Qns :→ Find Differential equation, of given system
→ Find transfer function
→ Draw Force - Voltage or Force to current analogy

mechanical s/m.

mechanical translation s/m — mechanical rotational s/m.

① Mechanical Translational system

In this mechanical system, object motion in straight line

x — displacement (m)

v — Velocity $\frac{dx}{dt}$ (m/s)

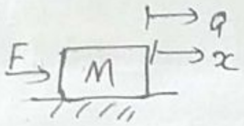
a — acceleration (m/s^2) $= \frac{dx^2}{dt} = \frac{dv}{dt}$.

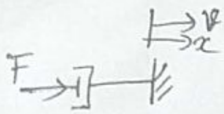
Based on this, different forces are

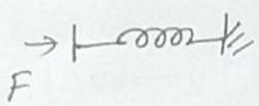
for mass (m)	Damper/Dashpot (B)	spring (k)
$F = ma$	$F = Bv$	$F = kx$.

Basic elements of mechanical systems are

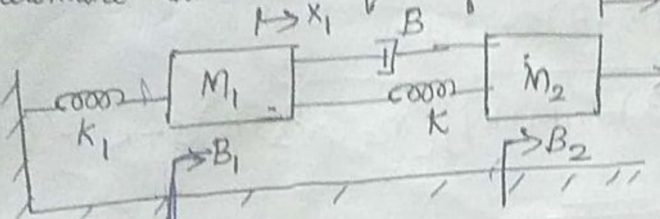
1) Mass (M) 2) Damper (Dashpot) (B) 3) spring (k)

① Mass  If we apply force on mass, there will be opposing force due to mass $\boxed{F_m = ma} = M\frac{d^2x}{dt^2}$

② Damper  If apply force F, displacement will be x with velocity v. so opposing force due to damper,
$$F_B = Bv = B\frac{dx}{dt}$$

③ spring (K)  Restoring force by spring $\boxed{F_k = kx}$

Write the differential equations governing the mechanical system and determine the transfer function.



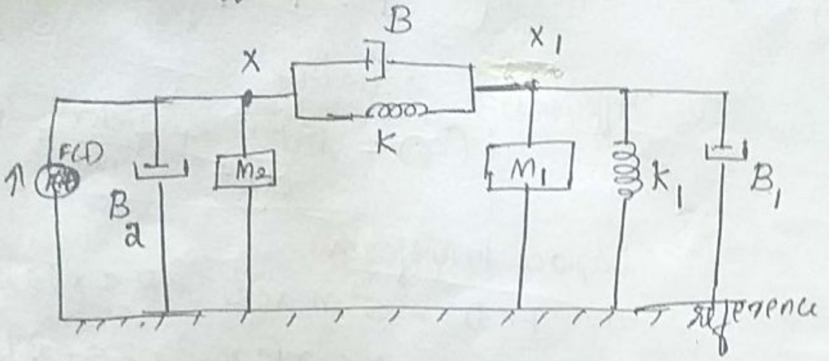
step 1: Find nodes = no. of displacement = 2

step 2: Connect mass M from node x with reference

connect spring k, ~~from~~ and damper B.

step 3: connect force F is applied.

step 4: write differential equations for corresponding node.



node 1

$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1) \quad -①$$

node 2

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + \frac{Bd(x_1 - x)}{dt} + K(x_1 - x) + K_1 x_1 = 0 \quad -②$$

equation ① & ② are the differential equations.

$\Rightarrow$ Find Laplace transform of eq^n ① & ② $\Rightarrow$

$$① \Rightarrow F(s) = M_2 s^2 X(s) + B_2 s X(s) + B s [X(s) - X_1(s)] + K[X(s) - X_1(s)]$$

$$F(s) = X(s)\left[M_2 s^2 + B_2 s + Bs + K\right] - X_1(s)[Bs + K] \quad -③$$

$$② \Rightarrow M_1 s^2 X_1(s) + B s (X_1(s) - X(s)) + K(X_1(s) - X(s)) + K_1 X_1(s) + B_1 s X_1(s) = 0$$

$$X_1(s)\left[M_1 s^2 + (B_1 + B)s + K + K_1\right] = X(s)\left[Bs + K\right]$$

$$X_1(s) = \frac{X(s)(Bs + K)}{M_1 s^2 + (B_1 + B)s + K_1 + K} \quad -④$$

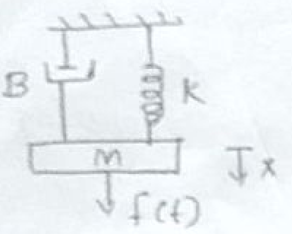
Substituting eq^n ④ in ③ $\Rightarrow$

$$F(s) = X(s)\left[M_2 s^2 + (B_2+B)s + k\right] - X(s)\,\frac{(Bs+k)^2}{M_1 s^2 + (B_1+B)s + K_1 + k}$$

