Qn: 1

Plot the Bode diagram for the following transfer function and obtain the gain and phase cross over frequencies

$$G(s) = \frac{10}{s(1+.4s)(1+.1s)}$$

Steps

- Take transfer function of the given s/m $G(s) \rightarrow$ std. form
- put $s = j\omega$ in the given transfer function

1) Magnitude plot
   1. Find corner frequencies
   2. Magnitude table
   3. choose ~~lowest & highest~~ starting freq at $\omega = .1$
   4. ~~Find the mag. at each freq.~~
   4) Draw plot in semilog graph.

2) Phase plot
   1. Find the phase (angle) of the given function $G(j\omega)$
   2. Substitute the value of ω & find the corresponding angles
   3. Draw Bodeplot on semilog graph

$$G(s) = \frac{10}{s(1+.4s)(1+.1s)} \rightarrow \text{std form} \Rightarrow (1+Ts)$$

① put $s = j\omega \Rightarrow G(j\omega) = \dfrac{10}{j\omega(1+.4j\omega)(1+.1j\omega)}$

Magnitude Plot

② Corner frequencies $\omega c_1 = \frac{1}{.4} = 2.5\,rad/s$

$\omega c_2 = \frac{1}{.1} = 10\,rad/s$

[The reciprocal of the co-efficient of $j\omega$ term is the corner freq].

③

Term	Corner freq rad/sec	Slope (db/dec)	Change in slope (total slope)
$\frac{10}{j\omega}$	–	−20	−20
$\frac{1}{1+j.4\omega}$	2.5 ω	−20	−20 + −20 = −40
$\frac{1}{1+j0.1\omega}$	10.	−20	−40 + −20 = −60°

40dB

$20\log\left(\frac{10}{2.5}\right) = 20\log 4 = 12dB$

$j\omega \rightarrow 20$ (slope)

$j\omega^2 \rightarrow 40dB/dec$

$j\omega^3 \rightarrow 60\,db/dec.$

$\frac{1}{j\omega} = -20dB/dec$

$\frac{1}{j\omega^2} \Rightarrow -40dB/dec$

$\frac{1}{j\omega^3} \Rightarrow -60dB/dec.$

$-40 \times \log\left(\frac{10}{2.5}\right) + 12dB$

-12.08 ~~40~~ dB

$-60° \times \log\left(\frac{100}{10}\right) + -12$

-72

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④ choose lowest & highest freq.

Let $\omega_l = .1$ $\quad \omega_{c_1} = 2.5$, $\omega_{c_2} = 10$ rad/sec. Let $\omega_h = 50$ rad/sec

$(\omega_l < \omega_{c_1})$ $\qquad (\omega_h > \omega_{c_2})$.

Starting point,

20 log (First term) at $\omega = .1$ [First term $\frac{10}{j\omega}$ j is not considered]

