Stack Machines

Concepts of Programming Languages
Lecture 25

CAS CS 320

Outline

- Finish our demo implementation of HM^-
- Discuss **stack-based languages** and **stack machines**
- Demo an implementation of compiling arithmetic expressions

Practice Problem

```
fun x -> fun y -> fun z -> x z (y z)
```

Determine the principle type of the above expression

Solution

```
fun x -> fun y -> fun z -> x z (y z)
```

Recap

Recall: Principle Types

$$\Gamma \vdash e : \tau \dashv \mathscr{C}$$

The constraints $\mathscr{C}$ defined a *unification problem.* Given a most general unifier $\mathcal{S}$ we can get the "actual" type of e:

$$\text{principle}(\tau, \mathscr{C}) = \forall\alpha_1 \ldots \forall\alpha_k . \mathcal{S}\tau \text{ where } \text{FV}(\mathcal{S}\tau) = \{\alpha_1, \ldots, \alpha_k\}$$

i.e, the **principle type** of e (note: it may not exist). Every type we *could* give e is a *specialization* of $\forall\alpha_1, \ldots, \alpha_k . \mathcal{S}\tau$

Recall: HM⁻ (Typing Variables)

$$\frac{(x : \forall\alpha_1 . \forall\alpha_2 \ldots \forall\alpha_k . \tau) \in \Gamma \qquad \beta_1, \ldots, \beta_k \text{ are fresh}}{\Gamma \vdash x : [\beta_1/\alpha_1] \ldots [\beta_k/\alpha_k]\tau \dashv \varnothing} \text{ (var)}$$

If x is declared in Γ, then x can be given the type τ *with all free variables replaced by* ***fresh variables***

This is where the polymorphism magic happens

Fresh variables can be unified with anything

Recall: Putting everything together

input: program P (sequence of top-level let-expressions)

$\Gamma \leftarrow \varnothing$

FOR EACH top-level let-expression let $x = e$ in P:

1. *Constraint-based inference:* Determine τ and $\mathscr{C}$ such that $\Gamma \vdash e : \tau \dashv \mathscr{C}$ is derivable
2. *Unification:* Solve $\mathscr{C}$ to get a most general unifier $\mathcal{S}$ (**TYPE ERROR** if this fails)
3. *Generalization:* Quantify over the free variables in $\mathcal{S}\tau$ to get the principle type $\forall\alpha_1 \ldots \forall\alpha_k . \mathcal{S}\tau$ of e
4. Add $(x : \forall\alpha_1 \ldots \forall\alpha_k . \mathcal{S}\tau)$ to Γ

demo

(finishing up type inference)

Stack Machines

High-Level

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- instruction sets for virtual stack machines, e.g., JVM, CPython, Lua (not any more), OCaml bytecode interpreter

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- "usable" stack-oriented languages, e.g., Forth
- instruction sets for virtual stack machines, e.g., JVM, CPython, Lua (not any more), OCaml bytecode interpreter *(these aren't exactly programming languages)*

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Virtual machines are typically implemented as **bytecode interpreters,** where "programs" are streams of bytes and a command is represented as a byte (plus possibly some extra data)

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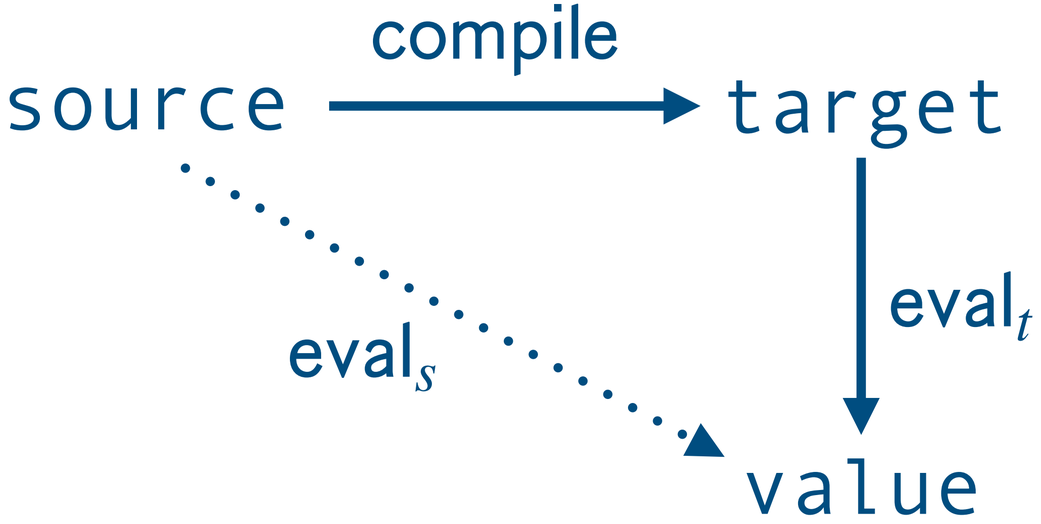
Portability: Any OS should be able to handle a stream of bytes, so the machine dependent part of our programming language can be simplified

