# Integration

$\frac{d}{dt}\mathbf{F}(t)=\mathbf{G}(t)$ $\int \mathbf{G}(t)dt=\mathbf{F}(t)+\mathbf{c}$ $\int_a^b \mathbf{G}(t)dt=[\mathbf{F}(t)]_a^b=\mathbf{F}(b)-\mathbf{F}(a)$

If $\mathbf{G}(t)=ti+t^2j+t^3k$ find $\int \mathbf{G}(t)dt$ and $\int_1^2 \mathbf{G}(t)dt$

$$\int \vec{G}(t)dt = \int (ti+t^2j+t^3k)dt = \frac{t^2}{2}i+\frac{t^3}{3}j+\frac{t^4}{4}k+\mathbf{c}$$

$$\int_1^2 \vec{G}(t)dt = \left[\frac{t^2}{2}i+\frac{t^3}{3}j+\frac{t^4}{4}k\right]_1^2 = \frac{3}{2}i+\frac{7}{3}j+\frac{17}{4}k$$

**Problem:** If $\mathbf{a}=xi-x^2j+(x-1)k$ and $\mathbf{b}=2x^2i+6xk$ find $\int_0^2 (\mathbf{a.b})dx$, $\int_0^2 (\mathbf{a}\mathrm{x}\mathbf{b})dx$ Ans 12, $-24i-\frac{40}{3}j+\frac{64}{5}k$

**Problem** The acceleration **a** of a particle at any time t is given by $\mathbf{a}=e^{-t}i-6(t+1)j+3\sin tk$. If the velocity **v** and displacement **r** are zero at time t=0 find **v** and **r** at any time.

$\frac{d\bar{v}}{dt}=\vec{a}$ $\vec{v}=\int \vec{a}dt$ $\vec{v}=(-e^{-t}i-6(\frac{t^2}{2}+t)j+3\cos tk)+\vec{c}$ at t=0, **v**=0 **c**=i+3k

$$\bar{v}=(1-e^{-t})i-(3t\quad+6t)j+(3-3\cos t)k$$

$\vec{v}=\frac{d\bar{r}}{dt}$ $\vec{r}=\int \vec{v}dt$ $r=(\qquad)+\vec{c}$ at t=0, r=0 **c**=-i

$$\bar{r}=(t-1+e^{-t})i-(t^3+3t^2)j+(3t+3\sin t)k$$

**Equation of Curve**

In ordinary calculus the equation of the curve is y=f(x),
In vector calculus the equation of the curve is **r=F(t)**

**Example** **r**=xi+yj x=cost, y=sint **r**=costi+sintj is the equation of the circle in vector calculus. **r=F(t)**

**Example r=**xi+yj   x=at$^2$,  y=2at        **r=** at$^2$i+2at j
is the equation of the parabola.            **r=F(**t**)**

## Three special types of integrals in vector calculus

1. **Line integral**
2. **Surface integral**
3. **Volume integral**

## 1. Line integral

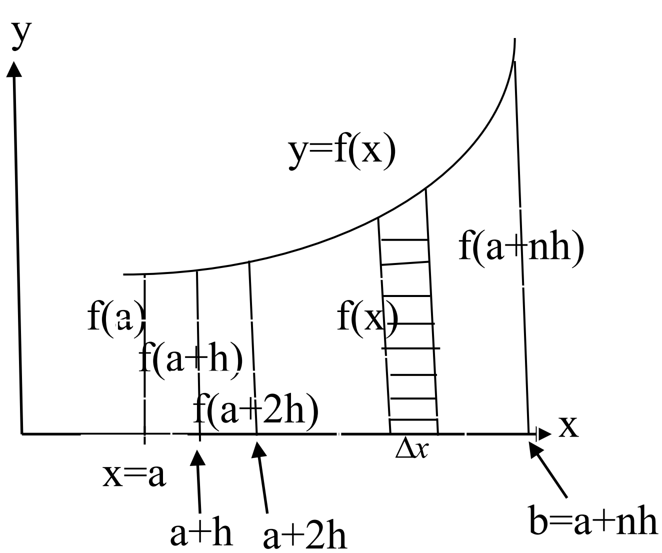
The line integrals are

1. $\int_C$ **A.**d**r**
2. $\int_C$ **A** x d**r,**
3. $\int_C$ $\phi$ d**r**

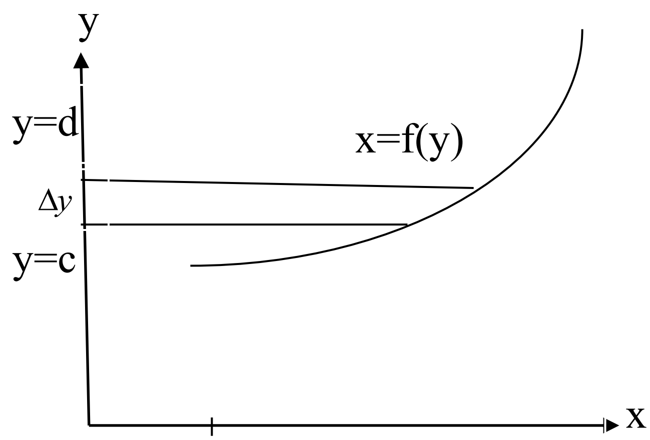
Explanation

Integration is usually done along x-axis, y-axis.

$\int_a^b f(x)dx$ $\int_a^b f(x)dx$




$\int f(x)dx$




$\int_a^b f(x)dx \int_a^b f(x)dx$

$\int f(y)dy$

. But integration is also done along any line and curve

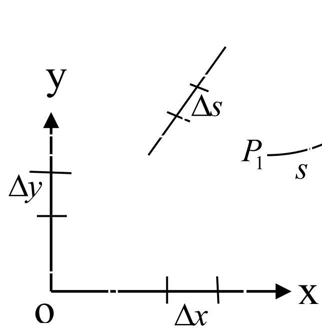



**fig.1**

$\int f(x)dx$ $\qquad$ $\int f(y)dy$, $\qquad$ $\int f(x,y)ds$ (line integral)

Three types of line integrals in vector calculus are

1. $\int_C$ **A.T**ds $\qquad$ or $\int_{P_1}^{P_2}$ **A.T**ds

2. $\int_C$ **A xT**ds $\qquad$ or $\int_{P_1}^{P_2}$ **A xT**ds

3. $\int_C \phi\, \mathbf{T} ds$ or $\int_{P_1}^{P_2} \phi\, \mathbf{T} ds$

where

$\mathbf{A}(x,y,z)=A_1(x,y,z)i+A_2(x,y,z)j+A_3(x,y,z)k$ is a vector function

**T**= unit tangent vector

s=length of the curve

$\Delta s$ =elementary length of the curve

Prove that d**r**= **T**ds

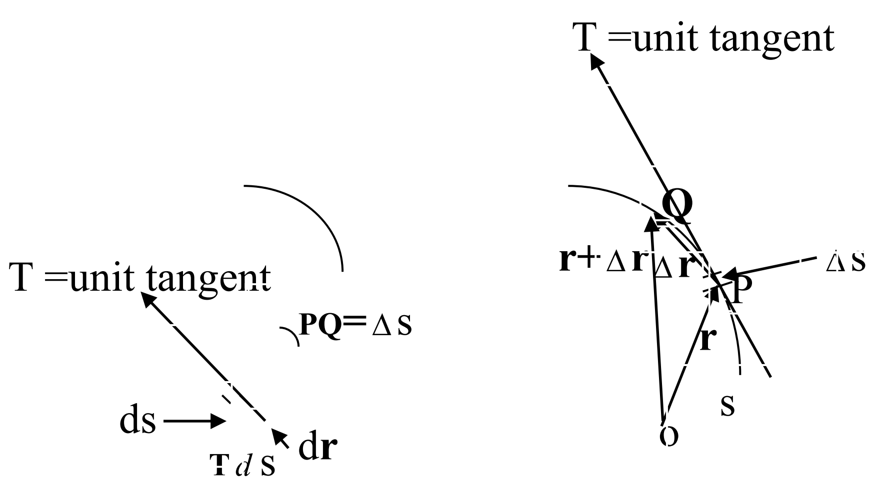



**fig.2**

we know $\vec{a} = a\hat{a}$

d**r**= **T**ds

d**r** is along the tangent and it is equal to **T**ds

$|\Delta \vec{r}| = \Delta s$ $|\vec{T}\Delta s| = \Delta s$

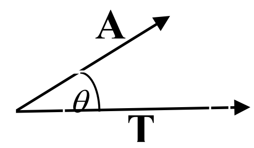
$\frac{d\vec{r}}{ds} = \vec{T}$ = unit tangent vector

So line integrals are

1. $\int_C \mathbf{A}.\mathbf{T}ds = \int_C \mathbf{A}.d\mathbf{r}$ or $\int_{P_1}^{P_2} \mathbf{A}.d\mathbf{r}$

