Lecture 0: Introduction

Yi, Yung (이융)

EE210: Probability and Introductory Random Processes
KAIST EE

August 27, 2022

Outline

- Course logistics
- Why this course?

Instructor

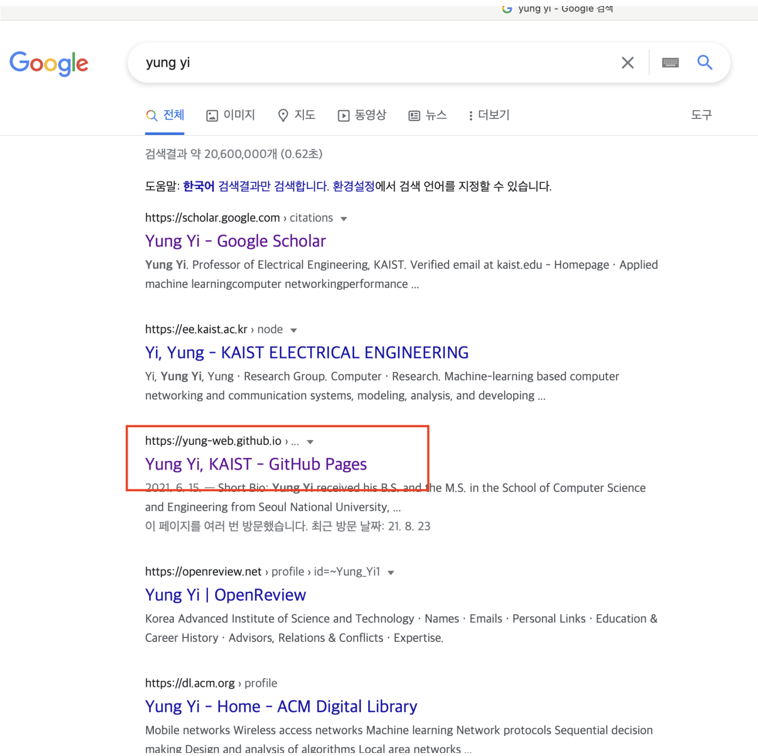
- Yi, Yung (이융)
- Office: N1, 810
- Homepage: `https://yung-web.github.io/home/`
- E-mail: `yiyung@kaist.edu`
- Computer Division
- In KAIST EE since 2008

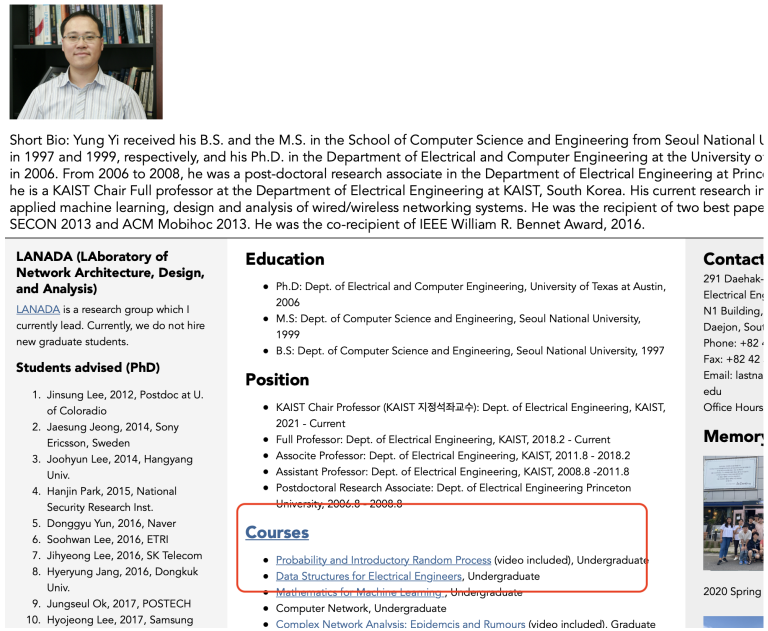
How to Run This Course

- All lecture videos have already been pre-recorded. Available in YouTube.
- non-real-time online ($\leq$50%) + real-time offline/online ($\geq$ 50%)
- non-real-time online: Watch and study anytime and anywhere you like.
- realtime offline/online: Watch lecture videos in the classroom or in the zoom, with asking and answering questions.
- No attendance check!

Accessing Lecture Videos and Slides

- Method 1: https://yung-web.github.io/home/courses/probability.html
- Method 2: (a) Type Yung Yi in the google, (b) visit his GitHub homepage, (c) find the links on Course.





