Key to Tutorial 2
Number Bases

Exercise 1

1. Convert the following numbers into their base-10 representations:

(1) $462_7 = 4\times7^2 + 6\times7 + 2 = 4\times49 + 42 + 2 = 196 + 44$ **= 240_{10}**
(2) $4BA_{12} = 4\times12^2 + 11\times12 + 10 = 4\times144 + 132 + 10 = 576 + 142 =$ **718_{10}**
(3) $11101101_2 = 128 + 64 + 32 + 8 + 4 + 1 =$ **237_{10}**
(4) $1022_3 = 3^3 + 2\times3 + 2 = 27 + 6 + 2 =$ **35_{10}**
(5) $377_8 = 3\times8^2 + 7\times8 + 7 = 3\times64 + 56 + 7 = 192 + 63 =$ **255_{10}**
(6) $BAC_{16} = 11\times16^2 + 10\times16 + 12 = 11\times256 + 160 + 12 = 2816 + 172 =$ **$2{,}988_{10}$**
(7) $12AD_{16} = 16^3 + 2\times16^2 + 10\times16 + 13 = 4{,}096 + 2\times256 + 160 + 13 = 4{,}269 + 512 =$ **$4{,}781_{10}$**

2. Convert the following base-10 numbers into the specified base. You should use the successive division method.

(1) 275 → base 2

275	/ 2 = 137	Remainder = **1**	
137	/ 2 = 68	Remainder = **1**	
68	/ 2 = 34	Remainder = **0**	
34	/ 2 = 17	Remainder = **0**	
17	/ 2 = 8	Remainder = **1**	
8	/ 2 = 4	Remainder = **0**	
4	/ 2 = 2	Remainder = **0**	
2	/ 2 = 1	Remainder = **0**	
1	/ 2 = 0	Remainder = **1**	**$275_{10} = 1\ 0001\ 0011_2$**

(2) 564 → base 2

564	/ 2 = 282	Remainder = **0**	
282	/ 2 = 141	Remainder = **0**	
141	/ 2 = 70	Remainder = **1**	
70	/ 2 = 35	Remainder = **0**	
35	/ 2 = 17	Remainder = **1**	
17	/ 2 = 8	Remainder = **1**	
8	/ 2 = 4	Remainder = **0**	
4	/ 2 = 2	Remainder = **0**	
2	/ 2 = 1	Remainder = **0**	
1	/ 2 = 0	Remainder = **1**	**$564_{10} = 10\ 0011\ 0100_2$**

(3) 687 → base 16

687	/ 16 = 42	Remainder = **15**
42	/ 16 = 2	Remainder = **10**
2	/ 16 = 0	Remainder = **2**

$\mathbf{687_{10} = 2AF_{16}}$

(4) 3,201 → base 16

3,201	/ 16 = 200	Remainder = **1**
200	/ 16 = 12	Remainder = **8**
12	/ 16 = 0	Remainder = **12**

$\mathbf{3{,}201_{10} = C81_{16}}$

(5) 4,321 → base 8

4,321	/ 8 = 540	Remainder = **1**
540	/ 8 = 67	Remainder = **4**
67	/ 8 = 8	Remainder = **3**
8	/ 8 = 1	Remainder = **0**
1	/ 8 = 0	Remainder = **1**

$\mathbf{4{,}321_{10} = 10341_8}$

Exercise 2

Quick conversion into a base-2^n representation:

(1) $AC7E_{16}$ → base 2
$\mathbf{AC7E_{16} = 1010\ 1100\ 0111\ 1110_2}$

(2) BCD_{16} → base 2
$\mathbf{BCD_{16} = 1011\ 1100\ 1101_2}$

(3) 1234_{16} → base 2
$\mathbf{1234_{16} = 0001\ 0010\ 0011\ 0100_2}$

(4) 5567_8 → base 2
$\mathbf{5567_8 = 101\ 101\ 110\ 111_2}$

(5) $ABCD_{16}$ → base 8 *(via base 2)*
$ABCD_{16} = 1010\ 1011\ 1100\ 1101_2$
$ABCD_{16} = 1\ 010\ 101\ 111\ 001\ 101_2$
$\mathbf{ABCD_{16} = 125715_8}$

(6) $2074_8 \rightarrow$ base 16 *(via base 2)*
$2074_8 = 010\ 000\ 111\ 100_2$
$2074_8 = 0100\ 0011\ 1100_2$
$\mathbf{2074_8 = 43C_{16}}$

(7) $1111100100101010_2 \rightarrow$ base 16
$\mathbf{1111\ 1001\ 0010\ 1010_2 = F92A_{16}}$

(8) $1110101100101010_2 \rightarrow$ base 8
$\mathbf{001\ 110\ 101\ 100\ 101\ 010_2 = 165452_8}$

Exercise 3

1. Work out the value of the base (b) so that the following identities are true:

(1) $132_b = 30_{10}$ **b > 3**

$b^2 + 3b + 2 = 30$
$b^2 + 3b - 28 = 0$

$\Delta = 3^2 - 4\times(-28) = 9 + 112 = 121$
$\cancel{b1} = (-3 - 11) / 2 = -14/2 = -7$
$b2 = (-3 + 11) / 2 = 8 / 2 = 4$

b = 4

(2) $2A_{16} = 36_b$ **b > 6**

$2\times16 + 10 = 3b + 6$
$3b = 32 + 10 - 6$
$3b = 36$

b = 12

(3) $22_b \times 21_b = 502_b$ **b > 5**

$(2b + 2)(2b + 1) = 5b^2 + 2$
$4b^2 + 2b + 4b + 2 = 5b^2 + 2$
$b^2 - 6b = 0$
$b(b - 6) = 0$

