


线代

行列式

$$1. f(x) = \begin{vmatrix} x+2 & -2 & 3 \\ 1 & 2x+3 & 4 \\ -2 & 4 & 3x+1 \end{vmatrix}$$

x^2 系数 23

$$(x+2)(2x+3)(3x+1)$$

$$= (2x^2 + 7x + 6)(3x+1)$$

$$= 6x^3 + 21x^2 + 18x + 2x^2 + 7x + 6$$

$$2. (a+1) + 2(a-2) + 3(a-1) = 0$$

$$6a - 6 = 0 \quad a = 1$$

3.

$$|A| = a$$

$$|B| = b$$

$$\begin{vmatrix} 0 & A \\ B & 0 \end{vmatrix} \begin{matrix} m \\ n \end{matrix}$$

$$(-1)^{mn} |A||B|$$

$$= (-1)^{mn} ab.$$

4.

$$[-A_1, -2A_2, 2A_1 + 3A_3 - 3A_5 + 2A_1]$$

$$= (A_1, A_2, A_3) \begin{pmatrix} -1 & 0 & 2 \\ -2 & 2 & 0 \\ 0 & 3 & -3 \end{pmatrix} = -2 \times 6 = 12 \checkmark$$

5.

$$A = (2, \gamma_1, \gamma_2)$$

$$B = (\beta_1, \gamma_1, \gamma_2)$$

$$|A| = 3$$

$$|B| = 4$$

$$|5A - 2B| = |(5 \cdot 2 - 2\beta_1, 5\gamma_1 - 2\gamma_2)|$$

$$= 9 \mid 52 - 2\beta, y_1, y_2 \mid$$

$$= 9 \left[\mid 52, y_1, y_2 \mid \quad -2 \mid \beta, y_1, y_2 \mid \right]$$

263.

6.

$$A \equiv \mathbb{R}^n$$

$$B \equiv \mathbb{R}^m$$

$$\mid A \mid = 2$$

$$\mid B \mid = 6$$

$$\left| - \begin{pmatrix} A^{-1} & \\ & 2B^T \end{pmatrix} \right|$$

$$= \begin{pmatrix} C & D \end{pmatrix} \begin{vmatrix} A^T \\ 2B^T \end{vmatrix} = -\frac{1}{2} \times 2^4 \times 6 = -48$$

7.

$$|C + A| = 0$$

$$A = B^T$$

$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Ax = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda_1 = 4 \quad \lambda_2 = 1$$

8 计算行列式

$$\begin{vmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{vmatrix} = 6 \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 48$$

9.

$$D = \begin{vmatrix} a^2 & (a+2)^2 & (a+4)^2 \\ b^2 & (b+2)^2 & (b+4)^2 \\ c^2 & (c+2)^2 & (c+4)^2 \end{vmatrix}$$

$$D = \begin{vmatrix} a^2 & 4a+4 & 8a+16 \\ b^2 & 4b+4 & 8b+16 \\ c^2 & 4c+4 & 8c+16 \end{vmatrix}$$

$$D = \begin{vmatrix} a^2 & a+1 & a+2 \\ b^2 & b+1 & b+2 \\ c^2 & c+1 & c+2 \end{vmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{vmatrix} a^2 & b^2 & c^2 \end{vmatrix}$$

$$\begin{pmatrix} a^2 & b^2 & c^2 \end{pmatrix} \begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{vmatrix} a^2 & b^2 & c^2 \end{vmatrix}$$

$$\begin{pmatrix} a^2 & b^2 & c^2 \end{pmatrix} \begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$$

$$\begin{pmatrix} a^2 & b^2 & c^2 \end{pmatrix} \begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$$

$$= -32 (c-a)(c-b)(b-a)$$