

IE 360 PROJECT REPORT

Solar Power Forecasting



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1. Introduction

Time series forecasting is used to make predictions by building a model with the statistic we have. There are various time series forecasting models that can be used and the decision of the model that is going to be used depends on model performance. However, to be able to make better predictions, some steps should be followed and the results of each step should be analyzed. Time series forecasting has a large scope of usage. Not only for business decisions but also for social life related decisions, time series forecasting has been used. As mentioned, one of the most popular areas of the time series is the business related decisions. Independent of the sector, there is always something to predict such as T-shirt sales for next summer or bread consumption for the next week.

In this project we are expected to predict the production of the solar energy santral KIVANÇ II GES. It is an energy santral located at the Mersin's province Gülnar. It is known to be the 6th biggest solar energy santral in Turkey. Due to the big production capacity of the santral, KIVANÇ II GES is an important energy supplier.

The production amount of solar energy santrals depends on various things, some of which are the climate of the area it is located, and geological structure. In the project file, we are provided location of the santral with the coordinates, weather related variables and their values for every hour of the given days. These variables are described briefly below:

- **TEMP:** Temperature at the provided location
- **REL_HUMIDITY:** Relative humidity at the provided location
- **DSWRF:** Downward shortwave radiation flux
- **CLOUD_LOW_LAYER:** Total cloud cover data (in terms of percentage) for low-level type of clouds

With the given data and the insights we have about the problem, we have decided to try different models to be able to make better interpretations. Linear regression, ARIMA, SARIMA and decomposition models are built with various parameter selections. Final approach decision is made by looking at models' error related results. Adjusted R^2 , AICc and Residual Standard Error are some of the error measurements used.

Approach

Before determining which model to apply to the data and with which parameters we will work, we analyzed the characteristics of the data visually. We made a separate production plot for each hour and observed the behavior of them. We followed two different approaches in the models we implemented. One of them is to take the data as a whole and apply a model to a dataset with seasonality on both day and hour basis. The other is to focus on the trend of each hour by splitting the data into each hour.

The models we decided to make predictions were linear regression, ARIMA, SARIMA and decomposition together with remainder regression and ARIMA.

We used the WMAPE measure which is also used in the submission phase to evaluate the models. In the submission phase, we sometimes switched to different models on an hourly basis according to the performance of our models, and sometimes we averaged the outputs from these models.

Literature

Day-Ahead Hourly forecasting power generation studies were examined for our project. Although some machine learning algorithms are generally used in this field, the time series regression method has also been used frequently. In addition, the ensemble method, that is, combining the obtained forecasts is mostly used. The parameters given to these algorithms are usually weather data. In the linear regression models we created, we created simpler models by taking the average of the weather data given for different latitudes and longitudes. The sources we benefited while doing our project are

1. *Day-Ahead Hourly Forecasting of Power Generation from Photovoltaic Plants* by Lorenzo Gigoni, Alessandro Betti, Emanuele Crisostomi, Alessandro Franco, Mauro Tucci, Fabrizio Bizzarri, Debora Mucci
2. *A benchmark of statistical regression methods for short-term forecasting of photovoltaic electricity production, part I: Deterministic forecast of hourly production* by M. Zamo, O. Mestre, P. Arbogast, O. Pannekoucke

2. Different Models

2.1 TIME SERIES LINEAR REGRESSION

Linear regression, especially on the basis of time series regression, is used to predict the value of a variable in the future. As the name suggests, this model tries to find a linear relationship between the dependent variable to be estimated and various independent input variables. Analyzing time-oriented data and forecasting future values of a time series are among the most important problems that analysts face in many fields, ranging from finance and economics, to managing production operations, to the analysis of political and social policy sessions, to investigating the impact of humans and the policy decisions that they make on the environment [1].

In our project, we provided hourly solar power prediction for the next day. We put the data in wide format to work on different models easily. In order to see the seasonality of each hour, we created a separate data frame for each hour of the data and set up a separate linear regression model for each of these hours.

- **Missing values**

We observed that there are rows containing production with NA. In order to fill these productions, we determined the days whose production is NA and we made forecasts for each day and hour with the linear regression model, starting with the first NA and using the data up to that day. We used the same input variables for each day while creating this regression. We used each column in the data and additional "month" information to set up the model.

