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A simple model of the vertical distribution of electron concentration in the ionosphere

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Abstract—An idealized model of the vertical distribution of the electron concentration in the *E*- and *F*-regions of the ionosphere is developed in which the model parameters are given by simple empirical expressions in terms of the characteristics conventionally measured at ionospheric observatories. Differences in the heights with a given electron concentration, as indicated by the model and by true-height profile analyses, are usually 20 km or less.

1. INTRODUCTION

VERTICAL-incidence ionospheric sounders (ionosondes) are employed as probes of the *E*- and *F*-regions of the ionosphere, but unfortunately they do not directly give the distributions of electron concentration as a function of the true height, but in terms of the virtual height of reflection of the probing waves. True-height data are needed for morphological studies and for the investigation of radio-propagation phenomena. A number of procedures have been developed to convert virtual- to true-height profiles (e.g. SCHMERLING, 1967; TITHERIDGE, 1967; PAUL, 1967; BECKER, 1967), but these are complicated, liable to errors because of missing portions of the ionosonde records (ionograms) and time-consuming, even with the aid of modern computers. Thus, such methods are uneconomical for the production of statistically representative quantities of data. It would clearly be useful if an idealized model could be developed approximating to the real ionosphere and with its parameters readily determined from the internationally agreed ionospheric characteristics which are regularly measured each hour of the day at many stations throughout the world (PIGGOTT and RAWER, 1972). The most important true-height parameters, e.g. the height of maximum electron concentration and the thickness of the *F2*-layer would then be available. In this paper we introduce such a model. This model has been selected to be as flexible as possible by matching the numbers of independent variable parameters and measured ionospheric characteristics. It is emphasized that it is not the only model that may be constructed with these constraints, nor necessarily the optimum model, but one which should prove accurate and useful for a number of applications.

The principle of using a model representation of the ionosphere is not new. In the past, models have been developed which assume a number of discrete ionospheric layers, each with a parabolic or quasi-parabolic law of variation of electron

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concentration with height. One such model employed by the International Radio Consultative Committee (CCIR) for field-strength prediction purposes (CCIR, 1970) is an adaptation of an earlier model of Lucas and Haydon (1966). In this model the E-layer has a parabolic form with fixed height of maximum ionization and fixed semi-thickness for all times and at all places. The $F2$-layer is also a parabolic layer with a height of maximum ionization deduced from $M(3000)F2$ using the formula of Shimazaki (1955). The semi-thickness of the $F2$-layer is derived from a base height given in terms of the ionospheric characteristic $h'F$. ($h'F$ is the minimum observed virtual height of reflection from the F-layer as a whole. It corresponds to reflection from the $F1$-region when there is an $F1$ discontinuity on the ionogram and to reflection from the $F2$-layer at all other times.) Thus the layer semi-thickness thereby determined can exceed the true $F2$-layer semi-thickness in the daytime when echoes are received from the $F1$-region. In the model there is always an ionization void between the top of the E-layer and the base of the F-layer. An improved model incorporating a parabolic $F1$-layer of fixed height of maximum ionization and semi-thickness in place of this void has recently been developed in the U.S.A. (Haydon, 1972). A model based on E-, $F1$- and $F2$-layers of α-Chapman form has been derived by Decker (1972), and examples given to show generally good agreement with measured profiles. There are however limitations in the implementation of this latter model because large numbers of polynomial coefficients must be used to generate the empirical relationships between the model parameters and the ionospheric characteristics, so that for effective use a computer must be employed.

All the above models give marked ionization valleys between the different layers, whereas rocket and incoherent scatter data reveal that there is no valley between the $F1$- and $F2$-layers and that the valley between the E- and $F1$-layers, if present, is usually relatively shallow. The new model presented here is claimed to give an improved match to measured ionization profiles in the $F1$-region; also by allowing for the effects of underlying ionization on the measured ionospheric characteristics, it provides a more accurate representation of the $F2$-layer height of maximum ionization and semi-thickness. There are additional advantages in that its parameters are derived by simple empirical equations, and analytic expressions are available for wave behaviour in the model.

Details of the model are given in Section 2 and the determination of the $F2$-layer height of maximum ionization and semi-thickness is discussed in Section 3. Comparisons are made in Section 4 between the model and true-height profile results obtained at a number of different locations over a wide range of conditions.

2. The New Model

Figure 1 gives the adopted model. It consists of:

(i) a parabolic E-layer below its height of maximum electron concentration hmE, with semi-thickness ymE. hmE is taken as constant at 110 km and ymE as 20 km;

(ii) a parabolic $F2$-layer with height of maximum electron concentration $hmF2$ and semi-thickness $ymF2$. $hmF2$ and $ymF2$ are determined from the empirical expressions given in Section 3; and

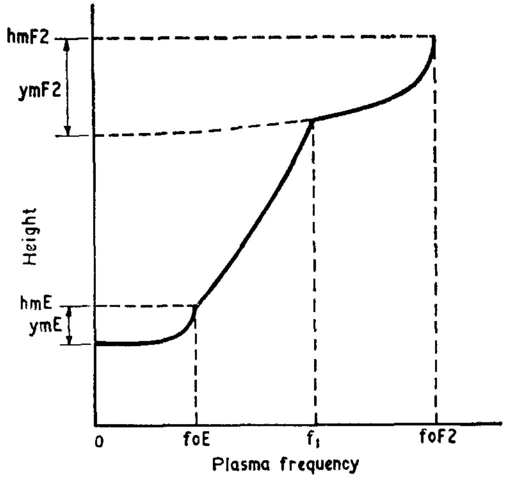


Fig. 1. The model. This consists of (i) a parabolic E-layer, (ii) a linear increase of electron concentration with height in the $F1$-region, and (iii) a parabolic $F2$-layer. The ordinate scale is taken as being linear in height and the abscissa as linear in frequency.

