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Module VI

Discrete control system fundamentals: overview of z-transform - State space representation for Discrete time systems Data sampled control systems, sampling theorem, sample & hold, Open & closed sampled data systems
State space analysis: Solving discrete time state space equations pulse transform domain Discretisation of continous state space equations, stability analyses of discrete time systems - Jury's test

Overview of z-transform

Let $f(k)$ — Discrete time signal / sequence

$$\boxed{F(z) = \sum_{k=\infty}^{\infty} f(k) z^{-k}}$$

signal $f(k)$	z transform $F(z)$
$\delta(k)$	1
$u(k)$	$\frac{z}{z-1}$
a^k	$\frac{z}{z-a}$
$k \cdot a^k$	$\frac{az}{(z-a)^2}$
$k^2 a^k$	$\frac{az(z+a)}{(z-a)^3}$
$(k+1) a^k$	$z^2 / (z-a)^2$
$\frac{(k+1)(k+2)}{2!} a^k$	$\frac{z^3}{(z-a)^3}$
$\frac{a^k}{k!}$	$e^{az^{-1}}$
e^{at}	$\frac{z}{z-e^{at}}$
e^{-at}	$\frac{z}{z-e^{-at}}$
$\sin \omega kT$	$(z \sin \omega T) / z^2 - 2z \cos \omega T + 1$
$\cos \omega kT$	$\frac{z(z-\cos\omega T)}{z^2 - 2z\cos\omega T + 1}$

$$kTe^{-akT} \rightarrow \frac{zTe^{-aT}}{(z-e^{-aT})^2}$$

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Stability analysis of discrete time systems - Jury's stability test.

Jury's stability test is used to determine whether the roots of the characteristic polynomial lie within a unit circle or not.

Let $F(z)$ be n^{th} order chara. polynomial

$$F(z) = a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0 = 0$$

The necessary conditions to be satisfied for the stability of the system with chara. poly, $F(z)$ are

$$F(1) > 0 \ \& \ (-1)^n F(-1) > 0$$

The sufficient condition for stability can be established by the method,

ROW	z^0	z^1	z^2	$\dots$	z^n
1	a_0	a_1	a_2		a_n
2	a_n	a_{n-1}	a_{n-2}	$\dots$	a_0
3	b_0	b_1	$\dots$		b_{n-1}
4	b_{n-1}	b_{n-2}			b_0
5	c_0	c_1	$\dots$	c_{n-2}	
6	c_{n-2}	c_{n-3}	$\dots$	c_0	
$\vdots$					

$$b_1 = \begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix}$$

$$\boxed{b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}}$$

$$b_0 = \begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix}$$

$$c_0 = \begin{vmatrix} b_0 & b_{n-1} \\ b_{n-1} & b_0 \end{vmatrix}$$

$$c_1 = \begin{vmatrix} b_0 & b_{n-2} \\ b_{n-1} & b_1 \end{vmatrix}$$

$$c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & k \end{vmatrix}$$

* The First column elements of the table are used to check the following sufficient condition for stability of the s/m.

$|a_0| < |a_n|$

$|b_0| > |b_{n-1}|$

$|c_0| > |c_{n-2}|$

$|d_0| > |d_{n-3}|$

$\vdots$

$|r_0| > |r_2|$

If the necessary & sufficient conditions are satisfied, then all the poles of the s/m lies inside the unit circle in z-plane
$\therefore$ s/m is stable.
$\Rightarrow$ If even one condition is not satisfied, s/m is unstable

① check for stability of the sampled data control systems represented by the following characteristic equation.

$2z^4+8z^3+12z^2+5z+1=0$.

$F(z)=2z^4+8z^3+12z^2+5z+1$

$F(1)=2\times1^4+8\times1^3+12\times1^2+5\times1+1$

$=2+8+12+5+1$

necessary condn $=\underline{\underline{28>0}}$.

$(-1)^n F(-1) = (-1)^4\left[2\times(-1)^4+8\times(-1)^3+12\times(-1)^2+5\times-1+1\right]$

$=2-8+12-5+1$

$=\underline{\underline{2>0}}$

Here $n=4$

No. of rows $=2n-3=2\times4-3=\underline{\underline{5}}$

Rows	z^0	z^1	z^2	z^3	z^4
1	1	5	12	8	2
2	2	8	12	5	1
3	-3	-11	-12	-2	
4	-2	-12	-11	-3	
5	5	9	14		

Rows	z^0	z^1	z^2	z^3	z^4
1	a_0	a_1	a_2	a_3	a_4
2	a_4	a_3	a_2	a_1	a_0
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2		

$b_k=\begin{vmatrix}a_0 & a_{n-k}\\ a_n & a_k\end{vmatrix}$ $\quad b_0=\begin{vmatrix}a_0 & a_4\\ a_4 & a_0\end{vmatrix}=\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}$

$=\underline{\underline{-3}}$

$b_1=\begin{vmatrix}a_0 & a_3\\ a_4 & a_1\end{vmatrix}=\begin{vmatrix}1 & 8\\ 2 & 5\end{vmatrix}=5-16=\underline{\underline{-11}}$

