General Trees (Arbres généraux)

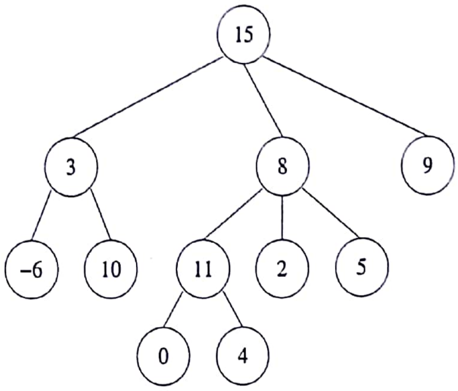


Figure 1: General Tree T_1

1 Measures

Exercise 1.1 (Size)

1. Give the definition of the size of a tree.
2. Write a function that returns the size of a tree, for both implementations:
   (a) by tuples (each node contains a tuples of children);
   (b) left child - right sibling (as a binary tree).

Exercise 1.2 (Height)

1. Give the definition of the height of a tree.
2. Write a function that returns the height of a tree, for both implementations:
   (a) by tuples;
   (b) left child - right sibling.

Exercise 1.3 (External Path Length)

1. Give the definition of external path length of a tree.
2. Write a function that returns the external path length of a tree, for both implementations:
   (a) by tuples;
   (b) left child - right sibling.

2 Traversals

Exercise 2.1 (Depth First Traversal)

1. Give the principle of a depth-first traversal for a general tree.
2. List elements in prefix and suffix orders for the depth-first traversal of the tree in figure 1. What other action can be done when visiting a node ?
3. Write a template depth-first traversal algorithm (insert node actions as comments) for both implementations:
   (a) by tuples;
   (b) left child - right sibling.

Exercise 2.2 (Breadth First Traversal)

1. Give the principle of a breadth-first traversal for a tree.
2. How can we detect level changes during the traversal?
3. Write a function that computes the width of a tree, for both implementations:
   (a) by tuples;
   (b) left child - right sibling.

3 Applications

Exercise 3.1 (Equality – *C3 - 2016*)

Write the function `same(`T`, `B`)` that tests whether T, a general tree in "classical" representation, and B, a general tree in *first child - right sibling* representation, are identical. That is, they contain same values in same nodes.

Exercise 3.2 (Average Arity of a General Tree – *C3 - 2016*)

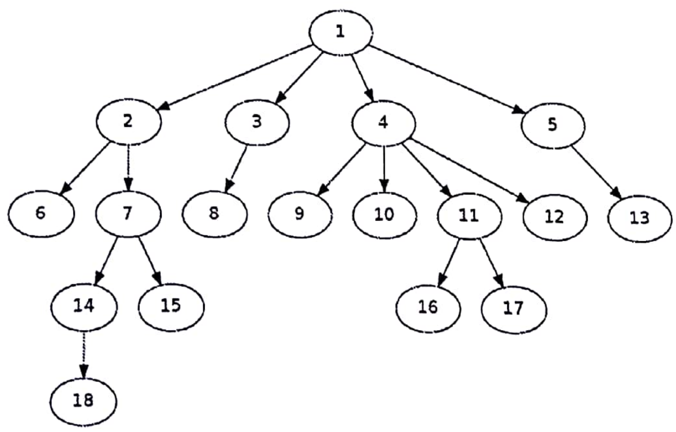


Figure 2: General tree

We now study the average arity (number of children for a node) in a general tree. We define the average arity as the sum of the number of children per node divided by the number of internal node (without leaves).

For example, in the tree from figure 2, there's 8 internal nodes and the sum of the number of children per node is 17 (check the arrows), thus the average arity is: $17/8 = 2,125$.

Write a function that returns the average arity of a general tree (only one traversal has to be done), for both implementations:

(a) by tuples; (b) left child - right sibling.

