Key to Tutorial 1
Binary Integers

Exercise 1

Without using the successive division method, convert the decimal integers below into their natural binary equivalents. From these binary representations, deduce their hexadecimal values.
28_{10}, 129_{10}, 147_{10}, 255_{10}.

Start with the most significant bit (MSB) and set each binary digit according to the sum of their weight.

		128	64	32	16	8	4	2	1
$\mathbf{28_{10}}$	→	**0**	**0**	**0**	**1**	**1**	**1**	**0**	**0**
$\mathbf{129_{10}}$	→	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**1**
$\mathbf{147_{10}}$	→	**1**	**0**	**0**	**1**	**0**	**0**	**1**	**1**
$\mathbf{255_{10}}$	→	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

To convert a binary number (base 2) into its hexadecimal form (base 16), we can split the binary number into groups of 4 bits ($2^4 = 16$) and replace each of these groups by its hexadecimal equivalent.

$\mathbf{28_{10}}$ = $0001\ 1100_2$ = $\mathbf{1C_{16}}$
$\mathbf{129_{10}}$ = $1000\ 0001_2$ = $\mathbf{81_{16}}$
$\mathbf{147_{10}}$ = $1001\ 0011_2$ = $\mathbf{93_{16}}$
$\mathbf{255_{10}}$ = $1111\ 1111_2$ = $\mathbf{FF_{16}}$

Exercise 2

1. Are the following binary numbers even or odd?
   11000010_2, 10010100_2, 11101111_2, 10000011_2, 10101000_2

   The last digit of an even number is 0: $\mathbf{11000010_2}$**,** $\mathbf{10010100_2}$**,** $\mathbf{10101000_2}$

2. Which ones are divisible by 4, 8 or 16?

   - The last two digits of a number divisible by 4 are ‘00’: $\mathbf{10010100_2}$**,** $\mathbf{10101000_2}$
   - The last three digits of a number divisible by 8 are ‘000’: $\mathbf{10101000_2}$
   - The last four digits of a number divisible by 16 are ‘0000’: **none of the numbers above**

3. Divide each number by 2, 4 and 8. Write down their quotients and remainders.

	11000010		**10010100**		**11101111**		**10000011**		**10101000**	
	Q	**r**	**Q**	**r**	**Q**	**r**	**Q**	**r**	**Q**	**r**
/2	1100001	0	1001010	0	1110111	1	1000001	1	1010100	0
/4	110000	10	100101	00	111011	11	100000	11	101010	00
/8	11000	010	10010	100	11101	111	10000	011	10101	000

Q = quotient; r = remainder

4. What operations could be performed instead of division in order to obtain the quotient and remainder of a binary number divided by 2^n?

   - **Quotient**: a **logical right shift by *n* bits** on the binary number could be performed.
   - **Remainder**: a **logical AND between 2^n – 1** and the binary number could be performed.

   Bitwise operations (e.g. shifts, AND, OR, etc.) are significantly faster to perform for a microprocessor than divisions.

5. What operation could be performed instead of multiplication in order to multiply any binary number by a power of 2?

   A logical left shift by *n* bits is equivalent to a multiplication by 2^n.

6. What operations could be performed instead of multiplication in order to multiply any binary number by 3 or 10?

   - ***3n = 2n + n***
     A multiplication by 3 can be performed by a logical left shift by 1 bit and an addition.

   - ***10n = 8n + 2n***
     A multiplication by 10 can be performed by a logical left shift by 3 bits, a logical left shift by 1 bit and an addition.

   If the multiplier is known, it can be split up in order to perform only logical shifts and additions, which are significantly faster to perform for a microprocessor.

Exercise 3

Work out the decimal range of signed and unsigned binary numbers for each of the following word lengths: 4, 8, 16, 32 and *n* bits.

Bits	Unsigned	Signed
4	$0 \rightarrow 15$	$-8 \rightarrow 7$
8	$0 \rightarrow 255$	$-128 \rightarrow 127$
16	$0 \rightarrow 65535$	$-32768 \rightarrow 32767$
32	$0 \rightarrow 2^{32} - 1$	$-2^{31} \rightarrow 2^{31} - 1$
n	$0 \rightarrow 2^{n} - 1$	$-2^{n-1} \rightarrow 2^{n-1} - 1$

Exercise 4

Given the following numbers: 11111111_2 et 10110110_2.