2. $\int_C \mathbf{A}\times\mathbf{T}ds = \int_C \mathbf{A}\times d\mathbf{r}$, or $\int_{P_1}^{P_2} \mathbf{A}\times d\mathbf{r}$

3. $\int_C \phi\mathbf{T}ds = \int_C \phi\, d\mathbf{r}$ or $\int_{P_1}^{P_2} \phi\, d\mathbf{r}$




$\vec{A}.\vec{T} = A.1.\cos\theta = A\cos\theta$

$= \tan gential\ component\ of\ \vec{A}$

$\int_C \mathbf{A}.\mathbf{T}ds = \int_C \mathbf{A}.d\mathbf{r}$ is also called the line integral of the tangential component of **A**

$d\mathbf{r}=dx\mathbf{i}+dy\mathbf{j}+dz\mathbf{k}$

$\int_C \mathbf{A}.d\mathbf{r} = \int_C (A_1dx+A_2dy+A_3dz) = \int_{P_1}^{P_2} (A_1dx+A_2dy+A_3dz)$

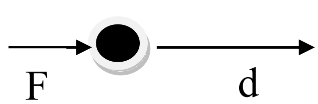
==================================================

**Physical meaning of line integral**

1. $\int_C \mathbf{A}.\mathbf{T}ds$ or $\int_{P_1}^{P_2} \mathbf{A}.\mathbf{T}ds$

If **A** is the force **F** on a particle moving along C this line integral represents the work done by the force **F**

Work=Force x displacement=Fd




W=F$\cos\theta$x d=**F.d** F$\cos\theta$ is the effective force

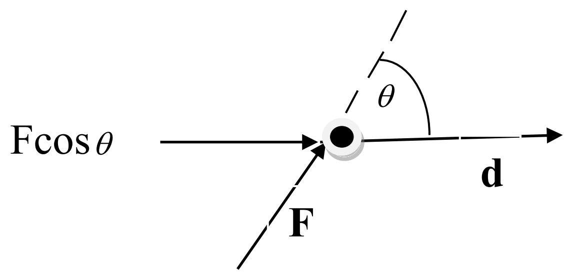



**fig.3**

**F**=2i+3j+4k   constant vector
**F**=-3x$^2$i+5xyj+2zk  variable vector

If a body moves along a curve by a variable force how can we measure the work done.
We consider a curve with elementary length $\Delta s$ then
**T**ds= d**r**

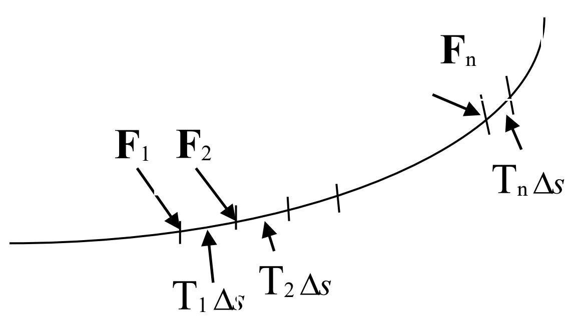
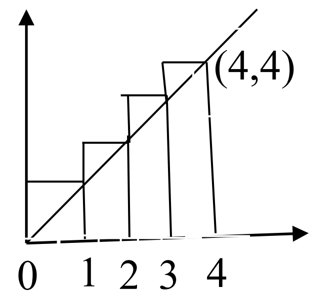


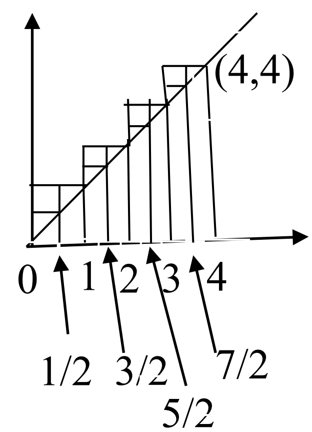

Fig 4

$$\text{Total work} = \mathbf{F}_1.\mathbf{T}_1\Delta s + \mathbf{F}_2.\mathbf{T}_2\Delta s + \mathbf{F}_3.\mathbf{T}_3\Delta s + \ldots\ldots\ldots\ldots \mathbf{F}_n.\mathbf{T}_n\Delta s$$
$$= {}_{Lt\,\Delta s\to 0}\sum \mathbf{F.T}\,\Delta s$$
$$= \int_C \mathbf{F.T}ds$$
$$= \int_C \mathbf{F}.d\mathbf{r}$$

Hence **W**= $\int_C$ **F**.d**r**

---




**Explanation of** $\underset{\Delta x \to 0}{Lt} \sum f(x)\Delta x = \int_a^b f(x)dx$

Area under the curve y=x from x=0 to x=4 A=$\frac{1}{2}.4.4 = 8$

A=$\int_0^4 xdx = \left[\frac{x^2}{2}\right]_0^4 = 8$

$\sum f(x)\Delta x =$ 1x1+2x1+3x1+4x1=10 $\Delta x$=1

$$\sum f(x)\Delta x = \frac{1}{2}x\frac{1}{2}+1x\frac{1}{2}+\frac{3}{2}x\frac{1}{2}+2x\frac{1}{2}+\frac{5}{2}x\frac{1}{2}+3x\frac{1}{2}+\frac{7}{2}x\frac{1}{2}+4x\frac{1}{2}=9 \qquad \Delta x=\frac{1}{2}$$

$$\sum f(x)\Delta x = \frac{1}{4}x\frac{1}{4}+\frac{2}{4}x\frac{1}{4}+\frac{3}{4}x\frac{1}{4}+\frac{4}{4}x\frac{1}{4}+\frac{5}{4}x\frac{1}{4}+\frac{6}{4}x\frac{1}{4}+\frac{7}{4}x\frac{1}{4}+\frac{8}{4}x\frac{1}{4}+ \qquad \Delta x=\frac{1}{4}$$

$$\frac{9}{4}x\frac{1}{4}+\frac{10}{4}x\frac{1}{4}+\frac{11}{4}x\frac{1}{4}+\frac{12}{4}x\frac{1}{4}+\frac{13}{4}x\frac{1}{4}+\frac{14}{4}x\frac{1}{4}+\frac{15}{4}x\frac{1}{4}+\frac{16}{4}x\frac{1}{4}=\frac{16x17}{32}=8.5$$

$$\Delta x=\frac{1}{4}$$

$$\sum f(x)\Delta x = 8.25 \qquad \Delta x=\frac{1}{8} \qquad \frac{32x33}{128}$$

$$\sum f(x)\Delta x = 8.125 \qquad \Delta x=\frac{1}{16} \qquad \frac{64x65}{512}$$

$$\sum f(x)\Delta x = 8.0625 \qquad \Delta x=\frac{1}{32} \qquad \frac{128x129}{2048}$$

$$\sum f(x)\Delta x = 8 \qquad \Delta x \to 0$$

$$\underset{\Delta x\to 0}{Lt}\sum f(x)\Delta x = 8 = \int_0^4 x\,dx$$

$$\underset{\Delta x\to 0}{Lt}\sum f(x)\Delta x = \int_a^b f(x)\,dx$$

---

## Evaluation of line integral

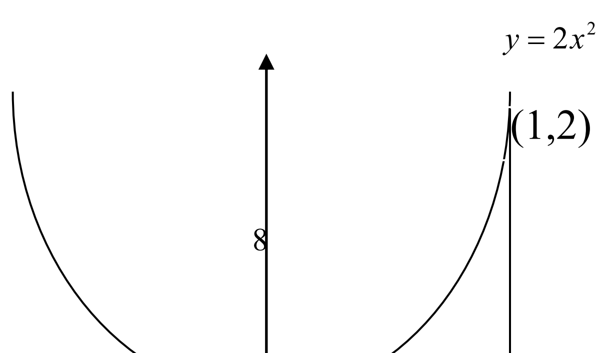
**[open curve problem]**

**Example 1.** Evaluate the line integral $\int_C$ **F**.d**r** where

**F**=-3x$^2$i+5xyj and C is the curve y=2x$^2$ in the xy plane from (0,0) to (1,2).

**OR,**

Find the work done in moving a particle in the force field **F**=-3x$^2$i+5xyj along the curve y=2x$^2$ in the xy plane from (0, 0) to (1, 2).



