组会报告

论文复现: On the Coexistence Between Full-Duplex and NOMA

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System Model

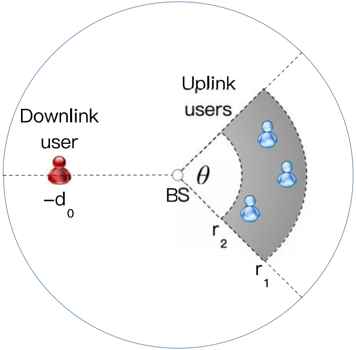

Figure: 1

1 个 DL user 位于 $(-d_0, 0)$,
M 个 UL users 均匀分布于 $\mathrm{ann}(\mathsf{BS}, r_2, r_1; -\frac{\theta}{2}, \frac{\theta}{2})$
BS 接收信号 $y_{\mathsf{BS}} = \sqrt{P_{\mathsf{U}}} \sum_{m=1}^{M} h_m s_m + h_{\mathsf{SI}} \sqrt{P_{\mathsf{SI}}} s_0 + n_{\mathsf{BS}}$.
DL 接收信号 $y_{\mathsf{D}} = \sqrt{P_{\mathsf{U}}} \sum_{m=1}^{M} g_m s_m + h_0 \sqrt{P_{\mathsf{BS}}} s_0 + n_{\mathsf{D}}$.

模型假设:

- UL users 的发射功率 P_{U} 均相同,
- $h_{\mathsf{SI}} \sim \mathcal{CN}(0, 1)$,
- $h_m \sim$ Rayleigh fading.

Performance Analysis

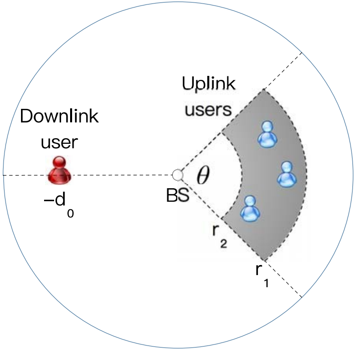

Figure: 1

1. Uplink sum rates
   - NOMA, FD

$$R^{\mathsf{U}}_{\mathsf{FD}} = \log\left(1 + \frac{\sum_{m=1}^{M} P_{\mathsf{U}} \left|h_m\right|^2}{P_{\mathsf{SI}} \left|h_{\mathsf{SI}}\right|^2 + 1}\right), \tag{1}$$

   - NOMA, HD

$$R^{\mathsf{U}}_{\mathsf{HD}} = \frac{1}{2}\log\left(1 + \sum_{m=1}^{M} P_{\mathsf{U}} \left|h_m\right|^2\right), \tag{2}$$

   - OMA,HD

$$R^{\mathsf{U}}_{\mathsf{OMA}} = \frac{1}{2M}\sum_{m=1}^{M}\log\left(1 + P_{\mathsf{U}} \left|h_m\right|^2\right). \tag{3}$$

Performance Analysis

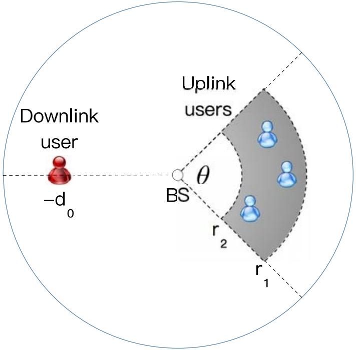

Figure: 1

2 Downlink sum rates

- NOMA, FD

$$R^{\mathsf{D}}_{\mathsf{FD}} = \log\left(1 + \frac{P_{\mathsf{BS}}\left|h_0\right|^2}{P_{\mathsf{U}}\sum_{m=1}^{M}\left|g_m\right|^2 + 1}\right), \qquad (4)$$

