

## The *design space* of data structures

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<https://bu-disc.github.io/CS561/>

## data structures

b+ trees

hash tables

zonemaps

radix trees

bitmap indexes

are in the core of:



database systems

file systems

operating systems

machine learning systems

systems for data science

how to decide which one to use?

workload (access patterns)



current focus

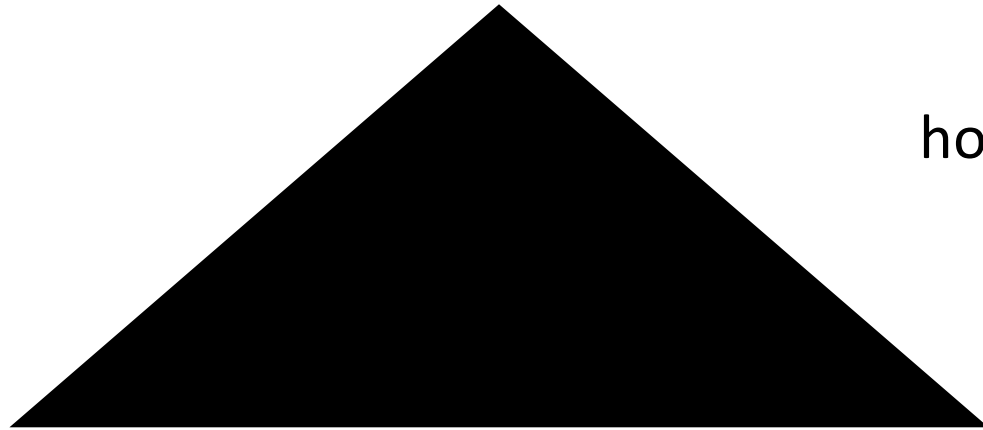
next →

hardware (memory/storage/network/compute)

how to decide how to *design* a data structure?

break it down to *design dimensions*

# how to break down the *design* in independent *dimensions*?



how to physically organize the data?

how to search through the data?

can I accelerate search through metadata?

multiple levels of nested organization?

how to update or add new data?

how to exploit additional memory/storage?

should the above decisions be applied eagerly or lazily?

# how to break down the *design* in independent *dimensions*?



global data organization

how to search through the data?

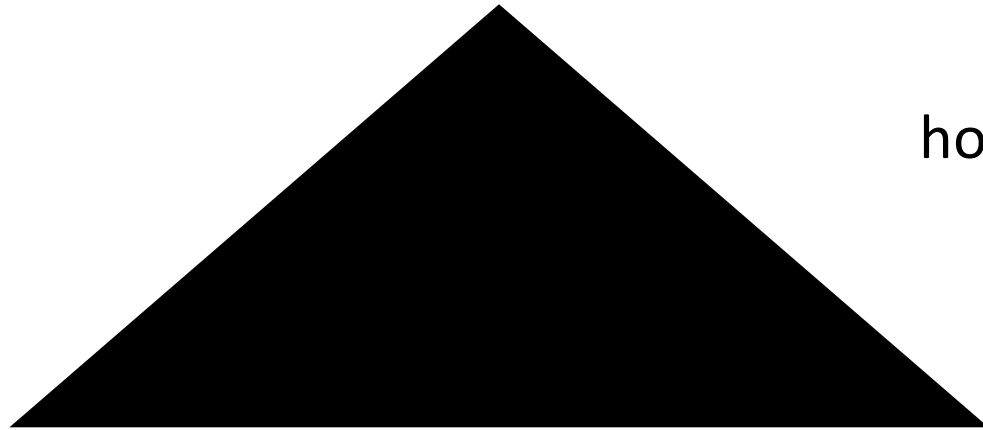
can I accelerate search through metadata?

multiple levels of nested organization?

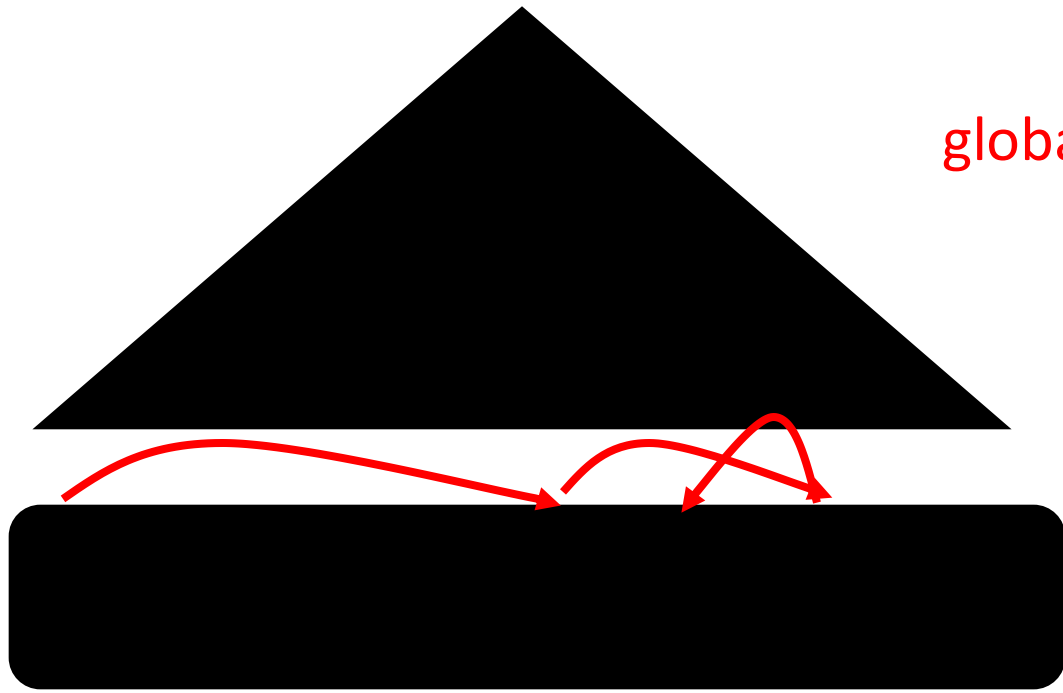
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# how to break down the *design* in independent *dimensions*?



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global search algorithm

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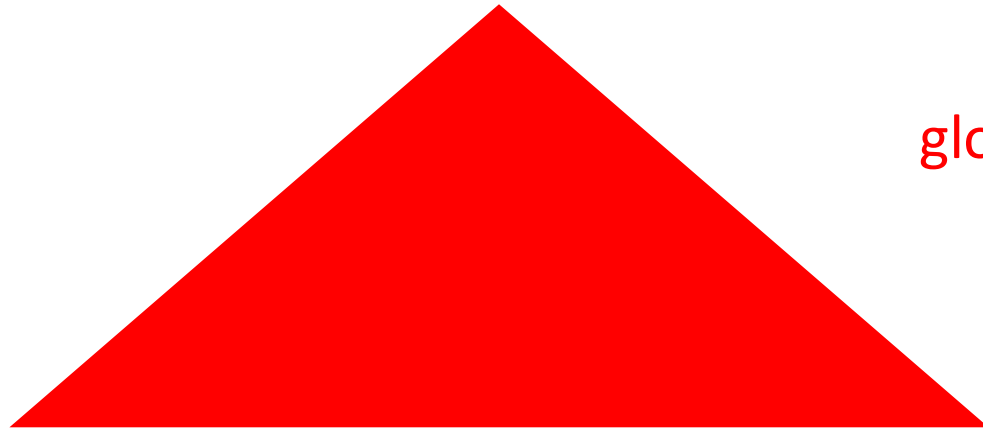
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global data organization

global search algorithm

metadata for searching

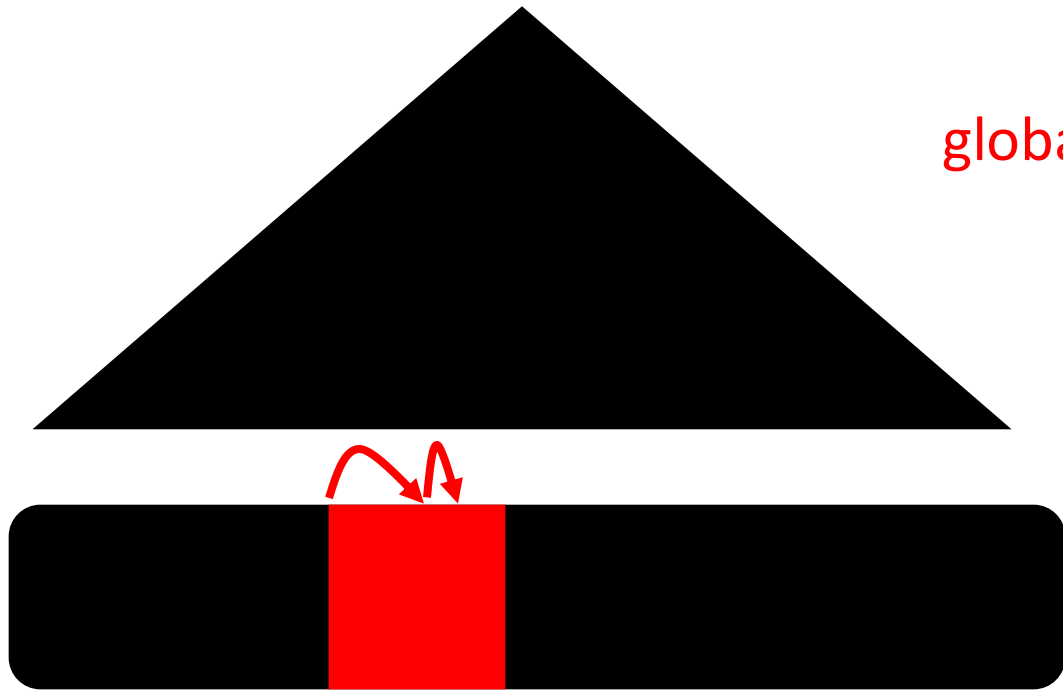
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how to break down the *design* in independent *dimensions*?



global data organization

global search algorithm

metadata for searching

local data organization & search algorithm

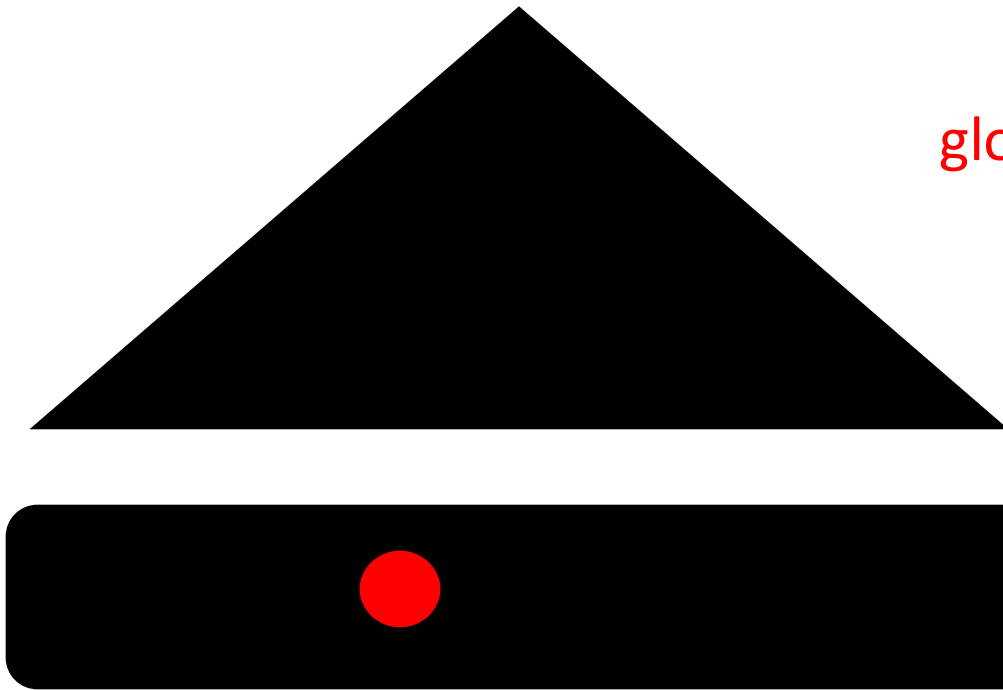
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global data organization

global search algorithm

metadata for searching

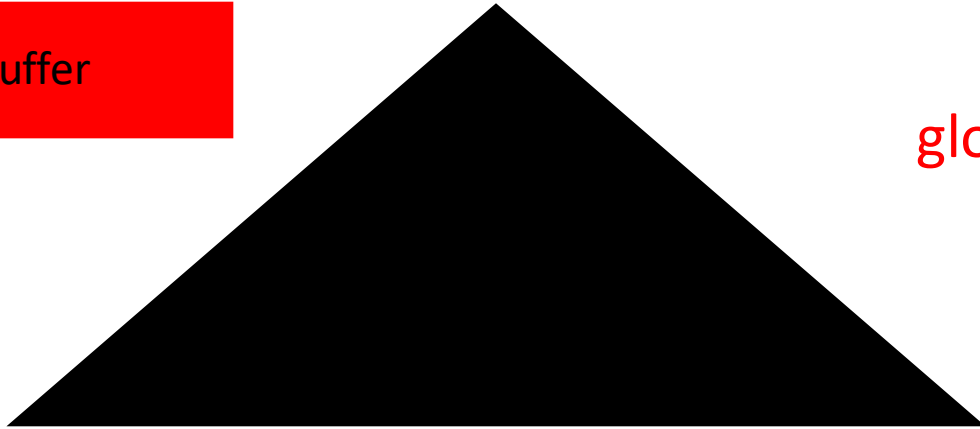
local data organization & search algorithm

modification policy

how to exploit additional memory/storage?

should the above decisions be applied eagerly or lazily?

# how to break down the *design* in independent *dimensions*?



global data organization

global search algorithm

metadata for searching

local data organization & search algorithm

modification policy

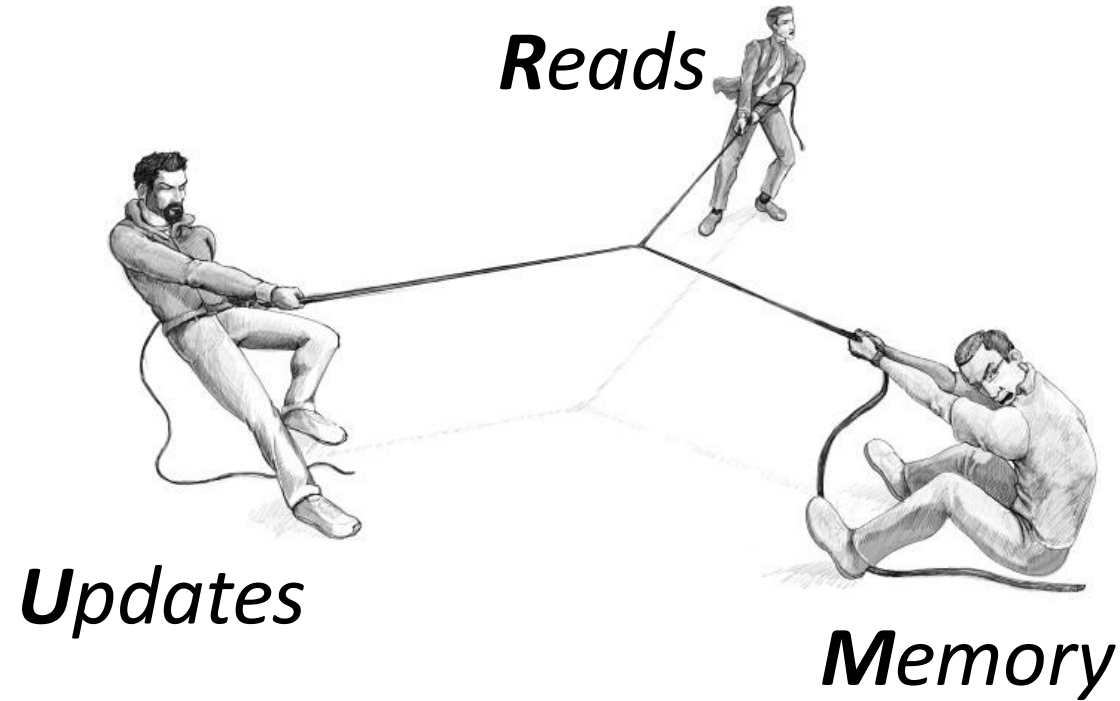
batching via buffering

should the above decisions be applied eagerly or lazily?