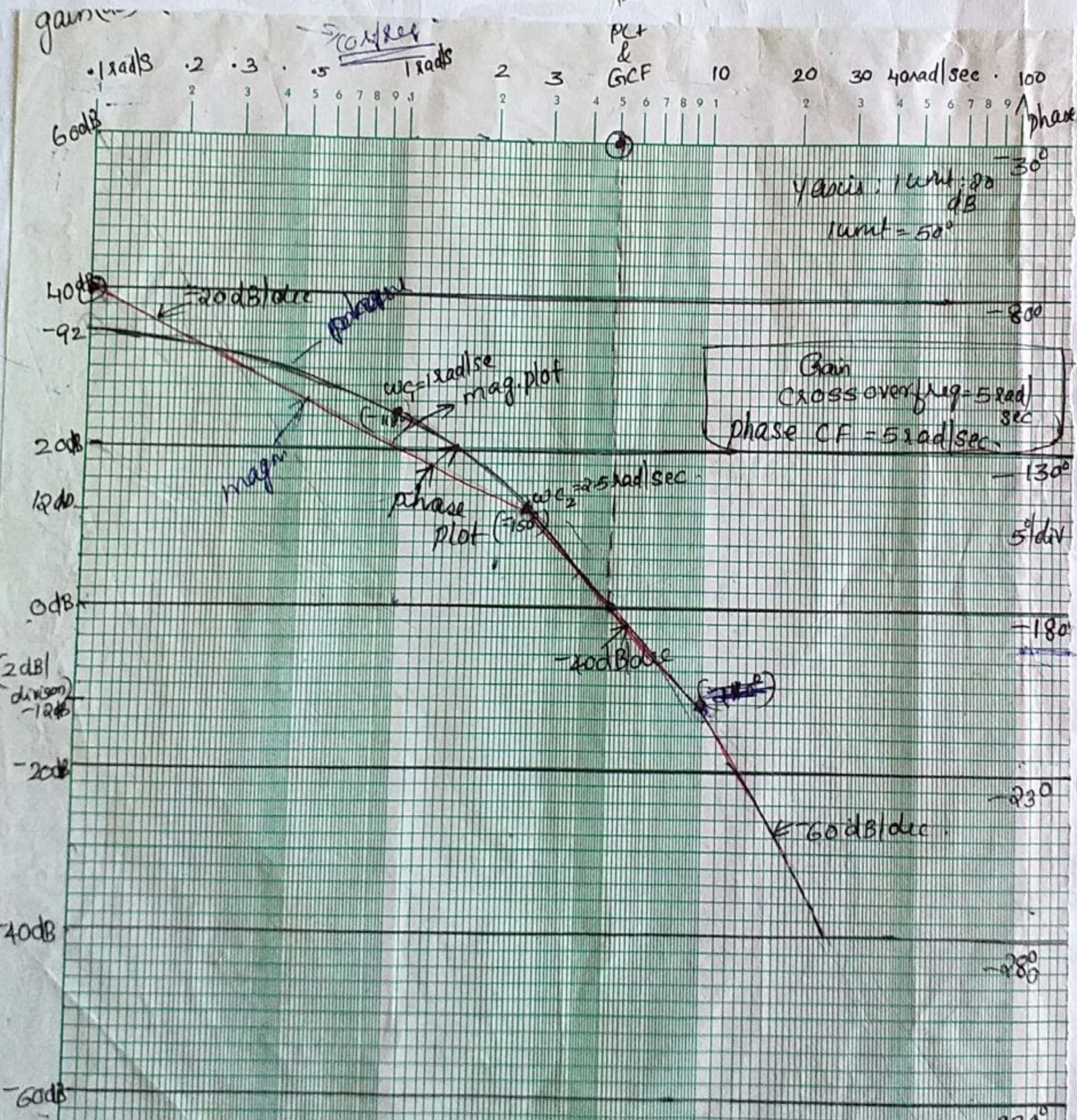
$\therefore 20 \log\left(\frac{10}{.1}\right) = \underline{40dB}$

Phase plot $\phi = -90^\circ - \tan^{-1} .4\omega - \tan^{-1} .1\omega$

Result

Gain & Phase cross over freq = 5 rad/sec

ω	$\phi = \angle G(j\omega)H(j\omega) = -90 - \tan^{-1}.4\omega - \tan^{-1}.1\omega$
.1	-92
1	-118
2.5	-150
4	-170
10	-210
20	-236.



$G(s) = \frac{10}{s(1+.4s)(1+.1s)}$

plot Bode plot & Find Gain & phase cross over frequencies

③ Draw the Bodeplot & hence find gain cross over frequency, phase cross over freq, gain margin, phase margin & stability of the system.

$$G(s)H(s) = \frac{20}{s(1+s)(10+s)}$$

Step 1: Standard form ⇒

$$G(s)H(s) = \frac{20}{s(1+s)\,10\left(1+\frac{s}{10}\right)}$$

$$= \frac{2}{s(1+s)\left(1+\frac{s}{10}\right)}. \qquad \left[\begin{array}{c}\text{std form} \\ (1+Ts)\end{array}\right]$$

Step 2: Substitute $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{2}{j\omega(1+j\omega)\left(1+\frac{j\omega}{10}\right)}.$$

Step 3: Table for Gain Plot.