(iii) a linear increase of electron concentration with height between hmE and the point on the parabolic $F2$-layer where the plasma frequency, f_1, is $1{\cdot}7\,foE$. This value was derived empirically to give results consistent with measured F-layer heights. Thus the model is specified in terms of the four variable parameters $foF2$, foE, $hmF2$ and $ymF2$.

In particular no $F1$-layer characteristics are used and the ionization in the $F1$ height region is taken as being given entirely in terms of the E- and $F2$-layer ionization parameters. Measured true-height profiles rarely show marked $F1$-layer discontinuities and many of the inflexions and cusps seen on ionograms which are scaled as $F1$-layer characteristics result from only minor fluctuations in electron concentration (PAUL, 1967). The new model has other advantages over models based on $foF1$ in that there is no abrupt change when that parameter ceases to be scaled.

3. THE $F2$-LAYER PARAMETERS

3.1 *General*

Three $F2$-layer ionospheric characteristics are normally available from ionograms; these are $foF2$, $M(3000)F2$ and $h'F,F2$. ($h'F,F2$ is the minimum observed virtual height of reflection from the $F2$-layer. It is equal to $h'F$ when there is no $F1$-layer discontinuity on the ionogram and to $h'F2$ at all other times.) In this section use is made of these characteristics together with foE to determine the parameters $hmF2$ and $ymF2$.

3.2 $hmF2$

For a parabolic $F2$-layer with no underlying ionization and in the absence of the Earth's magnetic field, the virtual height of reflection at a frequency of $0{\cdot}834\,foF2$, designated $hpF2$, is numerically equal to $hmF2$. SHIMAZAKI (1955) investigated the relationship between $hpF2$ and $M(3000)F2$, both for model layers and from observational data and concluded that it is relatively insensitive to the $F2$-layer semi-thickness. (This finding is consistent with the results of earlier theoretical investigations of the relationship between height of maximum ionization and maximum-usable-frequency factor for a single parabolic model ionosphere given by APPLETON and BEYNON (1947).) Shimazaki proposed the expression:

$$hpF2 = \frac{1490}{M(3000)F2} - 176 \text{ km} \tag{1}$$

When however underlying ionization is present $M(3000)F2$ is reduced and $hpF2$ increased. WRIGHT and MCDUFFIE (1960) concluded from an examination of many ionograms that $hpF2$ deduced from equation (1) is approximately equal to $hmF2$ at night; in the daytime at temperate latitudes $hmF2$ differs from $hpF2$ but may be derived from an expression of similar form to equation (1), with different numerical constants; further, that by day at high latitudes, neither the original nor the revised equation is adequate.

Clearly it is preferable to allow for the actual amount of underlying ionization present. In the ionospheric model developed by LUCAS and HAYDON (1966), $hpF2$ is assumed given by equation (1) and $hmF2$ is evaluated as

$$hmF2 = hpF2 - \Delta h \tag{2}$$

where Δh is computed as the retardation in the full parabolic model E-layer on a frequency of $0{\cdot}834 foF2$. This correction term is very small—typically 5 km—whereas in practice the total retardation due to ionization below the $F2$-layer can amount to 50 km or more. This is because the retardation in the region between the E- and $F2$-layers greatly exceeds that in the E-layer alone. In the CCIR model ymE is empirically increased to 30 km in the expression for Δh to try to compensate for this additional retardation, but the correction is still grossly inadequate. Thus both models give $F2$-layer heights which are too large.

A different procedure for determining $hmF2$ has been adopted in the present analysis based on a more realistic correction for underlying ionization. Appropriate equations for our model (see Appendix 1) have been used to synthesize ionograms with a range of different $hmF2$, $foF2/foE$ and $ymF2/hoF2$, where $hoF2 = hmF2 - ymF2$. $M(3000)F2$ has then been scaled from these ionograms by the conventional transmission-curve method with an overlay based on the standard international factors for ionospheric observatories (PIGGOTT and RAWER, 1972). $M(3000)F2$ is found to be relatively insensitive to $ymF2/hoF2$ over a considerable range of $F2$-layer semi-thicknesses. This is consistent with the results of Shimazaki. Taking the values for $ymF2/hoF2 = 0{\cdot}4$ as representative, Fig. 2 shows the variation of $M(3000)F2$ with $hmF2$ for different x, where $x = foF2/foE$. Results are quoted for the range of x where the $M(3000)F2$ overlay can be fitted to the synthesized ionograms, i.e. for x above about 1·7–2·0, depending on the $F2$-layer height. (In the limit for $x = 1{\cdot}7$, $f_1 = foF2$ and we have an entirely linear $F1/F2$-layer.) Also included in the figure for comparison are the relationships given by Shimazaki (equation (1)) and by WRIGHT and MCDUFFIE (1960). The Shimazaki expression is in close agreement with the model results when x is large (i.e. night-time), particularly over the middle

