

# Engineering Computation Project

## Modelling Wireless Communication Channels

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## 1 Introduction

This report explores the properties of various orthogonal function sets and their applicability to generating fits to real-life data.

## 2 Orthogonality

Orthogonality is a crucial mathematical concept and extends from the well-known geometrical application (vectors) to the analysis of functions.

We can define a set of orthogonal functions  $g_0, g_1, \dots, g_n$  whose linear combination can be used to fit an arbitrary function  $f$ :

$$f(x) = a_0g_0(x) + a_1g_1(x) + a_2g_2(x) + \dots + a_ng_n \quad (2.1)$$

We then define an inner product operation, for which, conveniently:

$$\langle g_n, g_m \rangle = 0 \quad \text{if } n \neq m \quad (2.2)$$

So that if we compute the inner product of both LHS and RHS with some arbitrary  $g_k$ , like:

$$\langle f, g_k \rangle = a_0\langle g_0, g_k \rangle + a_1\langle g_1, g_k \rangle + \dots + a_k\langle g_k, g_k \rangle + \dots + a_n\langle g_n, g_k \rangle \quad (2.3)$$

which lets us calculate the  $a_k$  coefficient as:

$$a_k = \frac{\langle f, g_k \rangle}{\langle g_k, g_k \rangle} \quad (2.4)$$

This result is crucial, as it demonstrates that the coefficients are independent of the size of the orthogonal basis used for the investigation i.e. if more functions are decided to be added, old coefficients do not have to be recomputed.

Investigation of orthogonal function fitting can be started with analysing the Fourier series. The set of sines and cosines making up the basis turn out to be orthogonal under integral operation over one period. Even though useful, Fourier series is quite a basic method for function fitting and will not be analysed in detail in this report. We will instead delve into the investigation of two particularly relevant orthogonal basis functions - Gram-Schmidt and Laguerre.

## 3 Gram-Schmidt functions

### 3.1 Process

The Gram-Schmidt process stems from a very simple logic for generating orthogonal functions - choosing a linearly independent basis, and adjusting the consecutive functions appropriately, making sure that every subsequent function is orthogonal to the previous one.

Hence having the linearly independent basis  $v_0, v_1, v_2, \dots, v_n$  we can compute the orthogonal functions  $g_0, g_1, g_2, \dots, g_n$  using:

$$g_0(x) = v_0(x) \quad (3.1)$$

$$g_1(x) = v_1(x) - e_{10}g_0(x) \quad (3.2)$$

$$g_2(x) = v_2(x) - e_{20}g_0 - e_{21}g_1(x) \quad (3.3)$$

$$g_3(x) = v_3(x) - e_{30}g_0 - e_{31}g_1(x) - e_{32}g_2(x) \quad (3.4)$$

As noted before, using the orthogonality property we can now compute the  $e$  coefficients:

$$g_n(x) = v_n(x) - e_{n0}g_0 - \dots - e_{nm}g_m(x) - \dots - e_{n(n-1)}g_{n-1}(x) \quad (3.5)$$

Taking the inner products:

$$\langle g_n, g_m \rangle = \langle v_n, g_m \rangle - e_{n0}\langle g_0, g_m \rangle - \dots - e_{nm}\langle g_m, g_m \rangle - \dots - e_{n(n-1)}\langle g_{n-1}, g_m \rangle \quad (3.6)$$

$$0 = \langle v_n, g_m \rangle - 0 - \dots - e_{nm}\langle g_m, g_m \rangle - \dots - 0 \quad (3.7)$$

$$\Rightarrow e_{nm} = \frac{\langle v_n, g_m \rangle}{\langle g_m, g_m \rangle} \quad (3.8)$$

We can then divide each function by its norm (square root of its inner product with itself) in order to achieve an orthonormal set.

### 3.2 Monomial basis

The independent basis chosen is a set of monomials  $v_0 = 1, v_1 = x, v_2 = x^2, \dots, v_n = x^n$  which will be used to produce an orthogonal basis under operation  $\int_0^\infty g_n g_m e^{-x} dx$ .

### 3.3 How to Make Sections and Subsections

Use section and subsection commands to organize your document. L<sup>A</sup>T<sub>E</sub>X handles all the formatting and numbering automatically. Use `ref` and `label` commands for cross-references.

### 3.4 How to Make Lists

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**Word** Definition

**Concept** Explanation

**Idea** Text

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