Tutorial 4
Encoded Integers

Exercise 1: Signed Numbers

1. Encode the following negative integers into 8-bit signed words:
   - –1
   - –29
   - –40
   - –127
   - –128

2. Write down the hexadecimal form of the 8-bit signed binary words used to encode the smallest and largest values of positive and negative integers.

Exercise 2: 8-bit Signed Operations

1. Perform the following 8-bit binary operations (the two operands and the result are 8 bits wide). Then, convert the result into unsigned and signed decimal values. If an overflow occurs, write down 'ERROR' instead of the decimal value.

Operation	Binary Result	Decimal Value	
		Unsigned	Signed
11110101 + 11111010			
11101000 – 11000110			
01011110 – 10011110			
01111110 + 00000101			
11001011 – 00011010			
10000000 + 11111010			
10000011 – 00001010			

2. When it comes to adding 8-bit signed binary words, find a quick way to detect any overflow by comparing the sign bit of the result and those of both operands. Be careful not to confuse this overflow with the carry, which is an unsigned overflow (the 9^{th} bit of the result).

Exercise 3: Gray Code

1. Write down the Gray code numbers from 0 to 15 and deduce the Gray code of 17, 24 and 31.

2. To encode a binary number (*Xb*) into a Gray code number (*Xg*), we can apply the following method:

   Assuming that:
   - $Xb = b_n\ b_{n-1}\ b_{n-2} \ldots b_1\ b_0$
   - $Xg = g_n\ g_{n-1}\ g_{n-2} \ldots g_1\ g_0$

   We have:
   - $g_n = b_n$
   - $g_{n-1} = 0$ if and only if $b_n = b_{n-1}$
   - $g_{n-2} = 0$ if and only if $b_{n-1} = b_{n-2}$
   - $g_0 = 0$ if and only if $b_0 = b_1$

   In other words, two consecutive bits are compared: if they are equal, we write down '0'; otherwise, we write down '1'.

   Example: $11001011_2 \rightarrow 10101110_{Gray}$

   Convert the following decimal numbers into Gray code numbers: 42, 109, 128.

3. To encode a Gray code number into a binary number, you can apply the following method:

   Count the number of 1s from the most significant bit up to the same position as the bit we are looking for: if the number of 1s is even, the value of the bit is 0; otherwise, it is 1.

   Example: $1011_{Gray} \rightarrow 1101_2$

   Convert the following Gray code numbers into decimal numbers:
   - 11001000
   - 11101011
   - 10000001

Exercise 4: Binary Coded Decimal (BCD)

Convert the following decimal numbers into BCD numbers: 421; 404; 3,009; 7,408.