⇒ Find corner frequencies

⇒ The reciprocal of the co-efficient of $j\omega$ term in $(1+j\omega)$ is the corner frequency. $\omega_{c_1} = \frac{1}{1} = 1$ $\quad \omega_{c_2} = \frac{1}{\cdot 1} = 10$

term	CF	slope	Total slope
$\frac{2}{j\omega}$	—	-20	-20 dB/dec ⇒ $20\log\left(\frac{2}{\cdot 1}\right) = 26$ dB
$\frac{1}{1+j\omega}$	$1/1 = 1$	-20	-40 dB/dec ⇒ $20\log\left(\frac{2}{1}\right) = \underline{6\text{dB}}$
$\frac{1}{1+\cdot 1 j\omega}$	$1/\cdot 1 = 10$	-20	-60 " ⇒ $-40 \times \log\left(\frac{10}{1}\right) + 6\text{dB} = \underline{-36\text{dB}}$

$-60 \times \log\left(\frac{50}{10}\right) + -36 =$

slope

$j\omega \to 20 \qquad 1/j\omega \to -20$

$j\omega^2 \to 40 \qquad \frac{1}{j\omega^2} \to -40$

$j\omega^3 \to 60 \qquad \frac{1}{j\omega^3} \to -60.$

Step 4: Finding starting point.

$20\log$ (first term) at $\omega = \cdot 1$ rad/sec $\quad$ [First term $\frac{2}{j\omega}$) j is not considered]

$20\log\frac{2}{\omega} = 20\log\frac{2}{\cdot 1} = 26\text{dB}$ starting pt.

Step 5: Phase plot

ω	$\angle G(j\omega)H(j\omega)$ $-90 - \tan^{-1}(\omega) - \tan^{-1}(\omega/10)$
$\cdot 1$	-96.28
$\cdot 3$	-108.41
$\cdot 5$	$-120°$
1	$-140°$
1.4	-152.4
3	-178.2
5	$-195.2°$
30	$-249.6°$

To calculate in degree mode of calculator.

phase

$j\omega \to 90° \qquad 1/j\omega \to -90°$

$j\omega^2 \to 180° \qquad \frac{1}{(j\omega)^2} \to -180°$

$j\omega^3 \to 270° \qquad 1/(j\omega^3) \to -270.$

* If it is in $(a+jb)$ form, then find $\tan^{-1}(b/a)$

* The Point where 0dB line intersects the gain plot, the corresponding frequency value will be gain cross over frequency (gcf) ⇒ ω_{gc}
* The point where -180° line intersect the phase plot, the corresponding freq. value will be phase cross over freq. (pcf) ⇒ ω_{pc}

phase margin:-

The line plots from gcf to the phasplot is called phase margin

[During ω_{gc}, magnitude difference with 0dB]