# how to break down the *design* in independent *dimensions*?



# data structure designs navigate a three-way tradeoff





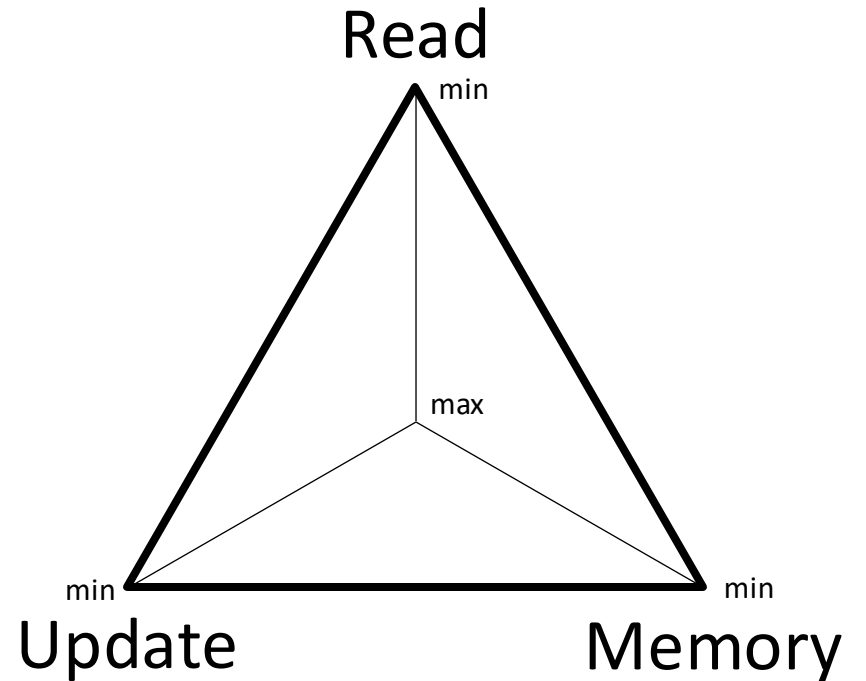
# The RUM Conjecture

every access method has a (quantifiable)

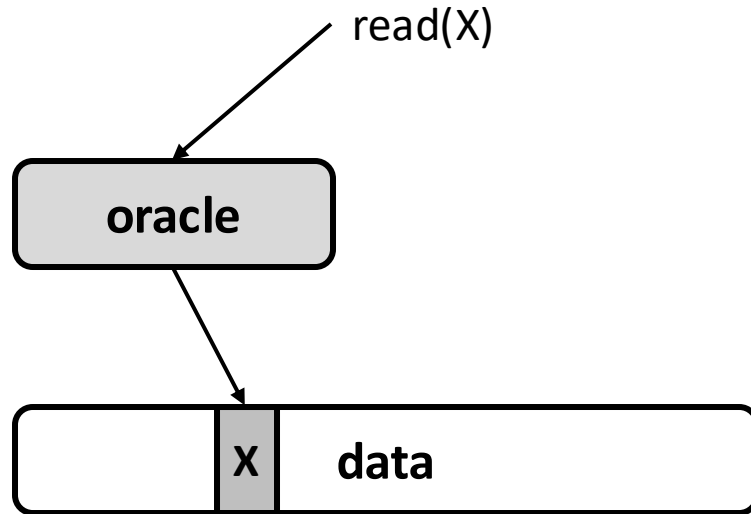
- read overhead
- update overhead
- memory overhead

the three of which form a competing triangle

*we can optimize for two of  
the overheads at the  
expense of the third*

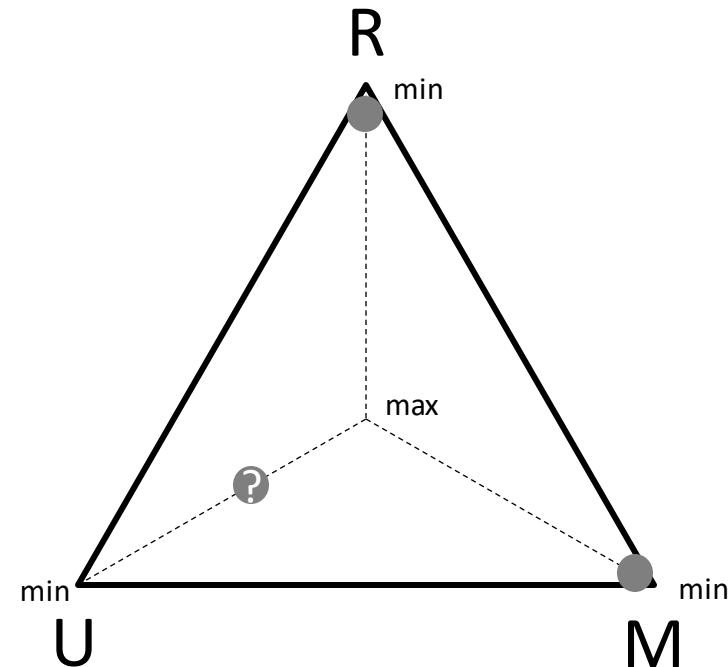


# what would be an **optimal read** behavior?

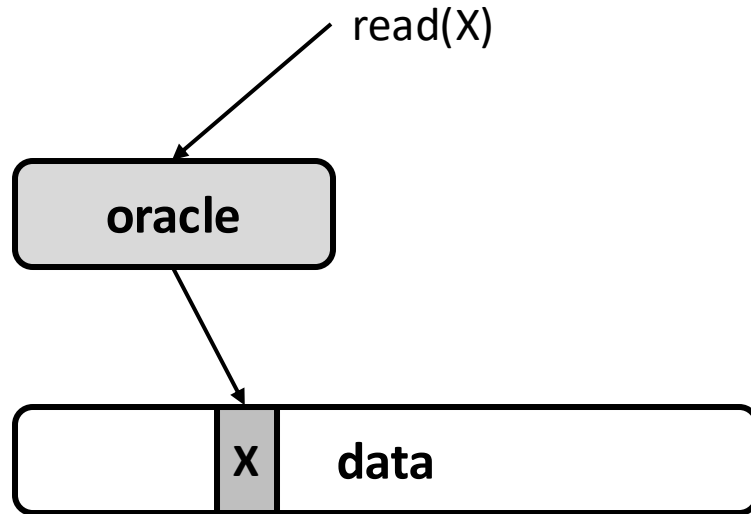


*read(x)* accesses only the bytes of object X

how *free* can an oracle be?

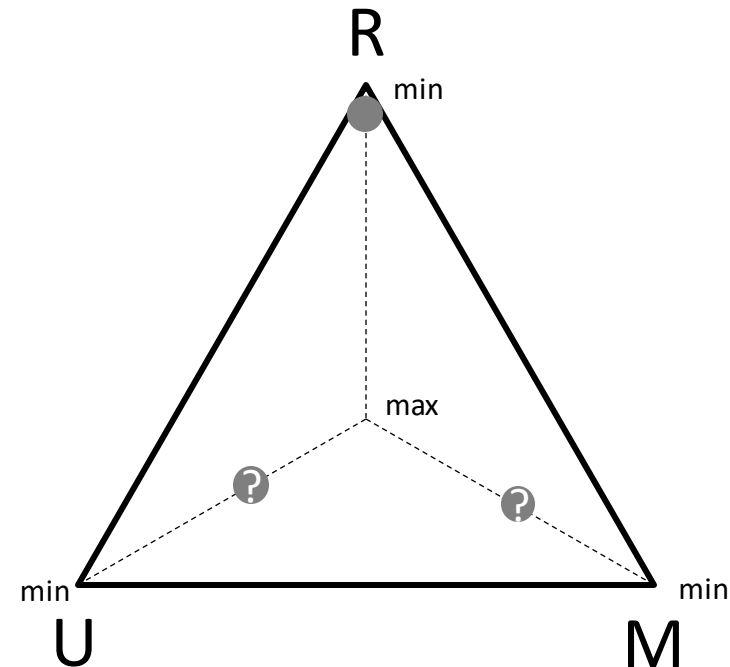


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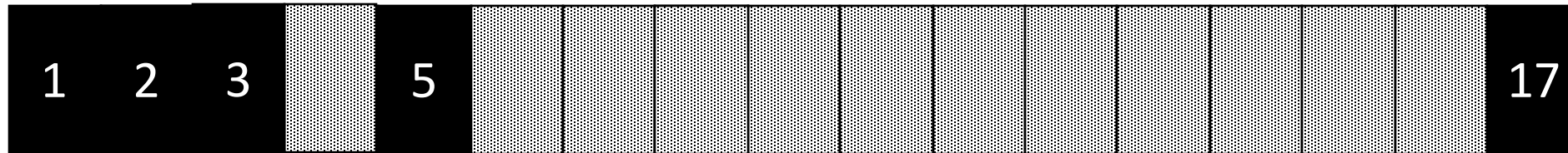


*read(x)* accesses only the bytes of object X

how *free* can an oracle be?



what would be an **optimal read** behavior?



update 17 -> 3

minimum read overhead

bound update overhead

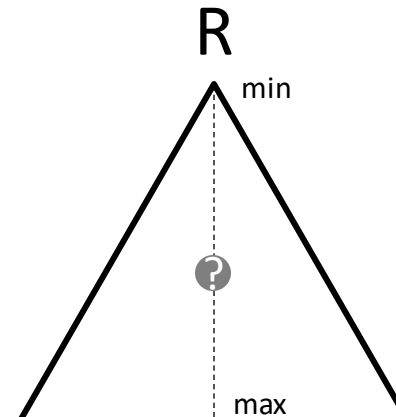
unbounded memory overhead



# what would be an **optimal update** behavior?

always *append*, and on update *invalidate*

*update (X)* changes the minimal number of bytes



what about reads?

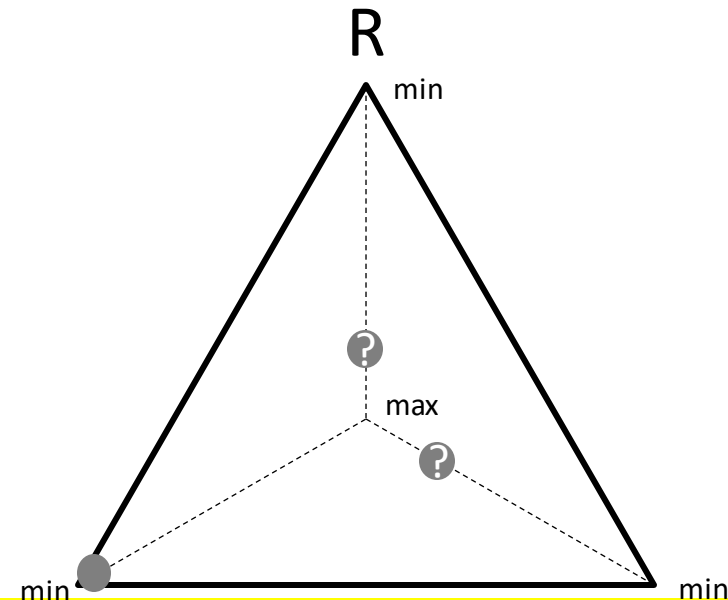
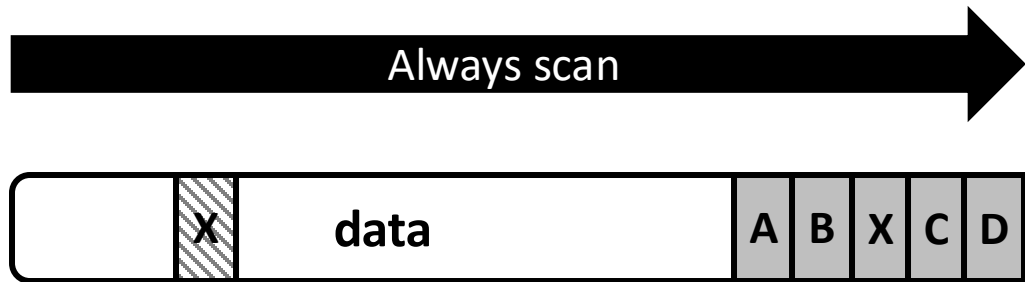


more data?

# what would be an **optimal update** behavior?

always *append*, and *invalidate* on update

*update (X)* changes the minimal number of bytes

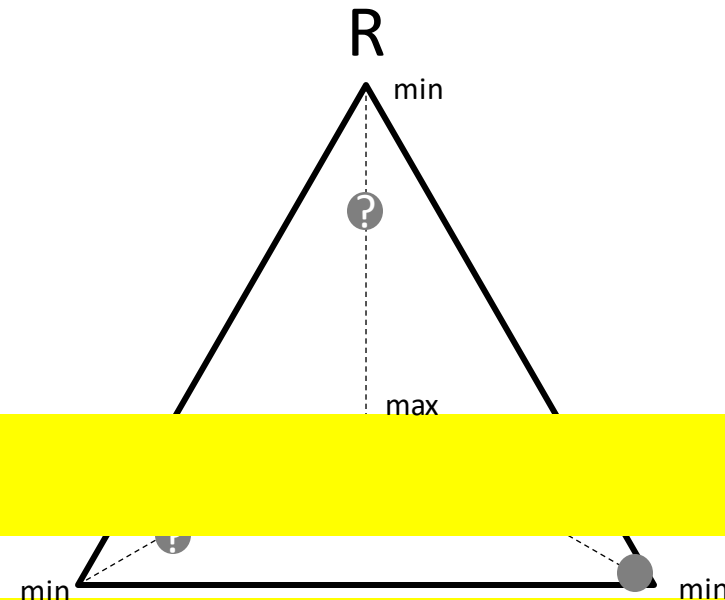
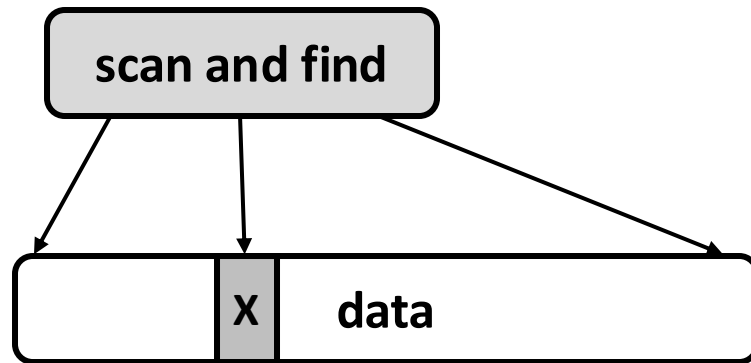


higher read and memory overhead

# what would be an **optimal memory** overhead?

*no metadata whatsoever*, would result in the smallest memory footprint

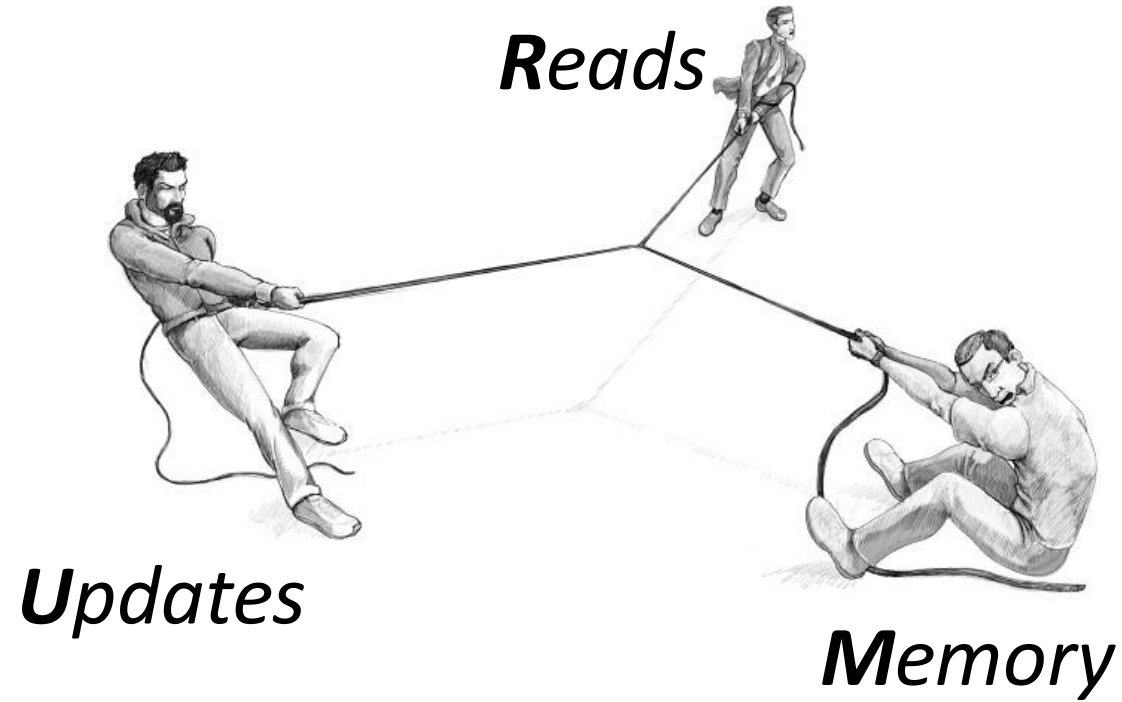
*scan and in-place updates*



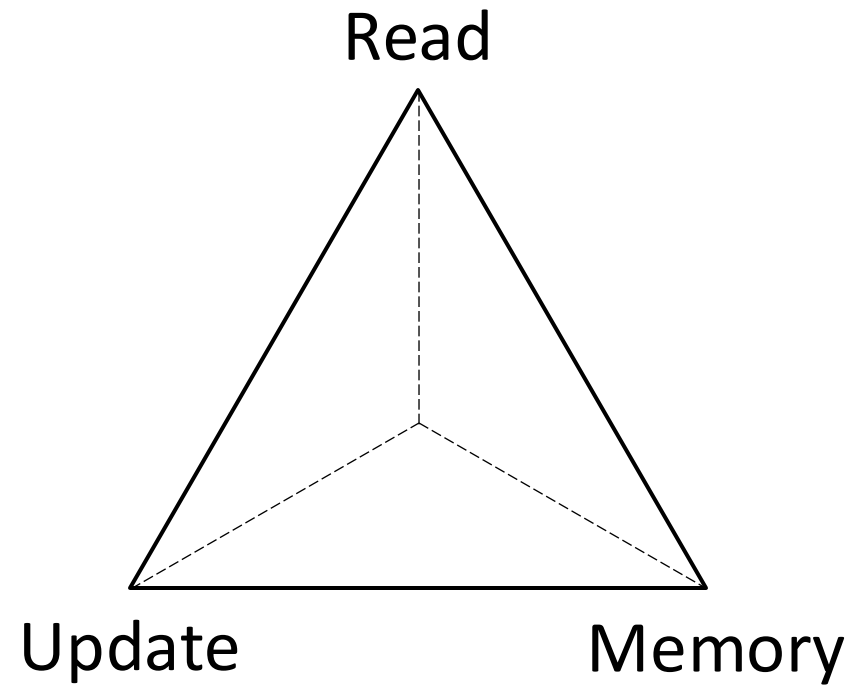
No!

do we need to reach the optimal(s)?