Using KLMS and Campuswire

- KLMS: All notifications and announcements (also sent to you via email)
- KLMS: Homework upload
- KLMS: Score upload and all the grade-related things

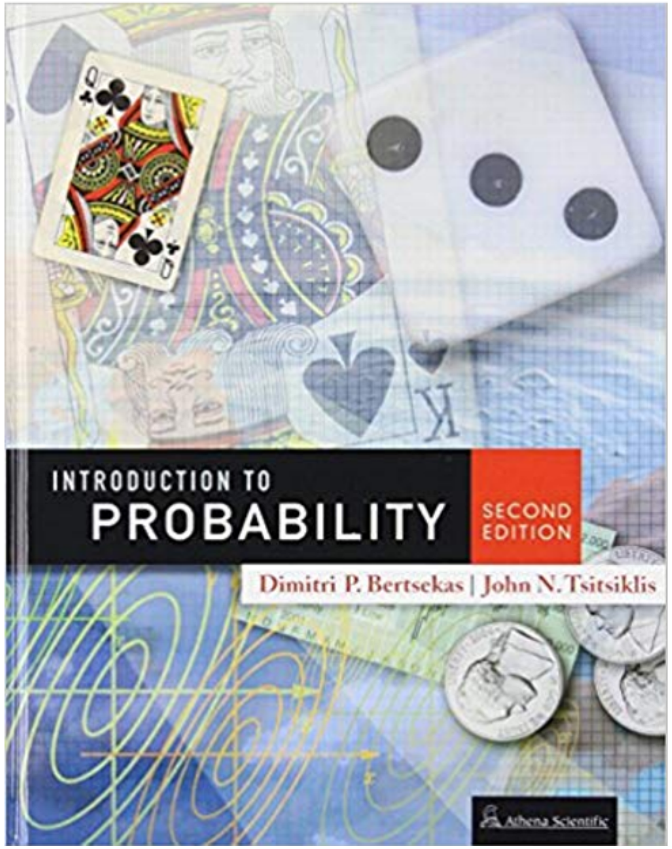
- Campuswire: All Questions (course contents, logistics, etc). Should be in English.

- NOT individual emails to the instructor or the TAs
- Emails to the instructor, Prof. Yung Yi, are allowed for handling private situations.

Campuswire Demonstration

Textbook

- Introduction to Probability (2nd edition)
  - MIT course textbook
  - Dimitri P. Bertsekas and John N. Tsitsiklis
- You can order it from Yes24, Aladin, Kyobo
  - Yes24: `http://www.yes24.com/Product/Goods/3995311`
  - Aladin: `https://www.aladin.co.kr/shop/wproduct.aspx?ItemId=12945615`
  - Kyobo: `http://www.kyobobook.co.kr/product/detailViewEng.laf?ejkGb=ENG&mallGb=ENG&barcode=9781886529380&orderClick=LAG&Kc=`



Textbook Homepage

- `http://athenasc.com/probbook.html`
- Solutions for all problems (so you have all solutions for your homework)
- Links to the old MIT courses
- You can find the urls (2006, 2010, 2013) for the MIT lectures based on the same textbook, where there are many useful resources (recitation problems, homework problems, old exam problems, etc)
- Some of my lecture slides are based on theirs, but my slides are largely modified/reorganized/edited in many places for our purpose.

Course Contents (Guided Schedule for Your Study)

1. Probabilistic model (0.5 week)
2. Conditioning and Independence (0.5 week)
3. Random Variable, Part I (Discrete Random Variable) (1.5 week)
4. Random Variable, Part II (Continuous Random Variable) (1.5 week)
5. Random Variable, Part III (Advanced Topic on Random Variable) (1.5 week)
6. Limit of Scaled Sum of Random Variables: Central Limit Theorem and Weak Law of Large Numbers (1.5 week)
7. Random Process: Bernoulli and Poisson Processes (2 week)
8. Random Process: Markov Chain (2 week)
9. Introduction to Statistical Inference (2 week)

Exams and Homeworks

- 2 Exams (mid-term and final)
- Homeworks
  - All problems are from exercise problems in the textbook.
  - We do NOT check whether you copy your solution from the problem solutions or not.
- 9 Homeworks for each of 9 chapters.

