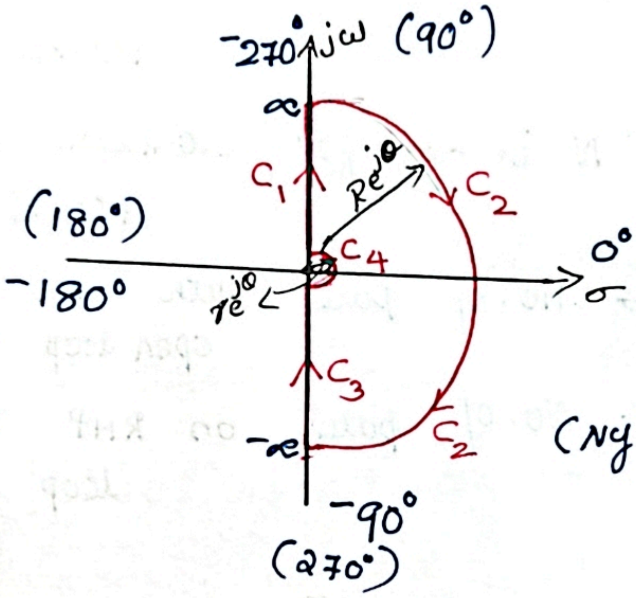
# Nyquist Plot.

- Find the stability of closed loop system using open loop transfer function.
- It has 4 parts. – $C_1, C_2, C_3, C_4$




(Nyquist contour)

- Each part of the Nyquist contour is used to map / convert s-plane in to G-H plane.
- For part 1 → $s = j\omega$ (Substitute) → Thisis is polar plot

  part 2 → varies from $+\infty$ to $-\infty$,
  Radius $Re^{j\theta}$, $R \to \infty$
  $S = \lim_{R \to \infty} Re^{j\theta}$, $\theta$ varies from $90°$ to $-90°$
  $G(s)H(s) = \lim_{R \to \infty} G(Re^{j\theta})H(Re^{j\theta})$

  part 3 → substitute $s = -j\omega$ → Inverse of polar plot

  part 4 → having radius $re^{j\theta}$, $r \to 0$
  $\theta$ varies from $-90°$ to $90°$
  $G(s)H(s) = \lim_{r \to 0} G(re^{j\theta})H(re^{j\theta})$

$C_4$ exist, if there is a pole in the origin

## Nyquist Criterion

- For a closed loop system to be stable, $N = P - Z$

$N \rightarrow$ No. of encirclement of point $-1+j0$

N is +ve for anticlockwise encirclement around $(-1, 0)$

N is -ve for clockwise encirclement around $(-1, 0)$

$P \rightarrow$ no. of poles with +ve real part (RHP) in open loop system.

$Z$ – No. of poles on RHP in closed loop system.

① Draw Nyquist plot and check the stability of the system.

$$G(s) = \frac{s+2}{(s+1)(s-1)}$$

Step 1 :- standard form $(1+Ts)$

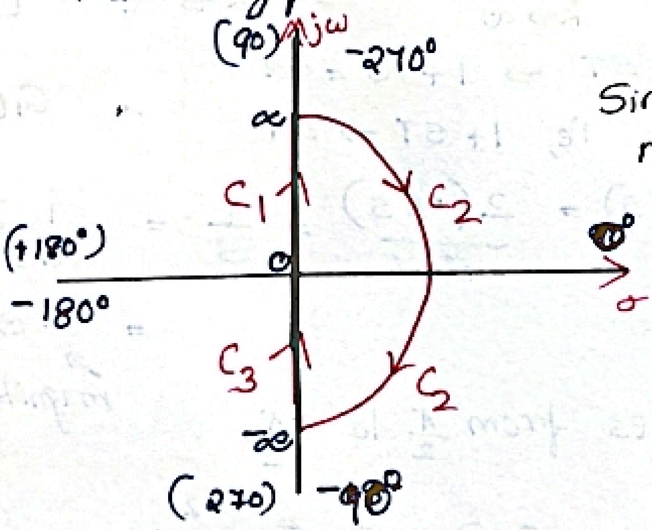
$$G(s) = \frac{2(1+0.5s)}{(1+s)(-1+s)} \quad -①$$

Step 2 :- Find no. of poles P on RHP

$$P = 1 \quad (\because s=1)$$

$\therefore$ Open loop system is unstable.

