2D U(1) Gauge Slab Action

1 Non-compact Gaussian Action

Here is the 2d L^2 action with $L_s + 1$ slices: $s = 0, \pm 1, \cdots L_s/2$.

$$S = \frac{1}{4}\sum_{x,s}\sum_{\mu,\nu} F_{\mu\nu}(x,s)F_{\mu\nu}(x,s) + \frac{1}{2}\sum_{\mu,\nu} E_\mu(x,s)E_\mu(x,s) \tag{1}$$

where

$$F_{\mu\nu}(x,s) = \Delta_\mu\theta_\nu(x,s) - \Delta_\nu\theta_\mu(x,s) = (\theta_\nu(x+\mu,s) - \theta_\nu(x,s)) - (\theta_\mu(x+\nu,s) - \theta_\mu(x,s))$$

and

$$E_\mu(x,s) = \Delta_s\theta_\mu(x,s) = \theta_\mu(x,s+1) - \theta_\mu(x,s) \tag{2}$$

Therefore,

$$S = \frac{1}{2}\sum_{x,s}\sum_{\mu<\nu}((\theta_\nu(x+\mu,s)-\theta_\nu(x,s))-(\theta_\mu(x+\nu,s)-\theta_\mu(x,s)))^2+\frac{1}{2}\sum_{x,s}\sum_{s=-L_s/2}^{L_s/2-1}(\theta_\mu(x,s+1)-\theta_\mu(x,s))^2 \tag{3}$$

We can go to momentum space by a unitary transformation :

$$\theta_\mu(x,s) = \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi} d^2k e^{ixk}\widetilde{\theta}_\mu(k,s) \quad \text{and} \quad \widetilde{\theta}_\mu(k,s) = \frac{1}{L}\sum_{x\in Z} e^{-ixk}\theta_\mu(x,s) \tag{4}$$

and

$$\Delta_\mu\theta_\nu(x,s) = \frac{1}{L}\sum_k (e^{ik_\mu} - 1)e^{ixk}\widetilde{\theta}_\nu(k,s) \tag{5}$$

or defining $(e^{ik_\mu} - 1) = i\hat{k}_\mu$ this gives,

$$\begin{aligned} S &= \frac{1}{2}\sum_{k,s}\sum_{\mu<\nu}[\hat{k}^*_\mu\widetilde{\theta}^*_\nu(k,s) - \hat{k}^*_\nu\widetilde{\theta}^*_\mu(k,s)][\hat{k}_\mu\widetilde{\theta}_\nu(k,s) - \hat{k}_\nu\widetilde{\theta}_\mu(k,s)] \\ &+ \frac{1}{2}\sum_{s=-L_s/2}^{L_s/2-1}\sum_{k,\mu}(\widetilde{\theta}^*_\mu(k,s+1) - \widetilde{\theta}^*_\mu(k,s))(\widetilde{\theta}_\mu(k,s+1) - \widetilde{\theta}_\mu(k,s)) \end{aligned} \tag{6}$$

(Note in 2D $\mu = x$, and $\nu = y$ so there is no sum at all!) The quadratic form is

$$S = \frac{1}{2}\widetilde{\theta}^*_\mu(k,s)M_{\mu\nu}(k)\theta_\nu(k,s) - \frac{1}{2}[\widetilde{\theta}^*_\mu(k,s)\theta_\mu(k,s+1) + \widetilde{\theta}^*_\mu(k,s+1)\theta_\mu(k,s)] \tag{7}$$

Since it is of course diagonal in k, the sum over k implicit.

We now integrate all **but** the zero-th central slice. Formally separating calling the thetas on the midels slide $\widetilde{\theta}(k,0) \equiv \widetilde{\theta}_\mu(0)$ we don the integral for over the others $\Theta_{\mu s}(k) = \widetilde{\theta}_\mu(k, s \neq 0)$'s; to get the effective action in $k - space$:

$$
\begin{aligned}
& e^{-S_{eff}} \\
= \; & e^{-\frac{1}{2}\widetilde{\theta}^*_\mu(k,0) M_{\mu\nu}(k) \widetilde{\theta}_\nu(k,0)} \\
\times \; & \int d^2\Theta_{\mu s}(k) e^{-\frac{1}{2}\Theta^\dagger_{\mu s}(k) G^{ss'}_{\mu\nu}(k) \Theta_{s'\nu}(k) + \frac{1}{2}[\widetilde{\theta}^*_\mu(k,0)(\Theta_{1,\mu}(k) + \Theta_{-1,\mu}(k)) + (\Theta^\dagger_{1,\mu} + \Theta^\dagger_{-1,\mu})\widetilde{\theta}_\mu(k,0)]} \\
= \; & e^{-\frac{1}{2}\widetilde{\theta}^*_\mu(k,0) M_{\mu\nu}(k) \widetilde{\theta}_\nu(k,0) + \frac{1}{2}\widetilde{\theta}^*_\mu(k,0)([1/G(k)]^{11}_{\mu\nu}(k) + [1/G(k)]^{-1-1}_{\mu\nu}(k)]\widetilde{\theta}_\nu(k,0)}
\end{aligned} \tag{8}
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