Question 1

计算当 $n \to \infty$ 时, 序列

$$\left\{\cos\left(\frac{\pi}{2n+1}\right)\cos\left(\frac{2\pi}{2n+1}\right)\cdots\cos\left(\frac{n\pi}{2n+1}\right)\right\}$$

的等价无穷小.

证明. 令

$$a_n = \cos\left(\frac{\pi}{2n+1}\right)\cos\left(\frac{2\pi}{2n+1}\right)\cdots\cos\left(\frac{n\pi}{2n+1}\right).$$

知

$$\lim_{n\to\infty} 2^n a_n = 1.$$

因此, 所求的等价无穷小是 $1/2^n$. □