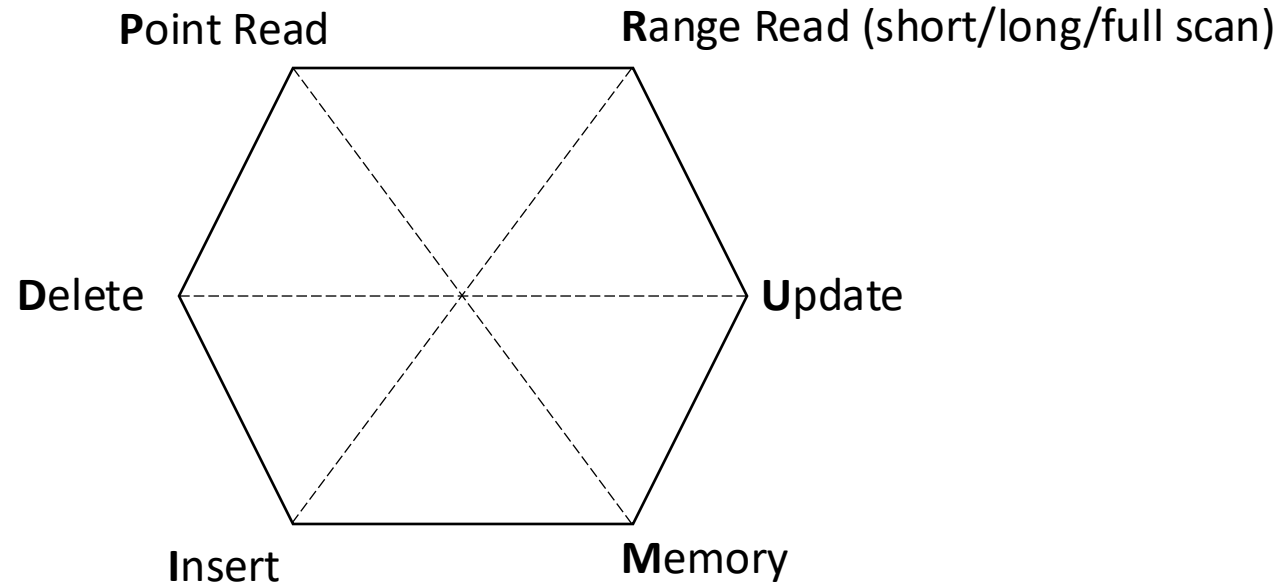
are there only three overheads?



**are there only three overheads?**



# are there only three overheads?



**PyRUMID overheads**

## data structures *design dimensions and their values*

global data organization

global search algorithm

metadata for searching

local data organization & search algorithm

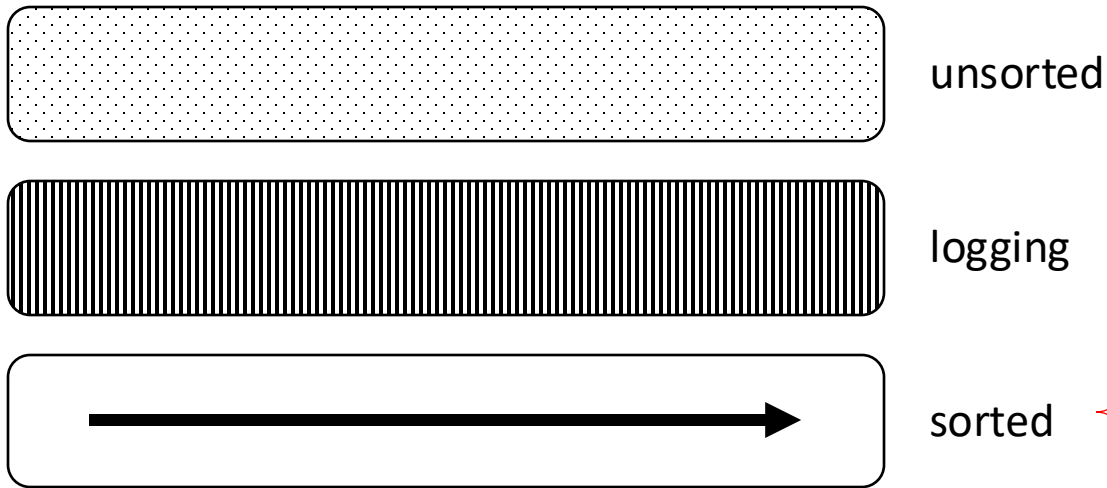
modification policy

batching via buffering

adaptivity

# global data organization

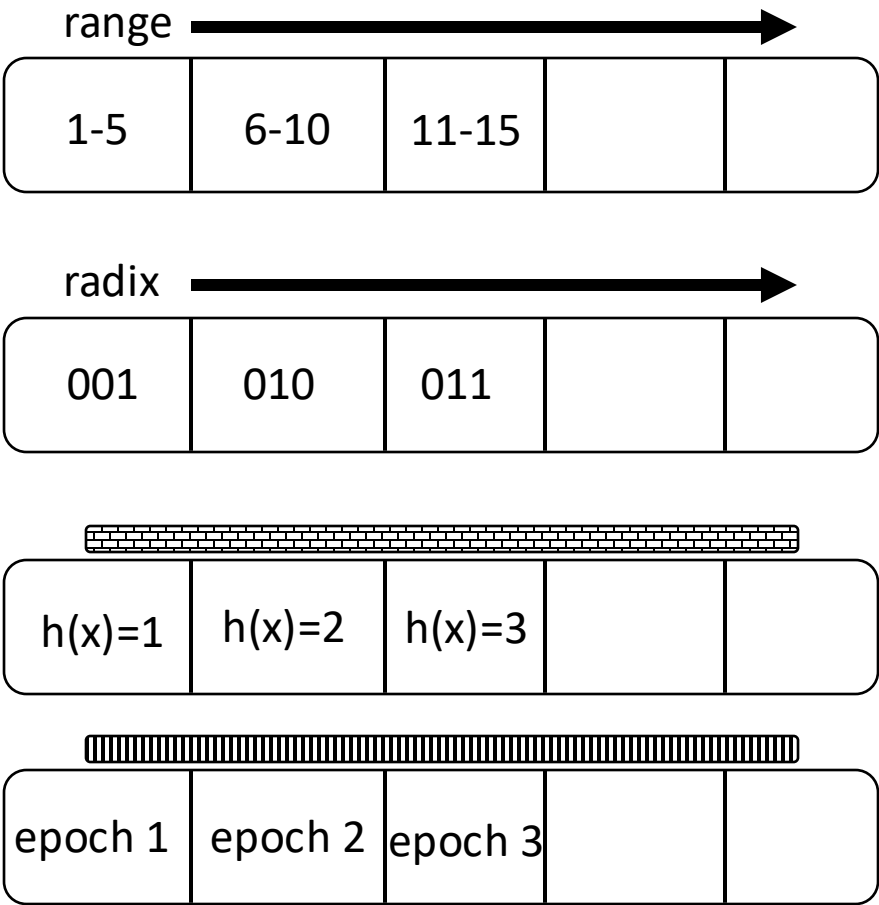
*key-level*



another decision to be made for each partition

hash partitioning  
partitioning logging

*partition-level*





# global search algorithm



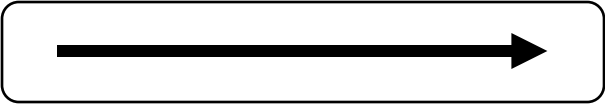
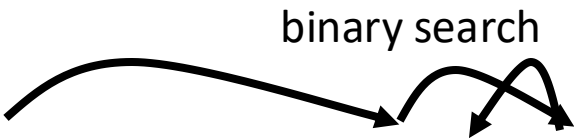
data organizations that can use it?

comments

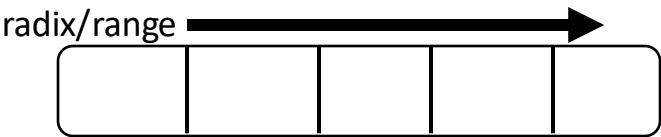


any data organization

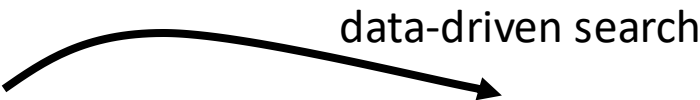
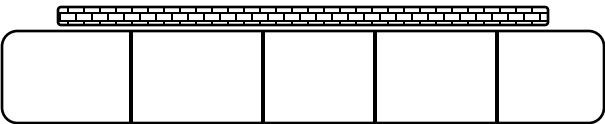
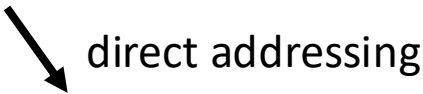
more suited for long range queries



point or range queries



more suited for point queries

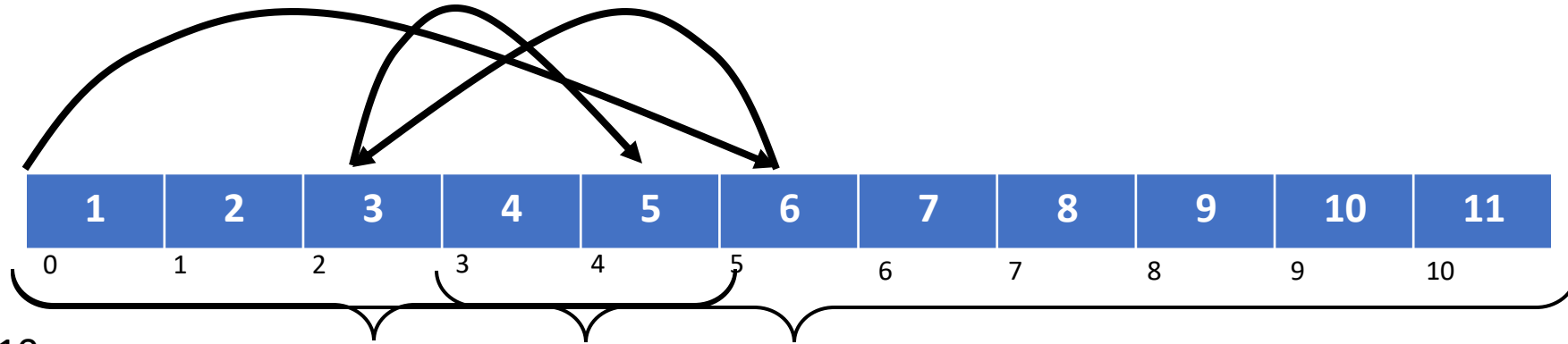


better match the data  
example: **interpolation search**



# Binary vs interpolation search

search for x=5



low = 0; high = 10;

mid = low + (high - low) / 2 = 5

val[mid] = val[5] = 6; so  $x < \text{val}[\text{mid}] \rightarrow \text{high} = \text{mid} - 1 = 4$

low = 0; high = 4;

mid = low + (high - low) / 2 = 2

val[mid] = val[2] = 3; so  $x > \text{val}[\text{mid}] \rightarrow \text{low} = \text{mid} + 1 = 3$

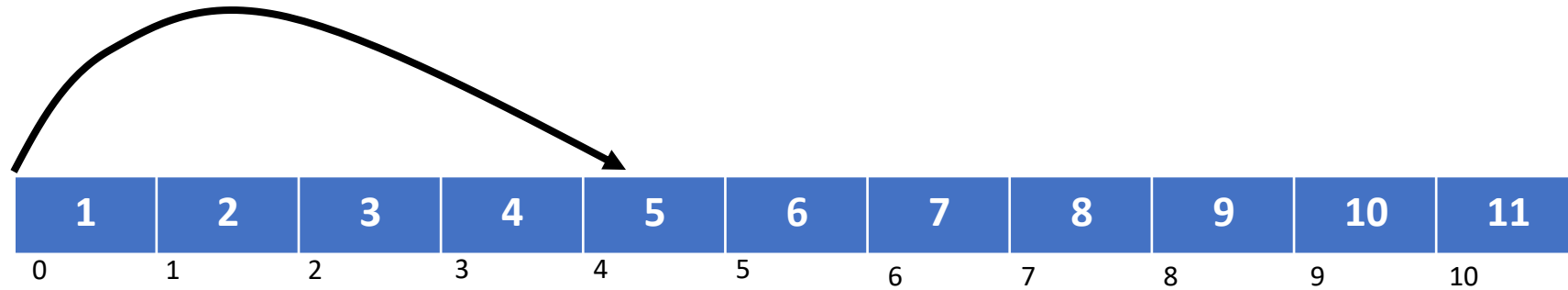
low = 3; high = 4;

mid = low + (high - low) / 2 = 3.5 (rounding to 4)

val[mid] = val[4] = 5; so  $x == \text{val}[\text{mid}] \rightarrow \text{success!!}$

# Binary vs interpolation search

search for x=5



low = 0; high = 10;

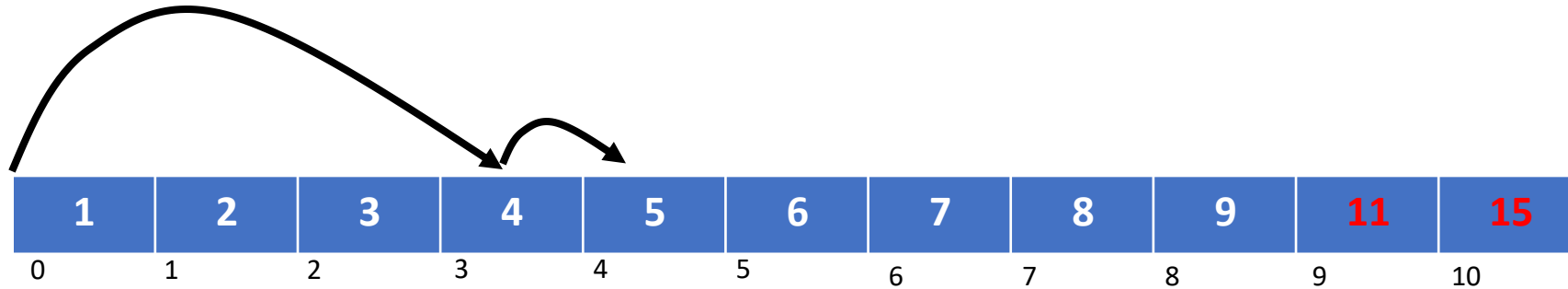
$mid = low + ((x - val[low]) * (high - low) / (val[high] - val[low])) = (5 - 1) * (10 - 0) / (11 - 1) = 4$

$val[mid] = val[4] = 5 \rightarrow$  **success!**

does it always need 1 hop?

