Formal Grammar

Concepts of Programming Languages
Lecture 11

CAS CS 320

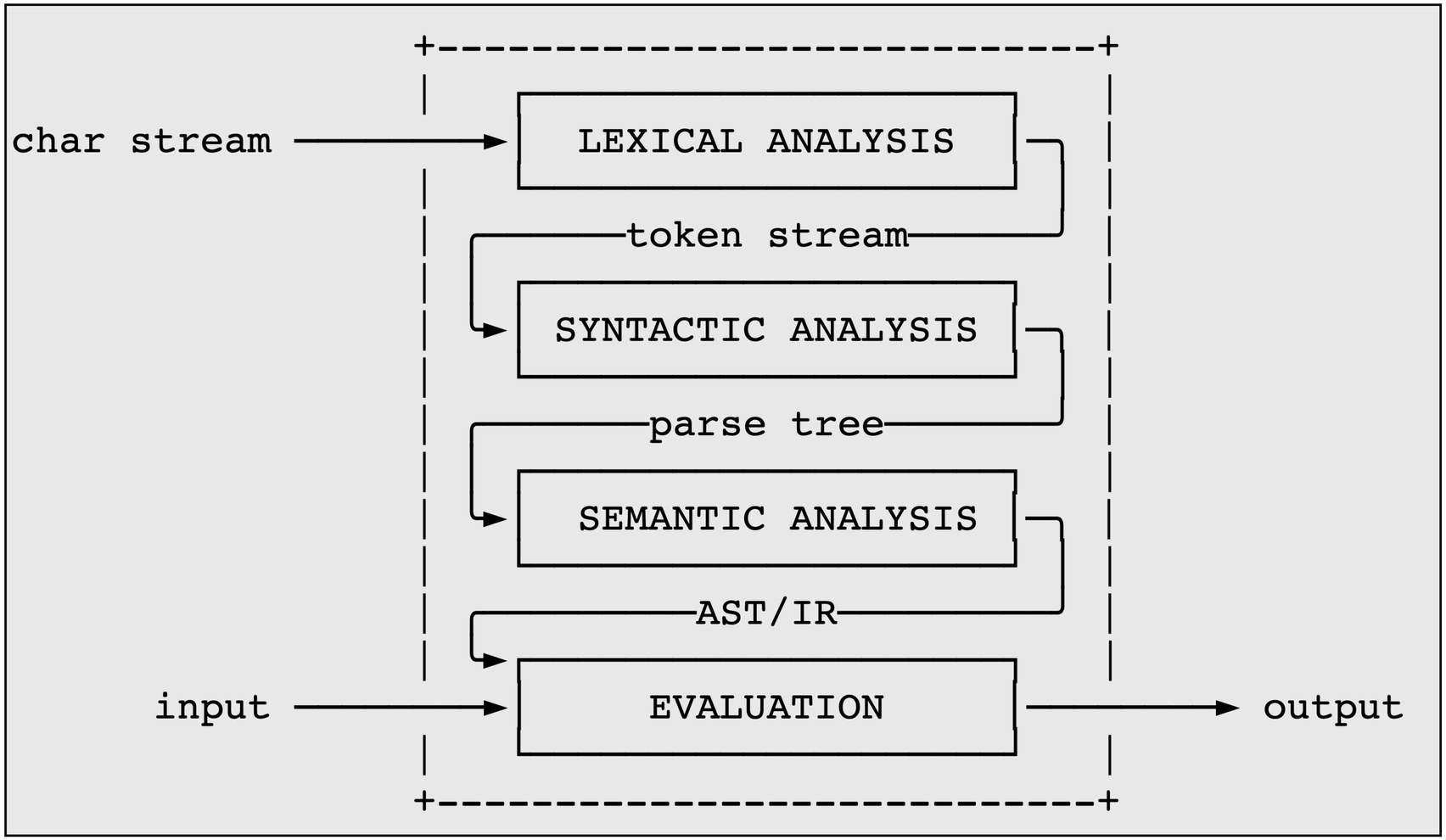
Outline

Discuss briefly the **interpretation pipeline**, and how it will look in the context of this course

Introduce **formal grammars**, a mathematical framework for thinking about syntax and parsing

The Interpretation Pipeline

The Picture


char stream
LEXICAL ANALYSIS
token stream
SYNTACTIC ANALYSIS
parse tree
SEMANTIC ANALYSIS
AST/IR
input
EVALUATION
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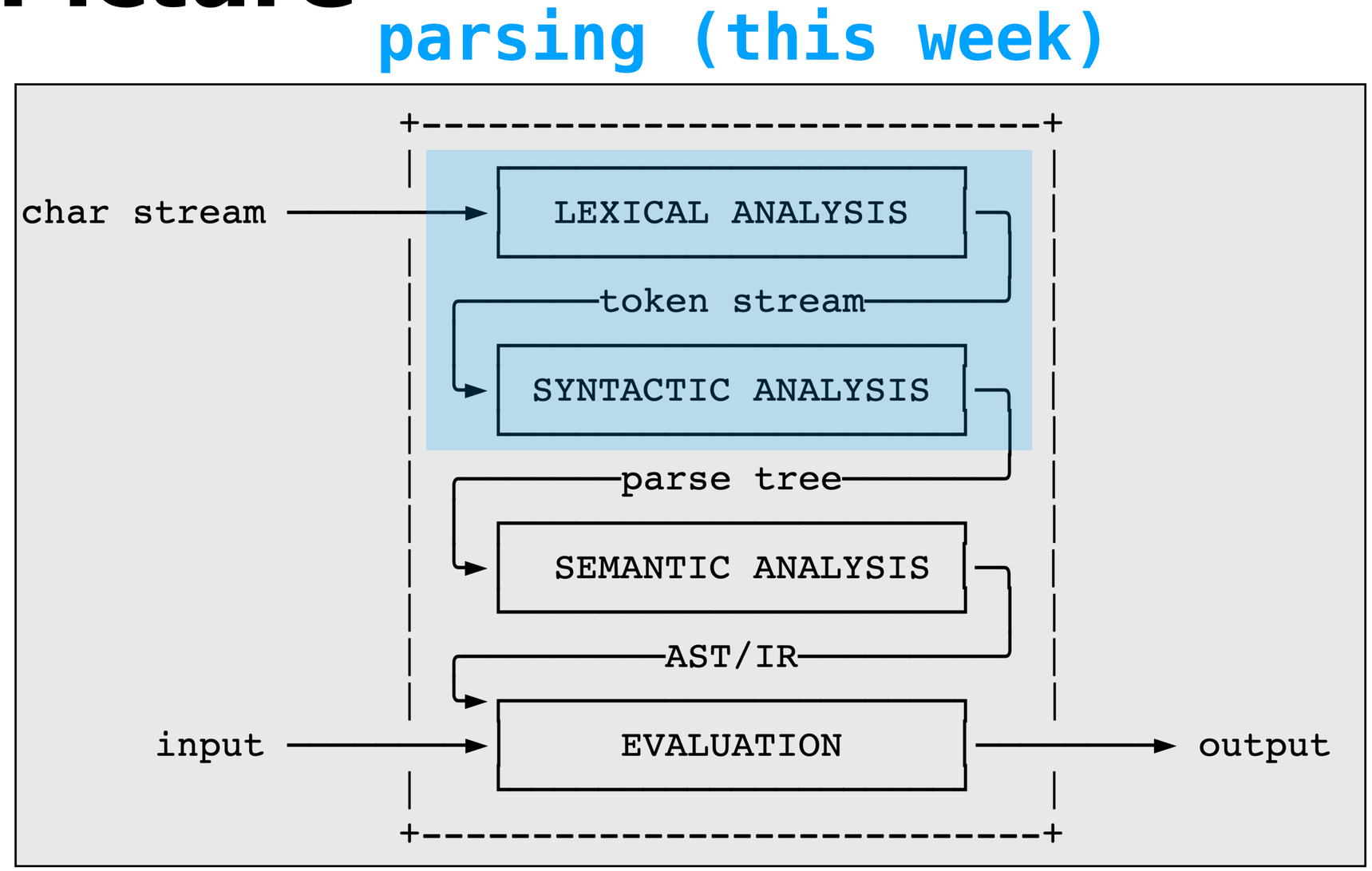
The Picture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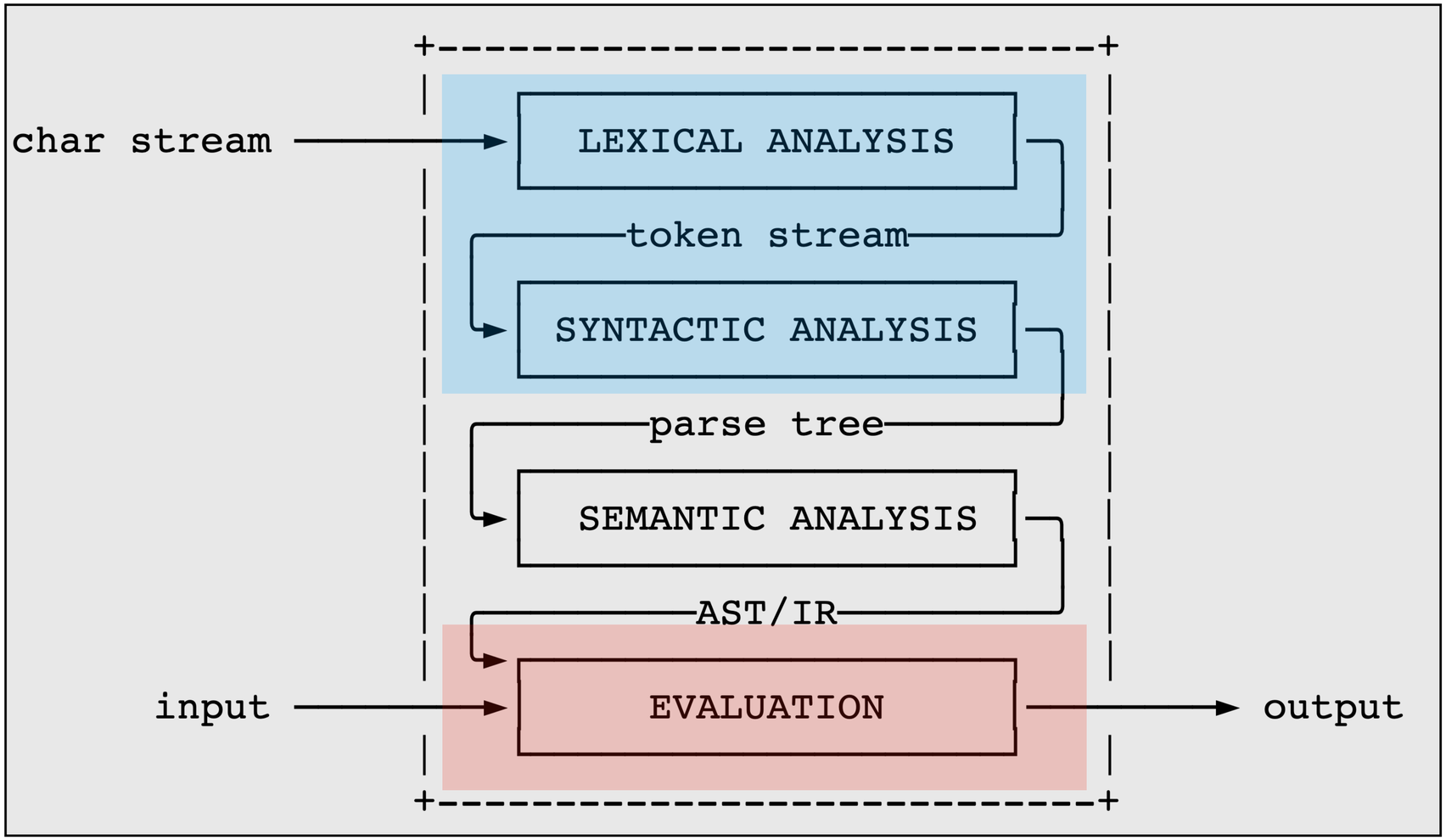
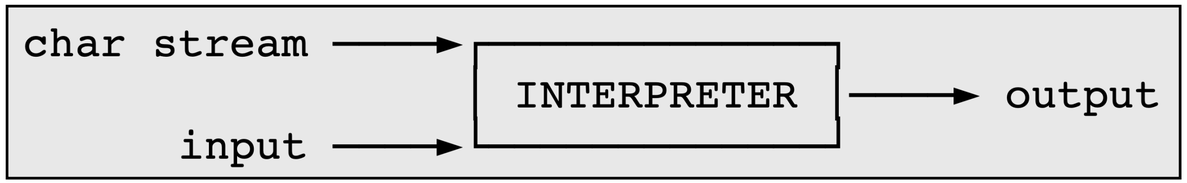
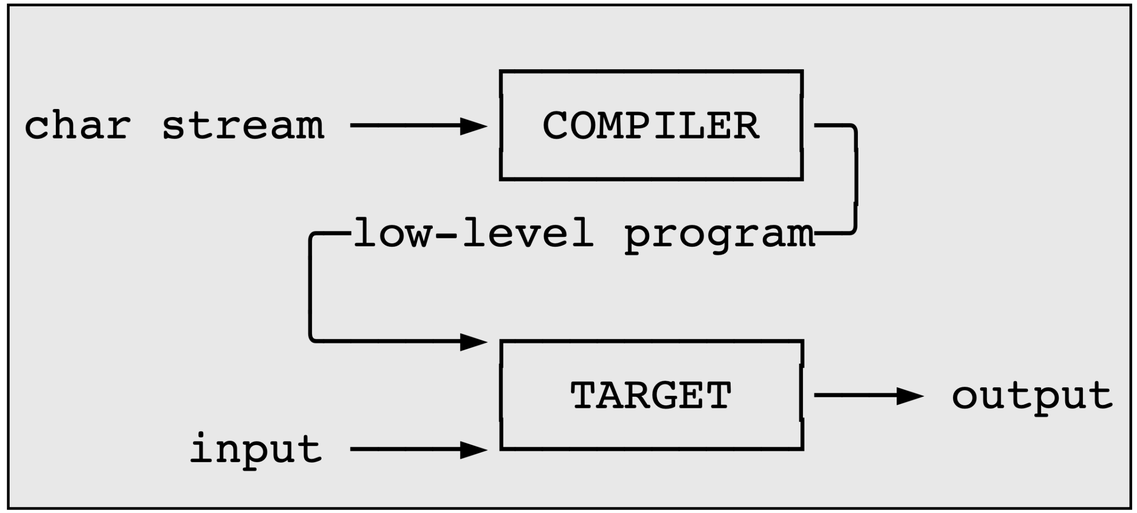


semantics (after the break)

