博弈论第六次作业

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[Title]:

In the social choice problem, consider the case which allows the use of "money": For each voter i, suppose its valuation function is $v_i:A\to R_+$. If the final winner is $a\in A$, the social welfare is $\sum v_i(a)$.

We want to design a mechanism which maximizes social welfare, and we can ask each voter to pay some money. Use VCG technique to design a truthful mechanism. Your mechanism should include an allocation rule and a payment rule.

[答]: 由题意可知,这是一个多参数机制环境. 对于投票者 i,会对每一个 $a \in A$ 给出估值 $v_i(a)$. 同时,在该机制是诚实的设定下,可知对每一位投票者报价时都会给出自己真实的估值,即 b = v.

因此,在社会福利最大化的要求之下,我们定义分配规则 x 为:

$$x(v) = \underset{a \in A}{\operatorname{argmax}} \sum_{i} v_i(a)$$

再由 VCG 机制的定义给出支付规则 p 为:

$$p_i(v) = \max_{a \in A} \sum_{j \neq i} v_j(a) - \sum_{j \neq i} v_j(a*)$$

其中 a* = x(v), 是分配规则得到的结果.