Step 3 :- draw nyquist contour




Since there is no poles at origin, nyquist contour has 3 parts $C_1$, $C_2$ & $C_3$.

Step 4 :- consider $C_1$,

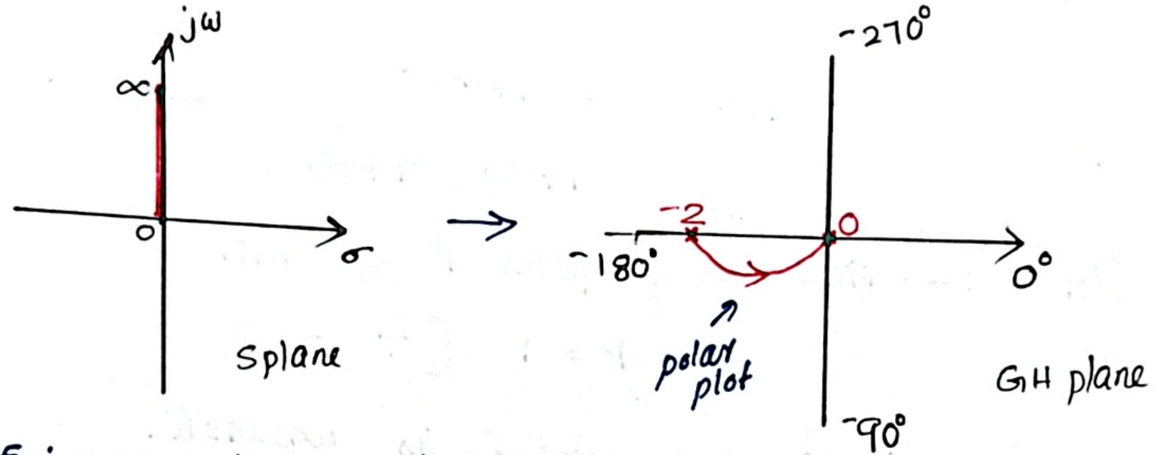
subsitute $s = j\omega$ in $G(s)$ $-①$

$$\therefore G(j\omega) = \frac{2(1+.5j\omega)}{(1+j\omega)(-1+j\omega)}$$

$$|G(j\omega)| = \frac{2\sqrt{1+(.5\omega)^2}}{\sqrt{1+\omega^2}\sqrt{1+\omega^2}}$$

$$\angle G(j\omega) = \tan^{-1}(.5\omega) - \tan^{-1}\omega - \tan^{-1}(-\omega)$$

| $\omega$ | 0 | $\infty$ |
|---|---|---|
| Magnitude | 2 | 0 |
| angle $\phi$ | $-180^\circ$ | $-90^\circ$ |




Step 5 :- Consider part $C_2$

$$S = \lim_{R\to\infty} R e^{j\theta} \quad ; \quad R \to \infty \qquad \boxed{S = \infty e^{j\theta}}$$

$$1 + ST \to 1 + \infty = \infty$$

$$G(s) = \frac{2(1 + \cdot 5s)}{(1+s)(-1+s)}$$

ie, $1 + ST \to ST$

$$\therefore G(S) = \frac{2(\cdot 5 S)}{S \times S} = \frac{1}{S} = \frac{1}{\infty e^{j\theta}}$$

$$= 0 \cdot e^{-j\theta}$$

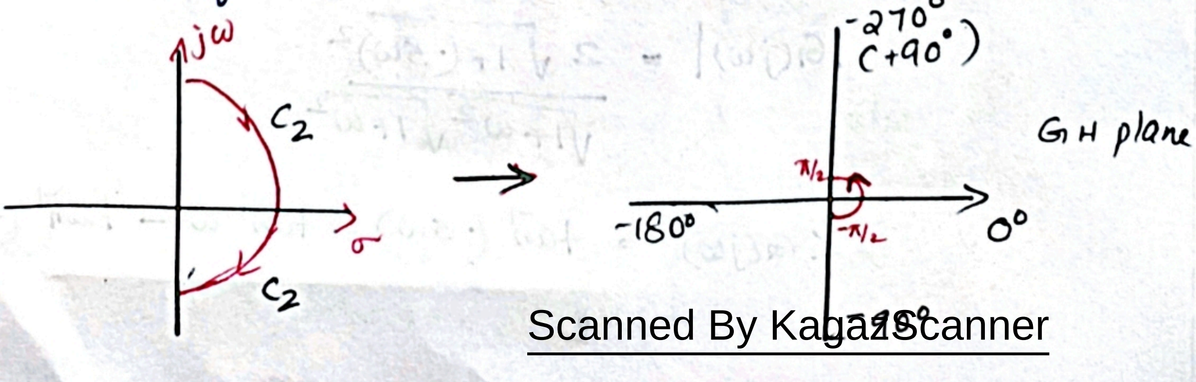
magnitude angle.

