Team Note of NLP

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Contents

1 DataStructure

1.1 Bipartite Union Find

Usage: Union-Find with friend, enemy relations

```
int P[_Sz], E[_Sz]; // Parent, Enemy, iota(P, P+_Sz, 0); memset(E, -1, sizeof E);
int find(int v){} bool merge(int u, int v){}
int set_friend(int u, int v){ return merge(u, v); }
int set_enemy(int u, int v){
  int ret = 0;
  if(E[u] == -1) E[u] = v; else ret += merge(E[u], v);
  if(E[v] == -1) E[v] = u; else ret += merge(u, E[v]);
  return ret;
}
```

1.2 Erasable Priority Queue

```
template<typename T, T inf>
struct pq_set{ // for max heap, inf=-1e18, less operator
  priority_queue<T, vector<T>, greater<T>> in, out; // min heap, inf = 1e18
  pq_set(){ in.push(inf); }
  void insert(T v){ in.push(v); } void erase(T v){ out.push(v); }
  T top(){
    while(out.size() && in.top() == out.top()) in.pop(), out.pop(); return in.top();
  }
  bool empty(){
    while(out.size() && in.top() == out.top()) in.pop(), out.pop(); return in.top() == inf;
  }
};
```

1.3 Convex Hull Trick

Usage: call init() before use

```
struct Line{
  ll a, b, c; // y = ax + b, c = line index
  Line(ll a, ll b, ll c) : a(a), b(b), c(c) {}
  ll f(ll x){ return a * x + b; }
};
vector<Line> v; int pv;
void init(){ v.clear(); pv = 0; }
int chk(const Line &a, const Line &b, const Line &c) const {
  return (__int128_t)(a.b - b.b) * (b.a - c.a) <= (__int128_t)(c.b - b.b) * (b.a - a.a);
}
void insert(Line l){
  if(v.size() > pv && v.back().a == l.a){
    if(l.b < v.back().b) l = v.back(); v.pop_back();
  }
  while(v.size() >= pv+2 && chk(v[v.size()-2], v.back(), l)) v.pop_back();
  v.push_back(l);
}
p query(ll x){ // if min query, then v[pv].f(x) >= v[pv+1].f(x)
  while(pv+1 < v.size() && v[pv].f(x) <= v[pv+1].f(x)) pv++;
  return {v[pv].f(x), v[pv].c};
}
///// line container start (max query) /////
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
}; // (for doubles, use inf = 1/.0, div(a,b) = a/b)
struct LineContainer : multiset<Line, less<>> {
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a % b); } // floor
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p) isect(x, erase(y));
  }
  ll query(ll x) { assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

1.4 Persistent Segment Tree

Usage: call init(root[0], s, e) before use

```
struct PSTNode{
  PSTNode *l, *r; int v;
  PSTNode(){ l = r = nullptr; v = 0; }
};
PSTNode *root[101010];
PST(){ memset(root, 0, sizeof root); } // constructor
void init(PSTNode *node, int s, int e){
  if(s == e) return;
  int m = s + e >> 1;
  node->l = new PSTNode; node->r = new PSTNode;
  init(node->l, s, m); init(node->r, m+1, e);
}
void update(PSTNode *prv, PSTNode *now, int s, int e, int x){
  if(s == e){ now->v = prv ? prv->v + 1 : 1; return; }
  int m = s + e >> 1;
  if(x <= m){
    now->l = new PSTNode; now->r = prv->r;
    update(prv->l, now->l, s, m, x);
  }
  else{
    now->r = new PSTNode; now->l = prv->l;
    update(prv->r, now->r, m+1, e, x);
  }
  int t1 = now->l ? now->l->v : 0;
  int t2 = now->r ? now->r->v : 0;
  now->v = t1 + t2;
}
int kth(PSTNode *prv, PSTNode *now, int s, int e, int k){
  if(s == e) return s;
```

```
  int m = s + e >> 1, diff = now->l->v - prv->l->v;
  if(k <= diff) return kth(prv->l, now->l, s, m, k);
  else return kth(prv->r, now->r, m+1, e, k-diff);
}
```

1.5 Splay Tree, Link-Cut Tree

```
struct Node{
  Node *l, *r, *p;
  bool flip; int sz;
  T now, sum, lz;
  Node(){ l = r = p = nullptr; sz = 1; flip = false; now = sum = lz = 0; }
  bool IsLeft() const { return p && this == p->l; }
  bool IsRoot() const { return !p || (this != p->l && this != p->r); }
  friend int GetSize(const Node *x){ return x ? x->sz : 0; }
  friend T GetSum(const Node *x){ return x ? x->sum : 0; }
  void Rotate(){
    p->Push(); Push();
    if(IsLeft()) r && (r->p = p), p->l = r, r = p;
    else l && (l->p = p), p->r = l, l = p;
    if(!p->IsRoot()) (p->IsLeft() ? p->p->l : p->p->r) = this;
    auto t = p; p = t->p; t->p = this; t->Update(); Update();
  }
  void Update(){
    sz = 1 + GetSize(l) + GetSize(r); sum = now + GetSum(l) + GetSum(r);
  }
  void Update(const T &val){ now = val; Update(); }
  void Push(){
    Update(now + lz); if(flip) swap(l, r);
    for(auto c : {l, r}) if(c) c->flip ^= flip, c->lz += lz;
    lz = 0; flip = false;
  }
};
Node* rt;
Node* Splay(Node *x, Node *g=nullptr){
  for(g || (rt=x); x->p!=g; x->Rotate()){
    if(!x->p->IsRoot()) x->p->p->Push(); x->p->Push(); x->Push();
    if(x->p->p != g) (x->IsLeft() ^ x->p->IsLeft() ? x : x->p)->Rotate();
  }
  x->Push(); return x;
}
Node* Kth(int k){
  for(auto x=rt; ; x=x->r){
    for(; x->Push(), x->l && x->l->sz > k; x=x->l);
    if(x->l) k -= x->l->sz;
    if(!k--) return Splay(x);
  }
}
Node* Gather(int s, int e){ auto t = Kth(e+1); return Splay(t, Kth(s-1))->l; }
Node* Flip(int s, int e){ auto x = Gather(s, e); x->flip ^= 1; return x; }
Node* Shift(int s, int e, int k){
  if(k >= 0){
    k %= e-s+1; if(k) Flip(s, e), Flip(s, s+k-1), Flip(s+k, e);
  }
  else{
    k = -k; k %= e-s+1; if(k) Flip(s, e), Flip(s, e-k), Flip(e-k+1, e);
  }
  return Gather(s, e);
}
int Idx(Node *x){ return x->l->sz; }
//////////// Link Cut Tree Start ////////////
Node* Splay(Node *x){
  for(; !x->IsRoot(); x->Rotate()){
    if(!x->p->IsRoot()) x->p->p->Push(); x->p->Push(); x->Push();
    if(!x->p->IsRoot()) (x->IsLeft() ^ x->p->IsLeft() ? x : x->p)->Rotate();
  }
  x->Push(); return x;
}
void Access(Node *x){
  Splay(x); x->r = nullptr; x->Update();
  for(auto y=x; x->p; Splay(x)) y = x->p, Splay(y), y->r = x, y->Update();
}
int GetDepth(Node *x){ Access(x); x->Push(); return GetSize(x->l); }
Node* GetRoot(Node *x){
  Access(x); for(x->Push(); x->l; x->Push()) x = x->l; return Splay(x);
}
Node* GetPar(Node *x){
  Access(x); x->Push(); if(!x->l) return nullptr;
  x = x->l; for(x->Push(); x->r; x->Push()) x = x->r;
  return Splay(x);
}
void Link(Node *p, Node *c){ Access(c); Access(p); c->l = p; p->p = c; c->Update(); }
void Cut(Node *c){ Access(c); c->l->p = nullptr; c->l = nullptr; c->Update(); }
Node* GetLCA(Node *x, Node *y){
  Access(x); Access(y); Splay(x); return x->p ? x->p : x;
}
Node* Ancestor(Node *x, int k){
  k = GetDepth(x) - k; assert(k >= 0);
  for(;;x->Push()){
    int s = GetSize(x->l); if(s == k) return Access(x), x;
    if(s < k) k -= s + 1, x = x->r; else x = x->l;
  }
}
void MakeRoot(Node *x){ Access(x); Splay(x); x->flip ^= 1; }
bool IsConnect(Node *x, Node *y){ return GetRoot(x) == GetRoot(y); }
void PathUpdate(Node *x, Node *y, T val){
  Node *root = GetRoot(x); // original root
  MakeRoot(x); Access(y);  // make x to root, tie with y
  Splay(x); x->lz += val; x->Push();
  MakeRoot(root);    // Revert
  Node *lca = GetLCA(x, y);
  Access(lca); Splay(lca); lca->Push();
  lca->Update(lca->now - val);
}
T VertexQuery(Node *x, Node *y){
  Node *l = GetLCA(x, y); T ret = l->now;
  Access(x); Splay(l); if(l->r) ret = ret + l->r->sum;
  Access(y); Splay(l); if(l->r) ret = ret + l->r->sum;
  return ret;
}
Node* GetQueryResultNode(Node *u, Node *v){
  if(GetRoot(u) != GetRoot(v)) return 0;
  MakeRoot(u); Access(v); auto ret = v->l;
  while(ret->mx != ret->v){
```

```
    if (ret->l && ret->mx == ret->l->mx) ret = ret->l;
    else ret = ret->r;
  }
  Access(ret); return ret;
}
```

2 Geometry

2.1 Rotating Calipers

```
pair<Point, Point> RotatingCalipers(const vector<Point> &H){
  ll mx = 0; Point a, b;
  for(int i=0, j=0; i<H.size(); i++){
    while(j+1 < H.size() && CCW(O, H[i+1]-H[i], H[j+1]-H[j]) >= 0){
      if(ll now = D2(H[i], H[j]); mx < now) mx = now, a = H[i], b = H[j];
      j++;
    }
    if(ll now = D2(H[i], H[j]); mx < now) mx = now, a = H[i], b = H[j];
  }
  return {a, b};
}
```

2.2 Point in Convex Polygon

```
bool Check(const vector<Point> &v, const Point &pt){
  if(CCW(v[0], v[1], pt) < 0) return false; int l = 1, r = v.size() - 1;
  while(l < r){
    int m = l + r + 1 >> 1;
    if(CCW(v[0], v[m], pt) >= 0) l = m; else r = m - 1;
  }
  if(l == v.size() - 1) return CCW(v[0], v.back(), pt) == 0 && v[0] <= pt && pt <= v.back();
  return CCW(v[0], v[l], pt) >= 0 && CCW(v[l], v[l+1], pt) >= 0 && CCW(v[l+1], v[0], pt) >= 0;
}
```

2.3 Half Plane Intersection, Tangent of Convex Hull

Usage: Line : $ax + by + c = 0$

```
double CCW(p1, p2, p3); bool same(double a, double b); const Point o = Point(0, 0);
struct Line{
  double a, b, c; Line() : Line(0, 0, 0) {}
  Line(double a, double b, double c) : a(a), b(b), c(c) {}
  bool operator < (const Line &l) const {
    bool f1 = Point(a, b) > o, f2 = Point(l.a, l.b) > o;
    if(f1 != f2) return f1 > f2;
    double cw = CCW(o, Point(a, b), Point(l.a, l.b));
    return same(cw, 0) ? c * hypot(l.a, l.b) < l.c * hypot(a, b) : cw > 0;
  }
  Point slope() const { return Point(a, b); }
};
Point LineIntersect(Line a, Line b){
  double det = a.a*b.b - b.a*a.b, x = (a.c*b.b - a.b*b.c) / det, y = (a.a*b.c - a.c*b.a) / det;
  return Point(x, y);
}
bool CheckHPI(Line a, Line b, Line c){
  if(CCW(o, a.slope(), b.slope()) <= 0) return 0;
  Point v = LineIntersect(a, b); return v.x*c.a + v.y*c.b >= c.c;
}
vector<Point> HPI(vector<Line> v){
  sort(v.begin(), v.end());
  deque<Line> dq; vector<Point> ret;
  for(auto &i : v){
    if(dq.size() && same(CCW(o, dq.back().slope(), i.slope()), 0)) continue;
    while(dq.size() >= 2 && CheckHPI(dq[dq.size()-2], dq.back(), i)) dq.pop_back();
    while(dq.size() >= 2 && CheckHPI(i, dq[0], dq[1])) dq.pop_front();
    dq.push_back(i);
  }
  while(dq.size() > 2 && CheckHPI(dq[dq.size()-2], dq.back(), dq[0])) dq.pop_back();
  while(dq.size() > 2 && CheckHPI(dq.back(), dq[0], dq[1])) dq.pop_front();
  for(int i=0; i<dq.size(); i++){
    Line now = dq[i], nxt = dq[(i+1)%dq.size()];
    if(CCW(o, now.slope(), nxt.slope()) <= eps) return vector<Point>();
    ret.push_back(LineIntersect(now, nxt));
  }
  for(auto &[x,y] : ret) x = -x, y = -y;
  return ret;
}
template<bool GET_MAX=true> // max - upper hull, min - lower hull
Point GetPoint(const vector<Point> &hull, double dy, double dx){ // given slope
  if(hull.size() == 1) return hull.front();
  if(dx < 0) dx = -dx, dy = -dy;
  if(dx == 0) return GET_MAX == (dy > 0) ? hull.front() : hull.back();
  auto cmp = [&](double a, double b){ return GET_MAX ? a < b : a > b; };
  if(cmp((hull[1].y - hull[0].y) * dx, (hull[1].x - hull[0].x) * dy)) return hull.front();
  int l = 1, r = (int)hull.size() - 1;
  while(l < r){
    int m = (l + r + 1) / 2;
    if(cmp((hull[m].y - hull[m-1].y) * dx, (hull[m].x - hull[m-1].x) * dy)) r = m - 1;
    else l = m;
  }
  return hull[l];
}
int ConvexTangent(const vector<Point> &v, const Point &pt, int up=1){ //given outer point
  auto sign = [&](ll c){ return c > 0 ? up : c == 0 ? 0 : -up; };
  auto local = [&](Point p, Point a, Point b, Point c){
    return sign(CCW(p, a, b)) <= 0 && sign(CCW(p, b, c)) >= 0;
  }; // assert(v.size() >= 2);
  int n = v.size() - 1, s = 0, e = n, m;
  if(local(pt, v[1], v[0], v[n-1])) return 0;
  while(s + 1 < e){
    m = (s + e) / 2;
    if(local(pt, v[m-1], v[m], v[m+1])) return m;
    if(sign(CCW(pt, v[s], v[s+1])) < 0){ // up
      if(sign(CCW(pt, v[m], v[m+1])) > 0) e = m;
      else if(sign(CCW(pt, v[m], v[s])) > 0) s = m; else e = m;
    }
    else{ // down
      if(sign(CCW(pt, v[m], v[m+1])) < 0) s = m;
      else if(sign(CCW(pt, v[m], v[s])) < 0) s = m; else e = m;
    }
  }
  if(s && local(pt, v[s-1], v[s], v[s+1])) return s;
  if(e != n && local(pt, v[e-1], v[e], v[e+1])) return e;
  return -1;
}
```

```
int Closest(const vector<Point> &v, const Point &out, int now){
  int prv = now > 0 ? now-1 : v.size()-1, nxt = now+1 < v.size() ? now+1 : 0, res = now;
  if(CCW(out, v[now], v[prv]) == 0 && Dist(out, v[res]) > Dist(out, v[prv])) res = prv;
  if(CCW(out, v[now], v[nxt]) == 0 && Dist(out, v[res]) > Dist(out, v[nxt])) res = nxt;
  return res; // if parallel, return closest point to out
} // int point_idx =  Closest(convex_hull, pt, ConvexTangent(hull + hull[0], pt, +-1) % N);
```

2.4 K-D Tree