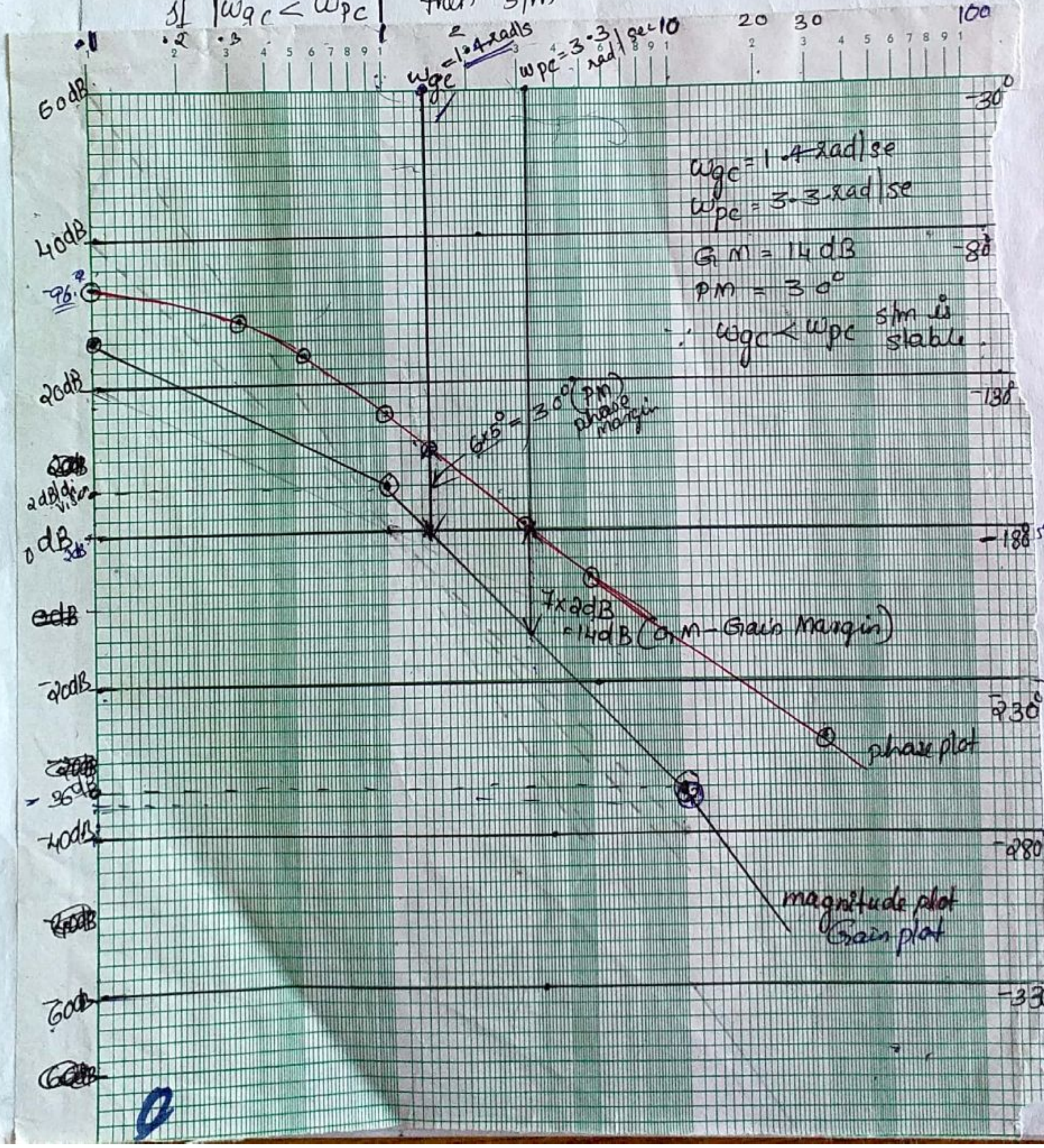
Gain Margin

The line plots from pcf to the gain plot is called Gain margin

[During ω_{pc}, phase having difference with -180]

Stability

If $\boxed{\omega_{gc} < \omega_{pc}}$ then s/m is stable



(4) Sketch Bode Plot for the following transfer function and determine the system gain k for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{ks^2}{(1+.2s)(1+.02s)}$$

⇒ Assume $k=1$

⇒ substitute $s=j\omega$

$$G(j\omega) = \frac{(j\omega)^2}{(1+.2j\omega)(1+.02j\omega)}$$

⇒ Magnitude plot (Gain Plot)

term	ω (rad/sec) corner freq	Slope	Total slope
$(j\omega)^2$	–	40 db/dec	40
$\frac{1}{1+.2j\omega}$	$\frac{1}{.2}=5$ ω_1	−20	20
$\frac{1}{1+.02j\omega}$	$\frac{1}{.02}=50$ ω_c	−20	0