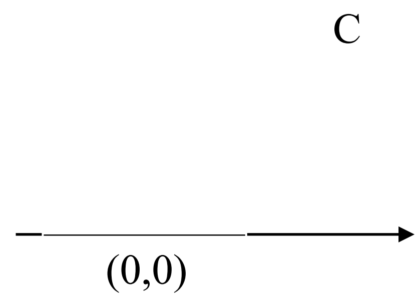



**Fig.5**

$$\int_C \mathbf{F}.d\mathbf{r}$$

$$=\int_C (-3x^2 i+5xyj).(\,dxi+dyj)$$

$$=\int_C (-3x^2dx+5xydy) \quad \text{along the curve C,} \; y=2x^2 \;\; dy=4xdx$$

$$=\int_0^1 (-3x^2dx+5x2x^2 4xdx)$$

$$=\int_0^1 (-3x^2dx+40x^4dx)$$

$$=\left[-x^3+8x^5\right]_0^1$$

$$=7$$

**Alternative method** **(**using dy)

$$\int_C \mathbf{F}.d\mathbf{r}$$

$$=\int_C (-3x^2 i+5xyj).(\,dxi+dyj) \qquad y=2x^2 \quad x^2=\frac{y}{2}$$

$$=\int_C (-3x^2dx+5xydy) \;\; \text{along the curve C,} \; x=\sqrt{\frac{y}{2}} \qquad dx=\frac{1}{2\sqrt{2}}\frac{1}{\sqrt{y}}dy$$

$$=\int_0^2 (-3\frac{y}{2}\frac{1}{2\sqrt{2}}\frac{1}{\sqrt{y}}dy+5\sqrt{\frac{y}{2}}\,ydy)$$

$$=\int_0^2 \left(-\frac{3}{4\sqrt{2}}\sqrt{y}dy+\frac{5}{\sqrt{2}}y^{\frac{3}{2}}dy\right)$$

$$=\left[-\frac{1}{2\sqrt{2}}y^{\frac{3}{2}}+\sqrt{2}y^{\frac{5}{2}}\right]_0^2$$

$$=-\frac{1}{2\sqrt{2}}2^{\frac{3}{2}}+\sqrt{2}2^{\frac{5}{2}}$$

=7

**Alternative method (**using parametric form)

$\int_C$ **F**.d**r**

**=**$\int_C$ (-3$x^2$i+5xyj).( dxi+dyj)

=$\int_C$ (-3$x^2$dx+5xydy)along the curve C, x=t y=2$t^2$ dx=dt dy=4tdt

=$\int_0^1$ (-3$t^2$dt+5t2$t^2$4tdt)

=$\int_0^1$ (-3$t^2$dt+40$t^4$dt)

$=\left[-t^3+8t^5\right]_0^1$

=7

==========================================

**[closed curve problem, W$\neq$0]**

**Example 2** Evaluate $\oint_C$ **F**.d**r** where **F**=(x-3y)i+(y-2x)j and C is the closed curve in the xy plane x=2cost, y=3sint from t=0 to t=$2\pi$.

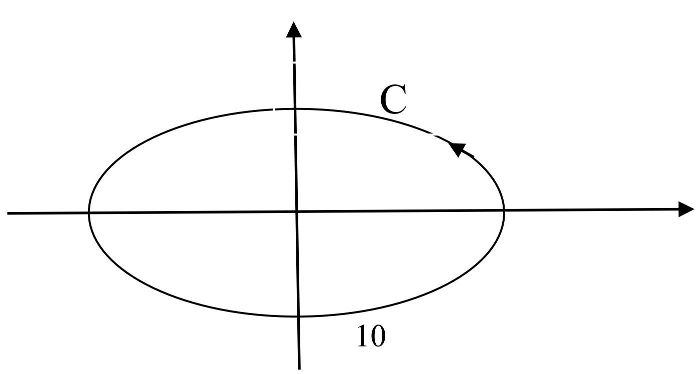


**fig.6**

$\oint_C$ **F**.d**r**= $\int_C$ [(x-3y)i+(y-2x)j].(dxi+dyj)

= $\oint_C$ (x-3y) dx+(y-2x) dy along the curve C, x=2cost, y=3sint

$= \int_0^{2\pi}$ (2cost-9sint)(-2sint)dt +(3sint-4cost)(3cost)dt

$= \int_0^{2\pi}$ (-4 sint cost+18sin$^2$t +9sintcost-12cos$^2$t)dt

$=6\pi$

**[closed curve problem, W=0]**

**Example 3** Evaluate $\oint_C$ **F**.d**r** where F=xi+yj and C is the closed curve in the xy plane x=2cost, y=3sint from t=0 to t=$2\pi$.

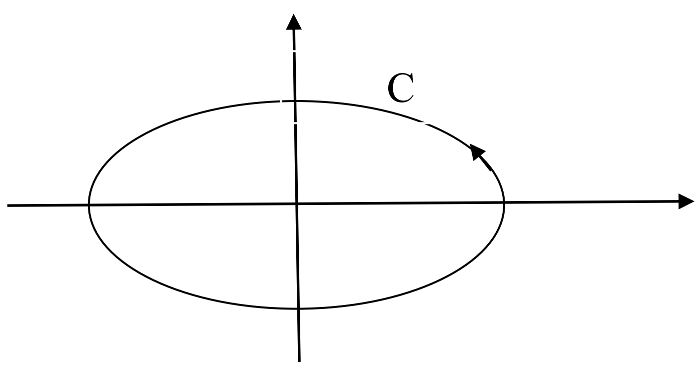



**fig.7**

$\oint_C$ **F**.d**r**= $\oint_C$ ( xi+yj).(dxi+dyj)

= $\oint_C$ x dx+y dy along the curve C, x=2cost, y=3sint

$$=\int_0^{2\pi} (2\cos t)(-2\sin t)dt +(3\sin t)(3\cos t)dt$$

$$=\int_0^{2\pi} (-4 \sin t \cos t+9\sin t\cos t)dt$$

$$=\int_0^{2\pi} 5\sin t\cos t\,dt$$

$=0$

[different path same result (end points same)]

**Example 4.** Evaluate $\int_C \mathbf{F}.d\mathbf{r}$ where F=yi+xj and C is

(a) the arc of $y=x^2-4$ from (2,0) to (4,12) in the xy plane.

(b) the portion of the x-axis from x=2 to x=4 and then the line x=4 from y=0 to y=12.

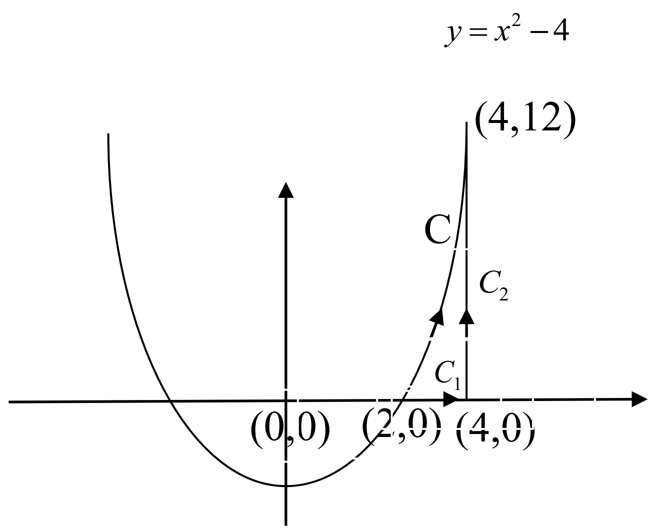



**fig.8**

**(a)** $\int_C \mathbf{F}.d\mathbf{r}=\int_C ( yi+xj).(dxi+dyj)$

$$= \int_C y\,dx + x\,dy$$

$$= \int_2^4 (x^2-4)dx + x2xdx$$ along the curve C, $y=x^2-4$ $dy=2xdx$

$=48$

**(b)** $\int_C \mathbf{F}.d\mathbf{r} = \int_C (y\mathbf{i}+x\mathbf{j}).(dx\mathbf{i}+dy\mathbf{j})$

$$= \int_C y\,dx + x\,dy$$

$$= \int_{C_1} y\,dx + x\,dy$$ along x-axis $y=0$, $dy=0$

$$+ \int_{C_2} y\,dx + x\,dy$$ along $x=4$, $dx=0$

$$= \int_2^4 0dx + x.0 + \int_0^{12} y.0 + 4dy$$

$=48$

**[different path different results (end points same)]**

**Example 5** Evaluate $\int_C \mathbf{F}.d\mathbf{r}$ where $F=xy\mathbf{i}+(x^2+y^2)\mathbf{j}$ and C is

(a)the arc of $y=x^2-4$ from (2,0) to (4,12) in the xy plane.

(b) the portion of the x-axis from x=2 to x=4 and then the line x=4 from y=0 to y=12.

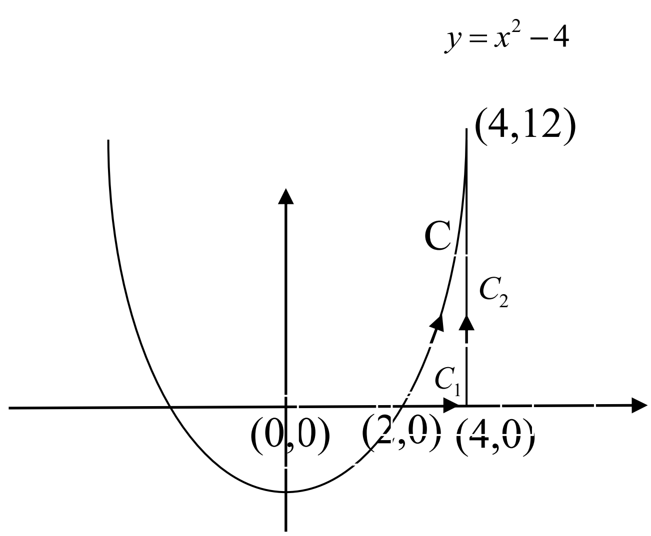



**fig.9**

**(a)** $\int_C$ **F**.d**r**=732 or 552 ?

**(b)** $\int_C$ **F**.d**r**=768 ok

**Two other line integrals:** $\int_C$ **Fx** d**r**, $\int_C \phi$ d**r**

Ex 6. Evaluate the line integral $\int_C$ **Fx dr** if **F**=xyi-yj+$x^2$k and C is the curve $x=t^3$, $y=2t$, $z=t^2$ from t=0 to t=1.