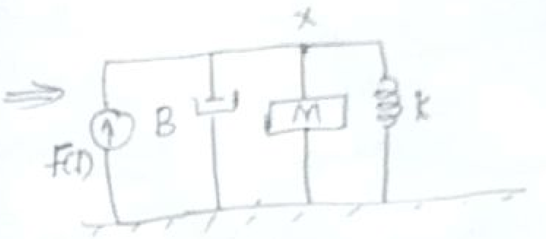
The transfer fn of the s/m is

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1+B)s + k_1 + k}{\left[M_1 s^2 + (B_1+B)s + [k_1+k]\right]\left[M_2 s^2 + (B_2+B)s + k\right] - (Bs+k)^2}$$

② Find transfer function of given mechanical system.



$\Rightarrow$



Differential equation

$$\boxed{f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx}$$

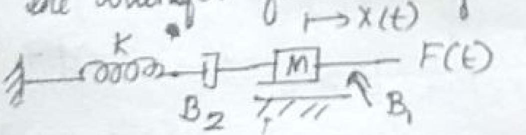
Laplace transform

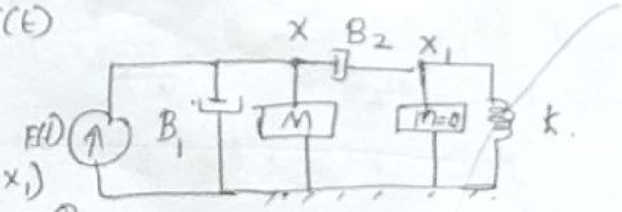
$$F(s) = s^2 M X(s) + B s X(s) + K X(s)$$

$$F(s) = X(s)\left[Ms^2 + Bs + k\right]$$

$$\boxed{\text{Transfer function } \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}}$$

③ Write the equations of motion in s-domain of the system and determine the transfer function of the system





$$F(t) = M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d}{dt}(x - x_1) \quad -①$$

$$0 = Kx_1 + B_2\frac{d}{dt}(x_1 - x) \quad -②$$

①&② are differential equation

Taking LT, $F(s) = Ms^2 X(s) + B_1 s X(s) + B_2 s\left(X(s) - X_1(s)\right)$

$$F(s) = X(s)\left[Ms^2 + (B_1+B_2)s\right] - X_1(s) B_2 s \quad -③$$

② ⇒ $0 = K X_1(s) + B_2 s\left(X_1(s) - X(s)\right)$

$B_2 s X(s) = X_1(s)\left[B_2 s + k\right] \Rightarrow X_1(s) = \frac{(B_2 s) X(s)}{B_2 s + K}$

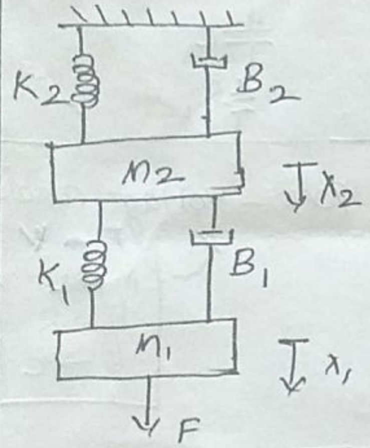
③ ⇒ $F(s) = X(s)\left[Ms^2 + (B_1+B_2)s\right] - \frac{(B_2 s)^2}{B_2 s + k} X(s)$

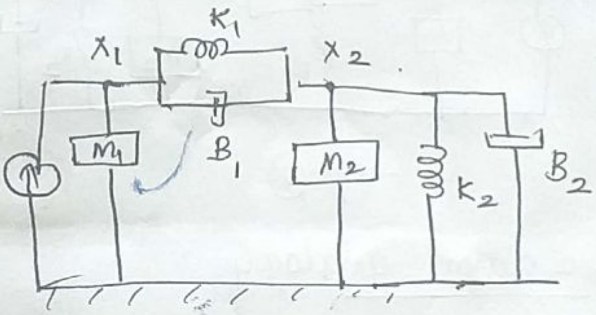
$$\boxed{\frac{X(s)}{F(s)} = \frac{B_2 s + K}{\left[Ms^2 + (B_1+B)s\right]\left[B_2 s + k\right] - (B_2 s)^2}} \Rightarrow \text{Transfer function.}$$

Conversion of Mechanical system to Electrical system.