Efficiency (sort of): They can be implemented in low-level languages, and so will generally be faster than the interpreters we build in this course (though not as fast as natively compiled code)

Looking Ahead: Compilation



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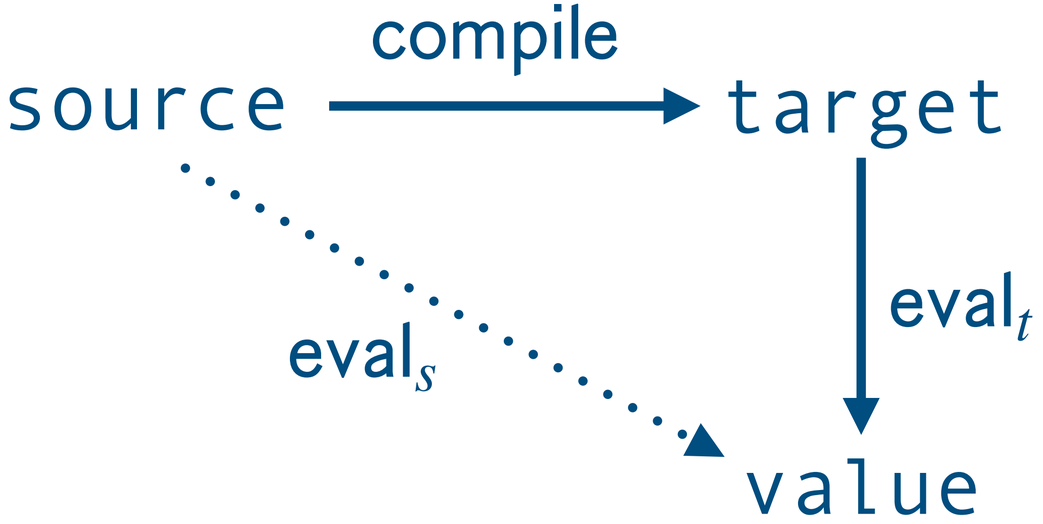

Compilation is the process of translating a program in one language to another, maintaining semantic behavior

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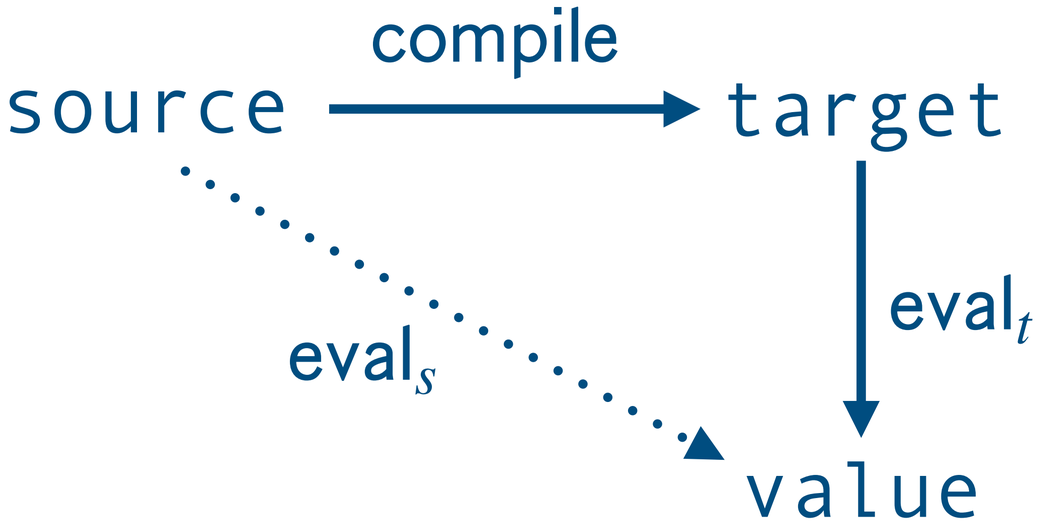