Along the curve C, **F**= xyi-yj+$x^2$k=$2t^4$i-2tj+$t^6$k

**r**=xi+yj+zk= $t^3$i+2tj+$t^2$k, d**r**=($3t^2$i+2j+2tk)dt

**Fxdr=(**$2t^4$i-2tj+$t^6$k)x($3t^2$i+2j+2tk)dt

$$=\begin{vmatrix} i & j & k \\ 2t^4 & -2t & t^6 \\ 3t^2 & 2 & 2t \end{vmatrix} dt$$

$$=[i(-4t^2-2t^6)+j(3t^8-4t^5)+k(4t^4+6t^3)]dt$$

$$\int_C \mathbf{Fxdr}= i\int_0^1 (-4t^2-2t^6)dt+j\int_0^1 (3t^8-4t^5)dt+k\int_0^1 (4t^4+6t^3)dt$$

$$=\frac{-34}{21}i+\frac{-1}{3}j+\frac{23}{10}k$$

Ex 7. Evaluate the line integral $\int_C \phi$ d**r** if $\phi$= xyz and C is the curve $x=t^3$, $y=2t$, $z=t^2$ from t=0 to t=1.

$\phi$= xyz=$2t^6$, **r**=xi+yj+zk= $t^3$i+2tj+$t^2$j, d**r**=($3t^2$i+2j+2tk)dt

$$\int_C \phi\,d\mathbf{r}=\int_0^1 2t^6(3t^2i+2j+2tk)dt$$

$$=i\int_0^1 6t^8dt+j\int_0^1 4t^6dt+k\int_0^1 4t^7dt$$

$$=\frac{6}{9}i+\frac{4}{7}j+\frac{1}{2}k$$

## Conservative Force field/ Conservative field

Spring, throw, work done is zero, force is conserved/preserved within body, called Conservative field of force.

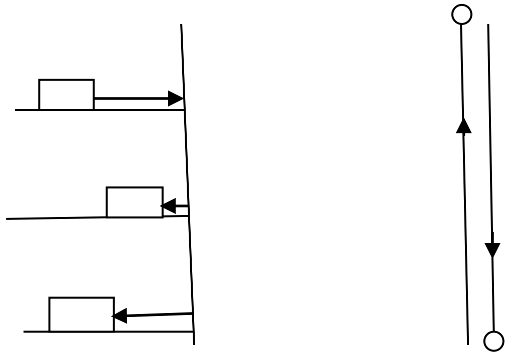
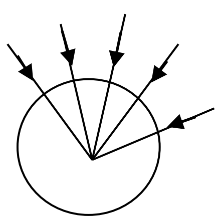


**Fig.10**

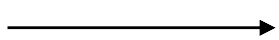
Gravitational field of force (**g**=-gk), electric field of force, magnetic field of force are all conservative field of force.



g=-gk

**fig.11**

Field of force-collection of so many vectors



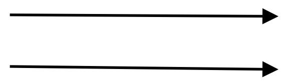


**fig.12**

Def. **Conservative field**- For a conservative field,
(i)Work done $\int_C \mathbf{F}.d\mathbf{r}=0$ around any closed curve C.

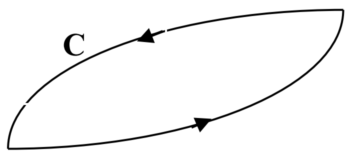



**fig.13**

**Ref. For (i) Example 3**

(ii)Work done $\int_C \mathbf{F}.d\mathbf{r}$ is independent of the path C joining $P_1$ and $P_2$. It depends on end points only

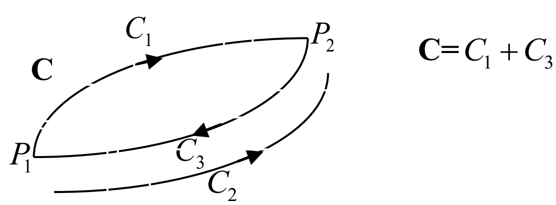



**fig. 14**

If $\mathbf{F}$ is conservative $\int_C \mathbf{F}.d\mathbf{r}=0$

Now $\int_C = \int_{C_1} + \int_{C_3} == \int_{C_1} - \int_{C_2} = 0$ $\qquad \int_{C_1} = \int_{C_2}$

**Ref. for (ii) Example 4**

## Find the condition for a force F to be conservative

If $\mathbf{F}=\nabla\phi$ where $\phi$ is a single valued scalar function then show that the work done in moving a particle from one point $P(x_1, y_1, z_1)$ in this field to another point $Q(x_2, y_2, z_2)$ is independent of the path joining the two points.

$$W=\int_P^Q \mathbf{F}.d\mathbf{r}$$

$$=\int_P^Q \nabla\phi.d\mathbf{r}$$

$$=\int_P^Q \left(i\frac{\partial\phi}{\partial x}+j\frac{\partial\phi}{\partial y}+k\frac{\partial\phi}{\partial z}\right).(dxi+dyj+dzk)$$

$$=\int_P^Q \left(\frac{\partial\phi}{\partial x}dx+\frac{\partial\phi}{\partial y}dy+\frac{\partial\phi}{\partial z}dz\right) \text{ Exact differential } \frac{\partial\phi}{\partial x}dx+\frac{\partial\phi}{\partial y}dy+\frac{\partial\phi}{\partial z}dz=d\phi$$

$$=\int_P^Q d\phi$$

$$=[\phi]_P^Q$$

$$=\phi(Q)-\phi(P)$$

$$=\phi(x_2,y_2,z_2)-\phi(x_1,y_1,z_1)$$

Work done $\int_P^Q \mathbf{F}.d\mathbf{r}$ is independent of the path C joining P and Q.

It depends on end points only

Therefore if $\mathbf{F}=\nabla\phi$ the force is conservative.

**We know that $\nabla \times \nabla\phi=0$**

**Force F is conservative if $\nabla \times \mathbf{F}=0$ (condition)**

Ex.8.(a)Show that F=yi+xj is a conservative force field

(b) Find the work done in moving an object in this field from (2,0) to (4,12)

(c)Find a Scalar potential $\phi$

(a) Curl **F**=0

Curl **F**= $\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & 0 \end{vmatrix}$ =i(    )+j(      )+(      )k=0

So F is Conservative

(b) W= $\int_P^Q$ **F.dr**

$= \int_{(2,0)}^{(4,12)} (\, yi+xj).(dxi+dyj)$

$= \int_{(2,0)}^{(4,12)} y\, dx+x\, dy$       [Exact differential]

$= \int_{(2,0)}^{(4,12)} d(xy) \;=\; [xy]_{(2,0)}^{(4,12)} = 48$

(c) As $\nabla$ x **F=0**,  **F** must be $\nabla\phi$

**F** $=\nabla\phi$

yi+xj = $i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y}$

$\frac{\partial\phi}{\partial x} = y$    ……. (1)

$\frac{\partial\phi}{\partial y} = x$    ……….(2)

Integrating (1) with respect to x

$\phi$=yx +f(y)  ………...(3)

$\frac{\partial\phi}{\partial y}$=x+$\frac{\partial f(y)}{\partial y}$    ……….(4)

Comparing (2) and (4) $\frac{\partial f(y)}{\partial y}$=0 ….. (5)

Integrating (5) with respect to y, f(y)= c

hence $\phi$=xy+c

**Alternative method**

$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy$

$d\phi = \text{ydx+xdy}$

Integrating

$$\phi = \int_{y\,constant} \text{ydx} + \int_{x\,term\,vanishes} xdy$$

$= \text{xy+c}$

**Alternative method**

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$$

=ydx+xdy

=d(xy)

Integrating

$\phi$=xy+c

**Alternative method**

$\phi$=yx +f(y)

$\phi$= xy+g(x)

f(y)=0

g(x)=0

hence $\phi$=xy+c

**Example 9.**

(a) Show that the Force

**F**=(-4x-3y+4z)i+(-3x+3y+5z)j+(4x+5y+3z)k is conservative.

(b)Find a scalar function $\phi$ so that **F**=$\nabla\phi$.