→ 20log (1st term) ω→.1 (j not exist)

⇒ 20log(1st term) at ω = 5 rad/s.
$20\log(5^2) = \underline{28dB}$

⇒ (Slope from ωc_1 to $\omega c_2 \times \log\frac{\omega c_2}{\omega c_1}$] + Mag. at ωc_1

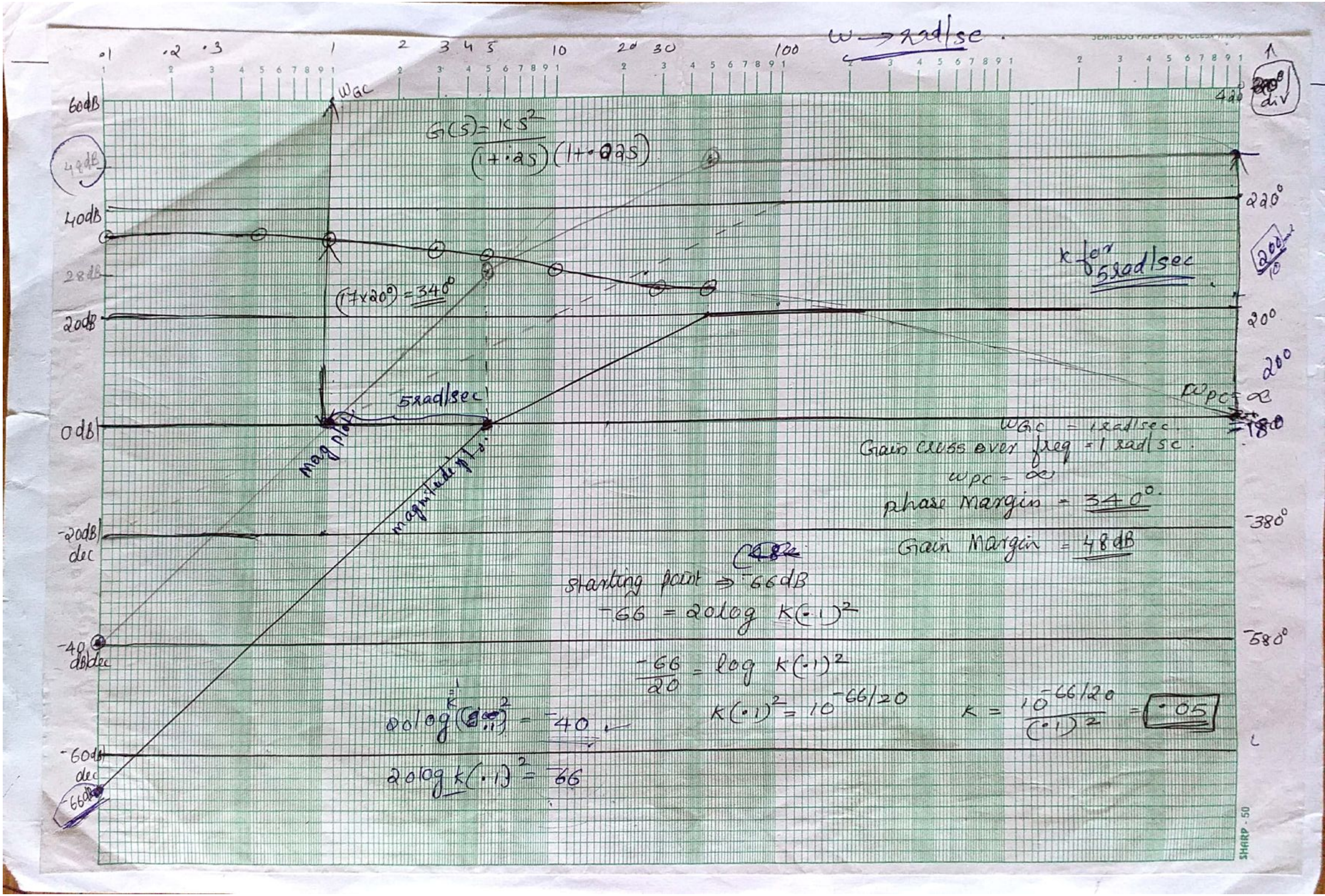
$20\times\log\frac{50}{5} + 28 = \underline{48dB}$

