

# Lecture 1: Probabilistic Model

Yi, Yung (이웅)

EE210: Probability and Introductory Random Processes  
KAIST EE

June 12, 2021

- (1) Probabilistic Model
  - Mathematical description of uncertain situations
  
- (2) Sample Space, Event, Probability Law
  - Elements of probability theory
  
- (3) Probability Axioms
  - 3 axioms for the completeness of a theory

- (1) Probabilistic Model
- (2) Sample Space, Event, Probability Law
- (3) Probability Axioms

**Modeling:** Understand reality with a simple (mathematical) model

- Experiment
  - Flip two coins
-

**Modeling:** Understand reality with a simple (mathematical) model

- Experiment
    - Flip two coins
  - Observation: a random outcome
    - for example,  $(H, H)$
-

**Modeling:** Understand reality with a simple (mathematical) model

- Experiment
    - Flip two coins
  - Observation: a random outcome
    - for example,  $(H, H)$
  - All outcomes
    - $\{(H, H), (H, T), (T, H), (T, T)\}$
-

**Modeling:** Understand reality with a simple (mathematical) model

- Experiment
    - Flip two coins
  - Observation: a random outcome
    - for example,  $(H, H)$
  - All outcomes
    - $\{(H, H), (H, T), (T, H), (T, T)\}$
- 

- **Our goal:** Build up a  for an experiment with random outcomes

**Modeling:** Understand reality with a simple (mathematical) model

- Experiment
  - Flip two coins
- Observation: a random outcome
  - for example,  $(H, H)$
- All outcomes
  - $\{(H, H), (H, T), (T, H), (T, T)\}$

- 
- **Our goal:** Build up a **probabilistic model** for an experiment with random outcomes



**Modeling:** Understand reality with a simple (mathematical) model

- Experiment
  - Flip two coins
- Observation: a random outcome
  - for example,  $(H, H)$
- All outcomes
  - $\{(H, H), (H, T), (T, H), (T, T)\}$

- 
- **Our goal:** Build up a **probabilistic model** for an experiment with random outcomes
  - **Probabilistic model?**
    - Assign a number to each outcome or a set of outcomes
    - Mathematical description of an uncertain situation

**Modeling:** Understand reality with a simple (mathematical) model

- Experiment
  - Flip two coins
- Observation: a random outcome
  - for example,  $(H, H)$
- All outcomes
  - $\{(H, H), (H, T), (T, H), (T, T)\}$

- 
- **Our goal:** Build up a **probabilistic model** for an experiment with random outcomes
  - **Probabilistic model?**
    - Assign a number to each outcome or a set of outcomes
    - Mathematical description of an uncertain situation
  - Which model is good or bad?

**Goal:** Build up a probabilistic model. Hmm... How?

The first thing:

**Question:**

**Goal:** Build up a probabilistic model. Hmm... How?

The first thing: What are the *elements* of a probabilistic model?

**Question:**

**Goal:** Build up a probabilistic model. Hmm... How?

The first thing: What are the *elements* of a probabilistic model?

## Elements of Probabilistic Model

1. All outcomes of my interest:

2. Assigned numbers to each outcome of  $\Omega$ :

**Question:**

**Goal:** Build up a probabilistic model. Hmm... How?

The first thing: What are the *elements* of a probabilistic model?

## Elements of Probabilistic Model

1. All outcomes of my interest:

2. Assigned numbers to each outcome of  $\Omega$ :

**Question:**

**Goal:** Build up a probabilistic model. Hmm... How?

The first thing: What are the *elements* of a probabilistic model?

## Elements of Probabilistic Model

1. All outcomes of my interest: **Sample Space  $\Omega$**

2. Assigned numbers to each outcome of  $\Omega$ : **Probability Law  $\mathbb{P}(\cdot)$**

**Question:**

**Goal:** Build up a probabilistic model. Hmm... How?

The first thing: What are the *elements* of a probabilistic model?

## Elements of Probabilistic Model

1. All outcomes of my interest: **Sample Space  $\Omega$**

2. Assigned numbers to each outcome of  $\Omega$ : **Probability Law  $\mathbb{P}(\cdot)$**

**Question:** What are the conditions of  $\Omega$  and  $\mathbb{P}(\cdot)$  under which their induced probability model becomes "legitimate"?



