Key to Tutorial 6
Karnaugh Maps

Exercise 1

Let us consider N, a number encoded in 3 bits (C, B, A). A is the least significant bit. Using Karnaugh maps, write down the most simplified expression of $S = f(N)$ for each of the following:

- $S = 1$ when $N \geq 3$

BA

	S	**00**	**01**	**11**	**10**
C	**0**	0	0	1	0
	1	1	1	1	1

$$\mathbf{S = C + B.A}$$

- $S = 1$ when $2 < N \leq 6$

BA

	S	**00**	**01**	**11**	**10**
C	**0**	0	0	1	0
	1	1	1	0	1

$$\mathbf{S = C.\overline{B} + C.\overline{A} + \overline{C}.B.A}$$
$$S = C.(\overline{B} + \overline{A}) + \overline{C}.B.A$$
$$S = C.(\overline{B.A}) + \overline{C}.B.A$$
$$S = C \oplus (B.A)$$

- $S = 1$ when $N = 1$, 3 or 5

BA

	S	**00**	**01**	**11**	**10**
C	**0**	0	1	1	0
	1	0	1	0	0

$$\mathbf{S = \overline{B}.A + \overline{C}.A}$$

- $S = 1$ when $N = 1$, 3 or 5 and S is undefined when $N = 0$ or 4

BA

	S	**00**	**01**	**11**	**10**
C	**0**	Φ	1	1	0
	1	Φ	1	0	0

$$\mathbf{S = \overline{B} + \overline{C}.A}$$

Exercise 2

We want to design a circuit that performs the two's complement operation. This circuit has three inputs (*C*, *B*, *A*) and three outputs (*C'*, *B'*, *A'*). *A* and *A'* are the least significant bits.

1. Write down the truth table for the three outputs.

C	B	A	C'	B'	A'
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	0
1	0	1	0	1	1
1	1	0	0	1	0
1	1	1	0	0	1

2. Write down their most simplified expressions.

A' = A (obvious solution)

BA

	B'	00	01	11	10
C	**0**	0	1	0	1
	1	0	1	0	1

$\mathbf{B' = \overline{B}.A + B.\overline{A}}$

$B' = B \oplus A$

BA

	C'	00	01	11	10
C	**0**	0	1	1	1
	1	1	0	0	0

$\mathbf{C' = \overline{C}.B + \overline{C}.A + C.\overline{B}.\overline{A}}$

$C' = \overline{C}.(B + A) + C.\overline{B}.\overline{A}$

$C' = \overline{C}.(B + A) + C.\overline{(B+A)}$

$C' = C \oplus (B + A)$

We want to design a circuit that converts a natural binary number into a Gray code number. This circuit has three inputs (*C*, *B*, *A*) and three outputs (*C'*, *B'*, *A'*). *A* and *A'* are the least significant bits.

3. Write down the truth table for the three outputs.

C	B	A	C'	B'	A'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	0	0

4. Write down their most simplified expressions.

BA

	A'	**00**	**01**	**11**	**10**
C	**0**	0	1	0	1
	1	0	1	0	1

A' = $\mathbf{\overline{B}.A + B.\overline{A}}$

A' = B⊕A

BA

	B'	**00**	**01**	**11**	**10**
C	**0**	0	0	1	1
	1	1	1	0	0

B' = $\mathbf{C.\overline{B} + \overline{C}.B}$

B' = C⊕B

C' = C (obvious solution)

Exercise 3

Let us consider N, a number encoded in 4 bits (D, C, B, A). A is the least significant bit. Using Karnaugh maps, write down the most simplified expression of $S = f(N)$ for each of the following:

- $S = 1$ when $N \geq 10$

BA

	S	00	01	11	10
DC	00	0	0	0	0
	01	0	0	0	0
	11	1	1	1	1
	10	0	0	1	1

$S = D.C + D.B$

- $S = 1$ when $N = 0, 4, 8, 10, 12$ or 14

BA

	S	00	01	11	10
DC	00	1	0	0	0
	01	1	0	0	0
	11	1	0	0	1
	10	1	0	0	1

$S = D.\overline{A} + \overline{B}.\overline{A}$

- $S = 1$ when $N = 0, 2, 5, 7, 8, 10, 13$ or 15

BA

	S	00	01	11	10
DC	00	1	0	0	1
	01	0	1	1	0
	11	0	1	1	0
	10	1	0	0	1

$S = C.A + \overline{C}.\overline{A}$

$S = \overline{C \oplus A}$

- $S = 1$ when $N = 2, 10, 11$ or 14

BA

	S	00	01	11	10
DC	00	0	0	0	1
	01	0	0	0	0
	11	0	0	0	1
	10	0	0	1	1

$S = D.\overline{C}.B + D.B.\overline{A} + \overline{C}.B.\overline{A}$

- $S = 1$ when $N = 2, 10, 11$ or 14 and S is undefined when $N = 6, 9, 13$ or 15

There are two equivalent possibilities:

		BA			
	S	**00**	**01**	**11**	**10**
DC	**00**	0	0	0	1
	01	0	0	0	Φ
	11	0	Φ	Φ	1
	10	0	Φ	1	1

$$\mathbf{S = D.B + B.\overline{A}}$$

or

		BA			
	S	**00**	**01**	**11**	**10**
DC	**00**	0	0	0	1
	01	0	0	0	Φ
	11	0	Φ	Φ	1
	10	0	Φ	1	1

$$\mathbf{S = D.A + B.\overline{A}}$$