**(a) F** is conservative if $\nabla$x**F=0**

$$\nabla \times \mathbf{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -4x - 3y + 4z & -3x + 3y + 5z & 4x + 5y + 3z \end{vmatrix} = 0$$

(b) As $\nabla$x **F=0,** **F** must be $\nabla\phi$

**F** $=\nabla\phi$

$(-4x-3y+4z)i+(-3x+3y+5z)j+(4x+5y+3z)k = i\frac{\partial\phi}{\partial x}+j\frac{\partial\phi}{\partial y}+k\frac{\partial\phi}{\partial z}$

$\frac{\partial\phi}{\partial x} = -4x-3y+4z$ ....... (1)

$\frac{\partial\phi}{\partial y} = -3x+3y+5z$ ..........(2)

$\frac{\partial\phi}{\partial z} = 4x+5y+3z$ .............(3)

Integrating (1) with respect to x

$\phi=-2x^2-3xy+4xz+f(y,z)$ .....(4)

$\frac{\partial\phi}{\partial y} = -3x+\frac{\partial f(y,z)}{\partial y}$ ..........(5)

$\frac{\partial\phi}{\partial z} = 4x+\frac{\partial f(y,z)}{\partial z}$ ............(6)

Comparing (2) and (5) $\frac{\partial f(y,z)}{\partial y}=3y+5z$ ..... (7)

Comparing (3) and (6) $\frac{\partial f(y,z)}{\partial z}=5y+3z$ ..... (8)

Integrating (7) with respect to y, $f(y, z)= \frac{3}{2}y^2+5zy+g(z)$

$\frac{\partial f(y,z)}{\partial z} = 5y+g'(z)$ ..... (9)

Comparing (8) and (9) $g'(z)=3z \quad g(z)=\frac{3}{2}z^2+c$

therefore $f(y, z)= \frac{3}{2}y^2+5yz+\frac{3}{2}z^2+c$

hence $\phi=-2x^2-3xy+4xz+\frac{3}{2}y^2+5yz+\frac{3}{2}z^2+c$

**Alternative method**

$d\phi=\frac{\partial\phi}{\partial x}dx+\frac{\partial\phi}{\partial y}dy+\frac{\partial\phi}{\partial z}dz$

$d\phi=(-4x-3y+4z)dx+(-3x+3y+5z)dy+(4x+5y+3z)dz$

Integrating

$$\phi=\int_{y,z\ constant}(-4x-3y+4z)dx+\int_{x\ term\ vanishes,z\ constant}(-3x+3y+5z)dy+\int_{x\ and\ y\ term\ vanishes}(4x+5y+3z)dz$$

$=-2x^2-3xy+4xz+\frac{3}{2}y^2+5yz+\frac{3}{2}z^2+c$

**Alternative method**

$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$

$d\phi =$(-4x-3y+4z)dx+(-3x+3y+5z)dy+(4x+5y+3z)dz

=-4xdx+3ydy+3zdz+-3ydx-3xdy+4zdx+4xdz+5zdy+5ydz

=4xdx+3ydy+3zdz-3(ydx+xdy)+4(zdx+xdz)+5(zdy+ydz)

=4xdx+3ydy+3zdz-3d(xy)+4d(xz)+5d(yz)

Integrating

$\phi=-2x^2+\frac{3}{2}y^2+\frac{3}{2}z^2-3xy+4xz+5yz+c$

**Alternative method**

$\phi=-2x^2-3xy+4xz+ \quad + \quad + \quad +f(y,z)$

$\phi= \quad -3xy+ \quad \frac{3}{2}y^2+ \; 5yz + \quad +g(z,x)$

$\phi= \quad 4xz+ \quad +5yz+\frac{3}{2}z^2 \; +h(x,y)$

$f(y,z)= \frac{3}{2}y^2+5yz+\frac{3}{2}z^2$

$g(z,x)= -2x^2+4xz+ \frac{3}{2}z^2$

$h(x,y)= -2x^2-3xy+ \frac{3}{2}y^2$

hence $\phi=-2x^2-3xy+4xz+\frac{3}{2}y^2+5yz+\frac{3}{2}z^2+c$

==========================================

Problem copied from VEC2

Ex. (a)Determine the constant a, b, c so that vector

**v**=(-4x-3y+az)i+(bx+3y+5z)j+(4x+cy+3z)k is irrotational.

(b)Find a scalar function $\phi$ so that **v**=$\nabla\phi$.

**v** is irrotational if $\nabla$x**v**=**0**

$$\nabla \times \mathbf{v}=\begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -4x-3y+az & bx+3y+5z & 4x+cy+3z \end{vmatrix}$$

$=(c-5)i-(4-a)j+(b+3)k=\mathbf{0}=0i+0j+0k$

$a=4, \quad b=-3, \quad c=5$

(b)As $\nabla \times \mathbf{v}=\mathbf{0}$, $\mathbf{v}$ must be $\nabla\phi$

$\mathbf{v}=\nabla\phi$

$(-4x-3y+4z)i+(-3x+3y+5z)j+(4x+5y+3z)k = i\dfrac{\partial\phi}{\partial x}+j\dfrac{\partial\phi}{\partial y}+k\dfrac{\partial\phi}{\partial z}$

$\dfrac{\partial\phi}{\partial x}= -4x-3y+4z \qquad \ldots\ldots. (1)$

$\dfrac{\partial\phi}{\partial y}= -3x+3y+5z \qquad \ldots\ldots\ldots.(2)$

$\dfrac{\partial\phi}{\partial z}=4x+5y+3z \qquad \ldots\ldots\ldots\ldots.(3)$

Integrating (1) with respect to x partially

$\phi=-2x^2-3xy+4xz+f(y,z) \quad \ldots..(4)$

$\dfrac{\partial\phi}{\partial y}=-3x+\dfrac{\partial f(y,z)}{\partial y} \qquad \ldots\ldots\ldots.(5)$

$\dfrac{\partial\phi}{\partial z}= 4x+\dfrac{\partial f(y,z)}{\partial z} \qquad \ldots\ldots\ldots\ldots.(6)$

Comparing (2) and (5) $\dfrac{\partial f(y,z)}{\partial y}=3y+5z \ldots..$ (7)

Comparing (3) and (6) $\dfrac{\partial f(y,z)}{\partial z}=5y+3z \ldots..$ (8)

Integrating (7) with respect to y, $f(y, z)= \dfrac{3}{2}y^2+5zy+g(z)$

$\dfrac{\partial f(y,z)}{\partial z}=5y+g'(z) \qquad \ldots..$ (9)

Comparing (8) and (9) $\quad g'(z)=3z \quad g(z)=\dfrac{3}{2}z^2+c$

therefore $f(y, z)= \dfrac{3}{2}y^2+5yz+\dfrac{3}{2}z^2+c$

hence $\phi=-2x^2-3xy+4xz+\dfrac{3}{2}y^2+5yz+\dfrac{3}{2}z^2+c$

**Alternative method**

$\mathbf{v}=\nabla\phi$

$$(-4x-3y+4z)i+(-3x+3y+5z)j+(4x+5y+3z)k = i\frac{\partial\phi}{\partial x}+j\frac{\partial\phi}{\partial y}+k\frac{\partial\phi}{\partial z}$$

$$\frac{\partial\phi}{\partial x}= -4x-3y+4z \qquad \ldots\ldots\ (1)$$

$$\frac{\partial\phi}{\partial y}= -3x+3y+5z \qquad \ldots\ldots\ldots(2)$$

$$\frac{\partial\phi}{\partial z}=4x+5y+3z \qquad \ldots\ldots\ldots\ldots(3)$$

$$\phi=-2x^2-3xy+4xz+f(y,z)$$

$$\phi= -3xy+\frac{3}{2}y^2+5yz+g(z,x)$$

$$\phi= 4xz + 5yz +\frac{3}{2}z^2 +h(x,y)$$

$$\phi=-2x^2-3xy+4xz+ \quad + \quad + \quad +f(y,z)$$

$$\phi= \quad -3xy+ \quad + \frac{3}{2}y^2 + 5yz + \quad +g(z,x)$$

$$\phi= \quad + \quad +4xz + \quad + 5yz + \frac{3}{2}z^2 \quad +h(x,y)$$

$$f(y,z)= \frac{3}{2}y^2+5yz+\frac{3}{2}z^2$$

$$g(z,x)= -2x^2+4xz+ \frac{3}{2}z^2$$

$$h(x,y)= -2x^2-3xy+ \frac{3}{2}y^2$$

hence $\phi=-2x^2-3xy+4xz+\frac{3}{2}y^2+5yz+\frac{3}{2}z^2+c$

==========================================

In fluid mechanics the line integral $\int_C$ **A**.d**r** is called the circulation of **A** about C where **A** represents the velocity of a fluid

Circulation= $C=\int_C \mathbf{v}.d\mathbf{r}$ [Work done=$W=\int_C \mathbf{F}.d\mathbf{r}$ ]

If no circulation (irrotational) $\int_C v.dr=0$  curlv=0

**E**x 10. Find the circulation of **A** round the curve C where $\mathbf{A}=(2x+y^2)i+(3y-4x)j$ and C is the curve $y=x^2$ from (0,0) to (1,1) and then $y^2=x$ from (1,1) to (0,0).

