

Lecture 1: Probabilistic Model

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EE210: Probability and Introductory Random Processes
KAIST EE

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- (1) Probabilistic Model
 - Mathematical description of uncertain situations

- (2) Sample Space, Event, Probability Law
 - Elements of probability theory

- (3) Probability Axioms
 - 3 axioms for the completeness of a theory

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- (2) Sample Space, Event, Probability Law
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Modeling: Understand reality with a simple (mathematical) model

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 - Flip two coins
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 - Mathematical description of an uncertain situation

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- **Our goal:** Build up a **probabilistic model** for an experiment with random outcomes
 - **Probabilistic model?**
 - Assign a number to each outcome or a set of outcomes
 - Mathematical description of an uncertain situation
 - Which model is good or bad?

Goal: Build up a probabilistic model. Hmm... How?

The first thing:

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The first thing: What are the *elements* of a probabilistic model?

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Elements of Probabilistic Model

1. All outcomes of my interest:

2. Assigned numbers to each outcome of Ω :

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Goal: Build up a probabilistic model. Hmm... How?

The first thing: What are the *elements* of a probabilistic model?

Elements of Probabilistic Model

1. All outcomes of my interest: **Sample Space Ω**
2. Assigned numbers to each outcome of Ω : **Probability Law $\mathbb{P}(\cdot)$**

Question: What are the conditions of Ω and $\mathbb{P}(\cdot)$ under which their induced probability model becomes "legitimate"?

- (1) Probabilistic Model
- (2) Sample Space, Event, Probability Law
- (3) Probability Axioms

1. Sample Space Ω

The set of all outcomes of my interest

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(1) Mutually exclusive

1. Toss a coin. What about this?
 $\Omega = \{H, T, HT\}$

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(not too concrete, not too abstract)

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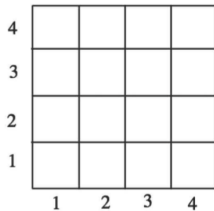
(b) The impact of the weather (rain or no rain) on the coin's behavior.

 $\implies \Omega = \{(H, R), (T, R), (H, NR), (T, NR)\},$

R(Rain), NR(No Rain).

- *Discrete case*: Two rolls of a tetrahedral die

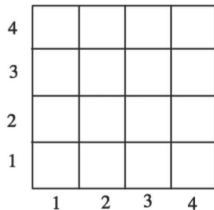
- $\Omega = \{(1, 1), (1, 2), \dots, (4, 4)\}$



4				
3				
2				
1				
	1	2	3	4

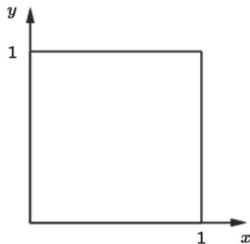
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- $\Omega = \{(1, 1), (1, 2), \dots, (4, 4)\}$



- *Continuous case*: Dropping a needle in a plain

- $\Omega = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x, y \leq 1\}$



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 - This is where probability meets set theory.
- Roll a dice. What is the probability of odd numbers?

$\mathbb{P}(\{1, 3, 5\})$, where $\{1, 3, 5\} \subset \Omega$ is an event.

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 - $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$
 - For two disjoint¹ events A and B , $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

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 - many others

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Probability Axioms: Version 1

- A1. **Nonnegativity**: $\mathbb{P}(A) \geq 0$ for any event $A \subset \Omega$
- A2. **Normalization**: $\mathbb{P}(\Omega) = 1$
- A3. **(Finite) additivity**: For two disjoint events A and B , $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

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- Note that coming up with the above axioms is far from trivial.

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

VIDEO PAUSE

1. For any event A , $\mathbb{P}(A) \leq 1$
2. $\mathbb{P}(\emptyset) = 0$
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3. If $A \subset B$, $\mathbb{P}(A) \leq \mathbb{P}(B)$

$$\mathbb{P}(B) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(B \setminus A) \stackrel{A1}{\geq} \mathbb{P}(A)$$

1. Specify the sample space
2. Specify a probability law
 - from my earlier belief, from data, from expert's opinion
3. Identify an event of interest
4. Calculate

Toss a (biased) coin

1. $\Omega = \{H, T\}$
2. $\mathbb{P}(\{H\}) = 1/4, \mathbb{P}(\{T\}) = 3/4,$
3. probability of head or tail
4. $1/4, 3/4$

- $\Omega = \{1, 2, 3, \dots\}, \mathbb{P}(\{n\}) = \frac{1}{2^n}, n = 1, 2, \dots$

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- $\mathbb{P}(\text{even numbers})?$

$$\begin{aligned}\mathbb{P}(\text{even}) &= \mathbb{P}(\{2, 4, 6, \dots\}) \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots = 1/3\end{aligned}$$

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- Is the above right? If not, why?

- $\Omega = \{1, 2, 3, \dots\}$, $\mathbb{P}(\{n\}) = \frac{1}{2^n}$, $n = 1, 2, \dots$
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- Is the above right? If not, why?
 - Wrong: **Finite** additivity axiom does not allow this.

Probability Axioms: Version 1

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A2. Normalization: $\mathbb{P}(\Omega) = 1$

A3. (Finite) additivity: For two disjoint events A and B , $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

Probability Axioms: Version 2

A1. Nonnegativity: $\mathbb{P}(A) \geq 0$ for any event $A \subset \Omega$

A2. Normalization: $\mathbb{P}(\Omega) = 1$

A3. **Countable additivity:** If A_1, A_2, A_3, \dots is an infinite sequence of disjoint events, then $\mathbb{P}(A_1 \cup A_2 \cup \dots) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \dots$.

- A narrow view: A branch of math
 - axioms \rightarrow theorems
 - Mathematicians work very hard to find the smallest set of necessary axioms (just like atoms in physics)

Anyway, we believe that probabilistic reasoning is very helpful to understand the world with many uncertain situations.

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- Frequencies: $\mathbb{P}(H) = 1/2$
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- Frequencies: $\mathbb{P}(H) = 1/2$
 - Understanding an uncertain situation: fractions of successes out of many experiments
- Beliefs: $\mathbb{P}(\text{He is reelected}) = 0.7$

Anyway, we believe that probabilistic reasoning is very helpful to understand the world with many uncertain situations.

Questions?

You build up the very basics of a probabilistic model.

What else do we need to build up?

- 1) Please explain what a probabilistic model is and why we need it.
- 2) What is the mathematical definition of event?
- 3) What are the key elements of the probabilistic model?
- 4) Please list up the probability axioms and explain them.
- 5) Why do we need countable additivity in the probability axioms?