2D-Ising

April 20, 2018

1 Calculation set-up

We are performing Monte-Carlo simulations of the 2D-Ising model. The hamiltonian is 'long range' in that it is not restricted to nearest neighbour interactions,

$$\mathcal{H} = \sum_{i \neq j} -J_{ij}\sigma_i, \sigma_j, \tag{1}$$

and we use the following definition of J_{ij},

$$J_{ij} = \frac{J_0}{(cosh((j_2 - j_1)\frac{\pi}{S_1}) - cos((i_2 - i_1)\frac{\pi}{S_1}))^{2+\sigma}}, \tag{2}$$

to make contact with a 2D system that has been radially quantized, i.e., to associate j with the temporal(dilatation) direction and i with the angle. σ governs the 'strength' of the long range interaction, and one can easily see that as $\sigma \to \infty$, the model becomes short range. We will be performing many simulations at various sizes, temperatures $(\frac{1}{J_0})$, and σ to locate the critical regions.

There is much speculation about the physics in the intermediate range of σ. More specifically, there is the idea by Slava et. al [CITE] that a second primary operator generated by an auxiliary field χ. We know that for a radially quantised system the correlation functions will obey

$$\langle \sigma(j_2, i_2)\sigma(j_1, i_1)\rangle = \frac{1}{(cosh((j_2 - j_1)\frac{\pi}{S_1}) - cos((i_2 - i_1)\frac{\pi}{S_1}))^{\Delta}}, \tag{3}$$

where S_1 is the circumference of the (AdS boundry) cylinder. The temporal (dilatation) dimension ought to be infinite, so we attempt to ameliorate finite volume effects by making the temporal extent 4 times the size of S_1. Ee wish to observe the various descendents $(\Delta + n)$ of the primary Δ, we must therefore expand (3) to isolate the descendants. We do this in the following fashion, where for brevity we use $t = (j_2 - j_1)\frac{\pi}{S_1}$ and $\theta = (i_2 - i_1)\frac{\pi}{S_1}$,

$$\begin{aligned} \langle \sigma(t, \theta)\sigma(0, 0)\rangle =& \frac{1}{2(e^t + e^{-t} - 2cos(\theta))^{\Delta}}, && (4) \\ =& \frac{e^{-t\Delta}}{2(1 + e^{-2t} - 2e^{-t}cos(\theta))^{\Delta}} && (5) \end{aligned}$$

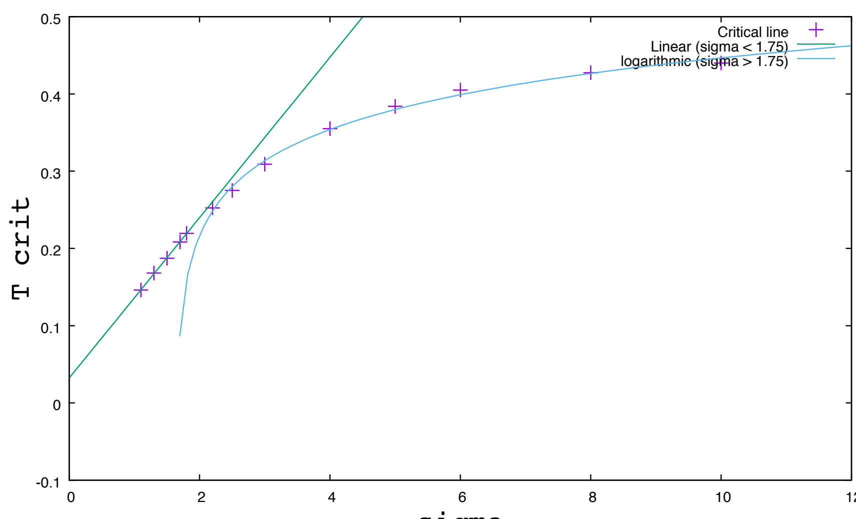
We now expand the denominator using e^{-t} as the expansion parameter. Mathematica says:

$$\langle\sigma(t,\theta)\sigma(0,0)\rangle = \frac{e^{-t\Delta}}{2} + \Delta cos(\theta)e^{-t(\Delta+1)} + \frac{\Delta(2\Delta+1)}{2}cos^2(\theta)e^{-t(\Delta+2)} + \frac{2}{3}cos(\theta)\Delta(\Delta+1)(2cos^3(\theta)(\Delta+2)-3)e^{-t(\Delta+3)} + \dots \tag{6}$$

and so on. Notice how the $(\Delta+3)$ term contains a linear and a cubic cosine term, making it difficult to perform a simple Fourier transform to project onto pure descendent states. We can, however, perform a multi-exponential fit to correlation function data. Consider a sum of 4 exponential decays, this would ordinarily comprise 8 free parameters (4 coefficients, 4 decay constants) but if we were to constrain those exponentials with known coefficients and known decay constants (as we have in (6)) then we can, with sufficient statistics, measure the scaling dimension and lattice spacing.

