

Unions and Products

Concepts of Programming Languages

Lecture 3

Practice Problem

Implement a function **`first_digit`** which takes an integer **`n`** as an input and returns the first digit of **`n`** (without converting to a string)

Outline

- » Discuss Formal Typing/Semantic Rules
- » Demonstrate how to organize data in OCaml in terms of products and unions types

Learning Objectives

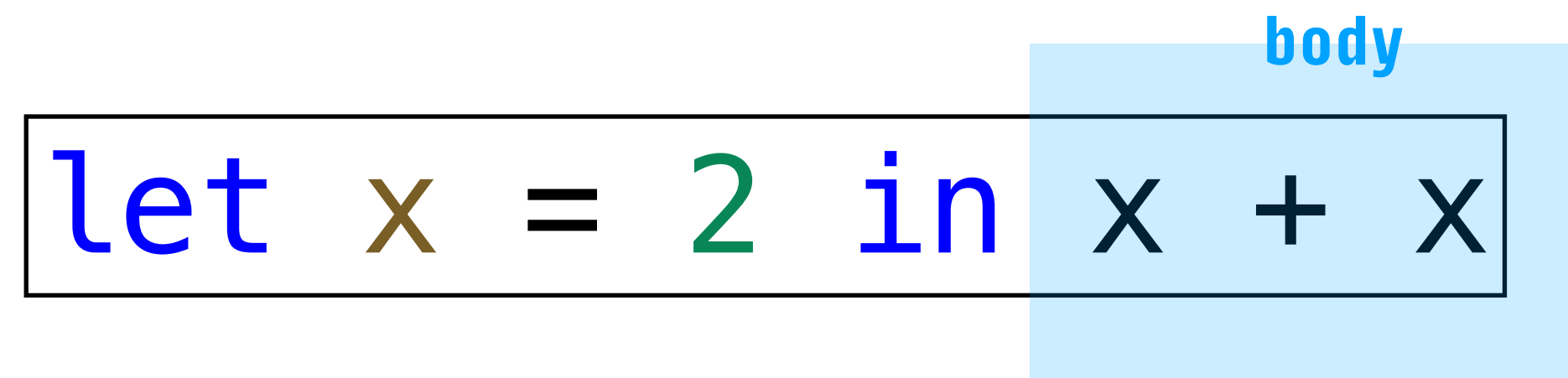
- » Read inference rules, i.e., translate mathematical notation to English and English to mathematical notation
- » Work with basic structured data in OCaml

Recap

Recall: Local Variables (Informal)

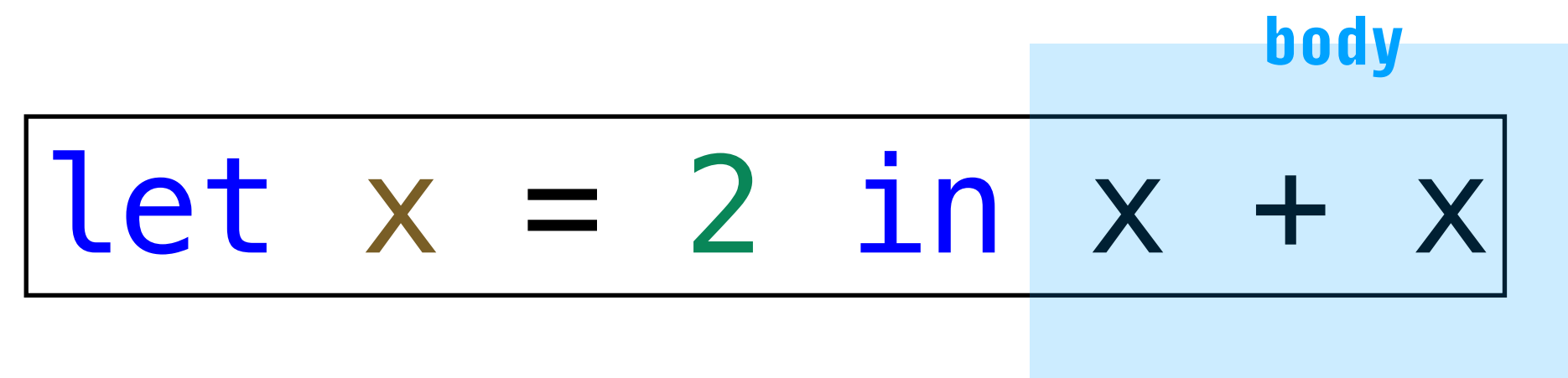
`let x = 2 in x + x`

body



The diagram illustrates the structure of a `let` expression. The code `let x = 2 in x + x` is shown. The `let` and `in` keywords are blue, `x` is brown, `=` is black, and `2` is green. A light blue rectangular box highlights the expression `x + x`, which is the body of the `let` binding. The word `body` is written in blue above the right side of this box.