$\theta$ varies from $\frac{\pi}{2}$ to $\frac{-\pi}{2}$

if $\theta = \frac{\pi}{2}$ ; $G(S) = 0 \cdot e^{-j\pi/2}$

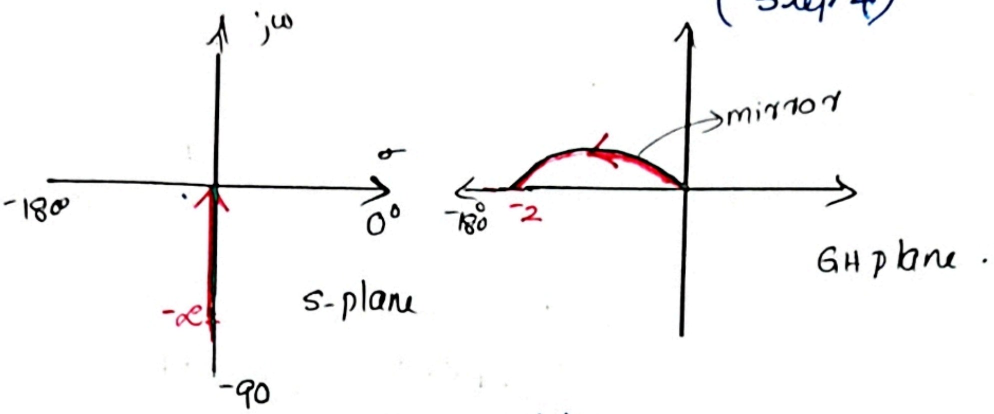
if $\theta = -\pi/2$ ; $G(S) = 0 \cdot e^{j\pi/2}$