$b_2=\begin{vmatrix}1 & 12\\ 2 & 12\end{vmatrix}=12-24=\underline{\underline{-12}}$

$b_3=\begin{vmatrix}1 & 5\\ 2 & 8\end{vmatrix}=8-10=\underline{\underline{-2}}$

$c_k=\begin{vmatrix}b_0 & b_{n-1-k}\\ b_{n-1} & b_k\end{vmatrix}$, $\quad c_0=\begin{vmatrix}b_0 & b_{n-1}\\ b_{n-1} & b_k\end{vmatrix}=\begin{vmatrix}-3 & -2\\ -2 & -3\end{vmatrix}$

$=\underline{9-4=5}$

$c_1=\begin{vmatrix}-3 & -12\\ -2 & -11\end{vmatrix}=33-24=9$

$c_2=\begin{vmatrix}-3 & -11\\ -2 & -12\end{vmatrix}=36-22=\underline{14}$

sufficient condition

$|a_0|<|a_n|$

$1<2\rightarrow$ satisfied.

$|b_0|>|b_{n-1}|$

$|-3|>|-2|$

$3>2\rightarrow$ satisfied

$|c_0|>|c_{n-2}|$

$|5|>|14|\rightarrow$ not satisfied

∴ The required necessary & sufficient conditions are not satisfied

∴ s/m is unstable.

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② $F(z) = z^4 - 1.7z^3 + 1.04z^2 - .268z + .024 = 0$

necessary condn

$F(1) = 1 - 1.7 + 1.04 - .268 + .024$

$= .096 > 0$

$(-1)^n F(-1) = 1 + 1.7 + 1.04 + .268 + .024 = 4.032 > 0$.

∴ satisfied

for sufficient condition   Here $n = 4$

∴ no. of rows $(2n-3) = 5$

Row	z^0	z^1	z^2	z^3	z^4
1	.024	−.268	1.04	−1.7	1
2	1	−1.7	1.04	−.268	.024
3	−.9994	1.693	−1.015	.2272	
4	.2272	−1.015	1.693	−.994	
5	.9472	−1.462	.6296		

	z^0	z^1	z^2	z^3	z^4
1	a_0	a_1	a_2	a_3	a_4
2	a_4	a_3	a_2	a_1	a_0
3	b_0	b_1	b_2	b_3	
4	b_3	b_2	b_1	b_0	
5	c_0	c_1	c_2		

$b_0 = \begin{vmatrix} .024 & 1 \\ 1 & .024 \end{vmatrix} = -.9994$

$b_1 = \begin{vmatrix} .024 & -1.7 \\ 1 & -.268 \end{vmatrix} = 1.6936$

$|a_0| < |a_n| = |a_0| < |a_4|$

$= |.024| < |1| \rightarrow$ satisfied

$|b_0| > |b_{n-1}| = |b_0| > |b_3|$

$= |-.9994| > |.2272| \rightarrow$ satisfied

$|c_0| > |c_{n-1}| = |.9472| > |.6296| \rightarrow$ satisfied

The necessary & sufficient conditions of stability are satisfied.
Hence the s/m is stable

U.Q
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③ $Q(z) = z^3 - 1.8z^2 + 1.05z - .20 = 0$

nessary condn

$F(1) = 1 - 1.8 + 1.05 - .20 = .5 > 0$

$(-1)^n F(-1) = (-1)^3 [-1 - 1.8 - 1.05 - .20] = 4.05 > 0$

sufficient condn.

$(2 \times 3 - 3) = 3$

Row	z^0	z^1	z^2	z^3
1	−.2	1.05	−1.8	1
2	1	−1.8	1.05	−.2
3	−.96	1.59	~~1.8~~ −.69	

$b_0 = \begin{vmatrix} -.2 & 1 \\ 1 & -.2 \end{vmatrix}$

$= .04 - 1 = -.96$

$b_1 = \begin{vmatrix} -.2 & -1.8 \\ 1 & 1.05 \end{vmatrix}$

$= -.21 + 1.8$

$= 1.59$

$|a_0| < |a_3|$

$|.2| < |1| \rightarrow$ satisfied

$b_2 = \begin{vmatrix} -.2 & 1.05 \\ 1 & -1.8 \end{vmatrix}$

$= 1.8$

$|b_0| > |b_2|$

$|-.96| > |-.69| \rightarrow$ satisfied

∴ s/m is stable

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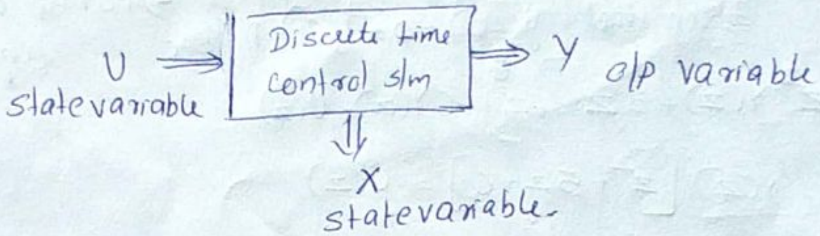
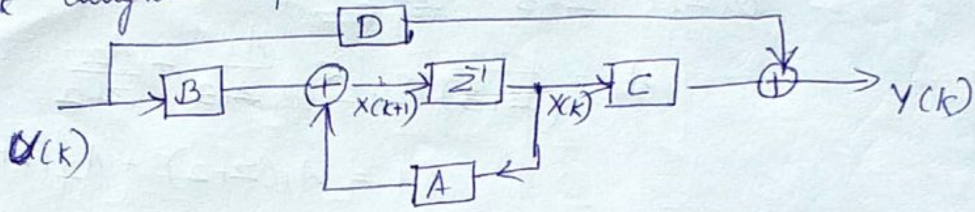
State space representation for Discrete time systems



