第八章 矩量法举例

加权余量法

- 给定边值问题的场方程（微分或积分方程）及边界条件统一表述为如下的算子方程

$$Lu = g \qquad u, g \in V$$

$$u|_{S1} = u_s(\mathbf{r}_b)$$

$$\left.\frac{\partial u}{\partial n}\right|_{S2} = q_s(\mathbf{r}_b)$$

- 离散化为矩阵

$$\int_v W_j L\tilde{u} dV = \int_v W_j g dV$$

左边等于

$$\int_v W_j L\left(\sum_{i=1}^{n} N_i u_i\right) dV$$

$$\int_v W_j \sum_{i=1}^{n} L(N_i) u_i dV = \sum_{i=1}^{n} u_i \int_v W_j L(N_i) dV$$

- 定义内积表达式

$$\int_v W_j L(N_i) dV \equiv \langle W_j, L(N_i) \rangle$$

右边等于 $\int_v W_j g dV \equiv \langle W_j, g \rangle$

- 加权余量式可简写成

$$\sum_{i=1}^{n} u_i \langle W_j, L(N_i) \rangle = \langle W_j, g \rangle \qquad (j = 1, 2, \cdots, n)$$

- 上式为含n个未知数的n个方程，可以用矩阵的形式来表示 $\{M\}\{u\} = \{g\}$

$$\{M\} = \begin{bmatrix} <W_1,L(N_1)> & <W_1,L(N_2)> & \cdots & <W_1,L(N_n)> \\ <W_2,L(N_1)> & <W_2,L(N_2)> & \cdots & <W_2,L(N_n)> \\ \vdots & \vdots & \vdots & \vdots \\ <W_n,L(N_1)> & <W_n,L(N_2)> & \cdots & <W_n,L(N_n)> \end{bmatrix}$$

$$M_{ji} = <W_j, L(N_i)>$$

$$\{u\} = \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{Bmatrix}, \{g\} = \begin{Bmatrix} <W_1, g> \\ <W_2, g> \\ \vdots \\ <W_n, g> \end{Bmatrix}$$

矩量法编程

$$M_{ji} = <W_j, L(N_i)>$$

- 网格剖分
- 基函数选择
- 权函数选择
- 编写基函数选择与方程算子结合的计算
- 编写权函数选择与基函数内积计算函数
- 矩阵填充计算

- 编写离散网格数据结构
- 编写离散点数据结构

```
struct StruPoint {
double x;
double y;
};
```

- 编写网格数据结构

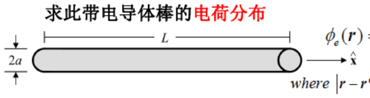
```
struct StruElemental {
int id;
double xs,xe,ys,ye;
StruPoint *pt;
}
```

- 编写基函数计算函数块
- 定义基函数类型
- enum BaseFunType{BASFUN_PULSE, BASFUN_TRI}
- 编写基函数计算的函数块
- double CmpBaseFun(BaseFunType type,double x)
- {
- …
- }

- 编写矩阵元素计算函数块
- 定义权函数类型
- enum WeightFunType{WGT_DELTA,WGT_GALERKIN}
- 矩阵元素计算类型
- double CmpMatrixElement(double (*L)(double
-),StruElemental *elemi,StruPoint *ptj,BaseFunType
- baseType,WeightFunType weightType)
- {
- double val;
- if (weightType==WGT_DELTA){
- val=L(elemi,ptj,baseType);
- }
- return val;
- }

带电导体棒的电场分布

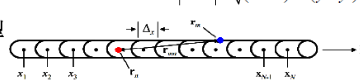
- 半径为a，长度为L的细长带电导体棒，给定电位 V_0 ，求此带电导体棒的电荷分布



$$\phi_e(\mathbf{r}) = \int_0^L \frac{q_e(x')}{4\pi\varepsilon|\mathbf{r}-\mathbf{r}'|}dx'$$

$$where\ |r-r'| = \sqrt{(x-x')^2+(y-y')^2}$$

- 导体棒的分段模型



- 由边界条件

$$\phi_e(\mathbf{r})|_{0<x<L,y=a} = \phi_0 = V_0$$

- 基函数展开

$$q_e(x') = \sum_{n=1}^{N} a_n \Pi_n(x') \qquad \Pi_n(x') = \begin{cases} 1 & x \in \Delta l_n \\ 0 & x \notin \Delta l_n \end{cases}$$

带电导体棒的电场分布

- 把基函数代入边界积分方程

$$V_0 = \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N} a_n \int_{(n-1)dx}^{ndx} \frac{1}{|\mathbf{r}-\mathbf{r}'|}dx' \qquad |r-r'| = \sqrt{(x-x')^2+(y-y')^2} = \sqrt{(x-x')^2+a^2}$$

- 点匹配法计算（权函数作用）

$$\langle \delta_m, V_0 \rangle = \frac{1}{4\pi\varepsilon}\sum_{n=1}^{N} a_n \int_0^L \delta(\mathbf{r}-\mathbf{r}_m)\int_{(n-1)dx}^{ndx} \frac{1}{|\mathbf{r}-\mathbf{r}'|}dx'd\mathbf{r}$$

$$4\pi\varepsilon V_e = \sum_{n=1}^{N} a_n \int_{(n-1)dx}^{ndx} \frac{1}{|\mathbf{r}_m-\mathbf{r}'|}dx', m=1,2,\ldots,N$$

