CS460: Intro to Database Systems

Class 5: Relational Algebra

Instructor: Manos Athanassoulis

https://bu-disc.github.io/CS460/

Up to now ...

we have been discussing how to:

(i) model the requirements

(ii) translate them into relational schema

today: execute queries relational algebra

Reminders

Relation

Schema: relation name, attributes (type & name)

Students(*sid*: string, *name*: string, *login*: string, *age*: integer, *gpa*: real)

Instance

a table containing rows of such columns

every relation instance is a <u>set</u> (all rows distinct)

Relational Algebra

Relational Query Languages

Selection & Projection

Union, Set Difference & Intersection

Cross product & Joins

Examples

Division

Relational Algebra

Relational Query Languages

Selection & Projection

Union, Set Difference & Intersection

Cross product & Joins

Examples

Division

Relational Query Languages

Query languages: manipulation and retrieval of data

Relational model supports simple, powerful QLs:

Strong formal foundation based on logic.

Allows for much optimization.

Query Languages != programming languages!

QLs not expected to be "Turing complete".

QLs not intended to be used for complex calculations.

QLs support easy, efficient access to large data sets.

Formal Relational Query Languages

Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:

<u>Relational Algebra</u>: More operational, very useful for representing execution plans.

Relational Calculus: Lets users describe what they want, rather than how to compute it. (Non-procedural, declarative.)

Understanding Algebra is key to understanding query processing!

Preliminaries

Query from a relation instance to a relation instance

input & output schema

different but fixed queries run over any legal instances output schema defined by the query constructs

attribute notation positional & name-field

Relational Algebra: 5 Basic Operations

- <u>Selection</u> (σ) Selects a subset of *rows* from relation (horizontal).
- <u>Projection</u> (π) Retains only wanted <u>columns</u> from relation (vertical).
- <u>Cross-product</u> (x) Allows us to combine two relations.
- <u>Set-difference</u> (–) Tuples in R_1 , but not in R_2 .
- <u>Union</u> (U) Tuples in R_1 and/or in R_2 .

each operation returns a relation : composability (Algebra is "closed")

Example Instances

sid	bid	<u>day</u>
22	101	10/10/16
58	103	11/12/16

Boats

<u>bid</u>	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

5,

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Relational Algebra

Relational Query Languages

Selection & Projection

Union, Set Difference & Intersection

Cross product & Joins

Examples

Division

Projection

Examples: $\pi_{age}(S_2)$; $\pi_{sname, rating}(S_2)$ retains only attributes that are in the "projection list"

schema of result:

fields of projection list (with the same names)

projection operator has to *eliminate duplicates* why we may have duplicates? why remove them?



Note: systems typically don't do duplicate elimination unless the user explicitly asks for it (Why not?)

Projection

\mathbf{S}	id	sname	rating	lag	ζe
2	8	yuppy	9	3:	5.0
3		lubber	8	5:	5.5
4	4	guppy	5	3	5.0
5	8	rusty	10	3	5.0
			-		_

 S_2

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$$\pi_{sname,rating}(S_{2})$$

Projection

si	d	sna	ıme	rat	ng	age
28	3	yu	ору	g		35.0
3	Ĺ	lut	ber	8		55.5
4	1	gu	рру	5	,)	35.0
5	3	rus	sty	1	0	35.0
			S ₂			

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$$\pi_{sname,rating}(S_{2})$$

age 35.0 55.5

$$\pi_{age}(S_2)$$

Selection (σ)

selects rows that satisfy a *selection condition*

result: has the same schema as the input relation

do we need to do <u>duplicate elimination</u>?



sid	sname	rating	age
28	yuppy	9	35.0
31	1 1 1	Q	555
<i>J</i> 1	luober	8	33.3
44	guppy	5	35.0
58	rusty	10	35.0

$$\sigma_{rating>8}(S_2)$$

Selection (σ)

selects rows that satisfy a *selection condition*

result: has the same schema as the input relation

do we need to do duplicate elimination?

si	\mathbf{d}	sname	rating	ag	e
2	8	yuppy	9	35	.0
3		lubber	8	55	5.5
4	1 †	guppy	5	3.	0.0
5	3	rusty	10	35	5.0
			((7 \	

sname	rating
yuppy	9
rusty	10

$$\pi_{sname,rating}(\sigma_{rating>8}(S_2))$$

Relational Algebra

Relational Query Languages

Selection & Projection

Union, Set Difference & Intersection

Cross product & Joins

Examples

Division

Union and Set-Difference

the <u>set</u> operations take two input relations which must be <u>union-compatible</u>

