数学分析公式结论总结 ——HYX_v1.0

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§ 0.常用公式表

三角函数

和差化积:

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

积化和差:

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin (\alpha + \beta) + \sin (\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin (\alpha + \beta) - \sin (\alpha - \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos (\alpha + \beta) + \cos (\alpha - \beta)]$$

$$\sin \alpha \sin \beta = -\frac{1}{2} [\cos (\alpha + \beta) - \cos (\alpha - \beta)]$$

半角公式:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

倍角公式:

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$$

$$1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}$$

$$\sec 2\alpha = \frac{\sec^2 \alpha + \csc^2 \alpha}{\csc^2 \alpha - \sec^2 \alpha} = \frac{\sec^2 \alpha \csc^2 \alpha}{\csc^2 \alpha - \sec^2 \alpha}$$

$$\csc 2\alpha = \frac{\sec^2 \alpha + \csc^2 \alpha}{2 \sec \alpha \csc \alpha} = \frac{\sec \alpha \csc \alpha}{2}$$

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

万能公式:

$$\sin lpha = rac{2 anrac{lpha}{2}}{1+ an^2rac{lpha}{2}} \ \cos lpha = rac{1- an^2rac{lpha}{2}}{1+ an^2rac{lpha}{2}} \ an lpha = rac{2 anrac{lpha}{2}}{1- an^2rac{lpha}{2}}$$

其他:

$$an lpha = rac{1}{\cot lpha}$$
 $\sin lpha = rac{1}{\csc lpha}$ $\cos lpha = rac{1}{\sec lpha}$

§ 1.数列极限

调和-几何-算术平均值不等式:

$$rac{n}{rac{1}{a_i}+rac{1}{a_i}+\cdots+rac{1}{a_i}} \leq \sqrt[n]{a_1a_2\cdots a_n} \leq rac{a_1+a_2+\cdots+a_n}{n} \quad (a_i>0, i=1,2,\cdots,n)$$

伯努利不等式:

$$(1+x)^n \geq 1+nx \quad (\forall x>-1, n\in \mathbb{N}^*)$$

柯西不等式:

$$(\sum_{i=1}^n a_i b_i)^2 \leq (\sum_{i=1}^n a_i^2)(\sum_{i=1}^n b_i^2)$$

二项式展开:

$$(a+b)^n = \sum_{k=0}^n C_n^k a^k b^{n-k}$$

因式分解:

$$a^{n} - b^{n} = (a - b)(a^{n-1} + a^{n-2}b + \dots + ab^{n-2} + b^{n-1})$$

闵科夫斯基不等式(可以通过几何意义来记忆):

$$egin{aligned} &(\sum_{i=1}^n{(a_i+b_i)^2})^{rac{1}{2}} \leq (\sum_{i=1}^n{a_i^2})^{rac{1}{2}} + (\sum_{i=1}^n{b_i^2})^{rac{1}{2}} \ &(\sum_{i=1}^n{(a_i+b_i)^p})^{rac{1}{p}} \leq (\sum_{i=1}^n{a_i^p})^{rac{1}{p}} + (\sum_{i=1}^n{b_i^p})^{rac{1}{p}} \end{aligned}$$

结论:

$$egin{aligned} &\lim_{n o\infty}n^{rac{1}{n}}=1\ &\lim_{n o\infty}rac{c^n}{n!}=0(c
eq0)\ &\lim_{n o\infty}rac{n^lpha}{c^n}=0(lpha>0,c>1) \end{aligned}$$

符号/定义:

- \forall : 任意选取 \exists : 存在 冒号: 满足的结论 $n!! = \begin{cases} 2 \cdot 4 \cdot 6 \cdots n & n \ mod \ 2 = 0 \\ 1 \cdot 3 \cdot 5 \cdots n & n \ mod \ 2 = 1 \end{cases}$
- 无穷小/无穷小量: 如果数列 a_n 的极限为零,那么称数列 a_n 为无穷小(量) 欧拉常数 (γ) : $1+\frac{1}{2}+\cdots+\frac{1}{n}-\ln n=\gamma+\epsilon(n),$ 其中 $\lim_{n\to\infty}\epsilon(n)=0$; 即调和级数与自然对数的差值的极
- 上/下确界: $sup E = \alpha, inf E = \beta$

数列极限定义:

$$orall \epsilon > 0, \exists N(\epsilon) \in \mathbb{N}^*, orall n > N: |a_n - a| < \epsilon$$

数列极限的保序性(取0时为保号性):

- (1)设 $\lim a_n = a, \alpha < a < eta$,则存在 $N \in \mathbb{N}^*$,使得当n > N时,有 $\alpha < a_n < eta$
- (2)设 $\lim_{n o\infty}a_n=a,\lim_{n o\infty}b_n=b$,且a< b,则存在 $N\in\mathbb{N}^*$,使得当n>N时,有 $a_n< b_n$
- (3)设 $\lim_{n o\infty}a_n=a,\lim_{n o\infty}b_n=b$,若存在 $N\in\mathbb{N}^*$,使得当n>N时,有 $a_n\leq b_n$,则 $a\leq b$

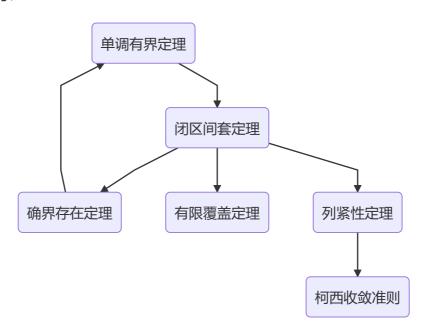
自然常数:

$$\lim_{n \to \infty} (1 + \frac{1}{n})^n = \lim_{n \to \infty} (1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}) = e$$

$$\lim_{n \to \infty} (1 + \frac{k}{n})^n = e^k, \qquad \lim_{n \to \infty} (1 - \frac{k}{n})^n = e^{-k} \quad (k \in \mathbb{N}^*)$$

$$x_n = (1 + \frac{1}{n})^n \text{ if } \text{if } \text{if } y_n = (1 + \frac{1}{n})^{n+1} \text{ if } \text{if } \text{$$

六大定理关系:



其他:

(1)对 $x \ge 0, y \ge 0, n \in \mathbb{N}^*$,有

$$(x+y)^n \ge x^n + y^n, \qquad (x^n + y^n)^{rac{1}{n}} \le x + y \ (x+y)^{rac{1}{n}} \le x^{rac{1}{n}} + y^{rac{1}{n}}, \qquad |x^{rac{1}{n}} - y^{rac{1}{n}}| \le |x-y|^{rac{1}{n}}$$

(2)

$$\lim_{n\to\infty}\left(a_1+a_2+\cdots+a_n\right)=s\Rightarrow \lim_{n\to\infty}\frac{a_1+2a_2+\cdots+na_n}{n}=0$$

(3)

$$n<\sqrt{(n-1)(n+1)}\Rightarrow rac{(2n-1)!!}{(2n)!!}<rac{1}{\sqrt{(2n+1)}}\quad (n\in\mathbb{N}^*)$$

(4)

$$\lim_{n \to \infty} \left(1 + \frac{1}{2^{\alpha}} + \dots + \frac{1}{n^{\alpha}} \right) = \begin{cases} \overline{\Lambda}$$
存在 $\alpha = 1$ 存在 $\alpha > 1$

№ 2.函数极限与连续