Mini-Project 2: The Environment Model

CAS CS 320: Principles of Programming Languages

Due April 17, 2025 by 8:00PM

In this project, you'll be building *another* interpreter for a subset of OCaml. It'll be your task to implement the following functions:

```
val desugar : prog -> expr
val type_of : expr -> (ty, error) result
val eval : expr -> value
val interp : string -> (value, error) result
```

The types used in the above signature appear in the module Utils. Your implementation of these functions should appear in the file interp2/lib/interp2.ml. Please read the following instructions completely and carefully.

Part 0: Parsing

This time around we're giving you the parser. We'll still give you the grammar. You should spend at bit of time looking through the lexer and parser to make sure you understand how it works.

```
< ::= {<toplet>}
<toplet> ::= let [rec] <var> {<arg>} : <ty> = <expr>
  <arg> ::= ( <var> : <ty> )
    \langle ty \rangle ::= int \mid bool \mid unit \mid \langle ty \rangle \rightarrow \langle ty \rangle \mid (\langle ty \rangle)
 <expr> ::= let [rec] <var> {<arg>} : <ty> = <expr> in <expr>
            if <expr> then <expr> else <expr>
               fun <arg> {<arg>} → <expr>
               <expr2>
\langle expr2 \rangle ::= \langle expr2 \rangle \langle bop \rangle \langle expr2 \rangle
           assert <expr3>
           <expr3> {<expr3>}
<expr3> ::= () | true | false
           <num> | <var>
            ( <expr> )
  <bop> ::= + | - | * | / | mod | < | <= | > | >= | = | <> | && | | |
 <num> ::= handled by lexer
   \langle var \rangle ::= handler by lexer
```

Operators and their associativity are presented in order of increasing precedence in the following table. Note that neither the function type arrow nor function application are "operators" but they're included in the table for completeness.

Operator	Associativity
	right
&&	right
<, <=, >, >=, =, <>	left
+, -	left
*, /, mod	left
function application	left
→	right

Part 1: Desugaring

You'll notice this time around that the kinds of expressions that we'll eventually type-check and evaluate aren't the same as programs that we write in our language. In other words, our *concrete* (or *surface-level*) syntax is not the same as our *abstract* syntax.

The process of translating surface-level syntax into abstract syntax is sometimes called *desugaring*.¹ The function desugar should translate a program of type prog to an expression of type expr according to the following rules.

▶ A sequence of top-level let-statements is shorthand for collection of nested let-expressions ending in a variable (the last defined name). That is, the surface-level syntax:

```
let [rec] x_1 {<arg>} : \tau_1 = e_1 let [rec] x_2 {<arg>} : \tau_2 = e_2 ... let [rec] x_k {<arg>} : \tau_k = e_k
```

is shorthand for:

```
\begin{array}{l} \texttt{let} \; [\texttt{rec}] \; x_1 \; \{\texttt{<arg>}\} \; : \; \tau_1 = e_1 \; \texttt{in} \\ \texttt{let} \; [\texttt{rec}] \; x_2 \; \{\texttt{<arg>}\} \; : \; \tau_2 = e_2 \; \texttt{in} \\ \dots \\ \texttt{let} \; [\texttt{rec}] \; x_k \; \{\texttt{<arg>}\} \; : \; \tau_k = e_k \; \texttt{in} \\ x_k \end{array}
```

The empty program is equivalent to the unit expression ().

▶ A let-expression with arguments is shorthand for a let expression with no arguments whose value is an anonymous function. That is, the surface-level syntax

```
let [rec] f ( x_1 : \tau_1 ) ... ( x_k : \tau_k ) : \tau = e_1 in e_2
```

is shorthand for

let [rec]
$$f: \tau_1 \rightarrow \ldots \rightarrow \tau_k \rightarrow \tau = \text{fun } (x_1: \tau_1) \ldots (x_k: \tau_k) \rightarrow e_1 \text{ in } e_2$$

▶ An anonymous function with more than one argument is shorthand for Curry-ed collection of single-argument anonymous functions. That is, the surface-level syntax

fun (
$$x_1 : \tau_1$$
) ... ($x_k : \tau_k$) $\rightarrow e$

is shorthand for

fun (
$$x_1:\tau_1$$
) \rightarrow ... fun \rightarrow ($x_k:\tau_k$) \rightarrow e

These rules should be applied until there are no more occurrences of let-statements, let-expressions with arguments, or anonymous functions with multiple arguments. Note that the type expr does not allow these constructs, so all this means is that you have to determine the valid translation from prog to expr.