(i) same number of fields

(ii) "corresponding" fields have the same type

for which, if any, is duplicate elimination required? (union/set-difference)

Union

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

5	_	
J	1	

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$$S_1 \cup S_2$$

Set Difference

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

sid	sname	rating	age
22	dustin	7	45.0

$$S_1 - S_2$$

 S_1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sid	sname	rating	age
28	yuppy	9	35.0
44	guppy	5	35.0

$$S_2 - S_1$$

Compound Operator: Intersection

in addition to the 5 basic operators
several additional <u>compound operators</u>
no new computational power, but useful shorthands
can be expressed solely with the basic ops

intersection takes two <u>union-compatible</u> relations

Q: How to express it using basic operators?



Intersection

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

 S_1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$$S_1 \cap S_2$$

Relational Algebra

Relational Query Languages

Selection & Projection

Union, Set Difference & Intersection

Cross product & Joins

Examples

Division

Cross-Product

 $S_1 \times R_1$: each row of S_1 paired with each row of R_1 how many rows in the result?

result schema has one field per field of S_1 and R_1 , with field names "inherited" (if possible)

may have a naming conflict:

both S₁ and R₁ have a field with the same name

in this case, can use the renaming operator:

$$\rho(C(1 \rightarrow sid_1, 5 \rightarrow sid_2), S_1 \times R_1)$$

Cross Product Example

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

sid	bid	day
22	101	10/10/16
58	103	11/12/16

 R_1

 S_1

Compound Operator: Join

Joins are compound operators : \times , σ , (sometimes) π

frequent type is "natural join" (often called "join")

 $R \bowtie S$ conceptually is:

compute R×S

select rows where attributes in both **R**, **S** have equal values **project** all unique attributes and one copy of the common ones

Note: Usually done much more efficiently than this Useful for putting *normalized* relations back together

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

sid	bid	day
22	101	10/10/16
58	103	11/12/16

 R_1

$$S_1$$

$$S_1 \bowtie R_1 =$$

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/16
58	rusty	10	35.0	103	11/12/16

 $S_1 \times R_1 =$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/16
22	dustin	7	45.0	58	103	11/12/16
31	lubber	8	55.5	22	101	10/10/16
31	lubber	8	55.5	58	103	11/12/16
58	rusty	10	35.0	22	101	10/10/16
58	rusty	10	35.0	58	103	11/12/16

 $S_1 \times R_1 =$

2

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/16
22	dustin	7	45.0	-58	103	11/12/16
21	lubber	Q	55.5	22		10/10/16
31	rubber	0	33.3		101	10/10/10
31	lubber	8	55.5	-58	103	11/12/16
FO		10				' '
58-	rusty	10	35.0	22	101	10/10/16
58	rusty	10	35.0	58	103	11/12/16

 $S_1 \times R_1 =$

2

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/16
22	dustin	7	45.0	-58	103	11/12/16
31	lubber	8	55.5	22	101	10/10/16
31_	lubber	8	55.5	-58	103	11/12/16
58	rusty	10	35.0	22	101	' '
58	rusty	10	35.0	58	103	11/12/16



$$S_1 \bowtie R_1 = \begin{bmatrix} \text{sid} & \text{sname} & \text{rating} & \text{age} & \text{bid} & \text{day} \\ 22 & \text{dustin} & 7 & 45.0 & 101 & 10/10/16 \\ 58 & \text{rusty} & 10 & 35.0 & 103 & 11/12/16 \end{bmatrix}$$

Other Types of Joins

condition join (or "theta-join")

$$R\bowtie_{c} S = \sigma_{c}(R \times S)$$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/16
31	lubber	8	55.5	58	103	11/12/16

$$S \bowtie S.sid < R.sid$$

result schema same as that of cross-product

may have fewer tuples than cross-product

Equi-Join: Special case: condition c contains only conjunction of equalities.