The state equation $\Rightarrow X(k+1) = A\,X(k) + B\,U(k)$

o/p equation $= C\,X(k) + D\,U(k)$

Block diagram representation.



solution of state equation

a) Homogeneous $X(k+1) = A\,X(k) + B\,U(k)$

$$zX(z) - zX(0) = AX(z) + BU(z) \qquad U(z) = 0$$

$$\therefore X(z)\left[zI - A\right] = zX(0)$$

$$\boxed{X(z) = \underbrace{(zI - A)^{-1}z}_{\text{state transition Matrix } (\phi(z))}X(0)}$$

$$\boxed{\therefore X(k) = \phi(k)\,X(0).}$$

non homogeneous

$$X(k+1) = A\,X(k) + B\,U(k)$$

$$zX(z) - z(X(0)) = AX(z) + B\,U(z)$$

$$X(z)\left[zI - A\right] = zX(0) + BU(z)$$

$$X(z) = (zI - A)^{-1}zX(0) + (zI - A)^{-1}BU(z)$$

$$\therefore \cancel{X(k) = \phi(k)X(0) + (zI - A)^{-1}BU(z)}$$

$$X(k) = z^{-1}\left\{(zI - A)^{-1}z\right\}X(0) + z^{-1}\left\{(zI - A)^{-1}BU(z)\right\}$$

state transition Mx $A^k = z^{-1}\left\{(zI - A)^{-1}z\right\}$

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① Solve the difference equation, a) Determine state model, b) State transition matrix

$Y(k+2) + 5Y(k+1) + 6Y(k) = U(k)$

Given that $y(0) = 0 = y(1)$, $T = 1\,sec$

c) For i/p $u(k) = 1;\ k \geq 1$, find o/p $Y(k)$

$$z^2 Y(z) + 5z Y(z) + 6Y(z) = U(z)$$

$$Y(z)\left[z^2 + 5z + 6\right] = U(z)$$

$$\frac{Y(z)}{U(z)} = \frac{1}{z^2+5z+6} = \frac{1}{(z+3)(z+2)}$$

By partial fraction expansion

$$\frac{Y(z)}{U(z)} = \frac{A_1}{z+3} + \frac{A_2}{z+2}$$

$$\frac{1}{(z+3)(z+2)} = \frac{A_1}{z+3} + \frac{A_2}{z+2}$$

$$1 = A_1(z+2) + A_2(z+3)$$

$$A_1\Big|_{(z=-3)} = -1 \qquad A_2\Big|_{z=-2} = 1$$

$$\therefore \frac{Y(z)}{U(z)} = \frac{-1}{z+3} + \frac{1}{z+2}.$$

$$Y(z) = \underbrace{\frac{-1\ U(z)}{z+3}}_{X_1(z)} + \underbrace{\frac{U(z)}{z+2}}_{X_2(z)} \Rightarrow Y(z) = X_1 + X_2.$$

$$X_1(z) = \frac{-1\ U(z)}{z+3} \qquad X_2(z) = \frac{U(z)}{z+2}$$

$$(z+3)X_1(z) = -U(z) \qquad (z+2)X_2(z) = U(z)$$

$$X_1(k+1) = -3X_1 - U \qquad X_2(k+1) = -2X_2 + U$$

$$\begin{bmatrix} X_1(k+1) \\ X_2(k+1) \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

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b) To find state transition matrix A^k

$$A^k = Z^{-1}\{(zI-A)^{-1}z\}$$

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$zI - A = z\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} z+3 & 0 \\ 0 & z+2 \end{bmatrix}$$

$$(zI-A)^{-1} = \frac{\begin{bmatrix} z+2 & 0 \\ 0 & z+3 \end{bmatrix}}{(z+2)(z+3)} = \begin{bmatrix} \frac{1}{z+3} & 0 \\ 0 & \frac{1}{z+2} \end{bmatrix}$$

$$A^k = Z^{-1}\{(zI-A)^{-1}z\} = Z^{-1}\begin{bmatrix} \frac{z}{z+3} & 0 \\ 0 & \frac{z}{z+2} \end{bmatrix}$$

$$= \begin{bmatrix} (-3)^k & 0 \\ 0 & (-2)^k \end{bmatrix}$$

$$\boxed{\therefore a^k \xrightarrow{Z} \frac{z}{z-a}}$$

c) To find $y(k)$ when the i/p is unit step.