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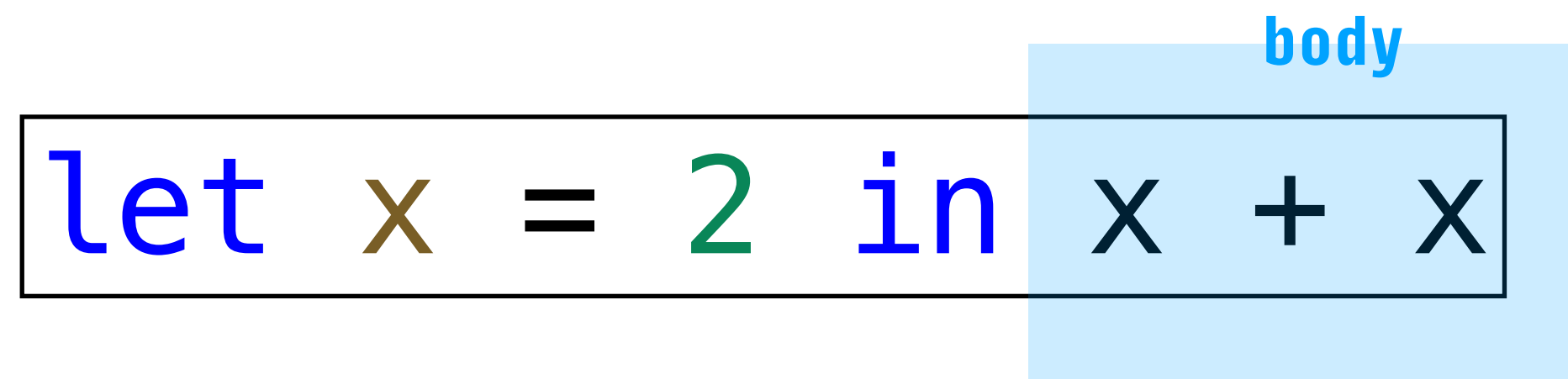


The diagram shows the code snippet `let x = 2 in x + x`. The words `let` and `in` are blue, `x` is brown, `=` is black, and `2` is green. A light blue rectangular background highlights the entire expression. A black rectangular box highlights the binding part `let x = 2 in`. The label `body` in blue text is positioned above the right side of the light blue background, indicating the body of the expression is `x + x`.

```
let x = 2 in x + x
```

syntax: `let VARIABLE = EXPRESSION in BODY`

Recall: Local Variables (Informal)

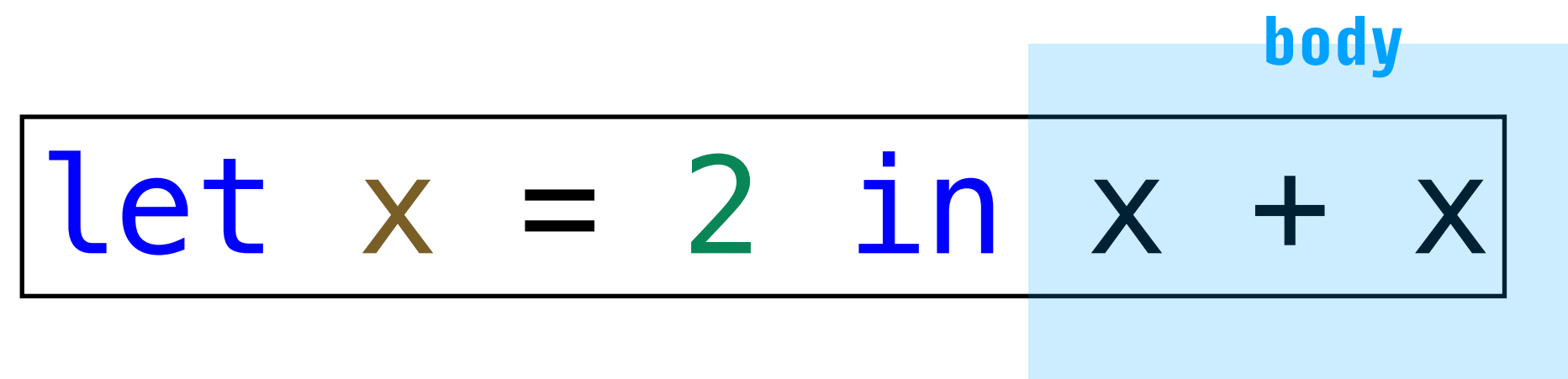


The diagram shows the code `let x = 2 in x + x`. The text is enclosed in a black rectangular box. The word `let` is blue, `x` is brown, `=` is black, `2` is green, and `in` is blue. The expression `x + x` is enclosed in a smaller black rectangular box, which is itself inside a larger light blue rectangular box. The word `body` is written in blue above the light blue box.

syntax: `let VARIABLE = EXPRESSION in BODY`

typing: the type is the same as that of BODY *given BODY is well-typed after substituting the VARIABLE in BODY*

Recall: Local Variables (Informal)



The diagram shows the code `let x = 2 in x + x` with syntax highlighting: `let` is blue, `x` is brown, `=` is black, `2` is green, `in` is blue, and `x + x` is black. A light blue rectangular box labeled "body" in blue text at the top right encloses the expression `x + x`.

syntax: `let VARIABLE = EXPRESSION in BODY`

typing: the type is the same as that of BODY *given BODY is well-typed after substituting the VARIABLE in BODY*

semantics: the is the same as the value of BODY *after substituting the VARIABLE in BODY*

Recall: A Note on Substitution

`let x = 2 in x + x` \longrightarrow `2 + 2`

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$$\boxed{\text{let } x = 2 \text{ in } x + x} \longrightarrow \boxed{2 + 2}$$

Formally, we write $[v/x]e$ to mean "substitute v for x in e ",
e.g., $[3/x](x + x)$ is the same as $3 + 3$

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Intuitively, substitution is simple: **replace the variable**

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Intuitively, substitution is simple: **replace the variable**

Turns out, this is **very hard** to do correctly, *it's subtle* and
a source of a lot of mistakes in PL implementations