- (1) Probabilistic Model
- (2) Sample Space, Event, Probability Law
- (3) Probability Axioms

# 1. Sample Space $\Omega$

The set of all outcomes of my interest

The set of all outcomes of my interest

(1) Mutually exclusive

1. Toss a coin. What about this?  
 $\Omega = \{H, T, HT\}$

The set of all outcomes of my interest

(1) Mutually exclusive

(2) Collectively exhaustive

1. Toss a coin. What about this?

$$\Omega = \{H, T, HT\}$$

2. Toss a coin. What about this?  $\Omega = \{H\}$

The set of all outcomes of my interest

- (1) Mutually exclusive
- (2) Collectively exhaustive
- (3) At the right granularity  
(not too concrete, not too abstract)

- 1. Toss a coin. What about this?  
 $\Omega = \{H, T, HT\}$
- 2. Toss a coin. What about this?  $\Omega = \{H\}$
- 3. (a) Just figuring out prob. of H or T.  
 $\implies \Omega = \{H, T\}$

The set of all outcomes of my interest

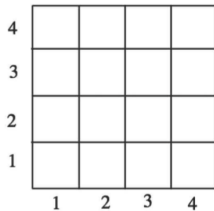
- (1) Mutually exclusive
- (2) Collectively exhaustive
- (3) At the right granularity  
(not too concrete, not too abstract)

- 1. Toss a coin. What about this?  
 $\Omega = \{H, T, HT\}$
- 2. Toss a coin. What about this?  $\Omega = \{H\}$
- 3. (a) Just figuring out prob. of H or T.  
 $\implies \Omega = \{H, T\}$   
  
(b) The impact of the weather (rain or no rain) on the coin's behavior.  
  
 $\implies \Omega = \{(H, R), (T, R), (H, NR), (T, NR)\},$   
  
R(Rain), NR(No Rain).



- *Discrete case:* Two rolls of a tetrahedral die

-  $\Omega = \{(1, 1), (1, 2), \dots, (4, 4)\}$

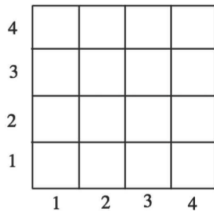


4				
3				
2				
1				
	1	2	3	4



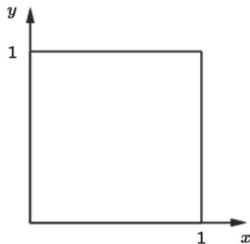
- *Discrete case:* Two rolls of a tetrahedral die

-  $\Omega = \{(1, 1), (1, 2), \dots, (4, 4)\}$



- *Continuous case:* Dropping a needle in a plain

-  $\Omega = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x, y \leq 1\}$





- Assign numbers to what? Each outcome?

- Assign numbers to what? Each outcome?
- What is the probability of dropping a needle at  $(0.5, 0.5)$  over the  $1 \times 1$  plane?

- Assign numbers to what? Each outcome?
- What is the probability of dropping a needle at  $(0.5, 0.5)$  over the  $1 \times 1$  plane?
- Assign numbers to each  of  $\Omega$

- Assign numbers to what? Each outcome?
- What is the probability of dropping a needle at  $(0.5, 0.5)$  over the  $1 \times 1$  plane?
- Assign numbers to each subset of  $\Omega$

- Assign numbers to what? Each outcome?
- What is the probability of dropping a needle at  $(0.5, 0.5)$  over the  $1 \times 1$  plane?
- Assign numbers to each subset of  $\Omega$
- a subset of  $\Omega$ :

- Assign numbers to what? Each outcome?
- What is the probability of dropping a needle at  $(0.5, 0.5)$  over the  $1 \times 1$  plane?
- Assign numbers to each subset of  $\Omega$
- a subset of  $\Omega$ : an event



- Assign numbers to what? Each outcome?
- What is the probability of dropping a needle at  $(0.5, 0.5)$  over the  $1 \times 1$  plane?
- Assign numbers to each subset of  $\Omega$
- a subset of  $\Omega$ : an event
- $\mathbb{P}(A)$ : Probability of an event  $A$ .
  - This is where probability meets set theory.