```
T GetDist(const P &a, const P &b){ return (a.x-b.x) * (a.x-b.x) + (a.y-b.y) * (a.y-b.y); }
struct Node{
  P p; int idx;
  T x1, y1, x2, y2;
  Node(const P &p, const int idx) : p(p), idx(idx), x1(1e9), y1(1e9), x2(-1e9), y2(-1e9) {}
  bool contain(const P &pt)const{ return x1 <= pt.x && pt.x <= x2 && y1 <= pt.y && pt.y <= y2; }
  T dist(const P &pt) const { return idx == -1 ? INF : GetDist(p, pt); }
  T dist_to_border(const P &pt) const {
    const auto [x,y] = pt;
    if(x1 <= x && x <= x2) return min((y-y1)*(y-y1), (y2-y)*(y2-y));
    if(y1 <= y && y <= y2) return min((x-x1)*(x-x1), (x2-x)*(x2-x));
    T t11 = GetDist(pt, {x1,y1}), t12 = GetDist(pt, {x1,y2});
    T t21 = GetDist(pt, {x2,y1}), t22 = GetDist(pt, {x2,y2});
    return min({t11, t12, t21, t22});
  }
};
template<bool IsFirst = 1> struct Cmp {
  bool operator() (const Node &a, const Node &b) const {
    return IsFirst ? a.p.x < b.p.x : a.p.y < b.p.y;
  }
};
struct KDTree { // Warning : no duplicate
  constexpr static size_t NAIVE_THRESHOLD = 16;
  vector<Node> tree;
  KDTree() = default;
  explicit KDTree(const vector<P> &v) {
    for(int i=0; i<v.size(); i++) tree.emplace_back(v[i], i); Build(0, v.size());
  }
  template<bool IsFirst = 1>
  void Build(int l, int r) {
    if(r - l <= NAIVE_THRESHOLD) return;
    const int m = (l + r) >> 1;
    nth_element(tree.begin()+l, tree.begin()+m, tree.begin()+r, Cmp<IsFirst>{});
    for(int i=l; i<r; i++){
      tree[m].x1 = min(tree[m].x1, tree[i].p.x); tree[m].y1 = min(tree[m].y1, tree[i].p.y);
      tree[m].x2 = max(tree[m].x2, tree[i].p.x); tree[m].y2 = max(tree[m].y2, tree[i].p.y);
    }
    Build<!IsFirst>(l, m); Build<!IsFirst>(m + 1, r);
  }
  template<bool IsFirst = 1>
  void Query(const P &p, int l, int r, Node &res) const {
    if(r - l <= NAIVE_THRESHOLD){
      for(int i=l; i<r; i++) if(p != tree[i].p && res.dist(p) > tree[i].dist(p)) res = tree[i];
    }
    else{
      const int m = (l + r) >> 1;
      const T t = IsFirst ? p.x - tree[m].p.x : p.y - tree[m].p.y;
      if(p != tree[m].p && res.dist(p) > tree[m].dist(p)) res = tree[m];
      if(!tree[m].contain(p) && tree[m].dist_to_border(p) >= res.dist(p)) return;
      if(t < 0){
        Query<!IsFirst>(p, l, m, res);
        if(t*t < res.dist(p)) Query<!IsFirst>(p, m+1, r, res);
      }
      else{
        Query<!IsFirst>(p, m+1, r, res);
        if(t*t < res.dist(p)) Query<!IsFirst>(p, l, m, res);
      }
    }
  }
  int Query(const P& p) const {
    Node ret(make_pair<T>(1e9, 1e9), -1); Query(p, 0, tree.size(), ret); return ret.idx;
  }
};
```

2.5 Dual Graph

```
constexpr int quadrant_id(const Point p){
  constexpr int arr[9] = { 5, 4, 3, 6, -1, 2, 7, 0, 1 };
  return arr[sign(p.x)*3+sign(p.y)+4];
}
pair<vector<int>, int> dual_graph(const vector<Point> &points, const vector<pair<int,int>>
&edges){
  int n = points.size(), m = edges.size();
  vector<int> uf(2*m); iota(uf.begin(), uf.end(), 0);
  function<int(int)> find = [&](int v){ return v == uf[v] ? v : uf[v] = find(uf[v]); };
  function<bool(int,int)> merge = [&](int u, int v){ return find(u) != find(v) &&
  (uf[uf[u]]=uf[v], true); };
  vector<vector<pair<int,int>>> g(n);
  for(int i=0; i<m; i++){
    g[edges[i].first].emplace_back(edges[i].second, i);
    g[edges[i].second].emplace_back(edges[i].first, i);
  }
  for(int i=0; i<n; i++){
    const auto base = points[i];
    sort(g[i].begin(), g[i].end(), [&](auto a, auto b){
      auto p1 = points[a.first] - base, p2 = points[b.first] - base;
      return quadrant_id(p1) != quadrant_id(p2) ? quadrant_id(p1) < quadrant_id(p2) :
      p1.cross(p2) > 0;
    });
    for(int j=0; j<g[i].size(); j++){
      int k = j ? j - 1 : g[i].size() - 1;
      int u = g[i][k].second << 1, v = g[i][j].second << 1 | 1;
      auto p1 = points[g[i][k].first], p2 = points[g[i][j].first];
      if(p1 < base) u ^= 1; if(p2 < base) v ^= 1;
      merge(u, v);
    }
  }
  vector<int> res(2*m);
  for(int i=0; i<2*m; i++) res[i] = find(i);
  auto comp = res; compress(comp);
  for(auto &i : res) i = IDX(comp, i);
  int mx_idx = max_element(points.begin(), points.end()) - points.begin();
  return {res, res[g[mx_idx].back().second << 1 | 1]};
}
```

2.6 Bulldozer Trick (Rotating Sweep Line)

```
struct Line{
  ll i, j, dx, dy; // dx >= 0
  Line(int i, int j, const Point &pi, const Point &pj)
    : i(i), j(j), dx(pj.x-pi.x), dy(pj.y-pi.y) {}
  bool operator < (const Line &l) const {
    return make_tuple(dy*l.dx, i, j) < make_tuple(l.dy*dx, l.i, l.j);
  }
  bool operator == (const Line &l) const {
    return dy * l.dx == l.dy * dx;
  }
};
void Solve(){
  sort(A+1, A+N+1); iota(P+1, P+N+1, 1);
  vector<Line> V; V.reserve(N*(N-1)/2);
  for(int i=1; i<=N; i++) for(int j=i+1; j<=N; j++) V.emplace_back(i, j, A[i], A[j]);
  sort(V.begin(), V.end());
  for(int i=0, j=0; i<V.size(); i=j){
    while(j < V.size() && V[i] == V[j]) j++;
    for(int k=i; k<j; k++){
      int u = V[k].i, v = V[k].j; // point id, index -> Pos[id]
      swap(Pos[u], Pos[v]); swap(A[Pos[u]], A[Pos[v]]);
      if(Pos[u] > Pos[v]) swap(u, v);
      // @TODO
    }
  }
}
```

2.7 Smallest Enclosing Circle

```
pt getCenter(pt a, pt b){ return pt((a.x+b.x)/2, (a.y+b.y)/2); }
pt getCenter(pt a, pt b, pt c){
  pt aa = b - a, bb = c - a;
  auto c1 = aa*aa * 0.5, c2 = bb*bb * 0.5, d = aa / bb;
  auto x = a.x + (c1 * bb.y - c2 * aa.y) / d;
  auto y = a.y + (c2 * aa.x - c1 * bb.x) / d;
  return pt(x, y);
}
Circle solve(vector<pt> v){
  pt p = {0, 0};
  double r = 0; int n = v.size();
  for(int i=0; i<n; i++) if(dst(p, v[i]) > r + EPS){
    p = v[i]; r = 0;
    for(int j=0; j<i; j++) if(dst(p, v[j]) > r + EPS){
      p = getCenter(v[i], v[j]); r = dst(p, v[i]);
      for(int k=0; k<j; k++) if(dst(p, v[k]) > r + EPS){
        p = getCenter(v[i], v[j], v[k]); r = dst(v[k], p);
      }
    }
  }
  return {p, r};
}
```

2.8 Delaunay Triangulation

```
using lll = __int128; // using T = ll; (if coords are < 2e4)
// return true if p strictly within circumcircle(a,b,c)
bool inCircle(P p, P a, P b, P c) {
    a -= p, b -= p, c -= p; // assert(cross(a,b,c)>0);
    lll x = (lll)norm(a)*cross(b,c)+(lll)norm(b)*cross(c,a)+(lll)norm(c)*cross(a,b);
    return x*(ccw(a,b,c)>0?1:-1) > 0;
} using Q = struct Quad*;
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point
struct Quad {
    bool mark; Q o, rot; P p;
    P F() { return r()->p; } Q r() { return rot->rot; }
    Q prev() { return rot->o->rot; } Q next() { return r()->prev(); }
};
Q makeEdge(P orig, P dest) {
    Q q[]{new Quad{0,0,0,orig}, new Quad{0,0,0,arb}, new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
    FOR(i,4) q[i]->o = q[-i & 3], q[i]->rot = q[(i+1) & 3]; return *q;
}
void splice(Q a, Q b) { swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o); }
Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p); splice(q, a->next()); splice(q->r(), b);
    return q;
}
pair<Q,Q> rec(const vP& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.bk);
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = cross(s[0], s[1], s[2]); Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }
#define H(e) e->F(), e->p
#define valid(e) (cross(e->F(),H(base)) > 0)
    Q A, B, ra, rb;
    int half = sz(s) / 2;
    tie(ra, A) = rec({all(s)-half}); tie(B, rb) = rec({sz(s)-half+all(s)});
    while ((cross(B->p,H(A)) < 0 && (A = A->next())) || (cross(A->p,H(B)) > 0 && (B =
    B->r()->o)));
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (inCircle(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); e = t; \
    }
    while (1) {
        DEL(LC, base->r(), o);  DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && inCircle(H(RC), H(LC)))) base = connect(RC, base->r());
        else base = connect(base->r(), LC->r());
    }
    return {ra, rb};
}
V<AR<P,3>> triangulate(vP pts) {
    sor(pts); assert(unique(all(pts)) == end(pts)); // no duplicates
    if (sz(pts) < 2) return {};
    Q e = rec(pts).f; V<Q> q = {e};
    while (cross(e->o->F(), e->F(), e->p) < 0) e = e->o;
```

```
#define ADD { Q c = e; do { c->mark = 1; pts.pb(c->p); \
  q.pb(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    int qi = 0; while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    V<AR<P,3>> ret(sz(pts)/3); FOR(i,sz(pts)) ret[i/3][i%3] = pts[i];
    return ret;
}
```

3 Graph

3.1 Euler Tour

```
// Not Directed / Cycle
constexpr int SZ = 1010;
int N, G[SZ][SZ], Deg[SZ], Work[SZ];
void DFS(int v){
  for(int &i=Work[v]; i<=N; i++) while(G[v][I]) G[v][i]--, G[i][v]--, DFS(i);
  cout << v << " ";
}
// Directed / Path
void DFS(int v){
  for(int i=1; i<=pv; i++) while(G[v][i]) G[v][i]--, DFS(i);
  Path.push_back(v);
}
void Get(){
  for(int i=1; i<=pv; i++) if(In[i] < Out[i]){ DFS(i); return; }
  for(int i=1; i<=pv; i++) if(Out[i]){ DFS(i); return; }
}
```

3.2 SCC + 2-SAT

Usage: CNF: (A or B) / alwaysTrue: A =¿ B / setValue / mostOne / exactlyOne

```
struct TwoSat{ // True(x) = x << 1, False(x) = x << 1 | 1, Inv(x) = x ^ 1
  int n; vector<vector<int>> g; vector<int> result;
  TwoSat(int n, int m = 0) : n(n), g(n+n) { if(!m) g.reserve(m+m); }
  int addVar(){ g.emplace_back(); g.emplace_back(); return n++; }
  void addEdge(int s, int e){ g[s].push_back(e); }
  void addCNF(int a, int b){ addEdge(Inv(a), b); addEdge(Inv(b), a); } // (A or B)
  void setValue(int x){ addCNF(x, x); } // (A or A)
  void addAlwaysTrue(int a, int b){ addEdge(a, b); addEdge(Inv(b), Inv(a)); } // A => B
  void addMostOne(const vector<int> &li){
    if(li.empty()) return; int t;
    for(int i=0; i<li.size(); i++){
      t = addVar(); addAlwaysTrue(li[i], True(t));
      if(!i) continue;
      addAlwaysTrue(True(t-1), True(t)); addAlwaysTrue(True(t-1), Inv(li[i]));
    }
  }
  vector<int> val, comp, z; int pv = 0;
  int dfs(int i){
    int low = val[i] = ++pv, x; z.push_back(i);
    for(int e : g[i]) if(!comp[e]) low = min(low, val[e] ?: dfs(e));
    if(low == val[i]){
      do{
        x = z.back(); z.pop_back(); comp[x] = low;
        if (result[x>>1] == -1) result[x>>1] = ~x&1;
      }while(x != i);
    }
    return val[i] = low;
  }
  bool sat(){
    result.assign(n, -1); val.assign(2*n, 0); comp = val;
    for(int i=0; i<n+n; i++) if(!comp[i]) dfs(i);
    for(int i=0; i<n; i++) if(comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
  vector<int> getValue(){ return move(result); }
};
```

3.3 BCC

Usage: call tarjan() before use

```
vector<int> G[MAX_V]; int In[MAX_V], Low[MAX_V], P[MAX_V];
void addEdge(int s, int e){ G[s].push_back(e); G[e].push_back(s); }
void tarjan(int n){ /// Pre-Process
  int pv = 0;
  function<void(int,int)> dfs = [&pv,&dfs](int v, int b){
    In[v] = Low[v] = ++pv; P[v] = b;
    for(auto i : G[v]){
      if(i == b) continue;
      if(!In[i]) dfs(i, v), Low[v] = min(Low[v], Low[i]); else Low[v] = min(Low[v], In[i]);
    }
  };
  for(int i=1; i<=n; i++) if(!In[i]) dfs(i, -1);
}
vector<int> cutVertex(int n){
  vector<int> res; array<char,MAX_V> isCut; isCut.fill(0);
  function<void(int)> dfs = [&dfs,&isCut](int v){
    int ch = 0;
    for(auto i : G[v]){
      if(P[i] != v) continue; dfs(i); ch++;
      if(P[v] == -1 && ch > 1) isCut[v] = 1; else if(P[v] != -1 && Low[i] >= In[v]) isCut[v]=1;
    }
  };
  for(int i=1; i<=n; i++) if(P[i] == -1) dfs(i);
  for(int i=1; i<=n; i++) if(isCut[i]) res.push_back(i);
  return move(res);
}
vector<PII> cutEdge(int n){
  vector<PII> res;
  function<void(int)> dfs = [&dfs,&res](int v){
    for(int t=0; t<G[v].size(); t++){
      int i = G[v][t]; if(t != 0 && G[v][t-1] == G[v][t]) continue;
      if(P[i] != v) continue; dfs(i);
      if((t+1 == G[v].size() || i != G[v][t+1]) && Low[i] > In[v]) res.emplace_back(min(v,i),
      max(v,i));
    }
  };
  for(int i=1; i<=n; i++) sort(G[i].begin(), G[i].end()); // multi edge -> sort
  for(int i=1; i<=n; i++) if(P[i] == -1) dfs(i);
  return move(res); // sort(all(res));
}
vector<int> BCC[MAX_V]; // BCC[v] = components which contains v
void vertexDisjointBCC(int n){ // allow multi edge, not allow self loop
```

```
  int cnt = 0; array<char,MAX_V> vis; vis.fill(0);
  function<void(int,int)> dfs = [&dfs,&vis,&cnt](int v, int c){
    vis[v] = 1; if(c > 0) BCC[v].push_back(c);
    for(auto i : G[v]){
      if(vis[i]) continue;
      if(In[v] <= Low[i]) BCC[v].push_back(++cnt), dfs(i, cnt); else dfs(i, c);
    }
  };
  for(int i=1; i<=n; i++) if(!vis[i]) dfs(i, 0);
  for(int i=1; i<=n; i++) if(BCC[i].empty()) BCC[i].push_back(++cnt);
}
void edgeDisjointBCC(int n){} // remove cut edge, do flood fill
```

3.4 Maximum Clique

```
int N, M; ull G[40], MX, Clique; // 0-index, adj list with bitset, O(3^{N/3})
void get_clique(int R = 0, ull P = (1ULL<<N)-1, ull X = 0, ull V=0){
  if((P|X) == 0){ if(R > MX) MX = R, Clique = V; return; }
  int u = __builtin_ctzll(P|X); ll c = P&~G[u];
  while(c){
    int v = __builtin_ctzll(c);
    get_clique(R + 1, P&G[v], X&G[v], V | 1ULL << v);
    P ^= 1ULL << v; X |= 1ULL << v; c ^= 1ULL << v;
  }
}
```

3.5 Tree Isomorphism