A Note on Compilation





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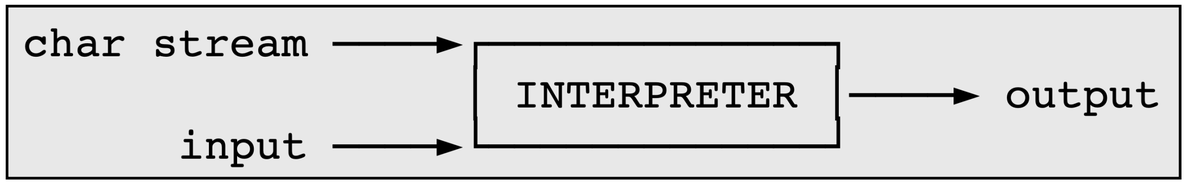

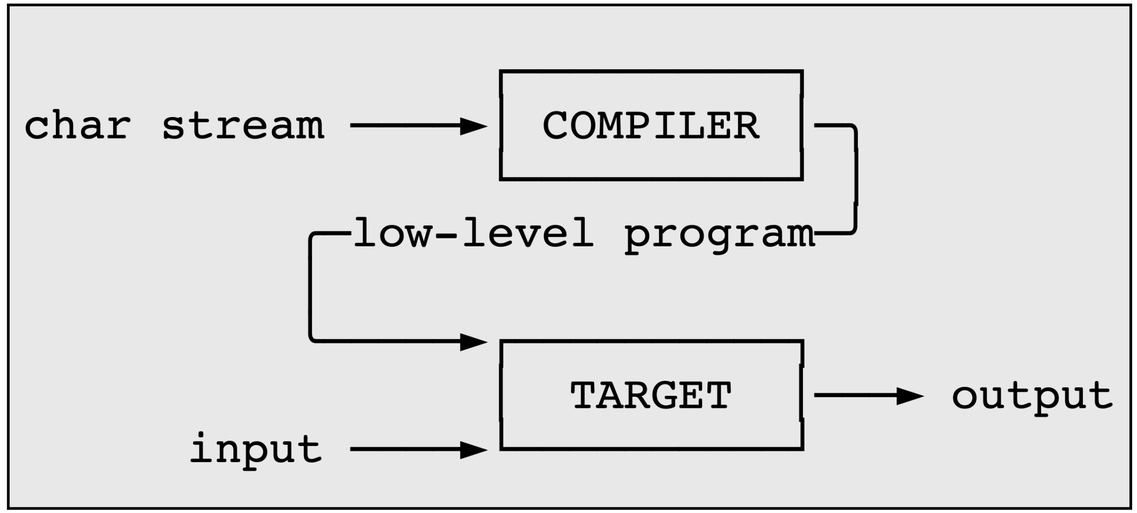

We will be building programs that directly read an evaluate programs (**interpreters**)

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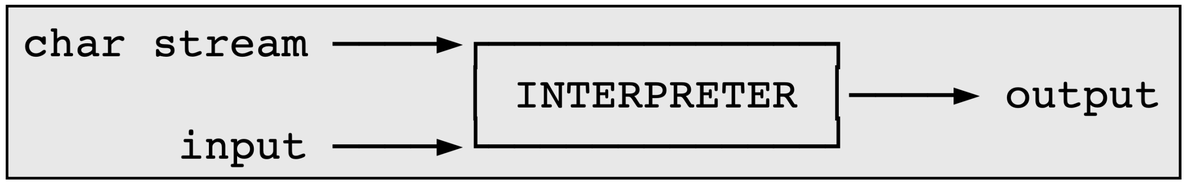
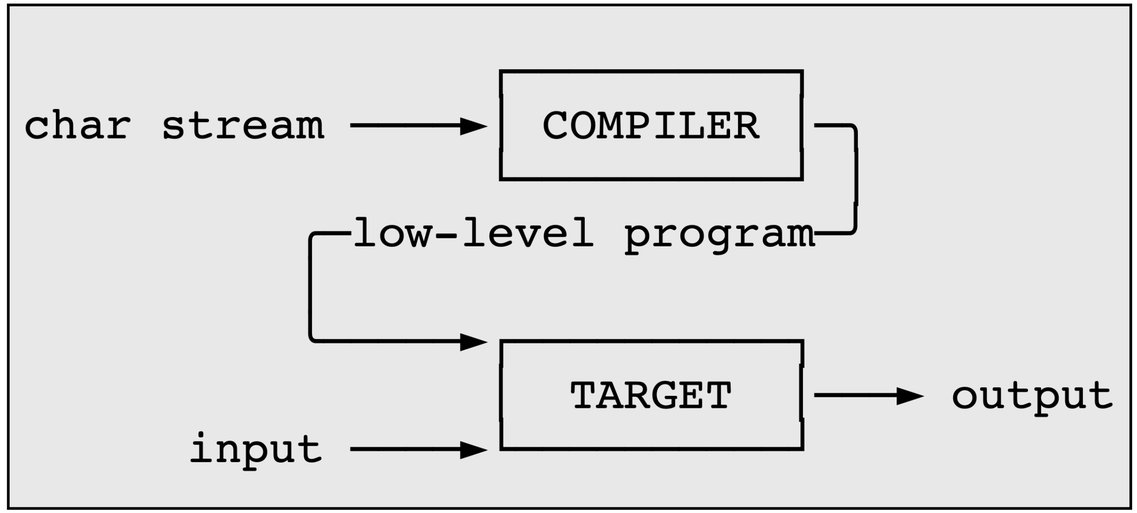




We will be building programs that directly read an evaluate programs (**interpreters**)

In a different course you may write a program which *translates* programs into another language which can then be evaluated elsewhere (**compilers,** we'll cover this briefly)

The Mini-Projects

There will be **three** mini-projects, each 2 weeks long.

For each project, you will build an interpreter.

You'll be given:

- the syntax
- the type rules (not in project 1)
- the semantics

Today

We need a formal language for describing the syntax of programming languages

This is part of the study of **formal language theory**

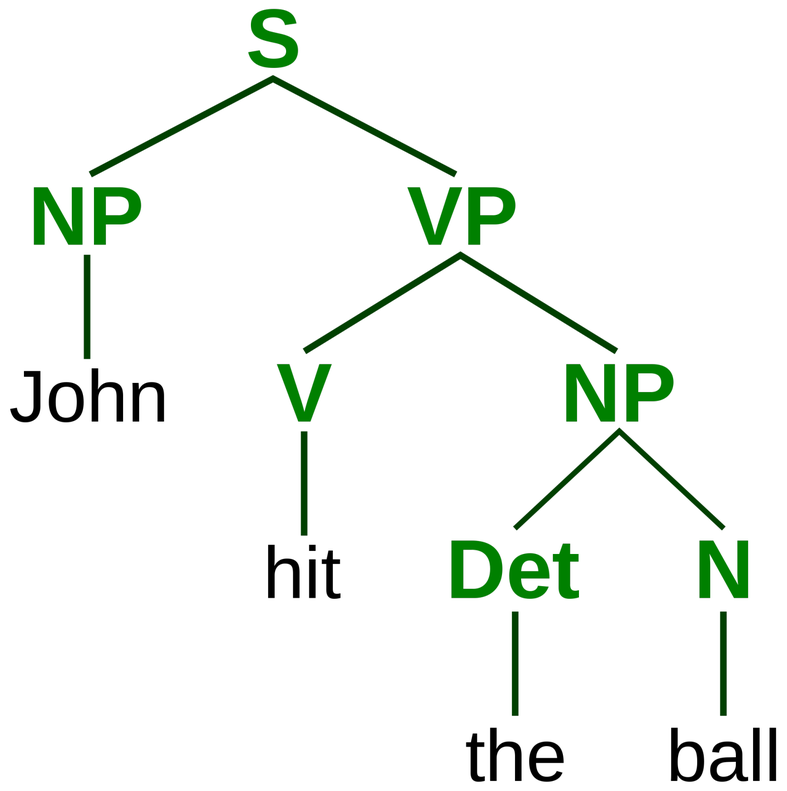
Nearly every PL out there (including OCaml) is described using **Backus–Naur Form (BNF) Grammars.**

Formal Grammar

What is Grammar?

John hit the ball

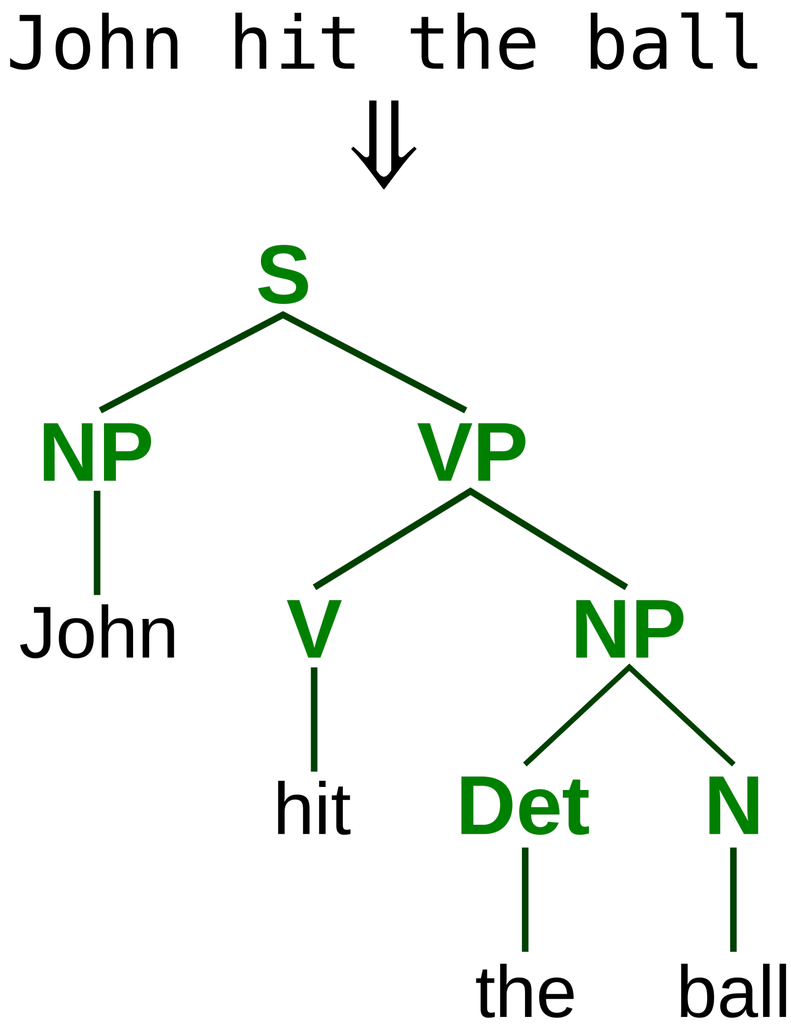
⇓



https://commons.wikimedia.org/wiki/File:ParseTree.svg

What is Grammar?

Grammar refers to the rules which govern what statements are well-formed



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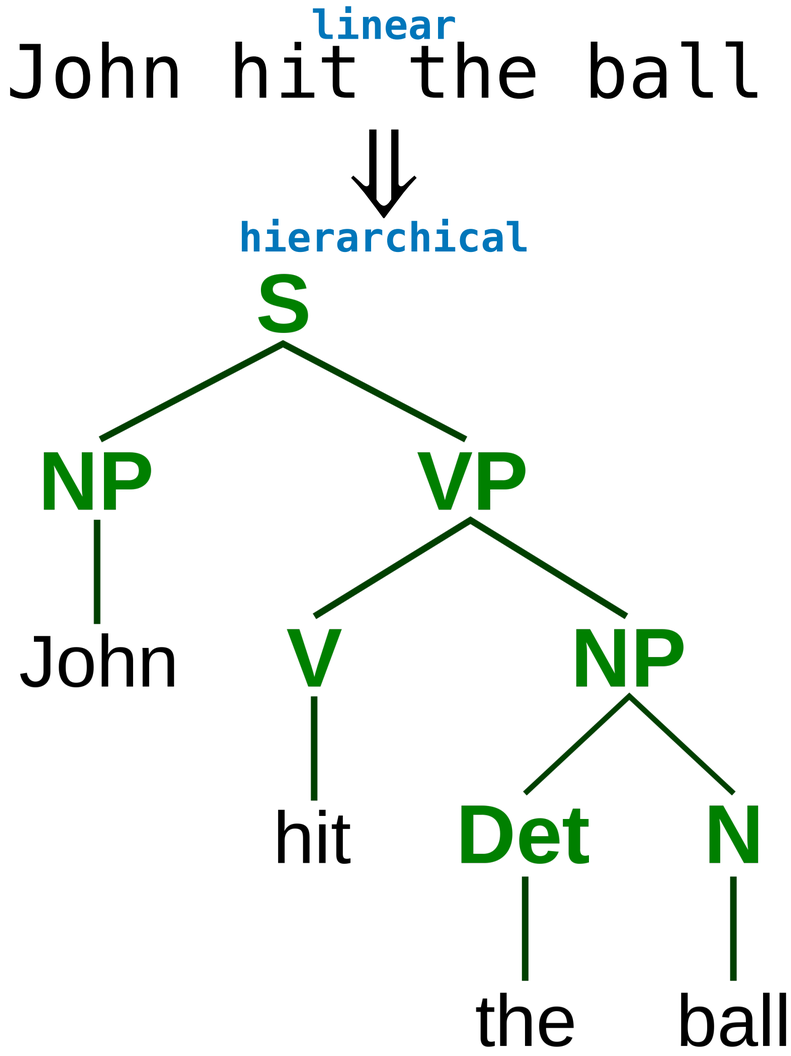
What is Grammar?