Recall: If-Expressions (Informal)

```
let abs x = if x > 0 then x else -x
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Syntax: if CONDITION then TRUE-CASE else FALSE-CASE

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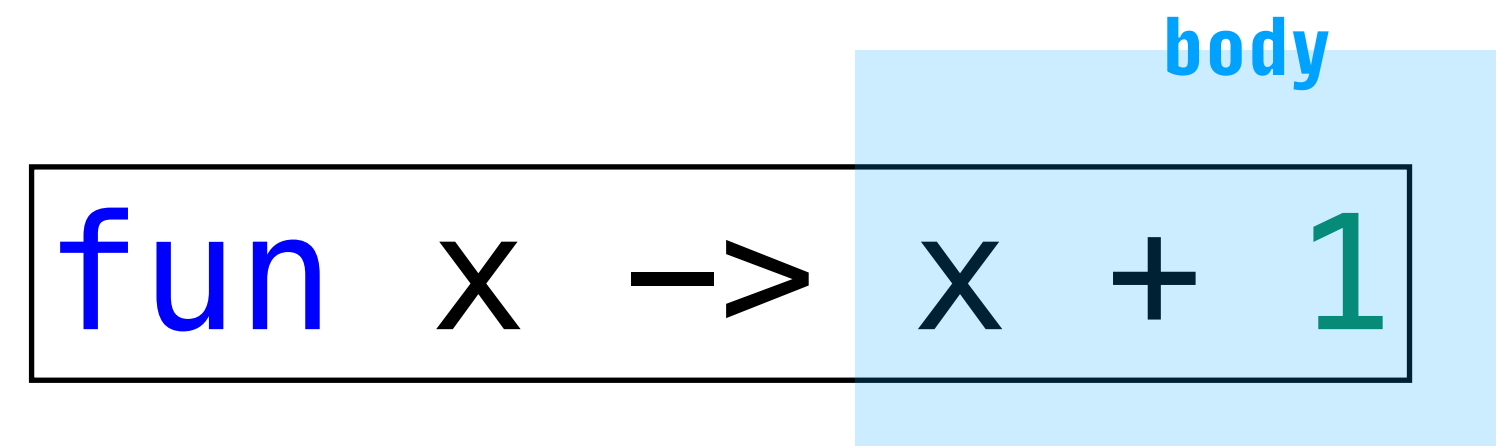
Semantics: If CONDITION holds, then we get the TRUE-CASE, otherwise we get the FALSE-CASE

Recall: Functions (Informal)

`fun x -> x + 1`

body

Recall: Functions (Informal)

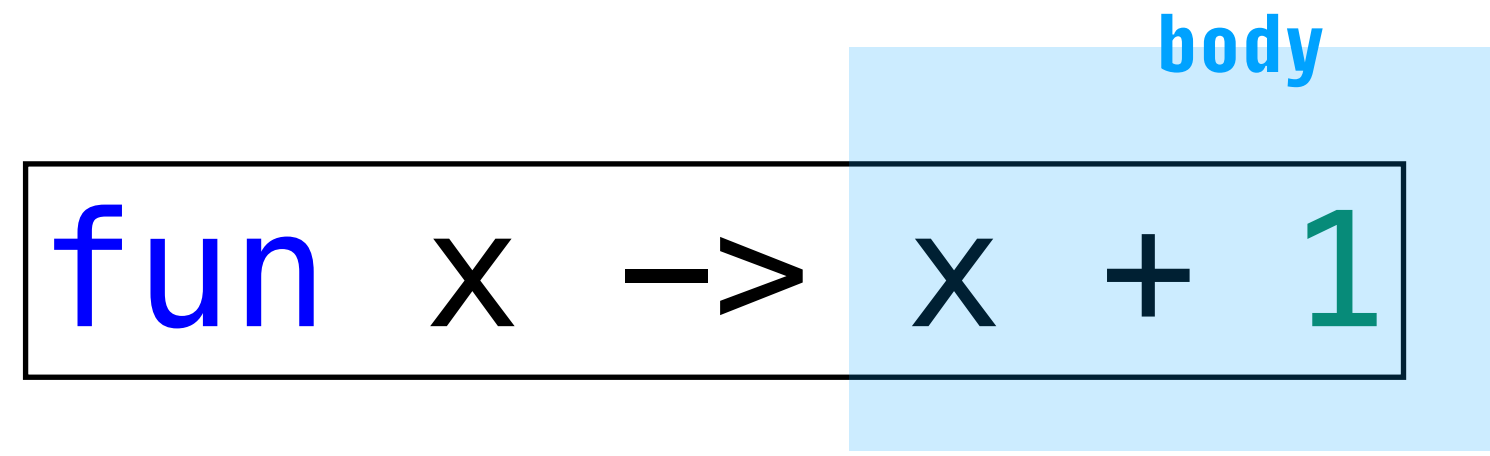


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Syntax: fun VAR-NAME -> EXPR

Recall: Functions (Informal)



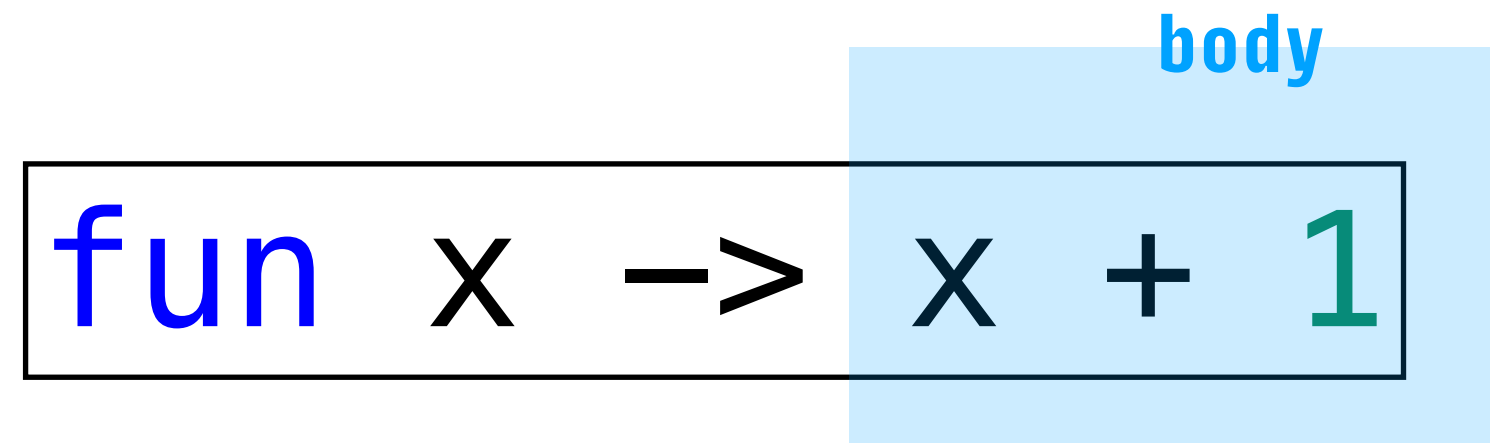
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Syntax: `fun VAR-NAME -> EXPR`

Typing: the type of a function is $T1 \rightarrow T2$ where $T1$ is the type of the input and $T2$ is the type of the output

Recall: Functions (Informal)



body

`fun x -> x + 1`

Syntax: `fun VAR-NAME -> EXPR`

Typing: the type of a function is `T1 -> T2` where `T1` is the type of the input and `T2` is the type of the output

Semantics: A function will evaluate to a special *function value* (printed as `<fun>` by UTop)

Recall: Curried Functions