- **Training the model**

We created different production plots for each hour to find out how much of the data to train. According to the plots that emerged as a result of this, we trained the entire data in some hours, while in some hours, we trained the data from 1 year ago until the day of prediction in order to obtain a pattern similar to the pattern in 2021.

- **Input variable selection**

The input variables we use for each hour have changed. We used the *ggpairs* function to decide which variable to add. We added the variables that have a linear relationship with production to the model. At the same time, adjusted R^2 was an important factor in which inputs we used. In addition to these, we sometimes add different lags of production to the model according to the autocorrelation plot of residuals.

- **Additionally added input variables:**

month: A column is added to account for the seasonality.

hour: A column is added to split the data into different hours.

trend: Calculated on a per day basis, different times of a day also got the same trend value.

avg_temp: Average temperature of different latitude, longitude.

avg_cloud: Average cloud cover data (in terms of percentage).

avg_dw: Average downward shortwave radiation flux.

avg_hum: Average relative humidity.

ma_y: Average of last 3 production levels (MA with m=3).

max_capacity: The maximum production level of the last 7 days has been determined as the production capacity of that hour of that day.

lag1_y: The value of the production level in lag 1.

lag14_y: The value of the production level in lag 14.

- **Pre-processing**

To reduce variability and make data stationary in some hours, logarithm and differencing transformations were applied for target variable production and regression was built by using them.

- **Post-processing**

Until the production level exceeds 35 in the actual data, if the results from the model are greater than 35, these results are replaced with 35. The model results were not changed when the actual values started to exceed 35, which we determined as the maximum capacity.

- **Hours that do not need a model**

After observing production plots of different hours, we saw that the production levels of $hour=0,1,2,3,4,21,22,23$ are strictly zero while $hour=20$ production fluctuates around zero. For that reason we decide not to build a model for these hours.

Table 1

HOURL	MODEL FORMULA
5	production ~ +max_capacity*trend+avg_cloud+avg_temp+avg_hum+month+lag1_y+lag14_y
6	production ~ +max_capacity*trend+avg_cloud+avg_temp+avg_hum+month+ma_y
7	production~+I(avg_cloud^2)+I(avg_temp^2)+I(avg_hum^2)+max_capacity*trend+ma_y+month
8	production ~ +avg_cloud+avg_dw+avg_hum+avg_temp+month+max_capacity*trend+ma_y+lag1_y
9	log_prod ~+trend +avg_temp+avg_hum+month+ma_y+max_capacity*trend+avg_hum+lag1_y
10	diff_prod ~+trend+month+avg_hum+avg_dw+avg_cloud+avg_temp+ma_y
11	production ~ma_y+month+avg_temp+avg_hum+avg_cloud+max_capacity+lag1_y
12	production ~trend+month+avg_temp+avg_hum+avg_cloud+max_capacity+lag1_y
13	production ~trend+month+avg_temp+avg_hum+lag1_y+ma_y
14	production ~month+avg_temp+avg_hum+ma_y+avg_temp+avg_dw+lag1_y
15	diff_prod ~ month+avg_hum+ma_y+avg_dw+lag1_y+max_capacity
16	production ~trend+month+avg_temp+lag1_y+ma_y+I(avg_hum^2)+avg_dw+max_capacity*avg_hum
17	production ~trend+month+avg_temp+lag1_y+ma_y+I(avg_hum^2)+avg_dw

hour	MODEL FORMULA
18	production ~month+avg_temp+lag1_y+ma_y+avg_dw+I(avg_dw^2)
19	production ~month+lag1_y+ma_y

Note: Since we had the production data until 2 days ago, and we made the forecast for the next day, there was a 3-day gap in between. The two days in between showed up as NA in the data. Since the NAs up to the day we made our forecast were filled with linear regression in pre-processing, errors accumulated. This is also the reason for the variance increase seen in the regression plots.

2.2 ARIMA

ARIMA is the abbreviation of the Autoregressive Integrated Moving Average. It is a statistical model used to predict future values based on the past values. In this project, the ARIMA model is used first to see whether it is a good model in terms of predicting the production of hourly solar power.