# Binary vs interpolation search

search for x=5



low = 0; high = 10;

mid = low + ((x - val[low]) \* (high - low) / (val[high] - val[low])) = (5-1)\*(10-0)/(15-1) = (rounding to) 3

val[mid] = val[3] = 4 ; so x > val[mid] → low = mid + 1 = 4

low = 4; high = 10;

mid = low + ((x - val[low]) \* (high - low) / (val[high] - val[low])) = 4 + (5-5)\*(10-4)/(15-5) = 4

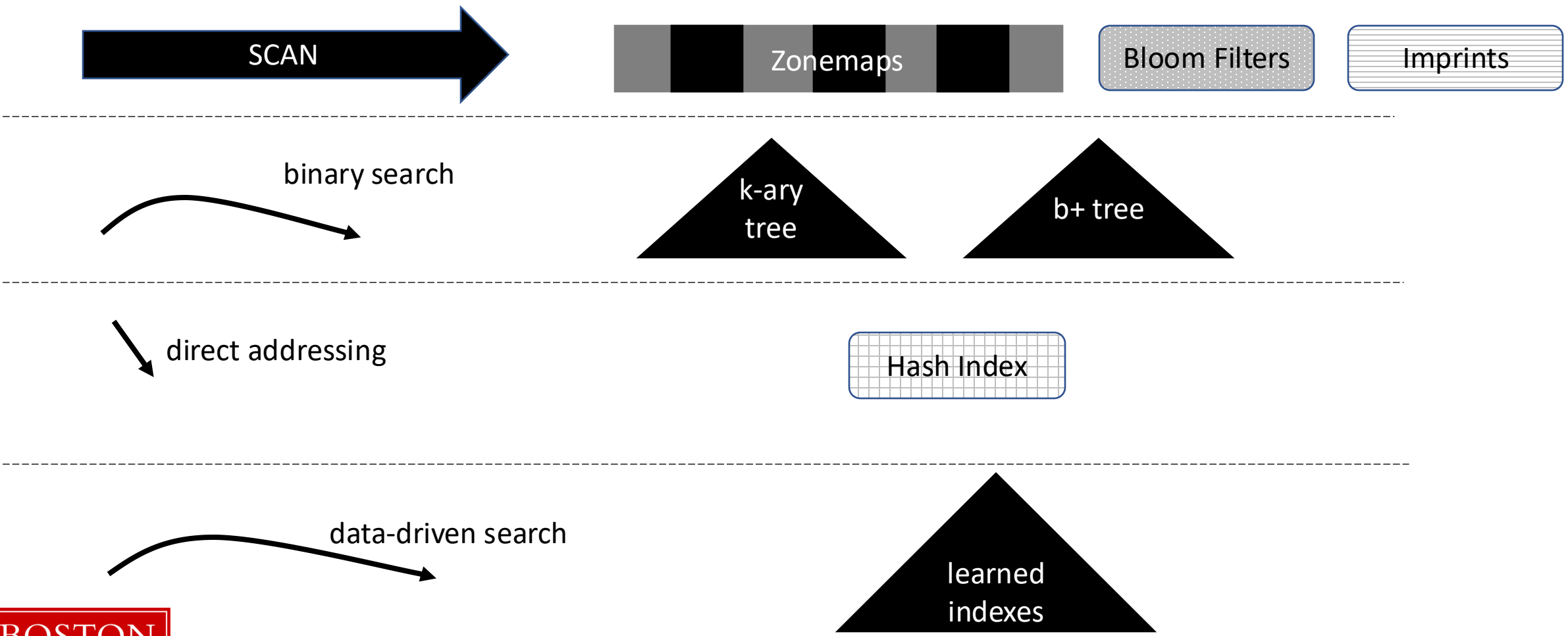
val[mid] = val[4] = 5 → success!

still better than binary!

works well with uniform distribution

# global search using metadata (indexing)

every search algorithm can be materialized and further optimized using indexing



# Imprints

similar to zonemaps

Z1: [32,72]	Z2: [13,45]	Z3: [1,10]	Z4: [21,99]	Z5: [28,35]	Z6: [5,12]
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storing a simplified histogram for each block

why?



it can capture better range queries and avoid useless overlap

Scan vs. Index

$$APS \text{ ratio} = \frac{\text{Index Cost}}{\text{Scan Cost}}$$

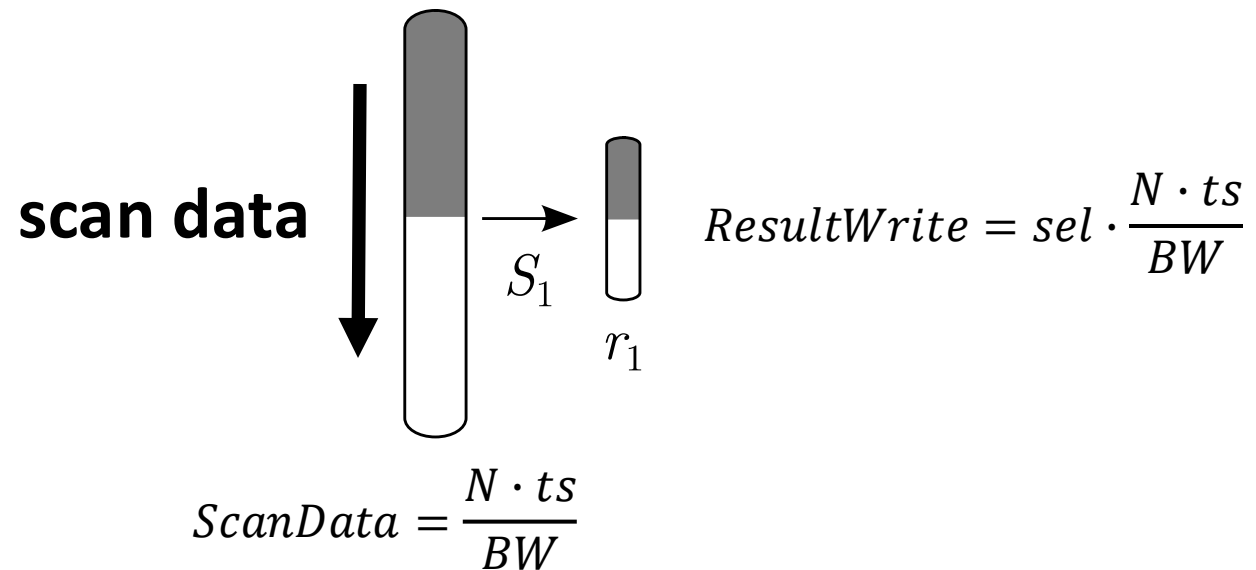
> 1 scan  
< 1 index

let us model



# access path selection *modeling*

$N$  rows  
 $ts$  tuple size  
 $BW$  memory bandwidth  
 $sel$  query selectivity



scan  
→

data

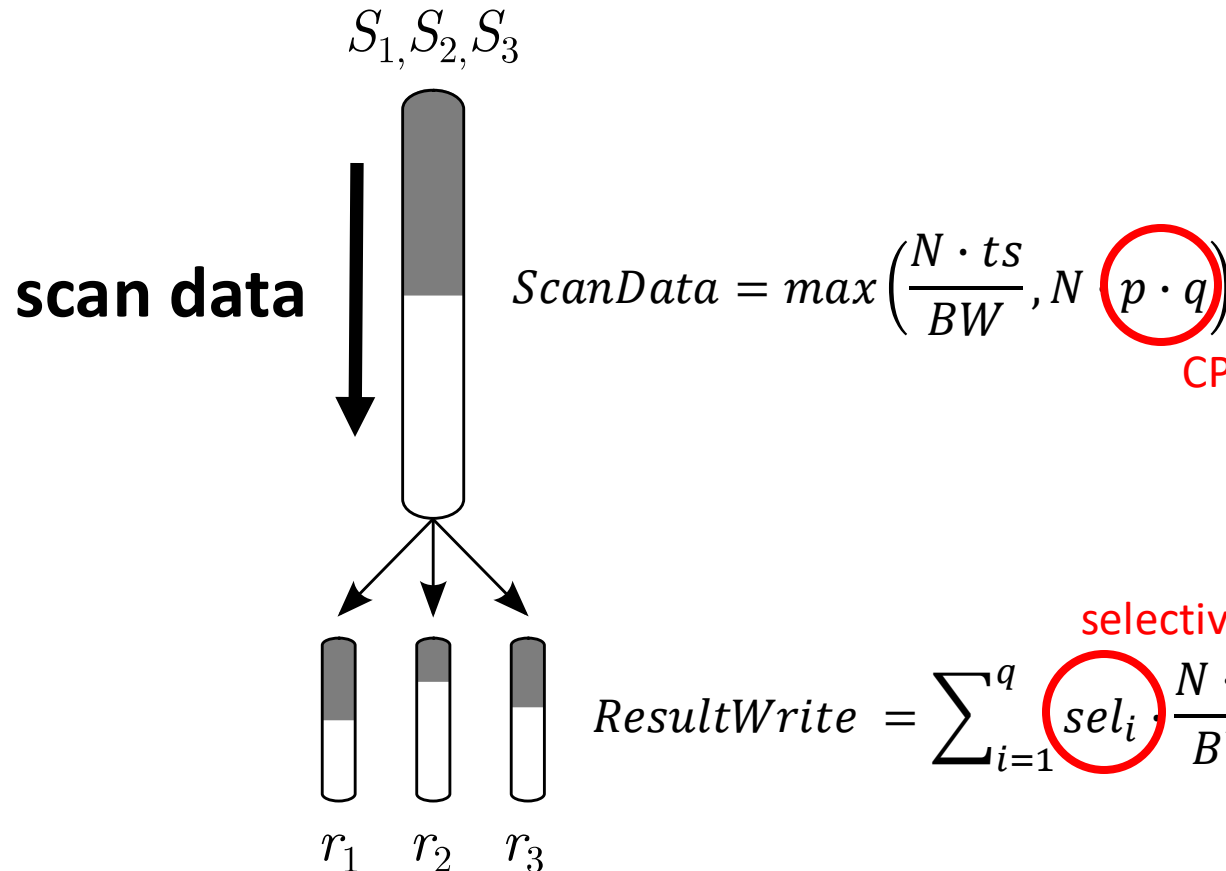
what if we have  $q$  queries?



# access path selection *modeling*

*q concurrent queries*

$N$	rows
$ts$	tuple size
$BW$	memory bandwidth
$sel$	query selectivity
$q$	queries
$p$	predicate evaluation cost (CPU)



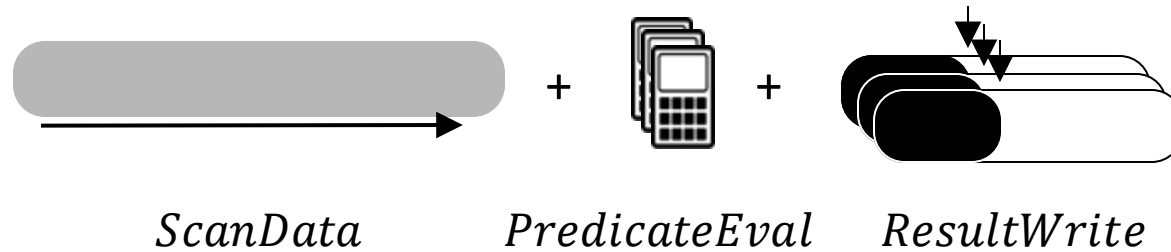
CPU cost per query (may be too high!)

selectivity per query

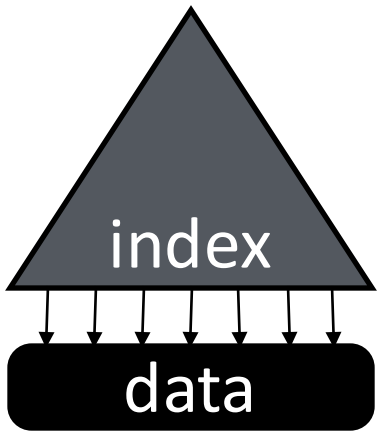
$$ResultWrite = \sum_{i=1}^q \text{selectivity per query} \cdot \frac{N \cdot ts}{BW} = S_q \cdot \frac{N \cdot ts}{BW}, \quad s.t. \quad S_q = \sum_{i=1}^q sel_i$$

total query selectivity (sum)

$$ScanCost = \max\left(\frac{N \cdot ts}{BW}, N \cdot p \cdot q\right) + S_q \cdot \frac{N \cdot ts}{BW}$$



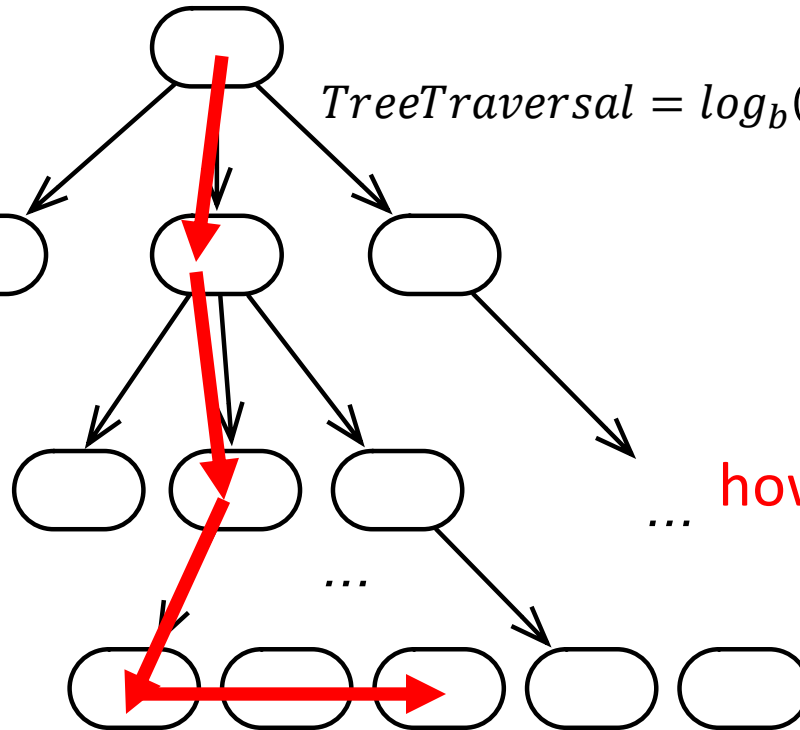
# access path selection *modeling*



**traverse tree**

$$TreeTraversal = \log_b(N) \cdot C_M$$

**traverse  
leaves+data**



**result write**



$N$	rows
$ts$	tuple size
$BW$	memory bandwidth
$sel$	query selectivity
$q$	queries
$p$	predicate evaluation cost (CPU)
$b$	branching factor (fanout)
$C_M$	cache miss latency
$its$	index tuple size

how to make index drop-in replacement?

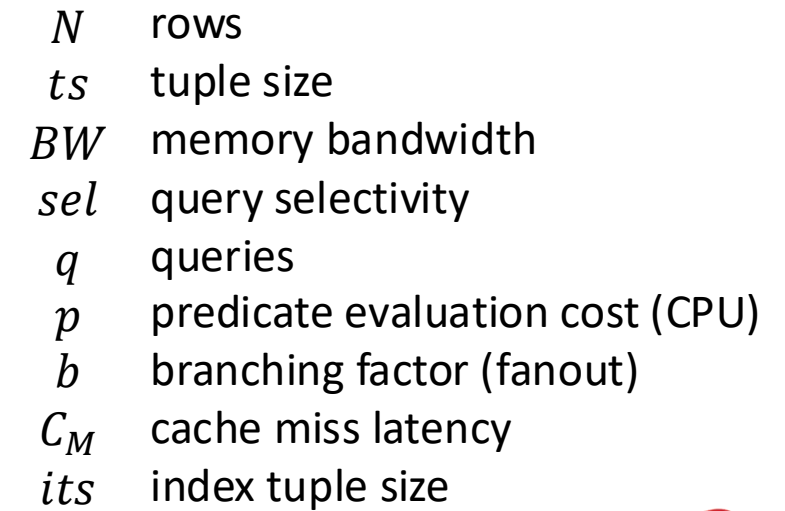


$$LeavesTraversal = sel \cdot \frac{N}{b} \cdot C_M$$

$$DataTraversal = sel \cdot \frac{N \cdot its}{BW}$$

$$ResultWrite = sel \cdot \frac{N \cdot ts}{BW}$$

A diagram illustrating an index structure. At the top is a gray triangle with a black outline, containing the word "index" in white. Below the triangle is a black rounded rectangle with a black outline, containing the word "data" in white. Seven black arrows point downwards from the base of the triangle to the top edge of the rectangle, representing the mapping from index entries to data records.