Force-Voltage Analogy		Force-current Analogy	
Mechanical s/m	**Electrical s/m**	**Mechanical s/m**	**Electrical s/m**
Force F	→ voltage V	Force F	→ Current I
Mass M	→ Inductance L	Mass M	→ capacitance C
Damping const B	→ Resistance R	Damping const. B	→ 1/resistance $\frac{1}{R}$
Spring const K	→ 1/capacitance $\frac{1}{C}$	Spring const K	→ 1/inductance $\frac{1}{L}$
Displacement X	→ charge q	Displacement X	→ Flux ϕ
Velocity $u = \frac{dx}{dt}$	→ Current $I = \frac{dq}{dt}$	Velocity $u = \frac{dx}{dt}$	→ voltage $V = \frac{d\phi}{dt}$

Question. Find Transfer function and draw Force-Voltage & Force-current analogy of given system.





Two nodes.

$$F = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} + K_1(x_1 - x_2) \quad (1)$$

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_1 \frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) \quad (2)$$

Take LT

$$(1) \Rightarrow F(s) = M_1 s^2 X_1(s) + B_1 s(X_1(s) - X_2(s)) + K_1(X_1(s) - X_2(s))$$

$$= X_1(s)\left[M_1 s^2 + B_1 s + K_1\right] - X_2(s)\left[B_1 s + K_1\right] \quad (3)$$

$$(2) \Rightarrow 0 = M_2 s^2 X_2(s) + B_2 s X_2(s) + K_2 X_2(s) + B_1 s\left[X_2(s) - X_1(s)\right] + K_1(X_2(s) - X_1(s))$$

$$= X_2(s)\left[M_2 s^2 + B_2 s + K_2 + B_1 s + K_1\right] - X_1(s)\left[B_1 s + K_1\right] \quad (4)$$

$$X_2(s) = \frac{X_1(s)[B_1 s + K_1]}{M_2 s^2 + B_2 s + K_2 + B_1 s + K_1} \quad \text{--- (5)}$$

(5) in (3)

(3) $\Rightarrow$

$$F(s) = X_1(s)[M_1 s^2 + B_1 s + K_1] - \frac{X_1(s)[B_1 s + K_1]^2}{M_2 s^2 + B_2 s + K_2 + B_1 s + K_1}$$

$$= X_1(s)\left[\frac{[M_2 s^2 + (B_1 + B_2)s + K_1 + K_2][M_1 s^2 + B_1 s + K_1] - (B_1 s + K_1)^2}{M_2 s^2 + B_2 s + K_1 + K_2 + B_1 s}\right]$$

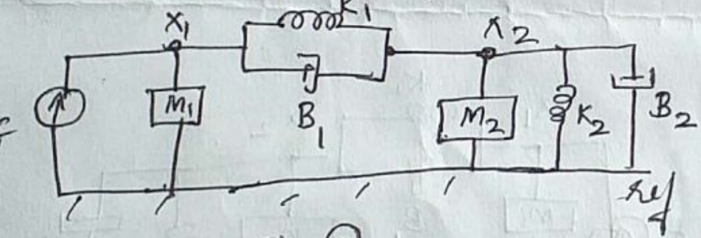
Transfer function,

$$\boxed{\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + (B_1 + B_2)s + K_1 + K_2}{(M_2 s^2 + (B_1 + B_2)s + K_1 + K_2)(M_1 s^2 + B_1 s + K_1) - (B_1 s + K_1)^2}}$$

Force - Voltage analogy

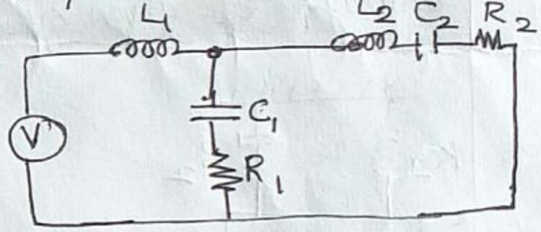
$F \to V$, $M \to L$, $B \to R$, $K \to \frac{1}{C}$, $x \to q$

series connection to parallel connection & parallel to series connection.