```
struct Tree{ // (M1,M2)=(1e9+7, 1e9+9), P1,P2 = random int array(sz >= N+2)
  int N; vector<vector<int>> G; vector<pair<int,int>> H; vector<int> S, C; // size,centroid
  Tree(int N) : N(N), G(N+2), S(N+2), H(N+2) {}
  void addEdge(int s, int e){ G[s].push_back(e); G[e].push_back(s); }
  int getCentroid(int v, int b=-1){
    S[v] = 1; // do not merge if
    for(auto i : G[v]) if(i!=b) if(int now=getCentroid(i,v); now<=N/2) S[v]+=now; else break;
    if(N - S[v] <= N/2) C.push_back(v); return S[v] = S[v];
  }
  int init(){
    getCentroid(1); if(C.size() == 1) return C[0];
    int u = C[0], v = C[1], add = ++N;
    G[u].erase(find(G[u].begin(), G[u].end(), v)); G[v].erase(find(G[v].begin(), G[v].end(),
    u));
    G[add].push_back(u); G[u].push_back(add); G[add].push_back(v); G[v].push_back(add);
    return add;
  }
  pair<int,int> build(const vector<ll> &P1, const vector<ll> &P2, int v, int b=-1){
    vector<pair<int,int>> ch; for(auto i : G[v]) if(i != b) ch.push_back(build(P1, P2, i, v));
    ll h1 = 0, h2 = 0; sort(ch.begin(), ch.end()); if(ch.empty()){ return {1, 1}; }
    for(int i=0; i<ch.size(); i++) h1=(h1+ch[i].first*P1[i])%M1, h2=(h2+ch[i].second*P2[i])%M2;
    return H[v] = {h1, h2};
  }
  int build(const vector<ll> &P1, const vector<ll> &P2){
    int rt = init(); build(P1, P2, rt); return rt;
  }
};
```

3.6 Bipartite Matching

```
vector<int> G[SzL]; void AddEdge(int s, int e){ G[s].push_back(e); }
int D[SzL], L[SzL], R[SzR];
bitset<SzL> Visit; bitset<SzL+SzR> Track;
void clear(){ for(int i=0; i<SzL; i++) G[i].clear(); Track.reset(); }
bool BFS(int N){
  bool ret = false;
  queue<int> Q; memset(D, 0, sizeof D);
  for(int i=1; i<=N; i++) if(L[i] == -1 && !D[i]) Q.push(i), D[i] = 1;
  while(Q.size()){
    int v = Q.front(); Q.pop();
    for(const auto &i : G[v]){
      if(R[i] == -1) ret = true;
      else if(!D[R[i]]) D[R[i]] = D[v] + 1, Q.push(R[i]);
    }
  }
  return ret;
}
bool DFS(int v){
  if(Visit[v]) return false; Visit[v] = true;
  for(const auto &i : G[v]){
    if(R[i] == -1 || !Visit[R[i]] && D[R[i]] == D[v] + 1 && DFS(R[i])){ L[v] = i; R[i] = v;
    return true; }
  }
  return false;
}
int Match(int N){
  int ret = 0; memset(L, -1, sizeof L); memset(R, -1, sizeof R);
  while(BFS(N)){
    Visit.reset(); for(int i=1; i<=N; i++) if(L[i] == -1 && DFS(i)) ret++;
  }
  return ret;
}
void DFS2(int v, int N){
  if(Track[v]) return; Track[v] = true;
  for(const auto &i : G[v]) Track[i+N] = true, DFS2(R[i], N);
}
pair<vector<int>, vector<int>> MinVertexCover(int N, int M){
  Match(N); for(int i=1; i<=N; i++) if(L[i] == -1) DFS2(i, N);
  vector<int> a, b;
  for(int i=1; i<=N; i++) if(!Track[i]) a.push_back(i);
  for(int i=N+1; i<=N+M; i++) if(Track[i]) b.push_back(i-N);
  return make_pair(a, b);
}
```

3.7 Push Relabel

```
template<typename flow_t> struct Edge {
  int u, v, r; flow_t c, f;
  Edge() = default;
  Edge(int u, int v, flow_t c, int r) : u(u), v(v), r(r), c(c), f(0) {}
};
template<typename flow_t, size_t _Sz> struct PushRelabel {
  using edge_t = Edge<flow_t>;
  int n, b, dist[_Sz], count[_Sz+1];
  flow_t excess[_Sz]; bool active[_Sz];
  vector<edge_t> g[_Sz]; vector<int> bucket[_Sz];
```

```
  void clear(){ for(int i=0; i<_Sz; i++) g[i].clear(); }
  void addEdge(int s, int e, flow_t x){
    g[s].emplace_back(s, e, x, (int)g[e].size());
    if(s == e) g[s].back().r++;
    g[e].emplace_back(e, s, 0, (int)g[s].size()-1);
  }
  void enqueue(int v){
    if(!active[v] && excess[v] > 0 && dist[v] < n){
      active[v] = true; bucket[dist[v]].push_back(v); b = max(b, dist[v]);
    }
  }
  void push(edge_t &e){
    flow_t fl = min(excess[e.u], e.c - e.f);
    if(dist[e.u] == dist[e.v] + 1 && fl > flow_t(0)){
      e.f += fl; g[e.v][e.r].f -= fl; excess[e.u] -= fl; excess[e.v] += fl; enqueue(e.v);
    }
  }
  void gap(int k){
    for(int i=0; i<n; i++){
      if(dist[i] >= k) count[dist[i]]--, dist[i] = max(dist[i], n), count[dist[i]]++;
      enqueue(i);
    }
  }
  void relabel(int v){
    count[dist[v]]--; dist[v] = n;
    for(const auto &e : g[v]) if(e.c - e.f > 0) dist[v] = min(dist[v], dist[e.v] + 1);
    count[dist[v]]++; enqueue(v);
  }
  void discharge(int v){
    for(auto &e : g[v]) if(excess[v] > 0) push(e); else break;
    if(excess[v] > 0) if(count[dist[v]] == 1) gap(dist[v]); else relabel(v);
  }
  flow_t maximumFlow(int _n, int s, int t){
    memset(dist, 0, sizeof dist); memset(excess, 0, sizeof excess);
    memset(count, 0, sizeof count); memset(active, 0, sizeof active);
    n = _n; b = 0;
    for(auto &e : g[s]) excess[s] += e.c;
    count[s] = n; enqueue(s); active[t] = true;
    while(b >= 0){
      if(bucket[b].empty()) b--;
      else{
        int v = bucket[b].back(); bucket[b].pop_back();
        active[v] = false; discharge(v);
      }
    }
    return excess[t];
  }
};
```

3.8 LR Flow

```
addEdge(t, s, inf) // 기존 싱크 -> 기존 소스 inf
addEdge(s, nt, l) // s -> 새로운 싱크 l
addEdge(ns, e, l) // 새로운 소스 -> e l
addEdge(a, b, r-l) // s -> e (r-l)
// ns -> nt의 max flow == l들의 합 확인
// maxflow : s -> t 플로우 찾을 수 있을 때까지 반복
```

3.9 Hungarian Method

```
// 1-based, only for minimum matching, maximum matching may get TLE
template<typename cost_t=int, cost_t _INF=0x3f3f3f3f>
struct Hungarian{
  int n; vector<vector<cost_t>> mat;
  Hungarian(int n) : n(n), mat(n+1, vector<cost_t>(n+1, _INF)) {}
  void addEdge(int s, int e, cost_t x){ mat[s][e] = min(mat[s][e], x); }
  pair<cost_t, vector<int>> run(){
    vector<cost_t> u(n+1), v(n+1), m(n+1);
    vector<int> p(n+1), w(n+1), c(n+1);
    for(int i=1,a,b; i<=n; i++){
      p[0] = i; b = 0; fill(m.begin(), m.end(), _INF); fill(c.begin(), c.end(), 0);
      do{
        int nxt; cost_t delta = _INF; c[b] = 1; a = p[b];
        for(int j=1; j<=n; j++){
          if(c[j]) continue;
          cost_t t = mat[a][j] - u[a] - v[j];
          if(t < m[j]) m[j] = t, w[j] = b;
          if(m[j] < delta) delta = m[j], nxt = j;
        }
        for(int j=0; j<=n; j++){
          if(c[j]) u[p[j]] += delta, v[j] -= delta; else m[j] -= delta;
        }
        b = nxt;
      }while(p[b] != 0);
      do{ int nxt = w[b]; p[b] = p[nxt]; b = nxt; }while(b != 0);
    }
    vector<int> assign(n+1); for(int i=1; i<=n; i++) assign[p[i]] = i;
    return {-v[0], assign};
  }
};
```

3.10 $O(V^3)$ Global Min Cut

```
int vertex, g[S][S], dst[S], chk[S], del[S];
void init(){
  memset(g, 0, sizeof g); memset(del, 0, sizeof del);
}
void addEdge(int s, int e, int x){ g[s][e] = g[e][s] = x; }
int minCutPhase(int &s, int &t){
  memset(dst, 0, sizeof dst);
  memset(chk, 0, sizeof chk);
  int mincut = 0;
  for(int i=1; i<=vertex; i++){
    int k = -1, mx = -1;
    for(int j=1; j<=vertex; j++) if(!del[j] && !chk[j])
      if(dst[j] > mx) k = j, mx = dst[j];
    if(k == -1) return mincut;
    s = t, t = k;
    mincut = mx, chk[k] = 1;
    for(int j=1; j<=vertex; j++){
      if(!del[j] && !chk[j]) dst[j] += g[k][j];
    }
  }
  return mincut;
}
int getMinCut(int n){
```

```
  vertex = n; int mincut = 1e9+7;
  for(int i=1; i<vertex; i++){
    int s, t;
    int now = minCutPhase(s, t);
    mincut = min(mincut, now); del[t] = 1;
    if(mincut == 0) return 0;
    for(int j=1; j<=vertex; j++){
      if(!del[j]) g[s][j] = (g[j][s] += g[j][t]);
    }
  }
  return mincut;
}
```

3.11 Gomory-Hu Tree

```
// 0-based, S-T cut in graph == S-T cut in gomory-hu tree (path minimum)
vector<Edge> GomoryHuTree(int n, const vector<Edge> &e){
    Dinic<int,100> Flow;
    vector<Edge> res(n-1); vector<int> pr(n);
    for(int i=1; i<n; i++, Flow.clear()){
        for(const auto &[s,e,x] : e) Flow.AddEdge(s, e, x); // bi-directed
        int fl = Flow.MaxFlow(pr[i], i);
        for(int j=i+1; j<n; j++){
            if(!Flow.Level[i] == !Flow.Level[j] && pr[i] == pr[j]) pr[j] = i;
        }
        res[i-1] = Edge(pr[i], i, fl);
    }
    return res;
}
```

3.12 Rectlinear MST

```
template<class T> vector<tuple<T, int, int>>
rectilinear_minimum_spanning_tree(vector<point<T>> a){
  int n = a.size();
  vector<int> ind(n);
  iota(ind.begin(), ind.end(), 0);
  vector<tuple<T, int, int>> edge;
  for(int k=0; k<4; k++){
    sort(ind.begin(), ind.end(), [&](int i,int j){return a[i].x-a[j].x < a[j].y-a[i].y;});
    map<T, int> mp;
    for(auto i: ind){
      for(auto it=mp.lower_bound(-a[i].y); it!=mp.end(); it=mp.erase(it)){
        int j = it->second; point<T> d = a[i] - a[j];
        if(d.y > d.x) break;
        edge.push_back({d.x + d.y, i, j});
      }
      mp.insert({-a[i].y, i});
    }
    for(auto &p: a) if(k & 1) p.x = -p.x; else swap(p.x, p.y);
  }
  sort(edge.begin(), edge.end());
  disjoint_set dsu(n);
  vector<tuple<T, int, int>> res;
  for(auto [x, i, j]: edge) if(dsu.merge(i, j)) res.push_back({x, i, j});
  return res;
}
```

3.13 $O((V+E)\log V)$ Dominator Tree

```
vector<int> DominatorTree(const vector<vector<int>> &g, int src){ // // 0-based
  int n = g.size();
  vector<vector<int>> rg(n), buf(n);
  vector<int> r(n), val(n), idom(n, -1), sdom(n, -1), o, p(n), u(n);
  iota(all(r), 0); iota(all(val), 0);
  for(int i=0; i<n; i++) for(auto j : g[i]) rg[j].push_back(i);
  function<int(int)> find = [&](int v){
    if(v == r[v]) return v;
    int ret = find(r[v]);
    if(sdom[val[v]] > sdom[val[r[v]]]) val[v] = val[r[v]];
    return r[v] = ret;
  };
  function<void(int)> dfs = [&](int v){
    sdom[v] = o.size(); o.push_back(v);
    for(auto i : g[v]) if(sdom[i] == -1) p[i] = v, dfs(i);
  };
  dfs(src); reverse(all(o));
  for(auto &i : o){
    if(sdom[i] == -1) continue;
    for(auto j : rg[i]){
      if(sdom[j] == -1) continue;
      int x = val[find(j), j];
      if(sdom[i] > sdom[x]) sdom[i] = sdom[x];
    }
    buf[o[o.size() - sdom[i] - 1]].push_back(i);
    for(auto j : buf[p[i]]) u[j] = val[find(j), j];
    buf[p[i]].clear();
    r[i] = p[i];
  }
  reverse(all(o)); idom[src] = src;
  for(auto i : o){ // WARNING : if different, takes idom
    if(i != src) idom[i] = sdom[i] == sdom[u[i]] ? sdom[i] : idom[u[i]];
  }
  for(auto i : o) if(i != src) idom[i] = o[idom[i]];
  return idom; // unreachable -> ret[i] = -1
}
```

3.14 $O(N^2)$ Stable Marriage Problem

```
// man : 1~n, woman : n+1~2n
struct StableMarriage{
  int n; vector<vector<int>> g;
  StableMarriage(int n) : n(n), g(2*n+1) { for(int i=1; i<=n+n; i++) g[i].reserve(n); }
  void addEdge(int u, int v){  g[u].push_back(v); } // insert in decreasing order of preference.
  vector<int> run(){
    queue<int> q; vector<int> match(2*n+1), ptr(2*n+1);
    for(int i=1; i<=n; i++) q.push(i);
    while(q.size()){
      int i = q.front(); q.pop();
      for(int &p=ptr[i]; p<g[i].size(); p++){
        int j = g[i][p];
        if(!match[j]){ match[i] = j; match[j] = i; break; }
        int m = match[j], u = -1, v = -1;
        for(int k=0; k<g[j].size(); k++){
          if(g[j][k] == i) u = k; if(g[j][k] == m) v = k;
        }
```

```
      if(u < v){
        match[m] = 0; q.push(m); match[i] = j; match[j] = i; break;
      }
    }
  }
  return match;
}
};
```

3.15 $O(VE)$ Vizing Theorem

```
// Graph coloring with (max-degree)+1 colors, O(N^2)
int C[MX][MX] = {}, G[MX][MX] = {}; // MX ~= 2500
void solve(vector<pii> &E, int N, int M){
  int X[MX] = {}, a, b;
  auto update = [&](int u){ for(X[u] = 1; C[u][X[u]]; X[u]++); };
  auto color = [&](int u, int v, int c){
    int p = G[u][v]; G[u][v] = G[v][u] = c;
    C[u][c] = v; C[v][c] = u; C[u][p] = C[v][p] = 0;
    if( p ) X[u] = X[v] = p; else update(u), update(v);
    return p; }; // end of function : color
  auto flip = [&](int u, int c1, int c2){
    int p = C[u][c1], q = C[u][c2];
    swap(C[u][c1], C[u][c2]);
    if( p ) G[u][p] = G[p][u] = c2;
    if( !C[u][c1] ) X[u] = c1; if( !C[u][c2] ) X[u] = c2;
    return p; }; // end of function : flip
  for(int i = 1; i <= N; i++) X[i] = 1;
  for(int t = 0; t < E.size(); t++){
    int u = E[t].first, v0 = E[t].second, v = v0, c0 = X[u], c = c0, d;
    vector<pii> L; int vst[MX] = {};
    while(!G[u][v0]){
      L.emplace_back(v, d = X[v]);
      if(!C[v][c]) for(a = (int)L.size()-1; a >= 0; a--) c = color(u, L[a].first, c);
      else if(!C[u][d])for(a=(int)L.size()-1;a>=0;a--)color(u,L[a].first,L[a].second);
      else if( vst[d] ) break;
      else vst[d] = 1, v = C[u][d];
    }
    if( !G[u][v0] ){
      for(;v; v = flip(v, c, d), swap(c, d));
      if(C[u][c0]){
        for(a = (int)L.size()-2; a >= 0 && L[a].second != c; a--);
        for(; a >= 0; a--) color(u, L[a].first, L[a].second);
      } else t--;
    }
  }
}
```

3.16 $O(E \log V)$ Directed MST