$\therefore$ Magnitude is 0 and angle varies from $-\pi/2$ to $\pi/2$

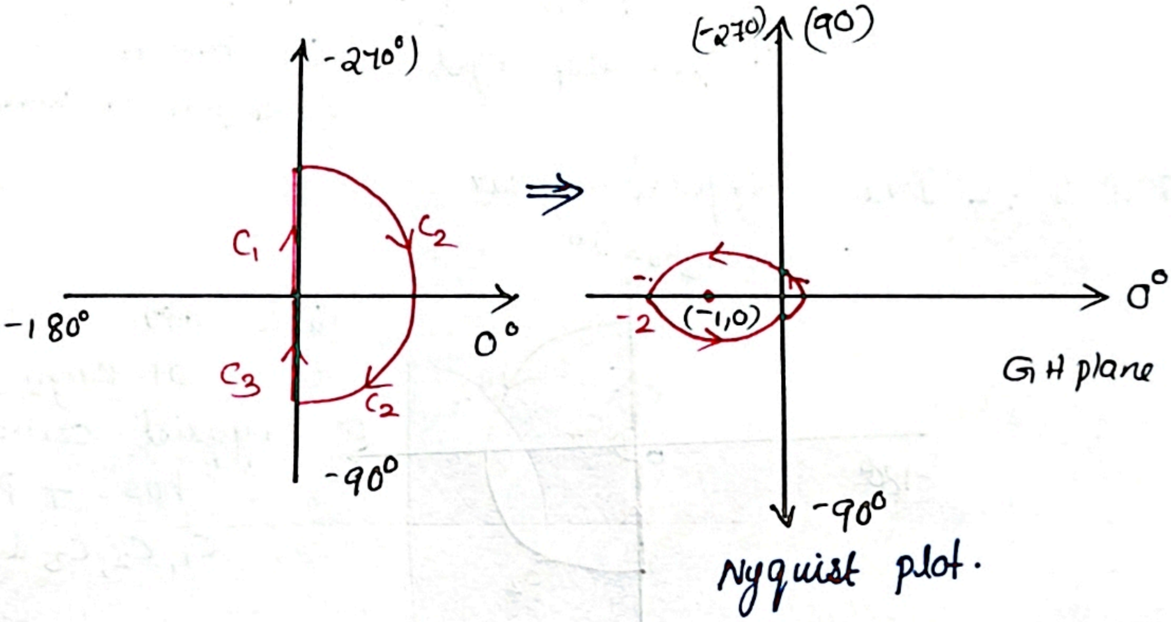


**Step 6 :** consider part $C_3 \Rightarrow s = -j\omega$

It is the inverse / mirror of polar plot (step 4)




**Step 7 :-** Combine all the plots




**Check for stability**

$N = P - Z$

$N \rightarrow$ no. of encirclement around $(-1, 0)$

$= 1$ (since encirclement is anticlockwise, N is +ve)

$P = 1$ (no. of RHP poles in OLS)

$\therefore \; Z = P - N$

$= 0$ (no. of RHP poles in Closed loop system)

$\Rightarrow$ Since there is no poles on the RHP of CLS, the system is stable

UQ: Aug21

② Draw Nyquist plot and find K for stability

$$G(s) = \frac{K}{s(s+2)(s+10)}$$

**step 1** :- standard form $(1+Ts)$

$$G(s) = \frac{K}{s(s+2)(s+10)} = \frac{K}{s \times 2(\frac{s}{2}+1) \times 10(\frac{s}{10}+1)}$$

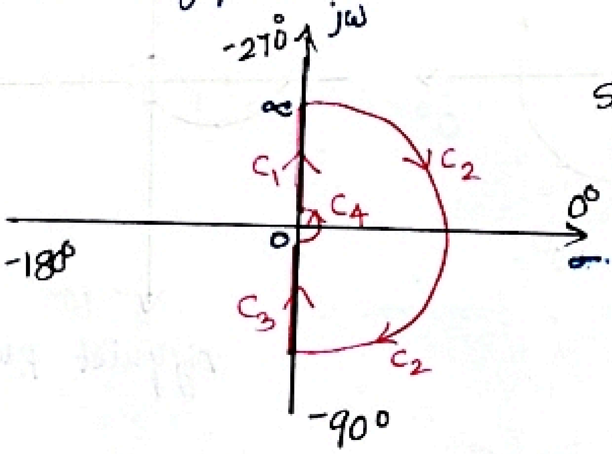
$$= \frac{.05\,K}{s(1+0.5s)(1+0.1s)}$$

**step 2** :- Find no. of poles P on RHP

$P=0$ $[\because s=0, -2, -10]$

$\therefore$ Open loop system is marginally stable (One pole on imag. axis)

**step 3** :- Draw Nyquist contour




Since there is a pole (s=0) at origin, nyquist contour has 4 parts $C_1, C_2, C_3$ & $C_4$.

**Step 4** :- consider part $C_1$,

substitute $s = j\omega$

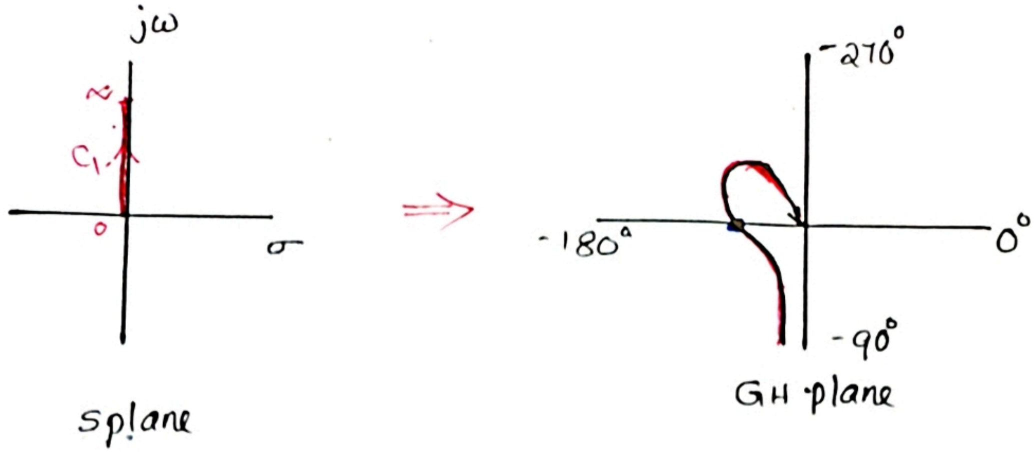
$$\therefore G(j\omega) = \frac{0.05K}{j\omega(1+0.5j\omega)(1+.01j\omega)}$$