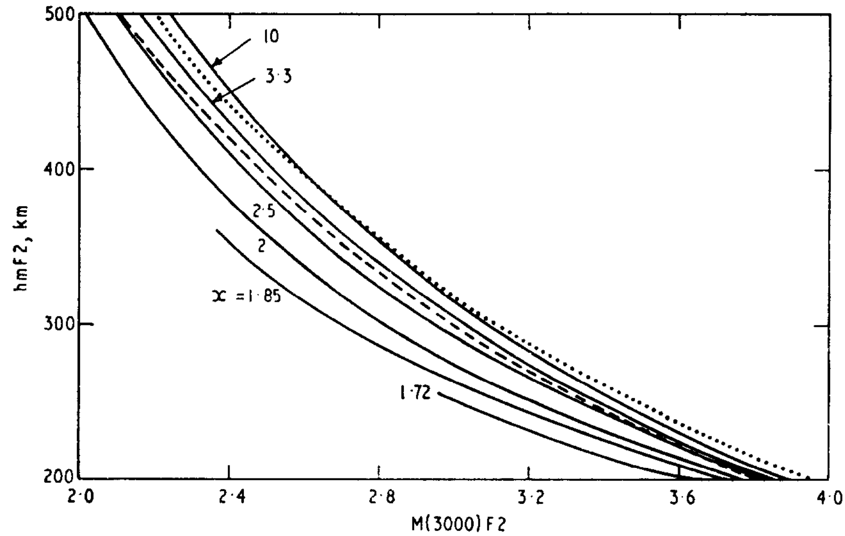


Fig. 2. $M(3000)F2$ for model ionospheres as function of $hmF2$ for range of $x = \dfrac{foF2}{foE}$. The curves relate to a fixed value of $\dfrac{ymF2}{hoF2} = 0{\cdot}4$. Also included are the empirical relationships given by SHIMAZAKI (1955) (dotted line), and by WRIGHT and McDUFFIE (1960) (dashed line).

height region considered; also, the Wright and McDuffie expression is consistent with the model results for x of the order of 3—a typical low- and middle-latitude daytime value.

For $hmF2$ between 200 and 500 km, which covers nearly all heights of $F2$-layer maximum ionization found in practice, and for the range of x which apply, the model results are consistent with the empirical expression

$$hmF2 = a[M(3000)F2]^b \text{ km} \tag{3}$$

with

$$a = 1890 - \frac{355}{(x - 1{\cdot}4)}$$

and

$$b = (2{\cdot}5x - 3)^{-2{\cdot}35} - 1{\cdot}6.$$

Equation (3) has been used to derive $hmF2$ for the model profiles and other analyses described in the later sections of this paper. However, the similarity in form of the curves of Fig. 2, given by the model and by Shimazaki's equation, suggests a simpler but more approximate empirical representation which may have application to manual calculations. This is:

$$hmF2 = \frac{1490}{M(3000)F2 + \Delta M} - 176 \tag{4}$$

with

$$\Delta M = \frac{0{\cdot}18}{x - 1{\cdot}4}.$$

In particular $\Delta M \to 0$ as x becomes large, giving Shimazaki's expression for the case of no underlying ionization. It is noted that the ΔM correction term for underlying ionization is more accurate than the fixed Δh allowance of equation (2) at constant x, since it leads to larger changes in $hmF2$ when the $F2$-layer is high and $M(3000)F2$ is small. This is in the correct sense, for there is then a thicker slab of underlying ionization giving more retardation.

Figure 3(a) shows the height errors, E, introduced by the use of equation (4) in place of the more exact expression of equation (3). It confirms that the approximate expression may be used for $x > 2$ and $hmF2 \leqslant 400$ km with results correct to within 10 km. Since $M(3000)F2$ is usually scaled to the nearest 0·05 these errors may be contrasted with the height uncertainties of 5 km for $hmF2 = 200$ and 15 km for $hmF2 = 500$ km associated with the uncertainties in $M(3000)F2$. Figure 3(b), deduced from the model results, shows the errors in $hmF2$ arising from the direct use of the uncorrected Shimazaki equation. At middle- and low-latitudes in the daytime with x in the range 2–3 and $hmF2$ around 300 km, $hmF2$ is typically overestimated by some 15–40 km; for the lower x that arise at higher latitudes in the summer, errors can exceed 60 km.

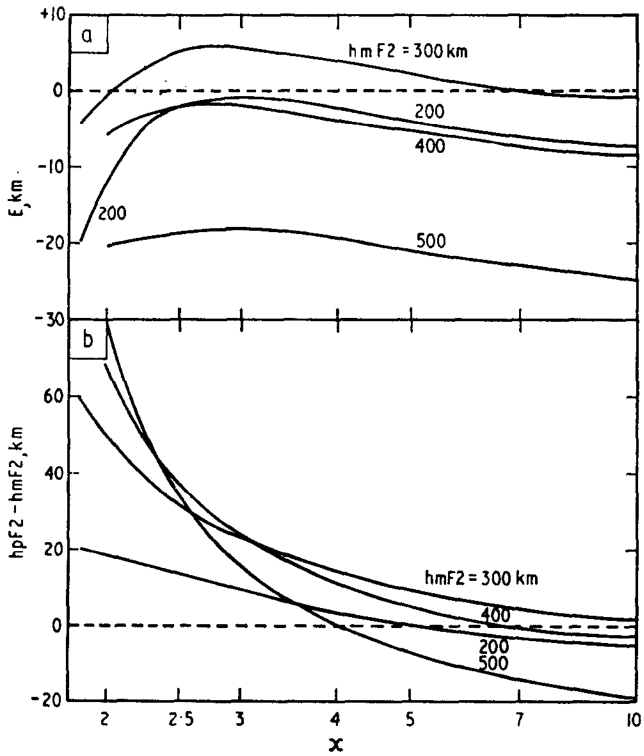


Fig. 3. (a) Error, E, in estimates of $hmF2$ given by the simplified expression of equation (4)—see text. Positive E corresponds to layer heights which are too large. (b) Error in estimates of $hmF2$ given by Shimazaki's relationship for $hpF2$ (see text, equation (1)).

3.3 $ymF2$

If there were no ionization below the $F2$-layer $h'F,F2$ would be approximately equal to $hoF2$. The model results indicate however that $h'F,F2$ increases rapidly as x is reduced, so that we must write:

$$ymF2 = hmF2 - (h'F,F2 - \Delta h') \tag{5}$$

with $\Delta h'$ a correction term.

