x_2 * x_3) \oplus (x_1 * x_2 * x_3')$

PCNF: Complement of PDNF

$(x_1 \oplus x_2 \oplus x_3) * (x_1 \oplus x_2 \oplus x_3)$

Express the following expression $x_1 \oplus x_2$ into Sum of Product Canonical form which involves the variable x_1, x_2, x_3

$x_1 \oplus x_2$

PDNF: $(x_1 \oplus x_2) * 1.$

$(x_1 \oplus x_2) * (x_3 \oplus x_3').$

$(x_1 * (x_3 \oplus x_3')) \oplus (x_2 * (x_3 \oplus x_3')).$

$= (x_1 * x_3) \oplus (x_1 * x_3') \oplus (x_2 * x_3) \oplus (x_2 * x_3')$

$= ((x_1 * x_3) * 1) \oplus ((x_2 * x_3') * 1) \oplus ((x_2 * x_3) * 1) \oplus ((x_2 * x_3') * 1)$

$= ((x_1 * x_3) * (x_2 \oplus x_2')) \oplus (((x_2 * x_3') * (x_1 \oplus x_1')) \oplus ((x_2 * x_3) * (x_1 \oplus x_1')) \oplus (x_2 * x_3' * (x_1 \oplus x_1'))$

$\Rightarrow [((x_1 * x_3) * (x_2 \oplus x_2')) \oplus ((x_2 * x_3') * (x_1 \oplus x_1')) \oplus ((x_2 * x_3) * (x_1 \oplus x_1')) \oplus ((x_2 * x_3') * (x_1 \oplus x_1'))$

$\Rightarrow (x_1 * x_3 * x_2) \oplus (x_1 * x_3 * x_2') \oplus (x_2 * x_3' * x_1) \oplus (x_2 * x_3' * x_1') \oplus (x_2 * x_3 * x_1)$

$\oplus (x_2 * x_3 * x_1') \oplus (x_2 * x_3' * x_1) \oplus (x_2 * x_3' * x_1')$.

$\Rightarrow (x_1 * x_2 * x_3) \oplus (x_2 * x_3' * x_1') \oplus (x_2 * x_3' * x_1) \oplus (x_1 * x_3 * x_2') \oplus$

$(x_2 * x_3 * x_1')$.