Grammar refers to the rules which govern what statements are well-formed

Grammar gives **linear** statements (in natural language or code) their **hierarchical** structure



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Grammar vs. Semantics

I taught my car in the refrigerator. 

vs.

My the car taught I refrigerator. 

Grammar vs. Semantics

I taught my car in the refrigerator. ✓

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Grammar is not (typically) interested in meaning, just structure

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(As we will see, it is useful to separate these two concerns)

Grammars for Programming Languages

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Formal grammars for PL
tell us which programs
are well-formed

Grammars for Programming Languages

```
# let f x = x + 1;;
val f : int -> int = <fun>
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Grammars for Programming Languages

Formal grammars for PL tell us which programs are well-formed

Well-formed programs don't need to be meaningful

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val f : int -> int = <fun>
# let rec x = x x x x ;;
Line 1, characters 14-15:
1 | let rec x = x x x x ;;
                  ^
Error: This expression has type ...
       but an expression was ex ...
       The type variable 'a occ ...
```

Grammars for Programming Languages

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Grammars for Programming Languages

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# let x = List.hd [];;
Exception: Failure "hd".
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Grammars for Programming Languages

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(In OCaml, well-formed programs are the ones we can type-check)

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# let rec f x = f x + 1 - 1;;
val f : 'a -> int = <fun>
# let x = List.hd [];;
Exception: Failure "hd".
# let x = ;;
Line 1, characters 8-10:
1 | let x = ;;
            ^^
Error: Syntax error
```

How do we formally represent well-formed sentences?

An Example

```
the cow jumped over the moon
```

An Example

the cow jumped over the moon

How do we know this a well-formed sentence?

An Example