Figure 4, produced from the model calculations, shows that $\Delta h'$ is normally very large; in practice 100–200 km, whereas no correction of such magnitude is applied in the CCIR model. Figure 4(a) demonstrates that $\Delta h'$ is relatively insensitive to $ymF2/hoF2$ over the range 0·2–0·6. Taking the values for $ymF2/hoF2 = 0{\cdot}4$ as representative, Fig. 4(b) reveals that $\Delta h'$ increases linearly with $hmF2$ at a fixed x. This can be predicted from equations (12) and (13)—see Appendix—assuming that $h'F,F2$ is measured at approximately the same wave frequency irrespective of the value of $hmF2$, and that this frequency is so low that the retardation within the $F2$-layer itself (the last term of equation (12)) may be neglected. $\Delta h'$

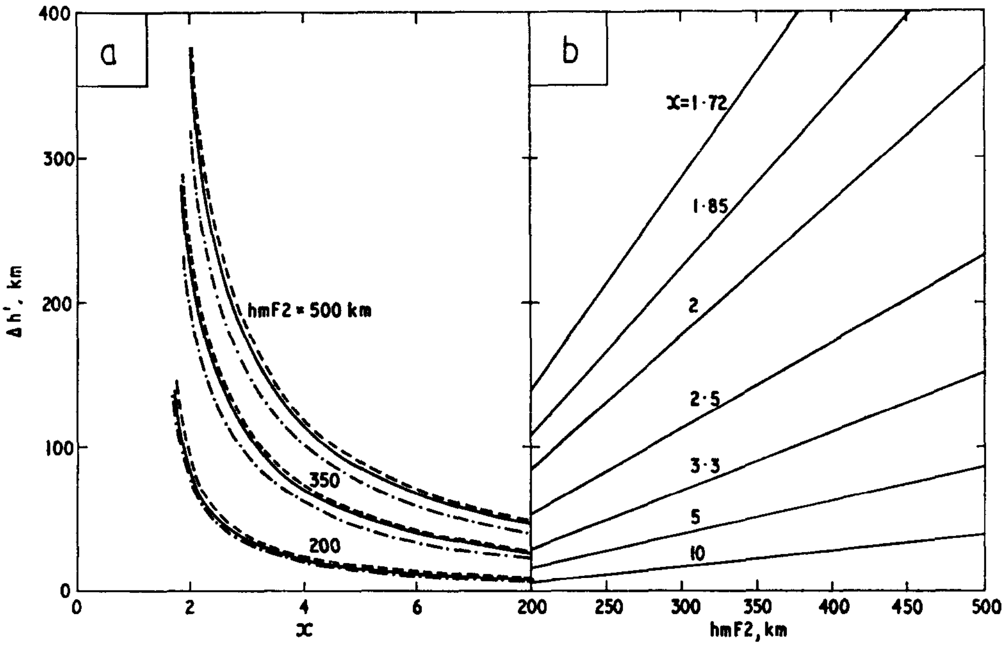


Fig. 4. (a) $\Delta h'$ for model ionospheres as function of x for range of $hmF2$ $\dfrac{ymF2}{hoF2} = 0{\cdot}2$ —.—.; 0·4 ———; 0·6 – – – –,
(b) $\Delta h'$ for model ionospheres as function of $hmF2$ for range of x. The curves relate to a fixed value of $\dfrac{ymF2}{hoF2} = 0{\cdot}4$.

is given empirically as:

$$\Delta h' = \left[\frac{0{\cdot}613}{x - 1{\cdot}33}\right]^{0{\cdot}86} (hmF2 - 104). \tag{6}$$

4. Comparisons with Experimental Data

Equations (3), (5) and (6) enable the new model to be specified for any ionosphere where *foF2, foE*, *M*(3000)*F2* and *h'F,F2* are given. In this section the models thus derived are compared with true height and *N*(*h*) profile results obtained from measurements recorded at a number of different locations and covering a wide range of conditions.

4.1 *Accuracy of model representation of hmF2 and ymF2*

As an illustration of the accuracy of the new model, Fig. 5 presents sample comparisons between *hmF2* and *ymF2* given by the model and the same parameters, here denoted as *haF2* and *yaF2*, respectively, given by the *hc*, *qc* true-height analysis procedure (Piggott and Rawer, 1972). The data relate to 75 selected high-quality ionograms recorded during 1967–1969 at Argentine Islands, where the regular variations of *hmF2* are very large. Approximately half of these ionograms are for the summer months of December and January, a quarter were recorded