¹You may have heard the term *syntactic sugar* with regards to convenient syntactic constructs in programming languages you've used.

Part 2: Type-Checking

An expression is well-typed if it is derivable according to the following rules. We write $\Gamma \vdash e : \tau$ to mean that e has type τ in the context Γ , where e is an $\langle \exp r \rangle$ after desurgaring, and τ is a $\langle ty \rangle$. In practice, you'll be implementing a type-inference algorithm, but because all function arguments and values are annotated with types, this is only slightly more difficult than type-checking (in mini-project 3, we'll take away the type-annotations, which will make the type-inference problem more difficult).

The function type_of, given an expression e of type expr, should evaluate to $\mathbb{O}k \tau$ if $\emptyset \vdash e : \tau$ is derivable. Otherwise, it should evaluate to Error err, where err is one of the following constructors of the type error.

- \triangleright UnknownVar x: The variable x is unbound.
- \triangleright IfTyErr τ_e τ_a : The else-case of an if-expression was expected to be type τ_e (based on the then-case) but was determined to be type τ_a .
- \triangleright IfCondTyErr τ_a : The condition of an if-expression was expected to be type bool but was determined to be type τ_a .
- \triangleright OpTyErrL \square τ_e τ_a : The left argument of an operator \square was expected to be type τ_e but was determined to be type τ_a .
- \triangleright OpTyErrR $\square \tau_e \tau_a$: The right argument of an operator \square was expected to be type τ_e but was determined to be type τ_a .
- ▷ FunArgTyErr τ_e τ_a : The argument of a function of type $\tau_e \rightarrow \tau$ was expected to be type τ_e but was determined to be type τ_a .
- \triangleright FunAppTyErr τ_a : An expression determined to be type τ_a was applied as a function but τ_a is not a function type.
- \triangleright LetTyErr τ_e τ_a : The binding of a let-expression was expected to be type τ_e (according to its annotation) but was determined to be type τ_a .
- \triangleright LetRecErr f: The binding of f in a recursive let-expression was found to be something other than an anonymous function.
- \triangleright AssertTyErr τ_a : The argument of an assert-expression was expected to be type bool but was determined to be type τ_a .

It'll be up to you to determine in which cases you should return a given error. If an expression is not well-typed, you should return the error corresponding to the *leftmost-innermost* subexpression which is ill-typed. In practice, this means **processing premises of a typing rules from left to right**.

Literals

$$\frac{}{\Gamma \vdash \text{() : unit}} \text{ (unit)} \qquad \frac{}{\Gamma \vdash \text{true : bool}} \text{ (true)} \qquad \frac{}{\Gamma \vdash \text{false : bool}} \text{ (false)} \qquad \frac{n \text{ is an integer literal}}{\Gamma \vdash n : \text{int}} \text{ (int)}$$

Variables

$$\frac{(x:\tau)\in\Gamma}{\Gamma\vdash x:\tau} \text{ (var)}$$

Operators

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 + e_2 : \mathbf{int}} \quad (\mathsf{add}) \qquad \frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 - e_2 : \mathbf{int}} \quad (\mathsf{sub}) \qquad \frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 * e_2 : \mathbf{int}} \quad (\mathsf{mul})$$

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 / e_2 : \mathbf{int}} \quad (\mathsf{div}) \qquad \frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 \bmod e_2 : \mathbf{int}} \quad (\mathsf{mod})$$