- Assign numbers to what? Each outcome?
- What is the probability of dropping a needle at  $(0.5, 0.5)$  over the  $1 \times 1$  plane?
- Assign numbers to each subset of  $\Omega$
- a subset of  $\Omega$ : an event
- $\mathbb{P}(A)$ : Probability of an event  $A$ .
  - This is where probability meets set theory.
- Roll a dice. What is the probability of odd numbers?

$\mathbb{P}(\{1, 3, 5\})$ , where  $\{1, 3, 5\} \subset \Omega$  is an event.

- (1) Probabilistic Model
- (2) Sample Space, Event, Probability Law
- (3) Probability Axioms

- Need to construct  $\mathbb{P}(\cdot)$  that naturally satisfies the intention of a probability theory designer just like you. What about the followings as starting points?

---

<sup>1</sup>Their intersection is empty.

- Need to construct  $\mathbb{P}(\cdot)$  that naturally satisfies the intention of a probability theory designer just like you. What about the followings as starting points?
  - $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

---

<sup>1</sup>Their intersection is empty.

- Need to construct  $\mathbb{P}(\cdot)$  that naturally satisfies the intention of a probability theory designer just like you. What about the followings as starting points?
  - $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$
  - $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$
  - $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$
  - For two disjoint<sup>1</sup> events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

---

<sup>1</sup>Their intersection is empty.

- Need to construct  $\mathbb{P}(\cdot)$  that naturally satisfies the intention of a probability theory designer just like you. What about the followings as starting points?
  - $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$
  - $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$
  - $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$
  - For two disjoint<sup>1</sup> events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$
  - $\mathbb{P}(\Omega) = 1$  (Why not  $\mathbb{P}(\Omega) = 10$ ?)
  - $\mathbb{P}(\emptyset) = 0$

---

<sup>1</sup>Their intersection is empty.

- Need to construct  $\mathbb{P}(\cdot)$  that naturally satisfies the intention of a probability theory designer just like you. What about the followings as starting points?
  - $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$
  - $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$
  - $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$
  - For two disjoint<sup>1</sup> events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$
  - $\mathbb{P}(\Omega) = 1$  (Why not  $\mathbb{P}(\Omega) = 10$ ?)
  - $\mathbb{P}(\emptyset) = 0$
  - If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$
  - many others

---

<sup>1</sup>Their intersection is empty.



- Surprisingly, we need just the following three rules (called **axioms**):

- Surprisingly, we need just the following three rules (called **axioms**):

## Probability Axioms: Version 1

A1. **Nonnegativity**:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

A2. **Normalization**:  $\mathbb{P}(\Omega) = 1$

A3. **(Finite) additivity**: For two disjoint events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

- Surprisingly, we need just the following three rules (called **axioms**):

## Probability Axioms: Version 1

A1. **Nonnegativity**:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

A2. **Normalization**:  $\mathbb{P}(\Omega) = 1$

A3. **(Finite) additivity**: For two disjoint events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

- No other things are necessary, and we can prove all other things from the above axioms.

- Surprisingly, we need just the following three rules (called **axioms**):

## Probability Axioms: Version 1

A1. **Nonnegativity**:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

A2. **Normalization**:  $\mathbb{P}(\Omega) = 1$

A3. **(Finite) additivity**: For two disjoint events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

- No other things are necessary, and we can prove all other things from the above axioms.
- Note that coming up with the above axioms is far from trivial.

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

VIDEO PAUSE

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

2.  $\mathbb{P}(\emptyset) = 0$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c)$$

2.  $\mathbb{P}(\emptyset) = 0$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c)$$

2.  $\mathbb{P}(\emptyset) = 0$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c) \implies \mathbb{P}(A) = 1 - \mathbb{P}(A^c)$$

2.  $\mathbb{P}(\emptyset) = 0$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$



A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c) \implies \mathbb{P}(A) = 1 - \mathbb{P}(A^c) \stackrel{A1}{\leq} 1$$

2.  $\mathbb{P}(\emptyset) = 0$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c) \implies \mathbb{P}(A) = 1 - \mathbb{P}(A^c) \stackrel{A1}{\leq} 1$$

2.  $\mathbb{P}(\emptyset) = 0$

$$\mathbb{P}(\Omega \cup \emptyset) \stackrel{A3}{=} \mathbb{P}(\Omega) + \mathbb{P}(\emptyset)$$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c) \implies \mathbb{P}(A) = 1 - \mathbb{P}(A^c) \stackrel{A1}{\leq} 1$$