1. Write down their decimal equivalents, assuming that they are 8-bit signed numbers.

- **11111111_2**
  The most significant bit is 1: the number is negative.
  Calculate the two's complement and convert the result into a decimal form:
  $(11111111_2)_{2C} = 00000000_2 + 1_2 = 1_2 = 1$
  Therefore, the decimal representation is –1.

- **10110110_2**
  The most significant bit is 1: the number is negative.
  Calculate the two's complement and convert the result into a decimal form:
  $(10110110_2)_{2C} = 01001001_2 + 1_2 = 01001010_2 = 64 + 8 + 2 = 74$
  Therefore, the decimal representation is –74.

2. Write down their decimal equivalents, assuming that they are 16-bit signed numbers.

- **11111111_2**
  The most significant bit is 0 ($\underline{\mathbf{0}}000000011111111_2$): the number is positive.
  Convert directly the binary number into its decimal form:
  $11111111_2 = 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 255$
  Therefore, the decimal representation is +255.

- **10110110_2**
  The most significant bit is 0 ($\underline{\mathbf{0}}000000011111111_2$): the number is positive.
  Convert directly the binary number into its decimal form:
  $10110110_2 = 128 + 32 + 16 + 4 + 2 = 182$
  Therefore, the decimal representation is +182.

Given the following negative number: -80_{10}.

3. Write down its binary and hexadecimal equivalents, assuming that it is an 8-bit signed number.

   Convert the absolute value of the number into its binary form: $80_{10} = 01010000_2$
   Calculate the two's complement: $(01010000_2)_{2C} = 10101111_2 + 1_2 = 10110000_2$
   Which gives: **10110000_2 for the binary form.**
   $B0_{16}$ for the hexadecimal form.

4. Write down its binary and hexadecimal equivalents, assuming that it is a 16-bit signed number.

   A simple sign extension is enough to convert an 8-bit signed number into a 16-bit signed number.
   Which gives: **$\underline{11111111}10110000_2$ for the binary form.**
   $\underline{FF}B0_{16}$ for the hexadecimal form.

Exercise 5

1. How many bits do the following values contain? Use a power-of-two notation.
   128 Kib, 16 Mib, 2 KiB, 512 GiB.

   It is known that:
   - 1 Ki = 2^{10}; 1 Mi = 2^{20}; 1 Gi = 2^{30}.
   - 1 byte = 8 bits = 2^3 bits.

   Therefore:
   - **128 Kib** = $2^7 \times 2^{10}$ bits = **2^{17} bits.**
   - **16 Mib** = $2^4 \times 2^{20}$ bits = **2^{24} bits.**
   - **2 KiB** = $2^1 \times 2^{10}$ bytes = $2^1 \times 2^{10} \times 2^3$ bits = **2^{14} bits.**
   - **512 GiB** = $2^9 \times 2^{30}$ bytes = $2^9 \times 2^{30} \times 2^3$ bits = **2^{42} bits.**

2. How many bytes do the following values contain? Use binary prefixes (Ki, Mi or Gi). Choose the most appropriate prefix so that the numerical value will be as small as possible.
   2 Mib, 2^{14} bits, 2^{26} bytes, 2^{32} bytes.

   - **2 Mib** = $2^1 \times 2^{20}$ bits = $2^1 \times 2^{20} / 2^3$ bytes = 2^{18} bytes = $2^8 \times 2^{10}$ bytes = **256 KiB**.
   - **2^{14} bits** = $2^{14} / 2^3$ bytes = 2^{11} bytes = $2^1 \times 2^{10}$ bytes = **2 KiB**.
   - **2^{26} bytes** = $2^6 \times 2^{20}$ bytes = **64 MiB**.
   - **2^{32} bytes** = $2^2 \times 2^{30}$ bytes = **4 GiB**.