Relational Algebra

Relational Query Languages

Selection & Projection

Union, Set Difference & Intersection

Cross product & Joins

Examples

Division

Examples

Reserves

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/16
58	103	11/12/16

Sailors

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Boats

bid	bname	color
101	Interlake	Blue
102	Interlake	Red
103	Clipper	Green
104	Marine	Red

Reserves (sid, bid, day)

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

Find names of sailors who have reserved boat #103

Solution 1:

$$\pi_{sname}((\sigma_{bid=103} \text{Reserves}) \bowtie Sailors)$$

Solution 2:

$$\pi_{sname}(\sigma_{bid=103}(Reserves \bowtie Sailors))$$

another solution?



Reserves (sid, bid, day)

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

Find names of sailors who have reserved a red boat

boat color only available in Boats; need an extra join:

$$\pi_{sname}((\sigma_{color=red}, Boats) \bowtie Reserves \bowtie Sailors)$$

a *more efficient* solution:

why more efficient?



$$\pi_{sname}(\pi_{sid}((\pi_{bid}\sigma_{color='red'},Boats)\bowtie Res)\bowtie Sailors)$$

a guery optimizer can find this given the first solution!

Reserves (sid, bid, day)

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

Find sailors who have reserved a red or a green boat

identify all red or green boats first
$$\rho \; (\textit{Tempboats}, (\sigma_{color = 'red' \lor color = 'green'}, \textit{Boats}))$$

then find sailors who have reserved one of these boats:

$$\pi_{sname}$$
(Temphoats \bowtie Reserves \bowtie Sailors)

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

Find sailors who have reserved a red and a green boat

Previous approach will not work! Why?

identify sailors who have reserved red boats

$$\rho$$
 (Tempred, $\pi_{sid}((\sigma_{color=red}, Boats)) \bowtie Reserves))$

sailors who have reserved green boats

$$\rho$$
 (Tempgreen, $\pi_{sid}((\sigma_{color=green}, Boats)) \bowtie Reserves))$

then find the intersection (sid is a key for Sailors)

$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$

More examples – Your turn! 71



- 1. Find (the name of) all sailors whose rating is above 9
- 2. Find all sailors who reserved a boat prior to November 1, 2016
- 3. Find (the names of) all boats that have been reserved at least once
- 4. Find all pairs of sailors with the same rating
- 5. Find all pairs of sailors in which the <u>older</u> sailor has a lower rating

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

(1) Find (the name of) all sailors whose rating is above 9



Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

(2) Find all sailors who reserved a boat prior to November 1, 2016



Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

(3) Find (the names of) all boats that have been reserved at least once

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

(4) Find all pairs of sailors with the same rating

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

(5) Find all pairs of sailors in which the older sailor has a lower rating



Relational Algebra

Relational Query Languages

Selection & Projection

Union, Set Difference & Intersection

Cross product & Joins

Examples

Division

Last Compound Operator: Division

useful for expressing "for all" queries like: "find sids of sailors who have reserved all boats"

for A/B attributes of B are subset of attributes of A may need to "project" to make this happen.

e.g., let A have 2 fields, x and y; B have only field y:

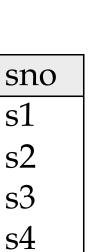
$$A/B = \{\langle x \rangle | \forall \langle y \rangle \in B(\exists \langle x, y \rangle \in A) \}$$

A/B contains all x tuples such that for every y tuple in B, there is an xy tuple in A

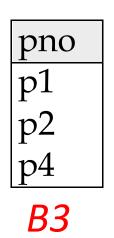
Examples of Division A/B

sno	pno
s1	p1
s1	p2)
s1	р3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

pno
p2
B1



pno p2 p4

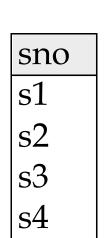


A/B1

Examples of Division A/B

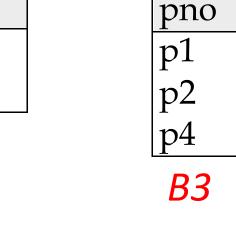
sno	pno
s1	p1
s1	p2
s1	p 3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4
Δ	

pno	
p2	
R1	



A/B1

pno	
p2	
p4	
<i>B2</i>	

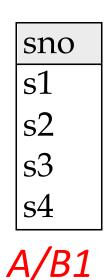


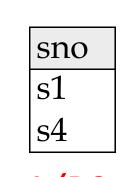
A/B2

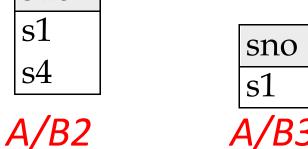
Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	Ď
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