Consider state equation

$$X(k+1) = AX(k) + BU(k).$$

taking z-transform

$$zX(z) - zX(0) = AX(z) + BU(z)$$

Assume $X(0) = 0$

$$\therefore zX(z) - AX(z) = BU(z)$$

$$\boxed{X(z) = (zI-A)^{-1}BU(z).}$$

given $u(k) = 1$ $\therefore$ $U(z) = Z(u(k)) = \frac{z}{z-1}$

$$(zI-A)^{-1} = \begin{bmatrix} \frac{1}{z+3} & 0 \\ 0 & \frac{1}{z+2} \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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$$\therefore X(z) = \begin{bmatrix} \frac{1}{z+3} & 0 \\ 0 & \frac{1}{z+2} \end{bmatrix} \begin{bmatrix} -1 \\ +1 \end{bmatrix} \frac{z}{z-1} = \begin{bmatrix} \frac{-1}{z+3} \\ \frac{+1}{z+2} \end{bmatrix} \left(\frac{z}{z-1} \right)$$

$$\therefore X(z) = \begin{bmatrix} \frac{-z}{(z-1)(z+3)} \\ \frac{+z}{(z-1)(z+2)} \end{bmatrix}$$

$$X(k) = z^{-1}\{X(z)\} = z^{-1} \begin{bmatrix} \frac{-z}{(z-1)(z+3)} \\ \frac{+z}{(z-1)(z+2)} \end{bmatrix}$$

1st term

$$\frac{1}{(z-1)(z+3)} = \left(\frac{1}{(z-1)(z+3)} \right)$$

$$= \left[\frac{A}{z-1} + \frac{B}{z+3} \right]$$

$$\frac{1}{(z-1)(z+3)} = \left(\frac{A}{z-1} + \frac{B}{z+3} \right)$$

$$1 = A(z+3) + B(z-1)$$

$$A = \frac{1}{4} \quad B = \frac{-1}{4}$$

$$\therefore \left(\frac{1}{4}\frac{z}{z-1} - \frac{1}{4}\frac{z}{z+3} \right)$$

$$x_1(k) = z^{-1}\left(\frac{1}{4}\frac{z}{z-1} - \frac{1}{4}\frac{z}{z+3} \right)$$

$$= \frac{1}{4}u(k) - \frac{1}{4}(-3)^k$$

2nd term

$$\frac{1}{(z-1)(z+2)} = \frac{A}{(z-1)} + \frac{B}{(z+2)}$$

$$1 = A(z+2) + B(z-1)$$

$$A = \frac{1}{3}, \quad B = \frac{-1}{3}$$

$$x_2(k) = z^{-1}\left\{ \frac{z}{(z-1)(z+2)} \right\}$$

$$= z^{-1}\left\{ \frac{1}{3}\frac{z}{z-1} - \frac{1}{3}\frac{z}{z+2} \right\}$$

$$= \frac{1}{3}u(k) - \frac{1}{3}(-2)^k$$

Response / o/p $y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -\left(\frac{1}{4}u(k) - \frac{1}{4}(-3)^k\right) \\ + \left[\frac{1}{3}u(k) - \frac{1}{3}(-2)^k\right] \end{bmatrix}$$

$$= \frac{-1}{4}u(k) + \frac{1}{4}(-3)^k + \left(\frac{1}{3}u(k) - \frac{1}{3}(-2)^k \right)$$

$$= +\frac{1}{4}(-3)^k - \frac{1}{3}(-2)^k + \frac{1}{12}u(k)$$

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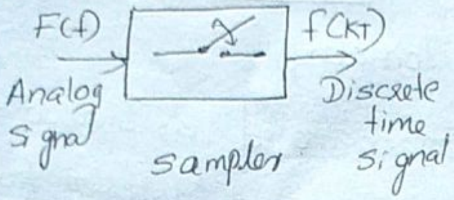
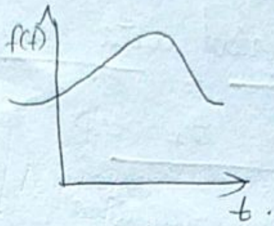
Sampling Process

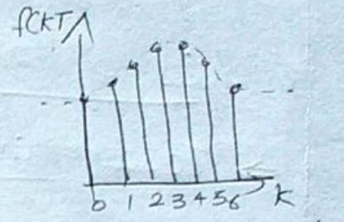
→ Sampling is the conversion of continuous-time signal into discrete time signal obtained by taking samples of continuous time signal at discrete time instants.



sampler



Analog s/l



Discrete signal.

Sampling theorem

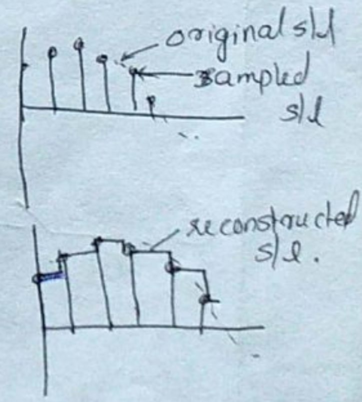
A band limited continuous time signal with highest freq. f_m hertz, can be uniquely recovered from its samples provided that the sampling rate F_s is greater than or equal to $2f_m$ samples/second.