$\mathbf{A}.d\mathbf{r} = (2x+y^2)dx+(3y-4x)dy$

Circulation= $\int_C \mathbf{A}.d\mathbf{r}=\int_{C_1} \mathbf{A}.d\mathbf{r}+\int_{C_2} \mathbf{A}.d\mathbf{r}$

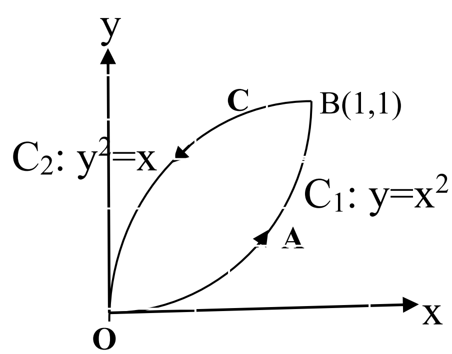



**Fig 15**

On $C_1$, OAB $y=x^2$ so that $dy=2xdx$

$\int_{C_1} \mathbf{A}.d\mathbf{r}=\int_{C_1} (2x+y^2)dx+(3y-4x)dy$

$=\int_0^1 (2x+x^4)dx+(3x^2-4x)2xdx=\frac{1}{30}$

On $C_2$ BCO $x=y^2$ so that $dx=2ydy$

$\int_{C_1} \mathbf{A}.d\mathbf{r}=\int_{C_1} (2x+y^2)dx+(3y-4x)dy$

$$=\int_1^0 (2y^2+y^2)2ydy+(3y-4y^2)dy=-\frac{5}{3}$$

$$\text{Circulation}=\int_C \mathbf{A}.d\mathbf{r}=\int_{C_1} \mathbf{A}.d\mathbf{r}+\int_{C_2} \mathbf{A}.d\mathbf{r}=\frac{1}{30}-\frac{5}{3}=-\frac{49}{30}$$

**E**x 11. Find the work done in moving a particle along the curve C in the force field $\mathbf{F}=(2x+y^2)i+(3y-4x)j$ where C is the curve $y=x^2$ from (0,0) to (1,1) and the $y^2=x$ from (1,1) to (0,0).

**Problem**

Determine the work done by the force of gravity **F** when a mass m is translated from the point $P(a_1, b_1, c_1)$ to the point $Q(a_2, b_2, c_2)$ along an arbitrary path C.

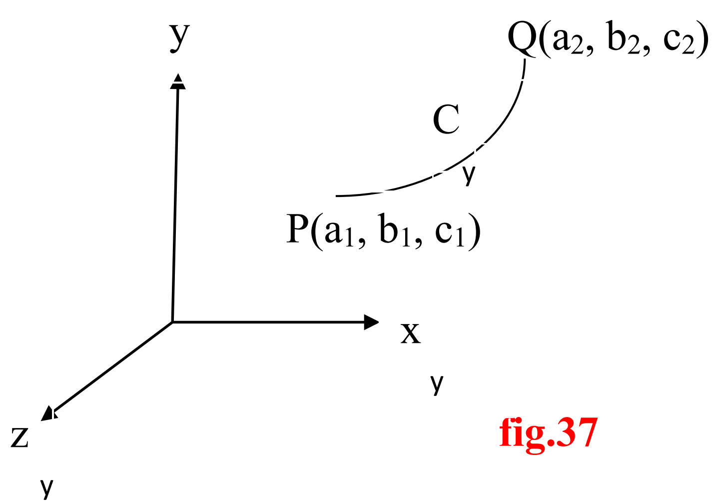



**fig.37**

Let F= Xi+Yj+Zk

The projections (components) of the force of gravity **F** on the coordinate axes are X=0, Y=0, Z=-mg

Hence, the desired work is

$$W=\int_C \mathbf{F}.d\mathbf{r}$$

$$= \int_{P}^{Q} (Xdx+Ydy+Zdz)$$

$$= \int_{(a_1\ b_1, c_1)}^{(a_2, b_2\ c_2)} (0dx+0dy +(-mg)dz$$

$$=mg(c_1-c_2)$$

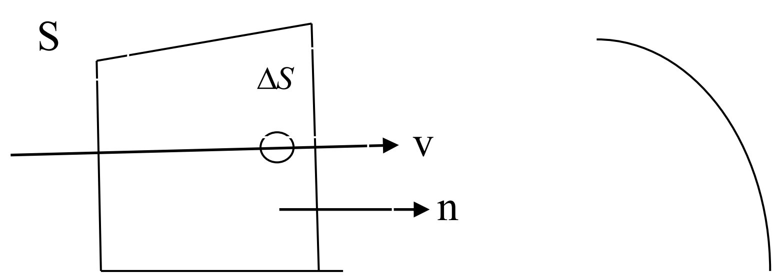
**09 -12-21 Quiz 1 explanation of projection ds-dxdy equation plane and its projection 1D 2D 3D**

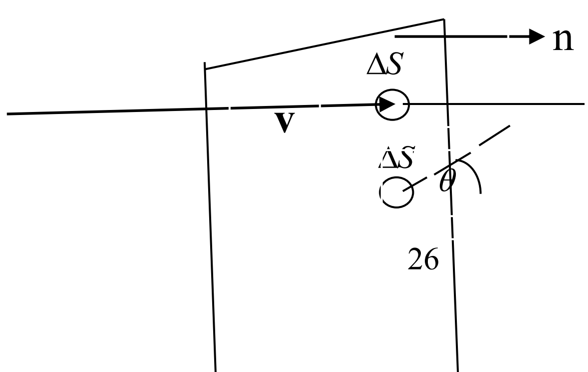
# Surface integral

**Def:** Different types of Surface integrals are (i) $\iint_S$ **A**.**n**dS (ii) $\iint_S \phi$ dS (iii) $\iint_S \phi$ **n**dS (iv) $\iint_S$ **A**xd**S**

Physical meaning of (i) $\iint_S$ **A**.**n**dS

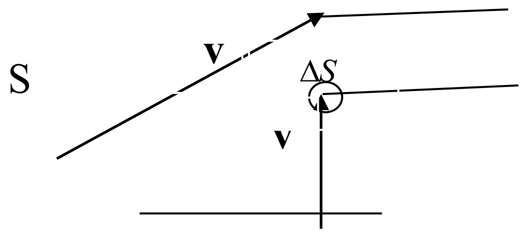
Suppose fluid is flowing through the surface $\Delta S$



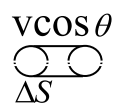

v= vcos0=vcos$\theta$=**v.n**

vcos$\theta$=**v.n**




$0 = v\cos\frac{\pi}{2} = v\cos\theta = \mathbf{v.n}$

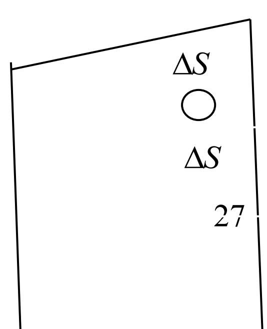
Flow is always along the normal to the surface.
Only component of **v** along the normal to the surface are taken into account




$dV = \Delta S\ v\cos\theta$

**fig.16**

Mass (quantity) of fluid passing through $\Delta S$ per unit time
$\rho dV = \rho v \cos\theta \Delta S = \rho\ \mathbf{v.n}\ \Delta S$ where **n** is outward drawn normal to the surface S.



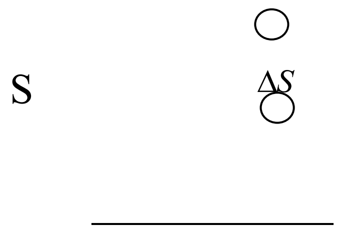



**fig.17**

Total fluid mass **Q** passing through S is

$\rho\,\mathbf{v_1.n_1}\,\Delta S + \rho\,\mathbf{v_2.n_2}\,\Delta S + \rho\,\mathbf{v_3.n_3}\,\Delta S \ldots\ldots\ldots\ldots\ldots\ldots\ldots + \rho\,\mathbf{v_n.n_n}\,\Delta S$

$$= {}_{Lt\,\Delta s \to 0} \sum \rho\,\mathbf{v.n}\,\Delta S$$

$$= \iint_S \rho\,\mathbf{v.n}dS$$

$$= \iint_S \rho\,\mathbf{v}.d\mathbf{S} \qquad\qquad \mathbf{n}dS = d\mathbf{S}$$

Hence **fluid flow** **Q**$= \iint_S \rho\,\mathbf{v}.d\mathbf{S}$

**Heat flow** $\iint_S \mathbf{q}.d\mathbf{S}$

**Current flow** $\iint_S \mathbf{J}.d\mathbf{S}$

In general Surface integral is $\iint_S \mathbf{A.n}dS$

It is a surface integral of the normal component of vector **A**

$\iint_S \mathbf{A.n}dS$ is the flux of A over S

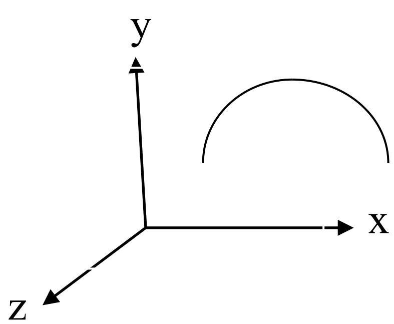
**Flux means flow**

**So surface integral is also called flux integral or flow integral**

**Evaluation of Surface integral**

**To evaluate the surface integral we are to** **project** **the surface on the coordinate planes.**

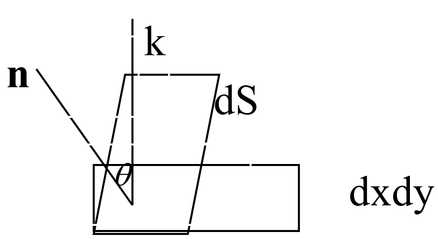
Projection of a surface on a plane becomes a plane.

