Key to Tutorial 7
Problems of Combinational Logic

Exercise 1 Polling Report

Four shop stewards (A, B, C, D) represent the following number of votes respectively: 100 votes, 150 votes, 250 votes and 175 votes. A proposal needs at least 50 % of the votes to be accepted. Write down the most simplified expression of a logic function (S) that is 1 when a proposal is accepted and 0 when it is rejected. Draw the circuit diagram.

Indication: 'A = 1' means that the A shop steward accepts a proposal and 'A = 0' means that he or she rejects it. The same goes for the other shop stewards.

First of all, let us determine the truth table of the S output:

A (100)	B (150)	C (250)	D (175)	S	Number of votes
0	0	0	0	0	0
0	0	0	1	0	175
0	0	1	0	0	250
0	0	1	1	1	425
0	1	0	0	0	150
0	1	0	1	0	325
0	1	1	0	1	400
0	1	1	1	1	575
1	0	0	0	0	100
1	0	0	1	0	275
1	0	1	0	1	350
1	0	1	1	1	525
1	1	0	0	0	250
1	1	0	1	1	425
1	1	1	0	1	500
1	1	1	1	1	675

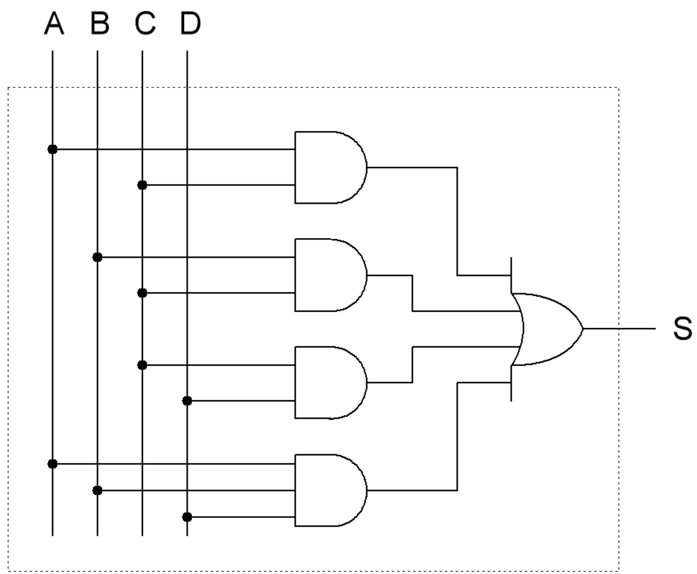
The total number of votes is 675 (100 + 150 + 250 + 175). Therefore, a proposal is accepted (S = 1) when the number of votes is equal to or greater than 338 (at least 50 % of the votes); otherwise it is rejected (S = 0).

Then, let us deduce the expression of *S* according to its Karnaugh map:

		CD			
	S	**00**	**01**	**11**	**10**
AB	**00**	0	0	1	0
	01	0	0	1	1
	11	0	1	1	1
	10	0	0	1	1

S = A.C + B.C + C.D + A.B.D

Finally, here is what the circuit diagram should look like:



<u>Exercise 2</u> Liquid Level

Let us consider two tanks: *R1* and *R2*. The liquid level of each tank is checked by two sensors: a high-level sensor (*A* for *R1*, *B* for *R2*) and a low-level sensor (*C* for *R1*, *D* for *R2*). The values of *A*, *B*, *C*, *D* are 1s when there is some liquid in front of the sensor; otherwise they are 0s. Three indicator lights (*V1*, *V2*, *V3*) are set according to the following conditions:

- *V1* = 1, if *R1* and *R2* are full.
- *V2* = 1, if *R1* and *R2* are empty.
- *V3* = 1, in any other cases.

Write down the truth tables and the most simplified expressions of the outputs. Draw the circuit diagram.

First of all, let us determine the truth table of the *V1*, *V2* and *V3* outputs:

A	B	C	D	V1	V2	V3	
0	0	0	0	0	1	0	
0	0	0	1	0	0	1	
0	0	1	0	0	0	1	
0	0	1	1	0	0	1	
0	1	0	0	Φ	Φ	Φ	← Don't care condition
0	1	0	1	0	0	1	
0	1	1	0	Φ	Φ	Φ	← Don't care condition
0	1	1	1	0	0	1	
1	0	0	0	Φ	Φ	Φ	← Don't care condition
1	0	0	1	Φ	Φ	Φ	← Don't care condition
1	0	1	0	0	0	1	
1	0	1	1	0	0	1	
1	1	0	0	Φ	Φ	Φ	← Don't care condition
1	1	0	1	Φ	Φ	Φ	← Don't care condition
1	1	1	0	Φ	Φ	Φ	← Don't care condition
1	1	1	1	1	0	0	

In this truth table, some conditions never occur. If a high-level sensor is 1, the low-level sensor of the same tank cannot be 0. In other words, if *A* is 1, *C* cannot be 0; and if *B* is 1, *D* cannot be 0.

The conditions where 'A = 1 and C = 0' and those where 'B = 1 and D = 0' are called 'don't care conditions' because they never occur. So, we do not care what their outputs are and we can set them to either 0 or 1. Outputs of don't care conditions are represented by the Greek character 'Φ' (phi), which means 0 or 1. (The 'x' character can also be used instead of the 'Φ' character.)

Then, let us deduce the expressions of *V1*, *V2*, *V3* according to their Karnaugh maps. **Don't care conditions are set to 0 or 1 in order to simplify the expression.**

		CD			
	V1	**00**	**01**	**11**	**10**
AB	**00**	0	0	0	0
	01	Φ	0	0	Φ
	11	Φ	Φ	1	Φ
	10	Φ	Φ	0	0

V1 = A.B

		CD			
	V2	**00**	**01**	**11**	**10**
AB	**00**	1	0	0	0
	01	Φ	0	0	Φ
	11	Φ	Φ	0	Φ
	10	Φ	Φ	0	0

V2 = $\overline{C}.\overline{D}$

		CD			
	V3	**00**	**01**	**11**	**10**
AB	**00**	0	1	1	1
	01	Φ	1	1	Φ
	11	Φ	Φ	0	Φ
	10	Φ	Φ	1	1

V3 = $\overline{A}.D + \overline{B}.C$

Finally, here is what the circuit diagram should look like: