组会报告

论文复现: On the Coexistence Between Full-Duplex and NOMA

Yong YANG

Beijing University of Posts and Telecommunications
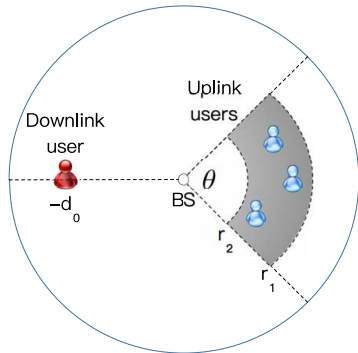`https://bupt-yy.github.io/`

May 6, 2021

SYSTEM MODEL

Figure: 1

1 个 DL user 位于 $(-d_0, 0)$,
M 个 UL users 均匀分布于 $\mathrm{ann}(\mathsf{BS}, r_2, r_1; -\frac{\theta}{2}, \frac{\theta}{2})$
BS 接收信号 $y_{\mathsf{BS}} = \sqrt{P_{\mathsf{U}}} \sum_{m=1}^{M} h_m s_m + h_{\mathsf{SI}} \sqrt{P_{\mathsf{SI}}} s_0 + n_{\mathsf{BS}}$.
DL 接收信号 $y_{\mathsf{D}} = \sqrt{P_{\mathsf{U}}} \sum_{m=1}^{M} g_m s_m + h_0 \sqrt{P_{\mathsf{BS}}} s_0 + n_{\mathsf{D}}$.

模型假设:

- UL users 的发射功率 P_{U} 均相同,
- $h_{\mathsf{SI}} \sim \mathcal{CN}(0, 1)$,
- $h_m \sim$ Rayleigh fading.
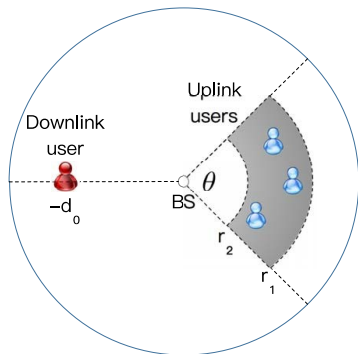
Figure: 1

1. Uplink sum rates
   - NOMA, FD

$$R_{\mathsf{FD}}^{\mathsf{U}} = \log\left(1 + \frac{\sum_{m=1}^{M} P_{\mathsf{U}} |h_m|^2}{P_{\mathsf{SI}} |h_{\mathsf{SI}}|^2 + 1}\right), \qquad (1)$$

   - NOMA, HD

$$R_{\mathsf{HD}}^{\mathsf{U}} = \frac{1}{2} \log\left(1 + \sum_{m=1}^{M} P_{\mathsf{U}} |h_m|^2\right), \qquad (2)$$

   - OMA, HD

$$R_{\mathsf{OMA}}^{\mathsf{U}} = \frac{1}{2M} \sum_{m=1}^{M} \log\left(1 + P_{\mathsf{U}} |h_m|^2\right). \qquad (3)$$
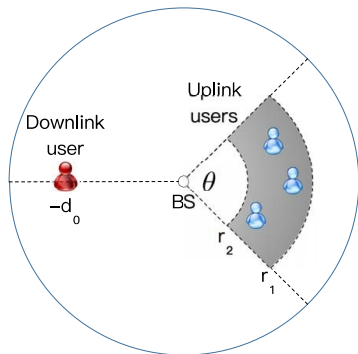
PERFORMANCE ANALYSIS

Figure: 1

2. Downlink sum rates

- NOMA, FD

$$R_{\mathsf{FD}}^{\mathsf{D}} = \log\left(1 + \frac{P_{\mathsf{BS}}\left|h_0\right|^2}{P_{\mathsf{U}}\sum_{m=1}^{M}\left|g_m\right|^2 + 1}\right), \qquad (4)$$

- NOMA, HD

$$R_{\mathsf{HD}}^{\mathsf{D}} = \frac{1}{2}\log\left(1 + P_{\mathsf{BS}}\left|h_0\right|^2\right), \qquad (5)$$

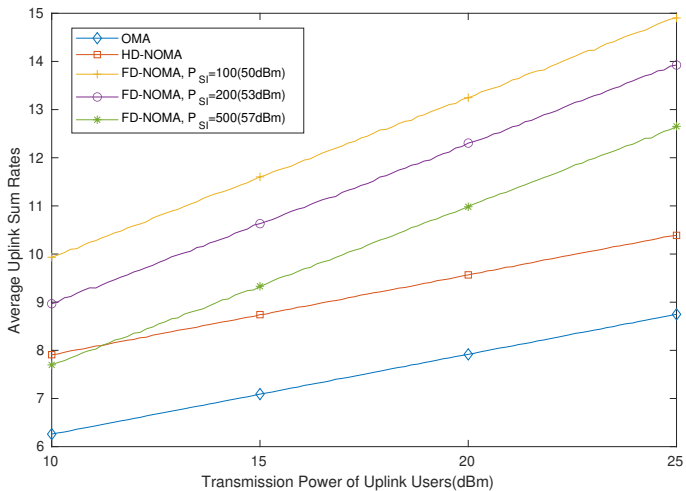$$\mathrm{distance}(\mathsf{DL\ user}, \mathsf{UL\ user}) \geqslant d_0.$$

Figure: 2.   The uplink sum rates achieved by the three transmission schemes. Noise power of uplink transmission is -61.5dBm, $\alpha = 4$, $r_1 = 10$, $r_2 = 0$ and $\theta = \frac{\pi}{8}$.