## how to make index drop-in replacement?



$$DataTraversal = sel \cdot \frac{N \cdot its}{BW}$$

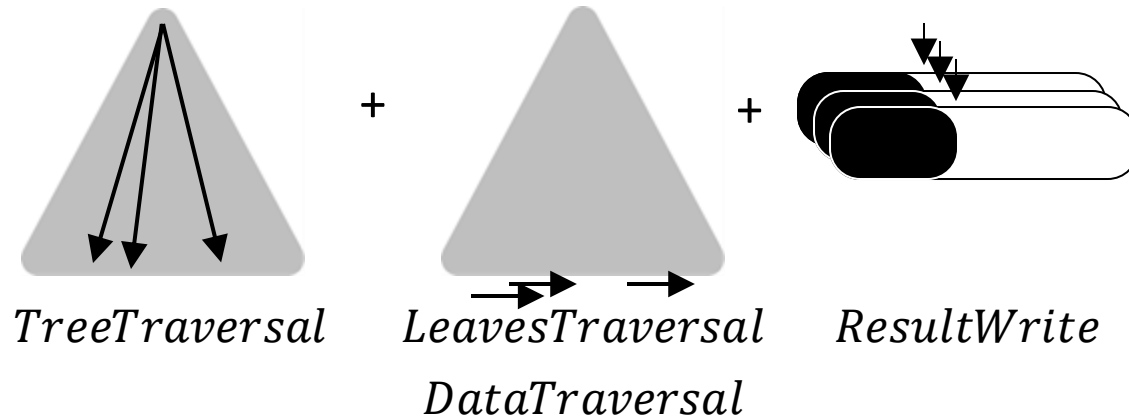
$$ResultWrite = sel \cdot \frac{N \cdot ts}{BW}$$

$$Sort = sel \cdot N \cdot \log_2(sel \cdot N) \cdot \frac{its}{BW} \quad \text{sort}$$

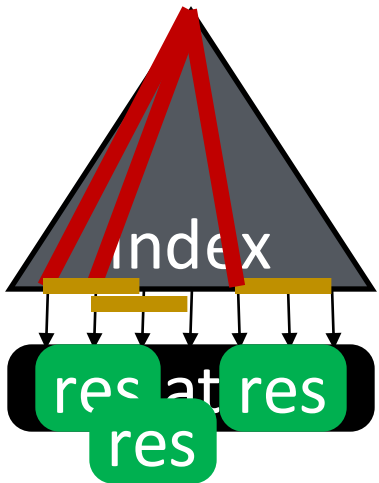
## result write



$$IndexCost = q \cdot \log_b(N) \cdot C_M + S_q \cdot \left( \frac{N}{b} \cdot C_M + \frac{N \cdot its}{BW} + \frac{N \cdot ts}{BW} \right)$$



# *new* access path selection



$$\text{APS ratio} = \frac{q \cdot \text{TreeTraversal} + S_q \cdot (\text{LeavesTraversal} + \text{DataTraversal} + \text{ResultWrite})}{\max(\text{ScanData}, \text{CPU cost of } q \text{ predicates}) + S_q \cdot \text{ResultWrite}}$$

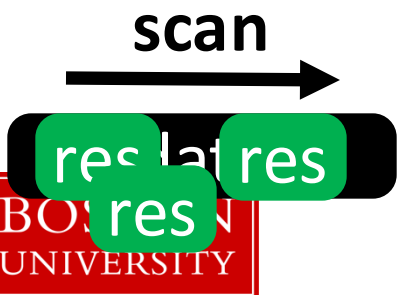
Diagram illustrating the APS ratio formula with annotations:

- index design** (red arrow) points to the  $q \cdot \text{TreeTraversal}$  term.
- data size & hardware** (yellow arrow) points to the  $S_q \cdot (\text{LeavesTraversal} + \text{DataTraversal} + \text{ResultWrite})$  term.
- data size (for result)** (green arrow) points to the  $S_q \cdot \text{ResultWrite}$  term.

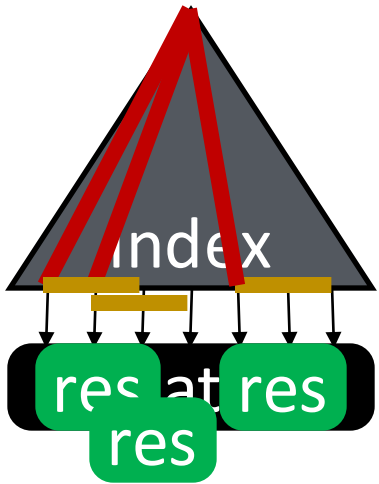
Dynamic Parameter

$q$  #concurrent read queries

$S_q$  sum of query selectivity of  $q$  queries  $S_q = \sum_{i=1}^q sel_i$



# *new* access path selection



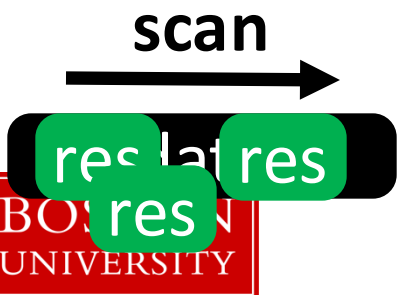
$N$	rows
$ts$	tuple size
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$sel$	query selectivity
$q$	queries
$p$	predicate evaluation cost (CPU)
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$its$	index tuple size

$$APS \text{ ratio} = \frac{q \cdot \log_b(N) \cdot C_M + S_q \cdot \left( \frac{N}{b} \cdot C_M + \frac{N \cdot its}{BW} + \frac{N \cdot ts}{BW} \right)}{\max \left( \frac{N \cdot ts}{BW}, q \cdot p \cdot N \right) + S_q \cdot \frac{N \cdot ts}{BW}}$$

## Dynamic Parameter

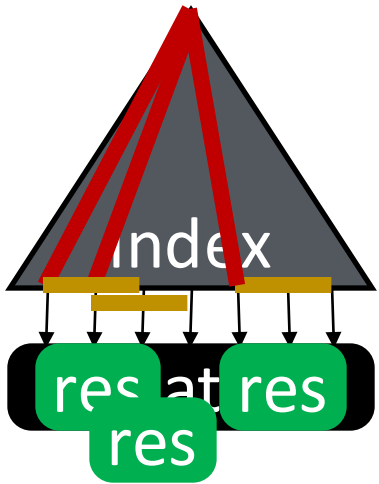
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# *new* access path selection



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$b$	branching factor (fanout)
$C_M$	cache miss latency
$its$	index tuple size

$$APS \text{ ratio} = \frac{q \cdot \left( \frac{\log_b(N)}{N} \cdot BW \cdot C_M + S_q \cdot \left( \frac{BW \cdot C_M}{b} + (ts + its) \right) \right)}{\max(ts \cdot q, p \cdot BW) + S_q \cdot ts}$$

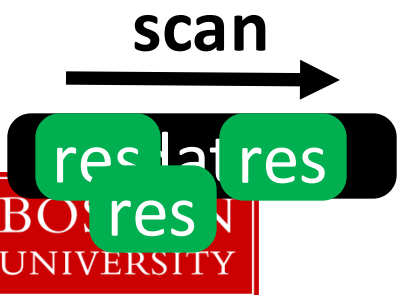
Diagram annotations for the APS ratio formula:

- index design** (purple arrows) points to  $\log_b(N)$  and  $b$ .
- hardware characteristics** (red circles) points to  $BW \cdot C_M$  and  $p \cdot BW$ .
- data layout** (yellow circles) points to  $(ts + its)$  and  $ts$ .

## Dynamic Parameter

$q$  #concurrent read queries

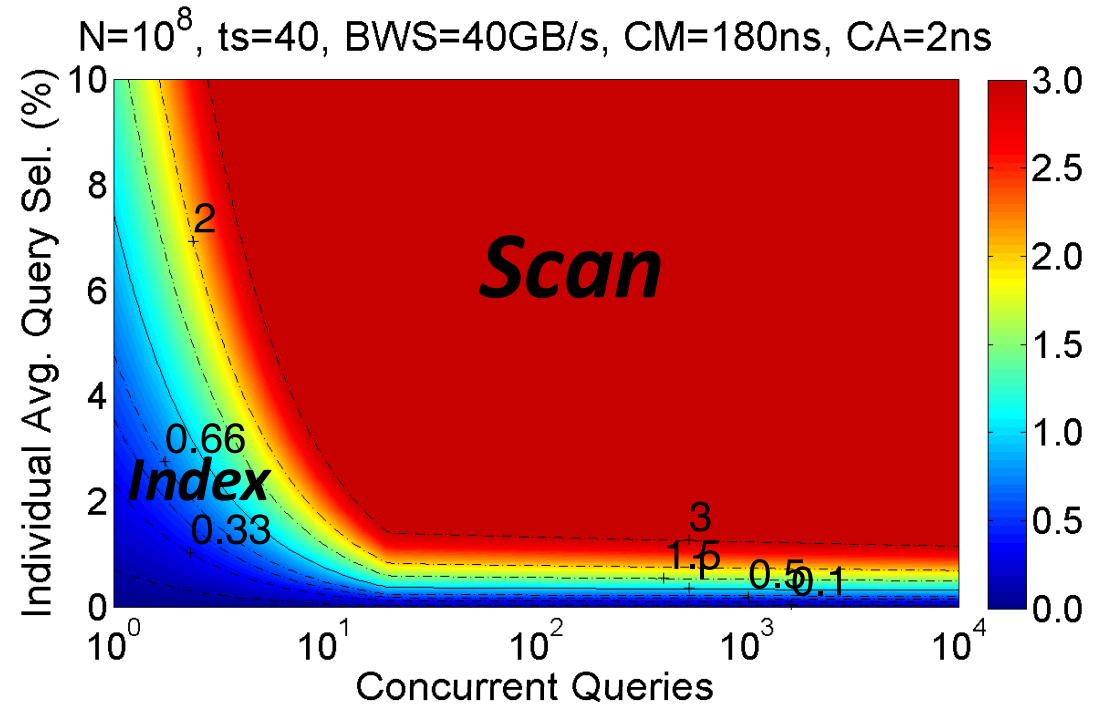
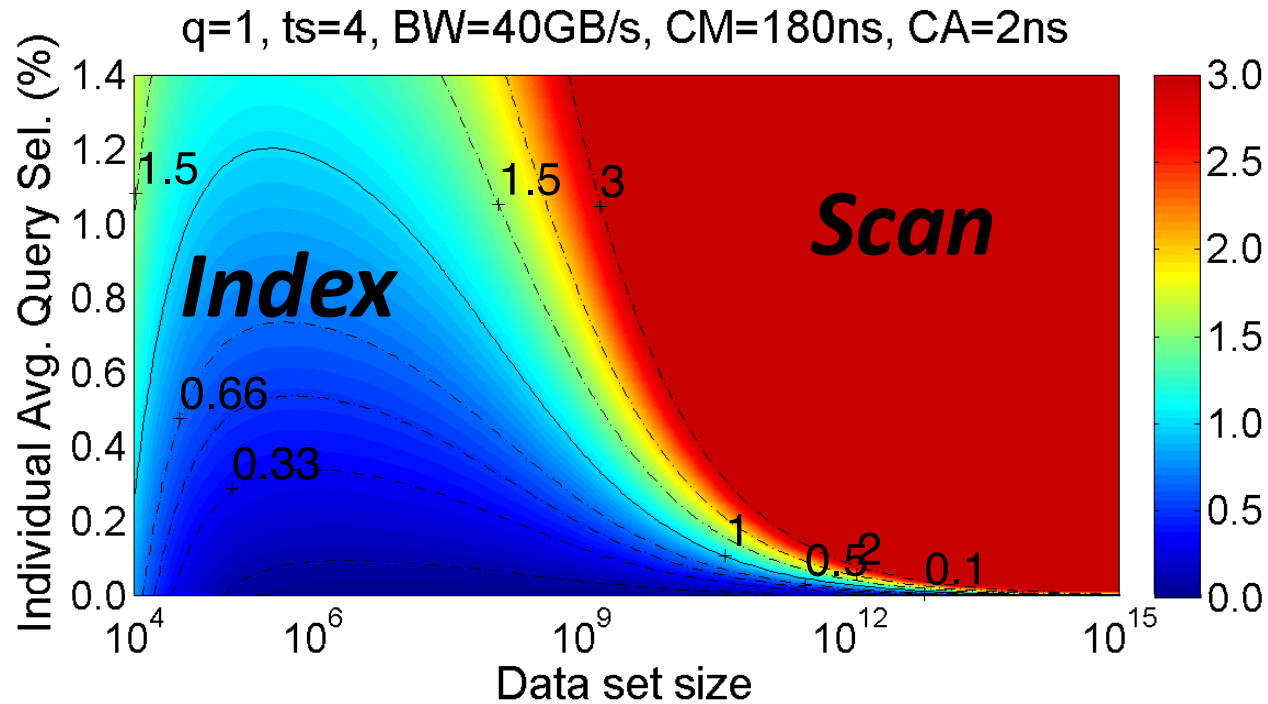
$S_q$  sum of query selectivity of  $q$  queries  $S_q = \sum_{i=1}^q sel_i$