ARIMA model consists of three parts:

- **AR-Autoregression:** In this kind of statistical model lagged values of the target variable are used to build the model.
- **I-Integrated:** Differencing is used to make times series stationary. Main reason behind making time series stationary is the fact that building a good model is not easy unless time series data is stationary. Difference between the value of the target variable and the previous one is taken and used as new data points.
- **MA-Moving Average:** Moving average takes residuals into account and search for their dependency by looking at their lagged values.

ARIMA combines these three parts and builds one single model with the parameters required.

These parameters are p,d and q and can be defined briefly as follows:

- **p:** order of the autoregressive part, the number of lag observations will be used in the model
- **q:** order of the moving average part, the moving average window will be used in the model
- **d:** degree of first differencing involved.

To build an ARIMA model parameters described above need to be specified or a function called *auto.arima* can be used.

In this project, to analyze every hour separately we have splitted data into 24 hours and for each hour in which there is production, we have built arima models. Following steps are applied to achieve a good model:

- **Visualization of the time series:** Before working on the data, the plot of the time series should be drawn to inspect data characteristics. The visualization of the time series provides some insights about successor steps to be followed. After plotting the time series, we have realized that the time series we have is not stationary and variance is changing over time.
- **Stationarity check of the data:** Stationary time series can be defined as the time series which does not depend on time and does not have variance changing over time. To build a better model one of the first things to consider is the stationarity of the time series. Before starting the model, we first applied one of the stationarity tests which is the KPSS test. The result we have found was higher than the critical values which implies that at the beginning time series for each hour was not stationary. To reduce variance of the time series and make time series stationary, we have used the Box-Cox algorithm. Once again the KPSS test is applied and the time series we found with the Box-Cox algorithm was not stationary again, thus we have applied another method used for eliminating non-stationarity of the time series.
- **Differencing:** Differencing is used to get rid of the variance around the mean which as a result leads to elimination of trend and seasonality. In short, a time series is obtained by taking the difference between the previous and the current observation. To see the differencing method effect, ACF and PACF are plotted and the results we have found were good enough to proceed with building an ARIMA model.
- **Auto.arima:** *Auto.arima* is used to find the parameters for the model. We have built different ARIMA models for each hour and the Table3 gives the final ARIMA models with the parameters.

Table 2

Hour	Best Model
All hours included	
0	ARIMA(0,0,0)
1	ARIMA(0,0,0)
2	ARIMA(0,0,0)
3	ARIMA(0,0,0)
4	ARIMA(0,0,0)
5	ARIMA(4,0,2)
6	ARIMA(0,0,2)
7	ARIMA(1,0,1)
8	ARIMA(0,0,2)
9	ARIMA(0,0,2)
10	ARIMA(1,0,1)
11	ARIMA(0,0,2)
12	ARIMA(1,0,1)
13	ARIMA(0,0,2)
14	ARIMA(1,0,1)
15	ARIMA(0,0,2)
17	ARIMA(0,0,1)
18	ARIMA(3,0,5)
19	ARIMA(0,0,3)
20	ARIMA(5,0,0)
21	ARIMA(0,0,0)
22	ARIMA(0,0,0)
23	ARIMA(0,0,0)

- Check Residuals:** After building the model, it is required to check residuals and they are expected to look like white noise. The residuals we have found were not following the exact white noise pattern however, since there was not considerable difference, we have continued with the models we have found.

2.3 SARIMA

An extension of arima, the sarima model, is used to model seasonality that the arima model does not take into account. The parameters for SARIMA(p,d,q)x(P,D,Q,s) model are as follows:

- **p and seasonal P**: indicate number of autoregressive terms (lags of the stationarized series)
- **d and seasonal D**: indicate differencing that must be done to stationarize series
- **q and seasonal Q**: indicate number of moving average terms (lags of the forecast errors)
- **s**: indicates seasonal length in the data

Since it is anticipated that the production of solar panels will vary from month to month or from season to season, the sarima model has also been applied in the project. **Approach**: SARIMA model can only model one seasonality. In case there is more than one seasonality in the data, wrapping is applied both to the whole data and to the data separately for each hour. The *auto.arima* function is used in both approaches, therefore parameters are determined automatically. Before applying the SARIMA, it was checked whether the data was stationary or not with the *KPSS* test. Since SARIMA takes seasonality into account, a prior differencing process is not applied to make the data stationary. We have made 3 days ahead production forecast because we have the production data until 2 days before the day we are and we make the forecast for the next day. When new data came in every day, all the days up to that day were used to train the model. So the expanding window approach was applied. According to results of the *auto.arima* function, the best models are as follows:

