矩阵:

1. $A = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} (2, 1, 1) = \begin{pmatrix} 2 & 1 & 1 \\ -2 & -1 & -1 \\ 4 & 2 & 2 \end{pmatrix}$

$\alpha = (1, -1, 2)^T \quad \beta = (2, 1, 1)^T$

$\beta^T \alpha = 3 \quad A^2 = \alpha\beta^T\alpha\beta^T$

$= 3\alpha\beta^T = 3A$

$A^n = 3^{n-1} A$

2.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad A^n - 2A^{n-1} = A^{n-1}(A - 2)$$

$$= A^{n-1}\begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

$$= A^{n-2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

3.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (A+3E)^{-1}(A^2 - 9E).$$

$= A - 3E.$

4. $A^2 - B^2 = (A+B)(A-B)$

$AB = BA.$

5. $|(\frac{1}{2}A)^{-1}| = |2A^{-1}| = 8\frac{1}{|A|} = 2.$

6. $|(\frac{1}{2}A^*)^{-1}| =$ $\quad AA^* = |A|$

$= 2^3 |(A^*)^{-1}| = 2^3 \frac{1}{|A^*|} = 2^3 \frac{1}{||A| A^{-1}|}$

$= 2^3 \frac{1}{4^3} |A|$

$= \frac{1}{2}$

7.

$|A^*| = 8$ $\qquad AA^* = |A|$ $\qquad A^{-1} = \frac{A^*}{|A|}$

$$|A^*| = \left| |A| A^{-1} \right| \qquad \left| 4 \frac{A^*}{|A|} - 3A^* \right|$$

$$= |A|^4 \frac{1}{|A|} |A|^3 = 8 \quad = \left| -A^* \right| = (-1)^4 8 = \cancel{-8}\ 8$$

$|A| = 2$ ✓

8.

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 3 & 2 \\ \cancel{2} & 0 & t \\ 1 & 2 & t+7 \end{pmatrix}$$

$\underline{AB = 0}$

$r(A) + r(B) \leq 3$

$r(B) \geq 1$

$r(A) \leq 2$

$$A = \begin{pmatrix} 1 & 3 & 3 \\ -1 & 3 & 2 \\ 2 & 0 & t \\ 1 & 9 & t+7 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 3 \\ 0 & 6 & 5 \\ 0 & -6 & t-6 \\ 0 & 6 & t+4 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 3 & 3 \\ 0 & 6 & 5 \\ 0 & 0 & t-1 \\ 0 & 0 & t-1 \end{pmatrix}$$

$t=1$

9.

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 2 & 0 \\ -1 & 2 & -2 \end{pmatrix} \qquad A^{-1} = \frac{A^*}{|A|}$$

$$A^{-1} = \frac{1}{6}\begin{pmatrix} 4 & -4 & -2 \\ -2 & 5 & 1 \\ -4 & 2 & -1 \end{pmatrix}$$

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$$A_1 = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} A_1 & \\ & A_2 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} A_1^{-1} & \\ & A_2^{-1} \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 5 & 2 & 1 & \\ 2 & 1 & & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & \frac{2}{5} & \frac{1}{5} & \\ 2 & 1 & & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & -\frac{2}{5} & 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cc|cc} 1 & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -2 & 5 \end{array}\right) \longrightarrow \left(\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & 5 \end{array}\right)$$

$$A_1^{-1} = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix} \qquad A_2^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{1}{2} \end{pmatrix}$$

4.

$$AA^* = |A| \qquad A^* = A^{-1}|A|$$

$$(A^*)^{-1} = \frac{1}{|A|} A$$

12.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} \qquad A^*\begin{pmatrix}1\\1\\1\end{pmatrix} + A^*\begin{pmatrix}1\\2\\1\end{pmatrix} \; E A^*\begin{pmatrix}1\\1\\3\end{pmatrix}$$

$$A = (\alpha_1, \alpha_2, \alpha_3).$$

$$|A| = 2$$

$$A^*A = 2E$$

$$A^*A = (A^*\alpha_1, A^*\alpha_2, A^*\alpha_3)$$

$$= \begin{pmatrix} 2 & & \\ & 2 & \\ & & 2 \end{pmatrix}$$

$$\cancel{\text{陈}} = 2\begin{pmatrix}1\\1\\1\end{pmatrix}$$

13.

$$AB = (E - aa^T)(E + \frac{1}{a} aa^T)$$

$$0 = \cancel{E^2} - aa^T + \frac{1}{a} aa^T - \frac{1}{a} a a^T a a^T$$

$$= \cancel{E}$$

$$a^T a = a^2$$

$$-aa^T + \frac{1}{a} aa^T - a\, a a^T = 0$$

$$-a - 1 + \frac{1}{a} = 0$$

$a + a - 1 = 0$

$a = \frac{-1 + \sqrt{5}}{2}$

14.

$r(AB) = r(A)$

$|B| \neq 0$

15.

$AB = 0$

$r(A) + r(B) \leq 3$

$r(A) < 3$

$r(A) = 2$.