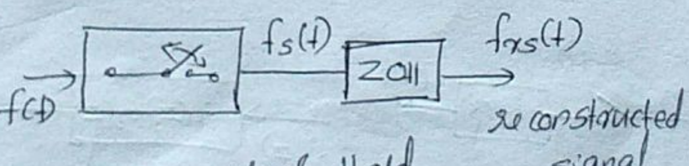
Sampler & Hold

In ADC process the hold circuit is used to hold the sample ~~unit~~ until the quantization and coding for the current sample is complete.

→ The simplest hold circuit is zero order hold (ZOH).

→ In ZOH circuits the signal is reconstructed such that the value of reconstructed signal for a sampling period is same as the value of last received sample.

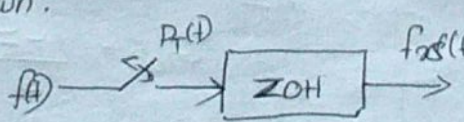




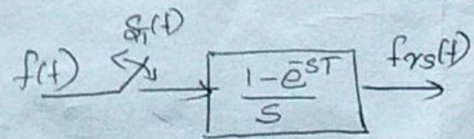
Sampler & Hold

Analysis of sampler & zero-order Hold

⇒ consider a pulse sampler with zero-order hold (ZOH) as shown.



pulse sampler with ZOH



equivalent rep. of pulse sampler with ZOH.

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⇒ ie, the o/p of pulse sampler with ZOH can be produced by impulse sampled $f(t)$ when passed through a transfer function $\boxed{G_0(s) = \frac{1-e^{-sT}}{s}}$.

Analysis of sampled data control system using z-transform.

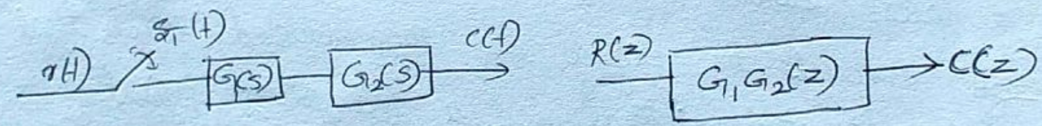
⇒ To determine the o/p in z-domain,

1. The pulse sampling is approximated as impulse sampling
2. The ZOH is replaced by a block with transfer function $G_0(s) = (1-e^{-sT})/s$.
3. When the input to a block is impulse sampled signal, then the z-transform of the o/p block can be obtained from the z-transform of the i/p and z-transform of the s-domain transfer function of the block.

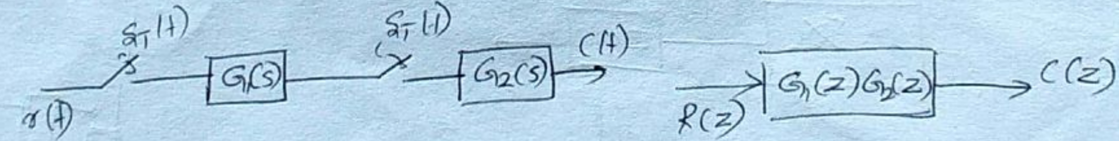
Open loop

① 

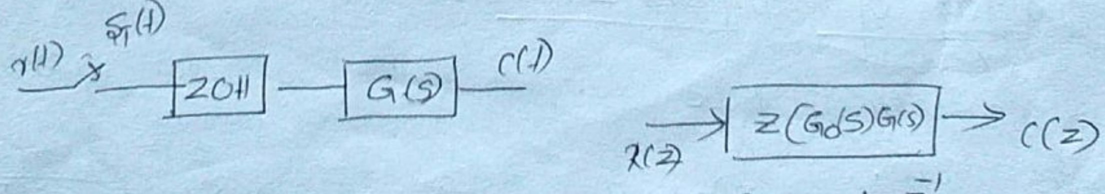
$$C(z) = G(z)R(z).$$

② 

$$C(z) = G_1G_2(z)R(z)$$

③ 

$$C(z) = G_1(z)\,G_2(z)\,R(z)$$

④ 

$$G_0 s = \frac{1-z^{-1}}{s}$$

$$\frac{1-e^{-sT}}{s} \Rightarrow \frac{1-z^{-1}}{s}. \quad \boxed{\begin{array}{l} e^{sT} = z \\ e^{-sT} = z^{-1} \end{array}}$$