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 < e_2 : \mathsf{bool}} \quad (\mathsf{lt}) \qquad \frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 < e_2 : \mathsf{bool}} \quad (\mathsf{lte}) \qquad \frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 > e_2 : \mathsf{bool}} \quad (\mathsf{gt})$$

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 > e_2 : \mathsf{bool}} \quad (\mathsf{gte}) \qquad \frac{\Gamma \vdash e_1 : \mathbf{int} \qquad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 = e_2 : \mathsf{bool}} \quad (\mathsf{neq})$$

$$\frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : \mathsf{bool}}{\Gamma \vdash e_1 : \mathsf{kil}} \quad (\mathsf{neq}) \qquad \frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : \mathsf{bool}}{\Gamma \vdash e_1 : \mathsf{kil}} \quad (\mathsf{or})$$

Conditionals

$$\frac{\Gamma \vdash e_1 : \mathsf{bool} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \mathsf{if} \ e_1 \ \mathsf{then} \ e_2 \ \mathsf{else} \ e_3 : \tau} \ (\mathsf{if})$$

Functions

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \mathbf{fun} \ (\ x : \tau_1 \) \stackrel{\bullet}{\rightarrow} e : \tau_1 \stackrel{\bullet}{\rightarrow} \tau_2} \ (\mathsf{fun}) \qquad \frac{\Gamma \vdash e_1 : \tau_2 \stackrel{\bullet}{\rightarrow} \tau \qquad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash e_1 e_2 : \tau} \ (\mathsf{app})$$

Let-Expressions

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \mathsf{let} \ x : \ \tau_1 = e_1 \ \mathsf{in} \ e_2 : \tau_2} \ \mathsf{(let)} \qquad \frac{e_1 = \mathsf{fun} \ (\ y : \ \tau \) \ \neg \ e}{\Gamma \vdash \mathsf{let} \ \mathsf{rec} \ \ x : \tau_1 \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2} \ \mathsf{(letRec)}$$

It's worth thinking about why we need this side-condition. The reason is a bit subtle, but in short we need e_1 to evaluate to an *unnamed* closure.

Assert

$$\frac{\Gamma \vdash e : \mathsf{bool}}{\Gamma \vdash \mathsf{assert} \ e : \mathsf{unit}} \ (\mathsf{assert})$$

Part 3: Evaluation

The evaluation of a program is given by the big-step operational semantics presented below. We write $\langle \mathcal{E}, e \rangle \Downarrow v$ to indicate that the expression e evaluates to the value v in the environment \mathcal{E} . We use the following notation for environments.

Notation	Description
Ø	empty environment
$\mathcal{E}[x \mapsto v]$	\mathcal{E} with x mapped to v
$\mathcal{E}(x)$	the value of x in \mathcal{E}

We take a value to be:

- \triangleright an integer (an element of the set \mathbb{Z}) denoted 1, -234, 12, etc.
- \triangleright a Boolean value (an element of the set \mathbb{B}) denoted \top and \bot
- ▷ unit, denoted •
- \triangleright a closure, denoted (\mathcal{E} , $s \mapsto e$), where \mathcal{E} is an environment, s is a name, and e is an expression. We'll write (\mathcal{E} , $\cdot \mapsto e$) for a closure without a name. For this project, we will only consider closures whose expressions are functions, so our VClos constructor is specific to this case.

The function eval, given an expression e of type expr, should evaluate to v in the case that $\langle \varnothing, e \rangle \Downarrow v$ is derivable according to the given semantics. There are two cases in which this function may raise one of the two exceptions.

- ▷ DivByZero, the second argument of a division (or mod) operator was 0
- ▷ AssertFail, an assertion within our language (not an OCaml assert) failed

It will be up to you to determine when to raise these exceptions.

Literals

Variables

$$\frac{x \text{ is a variable}}{\langle \ \mathcal{E} \ , \ x \ \rangle \Downarrow \mathcal{E}(x)} \text{ (varEval)}$$

Note that, because of type-checking, we don't need to verify that x appears in the environment \mathcal{E} .