```
struct Edge{
  int s, e; cost_t x;
  Edge() = default;
  Edge(int s, int e, cost_t x) : s(s), e(e), x(x) {}
  bool operator < (const Edge &t) const { return x < t.x; }
};
struct UnionFind{
  vector<int> P, S;
  vector<pair<int, int>> stk;
  UnionFind(int n) : P(n), S(n, 1) { iota(P.begin(), P.end(), 0); }
  int find(int v) const { return v == P[v] ? v : find(P[v]); }
  int time() const { return stk.size(); }
  void rollback(int t){
    while(stk.size() > t){
      auto [u,v] = stk.back(); stk.pop_back();
      P[u] = u; S[v] -= S[u];
    }
  }
  bool merge(int u, int v){
    u = find(u); v = find(v);
    if(u == v) return false;
    if(S[u] > S[v]) swap(u, v);
    stk.emplace_back(u, v);
    S[v] += S[u]; P[u] = v;
    return true;
  }
};
struct Node{
  Edge key;
  Node *l, *r;
  cost_t lz;
  Node() : Node(Edge()) {}
  Node(const Edge &edge) : key(edge), l(nullptr), r(nullptr), lz(0) {}
  void push(){
    key.x += lz;
    if(l) l->lz += lz;
    if(r) r->lz += lz;
    lz = 0;
  }
  Edge top(){ push(); return key; }
};
Node* merge(Node *a, Node *b){
  if(!a || !b) return a ? a : b;
  a->push(); b->push();
  if(b->key < a->key) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node* &a){ a->push(); a = merge(a->l, a->r); }

// 0-based
pair<cost_t, vector<int>> DirectMST(int n, int rt, vector<Edge> &edges){
  vector<Node*> heap(n);
  UnionFind uf(n);
  for(const auto &i : edges) heap[i.e] = merge(heap[i.e], new Node(i));
  cost_t res = 0;
  vector<int> seen(n, -1), path(n), par(n);
  seen[rt] = rt;
  vector<Edge> Q(n), in(n, {-1,-1, 0}), comp;
  deque<tuple<int, int, vector<Edge>>> cyc;
  for(int s=0; s<n; s++){
    int u = s, qi = 0, w;
    while(seen[u] < 0){
      if(!heap[u]) return {-1, {}};
```

```
      Edge e = heap[u]->top();
      heap[u]->lz -= e.x; pop(heap[u]);
      Q[qi] = e; path[qi++] = u; seen[u] = s;
      res += e.x; u = uf.find(e.s);
      if(seen[u] == s){ // found cycle, contract
        Node* nd = 0;
        int end = qi, time = uf.time();
        do nd = merge(nd, heap[w = path[--qi]]); while(uf.merge(u, w));
        u = uf.find(u); heap[u] = nd; seen[u] = -1;
        cyc.emplace_front(u, time, vector<Edge>{&Q[qi], &Q[end]});
      }
    }
    for(int i=0; i<qi; i++) in[uf.find(Q[i].e)] = Q[i];
  }
  for(auto& [u,t,comp] : cyc){
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.e)] = e;
    in[uf.find(inEdge.e)] = inEdge;
  }
  for(int i=0; i<n; i++) par[i] = in[i].s;
  return {res, par};
}
```

3.17 $O(E \log V + K \log K)$ K Shortest Path

```
int rnd(int l, int r){ /* return random int [l,r] */ }
struct node{
  array<node*, 2> son; pair<ll, ll> val;
  node() : node(make_pair(-1e18, -1e18)) {}
  node(pair<ll, ll> val) : node(nullptr, nullptr, val) {}
  node(node *l, node *r, pair<ll, ll> val) : son({l,r}), val(val) {}
};
node* copy(node *x){ return x ? new node(x->son[0], x->son[1], x->val) : nullptr; }
node* merge(node *x, node *y){ // precondition: x, y both points to new entity
  if(!x || !y) return x ? x : y;
  if(x->val > y->val) swap(x, y);
  int rd = rnd(0, 1);
  if(x->son[rd]) x->son[rd] = copy(x->son[rd]);
  x->son[rd] = merge(x->son[rd], y); return x;
}
struct edge{
  ll v, c, i; edge() = default;
  edge(ll v, ll c, ll i) : v(v), c(c), i(i) {}
};
vector<vector<edge>> gph, rev;
int idx;
void init(int n){ gph = rev = vector<vector<edge>>(n); idx = 0; }
void add_edge(int s, int e, ll x){
  gph[s].emplace_back(e, x, idx);
  rev[e].emplace_back(s, x, idx);
  assert(x >= 0); idx++;
}
vector<int> par, pae; vector<ll> dist; vector<node*> heap;
void dijkstra(int snk){ // replace this to SPFA if edge weight is negative
  int n = gph.size();
  par = pae = vector<int>(n, -1);
  dist = vector<ll>(n, 0x3f3f3f3f3f3f3f3f);
  heap = vector<node*>(n, nullptr);
  priority_queue<pair<ll,ll>, vector<pair<ll,ll>>, greater<>> pq;
  auto enqueue = [&](int v, ll c, int pa, int pe){
    if(dist[v] > c) dist[v] = c, par[v] = pa, pae[v] = pe, pq.emplace(c, v);
  }; enqueue(snk, 0, -1, -1); vector<int> ord;
  while(!pq.empty()){
    auto [c,v] = pq.top(); pq.pop(); if(dist[v] != c) continue;
    ord.push_back(v); for(auto e : rev[v]) enqueue(e.v, c+e.c, v, e.i);
  }
  for(auto &v : ord){
    if(par[v] != -1) heap[v] = copy(heap[par[v]]);
    for(auto &e : gph[v]){
      if(e.i == pae[v]) continue;
      ll delay = dist[e.v] + e.c - dist[v];
      if(delay < 1e18) heap[v] = merge(heap[v], new node(make_pair(delay, e.v)));
    }
  }
}
vector<ll> run(int s, int e, int k){
  using state = pair<ll, node*>; dijkstra(e); vector<ll> ans;
  priority_queue<state, vector<state>, greater<state>> pq;
  if(dist[s] > 1e18) return vector<ll>(k, -1);
  ans.push_back(dist[s]);
  if(heap[s]) pq.emplace(dist[s] + heap[s]->val.first, heap[s]);
  while(!pq.empty() && ans.size() < k){
    auto [cst, ptr] = pq.top(); pq.pop(); ans.push_back(cst);
    for(int j=0; j<2; j++) if(ptr->son[j])
      pq.emplace(cst-ptr->val.first + ptr->son[j]->val.first, ptr->son[j]);
    int v = ptr->val.second;
    if(heap[v]) pq.emplace(cst + heap[v]->val.first, heap[v]);
  }
  while(ans.size() < k) ans.push_back(-1);
  return ans;
}
```

3.18 Chordal Graph, Tree Decomposition

```
struct Set {
  list<int> L; int last;
  Set() { last = 0; }
};
struct PEO {
  int N;
  vector<vector<int> > g;
  vector<int> vis, res;
  list<Set> L;
  vector<list<Set>::iterator> ptr;
  vector<list<int>::iterator> ptr2;
  PEO(int n, vector<vector<int> > _g) {
    N = n; g = _g;
    for (int i = 1; i <= N; i++) sort(g[i].begin(), g[i].end());
    vis.resize(N + 1); ptr.resize(N + 1); ptr2.resize(N + 1);
    L.push_back(Set());
    for (int i = 1; i <= N; i++) {
      L.back().L.push_back(i);
      ptr[i] = L.begin(); ptr2[i] = prev(L.back().L.end());
```

```
    }
  }
  pair<bool, vector<int>> Run() {
    // lexicographic BFS
    int time = 0;
    while (!L.empty()) {
      if (L.front().L.empty()) { L.pop_front(); continue; }
      auto it = L.begin();
      int n = it->L.front(); it->L.pop_front();
      vis[n] = ++time;
      res.push_back(n);
      for (int next : g[n]) {
        if (vis[next]) continue;
        if (ptr[next]->last != time) {
          L.insert(ptr[next], Set()); ptr[next]->last = time;
        }
        ptr[next]->L.erase(ptr2[next]); ptr[next]--;
        ptr[next]->L.push_back(next);
        ptr2[next] = prev(ptr[next]->L.end());
      }
    }
    // PEO existence check
    for (int n = 1; n <= N; n++) {
      int mx = 0;
      for (int next : g[n]) if (vis[n] > vis[next]) mx = max(mx, vis[next]);
      if (mx == 0) continue;
      int w = res[mx - 1];
      for (int next : g[n]) {
        if (vis[w] > vis[next] && !binary_search(g[w].begin(), g[w].end(), next)){
          vector<int> chk(N+1), par(N+1, -1); // w와 next가 이어져 있지 않다면 not chordal
          deque<int> dq{next}; chk[next] = 1;
          while (!dq.empty()) {
            int x = dq.front(); dq.pop_front();
            for (auto y : g[x]) {
              if (chk[y] || y == n || y != w && binary_search(g[n].begin(), g[n].end(), y))
                continue;
              dq.push_back(y); chk[y] = 1; par[y] = x;
            }
          }
          vector<int> cycle{next, n};
          for (int x=w; x!=next; x=par[x]) cycle.push_back(x);
          return {false, cycle};
        }
      }
    }
    reverse(res.begin(), res.end());
    return {true, res};
  }
};
bool vis[200201]; // 배열 크기 알아서 수정하자.
int p[200201], ord[200201], P = 0; // P=정점 개수
vector<int> V[200201], G[200201]; // V=bags, G=edges
void tree_decomposition(int N, vector<vector<int> > g) {
  for(int i=1; i<=N; i++) sort(g[i].begin(), g[i].end());
  vector<int> peo = PEO(N, g).Run(), rpeo = peo;
  reverse(rpeo.begin(), rpeo.end());
  for(int i=0; i<peo.size(); i++) ord[peo[i]] = i;
  for(int n : rpeo) { // tree decomposition
    vis[n] = true;
    if (n == rpeo[0]) { // 처음
      P++; V[P].push_back(n); p[n] = P; continue;
    }
    int mn = INF, idx = -1;
    for(int next : g[n]) if (vis[next] && mn > ord[next]) mn = ord[next], idx = next;
    assert(idx != -1); idx = p[idx];
    // 두 set인 V[idx]와 g[n](visited ver)가 같나?
    // V[idx]의 모든 원소가 g[n]에서 나타나는지 판별로 충분하다.
    int die = 0;
    for(int x : V[idx]) {
      if (!binary_search(g[n].begin(), g[n].end(), x)) { die = 1; break; }
    }
    if (!die) { V[idx].push_back(n), p[n] = idx; } // 기존 집합에 추가
    else { // 새로운 집합을 자식으로 추가
      P++;
      G[idx].push_back(P); // 자식으로만 단방향으로 잇자.
      V[P].push_back(n);
      for(int next : g[n]) if (vis[next]) V[P].push_back(next);
      p[n] = P;
    }
  }
  for(int i=1; i<=P; i++) sort(V[i].begin(), V[i].end());
}
```

3.19 $O(V^3)$ General Matching

```
int N, M, R, Match[555], Par[555], Chk[555], Prv[555], Vis[555];
vector<int> G[555];
int Find(int x){ return x == Par[x] ? x : Par[x] = Find(Par[x]); }
int LCA(int u, int v){ static int cnt = 0;
  for(cnt++; Vis[u]!=cnt; swap(u, v)) if(u) Vis[u] = cnt, u = Find(Prv[Match[u]]);
  return u;
}
void Blossom(int u, int v, int rt, queue<int> &q){
  for(; Find(u)!=rt; u=Prv[v]){
    Prv[u] = v; Par[u] = Par[v=Match[u]] = rt; if(Chk[v] & 1) q.push(v), Chk[v] = 2;
  }
}
bool Augment(int u){
  iota(Par, Par+555, 0); memset(Chk, 0, sizeof Chk); queue<int> Q; Q.push(u); Chk[u] = 2;
  while(!Q.empty()){
    u = Q.front(); Q.pop();
    for(auto v : G[u]){
      if(Chk[v] == 0){
        Prv[v] = u; Chk[v] = 1; Q.push(Match[v]); Chk[Match[v]] = 2;
        if(!Match[v]){ for(; u; v=u) u = Match[Prv[v]], Match[Match[v]=Prv[v]] = v; return true;
        }
      }
      else if(Chk[v] == 2){ int l = LCA(u, v); Blossom(u, v, l, Q), Blossom(v, u, l, Q); }
    }
  }
  return 0;
}
void Run(){ for(int i=1; i<=N; i++) if(!Match[i]) R += Augment(i); }
```

3.20 $O(V^3)$ Weighted General Matching

```
namespace weighted_blossom_tree{
  #define d(x) (lab[x.u]+lab[x.v]-e[x.u][x.v].w*2)
  const int N=403*2; using ll = long long; using T = int; // sum of weight, single weight
  const T inf=numeric_limits<T>::max()>>1;
  struct Q{ int u, v; T w; } e[N][N]; vector<int> p[N];
  int n, m=0, id, h, t, lk[N], sl[N], st[N], f[N], b[N][N], s[N], ed[N], q[N]; T lab[N];
  void upd(int u, int v){ if (!sl[v] || d(e[u][v]) < d(e[sl[v]][v])) sl[v] = u; }
  void ss(int v){
    sl[v]=0; for(int u=1; u<=n; u++) if(e[u][v].w > 0 && st[u] != v && !s[st[u]]) upd(u, v);
  }
  void ins(int u){ if(u <= n) q[++t] = u; else for(int v : p[u]) ins(v); }
  void mdf(int u, int w){ st[u]=w; if(u > n) for(int v : p[u]) mdf(v, w); }
  int gr(int u,int v){
    if ((v=find(p[u].begin(), p[u].end(), v) - p[u].begin()) & 1){
      reverse(p[u].begin()+1, p[u].end()); return (int)p[u].size() - v;
    }
    return v;
  }
  void stm(int u, int v){
    lk[u] = e[u][v].v;
    if(u <= n) return; Q w = e[u][v];
    int x = b[u][w.u], y = gr(u,x);
    for(int i=0; i<y; i++) stm(p[u][i], p[u][i^1]);
    stm(x, v); rotate(p[u].begin(), p[u].begin()+y, p[u].end());
  }
  void aug(int u, int v){
    int w = st[lk[u]]; stm(u, v); if (!w) return;
    stm(w, st[f[w]]); aug(st[f[w]], w);
  }
  int lca(int u, int v){
    for(++id; u|v; swap(u, v)){
      if(!u) continue; if(ed[u] == id) return u;
      ed[u] = id; if(u = st[lk[u]]) u = st[f[u]]; // not ==
    }
    return 0;
  }
  void add(int u, int a, int v){
    int x = n+1; while(x <= m && st[x]) x++;
    if(x > m) m++;
    lab[x] = s[x] = st[x] = 0; lk[x] = lk[a];
    p[x].clear(); p[x].push_back(a);
    for(int i=u, j; i!=a; i=st[f[j]]) p[x].push_back(i), p[x].push_back(j=st[lk[i]]), ins(j);
    reverse(p[x].begin()+1, p[x].end());
    for(int i=v, j; i!=a; i=st[f[j]]) p[x].push_back(i), p[x].push_back(j=st[lk[i]]), ins(j);
    mdf(x, x); for(int i=1; i<=m; i++) e[x][i].w = e[i][x].w = 0;
    memset(b[x]+1, 0, n*sizeof b[0][0]);
    for (int u : p[x]){
      for(v=1; v<=m; v++) if(!e[x][v].w || d(e[u][v]) < d(e[x][v])) e[x][v] = e[u][v],e[v][x] =
      e[v][u];
      for(v=1; v<=n; v++) if(b[u][v]) b[x][v] = u;
    }
    ss(x);
  }
  void ex(int u){  // s[u] == 1
    for(int x : p[u]) mdf(x, x);
    int a = b[u][e[u][f[u]].u],r = gr(u, a);
    for(int i=0; i<r; i+=2){
      int x = p[u][i], y = p[u][i+1];
      f[x] = e[y][x].u; s[x] = 1; s[y] = 0; sl[x] = 0; ss(y); ins(y);
    }
    s[a] = 1; f[a] = f[u];
    for(int i=r+1; i<p[u].size(); i++) s[p[u][i]] = -1, ss(p[u][i]);
    st[u] = 0;
  }
  bool on(const Q &e){
    int u=st[e.u], v=st[e.v], a;
    if(s[v] == -1) f[v] = e.u, s[v] = 1, a = st[lk[v]], sl[v] = sl[a] = s[a] = 0, ins(a);
    else if(!s[v]){
      a = lca(u, v); if(!a) return aug(u,v), aug(v,u), true; else add(u,a,v);
    }
    return false;
  }
  bool bfs(){
    memset(s+1, -1, m*sizeof s[0]); memset(sl+1, 0, m*sizeof sl[0]);
    h = 1; t = 0; for(int i=1; i<=m; i++) if(st[i] == i && !lk[i]) f[i] = s[i] = 0, ins(i);
    if(h > t) return 0;
    while (true){
      while (h <= t){
        int u = q[h++];
        if (s[st[u]] != 1) for (int v=1; v<=n; v++) if (e[u][v].w > 0 && st[u] != st[v])
          if(d(e[u][v])) upd(u, st[v]); else if(on(e[u][v])) return true;
      }
      T x = inf;
      for(int i=n+1; i<=m; i++) if(st[i] == i && s[i] == 1) x = min(x, lab[i]>>1);
      for(int i=1; i<=m; i++) if(st[i] == i && sl[i] && s[i] != 1) x = min(x,
      d(e[sl[i]][i])>>s[i]+1);
      for(int i=1; i<=n; i++) if(~s[st[i]]) if((lab[i] += (s[st[i]]*2-1)*x) <= 0) return false;
      for(int i=n+1 ;i<=m; i++) if(st[i] == i && ~s[st[i]]) lab[i] += (2-s[st[i]]*4)*x;
      h = 1; t = 0;
      for(int i=1; i<=m; i++) if(st[i] == i && sl[i] && st[sl[i]] != i && !d(e[sl[i]][i]) &&
      on(e[sl[i]][i])) return true;
      for(int i=n+1; i<=m; i++) if(st[i] == i && s[i] == 1 && !lab[i]) ex(i);
    }
    return 0;
  }
  template<typename TT> pair<int,ll> run(int N, const vector<tuple<int,int,TT>> &edges){ //
  1-based
    memset(ed+1, 0, m*sizeof ed[0]); memset(lk+1, 0, m*sizeof lk[0]);
    n = m = N; id = 0; iota(st+1, st+n+1, 1); T wm = 0; ll r = 0;
    for(int i=1; i<=n; i++) for(int j=1; j<=n; j++) e[i][j] = {i,j,0};
    for(auto [u,v,w] : edges) wm = max(wm, e[v][u].w=e[u][v].w=max(e[u][v].w,(T)w));
    for(int i=1; i<=n; i++) p[i].clear();
    for(int i=1; i<=n; i++) for (int j=1; j<=n; j++) b[i][j] = i*(i==j);
    fill_n(lab+1, n, wm); int match = 0; while(bfs()) match++;
    for(int i=1; i<=n; i++) if(lk[i]) r += e[i][lk[i]].w;
    return {match, r/2};
  }
  #undef d
} using weighted_blossom_tree::run, weighted_blossom_tree::lk;
```

4 Math

4.1 Extend GCD, CRT, Combination