$$C(z) = (1-z^{-1})\, Z\left(\frac{G(s)}{s}\right) R(z)$$

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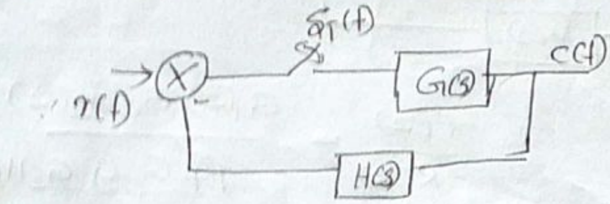
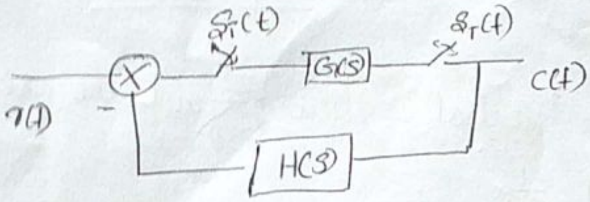
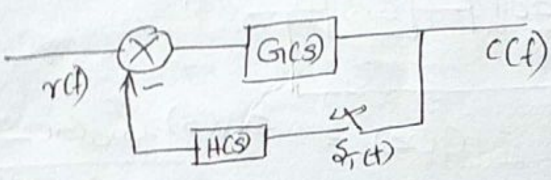
Closed loop sampled data control system → o/p in z-domain



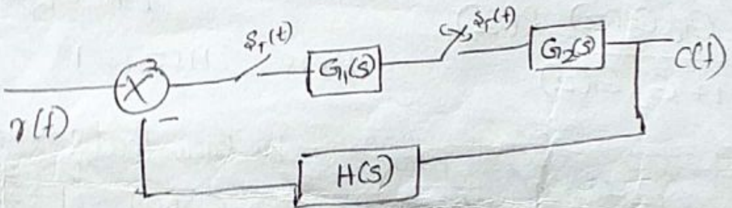
$$C(z) = \frac{G(z)\,R(z)}{1 + GH(z)}$$



$$C(z) = \frac{G(z)\,R(z)}{1 + G(z)\,H(z)}$$

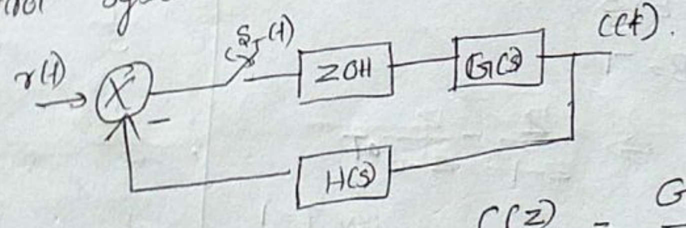


$$C(z) = \frac{G\,R(z)}{1 + GH(z)}$$



$$C(z) = \frac{G_1(z)\,G_2(z)\,R(z)}{1 + G_1(z)\,G_2H(z)}$$

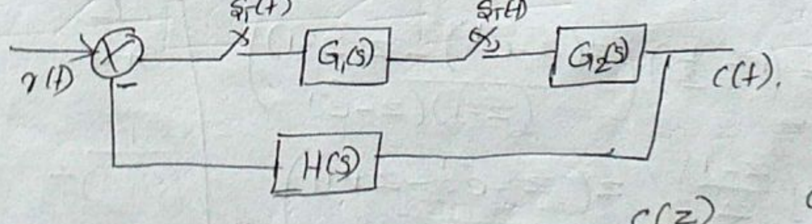
Qn:- 1) Find $C(z)/R(z)$ for the following CL sampled data control system. Assume all the samplers to be of impulse type.



Let ZOH → $G_0(s)$

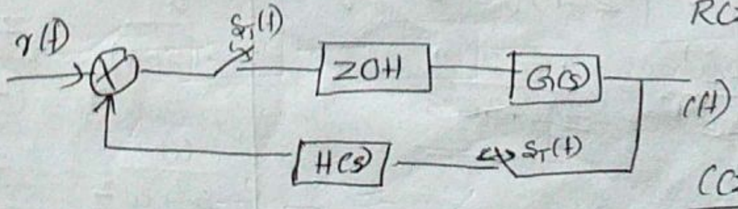
$$\frac{C(z)}{R(z)} = \frac{G_0G(z)}{1 + G_0GH(z)}$$

2)



$$\frac{C(z)}{R(z)} = \frac{G_1(z)\,G_2(z)}{1 + G_1(z)\,G_2H(z)}$$

3)



$$\frac{C(z)}{R(z)} = \frac{G_0G(z)}{1 + G_0G(z)\,H(z)}$$

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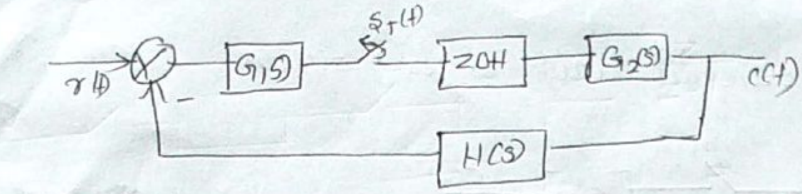
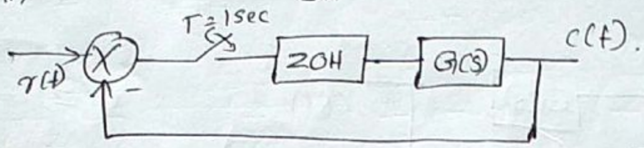
4)



$$\frac{C(z)}{R(z)} = \frac{G_1(z)\, G_0 G_2(z)}{1 + G_0 G_1 G_2 H(z)}$$

Qn: 2

For the sampled data control system, find the response to unit step i/p, where $G(s) = \frac{1}{s+1}$



$$C(z) = \frac{\cancel{G_0(s)}\, G_0 G(z)\, R(z)}{1 + G_0 G H(z)}$$

$$\therefore C(z) = \frac{G_0 G(z)\, R(z)}{1 + G_0 G(z)} \qquad \text{Here} \quad H(z) = 1$$

