线性方程组.

1. $AX=0$ 有非零解 $\Longleftrightarrow |A|=0$.

$$A=\begin{pmatrix}1&2&3\\0&4&a\\1&a&9\end{pmatrix}$$

$$\begin{vmatrix}1&2&3\\0&4&a\\0&a-2&6\end{vmatrix}=\begin{vmatrix}4&a\\a-2&6\end{vmatrix}$$

$$=24-(a^2-2a)=0$$

$$a^2-2a-24=0$$

$$(a-6)(a+4)=0$$

$$a<0$$

$$a=-4.$$

$$A^*A=|A|E=0.$$

$$r(A)=2 \qquad r(A^*)=1$$

A 即 A^* 的通解

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & -4 \\ 1 & -4 & 9 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & -4 \\ 0 & -6 & 6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^* A = 0.$$

$$X = c_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}.$$ c_1, c_2 为任意常数.

2.

A 的各行之和为 0

$r(A) = n-1$ $AX=0$ 的通解.

$$A\begin{pmatrix}1\\ \vdots\\ 1\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$$Ax=0$$

$$x=c_1\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$ (C为常数)

4. $\eta_1, \cdots \eta_s$

$$A(k_1\eta_1+k_2\eta_2\cdots+k_s\eta_s)=b$$

$$(k_1+\cdots k_s)b=b \qquad k_1+k_2\cdots+k_s=1.$$

5.

$$A=\begin{pmatrix}1&2&-2\\3&-1&k\\3&1&-1\end{pmatrix}$$

$$r(A)<3.$$

$$r(A)=2.$$

$$\begin{pmatrix} 1 & 2 & -2 \\ 0 & -7 & k+6 \\ 0 & -5 & 5 \end{pmatrix}$$

$$\frac{-7}{-5} = \frac{k+6}{5}$$

$$-35 = -5k - 30$$

$$k = 1$$

$r(A) = 2$

6.

$|B| = 0$

$r(A) = 3$

$$\alpha_1 + \alpha_2 = \begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}$$

$$\alpha_2 + \alpha_3 = \begin{pmatrix} -6 \\ 4 \\ 2 \\ -6 \end{pmatrix}$$

$$\alpha_3 - \alpha_1 = \begin{pmatrix} -3 \\ 5 \\ -1 \\ -12 \end{pmatrix}$$

$$A\alpha_1 = b$$

$$A\alpha_2 = b$$

$$A(\alpha_1 + \alpha_2) = 2b \qquad A\frac{(\alpha_1 + \alpha_2)}{2} = b.$$

通解 $k\begin{pmatrix} -3 \\ 5 \\ -1 \\ -12 \end{pmatrix} + \begin{pmatrix} 1 \\ -\frac{1}{2} \\ \frac{3}{2} \\ 1 \end{pmatrix}$

7. 方 组无解

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 3 & a+2 & 3 \\ 1 & a & -2 & 0 \end{array}\right) \longrightarrow \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & a & 1 \\ 0 & a-2 & -3 & -1 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -a & -1 \\ 0 & a-2 & -3 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -a & -1 \\ 0 & 0 & -3+a(a-2), & -1+(a-2) \end{pmatrix}$$

$a = -1$

8.

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -a_1 \\ 0 & 1 & 1 & 0 & a_2 \\ 0 & 0 & 1 & 1 & -a_3 \\ 1 & 0 & 0 & 1 & a_4 \end{array}\right)$$ 有解.

$r(A) = r(\bar{A})$.

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -a_1 \\ 0 & 1 & 1 & 0 & a_2 \\ 0 & 0 & 1 & 1 & -a_3 \\ 0 & -1 & 0 & 1 & a_4+a_1 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -a_1 \\ 0 & 1 & 1 & 0 & a_2 \\ 0 & 0 & 1 & 1 & -a_3 \\ 0 & 0 & 1 & 1 & a_4+a_1+a_2 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -a_1 \\ 0 & 1 & 1 & 0 & a_2 \\ 0 & 0 & 1 & 1 & -a_3 \\ 0 & 0 & 0 & 0 & \underline{a_4+a_1+a_2+a_3} = 0 \end{array}\right).$$

9.

A $m\times n$ 矩阵.

10. $A_{m\times n}$ 若 $r(A)=m$ 则 $AX=b$ 一定有解.

行满秩 $\to r(\bar{A})=m$.