2 Preliminary data

The first order of business is to identify (roughly) the critical temperature of a given lattice size and σ. We do this by examining the Binder cumulant, specific heat, and magnetic susceptibly for a range of temperatures. From (http://csml.northwestern.edu/resources/Reprints/prl7.pdf, figure 1) we can be assured that the Binder cumulant for longe range system with $\sigma > 1.75$ will be the same value at the critical point as it is for a short range system, thus allowing us to monitor the critical temperature with decreasing σ. This has been completed for one lattice size (16×64) shown in figure 1.



(a) 16x64.

Figure 1: Tc Vs Sigma

At the critical temperature, we have measured correlation function values along the cylinder in the temporal direction and attempted to plot to the constrained multi-exponential. A typical plot is given in figure 2 (16×64, $\sigma = 2.5$). It shows the (folded) correlation function value Vs time. Notice that the first term in (6) dominates the late times, but important contributions to the fit are made from the subsequent two terms. This 3 exponential fit was constrained with two parameters A, a the following fashion,

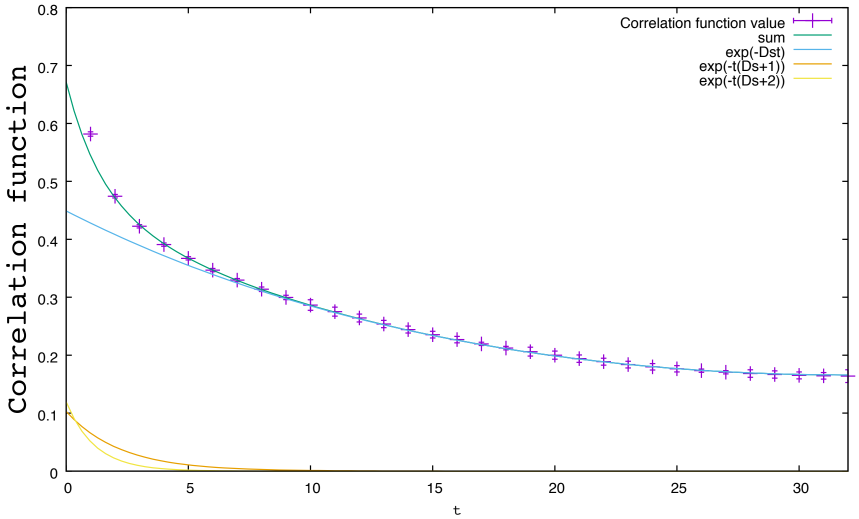


Figure 2: 16x64, $\sigma = 2.5$ at T_c: Fitting with three terms for the σ field only.

$$\begin{aligned} Correlation = & A\Bigg(\frac{e^{-\frac{a\Delta T}{2}}}{2} cosh\Big(\Delta\Big(t-\frac{T}{2}\Big)a\Big) \\ & + \Delta e^{-a(\Delta+1)Ta} cosh\Big((\Delta+1)\Big(t-\frac{T}{2}\Big)a\Big) \\ & + \Delta(2\Delta+1)e^{a(\Delta+2)Ta} cosh\Big((\Delta+2)\Big(t-\frac{T}{2}\Big)a\Big)\Bigg) \end{aligned} \tag{7}$$

where A is an overall coefficient and a is a temporal scaling parameter. We only omitted $t = 0, 1$ from the fit, and $\chi^2 = 0.14$. As $\sigma > 1.75$, we are still in the region where the Binder cumulant is the same as the short range theory. We allowed Δ to be a free parameter in the fit and we saw a value of 0.128, which is good agreement with 0.125 for this preliminary study.

We also attempted to fit data to,

$$\langle \sigma(t,\theta)\sigma(0,0)\rangle = \frac{C_\chi}{2(e^t + e^{-t} - 2cos(\theta))^{\Delta_\chi}} + \frac{C_\sigma}{2(e^t + e^{-t} - 2cos(\theta))^{\Delta_\sigma}} \tag{8}$$

where χ is the auxiliary field *a la* Slava and C_σ, C_χ are constants, also going up to the third term for both expansions, in figure 3, for the same data set shown in figure 2. We constrained the fit by asserting that $\Delta_\chi + \Delta_\sigma = 2$. The fit matches very well. Δ is again allowed to be a free parameter, and it evaluates this time to 0.114, which is reasonably good given the crudity of these calculations. The ratio of $C_\chi/C_\sigma = 0.034$ suggests that the effect (if any at this value of σ is very small indeed.