```
let f = fun x -> fun y -> fun z -> x + y + z
```

We should think of the above function as something which takes an input and returns **another function**

In other words, we *partially apply* the function

Recall: Application (Informally)

```
(fun x -> fun y -> x + y + 1) 3 2
```

Recall: Application (Informally)

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Syntax: FUNCTION-EXPR ARG-EXPR

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Recall: Application (Informally)

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(fun x -> fun y -> x + y + 1) 3 2
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Syntax: FUNCTION-EXPR ARG-EXPR

Typing: If FUNCTION-EXPR is of type $T1 \rightarrow T2$, and ARG-EXPR is of type $T1$, then the type is $T2$

Semantics: Substitute the value of ARG-EXPR into the body of FUNCTION-EXPR and evaluate that

Inference Rules

Note: Production Rules and Syntax

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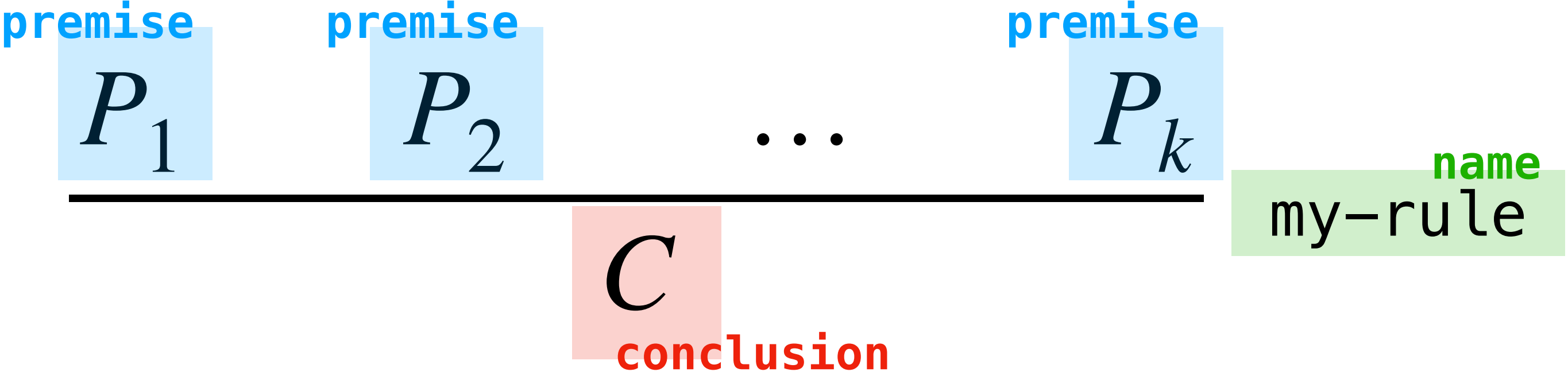
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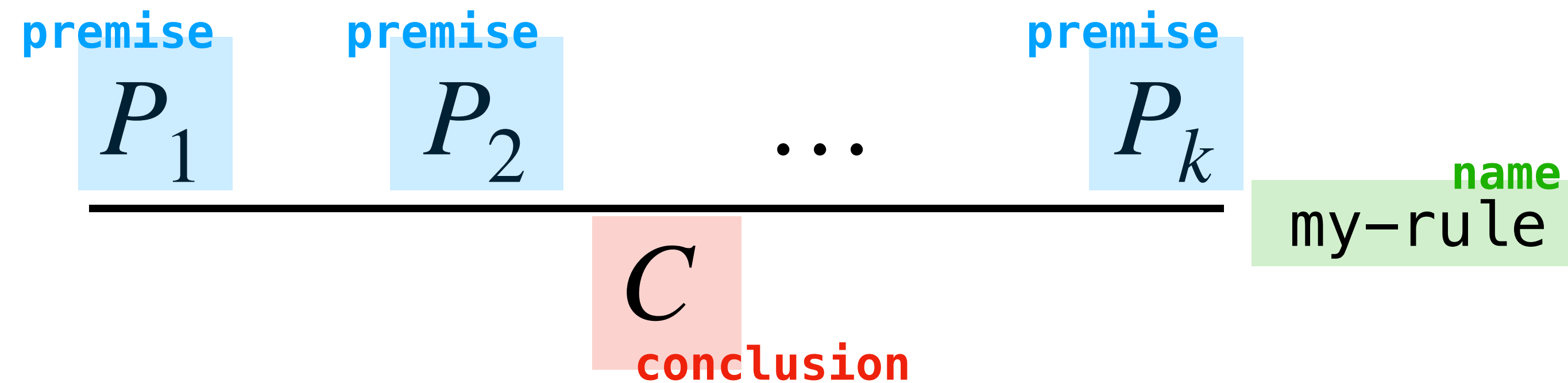
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We won't focus on this until the second half of the course but you should start to get comfortable with the syntax

Inference Rules

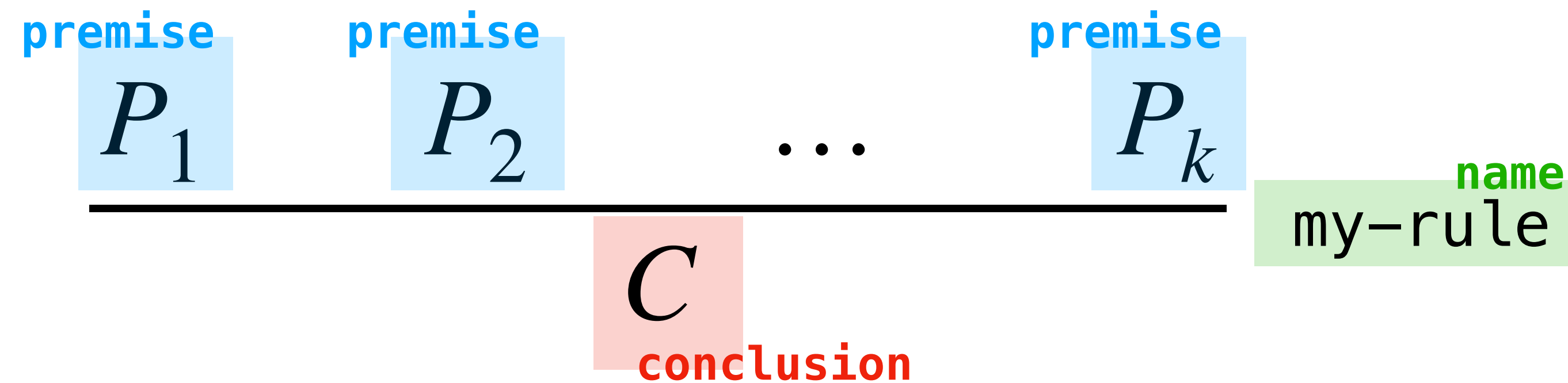


Inference Rules



Then general form of an inference rule has a collection of **premises** and a **conclusion**