- NOMA, HD

$$R^{\mathsf{D}}_{\mathsf{HD}} = \frac{1}{2}\log\left(1 + P_{\mathsf{BS}}\left|h_0\right|^2\right), \qquad (5)$$

$$\text{distance}(\mathsf{DL\ user}, \mathsf{UL\ user}) \geqslant d_0.$$

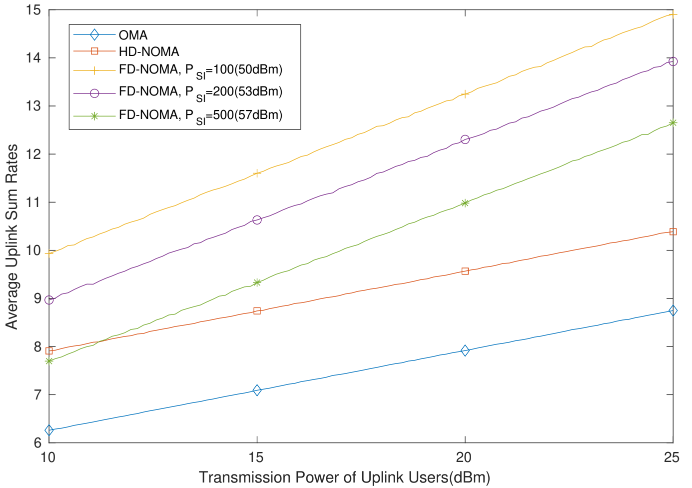

Figure: 2. The uplink sum rates achieved by the three transmission schemes. Noise power of uplink transmission is -61.5dBm, $\alpha = 4$, $r_1 = 10$, $r_2 = 0$ and $\theta = \frac{\pi}{8}$.

Main Result

1. $\Pr^{\mathsf{U}} \triangleq \Pr\left(R_{\mathsf{FD}}^{\mathsf{U}} \leqslant R_{\mathsf{HD}}^{\mathsf{U}}\right)$: FD yields worse performance than HD (NOMA, uplink).

$$\Pr^{\mathsf{U}} = \Pr\left(\sum_{m=1}^{M} |h_m|^2 \leqslant \frac{P_{\mathsf{SI}}^2 y^2 - 1}{P_{\mathsf{U}}}\right) \approx \cdots \text{ (Calculation)}, \tag{6}$$

其中 $y \sim \mathcal{E}(1)$, $|h_m|^2$ 有分布函数: $F_{|h_m|^2}(t) = 1 - \frac{2}{r_1^2 - r_2^2} \int_{r^2}^{r_1} \mathrm{e}^{-r^\alpha t} r \mathrm{d}r$.

2. $\Pr^{\mathsf{D}} \triangleq \Pr\left(R_{\mathsf{FD}}^{\mathsf{D}} \leqslant R_{\mathsf{HD}}^{\mathsf{D}}\right)$: FD yields worse performance than HD (NOMA, downlink).

$$\Pr^{\mathsf{D}} = \Pr\left(|h_0|^2 \leqslant \frac{P_{\mathsf{U}}^2 z^2 - 1}{P_{\mathsf{BS}}}\right) \approx \cdots \text{ (Calculation)}. \tag{7}$$

其中 $z = \sum_{m=1}^{M} |g_m|^2$

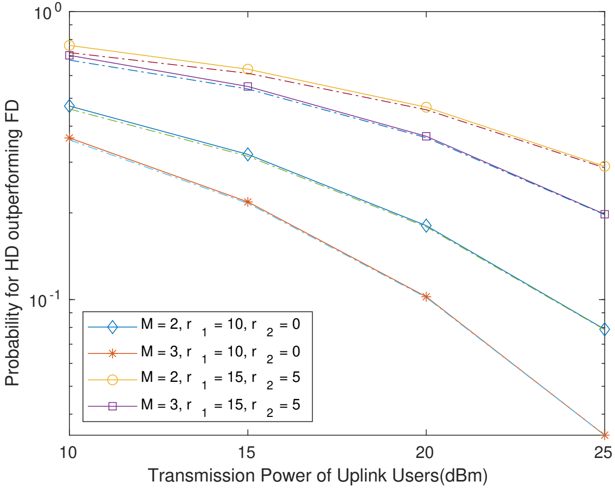

Figure: 3. The probability $P(R_{\text{FD}}^{\text{U}} \leqslant R_{\text{HD}}^{\text{U}})$ versus the user transmission power. $\alpha = 4$, $\theta = \frac{\pi}{8}$, and $P_{\text{SI}} = 200(53dBm)$.

