Introduction to Algorithms: 6.006
Massachusetts Institute of Technology
Instructors: Erik Demaine, Jason Ku, and Justin Solomon Problem Set 0

Problem Set 0

Name: BUAA-TYZ

Problem 0-1.

Known: $A = \{i + \binom{5}{i}, i \in \mathbb{Z}\ and\ 0 \leq i \geq 4\} and\ B = \{3i \mid i\ \in\ \{1, 2, 4, 5\}\}$

Enumerating i, we include that $A = \{1, 6, 12, 13, 9\}\ and\ B = \{3, 6, 12, 15\}$

(a) $A \cap B = \{6, 12\}$

(b) $A \cup B = \{1, 3, 6, 9, 12, 13, 15\} \Rightarrow |A \cup B| = 7$

(c) $A - B = \{1, 9, 13\} \Rightarrow |A - B| = 3$

Problem 0-2. Known: X be the random variable representing the number of heads seen after flipping a fair coin three times. Let Y be the random variable representing the outcome of rolling two fair six-sided dice and multiplying their values.

(a) The possible value of X is $\{0, 1, 2, 3\}$
$E[X] = \sum_{i=0}^{3} xp(x) = 0 * \frac{1}{8} + 1 * \frac{3}{8} + 2 * \frac{3}{8} + 3 * \frac{1}{8} = 1.5$

(b) The possible value of Y is $\{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\}$
$E[Y] = ... = 12.25$

(c) $E[X + Y] = E[X] + E[Y] = 13.75$

Problem 0-3. Known: $A = 6000/6 = 100 \quad and \quad B = 60\ mod\ 42 = 18$

(a) True

(b) False

(c) False

Problem 0-4. Prove by induction that $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2, \forall\ n \geq 1$

- Base case: $n = 1:$ Obviously.
- Suppose $n = k:$ Assume the formula is right. Then for $n = k + 1$, we have:

$$\sum_{i=1}^{k+1} i^3 = \sum_{i=1}^{k} i^3 + (k+1)^3 = \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3 = (k+1)^2 * (\frac{k^2}{4} + k + 1) = \left[\frac{(k+1)(k+2)}{2}\right]^2$$

- Conclusion: $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2, \forall\ n \geq 1$

Problem 0-5. Prove by induction that every connected undirected graph $G = (V, E)$ for which $|E| = |V| - 1$ is acyclic.

- Base case: $|E| = 1$: Two nodes and one edge, there is no cycle.
- Suppose that $|E| = k$: Assume this is right. Then we add one node and one edge, so this node can only be linked to one another node, which can't bring a cycle.
- Conclusion: Every connected undirected graph $G = (V, E)$ for which $|E| = |V| - 1$ is acyclic.

Problem 0-6. Submit your implementation to alg.mit.edu.

```python
def count_long_subarray(A):
    '''
    Input:  A     | Python Tuple of positive integers
    Output: count | number of longest increasing subarrays of A
    '''
    max_length_count = 0
    ##################
    # YOUR CODE HERE #
    ##################
    local_max_length = 1
    global_max_length = 1
    for i in range(1, length):
        if A[i] > A[i - 1]:
            local_max_length += 1
        else:
            local_max_length = 1
        if local_max_length == global_max_length:
            max_length_count += 1
        elif local_max_length > global_max_length:
            global_max_length = local_max_length
            max_length_count = 1
    return max_length_count
```