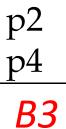
pno	
p2	
R1	





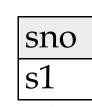


pno	
p2	
p4	
<i>B2</i>	



pno

p1



Expressing A/B Using Basic Operators

division is not essential op; just a shorthand

(true for joins, but so common that are implemented specially)

Idea: For *A/B*, compute all *x* <u>values that are not "disqualified"</u> by some *y* value in *B*

x value is disqualified if by attaching y value from B, we obtain an xy tuple that is not in A

Disqualified x values:
$$\pi_{\chi}((\pi_{\chi}(A) \times B) - A)$$

A/B:
$$\pi_{\chi}(A)$$
 — Disqualified x values

Expressing A/B: $\pi_{sno}(A) - \pi_{sno}((\pi_{sno}(A) \times B) - A)$

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

sno	pno
s1	p1
s1	p2
s1	p4
s2	p1
s2	p2
s2	p4
s3	p1
s3	p2
s3	p4
s4	p1
s4	p2
s4	p4

A

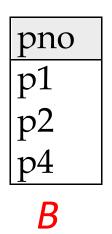
$$T1=\pi_{Sno}(A)\times B$$

Expressing A/B: $\pi_{sno}(A) - \pi_{sno}((\pi_{sno}(A) \times B) - A)$

pno
p1
p2
p3
p4
p1
p2
p2
p2
p4

sno	pno
s1	p1
_1	
31	p2
s1	p4
<u>.</u> 2	<u>n</u> 1
_	P
s2	p2
s2	p4
s3	p1
s3	p2
s3	p4
s4	p1
s4	12
s4	$\frac{1}{p4}$
	<u> </u>

sno	pno		
s2	p4		
₆ 2	n1		
	M		
s3	p4		
s4	p1		
T1-A			

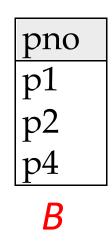


$$T2 = \pi \sqrt{T1-A}$$

Expressing A/B: $\pi_{sno}(A) - \pi_{sno}((\pi_{sno}(A) \times B) - A)$

		1	sno	pno
sno	pno		s1	101
s1	p1		<u>5</u> 1	122
s1	p2		s1	P - 124
s1	p3		s2	P 1 121
s1	p4		s2	p2
s2	p1		s2	p4
s2	p2		s3	p1
s3	p2		s3	122
s4	p2		s3	p4
s4	p4		s4	p1
		ļ	$\frac{51}{54}$	12 2
/	4		cA	PZ
			5/1	P'1

sno	pno		
s2	p4		
s3	12 1		
s3	p4		
s4	p1		
T1-A			



sno		
s1		sno
s2		s2
s 3	_	s3
s4		s4

$$\begin{bmatrix} s4 \\ T2 = \pi \\ sno \end{bmatrix} A/B = \pi \\ sno (A) - T2$$

Reserves (sid, bid, day)

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

Find the names of sailors who have reserved all boats

use division; schemas of the input relations to / must be carefully chosen (why?)

$$\rho \; (\textit{Tempsids}, (\pi_{\textit{sid,bid}} \text{Reserves}) \, / \, (\pi_{\textit{bid}} \textit{Boats}))$$

$$\pi_{\textit{sname}} (\textit{Tempsids} \bowtie \textit{Sailors})$$

To find sailors who have reserved all "Interlake" boats:

....
$$/\pi_{bid}(\sigma_{bname=Interlake}, Boats)$$

Reserves (sid, bid, day)

Sailors (sid, sname, rating, age)

Boats (bid, bname, color)

Find the names of sailors who have reserved all boats

use division; schemas of the input relations to / must be carefully chosen (why?)

$$\rho \; (\textit{Tempsids}, (\pi_{\textit{sid}, \textit{bid}} \text{Reserves}) \, / \, (\pi_{\textit{bid}} \textit{Boats})) \\ \pi_{\textit{sname}} \; (\textit{Tempsids} \bowtie \textit{Sailors})$$

what if we divided Reserves / $\pi_{bid}(Boats)$?