```
// ll gcd(ll a, ll b), ll lcm(ll a, ll b), ll mod(ll a, ll b)
tuple<ll,ll,ll> ext_gcd(ll a, ll b){ // return [g,x,y] s.t. ax+by=gcd(a,b)=g
  if(b == 0) return {a, 1, 0}; auto [g,x,y] = ext_gcd(b, a % b); return {g, y, x - a/b * y};
}
ll inv(ll a, ll m){ //return x when ax mod m = 1, fail -> -1
  auto [g,x,y] = ext_gcd(a, m); return g == 1 ? mod(x, m) : -1;
}
void DivList(ll n){ // {n/1, n/2, ... , n/n}, size <= 2 sqrt n
  for(ll i=1, j=1; i<=n; i=j+1) cout << i << " " << (j=n/(n/i)) << " " << n/i << "\n";
}
pair<ll,ll> crt(ll a1, ll m1, ll a2, ll m2){
  ll g = gcd(m1, m2), m = m1 / g * m2;
  if((a2 - a1) % g) return {-1, -1};
  ll md = m2/g, s = mod((a2-a1)/g, m2/g);
  ll t = mod(get<1>(ext_gcd(m1/g%md, m2/g)), md);
  return { a1 + s * t % md * m1, m };
}
pair<ll,ll> crt(const vector<ll> &a, const vector<ll> &m){
  ll ra = a[0], rm = m[0];
  for(int i=1; i<m.size(); i++){
    auto [aa,mm] = crt(ra, rm, a[i], m[i]);
    if(mm == -1) return {-1, -1}; else tie(ra,rm) = tie(aa,mm);
  }
  return {ra, rm};
}
struct Lucas{ // init : O(P), query : O(log P)
  const size_t P;
  vector<ll> fac, inv;
  ll Pow(ll a, ll b){ /* return a^b mod P */ }
  Lucas(size_t P) : P(P), fac(P), inv(P) {
    fac[0] = 1; for(int i=1; i<P; i++) fac[i] = fac[i-1] * i % P;
    inv[P-1] = Pow(fac[P-1], P-2); for(int i=P-2; ~i; i--) inv[i] = inv[i+1] * (i+1) % P;
  }
  ll small(ll n, ll r) const { return r <= n ? fac[n] * inv[r] % P * inv[n-r] % P : 0LL; }
  ll calc(ll n, ll r) const {
    if(n < r || n < 0 || r < 0) return 0;
    if(!n || !r || n == r) return 1; else return small(n%P, r%P) * calc(n/P, r/P) % P;
  }
};
template<ll p, ll e> struct CombinationPrimePower{ // init : O(p^e), query : O(log p)
  vector<ll> val; ll m;
  CombinationPrimePower(){
    m = 1; for(int i=0; i<e; i++) m *= p; val.resize(m); val[0] = 1;
    for(int i=1; i<m; i++) val[i] = val[i-1] * (i % p ? i : 1) % m;
  }
  pair<ll,ll> factorial(int n){
    if(n < p) return {0, val[n]};
    int k = n / p; auto v = factorial(k);
    int cnt = v.first + k, kp = n / m, rp = n % m;
    ll ret = v.second * Pow(val[m-1], kp % 2, m) % m * val[rp] % m;
    return {cnt, ret};
  }
  ll calc(int n, int r){
    if(n < 0 || r < 0 || n < r) return 0;
    auto v1 = factorial(n), v2 = factorial(r), v3 = factorial(n-r);
    ll cnt = v1.first - v2.first - v3.first;
    ll ret = v1.second * inv(v2.second, m) % m * inv(v3.second, m) % m;
    if(cnt >= e) return 0;
    for(int i=1; i<=cnt; i++) ret = ret * p % m;
    return ret;
  }
};
```

4.2 Partition Number

```
for(int j=1; j*(3*j-1)/2<=i; j++) P[i] += (j%2?1:-1)*P[i-j*(3*j-1)/2], P[i] %= MOD;
for(int j=1; j*(3*j+1)/2<=i; j++) P[i] += (j%2?1:-1)*P[i-j*(3*j+1)/2], P[i] %= MOD;
```

4.3 FloorSum

```
// sum of floor((A*i+B)/M) over 0 <= i < N in O(log(N+M+A+B))
ll FloorSum(ll N, ll M, ll A, ll B){ // 1 <= N,M <= 1e9, 0 <= A,B < M
  ll R = 0;
  if(A >= M) R += N * (N - 1) / 2 * (A / M), A %= M;
  if(B >= M) R += B / M * N, B %= M;
  ll Y = (A * N + B) / M, X = Y * M - B;
  if(Y == 0) return R;
  R += (N - (X + A - 1) / A) * Y;
  R += FloorSum(Y, A, M, (A - X % A) % A);
  return R;
}
```

4.4 XOR Basis(XOR Maximization)

```
// can use greedy maximize
//((staircase basis, basis coefficient),selected basis indices)
// staircase basis: has some good property
// basis coefficient and selected basis indices: for reconstruct
pair<vector<pair<ll,ll>>, vector<ll>> xor_basis(const vector<ll> &a) {
  vector<pair<ll,ll>> r(64, {-1, -1});  // descending
  vector<ll> bi;
  for(int i = 0; i < a.size(); i++) {
    ll x = a[i], xc = 0;
    for(auto [b, bc] : r)
      if(~b and x > (x ^ b)) x ^= b, xc ^= bc;
    if(x) r[63 - __lg(x)] = {x, xc ^ (1ll << bi.size())}, bi.push_back(i);
  }
  return {move(r), move(bi)};
} // for(auto i : r) mx = max(mx, mx ^ i.first);
```

4.5 Gauss Jordan Elimination

```
template<typename T> // return {rref, rank, det, inv}
tuple<vector<vector<T>>, T, T, vector<vector<T>>> Gauss(vector<vector<T>> a, bool square=true){
  int n = a.size(), m = a[0].size(), rank = 0;
  vector<vector<T>> out(n, vector<T>(m, 0)); T det = T(1);
  for(int i=0; i<n; i++) if(square) out[i][i] = T(1);
  for(int i=0; i<m; i++){
    if(rank == n) break;
    if(IsZero(a[rank][i])){
      T mx = T(0); int idx = -1; // fucking precision error
      for(int j=rank+1; j<n; j++) if(mx < abs(a[j][i])) mx = abs(a[j][i]), idx = j;
```

```
      if(idx == -1 || IsZero(a[idx][i])){ det = 0; continue; }
      for(int k=0; k<m; k++){
        a[rank][k] = Add(a[rank][k], a[idx][k]);
        if(square) out[rank][k] = Add(out[rank][k], out[idx][k]);
      }
    }
    det = Mul(det, a[rank][i]);
    T coeff = Div(T(1), a[rank][i]);
    for(int j=0; j<m; j++) a[rank][j] = Mul(a[rank][j], coeff);
    for(int j=0; j<m; j++) if(square) out[rank][j] = Mul(out[rank][j], coeff);
    for(int j=0; j<n; j++){
      if(rank == j) continue;
      T t = a[j][i]; // Warning: [j][k], [rank][k]
      for(int k=0; k<m; k++) a[j][k] = Sub(a[j][k], Mul(a[rank][k], t));
      for(int k=0; k<m; k++) if(square) out[j][k] = Sub(out[j][k], Mul(out[rank][k], t));
    }
    rank++;
  }
  return {a, rank, det, out};
}
```

4.6 Berlekamp + Kitamasa

Time Complexity: $O(NK + N \log mod), O(N^2 \log X)$

```
const int mod = 1e9+7; ll pw(ll a, ll b){ /* return a^b mod m */ }
vector<int> berlekamp_massey(vector<int> x){
  vector<int> ls, cur; int lf, ld;
  for(int i=0; i<x.size(); i++){
    ll t = 0;
    for(int j=0; j<cur.size(); j++) t = (t + 1ll * x[i-j-1] * cur[j]) % mod;
    if((t - x[i]) % mod == 0) continue;
    if(cur.empty()){ cur.resize(i+1); lf = i; ld = (t - x[i]) % mod; continue; }
    ll k = -(x[i] - t) * pw(ld, mod - 2) % mod;
    vector<int> c(i-lf-1); c.push_back(k);
    for(auto &j : ls) c.push_back(-j * k % mod);
    if(c.size() < cur.size()) c.resize(cur.size());
    for(int j=0; j<cur.size(); j++) c[j] = (c[j] + cur[j]) % mod;
    if(i-lf+(int)ls.size()>=(int)cur.size()){
      tie(ls, lf, ld) = make_tuple(cur, i, (t - x[i]) % mod);
    }
    cur = c;
  }
  for(auto &i : cur) i = (i % mod + mod) % mod; return cur;
}
int get_nth(vector<int> rec, vector<int> dp, ll n){
  int m = rec.size(); vector<int> s(m), t(m);
  s[0] = 1; if(m != 1) t[1] = 1; else t[0] = rec[0];
  auto mul = [&rec](vector<int> v, vector<int> w){
    int m = v.size();
    vector<int> t(2 * m);
    for(int j=0; j<m; j++) for(int k=0; k<m; k++){
      t[j+k] += 1ll * v[j] * w[k] % mod;
      if(t[j+k] >= mod) t[j+k] -= mod;
    }
    for(int j=2*m-1; j>=m; j--) for(int k=1; k<=m; k++){
      t[j-k] += 1ll * t[j] * rec[k-1] % mod;
      if(t[j-k] >= mod) t[j-k] -= mod;
    }
    t.resize(m); return t;
  };
  while(n){
    if(n & 1) s = mul(s, t);
    t = mul(t, t); n >>= 1;
  }
  ll ret = 0;
  for(int i=0; i<m; i++) ret += 1ll * s[i] * dp[i] % mod;
  return ret % mod;
}
int guess_nth_term(vector<int> x, ll n){
  if(n < x.size()) return x[n];
  vector<int> v = berlekamp_massey(x);
  if(v.empty()) return 0;
  return get_nth(v, x, n);
}
```

4.7 Miller Rabin + Pollard Rho

```
constexpr int SZ = 10'000'000; bool PrimeCheck[SZ+1]; vector<int> Primes;
void Sieve(){ memset(PrimeCheck, true, sizeof PrimeCheck); /* Sieve */ }
ull MulMod(ull a, ull b, ull c){ return (__uint128_t)a * b % c; }
// 32bit : 2, 7, 61
// 64bit : 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool MillerRabin(ull n, ull a){
  if(a % n == 0) return true;
  int cnt = __builtin_ctzll(n - 1);
  ull p = PowMod(a, n >> cnt, n);
  if(p == 1 || p == n - 1) return true;
  while(cnt--) if((p=MulMod(p,p,n)) == n - 1) return true;
  return false;
}
bool IsPrime(ll n){
  if(n <= SZ) return PrimeCheck[n];
  if(n <= 2) return n == 2;
  if(n % 2 == 0 || n % 3 == 0 || n % 5 == 0 || n % 7 == 0 || n % 11 == 0) return false;
  for(int p : {2, 325, 9375, 28178, 450775, 9780504, 1795265022}) if(!MillerRabin(n, p)) return
  false;
  return true;
}
ll Rho(ll n){
  while(true){
    ll x = rand() % (n - 2) + 2, y = x, c = rand() % (n - 1) + 1;
    while(true){
      x = (MulMod(x,x,n)+c) % n; y = (MulMod(y,y,n)+c) % n; y = (MulMod(y,y,n)+c) % n;
      ll d = __gcd(abs(x - y), n); if(d == 1) continue;
      if(IsPrime(d)) return d; else{ n = d; break; }
    }
  }
}
vector<pair<ll,ll>> Factorize(ll n){
  vector<pair<ll,ll>> v;
  int two = __builtin_ctzll(n);
  if(two > 0) v.emplace_back(2, two), n >>= two;
  if(n == 1) return v;
  while(!IsPrime(n)){
```

```
    ll d = Rho(n), cnt = 0; while(n % d == 0) cnt++, n /= d;
    v.emplace_back(d, cnt); if(n == 1) break;
  }
  if(n != 1) v.emplace_back(n, 1); return v;
}
```

4.8 Linear Sieve

```
// sp : 최소 소인수, 소수라면 0
// tau : 약수 개수, sigma : 약수 합
// phi : n 이하 자연수 중 n과 서로소인 개수
// mu : non square free이면 0, 그렇지 않다면 (-1)^(소인수 종류)
// e[i] : 소인수분해에서 i의 지수
vector<int> prime;
int sp[sz], e[sz], phi[sz], mu[sz], tau[sz], sigma[sz];
phi[1] = mu[1] = tau[1] = sigma[1] = 1;
for(int i=2; i<=n; i++){
  if(!sp[i]){
    prime.push_back(i);
    e[i] = 1; phi[i] = i-1; mu[i] = -1; tau[i] = 2; sigma[i] = i+1;
  }
  for(auto j : prime){
    if(i*j >= sz) break;
    sp[i*j] = j;
    if(i % j == 0){
      e[i*j] = e[i]+1; phi[i*j] = phi[i]*j; mu[i*j] = 0;
      tau[i*j] = tau[i]/e[i*j]*(e[i*j]+1);
      sigma[i*j] = sigma[i]*(j-1)/(pw(j, e[i*j])-1)*(pw(j, e[i*j]+1)-1)/(j-1);//overflow
      break;
    }
    e[i*j] = 1; phi[i*j] = phi[i] * phi[j]; mu[i*j] = mu[i] * mu[j];
    tau[i*j] = tau[i] * tau[j]; sigma[i*j] = sigma[i] * sigma[j];
  }
}
```

4.9 Power Tower

```
bool PowOverflow(ll a, ll b, ll c){
  __int128_t res = 1;
  bool flag = false;
  for(; b; b >>= 1, a = a * a){
    if(a >= c) flag = true, a %= c;
    if(b & 1){
      res *= a;
      if(flag || res >= c) return true;
    }
  }
  return false;
}
ll Recursion(int idx, ll mod, const vector<ll> &vec){
  if(mod == 1) return 1;
  if(idx + 1 == vec.size()) return vec[idx];
  ll nxt = Recursion(idx+1, phi[mod], vec);
  if(PowOverflow(vec[idx], nxt, mod)) return Pow(vec[idx], nxt, mod) + mod;
  else return Pow(vec[idx], nxt, mod);
}
ll PowerTower(const vector<ll> &vec, ll mod){ // vec[0]^(vec[1]^(vec[2]^(...)))
  if(vec.size() == 1) return vec[0] % mod;
  else return Pow(vec[0], Recursion(1, phi[mod], vec), mod);
}
```

4.10 Discrete Log / Sqrt

Time Complexity: Log : $O(\sqrt{P}\log P)$, $O(\sqrt{P})$ with hash set
Sqrt : $O(\log^2 P)$, $O(\log P)$ in random data