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Compilation can be a part of interpretation as well, like with **bytecode interpretation** (this is what OCaml does)

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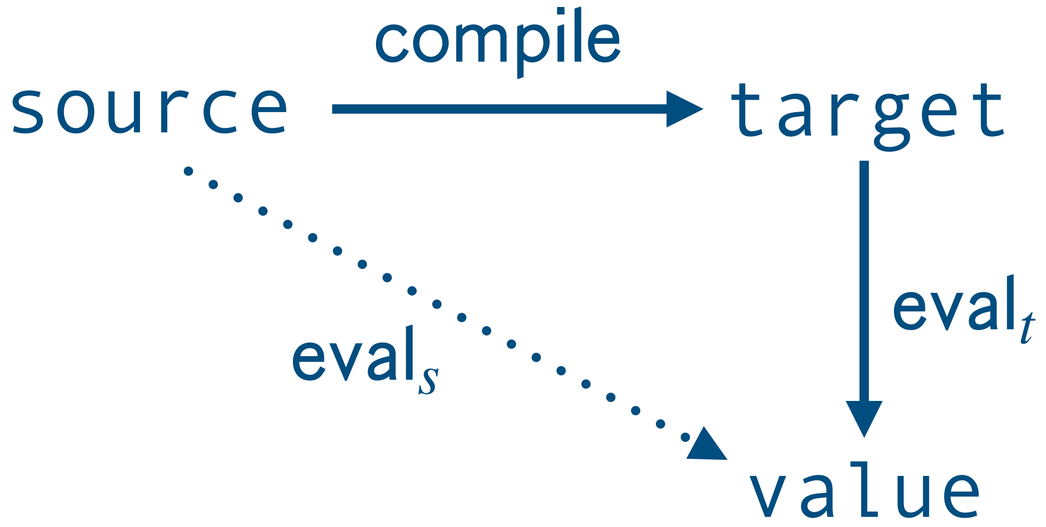

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Compilation can be a part of interpretation as well, like with **bytecode interpretation** (this is what OCaml does)

The simple case for today: *every arithmetic expression can be represented as an equivalent expression in reverse polish notation*

Stack-Based Arithmetic

Stack-Based Arithmetic (Syntax)

$$
\begin{aligned}
\texttt{<prog>} &::= \{\texttt{<com>}\} \\
\texttt{<com>} &::= \texttt{ADD} \mid \texttt{SUB} \mid \texttt{MUL} \mid \texttt{DIV} \mid \texttt{PUSH}\ \texttt{<num>} \\
\texttt{<num>} &::= \mathbb{Z}
\end{aligned}
$$

Stack-Based Arithmetic (Semantics)

$$\langle \mathcal{S}, P \rangle$$

A **value** is an integer ($\mathbb{Z}$)

A **configuration** is made up of a stack ($\mathcal{S}$) of values and a program (P) given by **<prog>**

Stack-Based Arithmetic (Semantics)

$$\frac{}{\langle m :: n :: \mathcal{S}, \texttt{ADD}\ P\rangle \longrightarrow \langle (m+n) :: \mathcal{S}, P\rangle}\texttt{(add)}$$

$$\frac{}{\langle m :: n :: \mathcal{S}, \texttt{SUB}\ P\rangle \longrightarrow \langle (m-n) :: \mathcal{S}, P\rangle}\texttt{(sub)}$$

$$\frac{}{\langle m :: n :: \mathcal{S}, \texttt{MUL}\ P\rangle \longrightarrow \langle (m\times n) :: \mathcal{S}, P\rangle}\texttt{(mul)}$$

$$\frac{n \neq 0}{\langle m :: n :: \mathcal{S}, \texttt{DIV}\ P\rangle \longrightarrow \langle (m/n) :: \mathcal{S}, P\rangle}\texttt{(div)}$$

$$\frac{}{\langle \mathcal{S}, \texttt{PUSH}\ n\ P\rangle \longrightarrow \langle n :: \mathcal{S}, P\rangle}\texttt{(div)}$$

Example (Evaluation)

```
PUSH 2 PUSH 3 SUB PUSH 4 MUL
```

demo

(stack machine)