⇒ starting pt = 20 log(1st term) at ω = .1 rad/s
$= 20\log(\omega^2) = 20\log(.1)^2 = \underline{-40dB}$

End pt at ω = 100 rad/s.
$0\times\log\frac{100}{50} + 48dB = \underline{48dB}$

⇒ phase Plot

ω (rad/sec)	$\phi = \angle G(j\omega)$ $= 180^0 - \tan^{-1}(.2\omega) - \tan^{-1}(.02)\omega$
.1	178.73
.5	173.7
1	167.5
3	145.6
5	129.3
10	105.3
30	68.4
50	50


ω → rad/sec
ωgc
G(s) = Ks² / (1+.2s)(1+.02s)
60dB
40dB
28dB
20dB
0dB
-20dB/dec
-40 dB/dec
-60dB/dec
-66dB
(17×20°) = 340°
5rad/sec
mag plot
magnitude plot
k for 5rad/sec
ωpc = ∞
220°
200°
180°
-380°
-580°
ωgc = 1rad/sec
Gain cross over freq = 1 rad/sec
ωpc = ∞
phase Margin = 340°
Gain Margin = 48dB
Starting point ⇒ -66dB
-66 = 20log K(.1)²
-66/20 = log K(.1)²
K(.1)² = 10^(-66/20)
K = 10^(-66/20) / (.1)² = .05
20log(K)(.1)² = -40
20log K(.1)² = -66

Q_2 $G(s) = \dfrac{50}{s(1+0.25s)(1+0.1s)}$

put $s = j\omega$

$$G(j\omega) = \frac{50}{j\omega(1+0.25j\omega)(1+0.1j\omega)}$$

terms	corner frequency (rad/sec)	slope (dB)	change in slope (dB)
$50/j\omega$		-20	
$\dfrac{1}{1+0.25j\omega}$	$\omega_{c1} = \dfrac{1}{0.25} = 4$	-20	-40
$\dfrac{1}{1+0.1j\omega}$	$\omega_{c2} = \dfrac{1}{0.1} = 10$	-20	-60

Let $\omega_L = 0.1$ rad/sec

$\omega_H = 50$ rad/sec

At $\omega = \omega_L$

$\omega_L = 0.1$ rad/sec

$|A| = 20 \log \left|\frac{50}{j\omega}\right|$ at $\omega = \omega_L$

$= 20 \log \frac{50}{0.1} = 53.97 \text{ dB} \approx \cancel{54 \text{ dB}}$

At $\omega = \omega_{c1}$

$\omega_{c1} = 4$ rad/sec

$|A| = 20 \log \left|\frac{50}{4}\right| = 21.94 \text{ dB} \; \cancel{22 \text{ dB}}$

At $\omega = \omega_{c2}$

$\omega_{c2} = 10$ rad/sec

$|A| = -40 \times \log \left|\frac{10}{4}\right| + 21.94$

$= 6.022$ dB

At $\omega = \omega_H$

$\omega_H = 50$ rad/sec

$|A| = -60 \times \log \left|\frac{50}{10}\right| + 6.022$

$= -35.916$ dB

ω (rad/sec)	|A| dB
0.1	53.97
4	21.94
10	6.022
50	-35.916

$\angle G(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(0.25j\omega) - \tan^{-1}(0.1\omega$