```
<article> cow jumped over the moon
```

An Example

<article> <noun> jumped over the moon

An Example

<noun-phrase> jumped over the moon

An Example

<noun-phrase> jumped over <article> moon

An Example

<noun-phrase> jumped over <article> <noun>

An Example

<noun-phrase> jumped over <noun-phrase>

An Example

<noun-phrase> jumped over <noun-phrase>

a thing jumped over a thing

An Example

<noun-phrase> jumped <prep> <noun-phrase>

An Example

<noun-phrase> jumped <prep-phrase>

An Example

<noun-phrase> <verb> <prep-phrase>

An Example

<noun-phrase> <verb-phrase>

An Example

<noun-phrase> <verb-phrase>

a thing did a thing

An Example

<sentence>

An Example

<sentence>

We know it's a sentence because it has the right kind of hierarchical structure

A Derivation

<sentence>
<noun-phrase> <verb-phrase>
<noun-phrase> <verb> <prep-phrase>
<noun-phrase> jumped <prep-phrase>
<noun-phrase> jumped <prep> <noun-phrase>
<noun-phrase> jumped over <noun-phrase>
<noun-phrase> jumped over <article> <noun>
<noun-phrase> jumped over <article> moon
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<article> <noun> jumped over the moon
<article> cow jumped over the moon
the cow jumped over the moon

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the cow jumped over the moon

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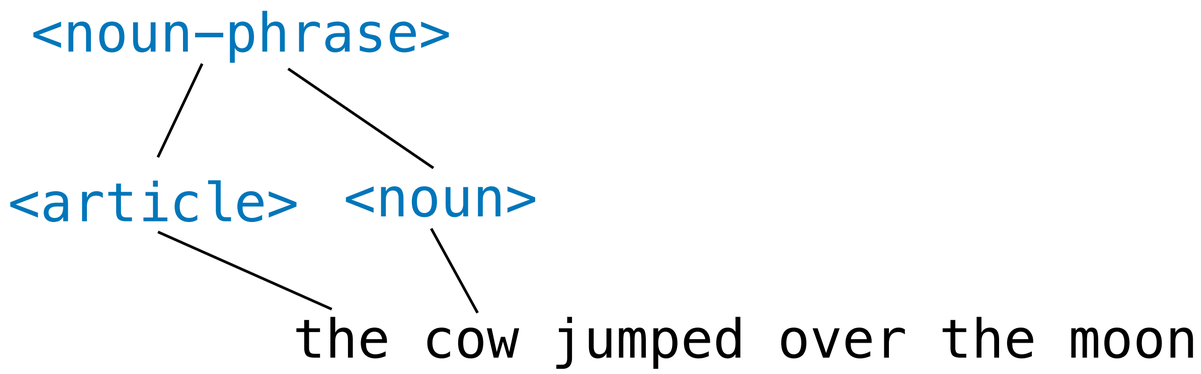

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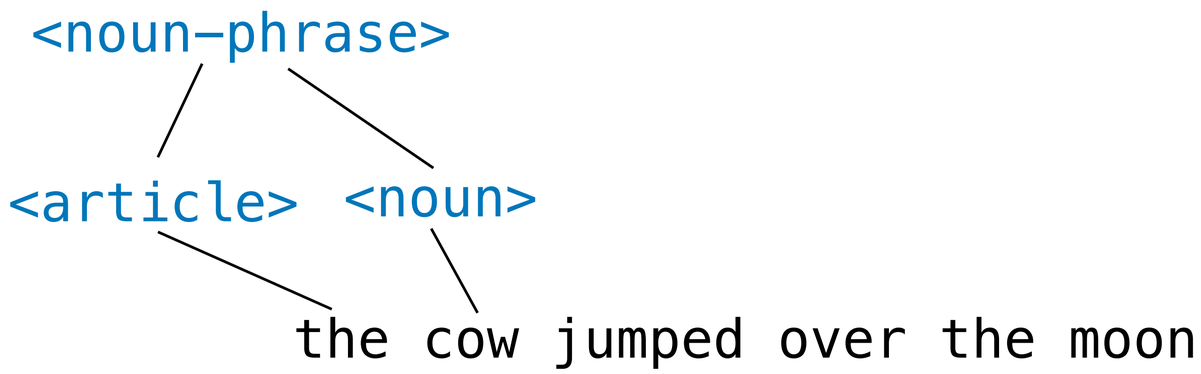

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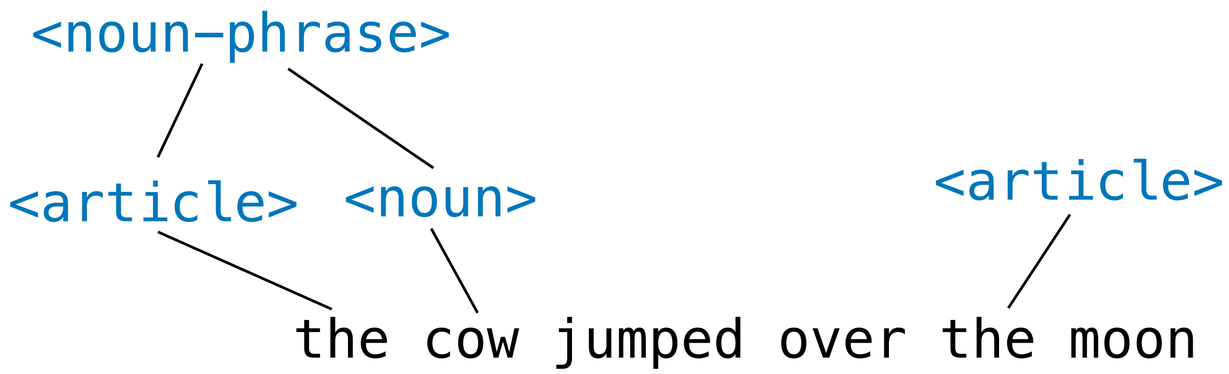

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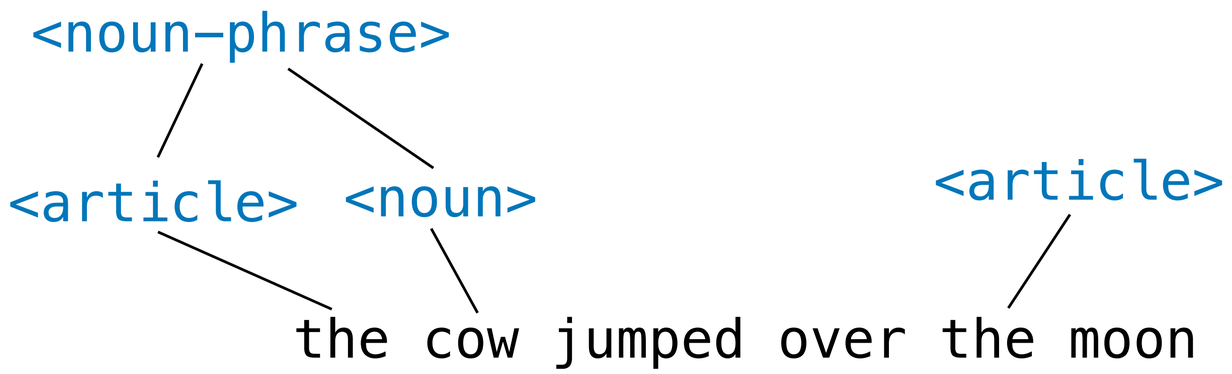

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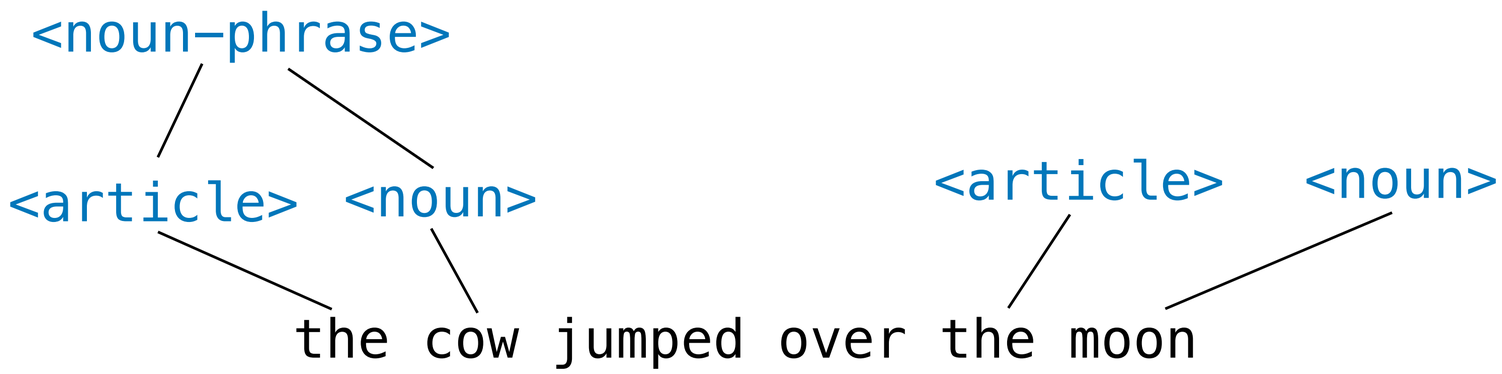

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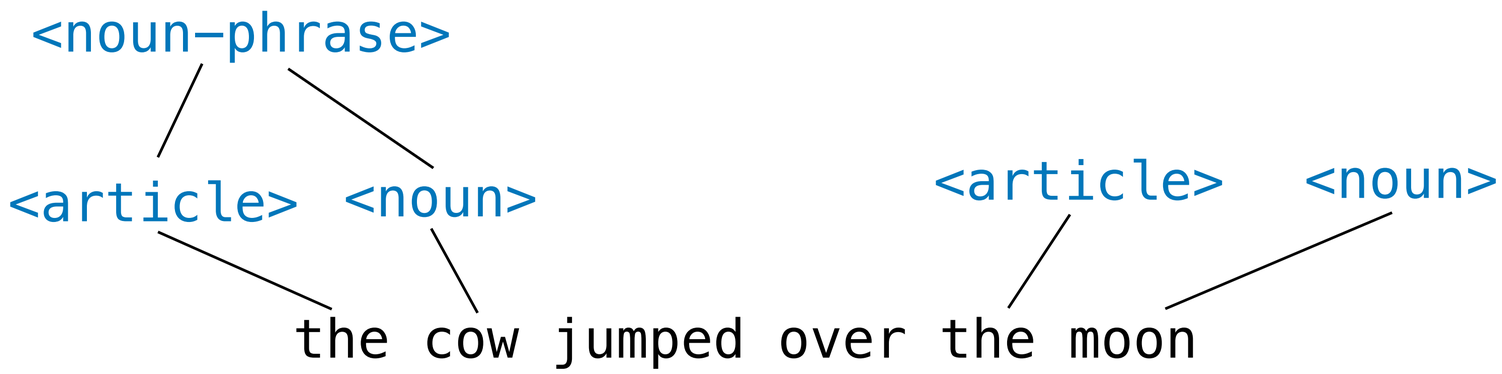

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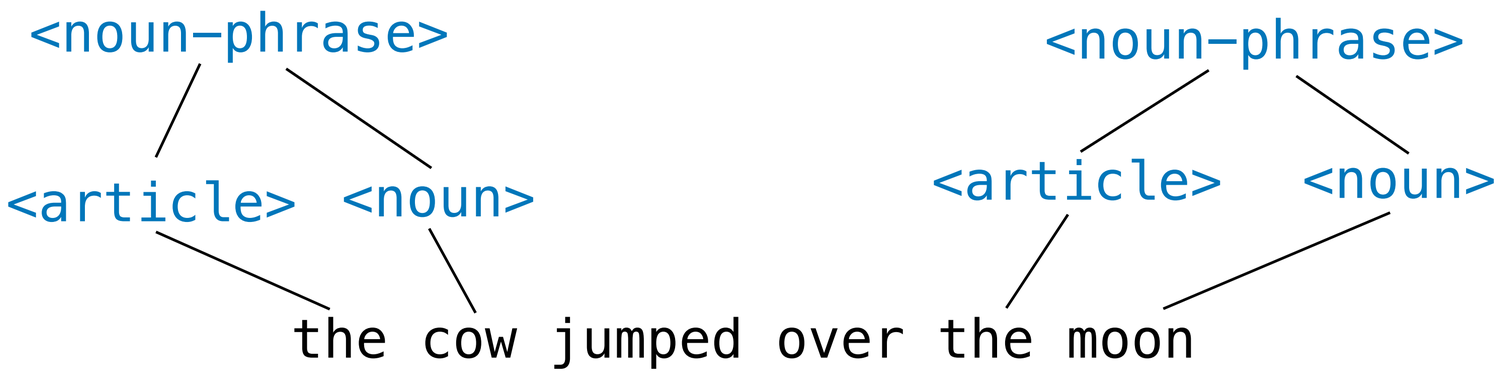