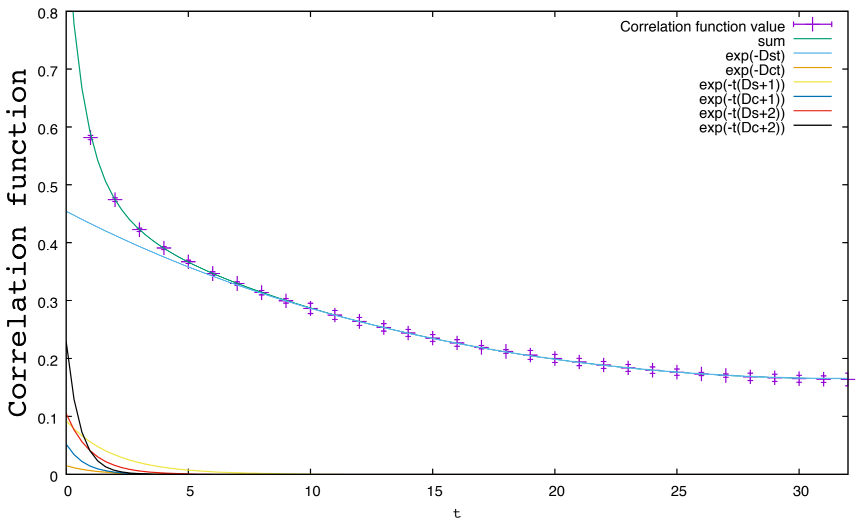


Figure 3: 16x64, $\sigma = 2.5$ at T_c: Fitting with three terms for the σ field and the χ field.

In order to see if moving to $\sigma < 1.75$ would reveal more information on the χ field, the above analysis was repeated for $\sigma = 1.3$. This time, we saw that $\Delta = 2.00$ with only the σ field used in the fit, and 0.194 with the χ field added. Both of these numbers significantly deviate from the Ising value of 0.125.

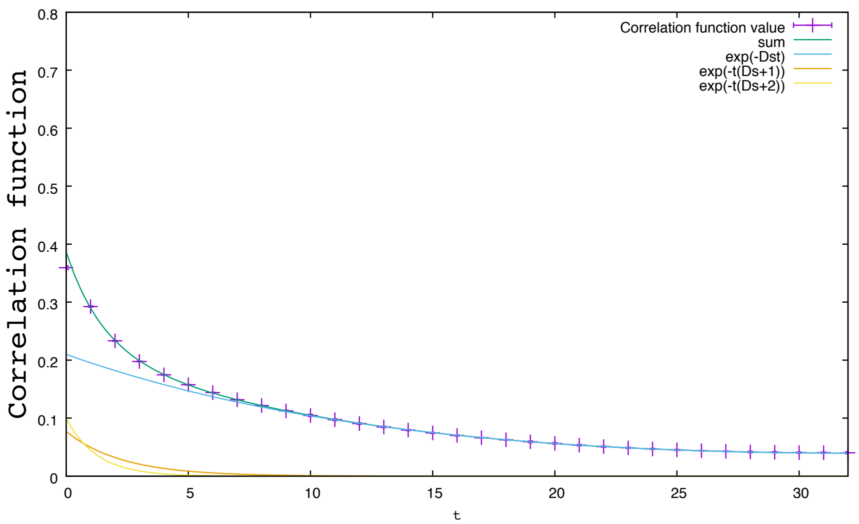


Figure 4: 16x64, $\sigma = 1.3$ at T_c: Fitting with three terms for the σ field.

3 Outlook

We can certainly identify the critical surface with relative ease, and the multi exponential plots are working well. The explicit contributions for the χ fields are very small indeed, at the order of the percent level, so exquisite data sets will be needed. If the χ information is in the early timeslices, we well need larger lattices.

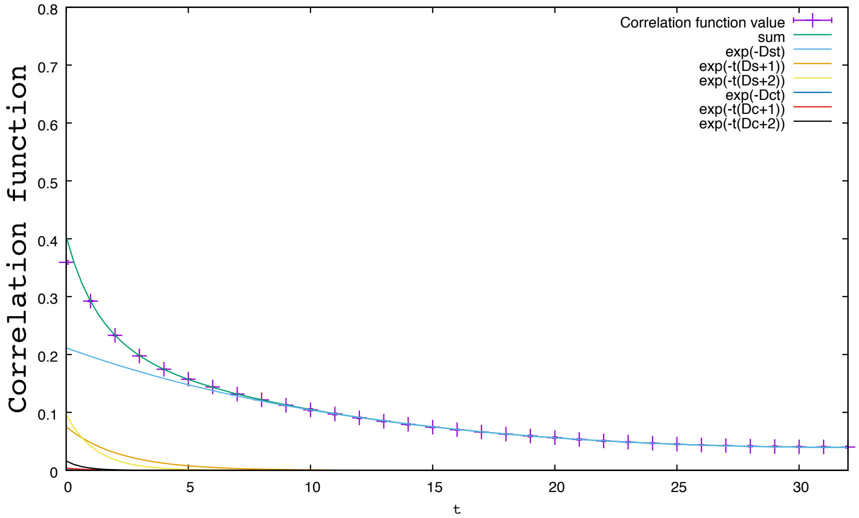


Figure 5: 16x64, $\sigma = 1.3$ at T_c: Fitting with three terms for the σ field and the χ field.

At the moment, we have serial, OMP, and GPU accelerated code for both the pure Ising long and short range, and for the long range ϕ^4. We have yet to implement ϕ^3 correlators, but this is a simple enough task and may reveal more information on the χ field.