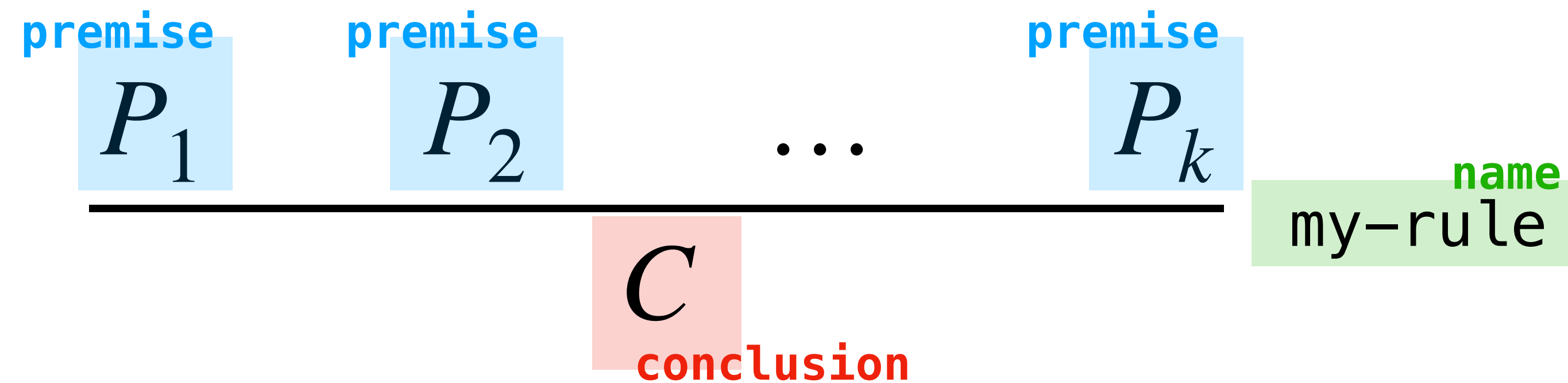
Inference Rules



Then general form of an inference rule has a collection of **premises** and a **conclusion**

There may be no premises, this is called an **axiom**

Inference Rules



We can read this as:

*If P_1 through P_k hold, then C holds (by **my-rule**)*

Typing Judgments

The diagram illustrates a typing judgment $\Gamma \vdash e : \tau$. The components are highlighted with colored boxes and labels: the context Γ is in a light blue box labeled "context" in blue; the expression e is in a light red box labeled "expression" in red; and the type τ is in a light green box labeled "type" in green. The symbols \vdash and $:$ are placed between the boxes.

A typing judgment is a compact way of representing the statement:

e is of type τ in the context Γ

A **typing rule** is an inference rule whose premises and conclusion are typing judgments

Recall: Integer Addition Typing Rule

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}} \text{ (addInt)}$$

If e_1 is an *int* (in any context Γ) and e_2 is an *int* then (in any context Γ) $e_1 + e_2$ is an *int* (in any context Γ)

Contexts

$\Gamma = \{ x : \text{int}, y : \text{string}, z : \text{int} \rightarrow \text{string} \}$

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A variable declaration $(x : \tau)$ says: "I declare that the variable x is of type τ "

Contexts

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A **context** is a set of **variable declarations**