Operators

$$\frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2}{\langle \mathcal{E} \,,\, e_1 + e_2 \,\rangle \Downarrow v_1 + v_2} \; (\mathsf{addEval}) \qquad \frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2}{\langle \mathcal{E} \,,\, e_1 - e_2 \,\rangle \Downarrow v_1 - v_2} \; (\mathsf{subEval})$$

$$\frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2}{\langle \mathcal{E} \,,\, e_1 * e_2 \,\rangle \Downarrow v_1 \times v_2} \; (\mathsf{mulEval}) \qquad \frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2 \quad v_2 \neq 0}{\langle \mathcal{E} \,,\, e_1 \,\rangle \nmid v_1 / v_2} \; (\mathsf{divEval})$$

$$\frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2 \quad v_2 \neq 0}{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_1 / v_2} \; (\mathsf{modEval})$$

$$\frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2 \quad v_1 < v_2}{\langle \mathcal{E} \,,\, e_1 \,\langle e_2 \,\rangle \Downarrow v_1 \quad \mathsf{mod} \,v_2} \; (\mathsf{ltTrue}) \qquad \frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2 \quad v_1 \geq v_2}{\langle \mathcal{E} \,,\, e_1 \,\langle e_2 \,\rangle \Downarrow \mathcal{I}} \; (\mathsf{ltFalse})$$

$$\frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2 \quad v_1 \leq v_2}{\langle \mathcal{E} \,,\, e_1 \,\langle e_2 \,\rangle \Downarrow \mathcal{I}} \; (\mathsf{ltFalse}) \; \frac{\langle \mathcal{E} \,,\, e_1 \,\rangle \Downarrow v_1 \quad \langle \mathcal{E} \,,\, e_2 \,\rangle \Downarrow v_2 \quad v_1 \geq v_2}{\langle \mathcal{E} \,,\, e_1 \,\langle e_2 \,\rangle \Downarrow \mathcal{I}} \; (\mathsf{ltFalse})$$

Conditionals

$$\frac{\langle \mathcal{E}, e_1 \rangle \Downarrow \top \quad \langle \mathcal{E}, e_2 \rangle \Downarrow v}{\langle \mathcal{E}, \text{ if } e_1 \text{ then } e_2 \text{ else } e_3 \rangle \Downarrow v} \text{ (ifTrue)} \qquad \frac{\langle \mathcal{E}, e_1 \rangle \Downarrow \bot \quad \langle \mathcal{E}, e_3 \rangle \Downarrow v}{\langle \mathcal{E}, \text{ if } e_1 \text{ then } e_2 \text{ else } e_3 \rangle \Downarrow v} \text{ (ifFalse)}$$

Functions

$$\frac{\langle \, \mathcal{E} \, , \, \mathsf{fun} \, (\, x \, : \, \tau \,) \, \rightarrow e \, \rangle \, \Downarrow \, (\, \mathcal{E} \, , \, \cdot \mapsto \mathsf{fun} \, (\, x \, : \, \tau \,) \, \rightarrow e \,)}{\langle \, \mathcal{E} \, , \, e_1 \, \rangle \, \Downarrow \, (\, \mathcal{E}' \, , \, \cdot \mapsto \mathsf{fun} \, (\, x \, : \, \tau \,) \, \rightarrow e \,)} \, \left(\, \mathcal{E} \, , \, e_2 \, \rangle \, \Downarrow \, v_2 \, \left(\, \mathcal{E}'[x \mapsto v_2] \, , \, e \, \rangle \, \Downarrow \, v \, \right)} \, \left(\mathsf{appEval} \right)$$

$$\frac{\langle \, \mathcal{E} \, , \, e_1 \, \rangle \Downarrow (\! \! \mathcal{E}' \, , \, s \mapsto \mathbf{fun} \, (\, x : \tau \,) \overset{\rightarrow}{\rightarrow} e \,) \qquad \langle \, \mathcal{E} \, , \, e_2 \, \rangle \Downarrow v_2 \qquad \langle \, \mathcal{E}'[s \mapsto (\! \! \! \mathcal{E}' \, , \, s \mapsto \mathbf{fun} \, (\, x : \tau \,) \overset{\rightarrow}{\rightarrow} e \,)][x \mapsto v_2] \, , \, e \, \rangle \Downarrow v}{\langle \, \mathcal{E} \, , \, e_1 e_2 \, \rangle \Downarrow v} \, (\mathsf{appRecEval})$$