Other QnA

KAIST EE

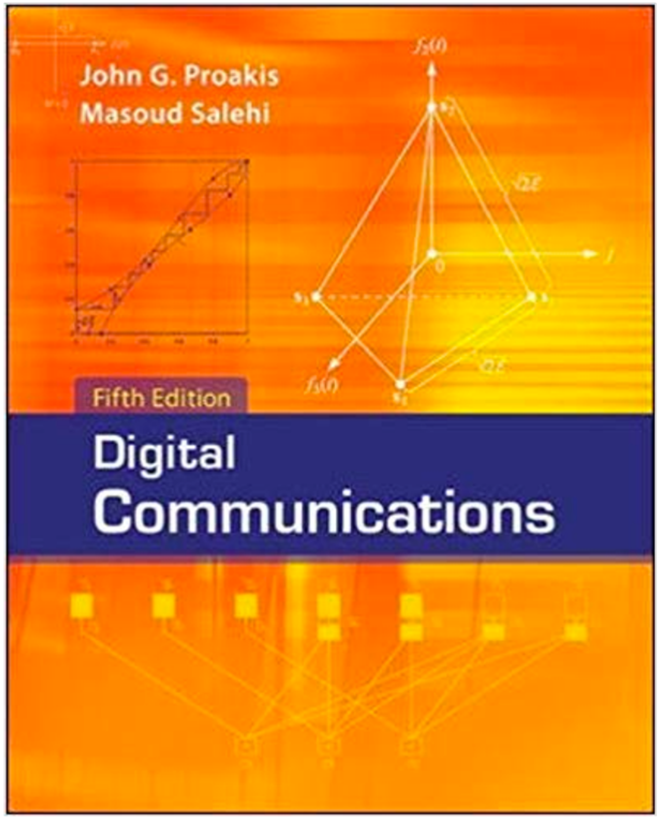
- Read ALL the emails.
- Need to buy the textbook?
  - Strongly recommend it. Taking a course is NOT just solving mid-term and final exam problems and getting a good grade.
- OK not to be present in the classroom? Yes.
- OK that my homework solutions is same as those in the solutions book? Yes.
- Can I ask for a personal meeting to ask questions or get other general advices? Sure. Send me an email.

Questions?

Why Probability?

- Many things are "probabilistic"
- Assume that you are a designer of the following engineering systems. Good design?
  - a web server
  - a communication device like mobile phones
  - an AI-based image classifier
- From an engineering point of view,
  - System input
  - Algorithms in systems
  - Analysis of systems

Textbook: Digital Communication



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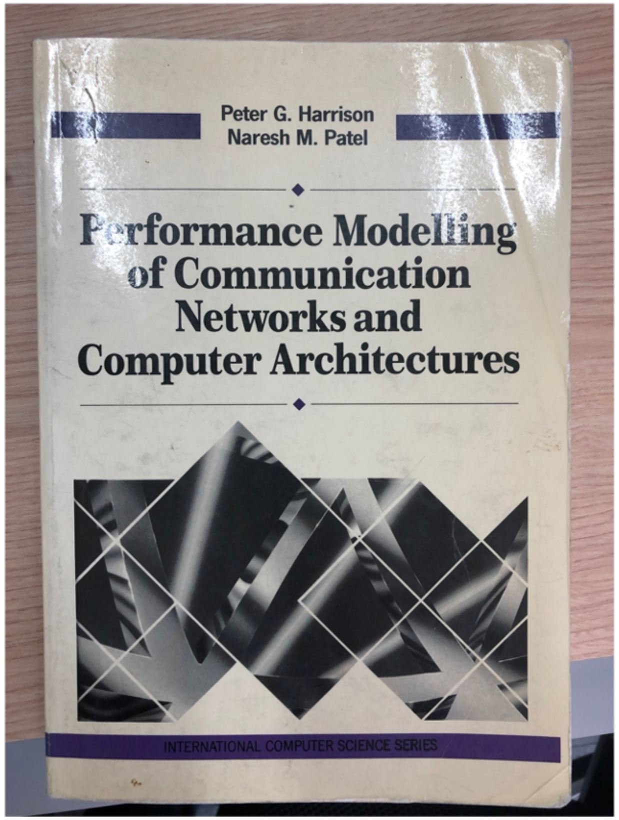
Textbook: Computer Networking