A variable declaration $(x : \tau)$ says: "I declare that the variable x is of type τ "

A context keeps track of all the types of variables in the "environment"

Example: Reading Typing Judgements

$\{b : \text{bool}\} \vdash \text{if } b \text{ then } 2 \text{ else } 3 : \text{int}$

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In English: *Given I declare that b is a bool , the expression $\text{if } b \text{ then } 2 \text{ else } 3$ is an int*

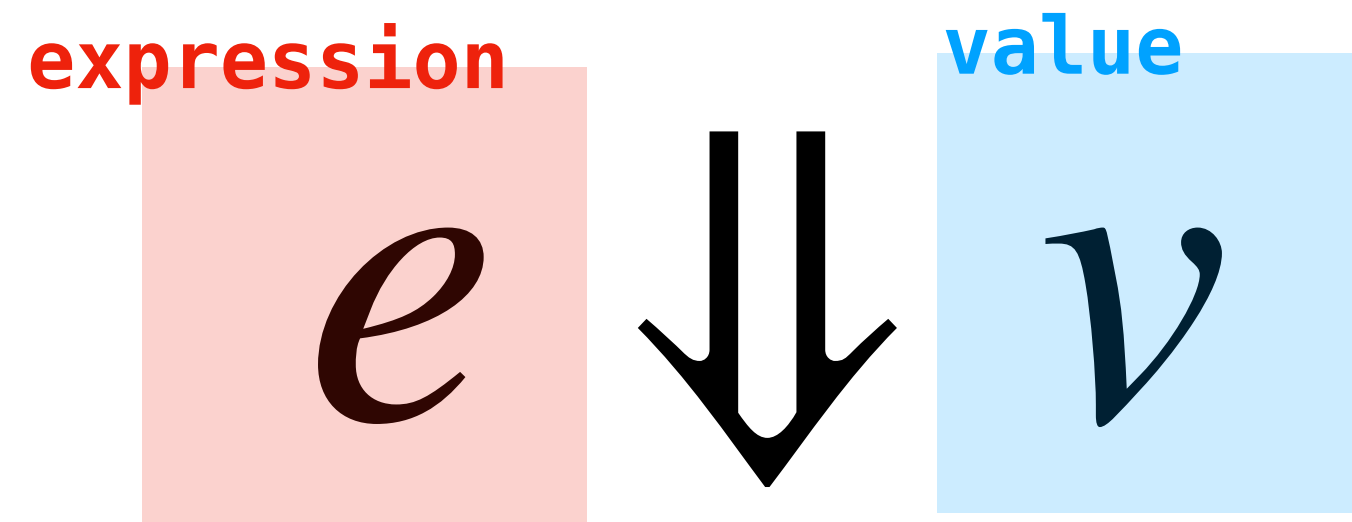
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In English: *Given I declare that b is a bool , the expression $\text{if } b \text{ then } 2 \text{ else } 3$ is an int*

The context allows us to determine the type of an expression *relative to the types of variables*

Semantic Judgements



A semantic judgment is a compact way of representing the statement:

The expression e evaluates to the value v

A **semantic rule** is an inference rule with semantic judgments

Recall: Integer Addition Semantic Rule

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 + e_2 \Downarrow v_1 + v_2} \text{ (evalInt)}$$

If e_1 evaluates to the (integer) v_1 and e_2 evaluates to the (integer) v_2 , then $e_1 + e_2$ evaluates to the (integer) $v_1 + v_2$

Example: Reading Semantic Judgments

`if 2 > 3 then 2 + 2 else 3` \Downarrow 3

In English: The expression `if 2 > 3 then 2 + 2 else 3` evaluates to the value 3

Note: Judgements are Statements