$$|G(j\omega)| = \frac{.05k}{\omega\sqrt{1+(.5\omega)^2}\sqrt{1+(.1\omega)^2}}$$

$$\angle Gj(\omega) = -90 - \tan^{-1}(0.5\omega) - \tan^{-1}(0.1\omega)$$

$\boxed{\tan^{-1}\infty = 90°}$

| $\omega$ | 0 | $\infty$ |
|---|---|---|
| $\lvert G(j\omega)\rvert$ | $\infty$ | 0 |
| $\angle G(j\omega)$ | $-90°$ | $-270°$ |




$\rightarrow$ To find intersect point on real axis, $\frac{.05k}{j\omega(1+.5j\omega)(1+.1j\omega)}$

$$= \frac{.05k}{(j\omega - .5\omega^2)(1+.1j\omega)}$$

$$= \frac{.05k}{j\omega - .1\omega^2 - .5\omega^2 - .05j\omega^3}$$

$$= \frac{.05k}{-.6\omega^2 + j\underbrace{\omega(1-.05\omega^2)}_{\Rightarrow \text{ imaginary value.}}}$$

$$Img = \omega(1-.05\omega^2) = 0$$

$$1 = .05\omega^2$$

$$\omega = 4.472 \text{ rad/s}.$$

$\Rightarrow$ real

$$\frac{.05K}{-.6\omega^2} = \frac{.05k}{-.6(4.472)^2} = -.000417k$$

**Step 5:** Consider part $C_2$

$S = \lim_{R\to\infty} R e^{j\theta}$ ; $\theta$ varies from $\frac{\pi}{2}$ to $-\pi/2$

$\because R = \infty$ $\boxed{S = \infty e^{j\theta}}$ ; $1+ST = \infty$ $[\because 1+\infty = \infty]$

$1+ST \Rightarrow ST$

$$\therefore G(s) = \frac{.05k}{S \times (.5S) \times (.1S)} = \frac{.05k}{.05S^3} = \frac{k}{S^3}$$

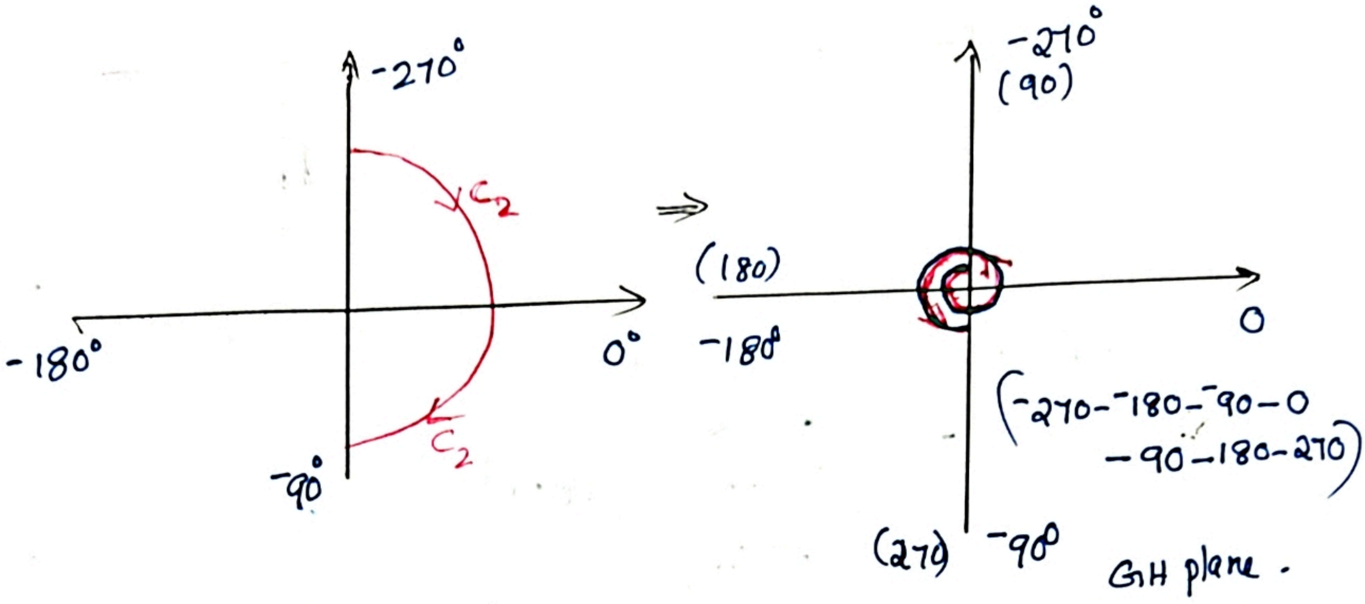
$$= \frac{k}{\infty e^{3j\theta}} = 0 \cdot e^{-3j\theta}$$