Let-Expressions

$$\frac{\langle \; \mathcal{E} \; , \; e_1 \; \rangle \Downarrow v_1 \qquad \langle \; \mathcal{E}[x \mapsto v_1] \; , \; e_2 \; \rangle \Downarrow v_2}{\langle \; \mathcal{E} \; , \; | \mathsf{let} \; x \; : \; \tau = e_1 \; \mathsf{in} \; e_2 \; \rangle \; \Downarrow v_2} \; (\mathsf{letEval})}$$

$$\frac{\langle \; \mathcal{E} \; , \; e_1 \; \rangle \; \Downarrow \; (\; \mathcal{E}' \; , \; \cdot \mapsto \mathsf{fun} \; (\; x \; : \; \tau \;) \; \rightarrow e \;) \qquad \langle \; \mathcal{E}[f \mapsto (\! \mid \mathcal{E}' \; , \; f \mapsto \mathsf{fun} \; (\; x \; : \; \tau \;) \; \rightarrow e \;)] \; , \; e_2 \; \rangle \; \Downarrow v_2}{\langle \; \mathcal{E} \; , \; \mathsf{let} \; \mathsf{rec} \; f \; : \; \tau_1 \; = e_1 \; \mathsf{in} \; e_2 \; \rangle \; \Downarrow v_2} \; (\mathsf{letRecEval})}$$

Because of the side condition in the (letRec) rule, the expression e_1 in the rule (letRecEval) is guaranteed to be an unnamed closure.

Assert

$$\frac{\langle \mathcal{E}, e \rangle \Downarrow \top}{\langle \mathcal{E}, \mathbf{assert} e \rangle \Downarrow \mathbf{()}}$$
 (assertEval)

In your implementation, if e does not evaluate to \top , then you should raise an assertion failure. This is a *side effect*, it's not reflected in the semantics proper.

Putting Everything Together

After you're done with parse, type_of, and eval, you should combines these into a single function called interp. This should, in essence, be function composition, except that if parse fails, interp should return Error ParseFail. Once you've implemented interp, you should be able to run:

```
dune exec interp2 filename
```

to execute code you've written in other files (replace filename with the name of the file which contains code you want to execute). Our language is subset of OCaml so you should be able to easily write programs, e.g., here is an implementation of sum_of_squares:

```
let sum_of_squares (x : int) (y : int) : int =
  let x_squared : int = x * x in
  let y_squared : int = y * y in
  x_squared + y_squared
let _ : unit = assert (sum_of_squares 3 (-5) = 34)
```

We can now use assert statements to test our own implementations! (As long as we've correctly implemented the semantics of assert)

Final Remarks

- ▶ There is a lot of repetition here, this is just the nature of implementing programming languages. So even though there is a lot of code to write, it should go pretty quickly. Despite this, it may be worthwhile to think about how to implement the interpreter without too much code replication.
- ▶ Test along the way. Don't try to write the whole interpreter and test afterwards.
- ➤ You must use exactly the same names and types as given at the top of this file. They must appear in the file interp2/lib/interp2.ml. If you don't do this, we can't grade your submission. You are, of course, allowed to add your own definitions.
- You are given a skeleton dune project to implement your interpreter. Do not change any of the given code. In particular, don't change the dune files or the utility files. When we grade your assignment we will assume this skeleton.
- ▶ We will not immediately release examples or the autograder. You should test yourself as best as you can first.

Good luck, happy coding.