2.  $\mathbb{P}(\emptyset) = 0$

$$\mathbb{P}(\Omega \cup \emptyset) \stackrel{A3}{=} \mathbb{P}(\Omega) + \mathbb{P}(\emptyset) \stackrel{A2}{=} 1 + \mathbb{P}(\emptyset)$$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c) \implies \mathbb{P}(A) = 1 - \mathbb{P}(A^c) \stackrel{A1}{\leq} 1$$

2.  $\mathbb{P}(\emptyset) = 0$

$$\mathbb{P}(\Omega \cup \emptyset) \stackrel{A3}{=} \mathbb{P}(\Omega) + \mathbb{P}(\emptyset) \stackrel{A2}{=} 1 + \mathbb{P}(\emptyset) \xrightarrow{\text{from 1.}} \mathbb{P}(\emptyset) = 0$$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c) \implies \mathbb{P}(A) = 1 - \mathbb{P}(A^c) \stackrel{A1}{\leq} 1$$

2.  $\mathbb{P}(\emptyset) = 0$

$$\mathbb{P}(\Omega \cup \emptyset) \stackrel{A3}{=} \mathbb{P}(\Omega) + \mathbb{P}(\emptyset) \stackrel{A2}{=} 1 + \mathbb{P}(\emptyset) \stackrel{\text{from 1.}}{\implies} \mathbb{P}(\emptyset) = 0$$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

$$\mathbb{P}(B) \stackrel{A3}{=}$$

A1: Nonnegativity, A2: Normalization, A3: Finite additivity

Prove the following properties using the axioms:

1. For any event  $A$ ,  $\mathbb{P}(A) \leq 1$

$$1 \stackrel{A2}{=} \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(A^c) \implies \mathbb{P}(A) = 1 - \mathbb{P}(A^c) \stackrel{A1}{\leq} 1$$

2.  $\mathbb{P}(\emptyset) = 0$

$$\mathbb{P}(\Omega \cup \emptyset) \stackrel{A3}{=} \mathbb{P}(\Omega) + \mathbb{P}(\emptyset) \stackrel{A2}{=} 1 + \mathbb{P}(\emptyset) \xrightarrow{\text{from 1.}} \mathbb{P}(\emptyset) = 0$$

3. If  $A \subset B$ ,  $\mathbb{P}(A) \leq \mathbb{P}(B)$

$$\mathbb{P}(B) \stackrel{A3}{=} \mathbb{P}(A) + \mathbb{P}(B \setminus A) \stackrel{A1}{\geq} \mathbb{P}(A)$$

1. Specify the sample space
2. Specify a probability law
  - from my earlier belief, from data, from expert's opinion
3. Identify an event of interest
4. Calculate

Toss a (biased) coin

1.  $\Omega = \{H, T\}$
2.  $\mathbb{P}(\{H\}) = 1/4, \mathbb{P}(\{T\}) = 3/4,$
3. probability of head or tail
4.  $1/4, 3/4$

- $\Omega = \{1, 2, 3, \dots\}, \mathbb{P}(\{n\}) = \frac{1}{2^n}, n = 1, 2, \dots$



- $\Omega = \{1, 2, 3, \dots\}$ ,  $\mathbb{P}(\{n\}) = \frac{1}{2^n}$ ,  $n = 1, 2, \dots$
- Is the above probability law legitimate? seems OK

- $\Omega = \{1, 2, 3, \dots\}$ ,  $\mathbb{P}(\{n\}) = \frac{1}{2^n}$ ,  $n = 1, 2, \dots$
- Is the above probability law legitimate? seems OK

$$\mathbb{P}(\Omega) = \frac{1}{2} + \frac{1}{2^2} + \dots = \frac{1/2}{1 - 1/2} = 1$$

- $\Omega = \{1, 2, 3, \dots\}$ ,  $\mathbb{P}(\{n\}) = \frac{1}{2^n}$ ,  $n = 1, 2, \dots$
- Is the above probability law legitimate? seems OK

$$\mathbb{P}(\Omega) = \frac{1}{2} + \frac{1}{2^2} + \dots = \frac{1/2}{1 - 1/2} = 1$$

