汤 1800

一、函数、极限、连续

(1) $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$

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① $\frac{\left(x + \frac{1}{3}x^3 + o(x^3)\right) - \left(x - \frac{x^3}{3!} + o(x^3)\right)}{x^3} = \frac{1}{2}$

② $\frac{\tan x(1 - \cos x)}{x^3} = \frac{\tan x}{x}\left(\frac{1-\cos x}{x^2}\right) = \frac{1}{2}$

(2) $\lim_{x\to 0} \frac{2^x + \sin 2x - 1}{x}$ $\quad \frac{0}{0}$

洛 x3k

$$\lim_{x \to 0} 2^x \ln 2 + 2\cos 2x = 2 + 2\ln 2$$

$$2^x = e^{x\ln 2} = x\ln 2 + 1 \quad x \to 0.$$

(3)

$$\frac{e^{x^2} + \cos x - 2}{x \arcsin 2x} = \frac{e^{x^2} - 1 + \cos x - 1}{2x^2}$$

$$= \frac{1}{2} + \left(-\frac{1}{4}\right) = \frac{1}{4}$$

(4).

$$\frac{e^x - e^{\arcsin x}}{x^3} = e^{\arcsin x} \frac{\left(e^{x - \arcsin x} - 1\right)}{x^3}.$$

$$= x - \arcsin x$$

x^3

洛 $= \dfrac{1-(1-x^2)^{-\frac{1}{2}}}{3x^2}$

$= -\frac{1}{3}\left[\lim_{x\to 0} \dfrac{(1-x^2)^{-\frac{1}{2}}-1}{x^2}\right]$

$= -\frac{1}{6}$

$(1+x)^{\alpha} \xrightarrow[x\to 0]{} \alpha x+1$

(5) $\lim_{x\to 0} \dfrac{\ln\frac{\sin x}{x}}{x^2} = \dfrac{0}{0}$

$$= \frac{\frac{\sin x}{x} - 1}{x^2} = \frac{\sin x - x}{x^3} = -\frac{1}{6}$$

(6)

$$\lim_{x \to 0} \frac{\sqrt{1+x\cos x} - \sqrt{1+x}}{x^3} = \frac{\frac{1}{2}(\cos x - 1)}{x^2} = -\frac{1}{4}$$

(7)

$$\lim_{x \to 0} \frac{(e^{2x}+1)^x - 1}{1-\cos x} = \frac{e^{x\ln(\frac{e^{2x}+1}{2})} - 1}{\frac{1}{2}x^2} = 2\lim_{x \to 0} \frac{\ln(\frac{e^{2x}+1}{2})}{x}$$

$$= \cancel{2} \frac{e^{2x}-1}{x} = 2$$

2. (1) $\lim\limits_{x\to 0}(1-2x^2)^{\frac{1}{x\sin x}}$ $\quad 1^{\infty}$

$$= e^{\frac{1}{x\sin x}\ln(1-2x^2)} = e^{\frac{-2x}{\sin x}} = e^{-2}.$$

(2) $\lim\limits_{x\to 0}(2^x+\sin x)^{\frac{1}{e^{2x}-1}} = e^{\frac{1}{e^{2x}-1}(2^x+\sin x-1)}$

$$= e^{\frac{\sin x}{2x}+\frac{2^x-1}{2x}} \quad \frac{2^x-1}{2x} \to \frac{2^x\ln 2}{2}$$

$$= e^{\frac{1}{2}+\frac{\ln 2}{2}}$$

3. $\lim\limits_{x\to\infty}\left[(x+1)\arctan x-\frac{\pi}{2}x\right]$

$$= (x\arctan x + \arctan x - \frac{\pi}{2}x)$$

$$- \frac{x(\arctan x - \frac{\pi}{2})}{\frac{1}{x}} + \arctan x$$

$$= \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} + \frac{\pi}{2} = \frac{\pi}{2} - 1$$

4. $$\lim_{x \to +\infty} (\sqrt{x+2\sqrt{x}} - \sqrt{x - \sqrt{x}})$$

$$= \frac{3\sqrt{x}}{\sqrt{x+2\sqrt{x}} + \sqrt{x - \sqrt{x}}} = 3\frac{1}{\sqrt{1+\frac{2}{\sqrt{x}}} + \sqrt{1-\frac{1}{\sqrt{x}}}} = \frac{3}{2}.$$

$$\lim_{x \to -\infty} \sqrt{x^2-2x+4} + x = \lim_{x \to -\infty} \frac{-2x+4}{\sqrt{x^2-2x+4} - x} = \frac{-2+\frac{4}{x}}{-\sqrt{1-\frac{2}{x}+\frac{4}{x^2}} - 1} = 1$$

5、

$$\lim_{x \to 0} \left[\frac{1}{\ln(1+x)} - \frac{1}{x} \right] = \frac{x - \ln(1+x)}{x\ln(1+x)} = \frac{1}{2}$$