```
// Given A, B, P, solve A^x === B mod P
ll DiscreteLog(ll A, ll B, ll P){
  __gnu_pbds::gp_hash_table<ll,__gnu_pbds::null_type> st;
  ll t = ceil(sqrt(P)), k = 1; // use binary search?
  for(int i=0; i<t; i++) st.insert(k), k = k * A % P;
  ll inv = Pow(k, P-2, P);
  for(int i=0, k=1; i<t; i++, k=k*inv%P){
    ll x = B * k % P;
    if(st.find(x) == st.end()) continue;
    for(int j=0, k=1; j<t; j++, k=k*A%P){
      if(k == x) return i * t + j;
    }
  }
  return -1;
}
// Given A, P, solve X^2 === A mod P
ll DiscreteSqrt(ll A, ll P){
  if(A == 0) return 0;
  if(Pow(A, (P-1)/2, P) != 1) return -1;
  if(P % 4 == 3) return Pow(A, (P+1)/4, P);
  ll s = P - 1, n = 2, r = 0, m;
  while(~s & 1) r++, s >>= 1;
  while(Pow(n, (P-1)/2, P) != P-1) n++;
  ll x = Pow(A, (s+1)/2, P), b = Pow(A, s, P), g = Pow(n, s, P);
  for(;; r=m){
    ll t = b;
    for(m=0; m<r && t!=1; m++) t = t * t % P;
    if(!m) return x;
    ll gs = Pow(g, 1LL << (r-m-1), P);
    g = gs * gs % P;
    x = x * gs % P;
    b = b * g % P;
  }
}
```

4.11 Simplex / LP Duality

```
//입력: Ax<=b, obj
//출력: maximize obj*x
//numeric stability is sensitive by M
//디버깅 노트
//1. T=f64 해보기(정수값만 나오는거같아도 중간에 유리수나올때 있음)
//2. M값 조절(답의 상한정도의 크기가 적절)
//듀얼후 리덕션한 결과값 primal로 복원하기
template<class T=f64,int M>
void dualize(Arr<Arr<T>> &a,Arr<T> &b,Arr<T>& obj){
  int m=sz(a), n=sz(a[0]);
  transpose(a),swap(b,obj);
  for(int i=0;i<n;i++){
```

```
    for(auto& j:a[i])j=-j;
    b[i]=-b[i];
  }
  for(auto& i:obj)i=-i;
}
template<class T=f64,int M>
tuple<T,Arr<T>,Arr<T>> simplex(Arr<Arr<T>>& a,Arr<T>& b,Arr<T>& obj){
  //return {maxval,argmax,dual_argmin}
  int m=sz(a),n=sz(a[0]),s=0;
  if(m>n){
    dualize<T,M>(a,b,obj);
    auto&& [x,y,z]=simplex<T,M>(a,b,obj);
    x*=-1;
    swap(y,z);
    return {move(x),move(y),move(z)};
  }
  func(void,elim,int r1,int r2,int c){//elim r2
    if(r1==r2){T x=a[r1][c]; for(auto& i:a[r1])i/=x;}
    else{
      T x=a[r2][c]/a[r1][c]; if(-eps<x&&x<eps)return;
      for(int i=0;i<n+s+m+2;i++)
        a[r2][i]-=x*a[r1][i];
    }
  };

  //make all b>=0
  Arr<char> geq(m);
  for(int i=0;i<m;i++)
    if(b[i]<0){
      for(auto& j:a[i])j=-j;
      for(auto& r:a)r.emplb(0);
      a[i][-1]=-1,b[i]=-b[i],geq[i]=true,s++;
    }

  //n vars, s slacks(-1), m slacks(1), 1 z, 1 b_value
  Arr<int> p(m);//행의 기본변수
  obj.resize(n+s+m+2);
  for(int i=0;i<m;i++)
    a[i].resize(n+s+m+2),a[i][p[i]=n+s+i]=1,a[i][-1]=b[i],obj[p[i]]=geq[i]?-M:0;

  //z=f(x) == z-f(x)=0
  for(auto &i:obj)i=-i;
  obj[-2]=1;
  a.emplb(obj);

  for(int i=0;i<m;i++)
    elim(i,m,p[i]);

  //now shape of a = (m+1)*(n+s+m+2)
  while(true){
    int ev=0,lvi=-1;
    for(int i=0;i<n+s+m;i++)
      ev=a[-1][ev]>a[-1][i]?i:ev;
    if(a[-1][ev]>-eps)break;
    for(int i=0;i<m;i++)
      if(a[i][ev]>eps and (!~lvi or a[i][-1]/a[i][ev]<a[lvi][-1]/a[lvi][ev]))
        lvi=i;
    if(!~lvi) throw "unbounded";
    for(int i=0;i<m+1;i++)elim(lvi,i,ev);
    p[lvi]=ev;
  }
  //if(?) throw "infeasible"
  Arr<T> ans(n+s+m+2);
  for(int i=0;i<m;i++)
    ans[p[i]]=a[i][-1];
  Arr<T> dual(m);
  for(int i=0;i<m;i++)
    dual[i]=a[-1][n+s+i]+(geq[i]?+M:0);
  return {a[-1][-1],ans,dual};
}
```

Simplex Example
Maximize $p = 6x + 14y + 13z$
Constraints
- $0.5x + 2y + z \le 24$
- $x + 2y + 4z \le 60$

Coding
- $n = 2, m = 3, a = \begin{pmatrix} 0.5 & 2 & 1 \\ 1 & 2 & 4 \end{pmatrix}, b = \begin{pmatrix} 24 \\ 60 \end{pmatrix}, c = [6, 14, 13]$

LP Duality & Example
tableu를 대각선으로 뒤집고 음수 부호를 붙인 답 = -(원 문제의 답)
- Primal : $n = 2, m = 3, a = \begin{pmatrix} 0.5 & 2 & 1 \\ 1 & 2 & 4 \end{pmatrix}, b = \begin{pmatrix} 24 \\ 60 \end{pmatrix}, c = [6, 14, 13]$
- Dual : $n = 3, m = 2, a = \begin{pmatrix} -0.5 & -1 \\ -2 & -2 \\ -1 & -4 \end{pmatrix}, b = \begin{pmatrix} -6 \\ -14 \\ -13 \end{pmatrix}, c = [-24, -60]$

공식
- Primal : $\max_x c^T x$, Constraints $Ax \le b, x \ge 0$
- Dual : $\min_y b^T y$,Constraints $A^T y \ge c, y \ge 0$

4.12 De Bruijn Sequence

```
// Create cyclic string of length k^n that contains every length n string as substring. alphabet
= [0, k - 1]
int res[10000000], aux[10000000]; // >= k^n
int de_bruijn(int k, int n) { // Returns size (k^n)
  if(k == 1) { res[0] = 0; return 1; }
  for(int i = 0; i < k * n; i++) aux[i] = 0;
  int sz = 0;
  function<void(int, int)> db = [&](int t, int p) {
    if(t > n) {
      if(n % p == 0) for(int i = 1; i <= p; i++) res[sz++] = aux[i];
    }
    else {
      aux[t] = aux[t - p]; db(t + 1, p);
      for(int i = aux[t - p] + 1; i < k; i++) aux[t] = i, db(t + 1, t);
    }
  };
  db(1, 1);
  return sz;
}
```

4.13 FFT, NTT, FWHT, Multipoint Evaluation, Interpolation

```
// 104,857,601   =  25 * 2^22 + 1, w = 3 | 998,244,353   = 119 * 2^23 + 1, w = 3
// 2,281,701,377 =  17 * 2^27 + 1, w = 3 | 2,483,027,969 =  37 * 2^26 + 1, w = 3
// 2,113,929,217 =  63 * 2^25 + 1, w = 5 | 1,092,616,193 = 521 * 2^21 + 1, w = 3
using real_t = double; using cpx = complex<real_t>;
void FFT(vector<cpx> &a, bool inv_fft=false){
  int N = a.size(); vector<cpx> root(N/2);
  for(int i=1, j=0; i<N; i++){
    int bit = N / 2;
    while(j >= bit) j -= bit, bit >>= 1;
    if(i < (j += bit)) swap(a[i], a[j]);
  }
  real_t ang = 2 * acos(-1) / N * (inv_fft ? -1 : 1);
  for(int i=0; i<N/2; i++) root[i] = cpx(cos(ang * i), sin(ang * i));
  /*
  NTT : ang = pow(w, (mod-1)/n) % mod, inv_fft -> ang^{-1}, root[i] = root[i-1] * ang
  XOR Convolution : set roots[*] = 1, a[j+k] = u+v, a[j+k+i/2] = u-v
   OR Convolution : set roots[*] = 1, a[j+k+i/2] += inv_fft ? -u : u;
  AND Convolution : set roots[*] = 1, a[j+k  ] += inv_fft ? -v : v;
  */
  for(int i=2; i<=N; i<<=1){
    int step = N / i;
    for(int j=0; j<N; j+=i) for(int k=0; k<i/2; k++){
        cpx u = a[j+k], v = a[j+k+i/2] * root[step * k];
        a[j+k] = u+v; a[j+k+i/2] = u-v;
      }
  }
  if(inv_fft) for(int i=0; i<N; i++) a[i] /= N; // skip for AND/OR convolution.
}
vector<ll> multiply(const vector<ll> &_a, const vector<ll> &_b){
  vector<cpx> a(all(_a)), b(all(_b));
  int N = 2; while(N < a.size() + b.size()) N <<= 1;
  a.resize(N); b.resize(N); FFT(a); FFT(b);
  for(int i=0; i<N; i++) a[i] *= b[i];
  vector<ll> ret(N); FFT(a, 1); // NTT : just return a
  for(int i=0; i<N; i++) ret[i] = llround(a[i].real());
  return ret;
}
vector<ll> multiply_mod(const vector<ll> &a, const vector<ll> &b, const ull mod){
  int N = 2; while(N < a.size() + b.size()) N <<= 1;
  vector<cpx> v1(N), v2(N), r1(N), r2(N);
  for(int i=0; i<a.size(); i++) v1[i] = cpx(a[i] >> 15, a[i] & 32767);
  for(int i=0; i<b.size(); i++) v2[i] = cpx(b[i] >> 15, b[i] & 32767);
  FFT(v1); FFT(v2);
  for(int i=0; i<N; i++){
    int j = i ? N-i : i;
    cpx ans1 = (v1[i] + conj(v1[j])) * cpx(0.5, 0);
    cpx ans2 = (v1[i] - conj(v1[j])) * cpx(0, -0.5);
    cpx ans3 = (v2[i] + conj(v2[j])) * cpx(0.5, 0);
    cpx ans4 = (v2[i] - conj(v2[j])) * cpx(0, -0.5);
    r1[i] = (ans1 * ans3) + (ans1 * ans4) * cpx(0, 1);
    r2[i] = (ans2 * ans3) + (ans2 * ans4) * cpx(0, 1);
  }
  vector<ll> ret(N); FFT(r1, true); FFT(r2, true);
  for(int i=0; i<N; i++){
    ll av = llround(r1[i].real()) % mod;
    ll bv = ( llround(r1[i].imag()) + llround(r2[i].real()) ) % mod;
    ll cv = llround(r2[i].imag()) % mod;
    ret[i] = (av << 30) + (bv << 15) + cv;
    ret[i] %= mod; ret[i] += mod; ret[i] %= mod;
  }
  return ret;
}
template<char op> vector<ll> FWHT_Conv(vector<ll> a, vector<ll> b){
  int n = max({(int)a.size(), (int)b.size() - 1, 1});
  if(__builtin_popcount(n) != 1) n = 1 << (__lg(n) + 1);
  a.resize(n); b.resize(n); FWHT<op>(a); FWHT<op>(b);
  for(int i=0; i<n; i++) a[i] = a[i] * b[i] % M;
  FWHT<op>(a, true); return a;
}
vector<ll> SubsetConvolution(vector<ll> p, vector<ll> q){ // N log^2 N
  int n = max({(int)p.size(), (int)q.size() - 1, 1}), w = __lg(n);
  if(__builtin_popcount(n) != 1) n = 1 << (w + 1);
  p.resize(n); q.resize(n); vector<ll> res(n);
  vector<vector<ll>> a(w+1, vector<ll>(n)), b(a);
  for(int i=0; i<n; i++) a[__builtin_popcount(i)][i] = p[i];
  for(int i=0; i<n; i++) b[__builtin_popcount(i)][i] = q[i];
  for(int bit=0; bit<=w; bit++) FWHT<'|'>(a[bit]), FWHT<'|'>(b[bit]);
  for(int bit=0; bit<=w; bit++){
    vector<ll> c(n); // Warning : MOD
    for(int i=0; i<=bit; i++) for(int j=0; j<n; j++) c[j] += a[i][j] * b[bit-i][j] % M;
    for(auto &i : c) i %= M;
    FWHT<'|'>(c, true);
    for(int i=0; i<n; i++) if(__builtin_popcount(i) == bit) res[i] = c[i];
  }
  return res;
}
vector<ll> Trim(vector<ll> a, size_t sz){ a.resize(min(a.size(), sz)); return a; }
vector<ll> Inv(vector<ll> a, size_t sz){
  vector<ll> q(1, Pow(a[0], M-2, M)); // 1/a[0]
  for(int i=1; i<sz; i<<=1){
    auto p = vector<ll>{2} - Multiply(q, Trim(a, i*2)); // polynomial minus
    q = Trim(Multiply(p, q), i*2);
  }
  return Trim(q, sz);
}
vector<ll> Division(vector<ll> a, vector<ll> b){
  if(a.size() < b.size()) return {};
  size_t sz = a.size() - b.size() + 1; auto ra = a, rb = b;
  reverse(ra.begin(), ra.end()); ra = Trim(ra, sz);
  reverse(rb.begin(), rb.end()); rb = Inv(Trim(rb, sz), sz);
  auto res = Trim(Multiply(ra, rb), sz);
  for(int i=sz-(int)a.size(); i>0; i--) res.push_back(0);
  reverse(res.begin(), res.end()); while(!res.empty() && !res.back()) res.pop_back();
  return res;
}
vector<ll> Modular(vector<ll> a, vector<ll> b){ return a - Multiply(b, Division(a, b)); }
vector<vector<ll>> PolynomialTree(const vector<ll> &x){
  int n = x.size(); vector<vector<ll>> tree(n*2-1);
  function<void(int,int,int)> build = [&](int node, int s, int e){
    if(e-s == 1){ tree[node] = vector<ll>{-x[s], 1}; return; }
    int m = s + (e-s)/2, v = node + (m-s)*2;
    build(node+1, s, m); build(v, m, e);
    tree[node] = Multiply(tree[node+1], tree[v]);
```

```
  }; build(0, 0, n); return tree;
}
vector<ll> MultipointEvaluation(const vector<ll> &a, const vector<ll> &x){ // n log^2 n
  if(x.empty()) return {}; if(a.empty()) return vector<ll>(x.size(), 0);
  int n = x.size(); auto tree = PolynomialTree(x); vector<ll> res(n);
  function<void(int,int,int,vector<ll>)> eval = [&](int node, int s, int e, vector<ll> f){
    f = Modular(f, tree[node]);
    if(e-s == 1){ res[s] = f[0]; return; }
    if(f.size() < 150){ for(int i=s; i<e; i++) res[i] = Evaluate(f, x[i]); return; }
    int m = s + (e-s)/2, v = node + (m-s)*2;
    eval(node+1, s, m, f); eval(v, m, e, f);
  }; eval(0, 0, n, a);
  return res;
}
vector<ll> Interpolation(const vector<ll> &x, const vector<ll> &y){ // n log^2 n
  assert(x.size() == y.size()); if(x.empty()) return {};
  int n = x.size(); auto tree = PolynomialTree(x);
  auto res = MultipointEvaluation(Derivative(tree[0]), x);
  for(int i=0; i<n; i++) res[i] = y[i] * Pow(res[i], M-2, M) % M; // y[i] / res[i]
  function<vector<ll>(int,int,int)> calc = [&](int node, int s, int e){
    if(e-s == 1) return vector<ll>{res[s]};
    int m = s + (e-s)/2, v = node + (m-s)*2;
    return Multiply(calc(node+1, s, m), tree[v]) + Multiply(calc(v, m, e), tree[node+1]);
  };
  return calc(0, 0, n);
}
```

4.14 Matroid Intersection