- $\mathbb{P}(\text{even numbers})?$

- $\Omega = \{1, 2, 3, \dots\}$ ,  $\mathbb{P}(\{n\}) = \frac{1}{2^n}$ ,  $n = 1, 2, \dots$
- Is the above probability law legitimate? seems OK

$$\mathbb{P}(\Omega) = \frac{1}{2} + \frac{1}{2^2} + \dots = \frac{1/2}{1 - 1/2} = 1$$

- $\mathbb{P}(\text{even numbers})?$

$$\begin{aligned}\mathbb{P}(\text{even}) &= \mathbb{P}(\{2, 4, 6, \dots\}) \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots = 1/3\end{aligned}$$

- $\Omega = \{1, 2, 3, \dots\}$ ,  $\mathbb{P}(\{n\}) = \frac{1}{2^n}$ ,  $n = 1, 2, \dots$
- Is the above probability law legitimate? seems OK

$$\mathbb{P}(\Omega) = \frac{1}{2} + \frac{1}{2^2} + \dots = \frac{1/2}{1 - 1/2} = 1$$

- $\mathbb{P}(\text{even numbers})?$

$$\begin{aligned}\mathbb{P}(\text{even}) &= \mathbb{P}(\{2, 4, 6, \dots\}) \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots = 1/3\end{aligned}$$

- Is the above right? If not, why?

- $\Omega = \{1, 2, 3, \dots\}$ ,  $\mathbb{P}(\{n\}) = \frac{1}{2^n}$ ,  $n = 1, 2, \dots$
- Is the above probability law legitimate? seems OK

$$\mathbb{P}(\Omega) = \frac{1}{2} + \frac{1}{2^2} + \dots = \frac{1/2}{1 - 1/2} = 1$$

- $\mathbb{P}(\text{even numbers})?$

$$\begin{aligned}\mathbb{P}(\text{even}) &= \mathbb{P}(\{2, 4, 6, \dots\}) \\ &= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots = 1/3\end{aligned}$$

- Is the above right? If not, why?
  - Wrong: **Finite** additivity axiom does not allow this.

## Probability Axioms: Version 1

A1. Nonnegativity:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

A2. Normalization:  $\mathbb{P}(\Omega) = 1$

A3. (Finite) additivity: For two disjoint events  $A$  and  $B$ ,  $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

## Probability Axioms: Version 2

A1. Nonnegativity:  $\mathbb{P}(A) \geq 0$  for any event  $A \subset \Omega$

A2. Normalization:  $\mathbb{P}(\Omega) = 1$

A3. **Countable additivity:** If  $A_1, A_2, A_3, \dots$  is an infinite sequence of disjoint events, then  $\mathbb{P}(A_1 \cup A_2 \cup \dots) = \mathbb{P}(A_1) + \mathbb{P}(A_2) + \dots$ .



- A narrow view: A branch of math
  - axioms  $\rightarrow$  theorems
  - Mathematicians work very hard to find the smallest set of necessary axioms (just like atoms in physics)

Anyway, we believe that probabilistic reasoning is very helpful to understand the world with many uncertain situations.

- A narrow view: A branch of math
  - axioms  $\rightarrow$  theorems
  - Mathematicians work very hard to find the smallest set of necessary axioms (just like atoms in physics)
- Frequencies:  $\mathbb{P}(H) = 1/2$ 
  - Understanding an uncertain situation: fractions of successes out of many experiments

Anyway, we believe that probabilistic reasoning is very helpful to understand the world with many uncertain situations.

- A narrow view: A branch of math
  - axioms  $\rightarrow$  theorems
  - Mathematicians work very hard to find the smallest set of necessary axioms (just like atoms in physics)
- Frequencies:  $\mathbb{P}(H) = 1/2$ 
  - Understanding an uncertain situation: fractions of successes out of many experiments
- Beliefs:  $\mathbb{P}(\text{He is reelected}) = 0.7$

Anyway, we believe that probabilistic reasoning is very helpful to understand the world with many uncertain situations.

Questions?

You build up the very basics of a probabilistic model.

What else do we need to build up?

- 1) Please explain what a probabilistic model is and why we need it.
- 2) What is the mathematical definition of event?
- 3) What are the key elements of the probabilistic model?
- 4) Please list up the probability axioms and explain them.
- 5) Why do we need countable additivity in the probability axioms?