# *new* access path selection

## *Modeling*

high selectivity → scan



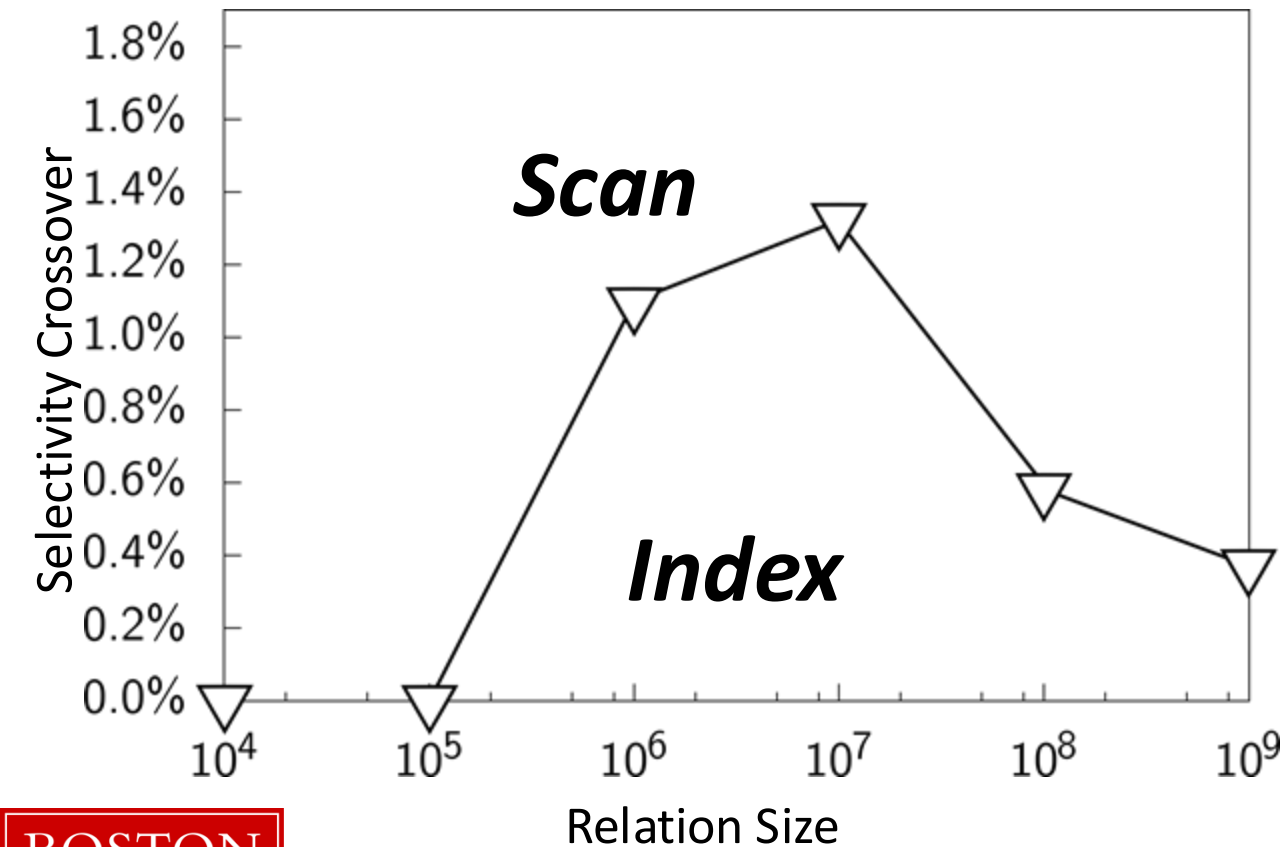
large data → scan

high concurrency matters → scan

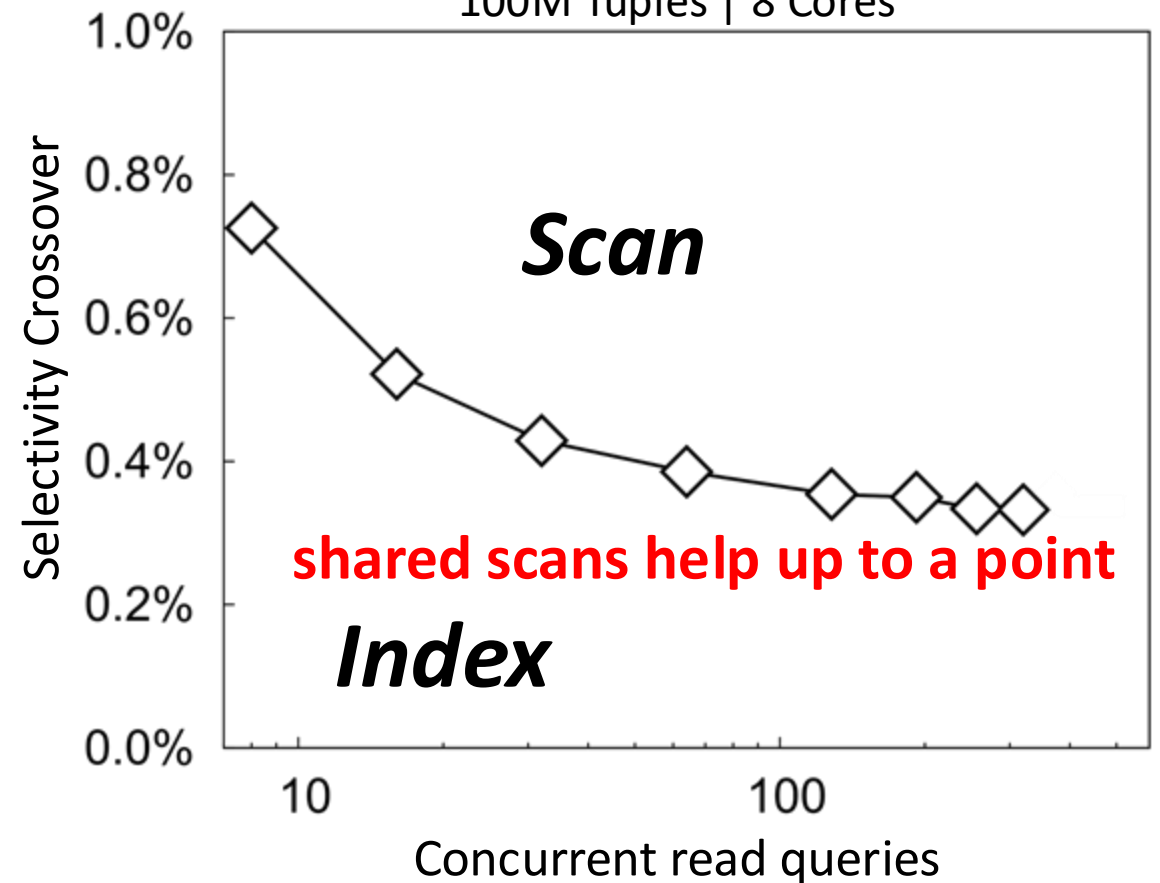
# *new* access path selection

## *Experiments*

8 Queries | 8 Cores



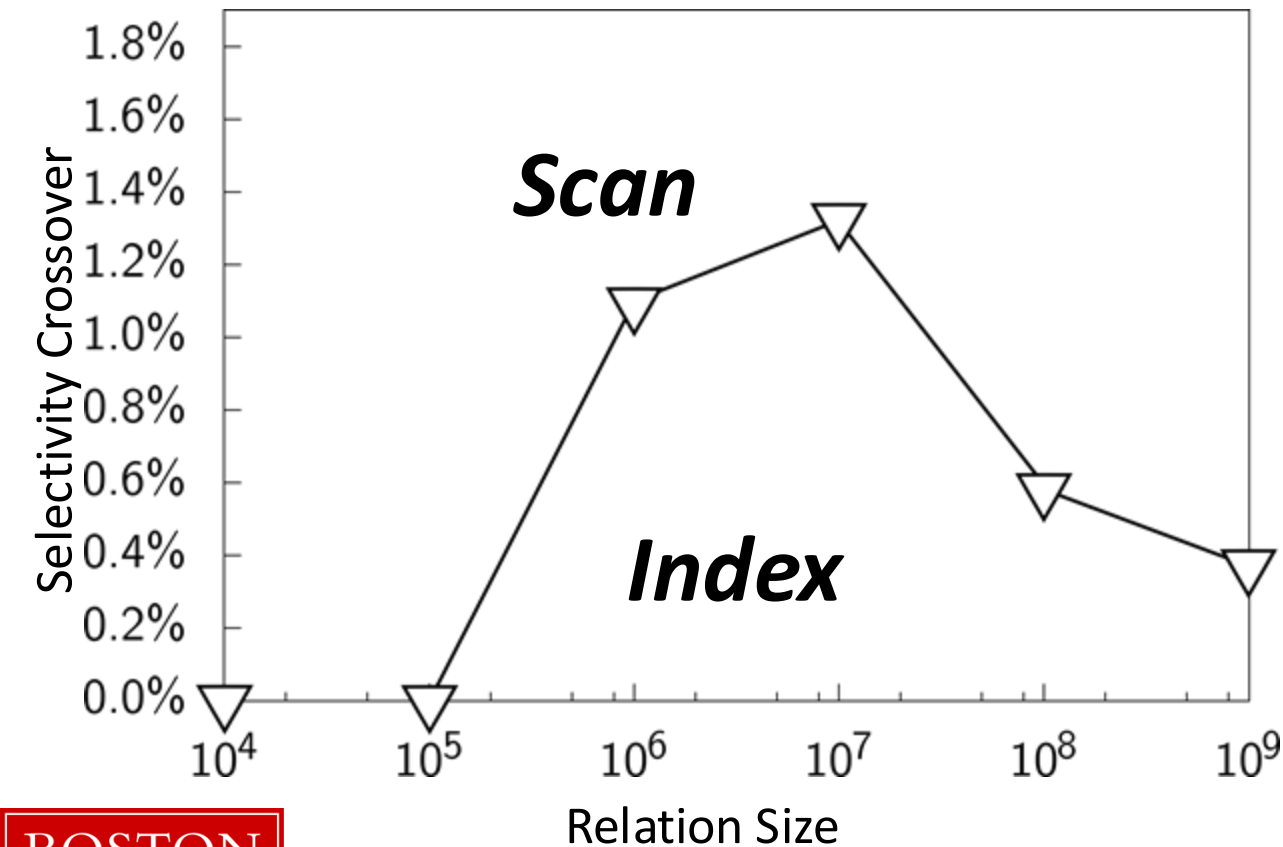
100M Tuples | 8 Cores



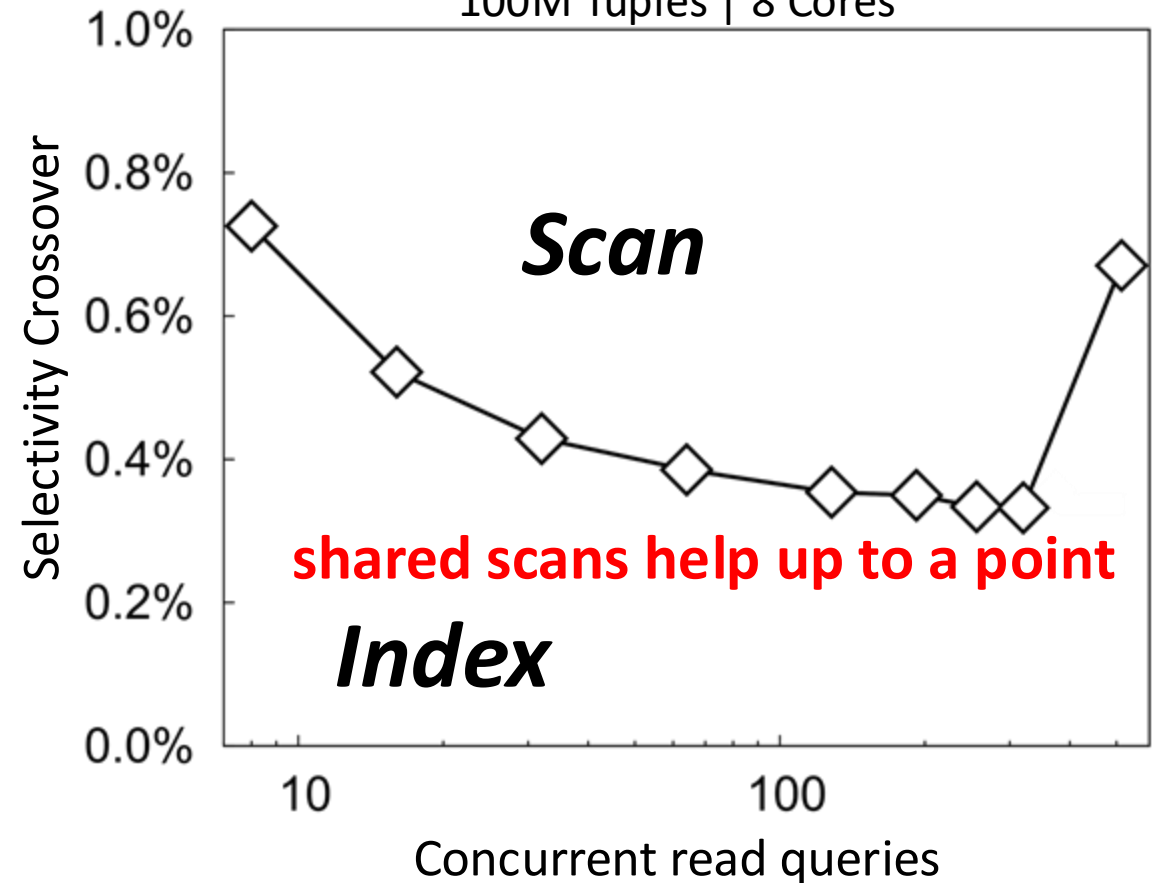
# *new* access path selection

## *Experiments*

8 Queries | 8 Cores



100M Tuples | 8 Cores

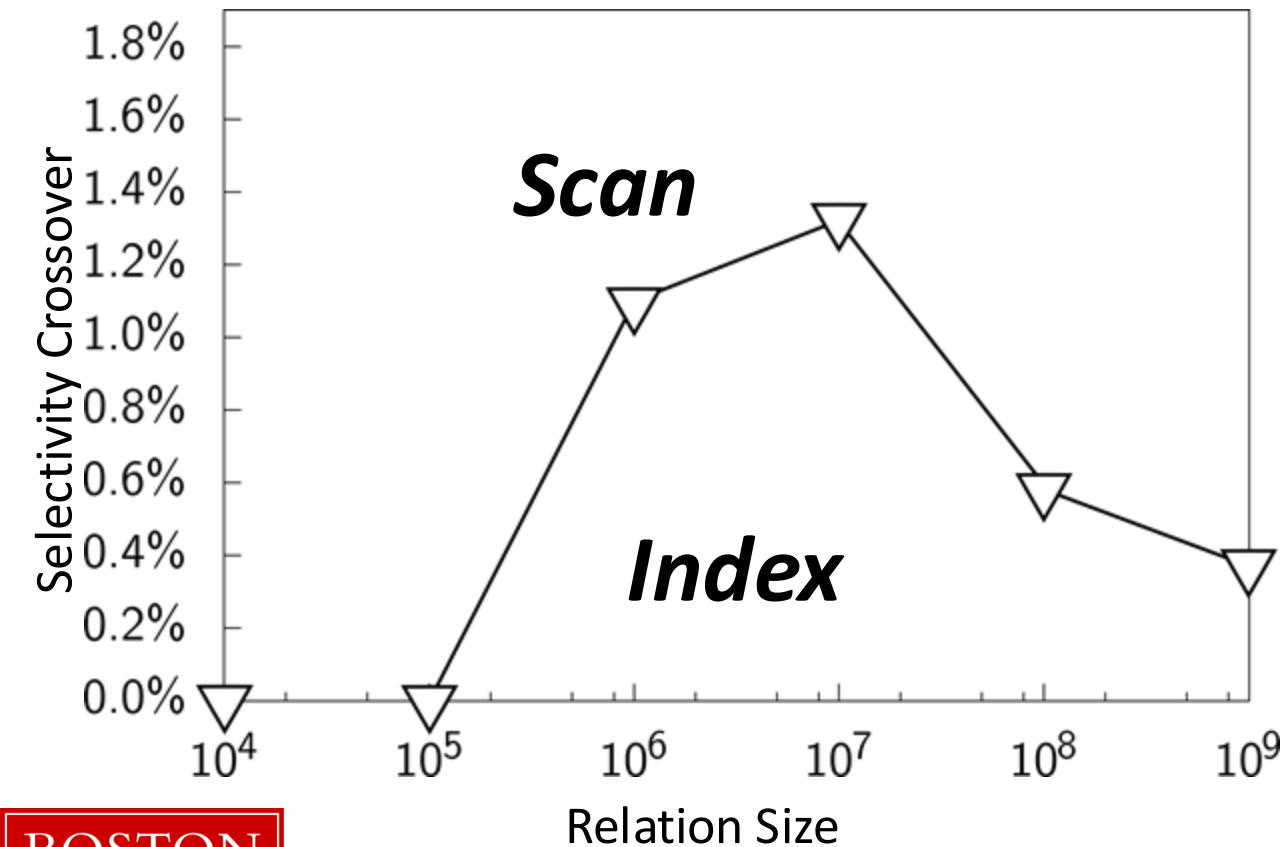


shared scans help up to a point

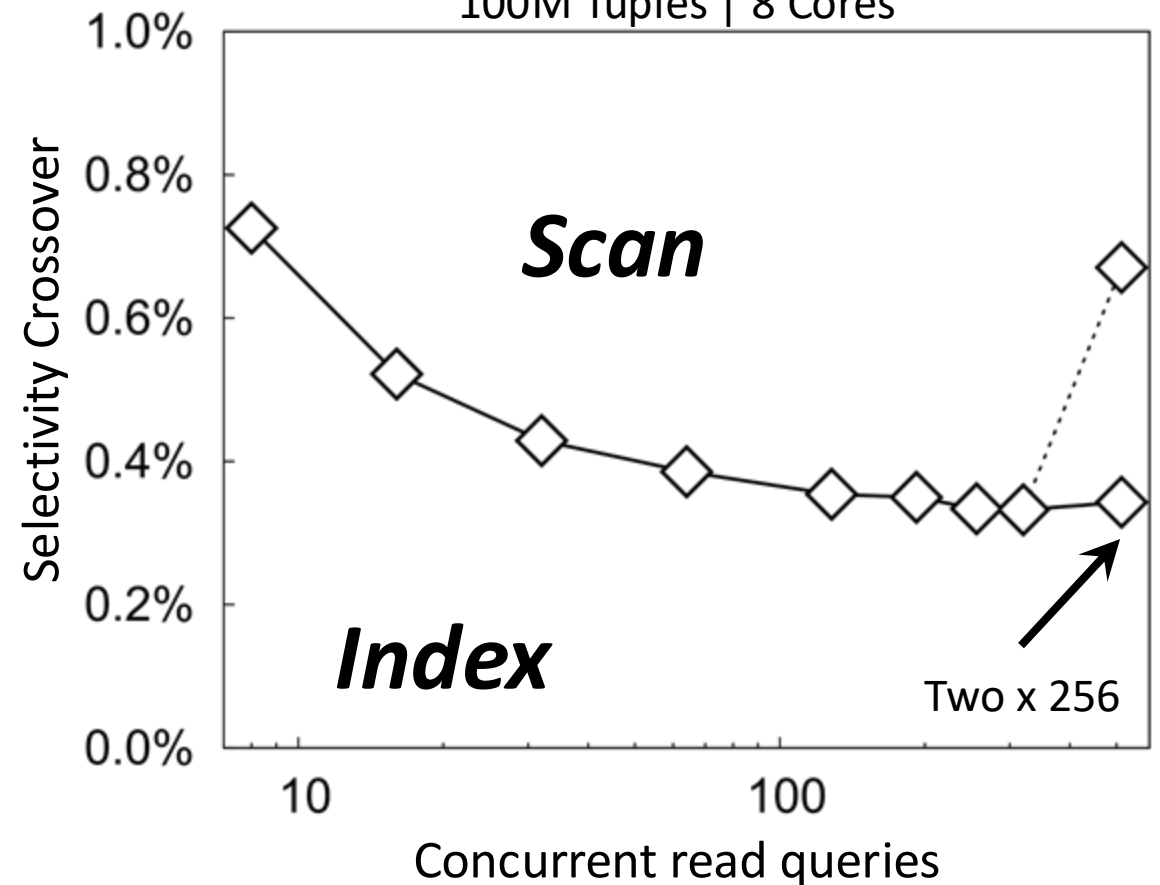
# *new* access path selection

## *Experiments*

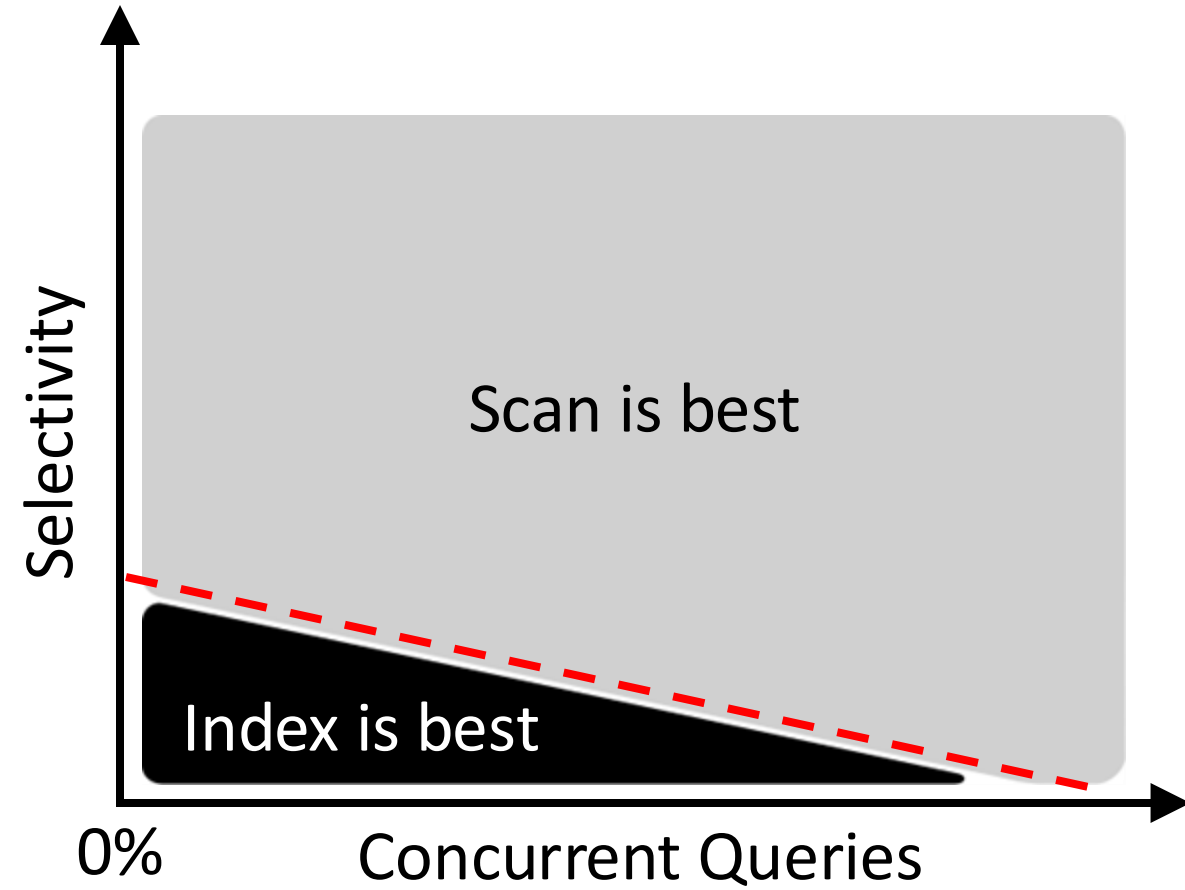
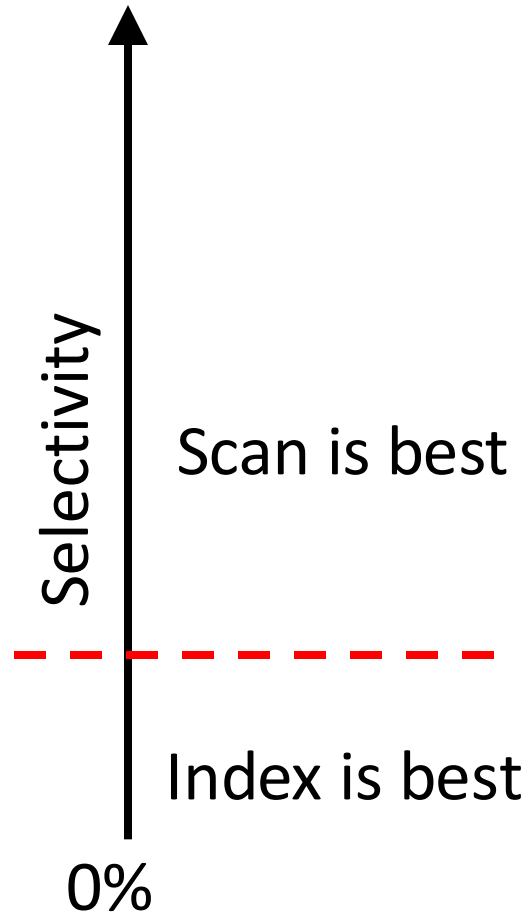
8 Queries | 8 Cores