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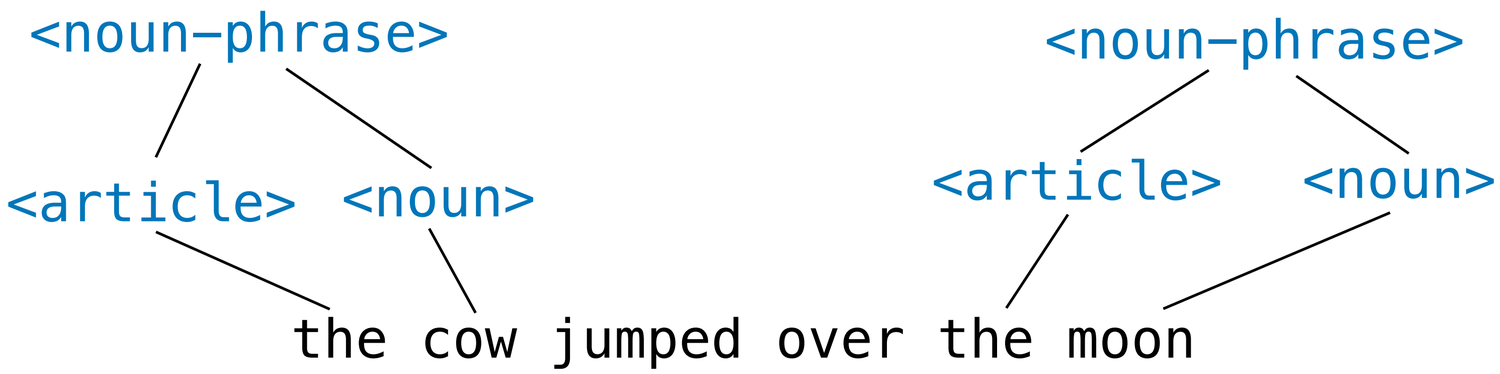

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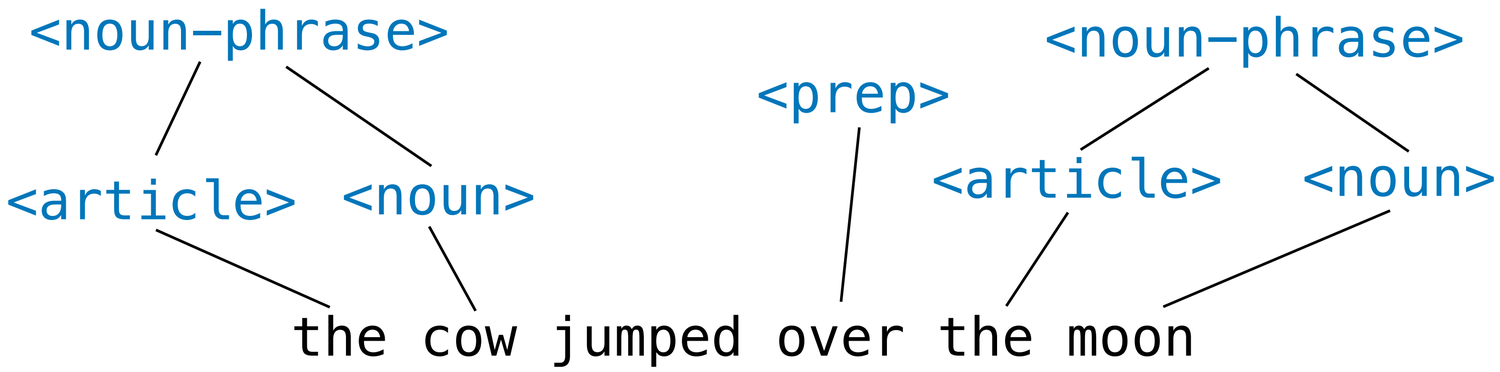

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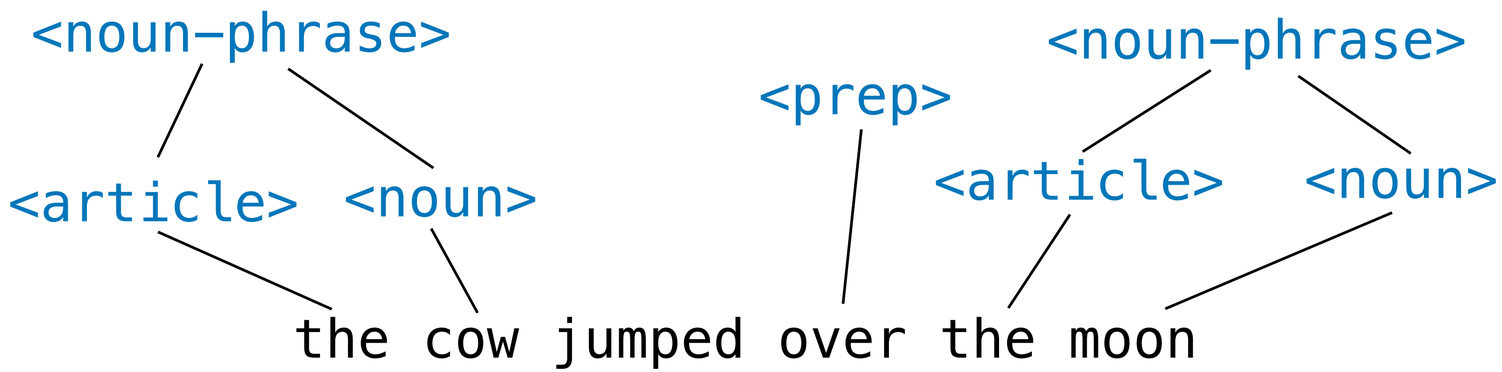

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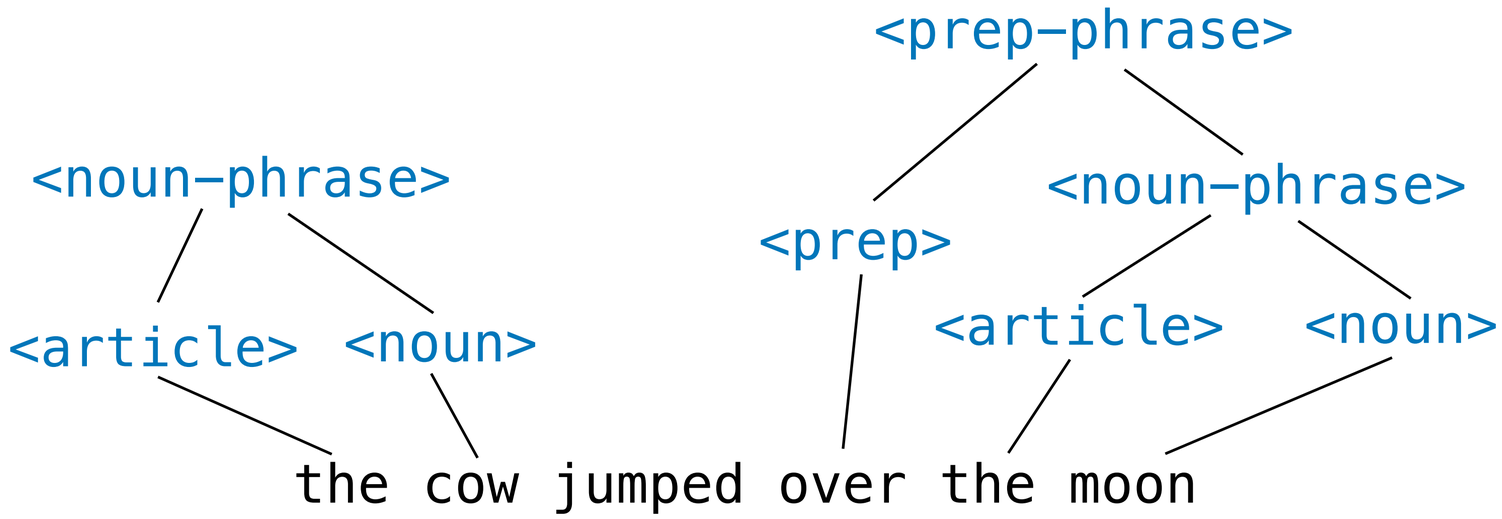

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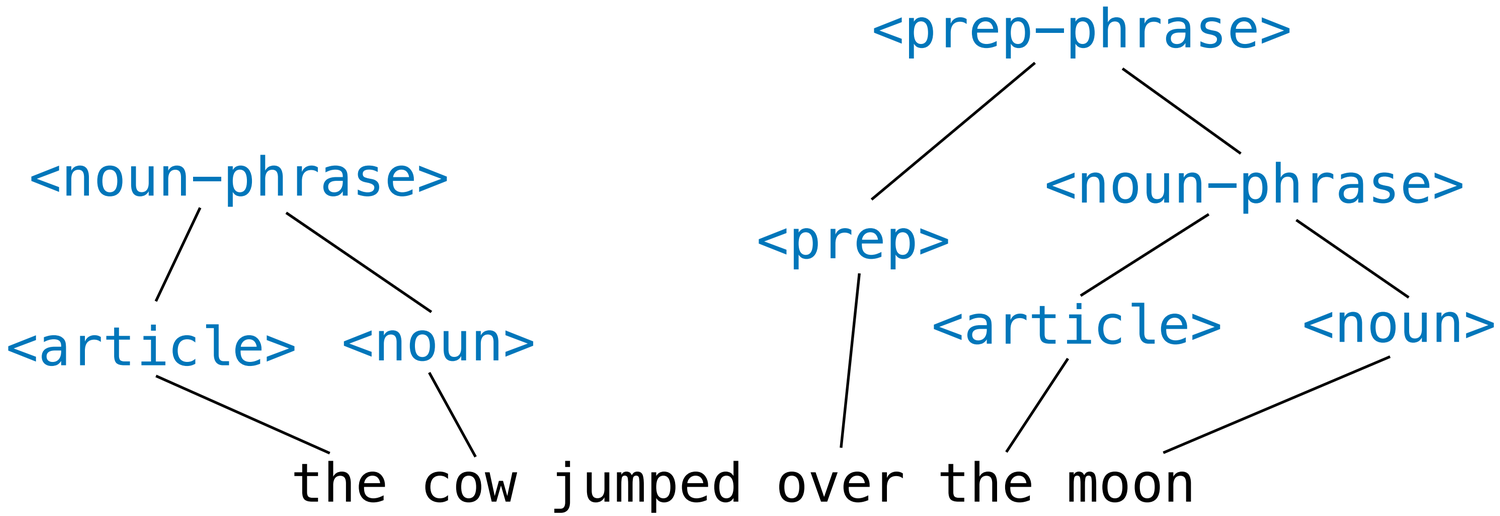

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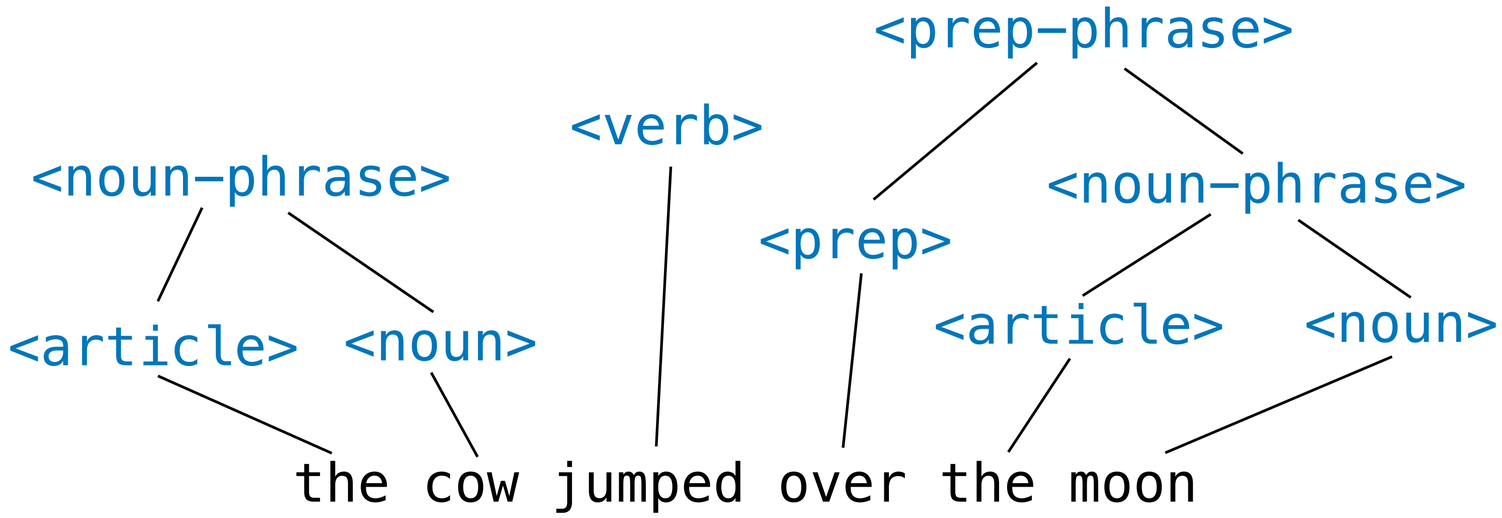

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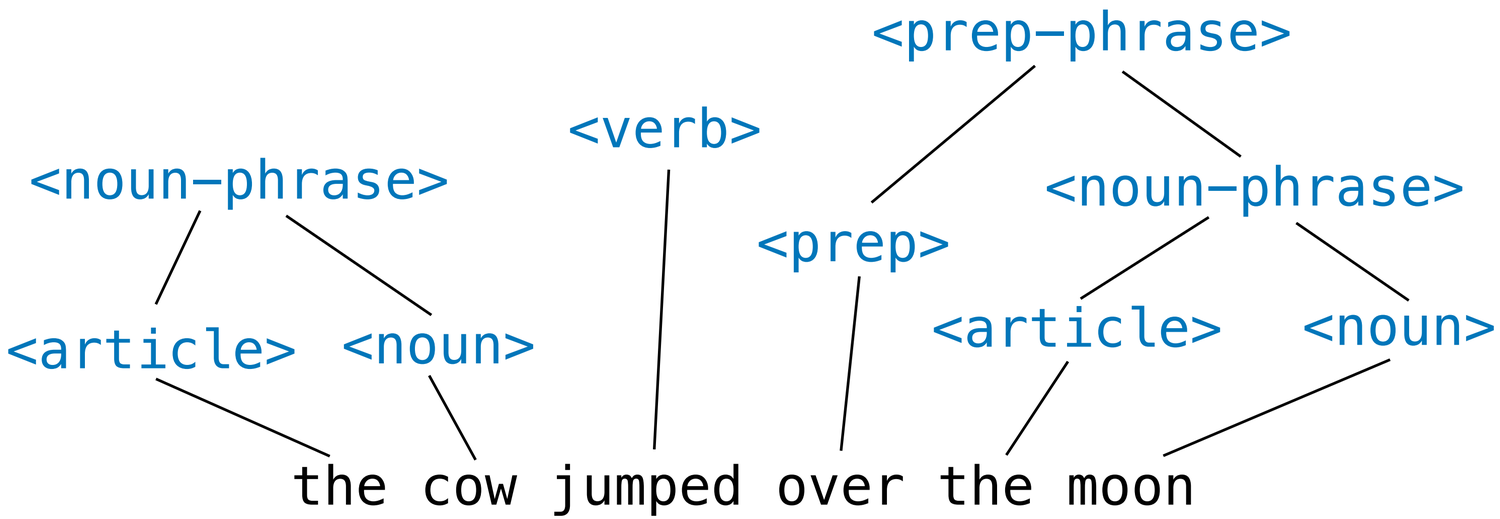

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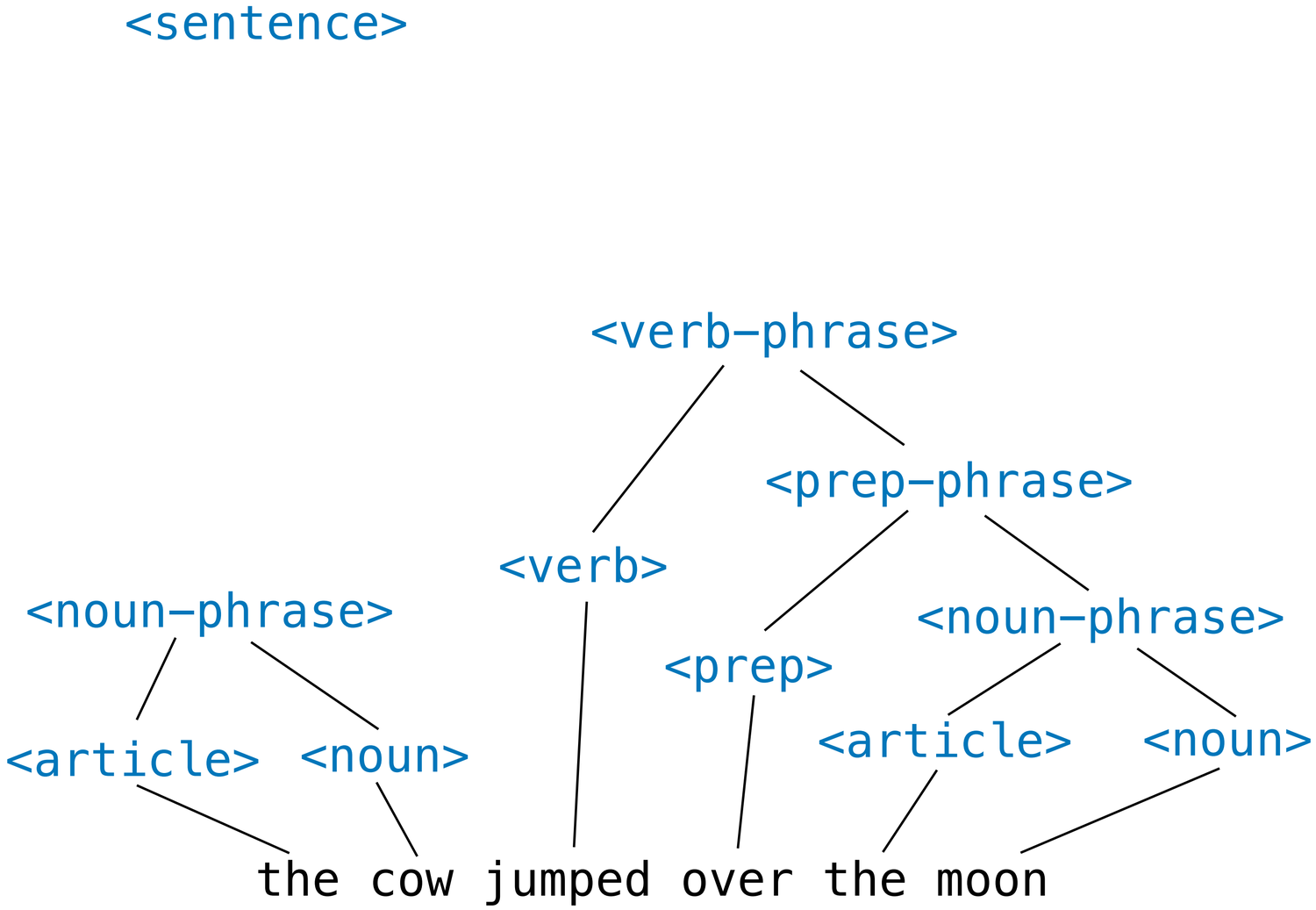

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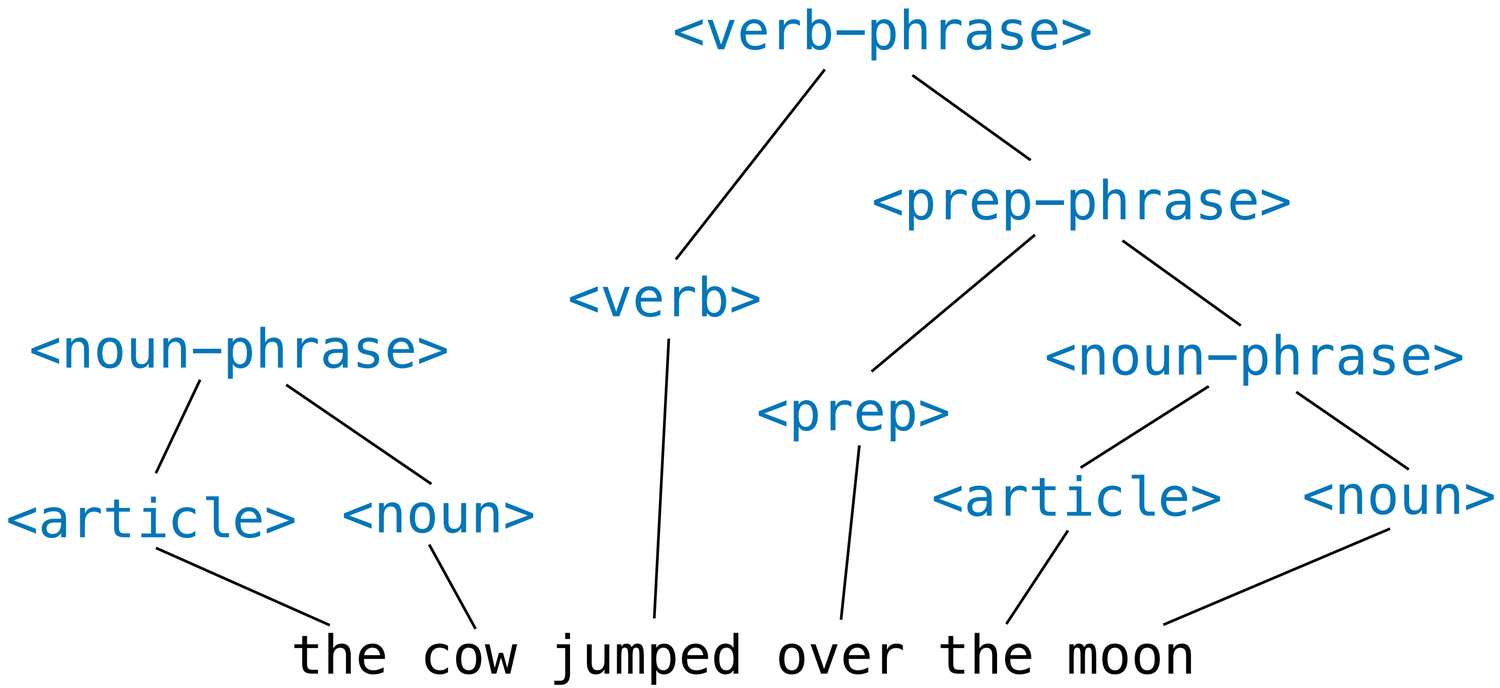

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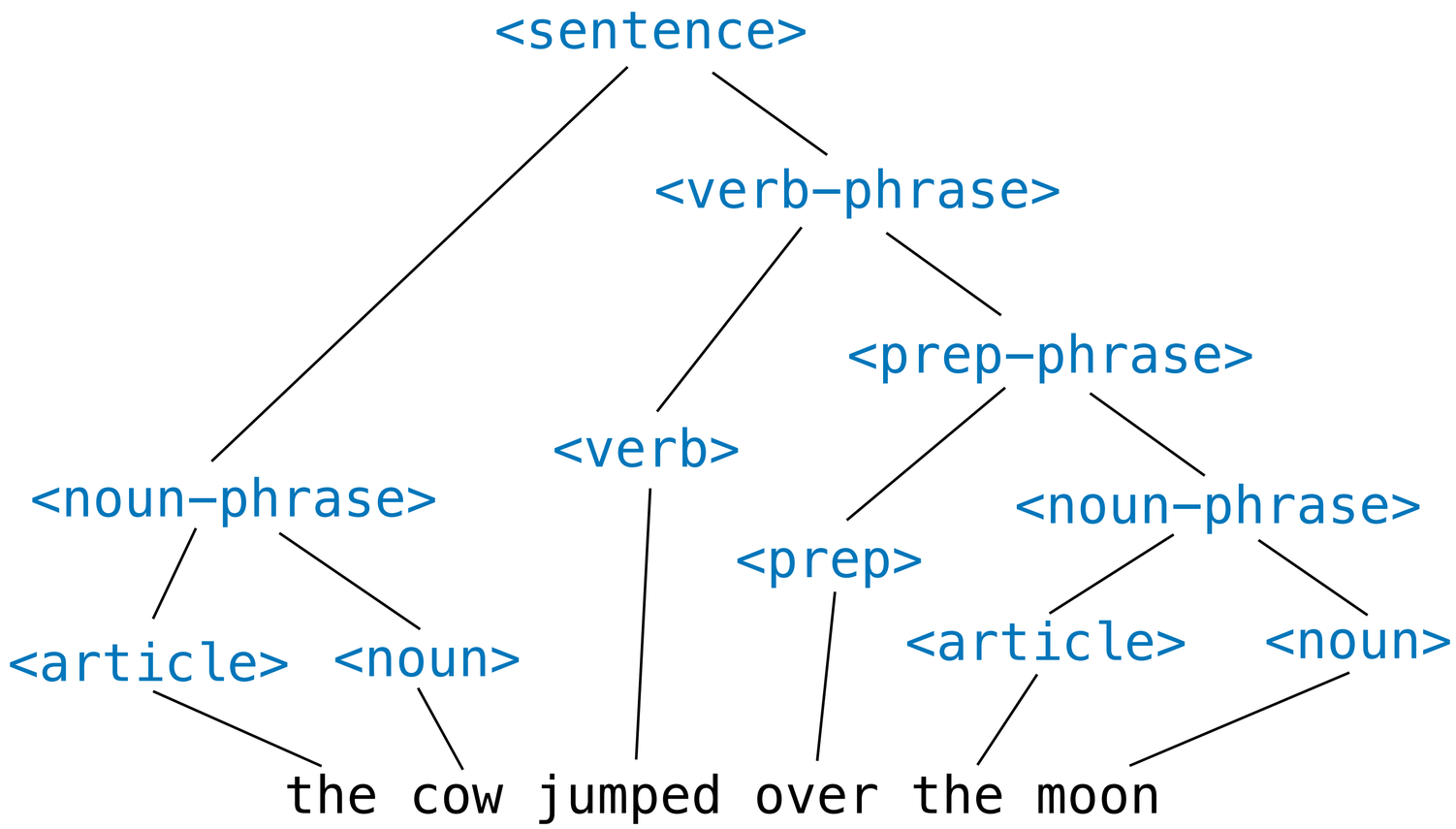
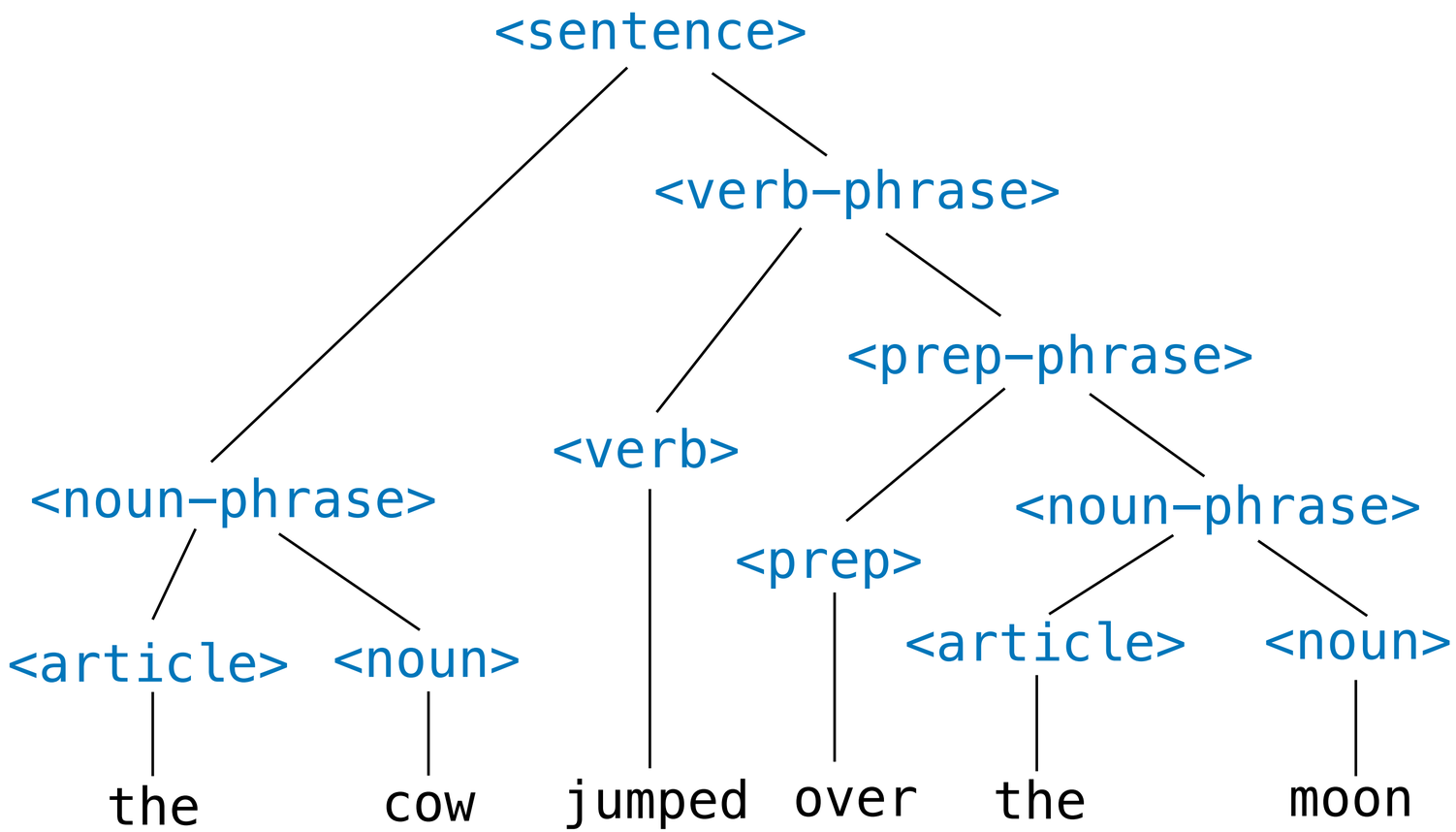


A Parse Tree



A derivation encodes hierarchical structure

Definitions (Symbols and Sentences)

<noun-phrase> jumped over <noun-phrase>

Definitions (Symbols and Sentences)

<noun-phrase> jumped over <noun-phrase>

A **grammar** is define in terms of a collection of symbols

Definitions (Symbols and Sentences)

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A **sentential form** is a sequence of terminal or nonterminal symbols

A **sentence** is a sequence of *only terminal* symbols

Production Rules

<non-term> ::= *sent-form1* | *sent-form2* | ...

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A **(BNF) production rule** describes what we can replace a non-terminal symbol with in a derivation

The "|" means: we can replace it with one or the other sentential forms on either side of the "|"

Recall: Let-Expressions (Syntax Rule)

<expr> ::= let <var> = <expr> in <expr>

If x is a valid variable name, and e_1 is a well-formed expression and e_2 is a well-formed expression then

let x = e_1 in e_2

is a well-formed expression

Example