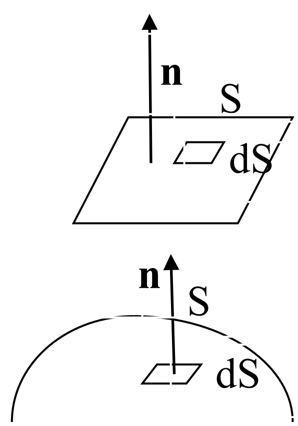


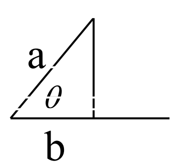

First we consider the projection on the xy plane

Elementary area in the xy plane is =dxdy

The angle between two planes is equal to the angle between their normals




Projection of a is $\quad b = a\cos\theta$

Fig. 18

Projection of dS is  $dxdy = dS\cos\theta = dS\mathbf{n}.k$

$$dS = \frac{dxdy}{n.k}$$

**Evaluation of surface integral**

$\iint_S$ A.ndS=$\iint_R$ A.n$\frac{dxdy}{|n.k|}$ projection on xy plane

$\iint_S$ A.ndS=$\iint_R$ A.n$\frac{dydz}{|n..i|}$ projection on yz plane

$\iint_S$ A.ndS=$\iint_R$ A.n$\frac{dzdx}{|n.j|}$ projection on zx plane

**Problem**

Example 12. Evaluate $\iint_S$ **F.n**dS where **F**=10zi+10j+3yk and S is the part of the plane 2x+3y+6z=12 which is located in the first octant.

Or,

Find the flux of **F**=10zi+10j+3yk through the surface S where S is the part of the plane 2x+3y+6z=12 which is located in the first octant.

Or,

Find the flow of fluid through the surface S where S is the part of the plane 2x+3y+6z=12 which is located in the first octant, if the fluid velocity is **v**=10zi+10j+3yk

Or,

Find the flow of heat through the surface S where S is the part of the plane 2x+3y+6z=12 which is located in the first octant, if the heat flux q=10zi+10j+3yk

Or,

Find the flow of current through the surface S where S is the part of the plane 2x+3y+6z=12 which is located in the first octant, if the current density J=10zi+10j+3yk

Solution:

Let the projection of the surface S on the xy plane be R

$\iint_S$ **F.n**dS= $\iint_R$ **F.n** $\frac{dxdy}{|n.k|}$

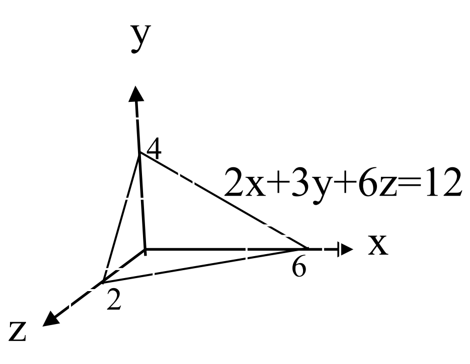
$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

**n**= $\frac{\nabla\phi}{|\nabla\phi|}$

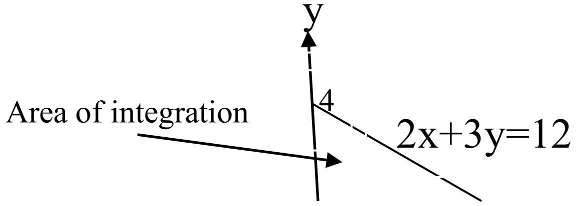
normal to the plane 2x+3y+6z=12

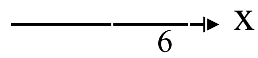
**n**= $\frac{2i+3j+6k}{7}$ **n.k**= $\frac{6}{7}$

**F.n**= $\frac{20z+30+18y}{7} = \frac{\frac{20}{6}(12-2x-3y)+30+18y}{7} = \frac{70-\frac{20}{3}x+8y}{7}$




$$\frac{x}{6}+\frac{y}{4}+\frac{z}{2}=1$$






$$\frac{x}{6}+\frac{y}{4}=1$$

$$A=\iint_R dxdy=\int_0^6\int_0^y dydx=\int_0^6 ydx=\int_0^6\frac{12-2x}{3}dx=12=A$$

$$A=\iint_R dxdy=\int_0^6\int_0^{\frac{12-2x}{3}} dydx=\int_0^6\frac{12-2x}{3}dx=12$$

projection of the plane on the xy plane will be a triangle

$$\iint_S \mathbf{F}.\mathbf{n}dS=\iint_R \mathbf{F}.\mathbf{n}\frac{dxdy}{|n.k|}=\iint_R\frac{70-\frac{20}{3}x+8y}{7}\frac{7}{6}dxdy$$

$$=\frac{1}{18}\iint_R(210-20x+24y)dxdy$$

$$=\frac{1}{18}\int_{x=0}^{6}\int_{y=0}^{(12-2x)/3}(210-20x+24y)dydx$$

$$=\frac{7}{4}\int_{x=0}^{6}\left[210y-20xy-12y^2\right]_0^{(12-2x)/3}dx$$

$$=\frac{7}{4}\int_{x=0}^{6}\left\{70(12-2x)-\frac{20}{3}x(12-2x)-\frac{4}{3}(12-2x)^2\right\}dx$$

……………………………………………………………

**Problem 5.21 p-116**

Example 13. Evaluate $\iint_S \phi\,\mathbf{n}dS$ where $\phi=xyz$ and S is the surface of the cylinder $x^2+y^2=16$ included in the first octant between z=0 and z=3.

Fig 5.8 of the book

$$\iint_S \phi\,\mathbf{n}dS=\iint_R \phi\bar{n}\frac{dxdz}{|n.j|}$$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

**n**= $\dfrac{\nabla\phi}{|\nabla\phi|}$

normal to the surface $x^2+y^2=16$ is

**n**= $\dfrac{2xi+2yj}{\sqrt{4x^2+4y^2}} = \dfrac{xi+yj}{4}$ **n.j**= $\dfrac{y}{4}$

projection of the surface on the xz plane will will be a rectangle.

$$\iint_S \phi\,\mathbf{n}dS = \iint_R \phi\bar{n}\frac{dxdz}{|n.j|} = \int_{z=0}^{3}\int_{x=0}^{4} xz(xi+yj)dxdz = \int_{z=0}^{3}\int_{x=0}^{4} x^2zi + xz\sqrt{16-x^2}\,j)dxdz$$

$$= i\int_{z=0}^{3}\left[\frac{x^3}{3}\right]_0^4 zdz - \frac{1}{3}j\int_{z=0}^{3}\left[\left(16-x^2\right)^{\frac{3}{2}}\right]_0^4 zdz$$

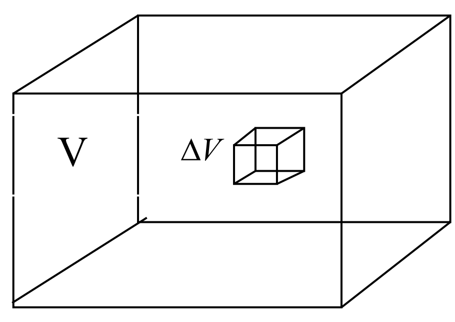
$$= \frac{64i}{3}\left[\frac{z^2}{2}\right]_0^3 + \frac{64j}{3}\left[\frac{z^2}{2}\right]_0^3$$

$$= 96i + 96j$$

# Volume Integral

Def: Volume integrals are (i) $\iiint_V \phi\, dV$ (ii) $\iiint_V \mathbf{A}\, dV$

Physical meaning of (i) $\iiint_V \phi\, dV$




Let $\rho(x,y,z)$ be the density of a body having volume V.

