

# Lecture 1: Probabilistic Model

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EE210: Probability and Introductory Random Processes  
KAIST EE

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- (1) Probabilistic Model
  - Mathematical description of uncertain situations
  
- (2) Sample Space, Event, Probability Law
  - Elements of probability theory
  
- (3) Probability Axioms
  - 3 axioms for the completeness of a theory

- (1) Probabilistic Model
- (2) Sample Space, Event, Probability Law
- (3) Probability Axioms

**Modeling:** Understand reality with a simple (mathematical) model

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  - Flip two coins
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- **Our goal:** Build up a **probabilistic model** for an experiment with random outcomes
  - **Probabilistic model?**
    - Assign a number to each outcome or a set of outcomes
    - Mathematical description of an uncertain situation
  - Which model is good or bad?

**Goal:** Build up a probabilistic model. Hmm... How?

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2. Assigned numbers to each outcome of  $\Omega$ : **Probability Law  $\mathbb{P}(\cdot)$**

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## Elements of Probabilistic Model

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2. Assigned numbers to each outcome of  $\Omega$ : **Probability Law  $\mathbb{P}(\cdot)$**

**Question:** What are the conditions of  $\Omega$  and  $\mathbb{P}(\cdot)$  under which their induced probability model becomes "legitimate"?



- (1) Probabilistic Model
- (2) Sample Space, Event, Probability Law
- (3) Probability Axioms

# 1. Sample Space $\Omega$

The set of all outcomes of my interest

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 $\Omega = \{H, T, HT\}$

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(not too concrete, not too abstract)

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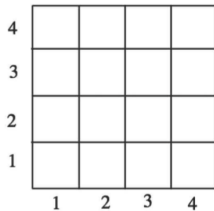
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 $\implies \Omega = \{H, T\}$   
  
(b) The impact of the weather (rain or no rain) on the coin's behavior.  
  
 $\implies \Omega = \{(H, R), (T, R), (H, NR), (T, NR)\},$   
  
R(Rain), NR(No Rain).



- *Discrete case:* Two rolls of a tetrahedral die

-  $\Omega = \{(1, 1), (1, 2), \dots, (4, 4)\}$

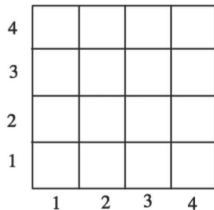


4				
3				
2				
1				
	1	2	3	4



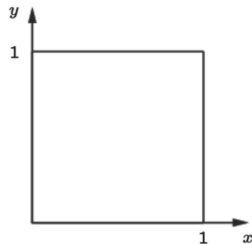
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- *Continuous case:* Dropping a needle in a plain

-  $\Omega = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x, y \leq 1\}$





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- Roll a dice. What is the probability of odd numbers?

$\mathbb{P}(\{1, 3, 5\})$ , where  $\{1, 3, 5\} \subset \Omega$  is an event.

- (1) Probabilistic Model
- (2) Sample Space, Event, Probability Law
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- Need to construct  $\mathbb{P}(\cdot)$  that naturally satisfies the intention of a probability theory designer just like you. What about the followings as starting points?

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  - If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$
  - many others

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## Probability Axioms: Version 1

- A1. **Nonnegativity**:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$
- A2. **Normalization**:  $\mathbb{P}(\Omega) = 1$
- A3. **(Finite) additivity**: For two disjoint events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

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- Note that coming up with the above axioms is far from trivial.

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

VIDEO PAUSE

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

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3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

$$\mathbb{P}(B) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(B \setminus A) \stackrel{A1}{\geq} \mathbb{P}(A)$$

1. Specify the sample space
2. Specify a probability law
  - from my earlier belief, from data, from expert's opinion
3. Identify an event of interest
4. Calculate

Toss a (biased) coin

1.  $\Omega = \{H, T\}$
2.  $\mathbb{P}(\{H\}) = 1/4, \mathbb{P}(\{T\}) = 3/4,$
3. probability of head or tail
4.  $1/4, 3/4$

- $\Omega = \{1, 2, 3, \dots\}, \mathbb{P}(\{n\}) = \frac{1}{2^n}, n = 1, 2, \dots$



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$$\begin{aligned}\mathbb{P}(\text{even}) &= \mathbb{P}(\{2, 4, 6, \dots\}) \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots = 1/3\end{aligned}$$

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- Is the above right? If not, why?

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- Is the above right? If not, why?
  - Wrong: **Finite** additivity axiom does not allow this.

## Probability Axioms: Version 1

A1. Nonnegativity:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

A2. Normalization:  $\mathbb{P}(\Omega) = 1$

A3. (Finite) additivity: For two disjoint events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

## Probability Axioms: Version 2

A1. Nonnegativity:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

A2. Normalization:  $\mathbb{P}(\Omega) = 1$

A3. **Countable additivity:** If  $A_1, A_2, A_3, \dots$  is an infinite sequence of disjoint events, then  $\mathbb{P}(A_1 \cup A_2 \cup \dots) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \dots$ .



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  - axioms  $\rightarrow$  theorems
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- Frequencies:  $\mathbb{P}(H) = 1/2$ 
  - Understanding an uncertain situation: fractions of successes out of many experiments
- Beliefs:  $\mathbb{P}(\text{He is reelected}) = 0.7$

Anyway, we believe that probabilistic reasoning is very helpful to understand the world with many uncertain situations.

Questions?

You build up the very basics of a probabilistic model.

What else do we need to build up?

- 1) Please explain what a probabilistic model is and why we need it.
- 2) What is the mathematical definition of event?
- 3) What are the key elements of the probabilistic model?
- 4) Please list up the probability axioms and explain them.
- 5) Why do we need countable additivity in the probability axioms?