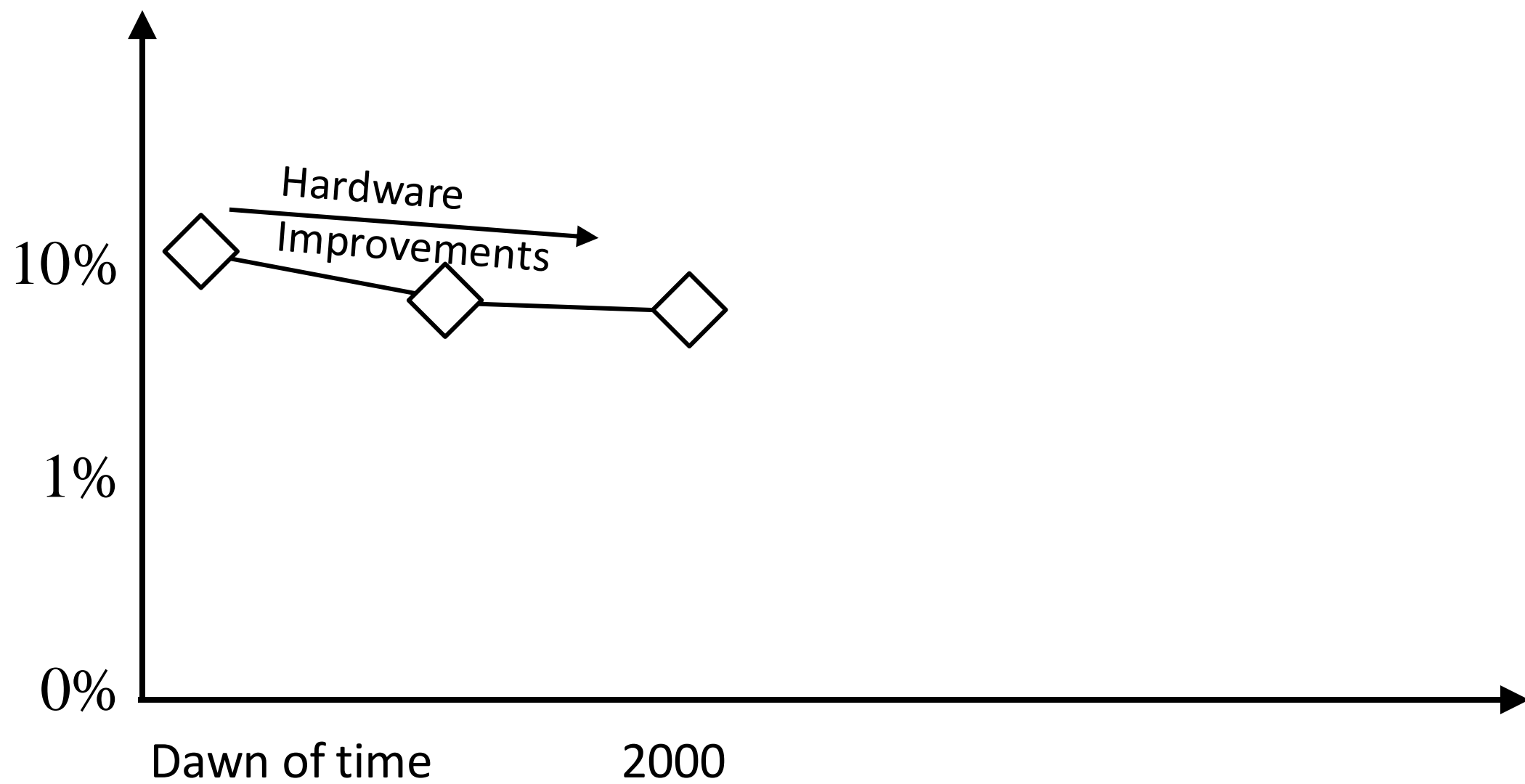


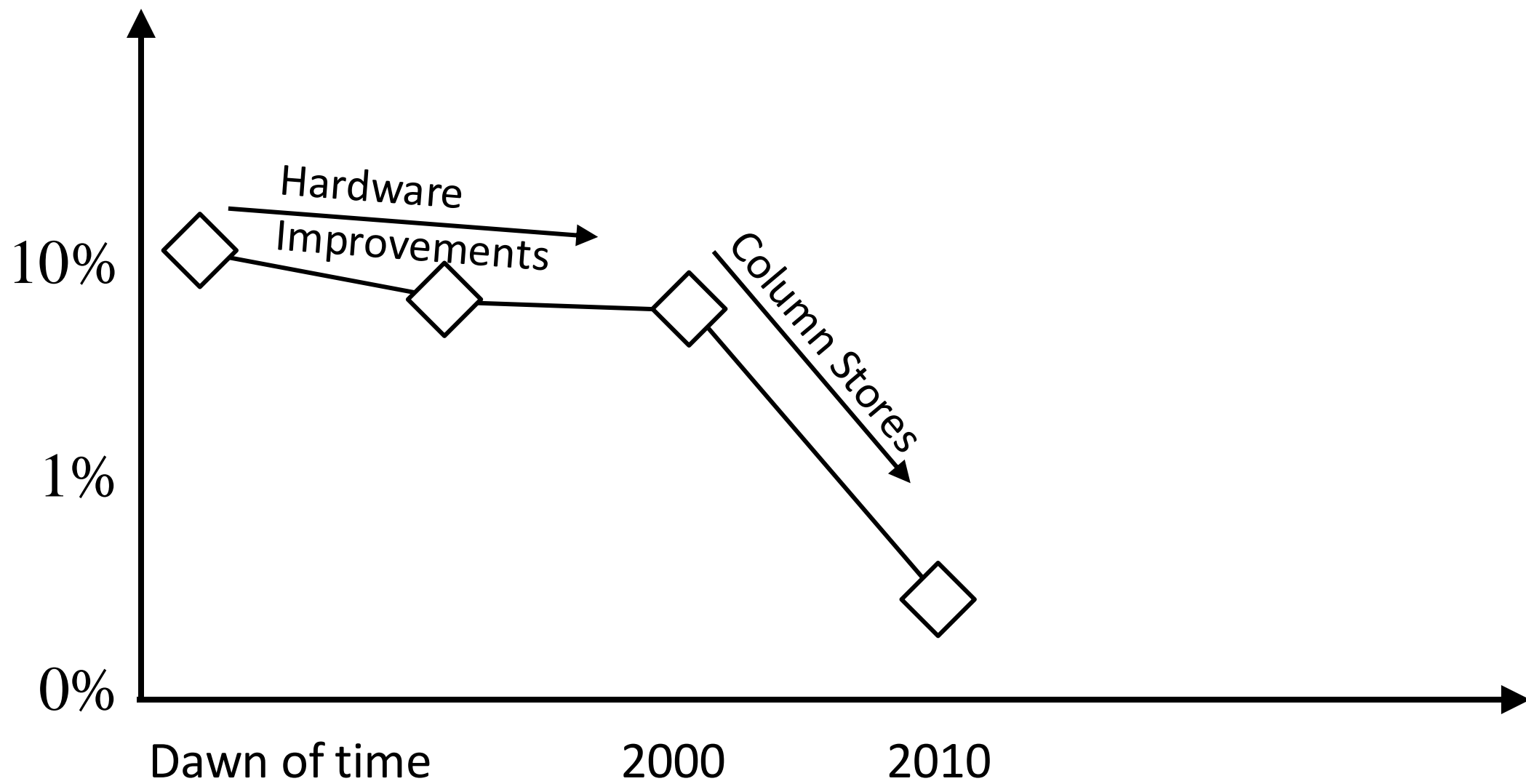
100M Tuples | 8 Cores



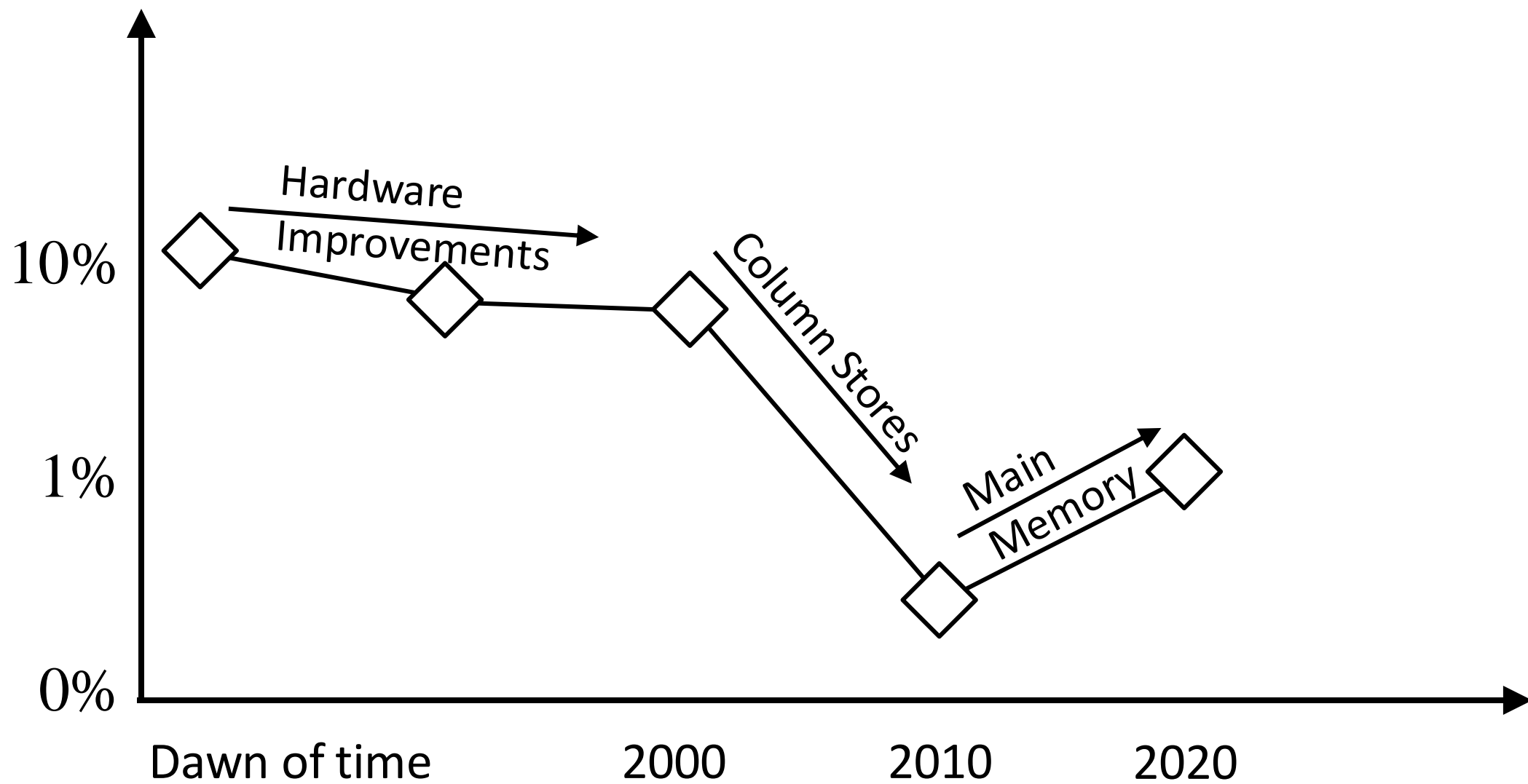
# *new* access path selection



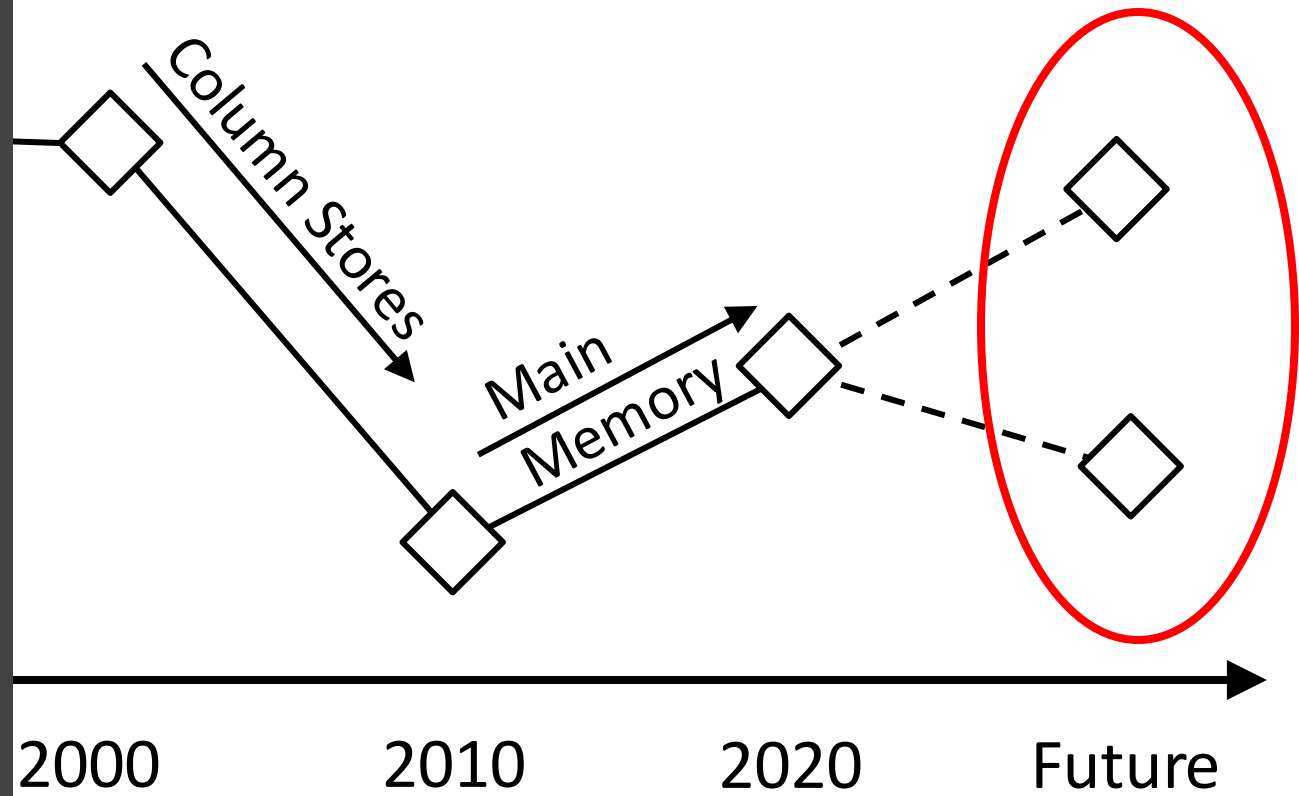








# “What-if” questions



# data structures *design dimensions and their values*

global data organization

global search algorithm

metadata for searching

local data organization & search algorithm

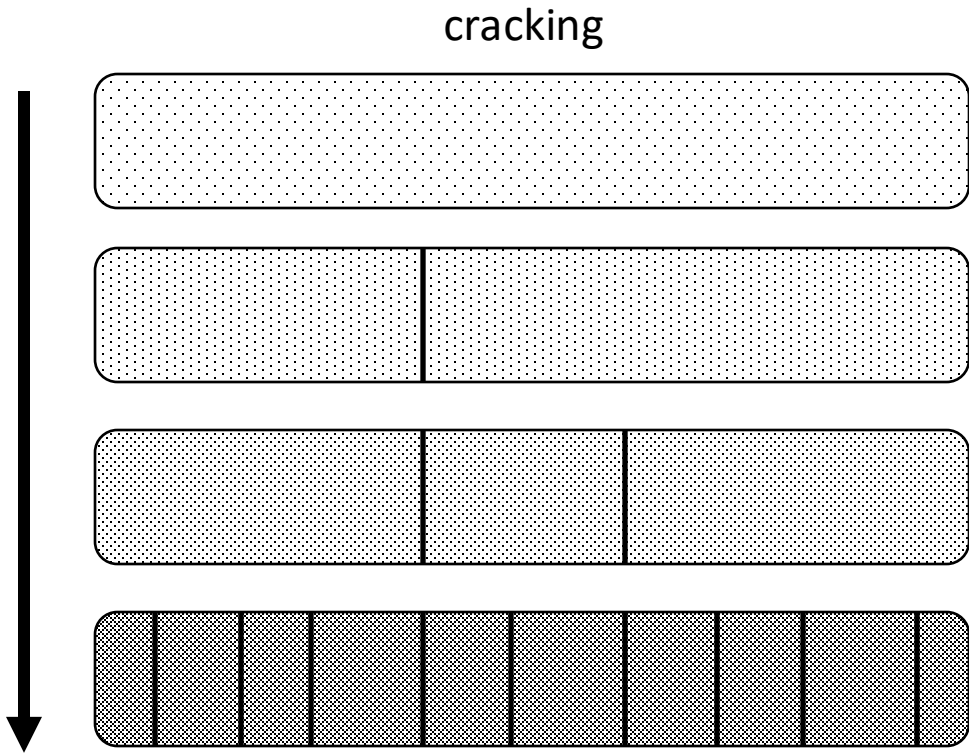
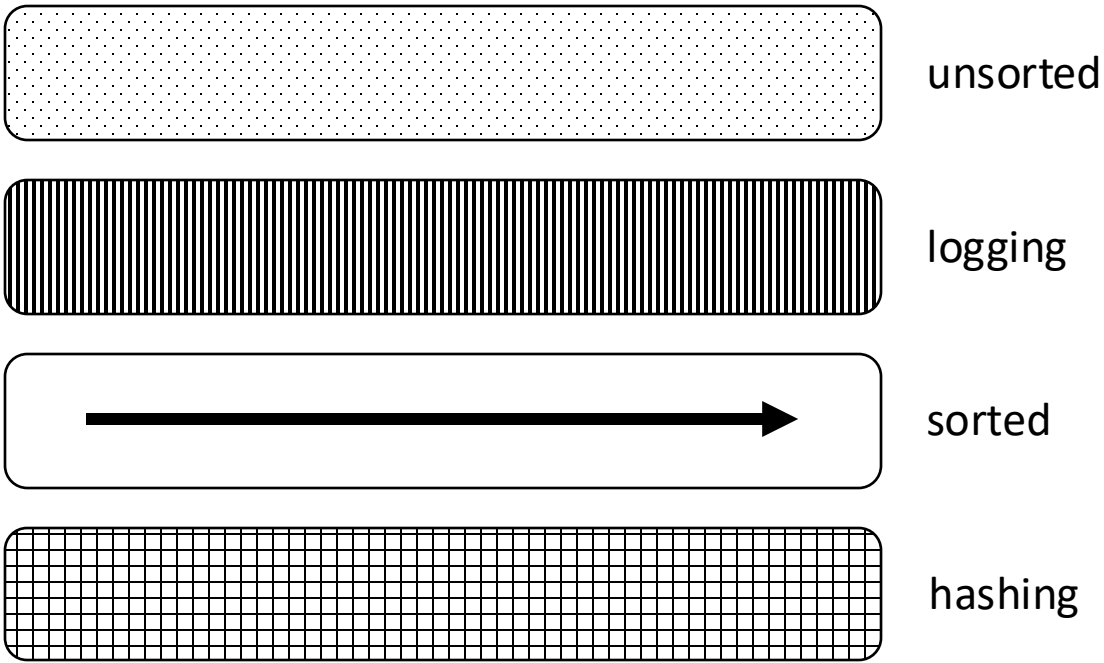
modification policy

batching via buffering

adaptivity

# local data organization

decision per partition



gradually from unsorted towards sorted

# local search algorithms



# modification policy (updates/deletes/inserts)



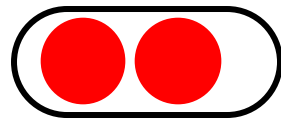
in-place

every update needs to find the “correct” position



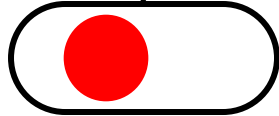
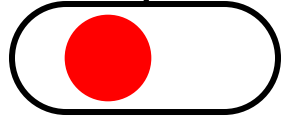
out-of-place

every read needs to search all data



deferred in-place

will eventually merge



how to break down *popular designs*  
to those design decisions?

# b+ trees



global data organization

global searching (algorithm or index)

local data organization

local search algorithm

modification policy

**Workload?** ?



point and range queries, modifications, and some scans

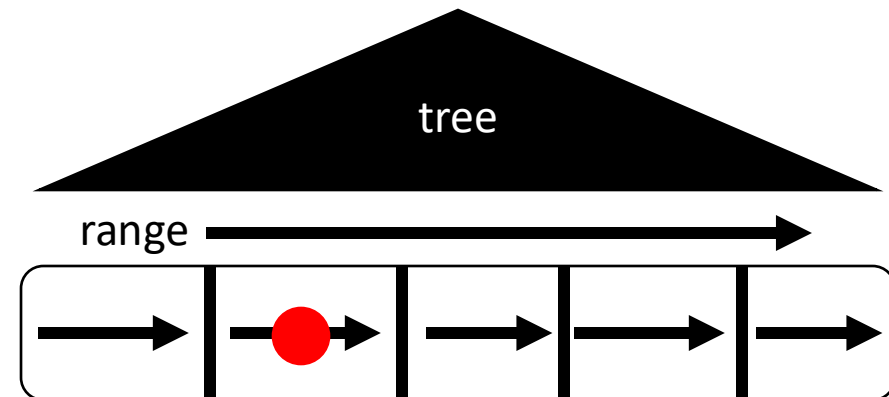
**range partitioning**

**search tree**

**sorted**

**binary search / scan**

**in-place**



# insert optimized b+ trees

global data organization

global searching (algorithm or index)

local data organization

local search algorithm

modification policy

Workload?



increased number of modifications



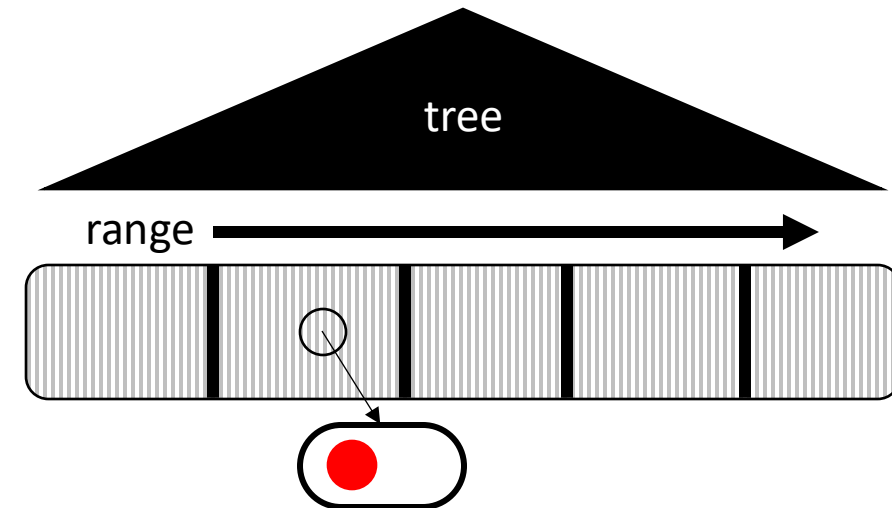
range partitioning

search tree

logging

binary search / scan

deferred in-place





# bounded disorder access method



global data organization

global searching (algorithm or index)

local data organization

local search algorithm

modification policy

