Assignment 4 Written Solutions

An OCaml Puzzle

$$\texttt{Foo (fun \_ → 42)}$$

Explanation. In rough terms, `baz e` applies the function carried by `e` to `e` itself. The trick is that the function `e` carries does not need to *use* its argument. This might be represented by the following collection of equalities.

$$
\begin{aligned}
\texttt{out} &= \texttt{baz (Foo (fun \_ → 42))} \\
&= \texttt{bar (Foo (fun \_ → 42)) (Foo (fun \_ → 42))} \\
&= \texttt{(fun \_ → 42) (Foo (fun \_ → 42))} \\
&= \texttt{42}
\end{aligned}
$$

Typing Derivation

$$
\cfrac{
  \cfrac{
    \cfrac{}{\varnothing \vdash \texttt{3 : int}}\text{intLit}
    \quad
    \cfrac{
      \cfrac{}{\{\texttt{b : int}\} \vdash \texttt{b : int}}\text{var}
      \quad
      \cfrac{}{\{\texttt{b : int}\} \vdash \texttt{b : int}}\text{var}
    }{\{\texttt{b : int}\} \vdash \texttt{b + b : int}}\text{intAdd}
  }{\varnothing \vdash \texttt{let b = 3 in b + b : int}}\text{let}
  \quad
  \cfrac{
    \cfrac{
      \cfrac{}{\{\texttt{a : int}\} \vdash \texttt{a : int}}\text{var}
      \quad
      \cfrac{}{\{\texttt{a : int}\} \vdash \texttt{true : bool}}\text{trueLit}
    }{\{\texttt{a : int}\} \vdash \texttt{(a, true) : int * bool}}\text{tuple}
  }{}
}{\varnothing \vdash \texttt{let a = let b = 3 in b + b in (a, true) : int * bool}}\text{let}
$$

Semantic Derviation

$$
\cfrac{
  \cfrac{\cfrac{}{\texttt{3} \Downarrow 3}\text{ iLE} \quad \cfrac{\cfrac{\cfrac{}{\texttt{3} \Downarrow 3}\text{ iLE} \quad \cfrac{}{\texttt{3} \Downarrow 3}\text{ iLE}}{\texttt{3 + 3} \Downarrow 6}\text{ iAE}}{\texttt{let b = 3 in b + b} \Downarrow 6}\text{ letEval} \quad \cfrac{\cfrac{}{\texttt{4} \Downarrow 4}\text{ iLE} \quad \cfrac{}{\texttt{6} \Downarrow 6}\text{ iLE}}{\texttt{4 * 6} \Downarrow 24}\text{ iME}
}{
  \texttt{let a = let b = 3 in b + b in 4 * a} \Downarrow 24
}\text{ letEval}
$$