- 计算电位系数（MOM矩阵元素）

$$z_{mn} = \int_{(n-1)dx}^{ndx} \frac{1}{|\mathbf{r}_m-\mathbf{r}'|}dx' = \int_{(n-1)dx}^{ndx} \frac{1}{\sqrt{(x_m-x')^2+a^2}}dx'$$

- 矩阵方程为

$$\begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1N} \\ z_{21} & z_{22} & \cdots & z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ z_{N1} & z_{N2} & \cdots & z_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{Bmatrix} 4\pi\varepsilon V_0 \\ 4\pi\varepsilon V_0 \\ \vdots \\ 4\pi\varepsilon V_0 \end{Bmatrix}$$

$$z_{mn} = \int_{(n-1)dx}^{ndx} \frac{1}{\sqrt{(x_m-x')+a^2}}dx'$$

$$= \log\left[\frac{(x_b-x_m)+\sqrt{(x_b-x_m)-a^2}}{(x_a-x_m)+\sqrt{(x_a-x_m)-a^2}}\right]$$

$$where\ x_b = ndx, x_a = (n-1)dx,$$

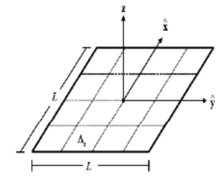
- 带电导体棒编程
- 定义导体棒的数据结构
- typedef struct StructWire{
- double lenx;
- double a;
- int nx;
- StruElemental *pElement;
- double (*L)(StruElemental *, StruPoint *);
- }StruMoM,*pStruMoM;

- 导体棒的算子函数

```
double CmpWireAlgo(StruElemental *elemi,StruPoint *ptj)
{
double val,vb2,va2,xa,xb,a2,xm;
xm=ptj->x;
xa=elemi->xs;
xb=elemi->xe;
a2=ptj->y-elemi->pt->y;
a2*=a2;
vb2=(xb-xm)*(xb-xm);
va2=(xa-xm)*(xa-xm);
val=log((vb2+sqrt(vb2-a2))/(va2+sqrt(va2-a2)));
return val;
}
```

导电平板的静电场

- 设正方形导电板，边长为L，位于Z=0平面上，中心点如图示，若导电平板电位V0，试求导电板上的电荷分布及电容



$$4\pi\varepsilon_0\phi(\boldsymbol{r})=\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\frac{q_e(x',y')}{\sqrt{(x-x')^2+(y-y')^2}}dx'dy'$$

- **1、首先分板为N个均匀小块△S，并选基函数为分域脉冲函数。**

$$q_e(\boldsymbol{r}')=\sum_{n=1}^{N}a_n\Pi_n(\boldsymbol{r}'),\Pi_n(\boldsymbol{r}')=\begin{cases}1 & \boldsymbol{r}\in\Delta S_n\\0 & \boldsymbol{r}\notin\Delta S_n\end{cases}$$

- **代入边界的积分方程** $4\pi\varepsilon_0\phi(\boldsymbol{r})|_S=4\pi\varepsilon_0V_0$

$$4\pi\varepsilon_0V_0=\sum_{n=1}^{N}a_n\iint_{S_n}\frac{1}{\sqrt{(x-x')^2+(y-y')^2}}dx'dy'$$

- **2、选权函数**

$$w_m=\delta(x-x_m)(y-y_m)$$

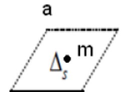
- **3、求内积**

$$z_{mn}=\langle w_m,L(q_{en})\rangle=\iint_S\delta(x-x_m)(y-y_m)L(q_{en})dxdy$$

$$=\iint_S\delta(x-x_m)(y-y_m)\left[\iint_{S_n}\frac{1}{\sqrt{(x-x')^2+(y-y')^2}}dx'dy'\right]dxdy$$

$$=\iint_{S_n}\frac{1}{\sqrt{(x_m-x')^2+(y_m-y')^2}}dx'dy'$$

- 4、矩阵主角元素计算
- 当m=n时 Integrant is a Singularity



$$z_{nn}=\iint_{S_n}\frac{1}{\sqrt{(x_m-x')^2+(y_m-y')^2}}dx'dy'$$

$$=\frac{2a}{\pi\varepsilon}\log(1+\sqrt{2})$$

$$m\neq n,z_{mn}=\iint_{S_n}\frac{1}{\sqrt{(x_m-x')^2+(y_m-y')^2}}dx'dy'$$

$$\approx\frac{\Delta S_n}{\sqrt{(x_m-x_n)^2+(y_m-y_n)^2}}$$

- **5、矩阵方程**

$$[z_{mn}][a_n]=[g_n]$$

$$[a_n]=[z_{mn}]^{-1}[g_n]$$

$$q_e=\sum_{n=1}^{N}a_n\Pi_n$$

- 导电平板编程
- 定义导体平板的数据结构

```
struct StructPlane: {
double lenx,leny;
int nx,ny;
StruElemental *pElement;
double (*L)(StruPoint *);
};
```

- 导体板的算子函数

```
double CmpPlaneAlgo(StruElemental  *elemi,StruPoint *ptj)
{ double ds,xn,yn,xm,ym,val,a;
if (ptj==elemi->pt)  {
a=elemi->xe-elemi->xs;
val=2*a*log(1.+sqrt(2.))/(PI*EPS0);
}else{
ds=(elemi->xe-elemi->xs)*(elemi->ye-elemi->ys);
xn=ptj->x;  yn=ptj->y;
xm=elemi->pt->x;  ym=elemi->pt->y;
val=ds/sqrt((xn-xm)*(xn-xm)+(yn-ym)*(yn-ym));
}
return val;}
```