Table 2

Hour	Best Model
All hours included	ARIMA(1,0,0)(2,1,0)[24] with drift
0	ARIMA(0,0,0)
1	ARIMA(0,0,0)
2	ARIMA(0,0,0)
3	ARIMA(0,0,0)
4	ARIMA(0,0,0)
5	ARIMA(1,1,5)

6	ARIMA(0,1,2)
7	ARIMA(1,1,1)(0,0,1)[30]
8	ARIMA(0,1,2)(0,0,1)[30]
9	ARIMA(0,1,2)(0,0,1)[30]
10	ARIMA(1,1,1)(0,0,1)[30]
11	ARIMA(0,1,2)(0,0,1)[30]
12	ARIMA(1,1,1)
13	ARIMA(1,1,1)
14	ARIMA(1,1,1)
15	ARIMA(0,1,2)
17	ARIMA(0,1,1)
18	ARIMA(2,1,3)
19	ARIMA(1,1,3)(0,0,1)[30]
20	ARIMA(0,1,1)
21	ARIMA(0,0,0)
22	ARIMA(0,0,0)
23	ARIMA(0,0,0)

2.4 MODELS ON THE REMAINDERS OF THE DECOMPOSITION

It is an alternative approach to decompose the time series into trend cycle, seasonal component and remainder component, then build models to predict the remainder component.

As the data are collected hourly, we expected them to have a seasonality of 24-row windows. That is, we expect high correlations between the same hours of different days. This is validated by the ACF of the data.

So the frequency parameter of the decompose function was set to 24, to apply decomposition.

After the series was decomposed and we were left with the remainder component to build a model for, separation of the data into hours was done again. Then the remainder series was separated accordingly and the obtained series were merged with their corresponding part of the data. Now we had twenty four data tables with the remainder from the composition added as columns. Then, likewise various models were built for the productions at each of the hours, now the models were fitted to predict the remainder components.

One thing that is important to notice is that although the remainder series was stationary before the partition into hours, considering the series separately for each hour made them non-stationary for most of the hours. To handle this situation, cube root transformations were applied, yet it stationarized the data only to some extent. So the results that were going to come from below models were not expected to work well. But for the sake of trying and as we were to apply aggregation to the models and do post processing, the models were still built.

Two models were fitted for each hour, one of which falls under the family of linear regression and the other is ARIMA, implemented by the `auto.arima` function.

Data visualization and input variable selection for linear regression:

The trend and hourly seasonality effects were already handled by the decomposition phase. So the series in hand were to be explained by i) the independent variables related to the weather and ii) still time series objects like lagged target variable/errors. So it needed to be evaluated as to which of these candidates to include in the ultimate models.

The shapes the data show is important in deciding in what form an input variable is to be added to the model to predict the target. In that sense, `ggpairs` was used while deciding which input variables to add to the model, like it is in the regular linear regression phase. When, for instance, an input variable appears to have a nonlinear relationship with the production, it would be wise to consider the second or third powers of that input variable. This worked relatively well in the case of hour 17, 18 and 19, where squared `avg_dw` performed as a good regressor.

A general method used was training a model with month input attribute along with the four columns related to weather as a baseline. Then the regressors that turned out to be not significantly informative were removed. On the other hand, the input variables that had high correlations with the target -and with the residuals- were added, and also their pairwise interactions were tried. For the lagged targets, the ACF and PACF graphs were checked.

After a model was trained, if it performed well, the process continued like this. The regressors that were not informative were removed and if needed, other regressors were added.

So a variety of models were trained iteratively by this process. Finally, the ones that met the below requirements best were selected as the final models.

Model evaluation for linear regression:

Checking the linear regression assumptions is vital. So, although many of the input variable combinations were tried and a diversity of R^2 or weighted mean absolute percentage error measures were considered, another significant criteria in the model selection phase was how much the model met the assumptions of the linear regression. So, for instance, even though a model performed better according to its adjusted R^2 value, if another model that performed slightly less in terms of the adjusted R^2 leaves residuals that are closer to having constant variance, the latter model is to be preferred. This convention was pursued in selecting amongst rival linear models.