We know that,

$$G_0(s) = \frac{(1 - z^{-1})}{s}$$

$$G(s) = \frac{1}{s+1} \text{ given}$$

$$\therefore G_0 G(z) = (1 - z^{-1})\, Z\left(\frac{1}{s(s+1)}\right)$$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$1 = A(s+1) + B \times s$$

$$A = 1, \quad B = -1$$

$$Z\left(\frac{1}{s} - \frac{1}{s+1}\right) \Rightarrow \frac{z}{z-1} - \frac{z}{z - e^{-T}}$$

here $T = 1$

$$\therefore G_0 G(z) = (1 - z^{-1})\left(\frac{z}{z-1} - \frac{z}{z - e^{-1}}\right)$$

$$= \left(1 - \frac{1}{z}\right)\left(\frac{z(z - e^{-1}) - z(z-1)}{(z-1)(z - e^{-1})}\right)$$

$$= \left(\frac{z-1}{z}\right)\left(\frac{z(z - e^{-1} - z + 1)}{(z-1)(z - e^{-1})}\right)$$

$$= \frac{1 - e^{-1}}{z - e^{-1}} = \frac{.632}{z - .368}$$

$H(s)$	$H(z)$
$\frac{1}{s}$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$
$\frac{1}{s+a}$	$\frac{z}{z - e^{-at}}$
$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z - e^{-aT})^2}$
$\frac{a}{s(s+a)}$	$\frac{z(1 - e^{-aT})}{(z-1)(z - e^{-sT})}$
$\frac{\omega}{s^2 + \omega^2}$	$\frac{z \sin\omega T}{z^2 - 2z\cos\omega T + 1}$
$\frac{s}{s^2 + \omega^2}$	$\frac{z(z - \cos\omega T)}{z^2 - 2z\cos\omega T + 1}$

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Given that i/p is unit step $\therefore R(z) = \frac{z}{z-1}$

$$\therefore C(z) = \frac{\left(\frac{.632}{z-.368}\right)\left(\frac{z}{z-1}\right)}{1+\left(\frac{.632}{z-.368}\right)} = \frac{.632\,z}{(z-1)(z-.368+.632)}$$

$$= \frac{.632\,z}{(z-1)(z+.264)}$$

$$\therefore \frac{C(z)}{z} = \frac{.632}{(z-1)(z+.264)}$$

By partial fraction,

$$\frac{C(z)}{z} = \frac{A}{(z-1)} + \frac{B}{z+.264} = \frac{.632}{(z-1)(z+.264)}$$

$$A = .5, \; B = -.5$$

$$\frac{C(z)}{z} = \frac{.5}{z-1} - \frac{.5}{z+.264}$$

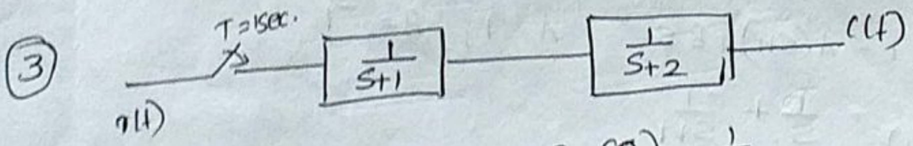
$$\boxed{\begin{aligned} Z(1) &= \frac{z}{z-1} \\ Z(a^k) &= \frac{z}{z-a} \end{aligned}}$$

$$C(z) = .5\,\frac{z}{z-1} - .5\,\frac{z}{z-(-.264)}$$

Taking Inverse z-transform,

$$c(k) = .5 - .5\,(-.264)^k \Rightarrow .5\left[1-(-.264)^k\right]$$

(3)



. Find $\frac{C(z)}{R(z)}$

$$G_1(s) = \frac{1}{S+1} \qquad G_2(s) = \frac{1}{S+2}$$

$$\frac{C(z)}{R(z)} = G_1G_2(z)$$

$$G_1G_2 = \frac{1}{(S+1)(S+2)}$$

By partial fraction,

$$\frac{1}{(S+1)(S+2)} = \frac{A}{S+1} + \frac{B}{S+2}$$

$$1 = A(S+2) + B(S+1) \; ; \quad A = 1, \quad B = -1$$

$$\frac{C(z)}{R(z)} = Z\left[\frac{1}{S+1} - \frac{1}{S+2}\right] = \left[\frac{z}{z-e^{-T}} - \frac{z}{z-e^{-2T}}\right] = \left[\frac{z}{z-e^{-1}} - \frac{z}{z-e^{-2}}\right]$$

$$\Rightarrow \boxed{\therefore \frac{C(z)}{R(z)} = \left[\frac{z}{z-.367} - \frac{z}{z-.135}\right]}$$

