Criteria to decide the nature of a series.

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Let (U_n) and (V_n) be positive sequences:

If (U_n) does not CV to 0, $\sum U_n$ is DV.

If $U_n \leq V_n$ (or $U_n = o(V_n)$):

$$\sum V_n \ CV \Rightarrow \sum U_n \ CV \sum U_n \ DV \Rightarrow \sum V_n \ DV$$

If $U_n \sim V_n$, $\sum U_n$ and $\sum V_n$ have the same nature.

Reference series

Riemann

$$\sum \frac{1}{n^\alpha} \ is \ CV \Leftrightarrow \alpha > 1$$

Geometric

$$\sum q^n \ is \ CV \Leftrightarrow |q| < 1$$

Using d'Alembert and Cauchy

$$or \quad \begin{array}{c|l|l} \frac{U_{n+1}}{U_n} \longrightarrow l & l > 1 & \sum U_n \ is \ DV \\ & l < 1 & \sum U_n \ is \ CV \\ \sqrt[n]{U_n} \longrightarrow l & l = 1 & not \ determined \end{array}$$