Table ?

HOURLY	MODEL FORMULA
5	remainder ~ month + avg_temp + avg_cloud + lag1_remainder
6	remainder ~ month + avg_temp + avg_cloud + avg_hum + lag1_remainder
7	remainder ~ month + avg_temp + avg_cloud + avg_dw + lag1_remainder
8	remainder ~ month + avg_temp + avg_cloud + avg_hum + avg_dw + lag1_remainder
9	remainder ~ month + avg_temp + avg_dw + lag1_remainder
10	remainder ~ month + avg_temp + avg_dw + lag1_remainder

11	remainder ~ month + avg_temp + avg_cloud + avg_dw + lag1_remainder + lag2_remainder + lag3_remainder
12	remainder ~ month + avg_temp + avg_cloud + lag1_remainder + lag2_remainder + lag3_remainder
13	remainder ~ month + avg_temp + avg_cloud + avg_dw
14	remainder ~ month + avg_temp + avg_hum + avg_dw
15	remainder ~ month + avg_dw + avg_temp + avg_hum + avg_dw*avg_temp
16	remainder ~ month + avg_dw + avg_hum*avg_temp
17	remainder ~ month + poly(avg_dw,2)
18	remainder ~ month + avg_temp + avg_hum + avg_temp*avg_hum + poly(avg_dw,2)
19	remainder ~ month + avg_temp + avg_cloud + poly(avg_hum,2) + avg_dw*avg_cloud

Predictions with decomposed data:

As the data were decomposed, after we obtained predictions from the linear regression and ARIMA models, we needed to add the trend cycle and seasonal components back. Because the decomposition was of additive type, the re-trending and re-seasonalizing processes were made by the addition of all, as follows:

$\hat{Y} = T_t + S_t + R_t$, where R_t is the cube of the forecast obtained from the model.

3. Model Comparison

ARIMA Model: ARIMA is a preferable method since it combines different models to build a better one to predict future values. It does not only consider the past values of the observations but also the error relations with the moving average. ARIMA works with stationary data which does not have trend and seasonality so we have tried to remove seasonality and trend by differencing. However, it does not consider the regressors' effect specifically, although some of the unexplained errors might have been explained by input variables. In the model evaluation part, we have used the Akaike Information Criterion to make interpretations about model performance. Although it was not the perfect model, results we have found were reasonable to use that model.

Time Series Linear Regression: It is a regression model which searches for relations between input variables and the target variable which is the production in this case. We had some weather related variables and since we believed that for each hour, the importance of the input variables may differ, we decided to build different models. However, finding the exact relation and required power transformation was not easy although we have tried various combinations based on logical dynamics. Model evaluations are done based on residual standard error and adjusted R^2 values.

SARIMA Model: Likewise ARIMA, SARIMA model does not consider external effects like the weather data we have in our data. The main difference between SARIMA and ARIMA is that it is developed for taking seasonality into account. When the SARIMA model is applied to the data not splitted into hours, it assigned seasonality to 24 as expected since we know that each hour has its own trend, which results in 24-hour seasonality. In some of the SARIMA models, which we applied separately for each hour, the seasonality parameter was assigned as 0 for some hours. This is because seasonality during the day is eliminated when we allocate each hour. Although the result of the SARIMA model is generally similar to the results of the ARIMA model, there are deviations in between. As in the ARIMA model, the Akaike Information Criterion is used for model evaluation purposes.

Models on The Remainders of The Decomposed Series: As it was discussed above, although the remainder series as a whole was stated to be stationary by the KPSS test, this stationarity melted down as the series was separated into hours. Moreover, the transformations that were tried did not help to overcome this problem but they provided only a limited improvement. So the remainder series, as a whole, was transformed by a cube-root function and then it was separated into hours. As expected, the variance in the residuals were not stable enough to suffice the requirements of the linear regression. This was also a problem for the ARIMA models. Throughout the submission phase, the weighted mean percentage errors were obtained by all of the alternative models and these two models on the remainder series were almost always the worst ones in terms of WMAPE. So we had a good reason to exclude them from the evaluation. There was, however, another relatively good enough reason to consider them. It was to decrease the variance of the predictions. So we gave a chance to them on each one of the days and included/excluded them considering their recent performances.