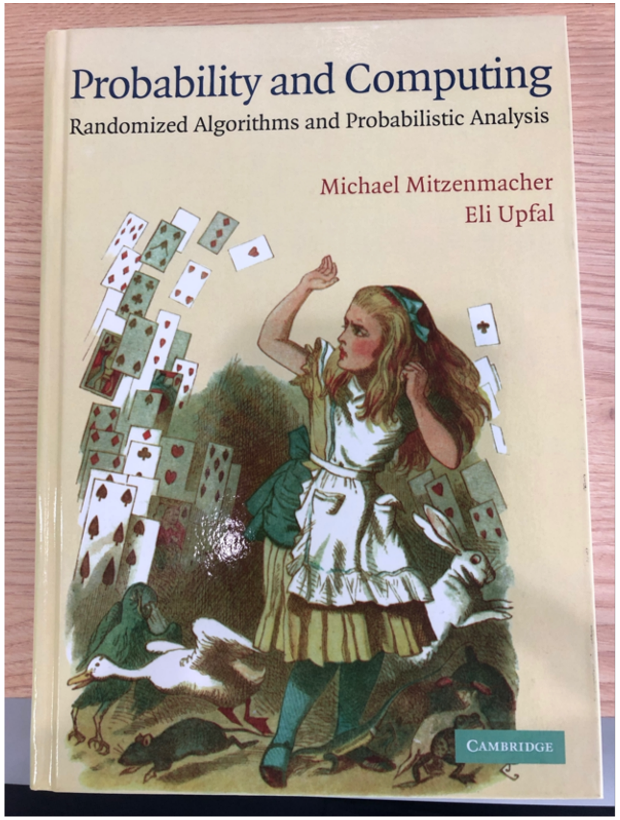
Preface

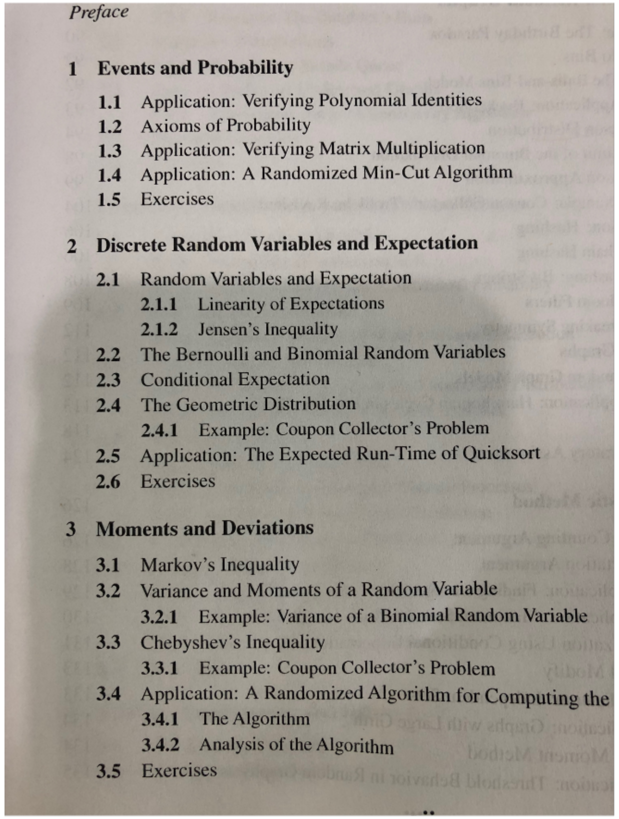
Chapter 1 Essentials of Probability Theory
- 1.1 Sample space, events and probability
- 1.2 Conditional probability
- 1.3 Independence
- Exercises

Chapter 2 Random Variables and Distributions
- 2.1 Probability distribution functions
- 2.2 Discrete random variables
- 2.3 Continuous random variables
- 2.4 Joint random variables
- 2.5 Conditional distributions
- 2.6 Independence and sums
- Exercises

Chapter 3 Expected Values and Moments
- 3.1 Expectation
- 3.2 Generating functions and transforms
- 3.3 Asymptotic properties
- Exercises