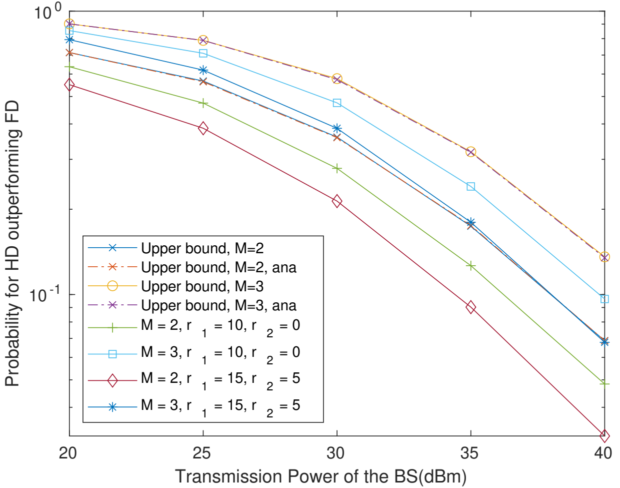

Figure: 4. The probability $P(R^{\mathsf{D}}_{\mathsf{FD}} \leqslant R^{\mathsf{D}}_{\mathsf{HD}})$ versus the BS transmission power. $\alpha = 4$, $\theta = \frac{\pi}{8}$, and $P_{\mathsf{U}} = 20\text{dBm}$, $d_0 = 100$.

Chebyshev-Gauss quadrature(approximation):

$$\int_{-1}^{1} f(x)\mathrm{d}x \approx \sum_{j=1}^{n} \frac{\pi}{n} f(x_j)\sqrt{1-x_j^2},\ x_j = \cos\left(\frac{2j-1}{2n}\pi\right). \tag{8}$$

Probability Facts:

- $X \sim \mathcal{CN}(0,1) \Rightarrow |X|^2 \sim \mathcal{E}(1)$.
- $X_1, \cdots X_n$ 与 X 独立同分布，有公共的概率密度 $f(x)$. 记 $S_n = \sum_{j=1}^{n} X_j$ 的概率密度函数为 $f_{S_n}(x)$. 若 $f(x)$ 的 Laplace 变换 $\psi(s) = \mathcal{L}(f(x)) = \mathbb{E}\mathrm{e}^{-sX}$ 收敛，则 $f_{S_n}(x)$ 的 Laplace 变换为 $[\psi(s)]^n$.
- $X \sim$ Rayleigh, i.e. $f_X(x) = \frac{2x}{\Omega}\mathrm{e}^{-\frac{x^2}{\Omega}}\mathbf{1}_{(x\geqslant 0)}$, $\Omega = \mathbb{E}X^2 \Rightarrow Y = X^2 \sim \mathcal{E}(1/\Omega)$.

Integral:

$$\begin{aligned}\int_0^\infty x^k \mathrm{e}^{-ax^2-bx}\mathrm{d}x &= \frac{1}{2}a^{-\frac{k}{2}-1}\Bigg(\sqrt{a}\Gamma\left(\frac{k+1}{2}\right)\,{}_1F_1\left(\frac{k+1}{2};\frac{1}{2};\frac{b^2}{4a}\right)\\ &\quad - b\Gamma\left(\frac{k}{2}+1\right)\,{}_1F_1\left(\frac{k}{2}+1;\frac{3}{2};\frac{b^2}{4a}\right)\Bigg)\\ &= \Gamma(k+1)\mathrm{e}^{b^2/8a}(2a)^{-(k+1)/2}D_{-k-1}(b/\sqrt{2a}).\end{aligned} \tag{9}$$

其中，${}_1F_1$ 表示 Kummer's confluent hypergeometric function:

$${}_1F_1(a;b;z) := \sum_{n=0}^{\infty}\frac{(a)_n}{(b)_n}\frac{z^n}{n!},\ (q)_n = \begin{cases}1, & n=0,\\ q(q+1)\cdots(q+n-1), & n>0.\end{cases} \tag{10}$$

$D_\nu(\cdot)$ 是 Parabolic cylinder functions(Whittaker & Watson):

$$D_\nu(z) = \begin{cases} \frac{1}{\sqrt{\pi}} 2^{\nu/2} \mathrm{e}^{-z^2/4} \Bigg(\cos\big(\frac{\pi}{2}\nu\big) \Gamma\Big(\frac{\nu+1}{2}\Big) \, {}_1F_1\Big(-\frac{\nu}{2}; \frac{1}{2}; \frac{z^2}{2}\Big) & \\ + \sqrt{2} z \sin\big(\frac{\pi}{2}\nu\big) \Gamma\Big(\frac{\nu}{2}+1\Big) \, {}_1F_1\Big(\frac{1}{2} - \frac{\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\Big)\Bigg), & \nu > 0, \\ \mathrm{e}^{-z^2/4}, & \nu = 0, \\ \frac{2^{(k-1)/2} \mathrm{e}^{-z^2/4}}{\Gamma(k+1)} \Bigg(\Gamma\big(\frac{k+1}{2}\big) \, {}_1F_1\Big(\frac{k+1}{2}; \frac{1}{2}; \frac{z^2}{2}\Big) & \\ - \sqrt{2} z \Gamma\big(\frac{k}{2}+1\big) \, {}_1F_1\Big(\frac{k}{2}+1; \frac{3}{2}; \frac{z^2}{2}\Big)\Bigg), & \nu = -k-1 < 0. \end{cases} \tag{11}$$