Exercise 3.3 (Serialization – *C3♯ - April 2017*)

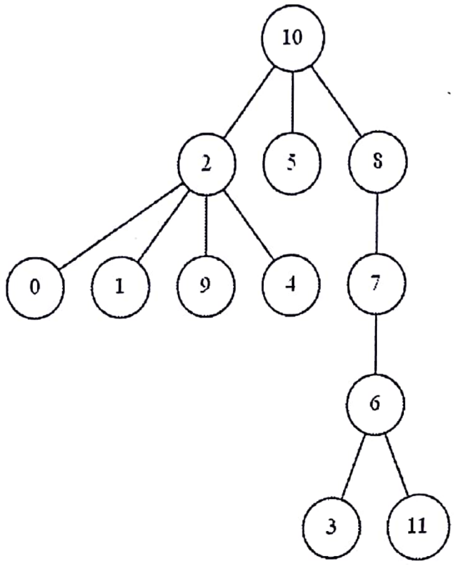


Figure 3: General Tree T_3

We shall now study an alternative representation for general trees: parent vectors. This implementation is linear and thus can be used to store trees in files (serialization.)

This representation is pretty simple. For each node of the tree, we associate a unique identifier: an integer in the range from 0 to the size of the tree - 1. We then build a vector where the cell i contains the identifier of the immediate parent for the node of identifier i. The parent of the root is -1.

1. Give the parent vector for the tree in figure 3.
2. Write a function which, from a tree, fills the corresponding parent vector (represented as a list in Python), for both implementations:
   (a) by tuples; (b) left child - right sibling.

Exercise 3.4 (Tuples ↔ left child - right sibling)

1. Write a function that builds, from a general tree with left child - right sibling implementation (*i.e.* a binary tree), its "by tuples" implementation.
2. Write the translation function for the other way.

Exercise 3.5 (List Representation)

Let A be the general tree $A = < o, A_1, A_2, ..., A_N >$. The following linear representation of A (o A_1 A_2 ... A_N) is called *list*.

1. (a) Give the linear representation of the tree in figure 1.
   (b) Draw the tree corresponding to the *list* `(12(2(25)(6)(-7))(0(18(1)(8))(9))(4(3)(11)))`.
2. Write the function that builds the linear representation (as a string) from a tree, for both implementations:
   (a) by tuples; (b) left child - right sibling.

   What has to be change to obtain an "abstract type" like representation ($A = < o, A_1, A_2, ..., A_N >$)?
3. Write the reciprocal function: that builds the tree (with both implementations) from the *list*.

Exercise 3.6 (dot format)

A tree can be represented as a list of links (like a graph): the *dot* format.

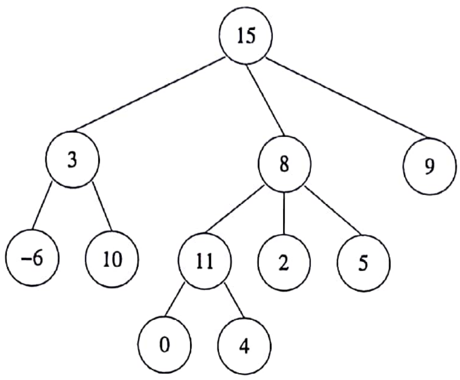
```
graph {
    15 -- 3;
    15 -- 8;
    15 -- 9;
    3 -- -6;
    3 -- 10;
    8 -- 11;
    8 -- 2;
    8 -- 5;
    11 -- 0;
    11 -- 4;
}
```

Another possibility:

```
graph {
    15 -- {3; 8; 9};
    3 -- {-6; 10};
    8 -- {11; 2; 5};
    11 -- {0; 4};
}
```

';' can be omitted.

If you want to see the graphical representation of your tree, use "Graphviz".
Warning: according to the order of links, the result will not be the same. **The order given here is the appropriate one for a tree.**



Write the functions that allow to:

- build the `.dot` file from a tree (for both implementations)
- and in return, build a tree (in both implementations) from a `.dot` file.