```
struct Matroid{
  virtual bool check(int i) = 0; // O(R^2N), O(R^2N)
  virtual void insert(int i) = 0; // O(R^3), O(R^2N)
  virtual void clear() = 0; // O(R^2), O(RN)
};
template<typename cost_t>
vector<cost_t> MI(const vector<cost_t> &cost, Matroid *m1, Matroid *m2){
  int n = cost.size();
  vector<pair<cost_t, int>> dist(n+1);
  vector<vector<pair<int, cost_t>>> adj(n+1);
  vector<int> pv(n+1), inq(n+1), flag(n); deque<int> dq;
  auto augment = [&]() -> bool {
    fill(dist.begin(), dist.end(), pair(numeric_limits<cost_t>::max()/2, 0));
    fill(adj.begin(), adj.end(), vector<pair<int, cost_t>>());
    fill(pv.begin(), pv.end(), -1);
    fill(inq.begin(), inq.end(), 0);
    dq.clear(); m1->clear(); m2->clear();
    for(int i=0; i<n; i++) if(flag[i]) m1->insert(i), m2->insert(i);
    for(int i=0; i<n; i++){
      if(flag[i]) continue;
      if(m1->check(i)) dist[pv[i]=i] = {cost[i], 0}, dq.push_back(i), inq[i] = 1;
      if(m2->check(i)) adj[i].emplace_back(n, 0);
    }
    for(int i=0; i<n; i++){
      if(!flag[i]) continue;
      m1->clear(); m2->clear();
      for(int j=0; j<n; j++) if(i != j && flag[j]) m1->insert(j), m2->insert(j);
      for(int j=0; j<n; j++){
        if(flag[j]) continue;
        if(m1->check(j)) adj[i].emplace_back(j, cost[j]);
        if(m2->check(j)) adj[j].emplace_back(i, -cost[i]);
      }
    }
    while(dq.size()){
      int v = dq.front(); dq.pop_front(); inq[v] = 0;
      for(const auto &[i,w] : adj[v]){
        pair<cost_t, int> nxt{dist[v].first+w, dist[v].second+1};
        if(nxt < dist[i]){
          dist[i] = nxt; pv[i] = v;
          if(!inq[i]) dq.push_back(i), inq[i] = 1;
        }
      }
    }
    if(pv[n] == -1) return false;
    for(int i=pv[n]; ; i=pv[i]){
      flag[i] ^= 1; if(i == pv[i]) break;
    }
    return true;
  };
  vector<int> res;
  while(augment()){
    int now = 0;
    for(int i=0; i<n; i++) if(flag[i]) now += cost[i];
    res.push_back(now);
  }
  return res;
}
```

5 String

5.1 KMP, Hash, Manacher, Z

```
vector<int> getFail(const container &pat){
    vector<int> fail(pat.size());
    // match: pat[0..j] and pat[j-i..i] is equivalent
    // ins/del: manipulate corresponding range to pattern starts at 0
    //      (insert/delete pat[i], manage pat[j-i..i])
    function<bool(int, int)> match = [&](int i, int j){ };
    function<void(int)> ins = [&](int i){ };
    function<void(int)> del = [&](int i){ };
    for(int i=1, j=0; i<pat.size(); i++){
        while(j && !match(i, j)){
            for(int s=i-j; s<i-fail[j-1]; s++) del(s);
            j = fail[j-1];
        }
        if(match(i, j)) ins(i), fail[i] = ++j;
    }
    return fail;
}
vector<int> doKMP(const container &str, const container &pat){
    vector<int> ret, fail = getFail(pat);
    // match: pat[0..j] and str[j-i..i] is equivalent
    // ins/del: manipulate corresponding range to pattern starts at 0
    //      (insert/delete str[i], manage str[j-i..i])
    function<bool(int, int)> match = [&](int i, int j){ };
```

```
    function<void(int)> ins = [&](int i){ };
    function<void(int)> del = [&](int i){ };
    for(int i=0, j=0; i<str.size(); i++){
        while(j && !match(i, j)){
            for(int s=i-j; s<i-fail[j-1]; s++) del(s);
            j = fail[j-1];
        }
        if(match(i, j)){
            if(j+1 == pat.size()){
                ret.push_back(i-j);
                for(int s=i-j; s<i-fail[j]+1; s++) del(s);
                j = fail[j];
            }
            else ++j;
            ins(i);
        }
    }
    return ret;
}
// 1e5+3, 1e5+13, 131'071, 524'287, 1'299'709, 1'301'021
// 1e9-63, 1e9+7, 1e9+9, 1e9+103
template<ll P, ll M> struct Hashing {
    vector<ll> H, B;
    void Build(const string &S){
        H.resize(S.size()+1);
        B.resize(S.size()+1);
        B[0] = 1;
        for(int i=1; i<=S.size(); i++) H[i] = (H[i-1] * P + S[i-1]) % M;
        for(int i=1; i<=S.size(); i++) B[i] = B[i-1] * P % M;
    }
    ll sub(int s, int e){
        ll res = (H[e] - H[s-1] * B[e-s+1]) % M;
        return res < 0 ? res + M : res;
    }
};
// # a # b # a # a # b # a #
// 0 1 0 3 0 1 6 1 0 3 0 1 0
vector<int> Manacher(const string &inp){
    int n = inp.size() * 2 + 1;
    vector<int> ret(n);
    string s = "#";
    for(auto i : inp) s += i, s += "#";
    for(int i=0, p=-1, r=-1; i<n; i++){
        ret[i] = i <= r ? min(r-i, ret[2*p-i]) : 0;
        while(i-ret[i]-1 >= 0 && i+ret[i]+1 < n && s[i-ret[i]-1] == s[i+ret[i]+1]) ret[i]++;
        if(i+ret[i] > r) r = i+ret[i], p = i;
    }
    return ret;
}
// input: manacher array, 1-based hashing structure
// output: set of pair(hash_val, length)
set<pair<hash_t,int>> UniquePalindrome(const vector<int> &dp, const Hashing &hashing){
    set<pair<hash_t,int>> st;
    for(int i=0,s,e; i<dp.size(); i++){
        if(!dp[i]) continue;
        if(i & 1) s = i/2 - dp[i]/2 + 1, e = i/2 + dp[i]/2 + 1;
        else s = (i-1)/2 - dp[i]/2 + 2, e = (i+1)/2 + dp[i]/2;
        for(int l=s, r=e; l<=r; l++, r--){
            auto now = hashing.get(l, r);
            auto [iter,flag] = st.emplace(now, r-l+1);
            if(!flag) break;
        }
    }
    return st;
}
//z[i]=match length of s[0,n-1] and s[i,n-1]
vector<int> Z(const string &s){
    int n = s.size();
    vector<int> z(n);
    z[0] = n;
    for(int i=1, l=0, r=0; i<n; i++){
        if(i < r) z[i] = min(r-i-1, z[i-l]);
        while(i+z[i] < n && s[i+z[i]] == s[z[i]]) z[i]++;
        if(i+z[i] > r) r = i+z[i], l = i;
    }
    return z;
}
```

5.2 Aho-Corasick

```
struct Node{
  map<char, Node*> ch; int terminal;
  Node() : terminal(-1) {}
  ~Node(){
    for(auto &i : ch) delete i.second;
    ch.clear();
  }
  void insert(const char *key, int num){
    if(*key == 0){ terminal = num; return; }
    if(!ch[*key]) ch[*key] = new Node();
    ch[*key]->insert(key+1, num);
  }
  Node *fail; vector<int> out;
};
void aho_getFail(Node *root){
  queue<Node*> q; q.push(root);
  root->fail = root;
  while(q.size()){
    Node *now = q.front(); q.pop();
    for(auto &i : now->ch){
      Node *ch = i.second;
      if(!ch) continue;
      if(root == now) ch->fail = root;
      else{
        Node *t = now->fail;
        while(t != root && !t->ch[i.first]) t = t->fail;
        if(t->ch[i.first]) t = t->ch[i.first];
        ch->fail = t;
      }
      ch->out = ch->fail->out;
      if(ch->terminal != -1) ch->out.push_back(ch->terminal);
      q.push(ch);
    }
```

```
  }
}
vector<p> aho_find(const string &s, Node *root){
  vector<p> ret; auto state = root;
  for(int i=0; i<s.size(); i++){
    while(state != root && !state->ch[s[i]]) state = state->fail;
    if(state->ch[s[i]]) state = state->ch[s[i]];
    for(int j=0; j<state->out.size(); j++){
      ret.emplace_back(i, state->out[j]);
    }
  }
  return ret;
}
```

5.3 $O(N \log N)$ SA + LCP

```
pair<vector<int>, vector<int>> SuffixArray(const string &s){ // O(N log N)
  int n = s.size(), m = max(n, 256);
  vector<int> sa(n), lcp(n), pos(n), tmp(n), cnt(m);
  auto counting_sort = [&](){
    fill(cnt.begin(), cnt.end(), 0);
    for(int i=0; i<n; i++) cnt[pos[i]]++;
    partial_sum(cnt.begin(), cnt.end(), cnt.begin());
    for(int i=n-1; i>=0; i--) sa[--cnt[pos[tmp[i]]]] = tmp[i];
  };
  for(int i=0; i<n; i++) sa[i] = i, pos[i] = s[i], tmp[i] = i;
  counting_sort();
  for(int k=1; ; k<<=1){
    int p = 0;
    for(int i=n-k; i<n; i++) tmp[p++] = i;
    for(int i=0; i<n; i++) if(sa[i] >= k) tmp[p++] = sa[i] - k;
    counting_sort();
    tmp[sa[0]] = 0;
    for(int i=1; i<n; i++){
      tmp[sa[i]] = tmp[sa[i-1]];
      if(sa[i-1]+k < n && sa[i]+k < n && pos[sa[i-1]] == pos[sa[i]] && pos[sa[i-1]+k] ==
      pos[sa[i]+k]) continue;
      tmp[sa[i]] += 1;
    }
    swap(pos, tmp); if(pos[sa.back()] + 1 == n) break;
  }
  for(int i=0, j=0; i<n; i++, j=max(j-1,0)){
    if(pos[i] == 0) continue;
    while(sa[pos[i]-1]+j < n && sa[pos[i]]+j < n && s[sa[pos[i]-1]+j] == s[sa[pos[i]]+j]) j++;
    lcp[pos[i]] = j;
  }
  return {sa, lcp};
}
auto [SA,LCP] = SuffixArray(S); RMQ<int> rmq(LCP);
vector<int> Pos(N); for(int i=0; i<N; i++) Pos[SA[i]] = i;
auto get_lcp = [&](int a, int b){
    if(Pos[a] > Pos[b]) swap(a, b);
    return a == b ? (int)S.size() - a : rmq.query(Pos[a]+1, Pos[b]);
};
vector<pair<int,int>> can; // common substring {start, lcp}
vector<tuple<int,int,int>> valid; // valid substring [string, end_l~end_r]
for(int i=1; i<N; i++){
  if(SA[i] < X && SA[i-1] > X) can.emplace_back(SA[i], LCP[i]);
  if(i+1 < N && SA[i] < X && SA[i+1] > X) can.emplace_back(SA[i], LCP[i+1]);
}
for(int i=0; i<can.size(); i++){
  int skip = i > 0 ? min({can[i-1].second, can[i].second, get_lcp(can[i-1].first,
  can[i].first)}) : 0;
  valid.emplace_back(can[i].first, can[i].first + skip, can[i].first + can[i].second - 1);
}
```

5.4 Bitset LCS

```
#include <x86intrin.h>
template<size_t _Nw> void _M_do_sub(_Base_bitset<_Nw> &A, const _Base_bitset<_Nw> &B){
  for(int i=0, c=0; i<_Nw; i++) c = _subborrow_u64(c, A._M_w[i], B._M_w[i], (ull*)&A._M_w[i]);
}
void _M_do_sub(_Base_bitset<1> &A, const _Base_bitset<1> &B){ A._M_w -= B._M_w; }
template<size_t _Nb> bitset<_Nb>& operator-=(bitset<_Nb> &A, const bitset<_Nb> &B){
  _M_do_sub(A, B); return A;
}
template<size_t _Nb> inline bitset<_Nb> operator-(const bitset<_Nb> &A, const bitset<_Nb> &B){
  bitset<_Nb> C(A); return C -= B;
}
char s[50050], t[50050];
int lcs(){ // O(NM/64)
  bitset<50050> dp, ch[26];
  int n = strlen(s), m = strlen(t);
  for(int i=0; i<m; i++) ch[t[i]-'A'].set(i);
  for(int i=0; i<n; i++){ auto x = dp | ch[s[i]-'A']; dp = dp - (dp ^ x) & x; }
  return dp.count();
}
```

5.5 Lyndon Factorization, Minimum Rotation

```
// factorize string into w1 >= w2 >= ... >= wk, wi is smallest cyclic shift of suffix.
vector<string> Lyndon(const string &s){ // O(N)
  int n = s.size(), i = 0, j, k;
  vector<string> res;
  while(i < n){
    for(j=i+1, k=i; i<n && s[k]<=s[j]; j++) k = s[k] < s[j] ? i : k + 1;
    for(; i<=k; i+=j-k) res.push_back(s.substr(i, j-k));
  }
  return res;
}
// rotate(v.begin(), v.begin()+min_rotation(v), v.end());
template<typename T> int min_rotation(T s){ // O(N)
  int a = 0, N = s.size();
  for(int i=0; i<N; i++) s.push_back(s[i]);
  for(int b=0; b<N; b++) for(int k=0; k<N; k++){
    if(a+k == b || s[a+k] < s[b+k]){ b += max(0, k-1); break; }
    if(s[a+k] > s[b+k]){ a = b; break; }
  }
  return a;
}
```

6 Misc

6.1 Ternary Search

```
while(s + 3 <= e){ // get minimum / when multiple answer, find minimum `s`
  T l = (s + s + e) / 3, r = (s + e + e) / 3;
  if(Check(l) > Check(r)) s = l; else e = r;
}
T mn = INF, idx = s;
for(T i=s; i<=e; i++) if(T now = Check(i); now < mn) mn = now, idx = i;
```

6.2 Monotone Queue Optimization

```
template<class T, bool GET_MAX = false> // D[i] = func_{0 <= j < i} D[j] + cost(j, i)
pair<vector<T>, vector<int>> monotone_queue_dp(int n, const vector<T> &init, auto cost){
  assert((int)init.size() == n + 1); // cost function -> auto, do not use std::function
  vector<T> dp = init; vector<int> prv(n+1);
  auto compare = [](T a, T b){ return GET_MAX ? a < b : a > b; };
  auto cross = [&](int i, int j){
    int l = j, r = n + 1;
    while(l < r){
      int m = (l + r + 1) / 2;
      if(compare(dp[i] + cost(i, m), dp[j] + cost(j, m))) r = m - 1; else l = m;
    }
    return l;
  };
  deque<int> q{0};
  for(int i=1; i<=n; i++){
    while(q.size() > 1 && compare(dp[q[0]] + cost(q[0], i), dp[q[1]] + cost(q[1], i)))
    q.pop_front();
    dp[i] = dp[q[0]] + cost(q[0], i); prv[i] = q[0];
    while(q.size() > 1 && cross(q[q.size()-2], q.back()) >= cross(q.back(), i)) q.pop_back();
    q.push_back(i);
  }
  return {dp, prv};
}
```

6.3 Aliens Trick

```
// 점화식에 min이 들어가는 경우: 구간을 쪼갤 때마다 +lambda
while(l <= r){
  ll m = l + r >> 1; [dp,cnt] = Solve(m);
  res = max(res, dp - k*m);
  if(cnt <= k) r = m - 1; else l = m + 1;
}
// 점화식에 max가 들어가는 경우: 구간을 쪼갤 때마다 +lambda
while(l <= r){
  ll m = l + r >> 1; [dp,cnt] = Solve(m);
  res = min(res, dp - k*m);
  if(cnt <= k) l = m + 1; else r = m - 1;
}
```

6.4 Slope Trick