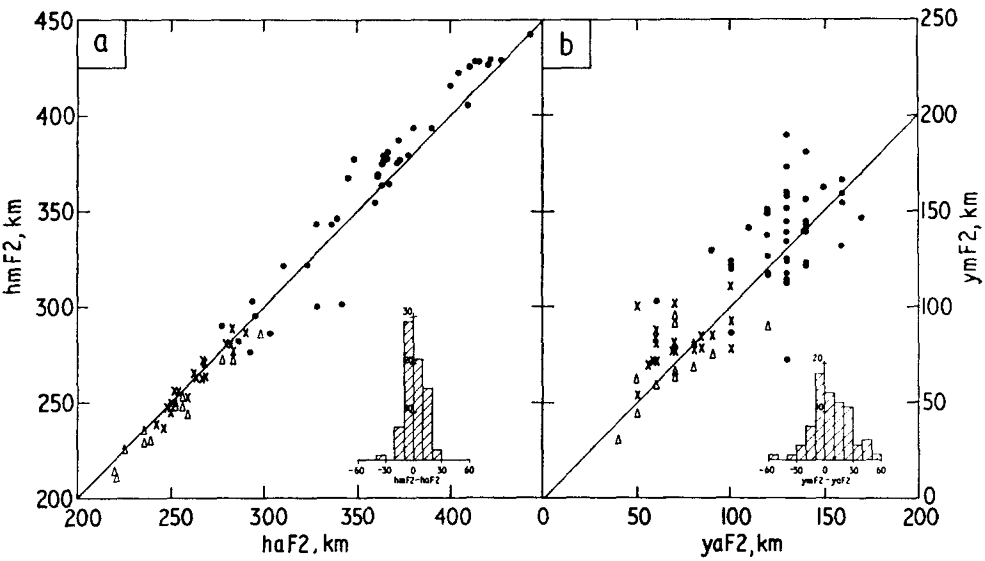


Fig. 5. Comparison of estimates of *F2*-layer height of maximum ionization and semi-thickness given from model relationship and from true-height profile analysis. (a) *hmF2* = height of maximum ionization given by equation (3). *haF2* = height of maximum ionization given by standard *hc*, *qc* analysis. —— line satisfying relationship *hmF2* = *haF2*. Inset is a histogram of the differences *hmF2* − *haF2*. (b) *ymF2* = semi-thickness given from equations (3), (5) and (6). *yaF2* = semi-thickness given by standard *hc*, *qc* analysis. —— line satisfying relationship *ymF2* = *yaF2*. Inset is a histogram of the differences *ymF2* − *yaF2*. The data relate to a sample of 75 ionograms recorded at Argentine Islands during 1967–1969 (see Appendix 2): • summer; x equinox; △ winter.

in September and the remainder relate to observations made in June and July. Table 1 presents the range of x, $ymF2/hoF2$ and $M(3000)F2$ represented by the data.

Table 1

Season	x	$\frac{ymF2}{hoF2}$	$M(3000)F2$
Summer	1·72–6·50	0·25–0·95	2·30–2·85
Equinox	2·50–5·60	0·24–0·69	3·10–3·45
Winter	2·40–5·20	0·22–0·67	3·15–3·80

The mean $hmF2$–$haF2$ is 2 km and the standard deviation of the sample about this mean is 9·5 km. The corresponding mean and standard deviation of $ymF2$–$yaF2$ are 8 and 19 km, respectively. These standard deviations are consistent with the measuring uncertainties in $hc(\pm 10$ km), $qc(\pm 5$ km) and $hmF2$ ($\pm$ 5 to $\pm$ 15 km, depending on $M(3000)F2$ and assuming a scaling accuracy of $\pm$ 0·05). The agreement between the model results and the true-height data over the range of heights involved (200–500 km) is good. There is no marked evidence of a systematic dependence of error on $hmF2$, but the slight tendency for $hmF2$ to exceed $haF2$ at the greater heights is noted. There are no obvious seasonal trends in the data despite the fact that the summer ionograms often showed distinct $F1$ cusp traces, whereas these features were absent during the winter and equinoxes. Thus the $F1$ ledge of ionization must be so small as to have negligible influence on the measured $F2$-layer characteristics and the absence of a specific $F1$-region parameter in the new model is justified.

Comparisons of model $hmF2$ and true height $haF2$ values for other locations and solar epochs are given in Fig. 6. Some data relate to true-height analyses undertaken by the present authors and some are derived from published values—Appendix 2 gives details of the data sources and their likely accuracy. Histograms of $hmF2$–$haF2$ are presented for samples representative of all seasons at two middle-latitude and one equatorial station. Table 2 shows the mean differences, together with the range in the data of $M(3000)F2$ and x. Agreement is good for Slough and Ibadan at sunspot minimum. The differences for Lindau and Slough at sunspot maximum, although somewhat larger, are still within acceptable limits for many applications. They arise in part from errors and uncertainties in the scaling of the ionospheric characteristics and in the true-height analysis techniques; also in part because at sunspot maximum there are more occasions when $F2$-layer heights are relatively large, and in such cases the use of the same fixed f_1 may not be strictly appropriate.

The dependence on layer height has been investigated in further comparisons for the equatorial stations of Ibadan and Singapore at sunspot maximum where $haF2$ sometimes exceeds 500 km. For Ibadan it is indeed found that $hmF2$–$haF2$ depends on $M(3000)F2$ (see Fig. 7). This figure indicates that below $M(3000)F2 = 2{\cdot}4$ the height difference rapidly becomes large. The Singapore data show a similar trend, but the range of $M(3000)F2$ giving significant systematic positive height differences is increased up to about $M(3000)F2 = 2{\cdot}7$. Close inspection of

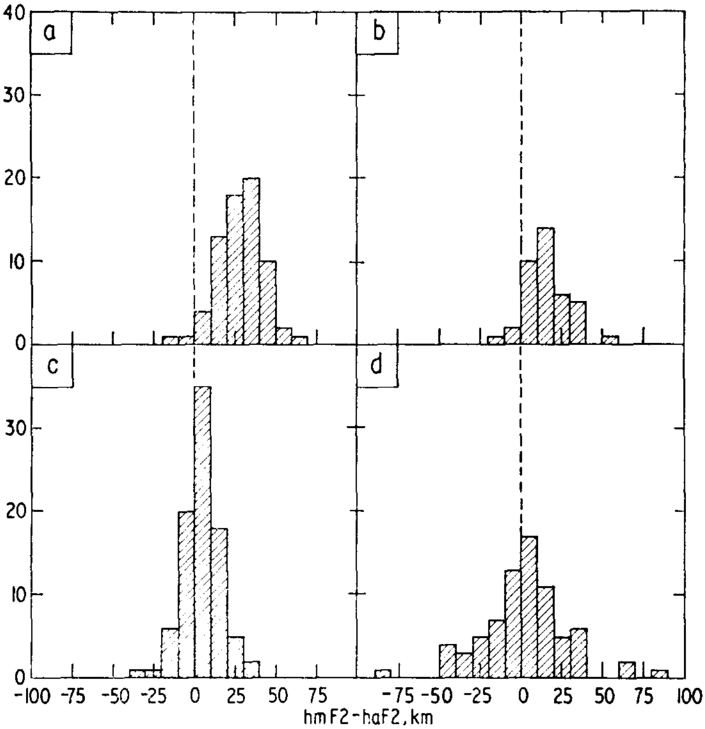


Fig. 6. Histogram of differences in estimates of $F2$-layer height of maximum ionization given from model relationship and from true-height profile analysis for range of stations and solar epochs.
$hmF2$ = height of maximum ionization given by equation (3).
$haF2$ = height of maximum ionization given by profile analysis (see Appendix 2).
(a) Slough—sample of 70 ionograms recorded during IGY;
(b) Lindau—sample of 39 ionograms recorded during IGY;
(c) Slough—sample of 88 ionograms recorded during IQSY;
(d) Ibadan—sample of 71 ionograms recorded during IQSY.
Each sample comprises approximately equal numbers of ionograms recorded in the different seasons.