```
{b : bool} ⊢ if b then 2 else 3 : string
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On Thursday: We will talk about **typing derivations**, which are used to demonstrate that expressions *actually* have their desired types in our PL

Note: Values are not Expressions

`if 2 > 3 then 2 + 2 else 3` \Downarrow 3

In this course, we will draw a distinction between values and expressions (note the font)

Example. We'll use regular numbers to represent integer values, and we'll use \top and \perp for the true and false Boolean values

Questions?

Expressions, Formally

Up Next

We'll formalize what we've seen so far:

» Let-expressions

» If-Expressions

» Functions

» Application

For now, just think of these as
formal descriptions of how our PL
behaves

Let-Expressions (Syntax Rule)

$\langle \text{expr} \rangle ::= \text{let } \langle \text{var} \rangle = \langle \text{expr} \rangle \text{ in } \langle \text{expr} \rangle$

If x is a valid variable name, and e_1 is a well-formed expression and e_2 is a well-formed expression then

$\text{let } x = e_1 \text{ in } e_2$

is a well-formed expression

Let-Expressions (Typing Rule)

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau} \text{ (let)}$$

If e_1 is of type τ_1 in the context Γ , and e_2 is of type τ in the context Γ with the variable declaration $(x : \tau_1)$ added to it, then

$\text{let } x = e_1 \text{ in } e_2$

is of type τ in the context Γ

Let-Expressions (Semantic Rule)

$$\frac{e_1 \Downarrow v_1 \quad [v_1/x]e_2 \Downarrow v}{\text{let } x = e_1 \text{ in } e_2 \Downarrow v} \text{ (letEval)}$$

If e_1 evaluates to v_1 and e_2 with v_1 substituted for x evaluates to v , then

$\text{let } x = e_1 \text{ in } e_2$

evaluates to v

If-Expressions (Syntax Rule)

$\langle \text{expr} \rangle ::= \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{expr} \rangle \text{ else } \langle \text{expr} \rangle$

If e_1 is a well-formed expression and e_2 is a well-formed expression and e_3 is a well-formed expression, then

$\text{if } e_1 \text{ then } e_2 \text{ else } e_3$

is a well-formed expression

If-Expressions (Typing Rule)

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \tau} \text{(if)}$$

If e_1 is of type `bool` in the context Γ and e_2 and e_3 are of type τ in the context Γ , then

`if` e_1 `then` e_2 `else` e_3

is of type τ in the context Γ

If-Expressions (Semantic Rule 1)

$$\frac{e_1 \Downarrow T \quad e_2 \Downarrow v_2}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v_2} \text{ (ifEvalTrue)}$$

If e_1 evaluates to T and e_2 evaluates to v_2 , then

$\text{if } e_1 \text{ then } e_2 \text{ else } e_3$

evaluates to v_2

If-Expressions (Semantic Rule 2)

$$\frac{e_1 \Downarrow \perp \quad e_3 \Downarrow v_3}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v_3} \text{ (ifEvalFalse)}$$

If e_1 evaluates to \perp and e_2 evaluates to v_2 , then

`if e_1 then e_2 else e_3`

evaluates to v_3

Functions (Syntax Rule)

$$\langle \text{expr} \rangle ::= \text{fun } \langle \text{var} \rangle \rightarrow \langle \text{expr} \rangle$$

If x is a valid variable name and e is a well-formed expression, then

$$\text{fun } x \rightarrow e$$

is a well-formed expression

Functions (Typing Rule)

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \text{fun } x \rightarrow e : \tau_1 \rightarrow \tau_2} \text{ (fun)}$$

If e has type τ_2 in the context Γ with the variable declaration $(x : \tau_1)$ added, then

$\text{fun } x \rightarrow e$

is of type $\tau_1 \rightarrow \tau_2$ in the context Γ

Functions (Semantic Rule)

$$\frac{}{\text{fun } x \rightarrow e \Downarrow \lambda x . e} \text{ (funEval)}$$

Under no premises, the expression

$\text{fun } x \rightarrow e$

evaluates to the function value $\lambda x . e$

Application (Syntax Rule)

$$\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{expr} \rangle$$

If e_1 is a well-formed expression and e_2 is a well-formed expression, then $e_1 e_2$ is a well-formed expression

Application (Typing Rule)

$$\frac{\Gamma \vdash e_1 : \tau_2 \rightarrow \tau \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 e_2 : \tau} \text{ (app)}$$

If e_1 has type $\tau_2 \rightarrow \tau$ under the context Γ and e_2 is of type τ_2 under the context Γ , then $e_1 e_2$ is of type τ under the context Γ

Application (Semantic Rule)

$$\frac{e_1 \Downarrow \lambda x . e \quad e_2 \Downarrow v_2 \quad [v_2/x]e \Downarrow v}{e_1 e_2 \Downarrow v} \text{(appEval)}$$

» e_1 evaluates to a function value $\lambda x . e$

» e_2 evaluates to v_2

» e with v_2 substituted for x evaluates to v

It follows that $e_1 e_2$ evaluates to v

Example

```
(let x = 2 in fun y -> x + y) (2 + 3)
```

Understanding Check

Offline, go back to the recap slides at the beginning and compare the formal and informal descriptions...

We'll see more typing
rules and semantic rules

We'll also give a written
reference for the rules we talk
about in class

Practice Problem

```
let k = fun x -> fun y -> x in  
let x = 3 + k k 2 3 in  
k x (k x)
```

What does the above expression evaluate to?

Products

Tuples

```
let point : float * float = (2.0, 3.0)
let student : string * int = ("Franco", 244342)
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Tuples are ordered unlabeled fixed-length heterogeneous collections of data

(I expect that these are familiar)

These are useful for returning multiple arguments from a function

Pattern Matching on Tuples

```
let hypotenuse (p : float * float) : float =  
  match p with  
  | (x, y) -> sqrt (x *. x +. y *. y)
```

There are no accessors for tuples

Instead we can use **pattern matching**

Pattern Matching in General

match *e* with

| *p* -> *o*

| ...

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Pattern Matching in General

```
match e with  
| p -> o  
| ...
```

A **pattern** is like a typed template for how a piece of data should look

A **match-expression** is a way of *destructing* any piece of data in OCaml

We *match* on an expression *e*, and check if the value of *e* *matches* with the pattern *p*

Note: Patterns are not Expressions

```
<expr> ::= match <expr> with  
        | <pattern> -> <expr>  
        | <pattern> -> <expr>  
        | ...
```

Patterns are similar to expressions, but with some key differences

They can be wildcards, they can be variables, there's a lot of options

We'll talk more about
patterns on Thursday

Advanced Pattern Matching

```
let hypotenuse ((x, y) : float * float) : float =  
    sqrt (x *. x +. y *. y)
```