Textbook: Algorithm and Computing





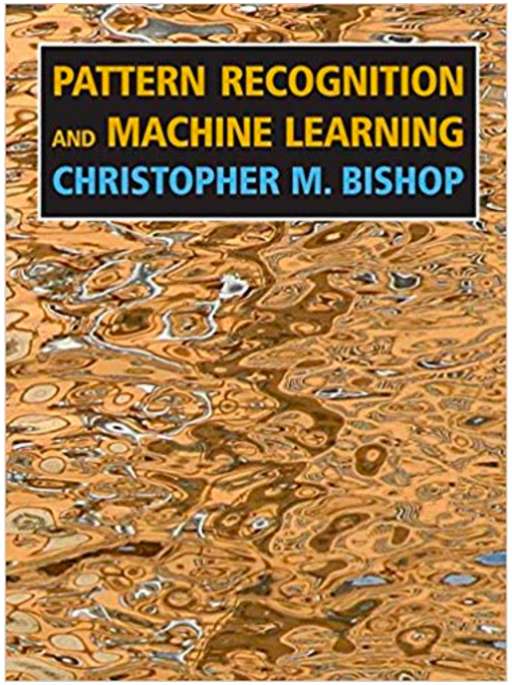
Preface

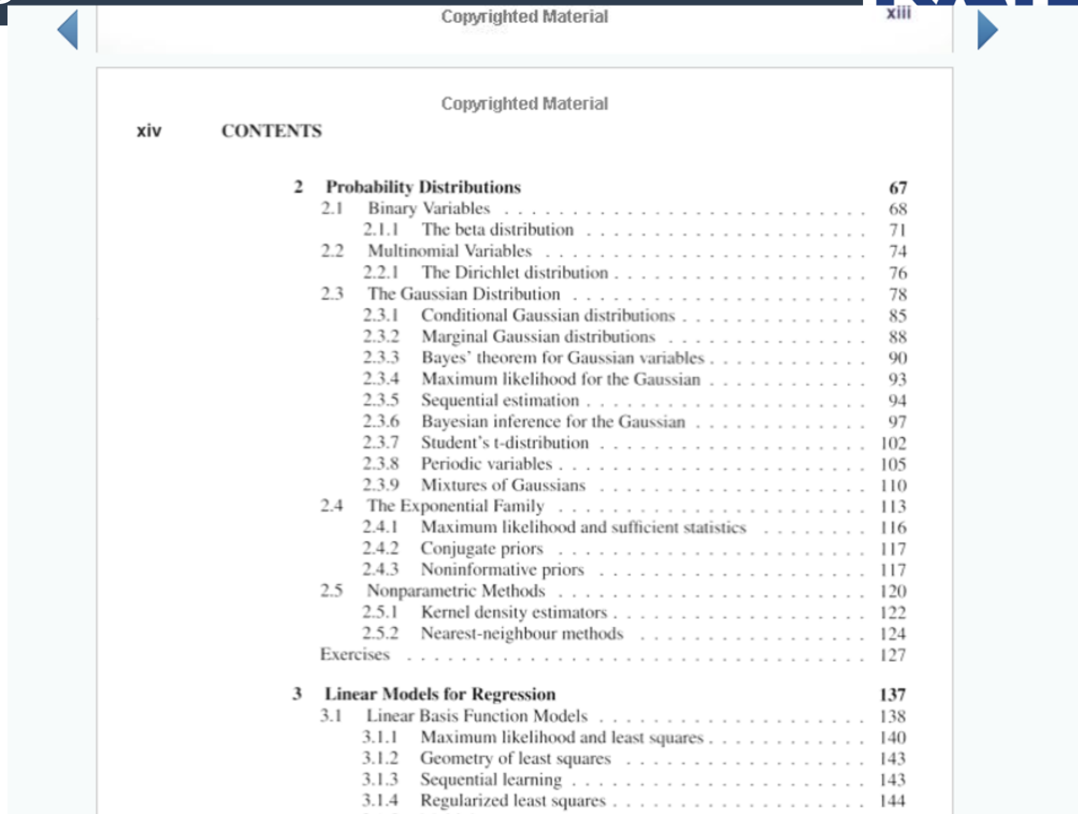
1 **Events and Probability**
- **1.1** Application: Verifying Polynomial Identities
- **1.2** Axioms of Probability
- **1.3** Application: Verifying Matrix Multiplication
- **1.4** Application: A Randomized Min-Cut Algorithm
- **1.5** Exercises

2 **Discrete Random Variables and Expectation**
- **2.1** Random Variables and Expectation
  - **2.1.1** Linearity of Expectations
  - **2.1.2** Jensen's Inequality
- **2.2** The Bernoulli and Binomial Random Variables
- **2.3** Conditional Expectation
- **2.4** The Geometric Distribution
  - **2.4.1** Example: Coupon Collector's Problem
- **2.5** Application: The Expected Run-Time of Quicksort
- **2.6** Exercises

3 **Moments and Deviations**
- **3.1** Markov's Inequality
- **3.2** Variance and Moments of a Random Variable
  - **3.2.1** Example: Variance of a Binomial Random Variable
- **3.3** Chebyshev's Inequality
  - **3.3.1** Example: Coupon Collector's Problem
- **3.4** Application: A Randomized Algorithm for Computing the
  - **3.4.1** The Algorithm
  - **3.4.2** Analysis of the Algorithm
- **3.5** Exercises

Textbook: Machine Learning





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These days, every area in CS and EE is directly or indirectly related to machine learning!

How to take this course? A designer's perspective

- Designer's perspective?
- In the year of 2022, suppose that unfortunately there is no theory of mathematically studying the *uncertainty* of some phenomena, events, etc.
- You have to design such a theory called "probability". How are you going to do it? Where are you going to start?
- You just have other basic mathematical theories such as set theory.
- You need to get used to the English terms on probability (e.g., sample space = 표본공간, probability density function = 확률밀도함수).
- We will take this exciting journey from the next lecture!

Questions?