Table 2

Station	Epoch	Mean $hmF2$–$haF2$ (km)	$M(3000)F2$	x
Slough	IGY	28	2·30–3·25	1·77–5·13
Lindau	IGY	16	2·15–3·00	1·72–4·96
Slough	IQSY	4	2·80–3·75	1·73–4·06
Ibadan	IQSY	2	2·25–3·45	1·72–4·84

the IQSY data for Ibadan reveals that these are completely consistent with the results for sunspot maximum but at sunspot minimum the number of occasions with $M(3000)F2 < 2{\cdot}4$ is small. The tendency for larger differences at the smaller $M(3000)F2$ may also be detected in the data for the other locations considered, but the effect is less pronounced.

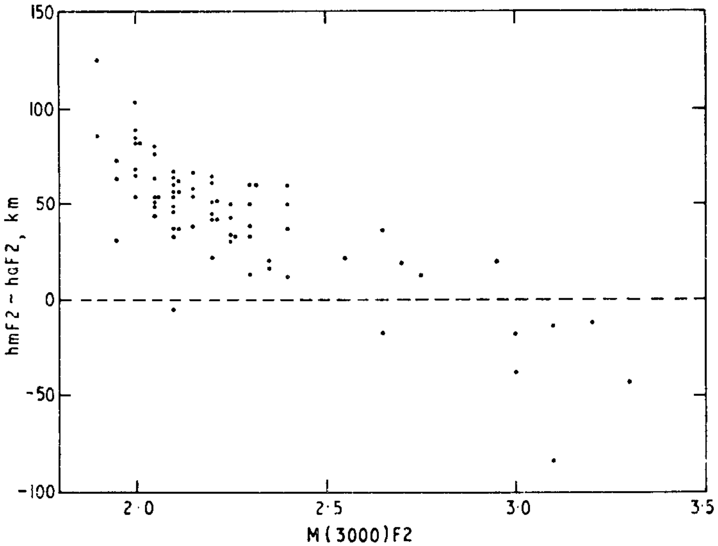
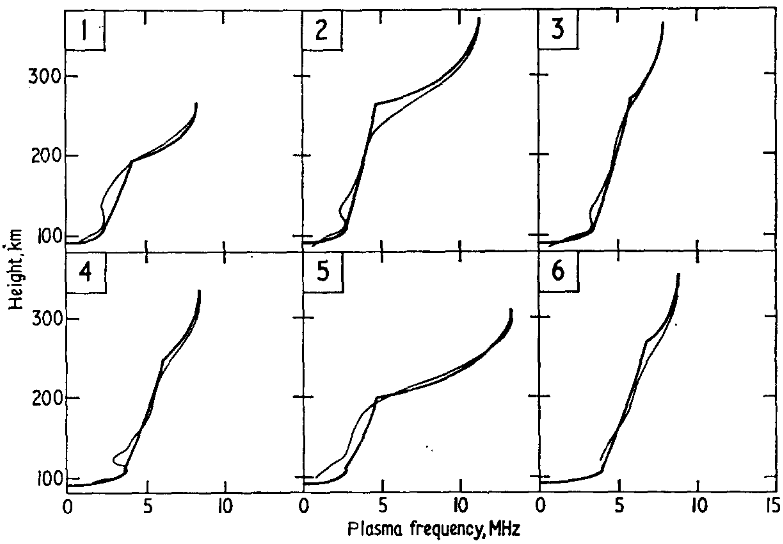


Fig. 7. Dependence on $M(3000)F2$ of difference in estimates of $F2$-layer height of maximum ionization given from model relationship and from true-height profile analysis.
$hmF2$ = height of maximum ionization given by equation (3).
$haF2$ = height of maximum ionization given by profile analysis (see Appendix 2).
The data relate to a sample of 75 ionograms recorded at Ibadan during 1957–1958.

Although the derived estimates of $hmF2$ are subject to some error, these errors are much less than those given by other existing prediction procedures. They may be regarded as unimportant, except when $M(3000)F2$ is small at equatorial stations. True-height profile analyses reveal that at low latitudes under such conditions $hmF2$ is generally of the order of 500 km or above and the electron concentration increases approximately linearly with height from a plasma frequency just above foE up to a frequency very close to $foF2$. This means that the $M(3000)F2$ overlays are fitted to the corresponding ionograms at plasma frequencies well below those where the $F2$-layer ionization has a parabolic form; hence the reduced agreement with the model results. Under these circumstances the significance of the scaled $M(3000)F2$ values is liable to misinterpretation since this characteristic is then determined by the gradient of the linear part of the profile; indeed there is some limiting gradient beyond which an overlay fit cannot be achieved. To derive a more representative model under such conditions, further comparative tests have been undertaken in which the parameter f_1 of the new model has been allowed to vary between $1{\cdot}7\,foE$ and the point of tangency to the parabolic section. It seems likely that a refinement could be introduced in which f_1 was an empirical function of $M(3000)F2$, so as to give differences between $hmF2$ and $haF2$ reduced to the same order as those for other latitudes and for higher $M(3000)F2$. It is questionable, however, whether the additional complexity of such a model would be justified and this aspect of the work is not considered further here. Errors arising from the use of the basic new model are regarded as sufficiently small for most applications.