4. Submission Phase

Date/Hour	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
05/24/2022	0.12	6.25	26.77	35	35	35	35	32.5	33.99	34.95	33	28.7	19.75	6.63	0.685
05/25/2022	0.12	5.25224	22.93531	35	32.15758	35	33.98846	32.98071	28.09268	28.26603	25.072	28.16508	19.72132	7.961824	0.851273
05/25/2022	0.12	5.25224	22.9331	35	32.15758	35	33.98846	32.98071	28.09268	28.26603	25.072	28.16508	19.72132	7.961824	0.851273
05/26/2022	0.15	7.1989	25.183	35	34.66	35	34.14	32.507	31.188	31.4684	32.941	28.607	15.61	8.109	0.897
05/27/2022	0.18	6.71	23.415	35	35	35	35	32.8	32.941	30.616	25.367	23.16	17.473	7.295	1.015
05/28/2022	0.18	7.21	26.83	35	35	35	35	35	35	33.983	30.367	30.161	21.269	9.295	1.115
05/29/2022	0.21	7.43021	25.12979	35	35	35	35	35	35	35	33.0754	25.66158	21.04575	9.365226	0.893089
05/30/2022	0.21	7.88175	26.63288	35	35	35	35	35	35	35	33.6345	30.14545	24.9845	10.67241	1.3025
05/31/2022	0.21	7.164	26.401	35	35	35	35	35	35	34.483	34.562	28.28	25.002	9.96	1.323
6.01.2022	0.28	6.99	25.96	35	35	36.8	38.59	38.26	37.96	38.02	38.05	37.53	24.02	9.33	1.32
6.02.2022	0.2	7.09	26.05	37.045	37.5	37.123	37.672	38.04	37.9	37.9	37.83	37.45	25.95	10.12	1.4

Before the submission phase, we have built our models and evaluated our models by looking at the WMAPE values for each. Final model is chosen by splitting data into training and test data sets. To lose as less observation as possible, we tried to keep our training data

set large enough while giving enough data to the test data set. We trained our model with these data points and tested them on the test data set. According to our results we have found, we decided to proceed with the one which performs better in terms of test error. Test error is an important measure to prevent the model from overfit.

During the submission phase, we have made some changes in the models we have been using. While inspecting the data, we have realized that 35 is a maximum point and although our model predicts slightly larger than 35 at some hours, we have reduced them to 35. Nonetheless, with the new data, we have realized that maximum capacity has increased and production amounts were more than 35 for some hours. As a result of this increase, instead of reducing their values, we kept the result obtained from the model or increased these results slightly.

Simultaneously, we were looking at the production amount for each hour of the last 7 days to be aware of sudden jumps in the production. If something unexpected happens in the production amount, we try to protect our model from such an outlier point. Besides, since averaging decreases the variance, after some point, we run the every model and then calculate their average as final submission values.

5. Conclusion

In this project, we have built different time series models to predict the production of the KIVANÇ 2 GES solar energy panel. There are various time series models that can be applied to the data however, we have focused on the ones we have covered during the lectures. Time series regression, ARIMA and SARIMA decomposition the models built on the remainders obtained by decomposition are the models we have worked on and tried to improve.

Each model was built by the following certain steps. While applying these steps, we got help from the plots. Visualization is one of the key steps of forecasting. We know that every model has advantages and disadvantages, so we tried to create as many reasonable models as we can to see model performance differences and to choose the best performing one.

WMAPE is the measurement of the model performance and the best result we obtained at the trial period was the outcome of the linear regression model. Linear regression models were built on every hour of the day and some of the input variables were added to the models. We began making predictions with the linear regression models, however, whenever we ended up with a relatively large WMAPE, we took the average of the predictions of the models. Besides, post processing is done for maximum capacity issues and in addition to these, with the help of the visualization, we have made slight adjustments.

During this submission period, we have tried to make better predictions by implementing different concepts which are in the scope of the IE360 lecture. We have realized that it is possible to build more than one model and improving a model is a dynamic process as long as there is enough data.

6. Conclusion

1- Introduction to Time Series Analysis and Forecasting, Montgomery, Jennings, and Kulahci, 2008, Wiley

2- <https://otexts.com/fpp2> (to be formatted)