June 2015

Final Exam S2

Ex. 1 Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 7 \\ -3 & -7 & -14 \end{pmatrix}$. Determine A^{-1}

Ex. 2 Expand in partial fractions of $\mathbb{R}(X)$ the following rational fractions:

1/ $F(X) = \dfrac{X^3 + X - 1}{(X-2)(X+3)}$

2/ $G(X) = \dfrac{1}{X^2(X-1)}$

3/ $H(X) = \dfrac{X+3}{(X-1)(X^2+1)^2}$

Ex. 3 Let $E = \mathbb{R}_2[X]$, $\begin{array}{rl} f : E & \to E \\ P & \to XP' - P \end{array}$

Let $\mathcal{B} = (1, X, X^2)$ be the standard basis of E and let $\mathcal{B}' = (1, X+2, (X-1)^2)$ be a family of E.

1/ Show that f is linear

2/ Give a basis of $\ker(f)$. Deduce that $\dim(\ker(f)) = 1$

3/ What is the dimension of $\mathrm{Im}(f)$? Give a basis of $\mathrm{Im}(f)$

4/ Is f injective? surjective? bijective?

5/ Determine $\mathrm{Mat}_{\mathcal{B}}(f)$

6/ Show that $\mathcal{B}'$ is a basis of E

7/ Determine $\mathrm{Mat}_{\mathcal{B}'}(f)$

8/ Determine $\mathrm{Mat}_{\mathcal{B}'\mathcal{B}}(f)$ and $\mathrm{Mat}_{\mathcal{B}\mathcal{B}'}(f)$

<u>Ex. 4</u> Let $n \in \mathbb{N}^*$, E be a $\mathbb{R}$-vs of dimension n and let $f \in \mathcal{L}(E)$ such that $f \circ f = f$

1/ Show that $\ker(f) \cap \mathrm{Im}(f) = \{0_E\}$

2/ Using the rank theorem, show that

$$E = \ker(f) \oplus \mathrm{Im}(f)$$