4.2 *Accuracy of model profiles*

It is not possible to show in this paper large numbers of sample comparisons between model and true-height electron-concentration profiles but 12 examples representing a wide range of profile shapes are given in Fig. 8. These have been selected solely on account of corresponding to high-quality ionograms so that the ionospheric characteristics and the true-height profiles should be relatively accurate. The examples cover a range of local times, seasons and solar epochs, but as in all the other comparisons with observational results presented in the paper,



(a)

Fig. 8. Comparison of model and true-height analysis electron-concentration profiles for range of sample occasions.

(a) High- and middle-latitude stations:

(1) Argentine Islands 1645 LT 29 September 1968;
(2) Argentine Islands 0530 LT 17 December 1968;
(3) Argentine Islands 1330 LT 31 December 1967;
(4) Lindau 1550 LT 21 July 1958;
(5) Lindau 1430 LT 10 December 1958;
(6) Slough 1200 LT 27 July 1957.

(b) Lower latitude and equatorial stations:

(7) Wallops Island 1520 LT 7 March 1970;
(8) Cape Zevgari 1000 LT 2 April 1972;
(9) Cape Zevgari 0800 LT 12 April 1972;
(10) Ibadan 0800 LT 6 June 1958;
(11) Ibadan 1000 LT 9 July 1958;
(12) Ibadan 0800 LT 9 July 1958.

For locations of recording stations and details of true-height analysis techniques employed see Appendix 2:

——— new model; ——— from true-height analysis.

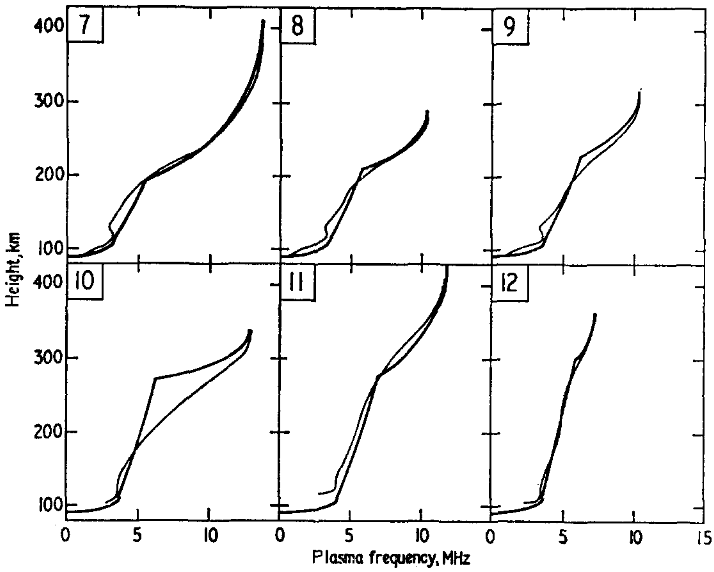


Fig. 8 (b)

are restricted to the daylight hours when foE can be measured directly; although the new model should also apply at night the accuracy of true-height profiles is then severely limited by uncertainties in foE.

Agreement between the profile and model results is generally good, the height differences corresponding to given plasma frequencies between the two sets of results usually not exceeding 20 km. The following features are of note:

(i) For many of the examples hmE chosen for the model is somewhat below that deduced by profile analysis. As explained in Appendix 2, the profile results at this height are liable to be 5–10 km too large;

(ii) Height differences at plasma frequencies corresponding to the lower portions of the $F2$-layer where the model profile is assumed to have a parabolic form are often less than the differences at the height of maximum ionization. For the model, errors in $hmF2$ lead to compensating errors in $ymF2$;

(iii) Profiles 1–4 and 7–9 incorporate an E-layer valley derived from an iterative match to the ordinary- and extraordinary-wave traces of the ionograms (see Appendix 2). The valleys indicated are relatively shallow in frequency and have little effect on the overall retardation of waves reflected from the $F2$-layer and thus on the parameters of the $F2$-layer model;

(iv) At equatorial stations daytime values of $h'F,F2$ derived in accordance with the stated definition are not consistent with data for other latitudes, because, as already noted, the profiles display a linear increase of electron concentration with height from hmE nearly to $hmF2$. Thus strictly $ymF2$ cannot be determined to enable the corresponding model profiles to be generated. However, numerical prediction maps of $h'F,F2$ are available (LEFTIN *et al.*, 1967), based for the equatorial region on the limited observational data when an $F1$-layer discontinuity could be detected. Use has been made of these numerical maps in the case of profiles 10 and 11 for Ibadan at sunspot maximum. In the example of profile 11 agreement between the model and the true-height results is very good. Profile 10 is an example where a larger f_1 would have resulted in some improvement of the model results. However, even for this extreme example, the height errors given by the model never exceed 65 km;

(v) The ionograms for profiles 3, 4, 7, 11 and 12 were recorded on magnetically disturbed days (magnetic disturbance index $\Sigma\, Kp \geqslant 30$). There is no evidence that the model is less accurate under such conditions.

5. Summary and Conclusions

A new model representation of the electron-concentration distribution of the E- and F-regions of the ionosphere has been developed and simple empirical equations derived to enable the parameters of the model to be generated in terms of the ionospheric characteristics which are normally scaled from ionograms.

Comparisons are presented with the results of true-height profile analyses and it is shown that for the most part discrepancies are 20 km or less, which is of the order of the uncertainties in the profile calculations and of the errors in the measured characteristics. At low latitudes when the height of maximum ionization of the $F2$-layer is around 500 km the new model tends to give layer heights which are too large by about 50 km. A means of modifying the model to minimize these errors is outlined but the added complexity may not be justified. The new model is shown to be more accurate than other models in current use.