$\uparrow$ magnitude ; $\theta \Rightarrow +\frac{\pi}{2}$ to $-\frac{\pi}{2}$

If $\theta = \frac{\pi}{2} \Rightarrow e^{-3j\frac{\pi}{2}} = -270°$

If $\theta = -\frac{\pi}{2} \Rightarrow e^{3j\frac{\pi}{2}} = 270°$

∴ magnitude is zero and angle from $-270°$ to $+270°$ through zero

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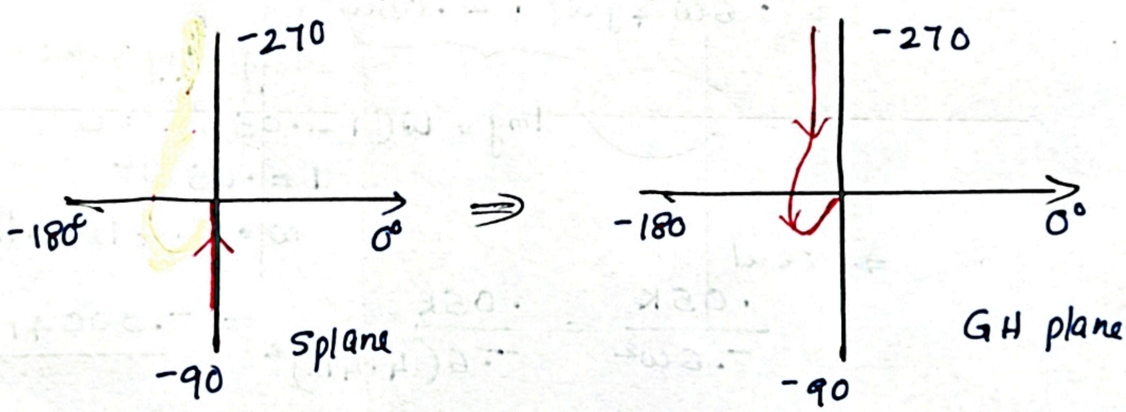