- Fig (a)

$\Rightarrow$



Force - Voltage analogy.

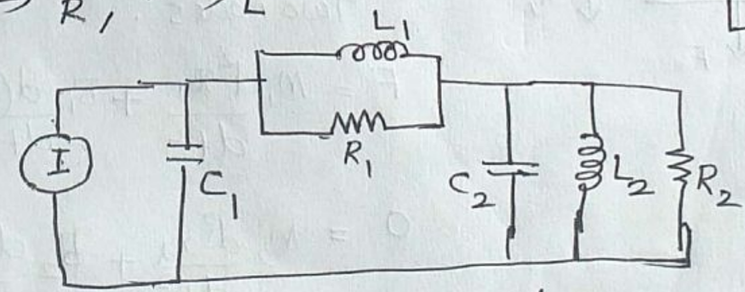
$F \to V$
$M \to L$
$B \to R$
$K \to C$

Force - current analogy

$F \to I$, $M \to C$, $B \to \frac{1}{R}$, $K \to \frac{1}{L}$

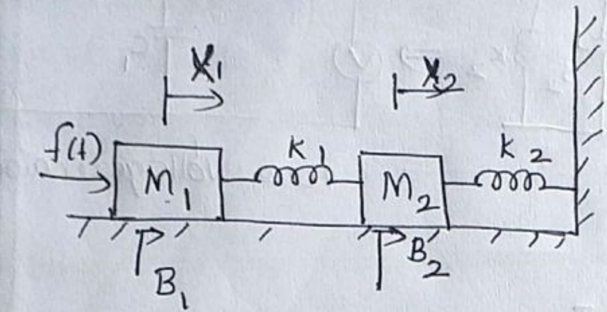
Fig (a) $\Rightarrow$

$F \to I$
$M \to C$
$B \to R$
$K \to L$



Force - current analogy.

Qn:- For the mechanical system shown in figure, Derive transfer function. Also draw force-voltage & Force-voltage analogous ckt.



step 1: Find nodes = no. of displacement = 2

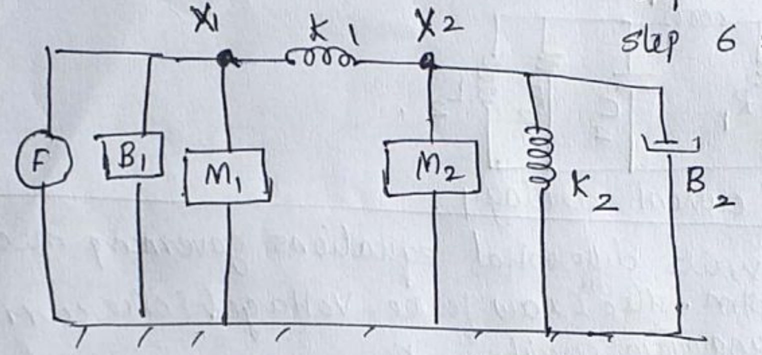
step 2: Connect mass to ref. connect B & K

step 3: Connect force F

step 4: Write diff. equations

step 5: Take Laplace transform

step 6: Find Transfer fn.



Differential equations are

consider node 1: $F = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1(x_1 - x_2)$ —①

node 2: $0 = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + k_2 x_2 + k_1(x_2 - x_1)$ —②.

LT

eqn ① $\Rightarrow F(s) = M_1 s^2 X_1(s) + B_1 s X_1(s) + K_1 X_1(s) - K_1 X_2(s)$

$= X_1(s)\left[M_1 s^2 + B_1 s + K_1\right] - K_1 X_2(s)$. —③

② $\Rightarrow 0 = M_2 s^2 X_2(s) + B_2 s X_2(s) + K_2 X_2(s) + K_1(X_2(s) - X_1(s))$

$= X_2(s)\left[M_2 s^2 + B_2 s + K_2 + K_1\right] - X_1(s) K_1$

$X_2(s) = X_1(s) \frac{K_1}{M_2 s^2 + B_2 s + K_1 + K_2}$ —④

④ in ③ $\Rightarrow$

$F(s) = X_1(s)\left[M_1 s^2 + B_1 s + K_1\right] - \frac{X_1(s) K_1^2}{M_2 s^2 + B_2 s + K_1 + K_2}$