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APPENDIX 1

Wave propagation at vertical incidence in the new model ionosphere

Analytical expressions for the true and virtual heights of reflection, h and h', respectively, for propagation at vertical incidence in the new model ionosphere are readily derived using relationships similar to those presented by BEYNON and THOMAS (1956). For a general frequency f giving reflection from one of the three regions E, $F1$ and $F2$, we have:

E-region: $f < foE$ with $\dfrac{f}{foE} = x_E$

$$h = hmE - ymE\sqrt{1 - x_E^2} \tag{7}$$

$$h' = (hmE - ymE) + x_E .\ ym_E .\ \operatorname{arctanh} x_E . \tag{8}$$

F1-region: $foE \leqslant f \leqslant f_1$ with $\dfrac{f}{f_1} = x_1;\ x_E \geqslant 1$

$$h = hmE + (h_1 - hmE) . \frac{x_1^2(x_E^2 - 1)}{(x_E^2 - x_1^2)} \tag{9}$$

$$h' = (hmE - ymE) + x_E .\ ymE.\ \operatorname{arccoth} x_E + 2(h_1 - hmE) . \frac{x_1^2 x_E\sqrt{x_E^2 - 1}}{(x_E^2 - x_1^2)} . \tag{10}$$

F2-region: $f_1 < f \leqslant foF2$ with $\dfrac{f}{foF2} = x_{F2};\ x_E > 1;\ x_1 > 1$

$$h = hmF2 - ymF2\sqrt{1 - x_{F2}^2} \tag{11}$$

$$h' = (hmE - ymE) + x_E .\ ymE.\ \operatorname{arccoth.} x_E + 2(h_1 - hmE) . \times \frac{x_1 x_E[x_1\sqrt{x_E^2 - 1} - x_E\sqrt{x_1^2 - 1}]}{(x_E^2 - x_1^2)} + x_{F2} .\ ymF2.\ \operatorname{arccosh}\left(\frac{hmF2 - h_1}{ymF2\sqrt{1 - x_{F2}^2}}\right) . \tag{12}$$

h_1 is the height of intersection of the $F1$ and $F2$ portions of the model corresponding to a plasma frequency f_1. It is given by

$$h_1 = hmF2 - ymF2\sqrt{1 - \left(\frac{f_1}{foF2}\right)^2} . \tag{13}$$

APPENDIX 2

True-height profile data

Table 3 lists the ionospheric observatories involved in the sample comparisons with the new model results and indicates the corresponding sources of true-height data used in the derivation of Figs. 5–8.

Table 3

Station	Latitude	Longitude	*haF2*; *yaF2* (Figs. 5–7)	$N(h)$ profiles (Fig. 8)
Argentine Islands	65·2°S	64·3°W	1	4
Lindau	51·6°N	10·1°E	5	4
Slough	51·5°N	0·5°W	IGY 3 IQSY 2	3
Wallops Island	37·8°N	75·5°W		4
Cape Zevgari	34·6°N	33·0°E		4
Ibadan	7·4°N	3·9°E	IGY 6 IQSY 2	3
Singapore	1·2°N	103·9°E	3	

(1) *hc*, *qc* analysis (Piggott and Rawer, 1972) undertaken by authors.
(2) *hc*, *qc* analysis using hourly values published by World Data Centre C.
(3) $N(h)$ profile analysis using the technique developed by Thomas *et al.* (1958); hourly values published by World Data Centre C.
(4) $N(h)$ profile analysis undertaken by authors using the technique developed by Jackson (1971).
(5) $N(h)$ profile analysis using the technique developed by Becker (1967). These data were kindly supplied by Dr. R. Eyfrig.
(6) $N(h)$ profile analysis using the technique developed by Kelso (1952); hourly values published by World Data Centre C.

In the *hc*, *qc* method of estimating *hmF2* and *ymF2* the true height where the plasma frequency is 0·95 *foF2* is first derived in terms of the virtual heights of reflection at 10 predetermined lower frequencies. The quarter thickness of the equivalent *F2*-layer parabolic distribution, *qc*, is then given by an overlay method and thus *hmF2* is readily deduced. For carefully selected ionograms the method leads to heights of maximum electron concentration and semi-thickness accurate to within ± 10 km, but when large numbers of routine soundings are analysed (as in (2) above) many will include cases where the virtual heights at more than one of the sampling frequencies are indeterminate and have to be estimated, so that errors can be greater. In such cases height accuracies cannot be guaranteed as better than ± 20 km.

The methods (3)–(5) here used for deriving $N(h)$ profiles all take account of the Earth's magnetic field. Method (6) ignores the field but has only been used at Ibadan where this has a very small effect because of the low dip angle of 6°. All four profile analysis methods assume a parabolic extrapolation of the profile from the region giving observable echoes up to *hmF2*. In methods (4) and (5), the electron concentration is assumed to increase monotonically with increase of height, but method (3) incorporates an *E*-region valley by iteratively minimizing the differences between the measured and synthesized extraordinary wave virtual-height profiles. The different methods of profile determination generally give results which differ from one another and probably depart from the true profile by up to ± 10 km over all heights. Thus for the purpose of comparison with the model results a zone of uncertainty of this magnitude must be assumed.

The model is seen from the examples of Fig. 8 to have *hmE* generally lower than the values indicated by the profile analyses. We have adopted 110 km because:

(i) all virtual heights scaled from ionograms are liable to be the order of 5 km or more too large because of pulse distortion in the ionosonde receiver (Lyon and Moorat, 1956);

(ii) *hmE* is often determined from portions of the ionogram trace just above *foE* where the group retardation is influenced by ionization above *hmE*;

(iii) when the phenomenon of 'cusp *Es*' is present and this degenerates into a high *Es* trace, the bounds of *hmE* may be readily determined from the values of *h'Es*. Such results support lower *hmE* than indicated by the profile analyses.