ω (rad/sec)	$-\tan^{-1}(\omega)$	$-\tan^{-1}(0.25\omega)$	$-\tan^{-1}(0.1\omega)$	φ
0.1	-90	-1.432	-0.57	-92.002
1	-90	-14.03	-5.71	-109.74
4	-90	-45	-14.03	-156.8
10	-90	-68.198	-45	-203.19
20	-90	-78.69	-78.69	-232.12
50	-90	-85.426	-84.28	-254.11

$\omega_{gc} = 12$ rad/sec

$\omega_{pc} = 7$ rad/sec

$GM = -12$ dB

$PM = 180 + -212 = -32$ dB

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So, the system is unstable.

Note 1:- If $G(s) = \frac{K}{s(1+0.1s)}$.

put $K=1$ and draw the bode plot. Find the condition of ω_{gc}. If $\omega_{gc} = 5$ rad/sec and $|A|$ in dB is 28 dB. shift magnitude plot by -28 dB to obtain the original plot.

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Second order

Eg:- $G(s) = \frac{75(1+0.2s)}{s(s^2+16s+100)}$

Second order general form $\Rightarrow s^2+2\delta\omega_n s+\omega_n^2$

$\omega_n^2 = 100$, $2\delta\omega_n = 16$

$\omega_n = 10$. $\delta = \frac{16}{2\times10} = \underline{\underline{0.8}}$

$$G(s) = \frac{75(1+0.2s)}{s\times100\left(1+\frac{16s}{100}+\frac{1}{100}s^2\right)}$$

$$= \frac{0.75(1+0.2s)}{s(1+0.16s+0.1s^2)}$$

put, $s = j\omega$.

$$G(j\omega) = \frac{0.75(1+0.2j\omega)}{j\omega(1+0.16j\omega+0.01(j\omega)^2)}$$

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$$G(j\omega) = \frac{0.75\,(1+j0.2\omega)}{j\omega\,(1-0.01\omega^2+0.16j\omega)}$$

Term	Corner Frequency	Slope	change in Slope (dB)
$0.75/j\omega$		-20	
$1+j0.2\omega$	$\omega c_1 = \frac{1}{0.2} = 5$	-20	0
$\frac{1}{1-0.01\omega^2+j0.16\omega}$	$\omega c_2 = \omega_n = 10$	-40	-40

$\omega_L = 0.5$ rad/sec

$\omega_H = 20$ rad/sec

At $\omega = \omega_L$

$\omega_L = 0.5$ rad/sec

$$|A| = 20\log\left|\frac{0.75}{0.5}\right| = \underline{\underline{3.5\text{ dB}}}$$

At $\omega = \omega c_1$

$\omega c_1 = 10$ rad/sec

$$|A| = 20\log\left|\frac{0.75}{5}\right| = -16.47 = \underline{\underline{-16.5\text{ dB}}}$$

At $\omega = \omega c_2$

$$= 0 + -16.5 = \underline{\underline{-16.5\text{ dB}}}$$

At $\omega = \omega_H$

$$= -40 \times \log\left|\frac{20}{10}\right| + -16.5 = \underline{\underline{-28.5\text{ dB}}}$$

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ω	|A|
0.5	3.5
5	−16.5
10	−16.5
20	−28.5

$$\angle G(j\omega) = \tan^{-1}(0.2\omega) - \tan^{-1}(\infty) - \tan^{-1}\left(\frac{0.16\omega}{1-0.0}\right.$$

ω(rad/sec)	$\tan^{-1}(0.2\omega)$	$-\tan^{-1}(\infty)$	$-\tan^{-1}\left(\frac{0.16\omega}{1-0.01\omega^2}\right)$	φ
0.1	1.14	−90	−0.91	−89.77
0.5	5.71	−90	−4.58	−88.87
1	11.30	−90	−9.18	−87.88
5	45	−90	−46.84	−91.84
10	63.43	−90	−90	−116
20	75.96	−90	−46.84+180 = 133.2	−148
~~50~~				

