

Assignment 3 Written Solutions

Typing Derivation to English

Suppose that f is a variable of type $\text{int} \rightarrow \text{bool}$. The expressions 2 and 3 are of type int under this assumption (i.e., in the context where f is declared to be of type $\text{int} \rightarrow \text{bool}$) by the `intLit` rule, so it must be that $2 + 3$ is of type int under this assumption as well by the `addInt` rule. The expression f is of type $\text{int} \rightarrow \text{bool}$ under this assumption by the `var` rule. And since f is a function and its input type matches the type of $2 + 3$, we know that $f (2 + 3)$ has type bool (i.e., the output type of f) under this assumption.

Note: This is a *top-down* reading of the derivation. A *bottom-up* reading, like what we did in lecture, is also fine.

Typing Derivation

$$\begin{array}{c}
 \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash f : \text{int} \rightarrow \text{bool}} \text{var} \quad \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash 2 : \text{int}} \text{intLit} \quad \frac{}{\{f : \text{int} \rightarrow \text{bool}, x : \text{int}\} \vdash \text{true} : \text{bool}} \text{trueLit} \\
 \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash f \ 2 : \text{bool}} \text{app} \quad \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash \text{fun } x \rightarrow \text{true} : \text{int} \rightarrow \text{bool}} \text{fun} \quad \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash f : \text{int} \rightarrow \text{bool}} \text{var} \\
 \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash \text{if } f \ 2 \text{ then } (\text{fun } x \rightarrow \text{true}) \text{ else } f : \text{int} \rightarrow \text{bool}} \text{if} \quad \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash 3 : \text{int}} \text{intLit} \\
 \frac{}{\{f : \text{int} \rightarrow \text{bool}\} \vdash (\text{if } f \ 2 \text{ then } (\text{fun } x \rightarrow \text{true}) \text{ else } f) \ 3 : \text{bool}} \text{app}
 \end{array}$$

Semantic Derivation

$$\begin{array}{c}
 \frac{}{\text{true} \Downarrow \top} \text{trueEval} \quad \frac{}{3 \Downarrow 3} \text{intLitEval} \quad \frac{}{\text{false} \Downarrow \perp} \text{falseEval} \quad \frac{}{\text{true} \Downarrow \top} \text{trueEval} \\
 \frac{}{\text{false} \parallel \text{true} \Downarrow \top} \text{orEvalFalse} \quad \frac{}{2 \Downarrow 2} \text{intLitEval} \\
 \frac{}{\text{if false} \parallel \text{true} \text{ then } 2 \text{ else } 1 \Downarrow 2} \text{ifEvalTrue} \\
 \frac{}{3 + (\text{if false} \parallel \text{true} \text{ then } 2 \text{ else } 1) \Downarrow 5} \text{addIntEval} \\
 \frac{}{\text{let } x = \text{true} \text{ in } 3 + (\text{if false} \parallel x \text{ then } 2 \text{ else } 1) \Downarrow 5} \text{letEval}
 \end{array}$$