Tutorial 8
Arithmetical Operations

Exercise 1: BCD Multiplication

Let us consider N, a 4-bit unsigned number: $N = DCBA$ (A is the least significant bit) and $N < 10$. We want to design a circuit that multiplies N by 2. The result should be encoded in a BCD form and so be made up of 2 digits: $H'G'F'E'$ for the tens column and $D'C'B'A'$ for the units column (E' and A' are the least significant bits).

Write down the truth tables and the most simplified expressions of the outputs.

Exercise 2: The Full Adder

Adding binary numbers (or numbers from any other bases) is very similar to adding decimal numbers. We add the digits of the same column to a possible carry from the previous column. We obtain a result and a possible carry that may be added to the next column.

A full adder is a circuit that adds two bits ($Ai + Bi$) and a possible carry (Ri_{-1}). It generates the sum (Si) and a possible carry (Ri).

We want to design a full adder.

1. Write down the truth tables of Si and Ri.
2. Write down the Karnaugh maps and the most simplified expressions of Si and Ri.
3. Draw the circuit diagram of the full adder.

Using four full adders, we want to design a 4-bit adder that performs a 4-bit addition of two numbers.

4. Draw the circuit diagram of the 4-bit adder.

Exercise 3: The Full Subtractor

A full subtractor is a circuit that subtracts two bits ($Ai - Bi$) and a possible borrow (Ri_{-1}). It generates the difference (Di) and a possible borrow (Ri).

We want to design a full subtractor.

1. Write down the truth tables of Di and Ri.
2. Write down the Karnaugh maps and the most simplified expressions of Di and Ri.
3. Draw the circuit diagram of the full subtractor.

Using four full subtractors, we want to design a 4-bit subtractor that performs a 4-bit subtraction of two numbers.

4. Draw the circuit diagram of the 4-bit subtractor.

Exercise 4: Adder–Subtractor

1. Design a circuit with one output (*S*) that inverts or not an input (*E*) according to the value of a control bit (*C*):
   - If $C = 0$, $S = E$
   - If $C = 1$, $S = \overline{E}$

2. Using the circuit above and a 4-bit adder (designed in a previous exercise), build an adder–subtractor that adds or subtracts two 4-bit numbers. This circuit has two 4-bit inputs (*A*, *B*), a control input (*C*), and a 4-bit output (*S*):
   - If $C = 0$, $S = A + B$
   - If $C = 1$, $S = A - B$