使用 MMA 软件检查计算结果:

In[1]:= $\int_0^{+\infty} x^k * E^{-a*x^2-b*x}\, dx$

$\int_0^{+\infty} x^k * E^{-a*x^2-b*x}\, dx$ − Gamma[$k+1$] $* E^{\frac{b^2}{8*a}} * (2*a)^{-\frac{k+1}{2}}$ * ParabolicCylinderD[$-k-1, \frac{b}{\sqrt{2*a}}$] // FullSimplify

Out[1]= $\frac{1}{2} a^{-1-\frac{k}{2}} \left(-b\, \text{Gamma}\left[1+\frac{k}{2}\right] \text{Hypergeometric1F1}\left[1+\frac{k}{2}, \frac{3}{2}, \frac{b^2}{4a}\right] + \sqrt{a}\, \text{Gamma}\left[\frac{1+k}{2}\right] \text{Hypergeometric1F1}\left[\frac{1+k}{2}, \frac{1}{2}, \frac{b^2}{4a}\right]\right)$ if Re[a] > 0 && Re[k] > −1

Out[2]= 0 if Re[a] > 0 && Re[k] > −1

原文 (4)、(21) 式中 $D_{-l}(\cdot)$ 的宗量都很大，直接计算会造成浮点异常．这里进行渐近展开，当 $z\to+\infty$ 时，有

$$
\mathrm{e}^{z^2/4}D_{-l}(z)=\begin{cases}
\frac{1}{z}-\frac{1}{z^3}+\frac{3}{z^5}-\frac{15}{z^7}+\frac{105}{z^9}-\frac{945}{z^{11}}+\frac{10395}{z^{13}}-\frac{135135}{z^{15}}+\cdots, & l=-1,\\
\frac{1}{z^2}-\frac{3}{z^4}+\frac{15}{z^6}-\frac{105}{z^8}+\frac{945}{z^{10}}-\frac{10395}{z^{12}}+\frac{135135}{z^{14}}+\cdots, & l=-2,\\
\frac{1}{z^3}-\frac{6}{z^5}+\frac{45}{z^7}-\frac{420}{z^9}+\frac{4725}{z^{11}}-\frac{62370}{z^{13}}+\frac{945945}{z^{15}}+\cdots, & l=-3,\\
\frac{1}{z^4}-\frac{10}{z^6}+\frac{105}{z^8}-\frac{1260}{z^{10}}+\frac{17325}{z^{12}}-\frac{270270}{z^{14}}+\cdots, & l=-4,\\
\frac{1}{z^5}-\frac{15}{z^7}+\frac{210}{z^9}-\frac{3150}{z^{11}}+\frac{51975}{z^{13}}-\frac{945945}{z^{15}}+\cdots, & l=-5,\\
\frac{1}{z^6}-\frac{21}{z^8}+\frac{378}{z^{10}}-\frac{6930}{z^{12}}+\frac{135135}{z^{14}}+\cdots, & l=-6,\\
\frac{1}{z^7}-\frac{28}{z^9}+\frac{630}{z^{11}}-\frac{13860}{z^{13}}+\frac{315315}{z^{15}}+\cdots, & l=-7,\\
\frac{1}{z^8}-\frac{36}{z^{10}}+\frac{990}{z^{12}}-\frac{25740}{z^{14}}+\cdots, & l=-8,\\
\frac{1}{z^9}-\frac{45}{z^{11}}+\frac{1485}{z^{13}}-\frac{45045}{z^{15}}+\cdots, & l=-9,\\
\frac{1}{z^{10}}-\frac{55}{z^{12}}+\frac{2145}{z^{14}}+\cdots, & l=-10,\\
\cdots, & \cdots.
\end{cases}\tag{12}
$$

THANKS