


3分 1800

一, 函数, 极限, 连续.

$$(1) \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

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$$\textcircled{1} \frac{\left(x + \frac{1}{6}x^3 + o(x^3)\right) - \left(x - \frac{x^3}{6!} + o(x^3)\right)}{x^3} = \frac{1}{2}.$$

$$\textcircled{2} \frac{\tan x (1 - \cos x)}{x^3} = \frac{\tan x}{x} \left(\frac{1 - \cos x}{x} \right) = \frac{1}{2}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{2^x + \sin 2x - 1}{x} \quad \frac{0}{0}$$

$$\cancel{\frac{1}{2} \times 3} \lim_{x \rightarrow 0} 2^x \ln 2 + 2 \cos 3x = 2 + 2 \ln 2$$

$$2^x = e^{x \ln 2} = x \ln 2 + 1 \quad x \rightarrow 0.$$

(3)

$$\frac{e^{x^2} + \cos x - 2}{x \arcsin x} = \frac{e^{x^2} - 1 + \cos x - 1}{2x^2}$$

$$= \frac{1}{2} + 0 = \frac{1}{2}$$

(4)

$$\frac{e^x - e^{\arcsin x}}{x^3} = e^{\arcsin x} \left(\frac{e^{x - \arcsin x} - 1}{x^3} \right)$$

$$= x - \arcsin x$$

$$\frac{x^3}{1 - (1-x^2)^{-\frac{1}{2}}} =$$

$$= -\frac{1}{3} \left[\lim_{x \rightarrow 0} \frac{(1-x^2)^{-\frac{1}{2}} - 1}{x^2} \right]$$

$$= -\frac{1}{6}$$

$$\lim_{x \rightarrow 0} (1+x)^2 \rightarrow 2x + 1$$

(5)

$$\lim_{x \rightarrow 0} \frac{\ln \frac{\sin x}{x}}{x^2} = \frac{0}{0}$$

$$= \frac{\frac{\sin x}{x} - 1}{x^2} = \frac{\sin x - x}{x^3} = \frac{1}{6}$$

c6)

$\lim_{x \rightarrow 0}$

$$\frac{\sqrt{1+x\cos x} - \sqrt{2+x}}{x^3} = \frac{\frac{1}{2}(\cos x - 1)}{x^2} = -\frac{1}{6}$$

c7)

$\lim_{x \rightarrow \infty}$

$$\frac{(e^{2x} + 1)^x - 1}{1 - \cos x} = \frac{e^{x \ln(\frac{e^{2x} + 1}{2})} - 1}{\frac{1}{2}x^2} - 1$$

$$\approx \lim_{x \rightarrow \infty} \frac{\ln(\frac{e^{2x} + 1}{2})}{x}$$

$$= \frac{e^{2x}}{e^{2x}} = 2$$

$$2. (1) \lim_{x \rightarrow 0} (1-x^2)^{\frac{1}{x \sin x}} \quad 1^\infty$$

$$= e^{\frac{1}{x \sin x} \ln(1-x^2)} = e^{\frac{-2x}{\sin x}} = e^{-2}.$$

$$\begin{aligned} (2) \lim_{x \rightarrow 0} (2^x + \sin x)^{\frac{1}{e^{2x}-1}} &= e^{\frac{1}{e^{2x}-1} (2^x + \sin x - 1)} \\ &= e^{\frac{\sin x}{2x} + \frac{2^x - 1}{2x}} \rightarrow \frac{2^x \ln 2}{2} \\ &= e^{\frac{1}{2} \ln 2} \end{aligned}$$

$$3. \lim_{x \rightarrow 0} \left[(x+1) \arcsin \frac{\tan x}{x} - \frac{5}{2}x \right]$$

$$= (\operatorname{arctan} x + \operatorname{arctan} x - \frac{2}{x})$$

$$= \frac{2(\operatorname{arctan} x - \frac{1}{x})}{\frac{1}{x^2}} + \operatorname{arctan} x$$

$$= \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} + \frac{2}{x} = \frac{4}{x} - 1$$

4.

$$\lim_{x \rightarrow \infty} (\sqrt{x+2\sqrt{x}} - \sqrt{x-\sqrt{x}})$$

$$= \frac{3\sqrt{x}}{\sqrt{x+2\sqrt{x}} + \sqrt{x-\sqrt{x}}} = 3 \frac{1}{\sqrt{1+\frac{2}{\sqrt{x}}} + \sqrt{1-\frac{1}{\sqrt{x}}}} = \frac{3}{2}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 - 2x + 4} - x}{\sqrt{x^2 - 2x + 4} - x} = \frac{-2x + 4}{\sqrt{x^2 - 2x + 4} - x} = \frac{-2 + \frac{4}{x}}{\sqrt{1 - \frac{2}{x} + \frac{4}{x^2}} - 1} = 1$$

5.

$$\lim_{x \rightarrow 0} \left[\frac{1}{\ln(1+x)} - \frac{1}{x} \right] = \frac{x - \ln(1+x)}{x \ln(1+x)} = \frac{1}{2}$$

(Note: The original image contains a crossed-out '5' and some additional scribbles below the limit expression.)

