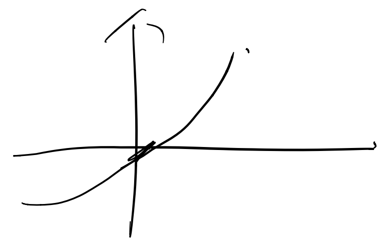
2021 数一

一、

1. $f(x) = \begin{cases} \dfrac{e^x - 1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

$\lim\limits_{x \to 0} f(x) =$



$\dfrac{f(x)}{x} = \dfrac{e^x - 1}{x^2}$

$\dfrac{f(x) - f(0)}{x - 0} = \dfrac{\dfrac{e^x - 1}{x} - 1}{x}$

$= \dfrac{e^x - 1 - x}{x}$

$\dfrac{e^x - (x+1)}{x} \quad x \to 0$

$= \dfrac{e^x - 1}{1} = 1$

D

2、

$f(x+1, e^x) = x(x+1)^2$

$f(x, x^2) = 2x^2 \ln x$

$f_1'(x+1, e^x) + e^x f_2'(x+1, e^x)$

$= (x+1)^2 + 2x(x+1)$

$f_1'(x, x^2) + 2x f_2'(x, x^2) = 4x\ln x + 2x$

$x=0$

$f_1'(1, 1) + f_2'(1, 1)$

$= 1$

$f_1'(1, 1) + 2f_2'(1, 1) = \cancel{2}$

$f_1'(1,1) = 0 \qquad f_2'(1,1) = 1$

$df(1,1) = f_1'(1,1)dx + f_2'(1,1)dy$

$= dy$.

3. 麦克劳林

$$f(x) = \sin x \cdot \frac{1}{1+x^2}$$

$$f(x) = \left(x - \frac{x^3}{3!} + o(x^3)\right)\left[1 - x^2 + o(x^3)\right]$$

$$= x - \frac{7}{6}x^3 + o(x^3)$$

A

4. B. 极限 → 积分.

5. C

6. $$k = \frac{[\alpha_2, \beta_1]}{[\beta_1, \beta_1]} = 1$$

$$\beta_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$l = \frac{[\alpha_3, \beta_1]}{[\beta_1, \beta_1]}$$

$$= \frac{5}{2}$$

$$l_2 = \frac{\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}}{4} = \frac{1}{2}$$

A

7. C

8. A: $P(A|B) = \frac{P(AB)}{P(B)} = P(A)$

$P(A|\bar{B}) = \frac{P(A\bar{B})}{P(\bar{B})} = P(A)$

B: $\frac{P(AB)}{P(B)} > P(A) \Rightarrow \frac{P(\bar{A}\bar{B})}{P(\bar{B})} > P(\bar{A})$

C: $P(A\bar{B}) = P(A(1-B))$

$= P(A - AB) = P(A) - P(AB)$

D ×

9. $\bar{X}-\bar{Y}$ 服从正态

$E(\bar{Z}) = E(\bar{X}-\bar{Y}) = E(\bar{X}) - E(\bar{Y}) = 0$

$D(\bar{Z}) = D(\bar{X}-\bar{Y})$

$= D(\bar{X}) + D(\bar{Y}) - \mathrm{cov}(\bar{X}, \bar{Y})$

$= \dfrac{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}{n}$. C

10. $\dfrac{\bar{X}-11.5}{\frac{1}{2}} \le \dfrac{11-11.5}{\frac{1}{2}} = 1-\varphi(1)$