$F(s) = X_1(s)\left[\frac{(M_2 s^2 + B_2 s + K_1 + K_2)(M_1 s^2 + B_1 s + K_1) - K_1^2}{M_2 s^2 + B_2 s + K_1 + K_2}\right]$

$$\boxed{\text{Transfer fn } \frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + B_2 s + K_1 + K_2}{(M_2 s^2 + B_2 s + K_1 + K_2)(M_1 s^2 + B_1 s + K_1) - K_1^2}}$$

Force to voltage analogy

$M \to L,\ B \to R,\ K \to C,\ F \to V$

series connections to $\parallel^{l}$ & $\parallel^{l}$ to series

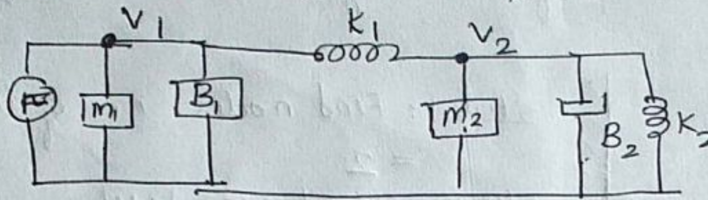
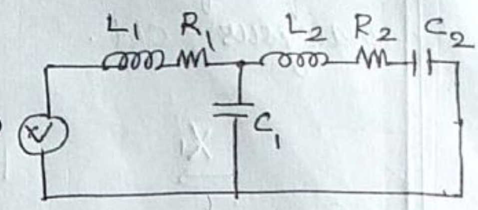


Fig (a)

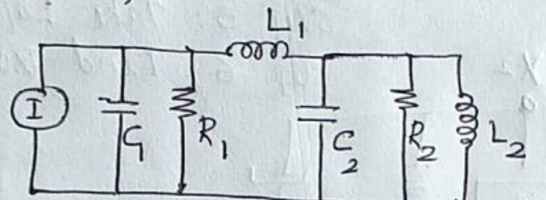


Force - Voltage analogy

Force to current analogy

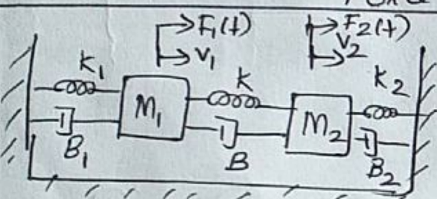
$M \to C,\ B \to R,\ K \to L,\ F \to I$

Fig (a) $\Rightarrow$

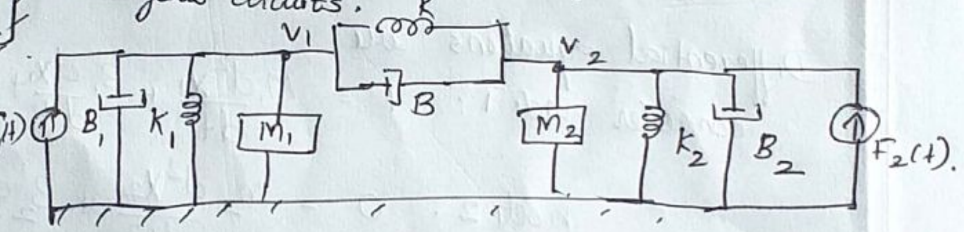


Force - current analogy.

Q:-



Write differential equations governing mechanical s/m. Also Draw force-Voltage & Force current analogous circuits.



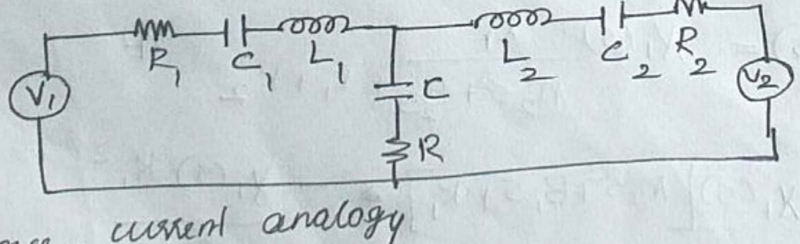
Differential equations

$$f_1(t) = M_1 \frac{dv_1}{dt} + B_1 v_1 + K_1 \int v_1\,dt + k\int (v_1 - v_2)\,dt + B(v_1 - v_2)$$

$$f_2(t) = M_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2\,dt + k\int (v_2 - v_1)\,dt + B(v_2 - v_1)$$

Force - Voltage analogy :- series to $\parallel^{l}$ & $\parallel^{l}$ to series

$M \to L,\ B \to R,\ K \to C,\ F \to V$



Force - current analogy

$M \to C,\ B \to R,\ K \to L,\ F \to I$