MAIN RESULT

1 $\Pr^{\mathsf{U}} \triangleq \Pr\left(R_{\mathsf{FD}}^{\mathsf{U}} \leqslant R_{\mathsf{HD}}^{\mathsf{U}}\right)$: FD yields worse performance than HD (NOMA, uplink).

$$\Pr^{\mathsf{U}} = \Pr\left(\sum_{m=1}^{M} |h_m|^2 \leqslant \frac{P_{\mathsf{SI}}^2 y^2 - 1}{P_{\mathsf{U}}}\right) \approx \cdots \text{ (Calculation)}, \tag{6}$$

其中 $y \sim \mathcal{E}(1)$, $|h_m|^2$ 有分布函数: $F_{|h_m|^2}(t) = 1 - \frac{2}{r_1^2 - r_2^2} \int_{r_2}^{r_1} e^{-r^\alpha t} r \mathrm{d}r$.

2 $\Pr^{\mathsf{D}} \triangleq \Pr\left(R_{\mathsf{FD}}^{\mathsf{D}} \leqslant R_{\mathsf{HD}}^{\mathsf{D}}\right)$: FD yields worse performance than HD (NOMA, downlink).

$$\Pr^{\mathsf{D}} = \Pr\left(|h_0|^2 \leqslant \frac{P_{\mathsf{U}}^2 z^2 - 1}{P_{\mathsf{BS}}}\right) \approx \cdots \text{ (Calculation)}. \tag{7}$$

其中 $z = \sum_{m=1}^{M} |g_m|^2$
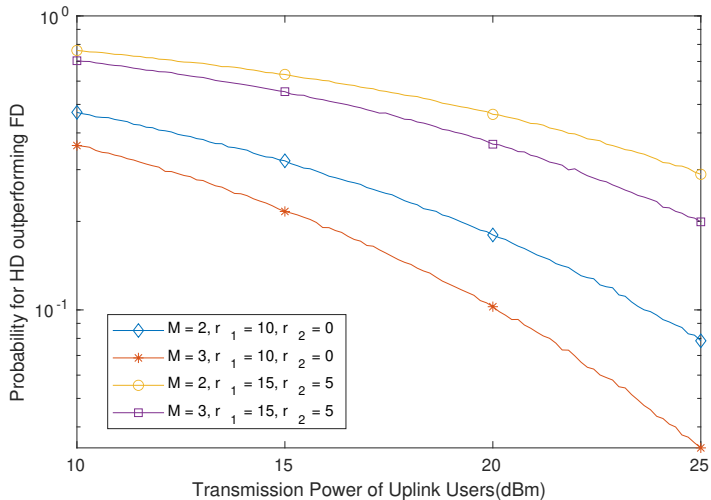
Figure: 3. The probability $P(R_{\mathsf{FD}}^{\mathsf{U}} \leqslant R_{\mathsf{HD}}^{\mathsf{U}})$ versus the user transmission power. $\alpha = 4$, $\theta = \frac{\pi}{8}$, and $P_{\mathsf{SI}} = 200(53dBm)$.
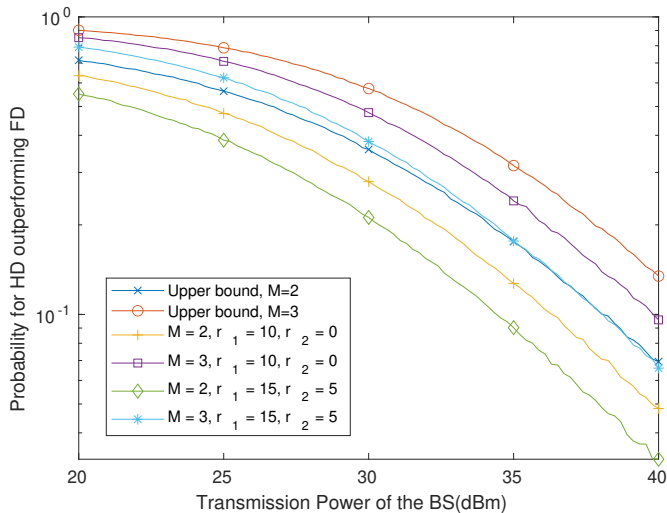
Figure: 4. The probability $P(R_{\mathsf{FD}}^{\mathsf{D}} \leqslant R_{\mathsf{HD}}^{\mathsf{D}})$ versus the BS transmission power. $\alpha = 4$, $\theta = \frac{\pi}{8}$, and $P_{\mathsf{U}} = 20\mathsf{dBm}$, $d_0 = 100$.