```
<sentence>     ::= <noun-phrase> <verb-phrase>

<verb-phrase>  ::= <verb> <prep-phrase>

<noun>         ::= cow | moon
```

BNF Grammar

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
<prep>        ::= over
```

BNF Grammar

A **BNF grammar** is defined by a collection of production rules and a **starting (nonterminal) symbol**

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
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A **BNF grammar** is defined by a collection of production rules and a **starting (nonterminal) symbol**

Note. We don't specify the symbols of a grammar, they are implicit in the rules

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
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```

BNF Grammar

A **BNF grammar** is defined by a collection of production rules and a **starting (nonterminal) symbol**

Note. We don't specify the symbols of a grammar, they are implicit in the rules

Note. We don't specify the start symbol, it's the left nonterminal symbol in the **first rule**

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase>
                | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow
                | moon
<verb>        ::= jumped
<prep>        ::= over
```

Example

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

Example

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
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```

tokens (terminal symbols)

Example

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<expr>  ::= <op1> <expr>
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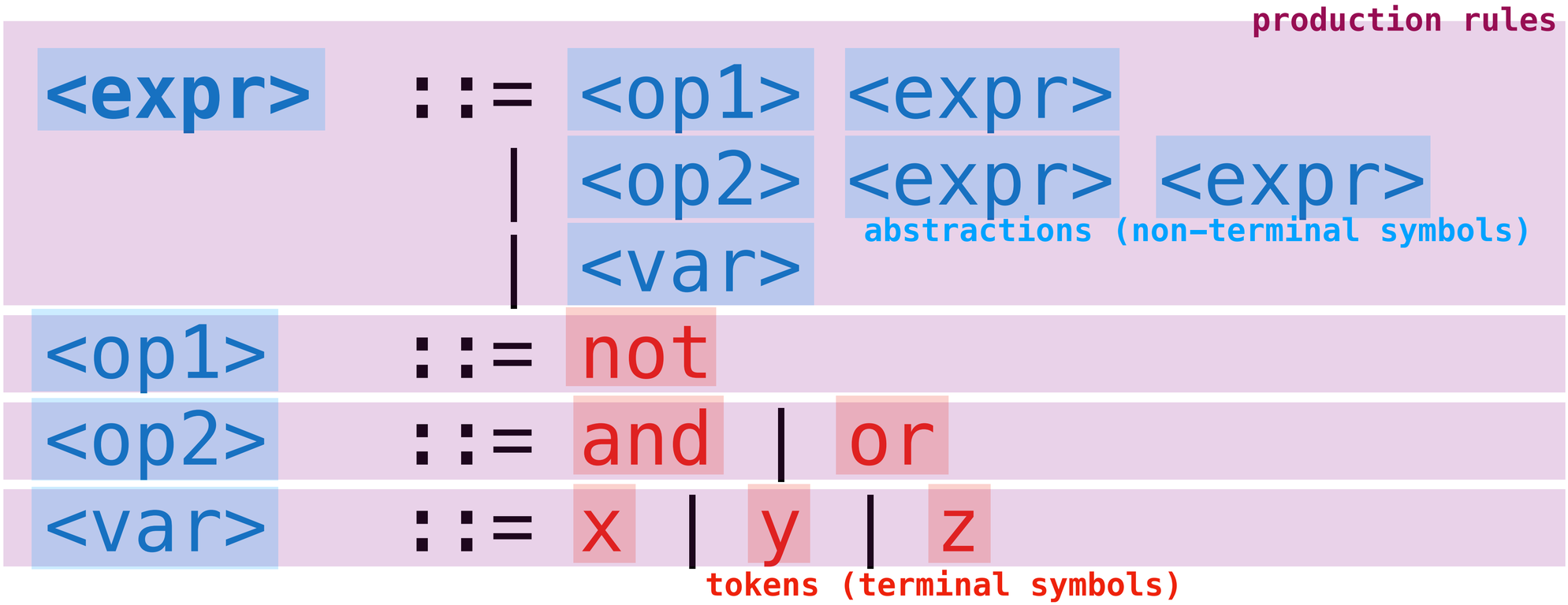
```

abstractions (non-terminal symbols)

tokens (terminal symbols)

Example


production rules
<expr> ::= <op1> <expr>
| <op2> <expr> <expr>
| <var>
abstractions (non-terminal symbols)
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Example

```
<sentence>    ::= <noun-phrase> <verb-phrase>
<verb-phrase> ::= <verb> <prep-phrase> | <verb>
<prep-phrase> ::= <prep> <noun-phrase>
<noun-phrase> ::= <article> <noun>
<article>     ::= the
<noun>        ::= cow | moon
<verb>        ::= jumped
<prep>        ::= over
```

What are the nonterminal and terminal symbols of this grammar?

Derivations and Parse Trees

Derivations and Parse Trees

Definition. A **derivation** is a sequence of sentential forms (beginning at the start symbol) in which each form is the result of **replacing a non-terminal symbol in the previous form** according to a production rule

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```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
and not x <var>
and not x y
```

Derivations and Parse Trees

```
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
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<expr>  ::= <op1> <expr>
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```

<expr>

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<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
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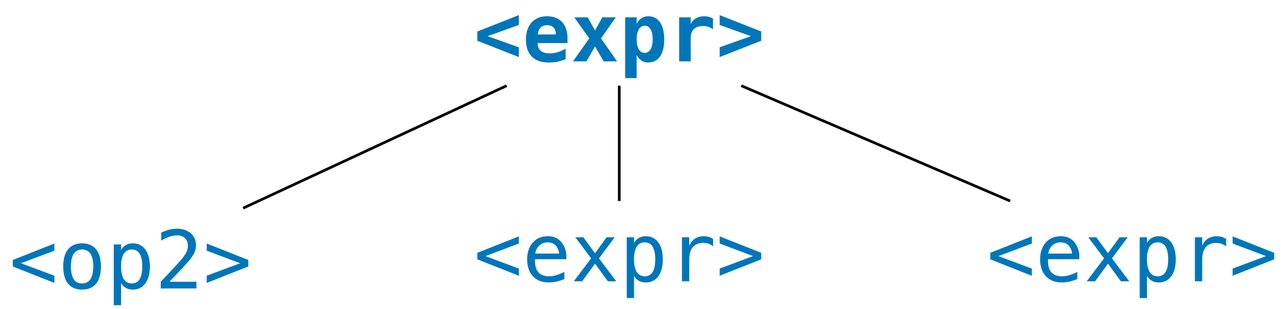
<op2>   ::= and | or
<var>   ::= x | y | z
```

<expr>
<op2> <expr> <expr>



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<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
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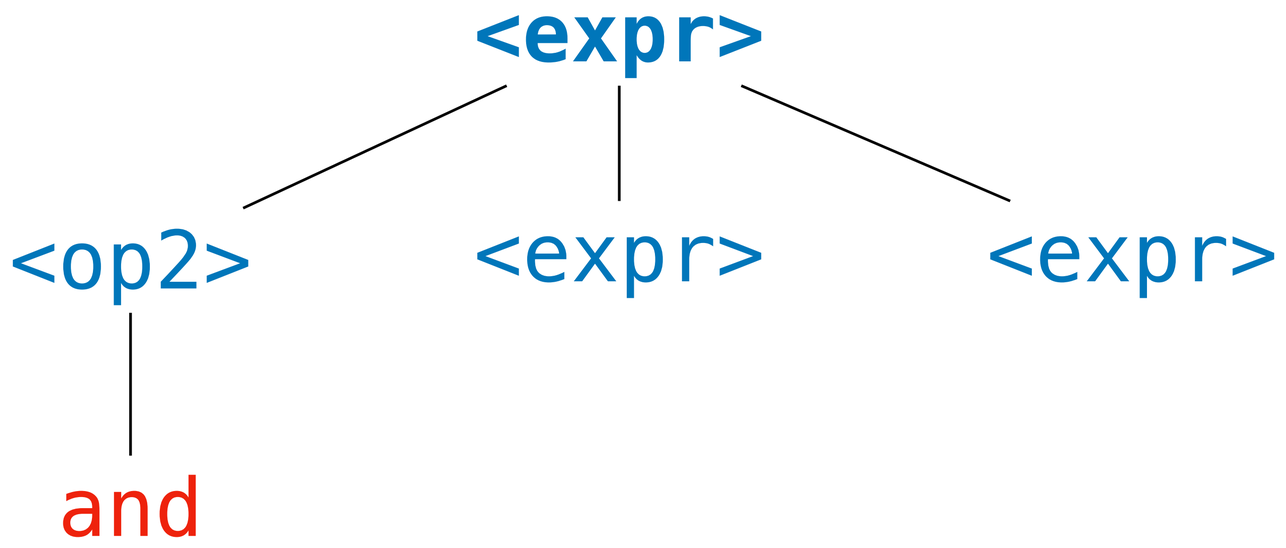
```