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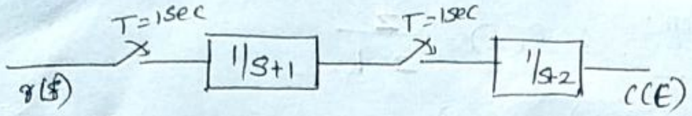
(4) 

Find $C(z)/R(z)$

$$\frac{C(z)}{R(z)} = G_1(z)G_2(z)$$

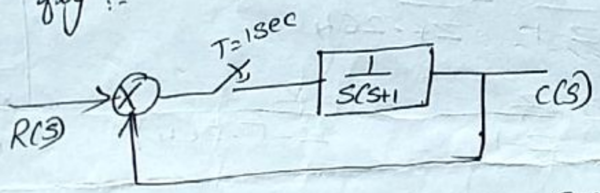
$$G_1(s) = \frac{1}{s+1} \qquad G_2(s) = \frac{1}{s+2}$$

$$G_1(z) = \frac{z}{z-e^{-T}} \qquad G_2(z) = \frac{z}{z-e^{-2T}}$$

$$\frac{C(z)}{R(z)} = \frac{z}{z-.367} * \frac{z}{z-.135}$$

$$\frac{C(z)}{R(z)} = \frac{z^2}{(z-.367)(z-.135)}$$

(5) Find the pulse transfer function for the error sampled s/m shown in fig :-



$$\frac{C(z)}{R(z)} = \frac{G(z)}{1+\cancel{G(z)H(z)}\ GH(z)} = \frac{G(z)}{1+G(z)}$$

$$\Rightarrow G(z) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$1 = A(s+1) + Bs \qquad A = 1, \quad B = -1$$

$$\frac{1}{s(s+1)} = \frac{1}{s} + \frac{-1}{s+1}$$

$$G(z) = \frac{z}{z-1} - \frac{z}{z-e^{-T}} \Rightarrow \frac{z}{z-1} - \frac{z}{z-.367}.$$

$$\therefore \frac{C(z)}{R(z)} = \frac{\frac{z}{z-1} - \frac{z}{z-.367}}{1 + \frac{z}{z-1} - \frac{z}{z-.367}} = \frac{z(z-.367) - z(z-1)}{(z-1)(z-.367) + z\left[z-.367 - (z-1)\right]}$$

$$= \frac{z(z-.367) - z(z-1)}{(z-1)(z-.367) + z\cdot 633}$$

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Find <u>z-domain transfer function</u> from s-domain transfer function

1. Determine $h(t)$ from $H(s)$, where $h(t) = L^{-1}(H(s))$
2. Determine the discrete sequence $h(kT)$ by replacing t by kT in $h(t)$.
3. Take z-transform of $h(kT)$, which is z-transfer function of the sm.

① $H(s) = \frac{a}{(s+a)^2}$.

1) $h(t) = L^{-1}[(H(s))] = L^{-1}\left(\frac{a}{(s+a)^2}\right] = a t e^{-at}$

2) substitute $t = kT$

$h(kT) = akT e^{-akT}$

3) z-transfer function $H(z) = Z(h(kT))$

$H(z) = Z(akT \cdot e^{-akT})$

$= \frac{a \cdot zT e^{-aT}}{(z - e^{-aT})^2}$

② $H(s) = \frac{s}{s^2+\omega^2}$

1) $h(t) = L^{-1}(H(s)) = L^{-1}\left(\frac{s}{s^2+\omega^2}\right) = \cos\omega t$.

2) put, $t = kT$ $= \cos\omega kT$

3) $H(z) = Z(\cos\omega kT) = \frac{z(z-\cos\omega T)}{z^2 - 2z\cos\omega T + 1}$

③ $H(s) = \frac{a}{s^2-a^2}$

1) $h(t) = L^{-1}(H(s))$

$= L^{-1}\left[\frac{a}{s^2-a^2}\right] = L^{-1}\left[\frac{a}{(s+a)(s-a)}\right]$

$\frac{a}{(s+a)(s-a)} = \frac{A_1}{s+a} + \frac{B_1}{s-a}$ $A_1 \rightarrow \frac{-1}{2}$

$B_1 \rightarrow \frac{1}{2}$

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$$h(t) = L^{-1}\left[\frac{-1}{2}\frac{1}{(s+a)} + \frac{1}{2(s-a)}\right]$$

$$= \frac{-1}{2}e^{-at} + \frac{1}{2}e^{at}$$

$$h(t) = \frac{e^{at} - e^{-at}}{2}$$

$$h(kT) = \frac{e^{+akT} - e^{akT}}{2} = \sinh akT$$

$$H(z) = \frac{z \sinh aT}{z^2 - 2z\cosh aT + 1}$$

$$\boxed{\sin\omega Tk \rightarrow \frac{z\sin\omega T}{z^2 - 2z\cos\omega T + 1}}$$

d) $H(s) = \dfrac{s+b}{(s+b)^2 + a^2}$ e) $H(s) = \dfrac{a}{(s+b)^2 + a^2}$

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