MATHEMATICS

Chebyshev-Gauss quadrature(approximation):

$$\int_{-1}^{1} f(x)\mathrm{d}x \approx \sum_{j=1}^{n} \frac{\pi}{n} f(x_j)\sqrt{1-x_j^2}, \ x_j = \cos\left(\frac{2j-1}{2n}\pi\right). \tag{8}$$

Probability Facts:

- $X \sim \mathcal{CN}(0,1) \Rightarrow |X|^2 \sim \mathcal{E}(1)$.
- $X_1, \cdots X_n$ 与 X 独立同分布, 有公共的概率密度 $f(x)$. 记 $S_n = \sum_{j=1}^{n} X_j$ 的概率密度函数为 $f_{S_n}(x)$. 若 $f(x)$ 的 Laplace 变换 $\psi(s) = \mathcal{L}(f(x)) = \mathbb{E}\mathrm{e}^{-sX}$ 收敛, 则 $f_{S_n}(x)$ 的 Laplace 变换为 $[\psi(s)]^n$.
- $X \sim$ Rayleigh, i.e. $f_X(x) = \frac{2x}{\Omega}\mathrm{e}^{-\frac{x^2}{\Omega}}\mathbf{1}_{(x\geqslant 0)}$, $\Omega = \mathbb{E}X^2 \Rightarrow Y = X^2 \sim \mathcal{E}(1/\Omega)$.

MATHEMATICS

Integral:

$$\int_0^\infty x^k e^{-ax^2-bx} dx = \frac{1}{2} a^{-\frac{k}{2}-1} \left(\sqrt{a}\, \Gamma\left(\frac{k+1}{2}\right) \, {}_1F_1\left(\frac{k+1}{2}; \frac{1}{2}; \frac{b^2}{4a}\right) \right.$$
$$\left. - b\Gamma\left(\frac{k}{2}+1\right) {}_1F_1\left(\frac{k}{2}+1; \frac{3}{2}; \frac{b^2}{4a}\right) \right)$$
$$= \Gamma(k+1)e^{b^2/8a}(2a)^{-(k+1)/2} D_{-k-1}(b/\sqrt{2a}). \tag{9}$$

其中, ${}_1F_1$ 表示 Kummer's confluent hypergeometric function:

$$ {}_1F_1(a;b;z) := \sum_{n=0}^\infty \frac{(a)_n}{(b)_n} \frac{z^n}{n!}, \ (q)_n = \begin{cases} 1, & n = 0, \\ q(q+1)\cdots(q+n-1), & n > 0. \end{cases} \tag{10}$$

MATHEMATICS

$D_\nu(\cdot)$ 是 Parabolic cylinder functions(Whittaker & Watson):

$$D_\nu(z) = \frac{1}{\sqrt{\pi}} 2^{\nu/2} \mathrm{e}^{-z^2/4} \bigg(\cos\Big(\frac{\pi}{2}\nu\Big) \Gamma\Big(\frac{\nu+1}{2}\Big) {}_1F_1\Big(-\frac{\nu}{2}; \frac{1}{2}; \frac{z^2}{2}\Big)$$
$$+ \sqrt{2} z \sin\Big(\frac{\pi}{2}\nu\Big) \Gamma\Big(\frac{\nu}{2}+1\Big) {}_1F_1\Big(\frac{1}{2}-\frac{\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\Big) \bigg), \quad \nu > 0. \tag{11}$$

使用 MMA 软件检查计算结果:

In[1]:= $\int_0^{+\infty} x^k * E^{-a*x^2-b*x} \, d\!l\, x$

$\int_0^{+\infty} x^k * E^{-a*x^2-b*x} \, d\!l\, x - \text{Gamma}[k+1] * E^{\frac{b^2}{8*a}} * (2*a)^{-\frac{k+1}{2}} * \text{ParabolicCylinderD}\left[-k-1, \frac{b}{\sqrt{2*a}}\right] \,/\!/ \text{FullSimplify}$

Out[1]= $\boxed{\frac{1}{2} a^{-1-\frac{k}{2}} \left(-b\, \text{Gamma}\left[1+\frac{k}{2}\right] \text{Hypergeometric1F1}\left[1+\frac{k}{2}, \frac{3}{2}, \frac{b^2}{4a}\right] + \sqrt{a}\, \text{Gamma}\left[\frac{1+k}{2}\right] \text{Hypergeometric1F1}\left[\frac{1+k}{2}, \frac{1}{2}, \frac{b^2}{4a}\right]\right)}$
$\quad$ if $\text{Re}[a] > 0 \,\&\&\, \text{Re}[k] > -1$

Out[2]= $\boxed{0 \;\; \text{if} \;\; \text{Re}[a] > 0 \,\&\&\, \text{Re}[k] > -1}$

THANKS