Workload?



mixed workload, without short range queries

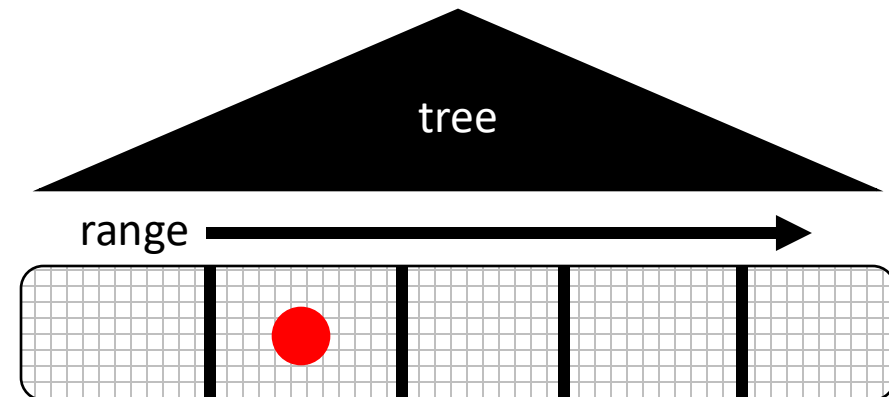
range partitioning

search tree

hashing

hashing

in-place



# Bloom-filter tree



global data organization

global searching (algorithm or index)

local data organization

local search algorithm

modification policy

**Workload?**



mixed workload, without short range queries

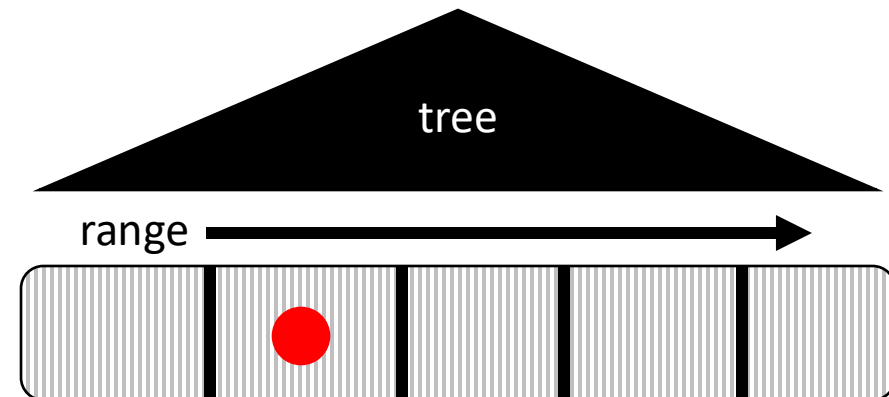
**range partitioning**

**search tree**

**logging**

**filtering**

**in-place**



# static hashing

global data organization

global searching (algorithm or index)

local data organization

local search algorithm

modification policy

Workload?



point queries and modifications



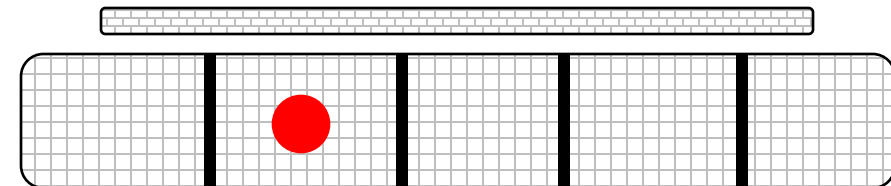
## hash partitioning

### direct addressing (hashing)

logging

scan

in-place



# scans with zonemaps



global data organization

none / logging

global searching (algorithm or index)

scan (with filters)

local data organization

n/a

local search algorithm

n/a

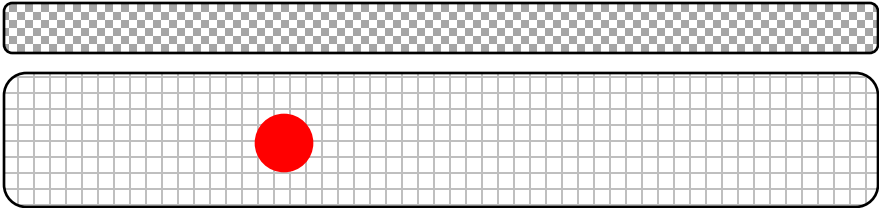
modification policy

in-place

Workload?



long range queries and modifications



# lsm-trees



global data organization

global searching (algorithm or index)

local data organization

local search algorithm

modification policy

**Workload?** ?



modification-heavy with point and range queries

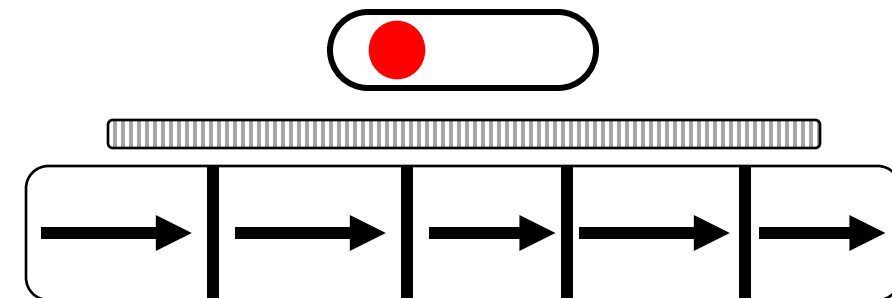
**partitioned logging**

**filter indexing**

**sorted**

**binary / data-driven search**

**out-of-place**



# lsm-hash



global data organization

global searching (algorithm or index)

local data organization

local search algorithm

modification policy

Workload?



modification-heavy with point queries and no range queries

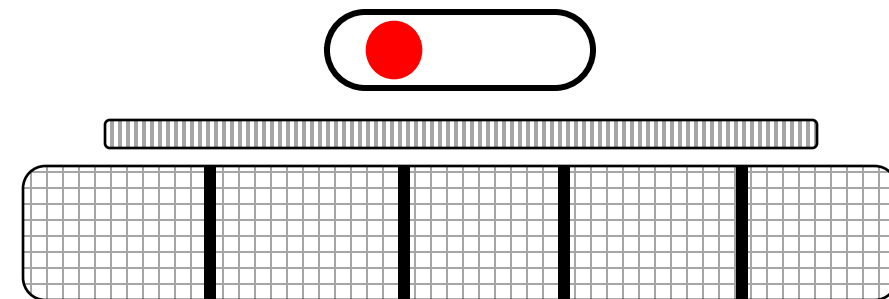
**partitioned logging**

**filter indexing**

**hashing**

**hashing**

**out-of-place**



## The *design space* of data structures

Prof. Manos Athanassoulis

<https://bu-disc.github.io/CS561/>