Frequency domain specification

Resonant peak :-

The maximum value of magnitude is known as resonant peak. Gives the information about the relative stability of the system

$$M_r = \frac{1}{2\delta\sqrt{1-\delta^2}}$$

Resonant frequency , ω_r

The frequency at which magnitude has maximum value of ω_r is large the time response is fast

$$\omega_r = \omega_n \sqrt{1-2\delta^2}$$

Bandwidth

Bandwidth is defined as the range of frequencies in which the magnitude of close loop does not drop - 3 dB.

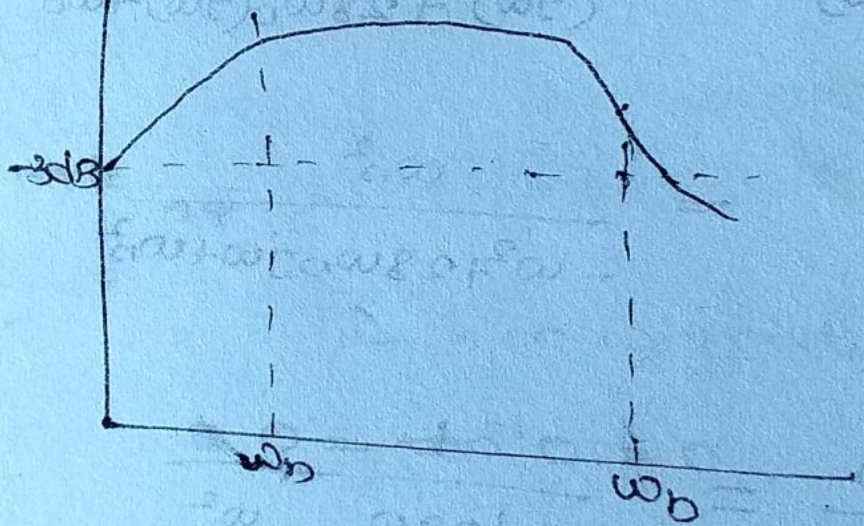
cutoff frequency, wb

The frequency at which the magnitu[de]

is 3dB below its zero frequency value.

cut off rate, φ

slope of the along magnitude curve



4.5 FREQUENCY RESPONSE PLOTS

Frequency response analysis of control systems can be carried either analytically or graphically. The various graphical techniques available for frequency response analysis are

1. Bode plot
2. Polar plot (or Nyquist plot)
3. Nichols plot
4. M and N circles
5. Nichols chart

The Bode plot, Polar plot and Nichols plot are usually drawn for open loop systems. From the open loop response plot the performance and stability of closed loop system are estimated. The M and N circles and Nichols chart are used to graphically determine the frequency response of unity feedback closed loop system from the knowledge of open loop response.

The frequency response plots are used to determine the frequency domain specifications, to study the stability of the systems and to adjust the gain of the system to satisfy the desired specifications.

4.6 BODE PLOT

The Bode plot is a frequency response plot of the transfer function of a system. A Bode plot consists of two graphs. One is a plot of the magnitude of a sinusoidal transfer function versus log ω. The other is a plot of the phase angle of a sinusoidal transfer function versus log ω.

The Bode plot can be drawn for both open loop and closed loop transfer function. Usually the bode plot is drawn for open loop system. The standard representation of the logarithmic magnitude of open loop transfer function of $G(j\omega)$ is $20 \log |G(j\omega)|$ where the base of the logarithm is 10. The unit used in this representation of the magnitude is the decibel, usually abbreviated db. The curves are drawn on semilog paper, using the log scale (abcissa)for frequency and the linear scale (ordinate) for either magnitude (in decibels) or phase angle (in degrees).

The main advantage of the bode plot is that multiplication of magnitudes can be converted into addition. Also a simple method for sketching an approximate log-magnitude curve is available.