<expr>
<op2> <expr> <expr>
and <expr> <expr>



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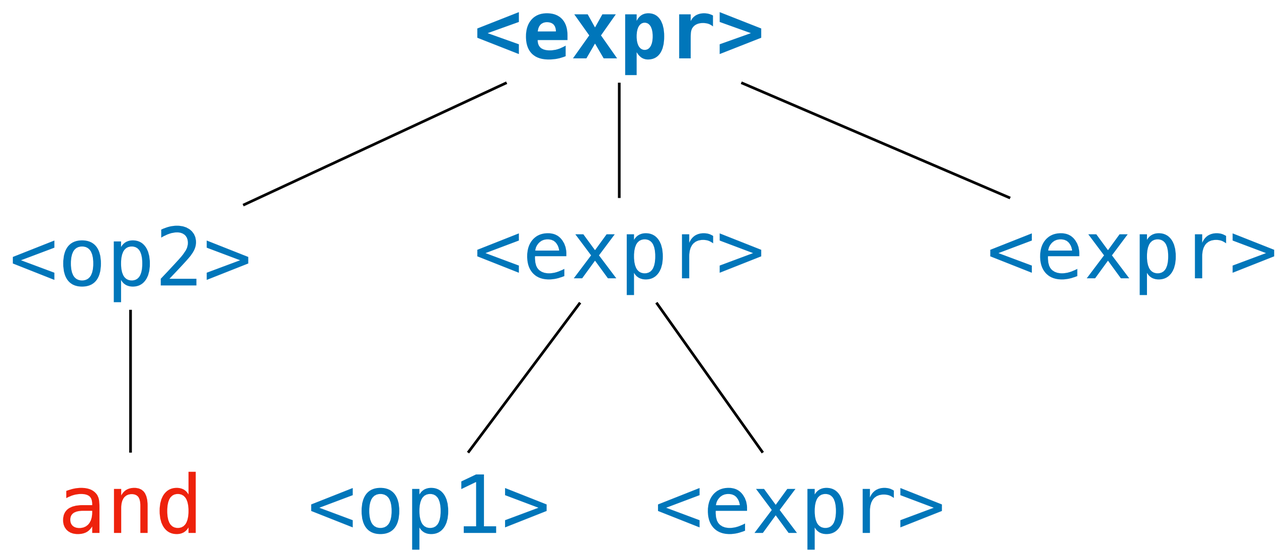
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
```



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<expr>  ::= <op1> <expr>
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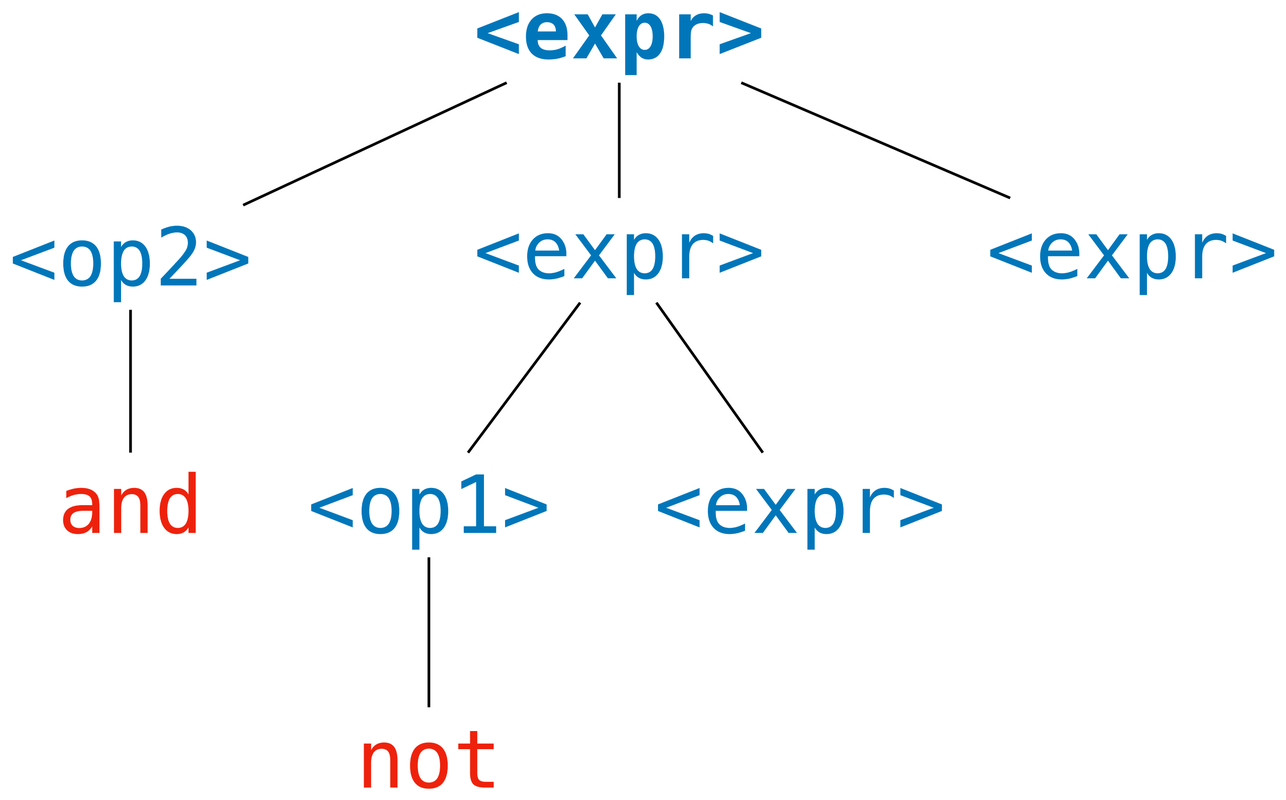
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
```



Derivations and Parse Trees

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<expr>  ::= <op1> <expr>
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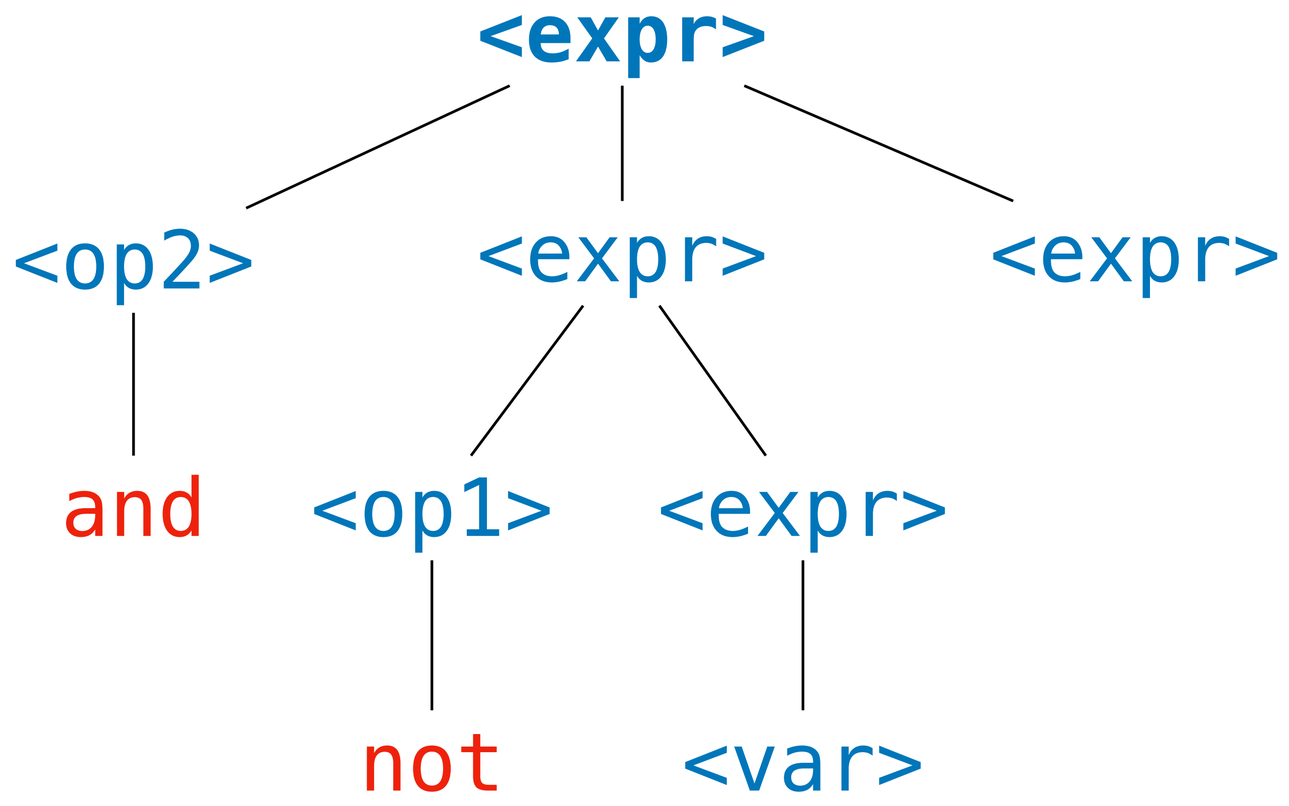
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
```



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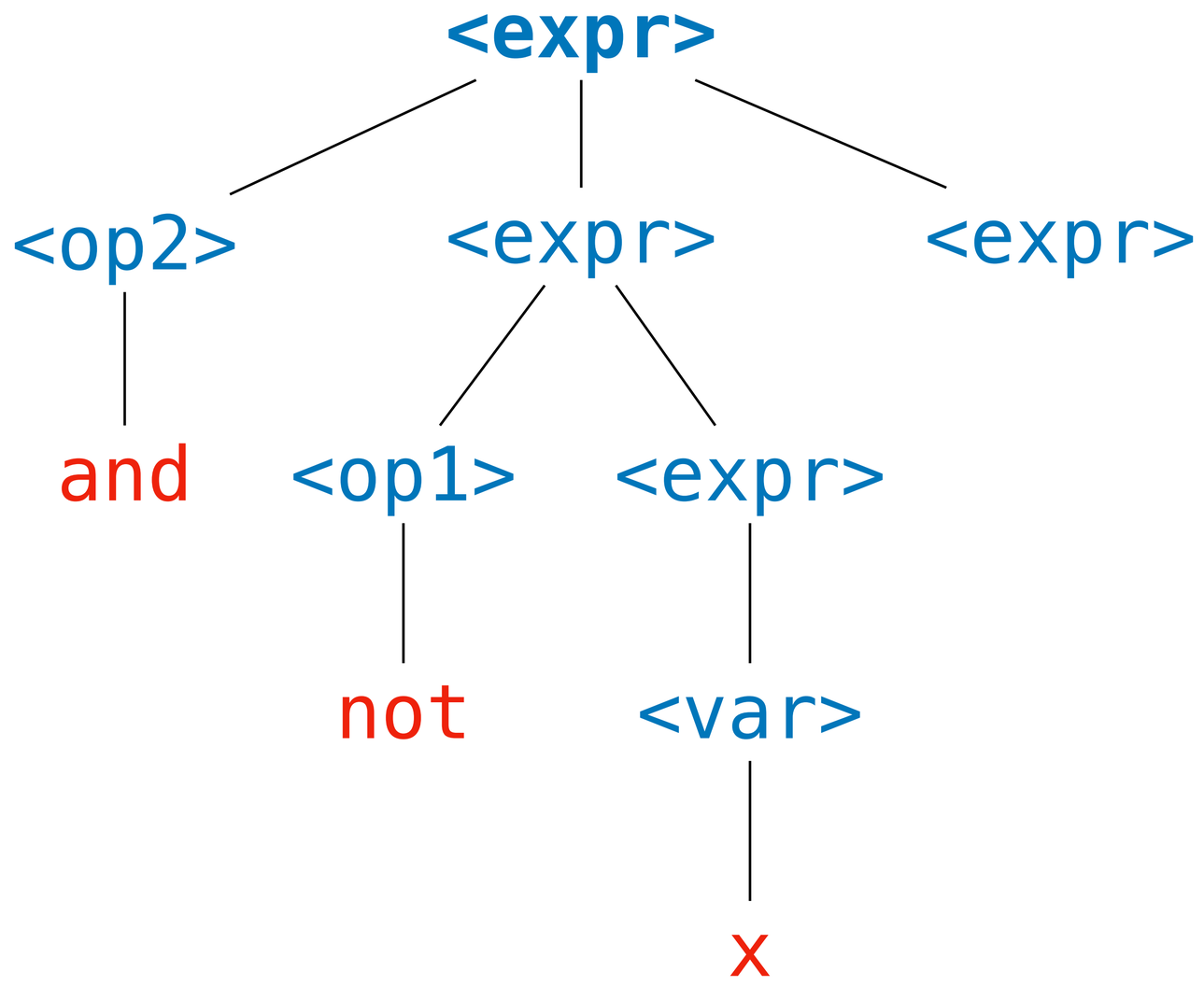
<expr>  ::= <op1> <expr>
          | <op2> <expr> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
```