And let $\Delta$V=$\Delta$x$\Delta$y$\Delta$z be the elementary volume then the mass of the elementary volume $\Delta$V is $\rho\,\Delta$V

Total Mass M= $\rho_1\,\Delta V + \rho_2\,\Delta V + \rho_3\,\Delta V + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \rho_n\,\Delta V$

$$= Lt_{\Delta V \to 0} \sum \rho\,\Delta V$$

$$= \iiint_V \rho\, dV$$

$$= \iiint_V \rho\, dxdydz$$

In general this volume integral is $\iiint_V \phi\, dV$

The other volume integral is $\iiint_V \mathbf{A}\, dV$

**Problem**

Example 14.Evaluate $\iiint_V f\,dV$ where $f = x^2 y$ and V is the region bounded by the planes 4x+2y+z=8, x=0, y=0, z=0.

$$\iiint_V fdV$$

$$= \iiint_V fdxdydz$$

$$= \int_{x=0}^{2} \int_{y=0}^{4-2x} \int_{z=0}^{8-4x-2y} x^2 y dz dy dx$$

$$= \int_{x=0}^{2} \int_{y=0}^{4-2x} \left[ x^2 y z \right]_0^{8-4x-2y} dy dx$$

$$= \int_{x=0}^{2} \int_{y=0}^{4-2x} x^2 y(8-4x-2y) dy dx$$

$$= \int_{x=0}^{2} \int_{y=0}^{4-2x} \left\{ x^2(8-4x)y - 2x^2 y^2 \right\} dy dx$$

$$= \int_{x=0}^{2} \left[ x^2(8-4x)\frac{y^2}{2} - 2x^2\frac{y^3}{3} \right]_0^{4-2x} dx$$

$$= \int_{x=0}^{2} \left\{ x^2(8-4x)\frac{(4-2x)^2}{2} - 2x^2\frac{(4-2x)^3}{3} \right\} dx$$

$$= \int_{x=0}^{2} \frac{1}{3}x^2(4-2x)^3\,dx$$

$$= \frac{128}{45}$$

Interpretation: Physically the result can be interpreted as the mass of the region V in which density $\phi$ varies according to the formula $\phi = x^2 y$

**Problem 5.26 Do it**

Example-15. Evaluate $\iiint_V$ **F** dV where **F**=2xzi-xj+y$^2$k and V is the region bounded by the surfaces x=0, y=0, y=6, z=x$^2$, z=4.

Since z=x$^2$, z=4,  x$^2$=4,  x=2

$$\iiint_V \mathbf{F}\,dV$$

$$= \int_0^2 \int_0^6 \int_{x^2}^4 (2xz\mathbf{i} - x\mathbf{j} + y^2\mathbf{k})\,dz\,dy\,dx$$

Example 16. Evaluate $\iiint_V$ **F** dV where **F**=2x$^2$i-xzj+y$^2$zk and V is the region bounded by the surfaces x=1, y=0, y=6, z=x$^2$, z=4.

Since z=x$^2$, z=4, x$^2$=4, x=2

$$\iiint_V \mathbf{F}\,dV$$

$$= \int_1^2 \int_0^6 \int_{x^2}^4 (2x^2\mathbf{i} - xz\mathbf{j} + y^2z\mathbf{k})\,dz\,dy\,dx$$

$$=i\int_1^2\int_0^6\int_{x^2}^4 2x^2\ dzdydx-j\int_1^2\int_0^6\int_{x^2}^4 xz\ dzdydx+k\int_1^2\int_0^6\int_{x^2}^4 y^2z\ dzdydx$$

$$=i\int_1^2\int_0^6 2x^2[z]_{x^2}^4\ dydx-j\int_1^2\int_0^6 x\left[\frac{z^2}{2}\right]_{x^2}^4 dydx+k\int_1^2\int_0^6 y^2\left[\frac{z^2}{2}\right]_{x^2}^4 dydx$$

$$=i\int_1^2\int_0^6 2x^2 (4-x^2)dydx-j\frac{1}{2}\int_1^2\int_0^6 x(16-x^4)\ dydx+k\frac{1}{2}\int_1^2\int_0^6 y^2(16-x^4)\ dydx$$

$$=i\int_1^2 2x^2 (4-x^2)\ [y]_0^6 dx-j\frac{1}{2}\int_1^2 x(16-x^4)\ [y]_0^6 dx+k\frac{1}{2}\int_1^2 (16-x^4)\left[\frac{y^3}{3}\right]_0^6 dx$$

$$=i\int_1^2 2x^2 (4-x^2)\ [y]_0^6 dx-j\frac{1}{2}\int_1^2 x(16-x^4)\ [y]_0^6 dx+k\frac{1}{2}\int_1^2 (16-x^4)\left[\frac{y^3}{3}\right]_0^6 dx$$

$$=i\int_1^2 12x^2 (4-x^2)\ dx-j\int_1^2 3x(16-x^4)\ dx+k\int_1^2 36\ (16-x^4)\ dx$$

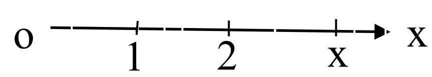
$$=i\int_1^2 (48x^2 -12x^4)\ dx-j\int_1^2 (48x -3x^5)\ dx+k\int_1^2 (576-36x^4)\ dx$$

$$=i\left[48\frac{x^3}{3}-12\frac{x^5}{5}\right]_1^2 - j\left[48\frac{x^2}{2}-3\frac{x^6}{6}\right]_1^2 +k\left[576x-36\frac{x^5}{5}\right]_1^2$$

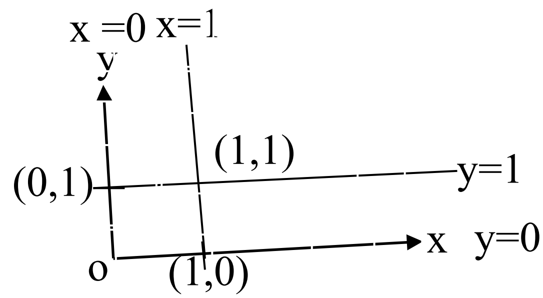
$$=i\frac{188}{5}-j\frac{81}{2}+k\frac{1764}{5}$$

Discussion:

1D




2D




General equation of straight line is Ax+By+C=0

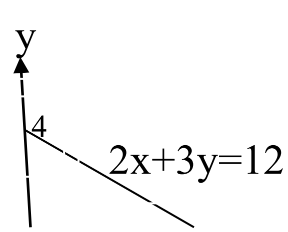
Intercept form of the equation is $\frac{x}{a}+\frac{y}{b}=1$

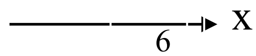
Equation of x axis is y=0

Equation of y axis is x=0

x=1 is a straight line parallel to y- axis

y=1 is a straight line parallel to x- axis



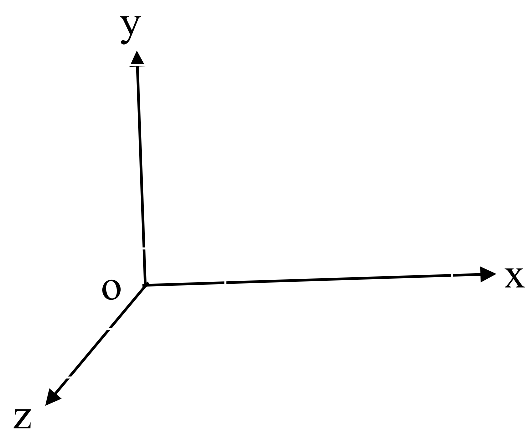



$$\frac{x}{6}+\frac{y}{4}=1$$

Equation of the circle is $x^2+y^2=16$

Equation of the straight line passing through two points $(x_1, y_1)$ and $(x_2, y_2)$ is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$

3D

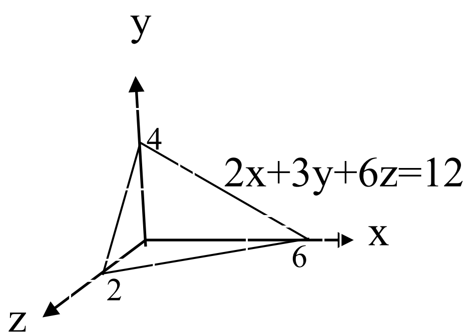



General equation of plane line is Ax+By+Cz+D=0

Intercept form of the equation is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Equation of xy plane is z=0
Equation of yz plane is x=0
Equation of zx plane is y=0

z=1 is a plane parallel to xy plane
x=1 is a plane parallel to yz plane
y=1 is a plane parallel to zx plane




$$\frac{x}{6}+\frac{y}{4}+\frac{z}{2}=1$$

Equation of the cylinder is $x^2+y^2=16$, $z=0$, $z=3$.
Equation of the cylinder is $y^2+z^2=16$, $x=0$, $x=3$.
Equation of the cylinder is $z^2+x^2=16$, $y=0$, $y=3$.
Equation of the parabolic sheet is $y^2=x$
Equation of the parabola is $y^2=x$, $z=0$

General equation of the straight line is
$A_1x++B_1y+C_1z+D_1=0=A_2x+B_2y+C_2z+D_2$

Equation of the straight line passing through two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$