$\cancel{b1} = 0$
$b2 = 6$

b = 6

2. Work out the smallest values of the bases (a and b) so that the following identities are true:

   (1) $101_a = 401_b$ **$a > 1$ and $b > 4$**

   $a^2 + 1 = 4b^2 + 1$
   $a^2 = 4b^2$
   $a = 2b$

   $\boldsymbol{b_{min} = 5}$
   $\boldsymbol{a_{min} = 10}$

   (2) $501_a = 50001_b$ **$a > 5$ and $b > 5$**

   $5a^2 + 1 = 5b^4 + 1$
   $a^2 = b^4$
   $a = b^2$

   $\boldsymbol{b_{min} = 6}$
   $\boldsymbol{a_{min} = 36}$

   (3) $12_a = 1002_b$ **$a > 2$ and $b > 2$**

   $a + 2 = b^3 + 2$
   $a = b^3$

   $\boldsymbol{b_{min} = 3}$
   $\boldsymbol{a_{min} = 27}$

Exercise 4

1. How can an even number be identified in an even base?

   Let us consider an even base b and a number of this base: $\sum_{i=0}^{n} a_i b^i$

   $b \equiv 0[2]$
   $b^i \equiv 0[2]$ **$i > 0$**
   $a_i b^i \equiv 0[2]$
   $\sum_{i=1}^{n} a_i b^i \equiv 0[2]$
   $a_0 + \sum_{i=1}^{n} a_i b^i \equiv a_0[2]$
   $\sum_{i=0}^{n} a_i b^i \equiv a_0[2]$

   Therefore, a number of an even base is even if and only if its rightmost digit (a_0) is even.

2. How can an even number be identified in an odd base?

   Let us consider an odd base b and a number of this base: $\sum_{i=0}^{n} a_i b^i$

   $b \equiv 1[2]$

   $b^i \equiv 1[2]$ $\quad\quad$ $\boldsymbol{i \geq 0}$

   $a_i b^i \equiv a_i [2]$

   $\sum_{i=0}^{n} a_i b^i \equiv \sum_{i=0}^{n} a_i [2]$

   Therefore, a number of an odd base is even if and only if the sum of its digit ($\sum_{i=0}^{n} a_i$) is even.

Exercise 5

1. Convert the following numbers into their base-10 representations:

   (1) 1101.011_2
   $= 2^3 + 2^2 + 1 + 2^{-2} + 2^{-3}$
   $= 8 + 4 + 1 + 0.25 + 0.125$
   $\mathbf{= 13.375_{10}}$

   (2) 123.42_8
   $= 8^2 + 2\times8 + 3 + 4\times8^{-1} + 2\times8^{-2}$
   $= 64 + 16 + 3 + 2^{-1} + 2^{-5}$
   $= 64 + 16 + 3 + 0.5 + 0.03125$
   $\mathbf{= 83.53125_{10}}$

   (3) $BAC.028_{16}$
   $= 11\times16^2 + 10\times16 + 12 + 2\times16^{-2} + 8\times16^{-3}$
   $= 11\times256 + 160 + 12 + 2^{-7} + 2^{-9}$
   $= 2{,}816 + 172 + 0.0078125 + 0.001953125$
   $\mathbf{= 2{,}988.009765625_{10}}$

2. Convert the following numbers into the specified base:

   (1) $164.76_{10} \rightarrow$ base 8 (3 digits after the point)

   164 / 8 = 20 Remainder = **4**
   20 / 8 = 2 Remainder = **4**
   2 / 8 = 0 Remainder = **2** $\quad$ $\mathbf{164_{10} = 244_8}$

   0.76 × 8 = **6**.08
   0.08 × 8 = **0**.64
   0.64 × 8 = **5**.12 $\quad$ $\mathbf{0.76_{10} \approx 0.605_8}$

   Therefore, $\mathbf{164.76_{10} \approx 244.605_8}$

(2) $24.42_{10} \rightarrow$ base 2 (7 digits after the point; why 7 digits?)

24 / 2 = 12	Remainder = **0**	
12 / 2 = 6	Remainder = **0**	
6 / 2 = 3	Remainder = **0**	
3 / 2 = 1	Remainder = **1**	
1 / 2 = 0	Remainder = **1**	$\mathbf{24_{10} = 1\ 1000_2}$

0.42 × 2 = **0**.84	
0.84 × 2 = **1**.68	
0.68 × 2 = **1**.36	
0.36 × 2 = **0**.72	
0.72 × 2 = **1**.44	
0.44 × 2 = **0**.88	
0.88 × 2 = **1**.76	$\mathbf{0.42_{10} \approx 0.0110101_2}$

Therefore, $\mathbf{24.42_{10} \approx 1\ 1000.0110101_2}$

In its binary representation, this number requires 7 digits after the point in order to get a precision greater than or equal to its decimal representation.

- Two digits after the point in the decimal representation give a precision of 1/100.
- Seven digits after the point in the binary representation give a precision of 1/128.

(3) $69.23_{10} \rightarrow$ base 16 (3 digits after the point)

69 / 16 = 4	Remainder = **5**	
4 / 16 = 0	Remainder = **4**	$\mathbf{69_{10} = 45_{16}}$

0.23 × 16 = **3**.68	
0.68 × 16 = **10**.88	
0.88 × 16 = **14**.08	$\mathbf{0.23_{10} \approx 0.3AE_{16}}$

Therefore, $\mathbf{69.23_{10} = 45.3AE_{16}}$

(4) $11011000111.010011011_2 \rightarrow$ base 16
$\mathbf{0110\ 1100\ 0111.0100\ 1101\ 1000_2 = 6C7.4D8_{16}}$

(5) $1011110100.1111011_2 \rightarrow$ base 8
$\mathbf{001\ 011\ 110\ 100.111\ 101\ 100_2 = 1364.754_8}$