# Key to Tutorial 1
# Mathematical Tools

## Exercise 1

1. Work out the following positive powers of two.
   - $2^0 = \mathbf{1}$
   - $2^1 = \mathbf{2}$
   - $2^2 = \mathbf{4}$
   - $2^3 = \mathbf{8}$
   - $2^4 = \mathbf{16}$
   - $2^5 = \mathbf{32}$
   - $2^6 = \mathbf{64}$
   - $2^7 = \mathbf{128}$
   - $2^8 = \mathbf{256}$
   - $2^9 = \mathbf{512}$
   - $2^{10} = \mathbf{1,024}$
   - $2^{11} = \mathbf{2,048}$
   - $2^{12} = \mathbf{4,096}$
   - $2^{13} = \mathbf{8,192}$
   - $2^{14} = \mathbf{16,384}$
   - $2^{15} = \mathbf{32,768}$
   - $2^{16} = \mathbf{65,536}$
   - $2^{17} = \mathbf{131,072}$
   - $2^{18} = \mathbf{262,144}$
   - $2^{19} = \mathbf{524,288}$
   - $2^{20} = \mathbf{1,048,576}$

**From now on, you have to know the first positive powers of two by heart (at least from $2^0$ to $2^{16}$).**

2. Work out the following negative powers of two (give the decimal expansions).
   - $2^{-1} = \mathbf{0.5}$
   - $2^{-2} = \mathbf{0.25}$
   - $2^{-3} = \mathbf{0.125}$
   - $2^{-4} = \mathbf{0.0625}$
   - $2^{-5} = \mathbf{0.03125}$
   - $2^{-6} = \mathbf{0.015625}$
   - $2^{-7} = \mathbf{0.0078125}$
   - $2^{-8} = \mathbf{0.00390625}$
   - $2^{-9} = \mathbf{0.001953125}$
   - $2^{-10} = \mathbf{0.0009765625}$

**From now on, you have to know the first negative powers of two by heart (at least from $2^{-1}$ to $2^{-5}$).**

## Exercise 2

Work out the following expressions. Give the results in their power-of-two forms.

1. $$\frac{2^{24}\cdot 2^{15}}{2^7} = 2^{24}\cdot 2^{15}\cdot 2^{-7} = 2^{32}$$

2. $$\frac{2^{17}\cdot(3{,}990+106)^3\cdot 32}{8\cdot 4^6} = \frac{2^{17}\cdot(4{,}096)^3\cdot 2^5}{2^3\cdot(2^2)^6} = \frac{2^{17}\cdot(2^{12})^3\cdot 2^5}{2^3\cdot 2^{12}} = \frac{2^{17}\cdot 2^{36}\cdot 2^5}{2^{15}} = \frac{2^{58}}{2^{15}} = 2^{43}$$

3. $$\frac{32^4\cdot 16^3\cdot 2^5}{(1{,}024^{-5}\cdot 16^8)^{-3}} = \frac{(2^5)^4\cdot(2^4)^3\cdot 2^5}{((2^{10})^{-5}\cdot(2^4)^8)^{-3}} = \frac{2^{20}\cdot 2^{12}\cdot 2^5}{(2^{-50}\cdot 2^{32})^{-3}} = \frac{2^{37}}{2^{150}\cdot 2^{-96}} = \frac{2^{37}}{2^{54}} = 2^{-17}$$

4. $$\frac{(8^4\cdot 4{,}096^{-2})\cdot(90+38)^{-7}}{(4^{-5}\cdot(2^{13}-2^{12}))^5\cdot 64^{-10}} = \frac{((2^3)^4\cdot(2^{12})^{-2})\cdot(128)^{-7}}{((2^2)^{-5}\cdot(2^{13}-2^{12}))^5\cdot(2^6)^{-10}} = \frac{(2^{12}\cdot 2^{-24})\cdot(2^7)^{-7}}{(2^{-10}\cdot 2^{12})^5\cdot 2^{-60}} = \frac{2^{-12}\cdot 2^{-49}}{2^{10}\cdot 2^{-60}} = \frac{2^{-61}}{2^{-50}} = 2^{-11}$$

5. $$\left(\frac{((8{,}192\cdot 2^5)^4\cdot 32{,}768^{-7})^3}{(8^{-3}\cdot 128)^{-4}\cdot(65{,}536-2^{15})^4}\right)^2 = \left(\frac{((2^{13}\cdot 2^5)^4\cdot(2^{15})^{-7})^3}{((2^3)^{-3}\cdot 2^7)^{-4}\cdot(2^{16}-2^{15})^4}\right)^2 = \left(\frac{(2^{72}\cdot 2^{-105})^3}{(2^{-9}\cdot 2^7)^{-4}\cdot(2^{15})^4}\right)^2 = \left(\frac{2^{-99}}{2^{68}}\right)^2 = 2^{-334}$$

## Exercise 3

Give the quotient and the remainder of the following Euclidean divisions.

1. 386 / 7 = **55 remainder 1**
2. 2,860 / 16 = **178 remainder 12**
3. 51,862 / 25 = **2,074 remainder 12**
4. 160,853 / 120 = **1,340 remainder 53**

Perform the following divisions and give the result with three digits after the point.

5. 521 / 14 = **37.214**
6. 632 / 15 = **42.133**

## Exercise 4

1. How many bits is a byte made up of?

   A byte is made up of **8 bits**.

2. Give the numerical values of the following binary prefixes: Ki, Mi, Gi and Ti. Use a power-of-two notation.
   - 1 Ki = $\mathbf{2^{10}}$
   - 1 Mi = $\mathbf{2^{20}}$
   - 1 Gi = $\mathbf{2^{30}}$
   - 1 Ti = $\mathbf{2^{40}}$

3. How many bits do the following values contain? Use a power-of-two notation.
   64 Kib, 128 Mib, 8 KiB, 256 GiB.

   - **64 Kib** = $2^6 \times 2^{10}$ bits = $\mathbf{2^{16}}$ **bits.**
   - **128 Mib** = $2^7 \times 2^{20}$ bits = $\mathbf{2^{27}}$ **bits.**
   - **8 KiB** = $2^3 \times 2^{10}$ bytes = $2^3 \times 2^{10} \times 2^3$ bits = $\mathbf{2^{16}}$ **bits.**
   - **256 GiB** = $2^8 \times 2^{30}$ bytes = $2^8 \times 2^{30} \times 2^3$ bits = $\mathbf{2^{41}}$ **bits.**

4. How many bytes do the following values contain? Use binary prefixes (Ki, Mi or Gi). Choose the most appropriate prefix so that the numerical value will be as small as possible.
   1 Mib, $2^{15}$ bits, $2^{23}$ bytes, $2^{30}$ bytes.

   - **1 Mib** = $2^{20}$ bits = $2^{20} / 2^3$ bytes = $2^{17}$ bytes = $2^7 \times 2^{10}$ bytes = **128 KiB**.
   - $\mathbf{2^{15}}$ **bits** = $2^{15} / 2^3$ bytes = $2^{12}$ bytes = $2^2 \times 2^{10}$ bytes = **4 KiB**.
   - $\mathbf{2^{23}}$ **bytes** = $2^3 \times 2^{20}$ bytes = **8 MiB**.
   - $\mathbf{2^{38}}$ **bytes** = $2^8 \times 2^{30}$ bytes = **256 GiB**.