```
//NOTE: f(x)=min{f(x+i),i<a}+|x-k|+m -> pf(k)sf(k)ab(-a,m)
//NOTE: sf_inc에 답구하는게 들어있어서, 반드시 한 연산에 대해 pf_dec->sf_inc순서로 호출
struct LeftHull{
  void pf_dec(int x){pq.empl(x-bias);}//x이하의 기울기들 -1
  int sf_inc(int x){//x이상의 기울기들 +1, pop된 원소 반환(Right Hull관리에 사용됨)
    if(pq.empty() or argmin()<=x)return x;
    ans+=argmin()-x;//이 경우 최솟값이 증가함
    pq.empl(x-bias);//x 이하 -1
    int r=argmin();pq.pop();//전체 +1
    return r;
  }
  void add_bias(int x,int y){bias+=x;ans+=y;}//그래프 x축 평행이동
  int minval(){return ans;}//최소값
  int argmin(){return pq.empty()?-inf<int>():pq.top()+bias;}//최소값 x좌표
  void operator+=(LeftHull& a){
    ans+=a.ans;
    while(sz(a.pq))pf_dec(a.argmin()), a.pq.pop();
  }
  int size()const{return sz(pq);}
// private:
  PQMax<int> pq;
  int ans=0,bias=0;
};
//NOTE: f(x)=min{f(x+i),a<i<b}+|x-k|+m -> pf(k)sf(k)ab(-a,b,m)
struct SlopeTrick{
  void pf_dec(int x){l.pf_dec(-r.sf_inc(-x));}
  void sf_inc(int x){r.pf_dec(-l.sf_inc(x));}
  void add_bias(int lx,int rx,int y){l.add_bias(lx,0),r.add_bias(-rx,0),ans+=y;}
  int minval(){return ans+l.minval()+r.minval();}
  pint argmin(){return {l.argmin(),-r.argmin()};}
  void operator+=(SlopeTrick& a){
    while(sz(a.l.pq)) pf_dec(a.l.argmin()),a.l.pq.pop();
    l.ans+=a.l.ans;
    while(sz(a.r.pq)) sf_inc(-a.r.argmin()),a.r.pq.pop();
    r.ans+=a.r.ans;
    ans+=a.ans;
  }
  int size()const{return l.size()+r.size();}
// private:
  LeftHull l,r;
  int ans=0;
};
//LeftHull 역추적 방법: 스텝i의 argmin값을 am(i)라고 하자. 스텝n부터 스텝1까지
ans[i]=min(ans[i+1],am(i))하면 된다. 아래는 증명..은 아니고 간략한 이유
//am(i)<=ans[i+1]일때: ans[i]=am(i)
//x[i]>ans[i+1]일때: ans[i]=ans[i+1] 왜냐하면 f(i,a)는 a<x[i]에서 감소함수이므로 가능한 최대로
오른쪽으로 붙은 ans[i+1]이 최적.
//스텝i에서 add_bias(k,0)한다면 간격제한k가 있는것이므로 ans[i]=min(ans[i+1]-k,x[i])으로 수정.
//LR Hull 역추적은 케이스나눠서 위 방법을 확장하면 될듯
```

6.5 Random, PBDS, Bit Trick

```
mt19937 rd((unsigned)chrono::steady_clock::now().time_since_epoch().count());
uniform_int_distribution<int> rnd_int(l, r); // rnd_int(rd)
uniform_real_distribution<double> rnd_real(0, 1); // rnd_real(rd)
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <ext/rope>
using namespace __gnu_pbds; //ordered_set : find_by_order(order), order_of_key(key)
using namespace __gnu_cxx; //crope : append(str), substr(s, e), at(idx)
template <typename T>
```

```
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
int __builtin_clz(int x);// number of leading zero
int __builtin_ctz(int x);// number of trailing zero
int __builtin_popcount(int x);// number of 1-bits in x
lsb(n): (n & -n); // last bit (smallest)
floor(log2(n)): 31 - __builtin_clz(n | 1);
floor(log2(n)): 63 - __builtin_clzll(n | 1);
long long next_perm(long long v){
  long long t = v | (v-1);
  return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(v) + 1));
}
int main2(){ return 0; }
int main(){
  size_t  sz = 1<<29;  // 512MB
  void* newstack = malloc(sz);
  void* sp_dest = newstack + sz - sizeof(void*);
  asm  __volatile__("movq %0, %%rax\n\t"
           "movq %%rsp , (%%rax)\n\t"
           "movq %0, %%rsp\n\t": : "r"(sp_dest): );
  main2();
  asm  __volatile__("pop %rsp\n\t");
  return  0;
}
```

6.6 Fast I/O, Fast Div/Mod, Hilbert Mo's

```
namespace io { // thanks to cgiosy
    const signed IS=1<<20;
    char I[IS+1],*J=I;
    inline void
    daer(){if(J>=I+IS-64){char*p=I;do*p++=*J++;while(J!=I+IS);p[read(0,p,I+IS-p)]=0;J=I;}}
    template<int N=10,typename T=int>inline T getu(){daer();T x=0;int k=0;do
    x=x*10+*J-'0';while(*++J>='0'&&++k<N);++J;return x;}
    template<int N=10,typename T=int>inline T geti(){daer();bool
    e=*J=='-';J+=e;return(e?-1:1)*getu<N,T>();}
    struct f{f(){I[read(0,I,IS)]=0;}}flu;
};
struct FastMod{ // typedef __uint128_t L;
  ull b, m;
  FastMod(ull b) : b(b), m(ull((L(1) << 64) / b)) {}
  ull reduce(ull a){ // can be proven that 0 <= r < 2*b
    ull q = (ull)((L(m) * a) >> 64), r = a - q * b;
    return r >= b ? r - b : r;
  }
};
inline int64_t hilbertOrder(int x, int y, int pow, int rotate) {
  if(pow == 0) return 0;
  int hpow = 1 << (pow-1), seg = (x<hpow) ? ( (y<hpow) ? 0 : 3 ) : ( (y<hpow) ? 1 : 2 );
  const int rotateDelta[4] = {3, 0, 0, 1}; seg = (seg + rotate) & 3;
  int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
  int nrot = (rotate + rotateDelta[seg]) & 3;
  int64_t subSquareSize = int64_t(1) << (2*pow - 2);
  int64_t ans = seg * subSquareSize, add = hilbertOrder(nx, ny, pow-1, nrot);
  ans += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1); return ans;
}
struct Query{
  int s, e, x; ll order; void init(){ order = hilbertOrder(s, e, 21, 0); }
  bool operator < (const Query &t) const { return order < t.order; }
};
```

6.7 DP Opt, Tree Opt, Well-Known Ideas

```
// Quadrangle Inequality : C(a, c)+C(b, d) ≤ C(a, d)+C(b, c)
// Monotonicity : C(b, c) ≤ C(a, d)
// CHT, DnC Opt(Quadrangle), Knuth(Quadrangle and Monotonicity)

// 크기가 A, B인 두 서브트리의 결과를 합칠 때 O(AB)이면 O(N^3)이 아니라 O(N^2)
// 각 정점마다 sum(2 ~ C번째로 높이가 작은 정점의 높이)에 결과를 구할 수 있으면 O(N^2)이 아니라 O(N)

// IOI 16 Alien(Lagrange Multiplier), IOI 11 Elephant(sqrt batch process)
// IOI 09 Region
// 서로소 합집합의 크기가 적당히 bound 되어 있을 때 사용
// 쿼리 메모이제이션 / 쿼리 하나에 O(A log B), 전체 O(N√Q log N)
```

6.8 Catalan, Burnside, Grundy, Pick, Hall, Simpson, Kirchhoff, Area of Quadrangle

- 카탈란 수
  1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012,742900
  $C_n = binomial(n*2, n)/(n+1)$;
  - 길이가 2n인 올바른 괄호 수식의 수
  - n + 1개의 리프를 가진 풀 바이너리 트리의 수
  - n + 2각형을 n개의 삼각형으로 나누는 방법의 수
- Burnside's Lemma
  - 수식
  G=(X,A): 집합X와 액션A로 정의되는 군G에 대해, $|A||X/A| = sum(|\text{Fixed points of a}|, \text{for all a in A})$
  X/A 는 Action으로 서로 변형가능한 X의 원소들을 동치로 묶었을때 동치류(파티션) 집합 - 풀어쓰기
  orbit: 그룹에 대해 두 원소 a,b와 액션f에 대해 f(a)=b인거에 간선연결한 컴포넌트(연결집합)
  orbit개수 = sum(각 액션 g에 대해 f(x)=x인 x(고정점)개수)/액션개수 - 자유도 치트시트
  회전 n개: 회전i의 고정점 자유도=gcd(n,i)
  임의뒤집기 n=홀수: n개 원소중심축(자유도 (n+1)/2)
  임의뒤집기 n=짝수: n/2개 원소중심축(자유도 n/2+1) + n/2개 원소안지나는축(자유도 n/2)
- 알고리즘 게임
  - Nim Game의 해법 : 각 더미의 돌의 개수를 모두 XOR했을 때 0 이 아니면 첫번째, 0 이면 두번째 플레이어가 승리.
  - Grundy Number : 어떤 상황의 Grundy Number는, 가능한 다음 상황들의 Grundy Number를 모두 모은 다음, 그 집합에 포함 되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러개의 state들로 나뉠 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.
  - Subtraction Game : 한 번에 k 개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를 k + 1로 나눈 나머지를 XOR 합하여 판단한다.
  - Index-k Nim : 한 번에 최대 k개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을 k + 1로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.
  - Misere Nim : 모든 돌 무더기가 1이면 N이 홀수일 때 후공 승, 그렇지 않은 경우 XOR 합 0이면 후공 승
- Pick's Theorem
  격자점으로 구성된 simple polygon이 주어짐. I 는 polygon 내부의 격자점 수, B 는 polygon 선분 위 격자점 수, A는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다. $A = I + B/2 - 1$

  ```
  // number of (x, y) : (0 <= x < n && 0 < y <= k/d x + b/d)
  ll count_solve(ll n, ll k, ll b, ll d) { // argument should be positive
    if (k == 0) {
      return (b / d) * n;
    }
  ```

```
    if (k >= d || b >= d) {
      return ((k / d) * (n - 1) + 2 * (b / d)) * n / 2 + count_solve(n, k % d, b % d, d);
    }
    return count_solve((k * n + b) / d, d, (k * n + b) % d, k);
}
```

- 홀의 결혼 정리 : 이분그래프(L-R)에서, 모든 L을 매칭하는 필요충분 조건 = L에서 임의의 부분집합 S를 골랐을 때, 반드시 (S의 크기) <= (S와 연결되어있는 모든 R의 크기)이다.
- Simpson 공식 (적분) : Simpson 공식, $S_n(f) = \frac{h}{3}[f(x_0) + f(x_n) + 4\sum f(x_{2i+1}) + 2\sum f(x_{2i})]$
  - $M = \max|f^4(x)|$이라고 하면 오차 범위는 최대 $E_n \le \frac{M(b-a)}{180}h^4$
- Kirchhoff's Theorem : 그래프의 스패닝 트리 개수
  - m[i][j] := -(i-j 간선 개수) (i ≠ j)
  - m[i][i] := 정점 i의 degree
  - res = (m의 첫 번째 행과 첫 번째 열을 없앤 (n-1) by (n-1) matrix의 행렬식)
- Tutte Matrix : 그래프의 최대 매칭
  - m[i][j] := 간선 (i,j)가 없으면 0, 있으면 $i < j?r : -r$, r은 $[0, P)$ 구간의 임의의 정수
  - $rank(m)/2$가 높은 확률로 최대 매칭
- 브라마굽타 : 원에 내접하는 사각형의 각 선분의 길이가 a, b, c, d일 때 사각형의 넓이 $S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$, $s = (a+b+c+d)/2$
- 브레치나이더 : 임의의 사각형의 각 변의 길이를 a, b, c, d라고 하고, 마주보는 두 각의 합을 2로 나눈 값을 θ라 하면, $S = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd \times cos^2\theta}$
- $g^0 + g^1 + g^2 + \cdots g^{p-2} \equiv -1 \pmod p$ iff $g = 1$, otherwise 0.

6.9 inclusive and exclusive, Stirling Number, Bell Number

- 공 구별 X, 상자 구별 O, 전사함수 : 포함배제 $\sum_{i=1}^{k}(-1)^{k-i} \times kCi \times i^n$
- 공 구별 O, 상자 구별 X, 전사함수 : 제 2종 스털링 수 $S(n,k) = k \times S(n-1,k) + S(n-1,k-1)$
  포함배제하면 $O(K \log N)$, $S(n,k) = 1/k! \times \sum_{i=1}^{k}(-1)^{k-i} \times kCi \times i^n$
- 공 구별 O, 상자 구별 X, 제약없음 : 벨 수 $B(n,k) = \sum_{i=0}^{k} S(n,i)$ 몇 개의 상자를 버릴지 다 돌아보기
  수식 정리하면 $O(\min(N,K)\log N)$에 됨. $B(n,n) = \sum_{i=0}^{n-1}(n-1)Ci \times B(i,i)$
  $B(n,k) = \sum_{j=0}^{k} S(n,j) = \sum_{j=0}^{k} 1/j! \sum_{i=0}^{j}(-1)^{j-i} jCi \times i^n = \sum_{j=0}^{k}\sum_{i=0}^{j}\frac{(-1)^{j-i}}{i!(j-i)!}i^n$
  $= \sum_{i=0}^{k}\sum_{j=i}^{k}\frac{(-1)^{j-i}}{i!(j-i)!}i^n = \sum_{i=0}^{k}\sum_{j=0}^{k-i}\frac{(-1)^j}{i!j!}i^n = \sum_{i=0}^{k}\frac{i^n}{i!}\sum_{j=0}^{k-i}\frac{(-1)^j}{j!}$

6.10 About Graph Matching(Graph with $|V| \le 500$)

- **Game on a Graph** : s에 토큰이 있음. 플레이어는 각자의 턴마다 토큰을 인접한 정점으로 옮기고 못 옮기면 짐. s를 포함하지 않는 최대 매칭이 존재함 ↔ 후공이 이김
- **Chinese Postman Problem** : 모든 간선을 방문하는 최소 가중치 Walk를 구하는 문제.
  Floyd를 돌린 다음, 홀수 정점들을 모아서 최소 가중치 매칭 (홀수 정점은 짝수 개 존재)
- **Unweighted Edge Cover** : 모든 정점을 덮는 가장 작은(minimum cardinality/weight) 간선 집합을 구하는 문제
  $|V| - |M|$, 길이 3짜리 경로 없음, star graph 여러 개로 구성
- **Weighted Edge Cover** : $sum_{v \in V}(w(v)) - sum_{(u,v) \in M}(w(u) + w(v) - d(u,v))$, $w(x)$는 x와 인접한 간선의 최소 가중치
- **NEERC'18 B** : 각 기계마다 2명의 노동자가 다뤄야 하는 문제.
  기계마다 두 개의 정점을 만들고 간선으로 연결하면 정답은 $|M| - |기계|$임. 정답에 1/2씩 기여한다는 점을 생각해보면 좋음.
- **Min Disjoint Cycle Cover** : 정점이 중복되지 않으면서 모든 정점을 덮는 길이 3 이상의 사이클 집합을 찾는 문제.
  모든 정점은 2개의 서로 다른 간선, 일부 간선은 양쪽 끝점과 매칭되어야 하므로 플로우를 생각할 수 있지만 용량 2짜리 간선에 유량을 1만큼 흘릴 수 있으므로 플로우는 불가능.
  각 정점과 간선을 2개씩((v, v'), $(e_{i,u}, e_{i,v})$)로 복사하자. 모든 간선 $e = (u,v)$에 대해 e_u와 e_v를 잇는 가중치 w짜리 간선을 만들고(like NEERC18), $(u, e_{i,u}), (u', e_{i,u}), (v, e_{i,v}), (v', e_{i,v})$를 연결하는 가중치 0짜리 간선을 만들자.
  Perfect 매칭이 존재함 ↔ Disjoint Cycle Cover 존재. 최대 가중치 매칭 찾은 뒤 모든 간선 가중치 합에서 매칭 빼면 됨.
- **Two Matching** : 각 정점이 최대 2개의 간선과 인접할 수 있는 최대 가중치 매칭 문제.
  각 컴포넌트는 정점 하나/경로/사이클이 되어야 함. 모든 서로 다른 정점 쌍에 대해 가중치 0짜리 간선 만들고, 가중치 0짜리 (v, v') 간선 만들면 Disjoing Cycle Cover 문제가 됨. 정점 하나만 있는 컴포넌트는 self-loop, 경로 형태의 컴포넌트는 양쪽 끝점을 연결한다고 생각하면 편함.

6.11 Checklist

- 비슷한 문제를 풀어본 적이 있던가?
- 단순한 방법에서 시작할 수 있을까? (Brute Force)
- 내가 문제를 푸는 과정을 수식화할 수 있을까? (예제를 직접 해결해보면서)
- 문제를 단순화할 수 없을까?
- 그림으로 그려볼 수 있을까?
- 수식으로 표현할 수 있을까?
- 문제를 분해할 수 있을까?
- 뒤에서부터 생각해서 풀 수 있을까?
- 순서를 강제할 수 있을까?
- 특정 형태의 답만을 고려할 수 있을까? (정규화)
- 구간을 통째로 가져간다 : 플로우 + 적당한 자료구조 $(i, i+1, k, 0), (s, e, 1, w), (N, T, k, 0)$
- a = b : a만 움직이기, b만 움직이기, 두 개 동시에 움직이기, 반대로 움직이기
- 말도 안 되는 것들을 한 번은 생각해보기 / "당연하다고 생각한 것" 다시 생각해보기
- Directed MST / Dominator Tree
- 일정 비율 충족 or 2 3개로 모두 커버 : 랜덤
- 확률 : DP, 이분 탐색(NYPC 2019 Finals C)
- 최대/최소 : 이분 탐색, 그리디(Prefix 고정, Exchange Argument), DP(순서 고정)