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<expr>  ::= <op1> <expr>
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<op1>   ::= not
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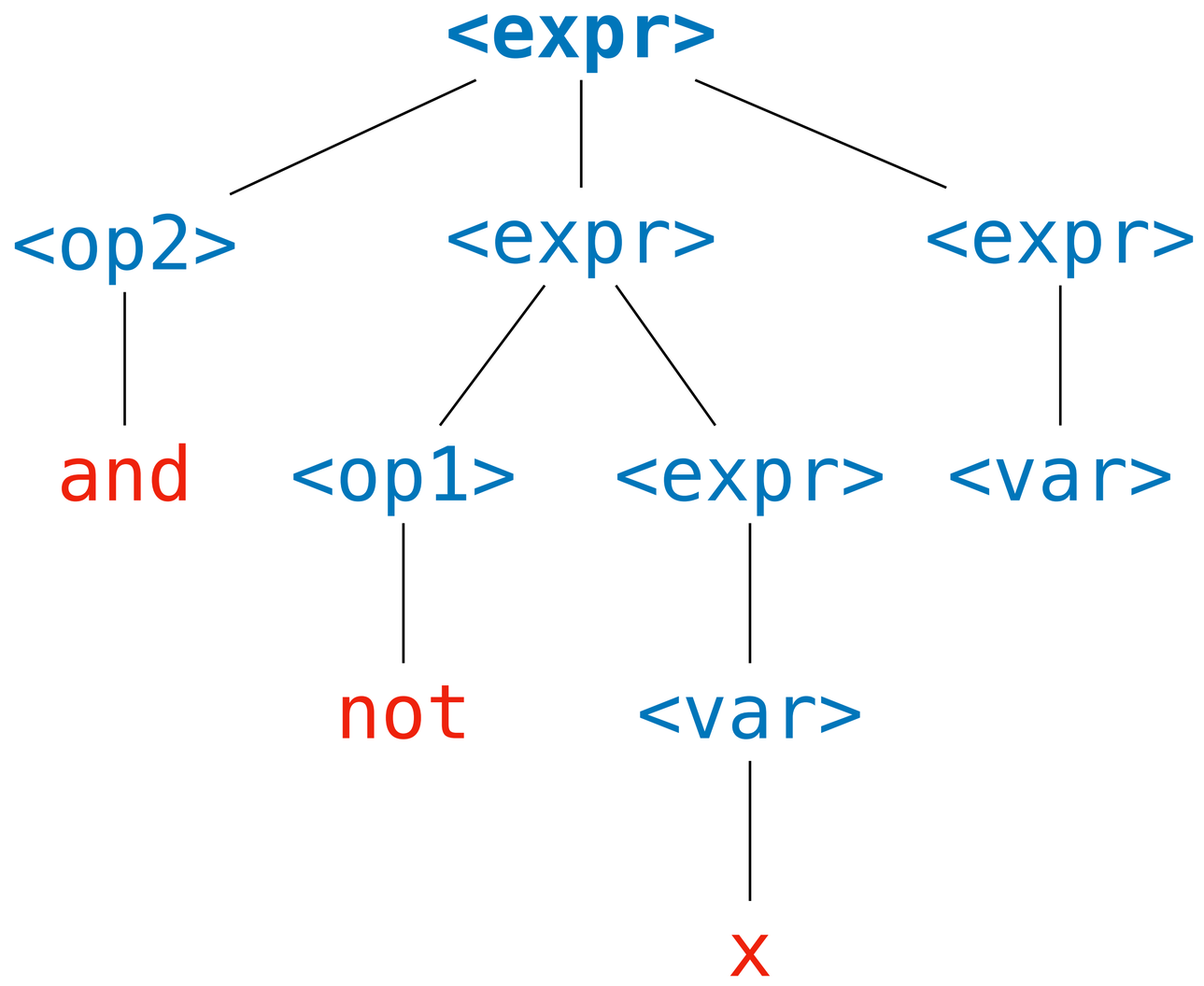
```

```
<expr>
<op2> <expr> <expr>
and <expr> <expr>
and <op1> <expr> <expr>
and not <expr> <expr>
and not <var> <expr>
and not x <expr>
and not x <var>
```



Derivations and Parse Trees

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<expr>  ::= <op1> <expr>
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<op1>   ::= not
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```

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<expr>
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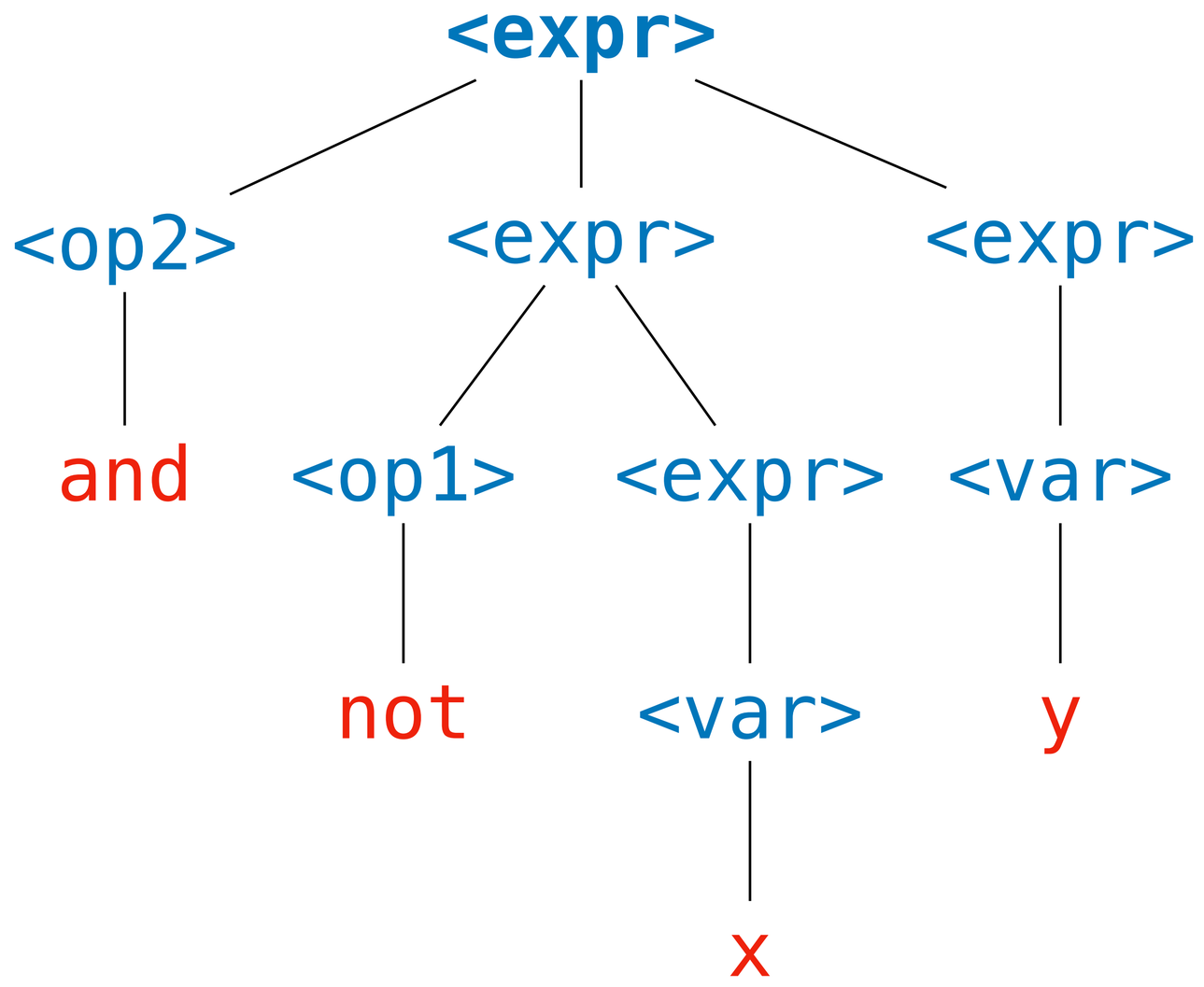
and not x y
```



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          | <op2> <expr> <expr>
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```

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<expr>
<op2> <expr> <expr>
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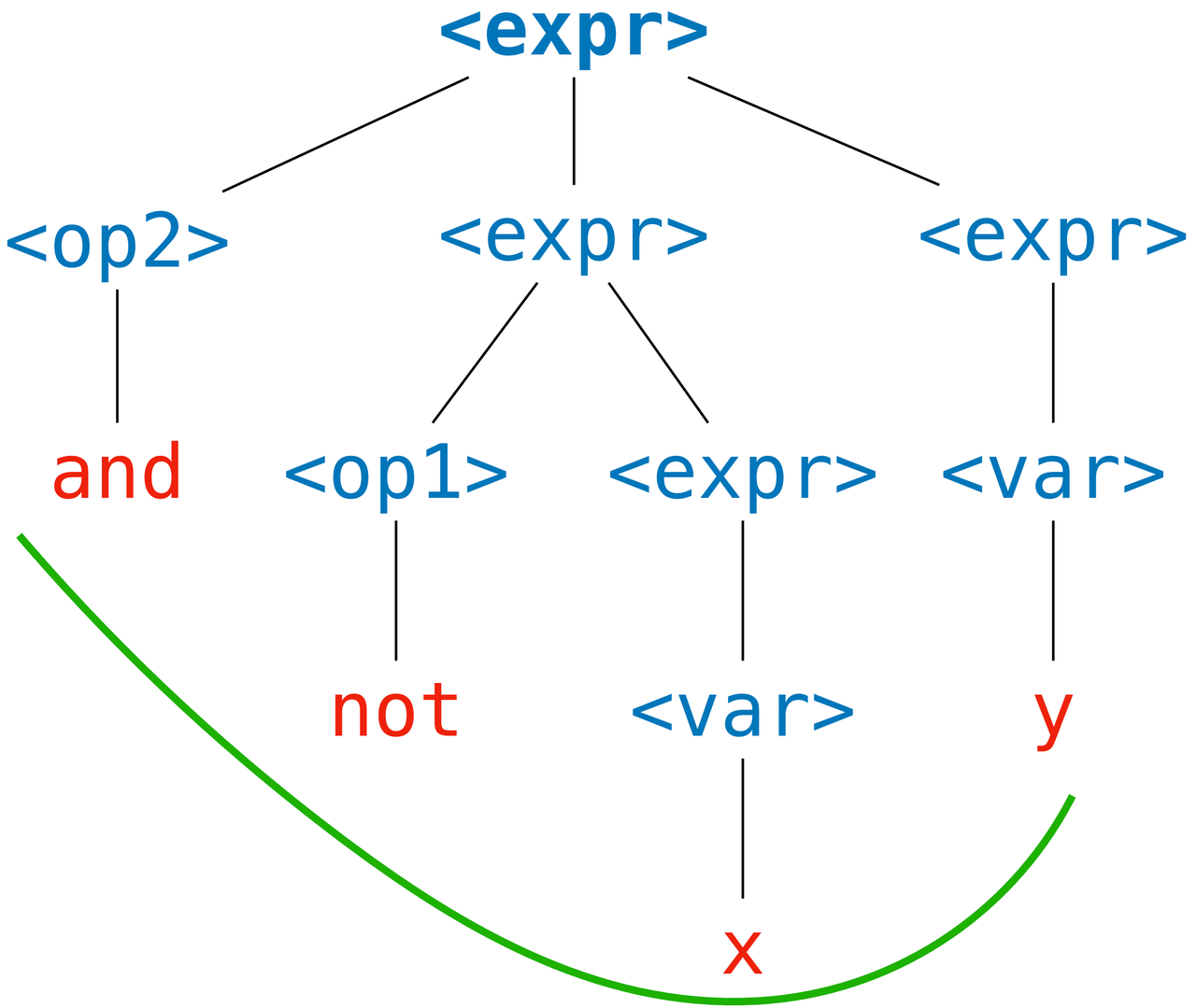
```



The point: parse trees and derivations represent the same hierarchical structure

Why do we care?

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We will parse **token streams** into **parse trees**

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We will parse **token streams** into **parse trees**

It is much easier to evaluate something hierarchical than something which is linear

Practice Problem

```
<expr>  ::= <op1> <expr>
          | <expr> <op2> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

*Give a derivation of **not x and y or z** in the above grammar, both as a sequence of sentential forms and as a parse tree*

*(In Python, if **x** and **y** and **z** are **True**, what does this expression evaluate to?)*

Answer

```
<expr>  ::= <op1> <expr>
          | <expr> <op2> <expr>
          | <var>
<op1>   ::= not
<op2>   ::= and | or
<var>   ::= x | y | z
```

not x and y or z

An Example from 320Caml

```
let x = 2 in if x = z then x else y
```

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How can we demonstrate that this is a well-formed expression?

An Example from 320Caml

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let x = 2 in if x = z then x else y
```

How can we demonstrate that this is a well-formed expression?

Answer: Well build a derivation/parse tree for it with the root **<expr>**!

Recall: Let-Expressions (Syntax Rule)

<expr> ::= let <var> = <expr> in <expr>

If x is a valid variable name, and e_1 is a well-formed expression and e_2 is a well-formed expression then

let x = e_1 in e_2

is a well-formed expression

Recall: If-Expressions (Syntax Rule)

<expr> ::= if <expr> then <expr> else <expr>

If e_1 is a well-formed expression and e_2 is a well-formed expression and e_3 is a well-formed expression, then

if e_1 then e_2 else e_3

is a well-formed expression

```
let x = 2 in if x = z then x else y
```

Summary

When we specify a PL (e.g., in the projects) you will be given a *BNF grammar*

You will need to know how to translate this into a parser

So you will need *practice reading BNF specifications*