Step 7: consider part $C_4$

⇒ This part is due to pole at origin

varies → $(-\pi/2$ to $\pi/2)$

$\boxed{s = r e^{j\theta}}$
$r \to 0$

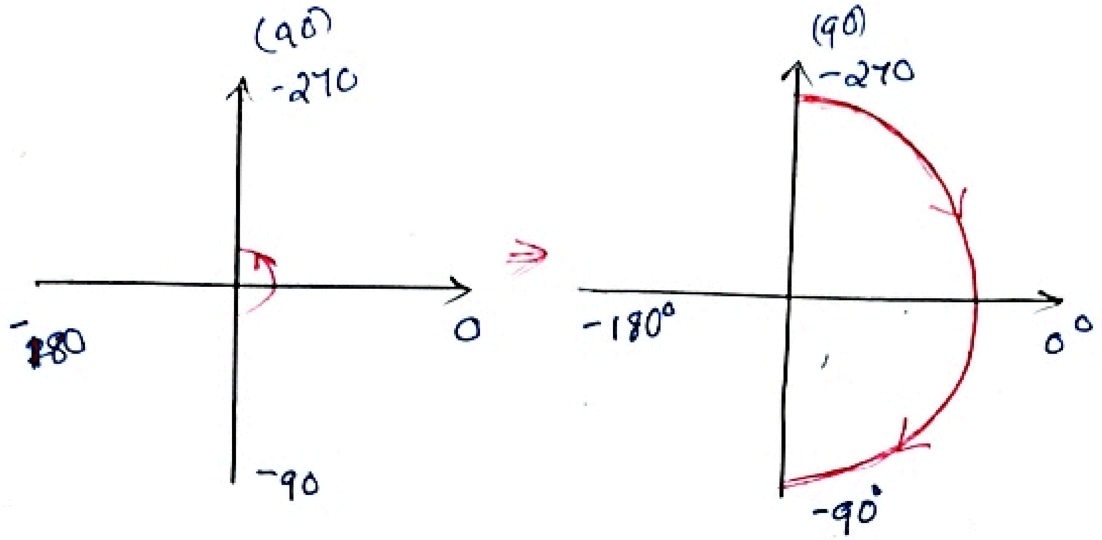
$s = \lim_{r \to 0} r e^{j\theta}$ ie, $1 + sT \simeq 1$

$G(s) = \dfrac{.05k}{s(1+.5s)(1+.1s)}$

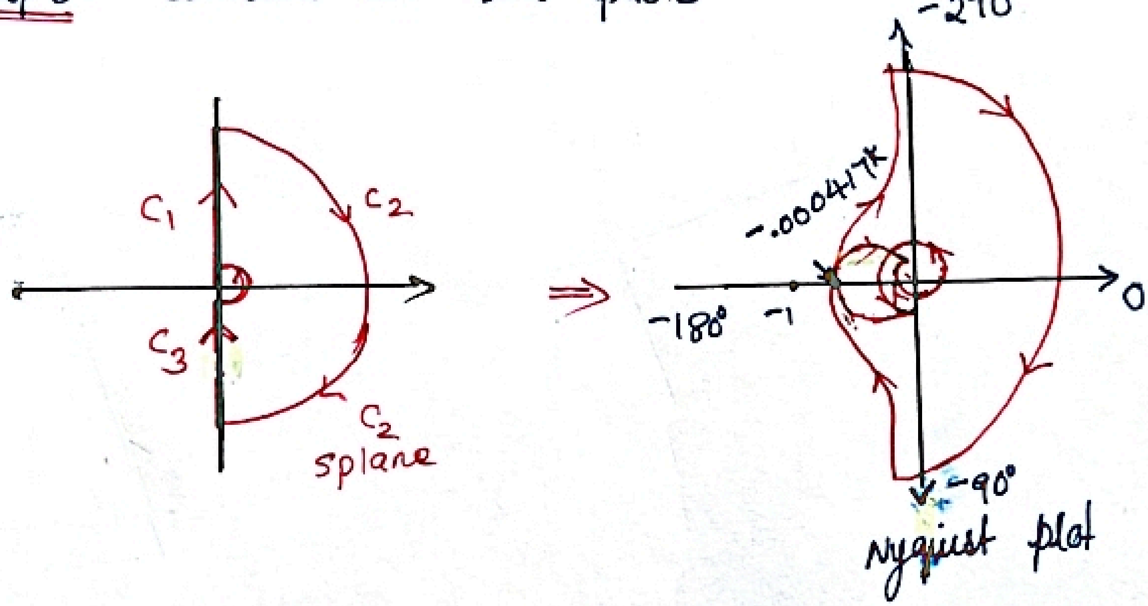
$G(s) = \dfrac{.05k}{s \times 1 \times 1} = \dfrac{.05k}{0 \times e^{j\theta}}$

$= \underset{\text{mag.}}{\infty} \, e^{\overset{\text{phase}}{-j\theta}}$

$-\pi/2$ to $\pi/2 \xrightarrow{\text{becomes}} \frac{\pi}{2}$ to $-\pi/2$




**Step 8:** combine all the plots




**Check for stability**

$$N = P - Z \qquad P = 0$$
$$N = 0,$$

For system stable,

$$-.000417k = -1$$
$$k = \frac{-1}{-.000417} = 240$$

the value of $k < 240$