Compiling Arithmetic Expressions

$$\texttt{n} \Longrightarrow \texttt{PUSH n}$$

$$e_1 \texttt{ + } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{ADD}$$

$$e_1 \texttt{ - } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{SUB}$$

$$e_1 \texttt{ * } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{MUL}$$

$$e_1 \texttt{ / } e_2 \Longrightarrow \mathscr{C}(e_2)\ \mathscr{C}(e_1)\ \texttt{DIV}$$

We need a procedure $\mathscr{C}$ for converting an arithmetic expression into a stack program. *Note the order!*

Example (Compilation)

```
4 * (2 – 3)
```

demo

(compiling arithmetic expressions)

Variables

Variables (Syntax)

$$
\begin{aligned}
\texttt{<prog>} &::= \{\texttt{<com>}\} \\
\texttt{<com>} &::= \texttt{ADD} \mid \texttt{SUB} \mid \texttt{MUL} \mid \texttt{DIV} \mid \texttt{PUSH}\ \texttt{<num>} \\
&\quad \mid \texttt{ASSIGN}\ \texttt{<var>} \mid \texttt{LOOKUP}\ \texttt{<var>} \\
\texttt{<num>} &::= \mathbb{Z} \\
\texttt{<var>} &::= \mathbb{I}
\end{aligned}
$$

Variables (Semantics)

$$\langle \mathcal{S}, \mathcal{E}, P \rangle$$

A **value** is an integer ($\mathbb{Z}$)

A **configuration** is made up of a stack $\mathcal{S}$ of values, an environment $\mathcal{E}$ (mapping of identifiers to values), and a program P given by **<prog>**

Variables (Semantics)

$$\frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{ADD}\ P\rangle \longrightarrow \langle (m+n) :: \mathcal{S}, \mathcal{E}, P\rangle}\texttt{(add)}$$

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$$\frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{ASSIGN}\ x\ P\rangle \longrightarrow \langle \mathcal{S}, \mathcal{E}[x \mapsto n], P\rangle}\texttt{(asn)}$$

$$\frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{LOOKUP}\ x\ P\rangle \longrightarrow \langle \mathcal{E}(x) :: \mathcal{S}, \mathcal{E}, P\rangle}\texttt{(lkp)}$$

Variables (Semantics)

basically the same

$$\frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{ADD}\ P\rangle \longrightarrow \langle (m+n) :: \mathcal{S}, \mathcal{E}, P\rangle}\text{(add)} \qquad \frac{}{\langle m :: n :: \mathcal{S}, \mathcal{E}, \texttt{SUB}\ P\rangle \longrightarrow \langle (m-n) :: \mathcal{S}, \mathcal{E}, P\rangle}\text{(sub)}$$

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Variables (Semantics)

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new rules

$$\frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{ASSIGN}\ x\ P\rangle \longrightarrow \langle \mathcal{S}, \mathcal{E}[x \mapsto n], P\rangle}\text{(asn)} \qquad \frac{}{\langle n :: \mathcal{S}, \mathcal{E}, \texttt{LOOKUP}\ x\ P\rangle \longrightarrow \langle \mathcal{E}(x) :: \mathcal{S}, \mathcal{E}, P\rangle}\text{(lkp)}$$

Example (Evaluation)

```
PUSH 2 ASSIGN x PUSH 3 ASSIGN y
LOOKUP x LOOKUP y ADD
```

Compiling Let-Expressions (Attempt)

$$\texttt{x} \quad \Longrightarrow \quad \texttt{LOOKUP}\ x$$

$$\texttt{let}\ x\ \texttt{=}\ e_1\ \texttt{in}\ e_2 \quad \Longrightarrow \quad \mathscr{C}(e_1)\ \texttt{ASSIGN}\ x\ \mathscr{C}(e_2)$$

Compiling Let-Expressions (Attempt)

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$$\texttt{let}\ x\ \texttt{=}\ e_1\ \texttt{in}\ e_2 \quad \Longrightarrow \quad \mathscr{C}(e_1)\ \texttt{ASSIGN}\ x\ \mathscr{C}(e_2)$$

Except this isn't quite right

Example

```
let y = 1 in
let x = let y = 2 in y in
y
```

Scoping

```
let y = 1 in
let x = let y = 2 in y in
y
```

The language we've just described is only good for compiling from languages with **dynamic scoping**

Next time. We'll add *closures* so that we can deal with lexical scoping (and functions)

Summary

Compilation is the process of translating a program in a source language into a program in a target language which preserves the semantics

Targeting a **virtual machine** can make the implementation of a language more portable and less complex

We'll need **closures** to deal with lexical scoping correctly