```
let hypotenuse (p : float * float) : float =  
    let (x, y) = p in  
    sqrt (x *. x +. y *. y)
```

Pattern matching can also be done implicitly in let-expression and function arguments!

And we can do all this
formally...

Tuples (Syntax Rule)

$$\langle \text{expr} \rangle ::= (\langle \text{expr} \rangle , \dots , \langle \text{expr} \rangle)$$

If e_1, \dots, e_n are well-formed expressions, then

$$(e_1 , \dots , e_n)$$

is a well-formed expression

Tuple (Typing Rule)

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma \vdash e_2 : \tau_2 \quad \dots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash (e_1 \textcolor{red}{,} \dots \textcolor{red}{,} e_n) : \tau_1 \textcolor{red}{*} \dots \textcolor{red}{*} \tau_n} \text{(tuple)}$$

If e_1, \dots, e_n are of type τ_1, \dots, τ_n , respectively, in the context Γ then

$$(e_1 \textcolor{red}{,} \dots \textcolor{red}{,} e_n)$$

is of type $\tau_1 \textcolor{red}{*} \dots \textcolor{red}{*} \tau_n$ in the context Γ

Tuple (Semantic Rule)

$$\frac{e_1 \Downarrow v_1 \quad \dots \quad e_n \Downarrow v_n}{(e_1 \text{ , } \dots \text{ , } e_n) \Downarrow (v_1 \text{ , } \dots \text{ , } v_n)} \text{ (tupleEval)}$$

If e_1, \dots, e_n evaluate to v_1, \dots, v_n , respectively, then

$$(e_1 \text{ , } \dots \text{ , } e_n)$$

evaluates to $(v_1 \text{ , } \dots \text{ , } v_n)$

Records

```
type point = { x_cord : float ; y_cord : float }  
let origin : point = { x_cord = 0. ; y_cord = 0. }
```

```
type user = {  
  name : string ;  
  email : string ;  
  num_posts : int ;  
}
```

Records are unordered labeled fixed-length heterogeneous collections of data

They are useful for organizing large collections of data (akin to database records)

Record Syntax

```
type record_ty =  
  {  
    field1 : ty1;  
    field2 : ty2;  
    ...  
    fieldn : tyn'  
  }
```

```
let record_expr : record_ty =  
  {  
    field1 = expr1;  
    field2 = expr2;  
    ...  
    fieldn : exprn;  
  }
```

For a record, we have to specify the type of each field

When we construct a record, every field must have a value

Accessors

```
type point = { x_cord : float ; y_cord : float }
```

```
let dist (p : point) (q : point) =  
  let xd = p.x_cord -. q.x_cord in  
  let yd = p.y_cord -. q.y_cord in  
  sqrt (xd *. xd +. yd *. yd)
```

Records support **dot-notation**

(we can also access records by pattern matching)

Record Updates

```
let new_post u : user =  
  { u with num_posts = u.num_posts + 1 }
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"u with number of posts incremented, keep everything else the same"

Data in functional languages are immutable. This returns a new record with the update

Unions

Simple Variants

```
type os = BSD | Linux | MacOS | Windows
```

A **simple variant** is a *user-defined* type for values of a fixed collection of possibilities

Type names are **lower_case** and Constructors names are **Upper_case**

Simple Variants

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type os = constructor BSD | Linux | MacOS | Windows
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Pattern Matching

```
let supported (sys : os) : bool =  
  match sys with  
  | BSD -> false  
  | _ -> true
```

We work with variants by **pattern matching**:

- » giving a pattern that a value can match with
- » writing what to do for each pattern

Pattern Matching

```
let supported (sys : os) : bool =  
  match sys with  
  constant pattern | BSD -> false  
  wildcard pattern | _ -> true
```

We work with variants by **pattern matching**:

- » giving a pattern that a value can match with
- » writing what to do for each pattern

Data-Carrying Variants

```
type linux_distro = Arch | Fedora | NixOS | Ubuntu
```

```
type os  
  = BSD of int * int  
  | Linux of linux_distro * int  
  | MacOS of int  
  | Windows of int
```

```
let supported (sys : os) : bool =  
  match sys with  
  | BSD (major , minor) -> major > 2 && minor > 3  
  | _ -> true
```

Variants can carry data, which allows us to represent more complex structures

Data-Carrying Variants

```
type linux_distro = Arch | Fedora | NixOS | Ubuntu
```

```
type os
  = BSD of int * int
  | Linux of linux_distro * int
  | MacOS of int
  | Windows of int
```

Note the syntax

```
let supported (sys : os) : bool =
  match sys with
  | BSD (major , minor) -> major > 2 && minor > 3
  | _ -> true
```

Variants can carry data, which allows us to represent more complex structures

Pro Tip: Named Data-Carrying Variants

```
type os
  = MacOS of {
      major : int ;
      minor : int ;
      patch : int
    }
  | ...

let support (sys : os) : bool =
  match sys with
  | MacOS info -> info.minor >= 14 && info.patch >= 1
    (* MacOS Sonoma 10.14.(1-3) *)
  | ...
```

Since we can carry *any* kind of data in a constructor, we can carry **records** to **name the parts** of our carried data.

Understanding Check

```
let area (s : shape) =  
  match s with  
  | Rect r -> r.base *. r.height  
  | Triangle { sides = (a, b) ; angle } -> Float.sin angle *. a *. b  
  | Circle r -> r *. r *. Float.pi
```

*Define the variant **shape** which makes this function type-check.*

Summary

Inference rules formally describe how the typing and semantics of a programming language work

Tuples and **records** allow us to group data

Variants allow us to organize data by *possible outcomes*