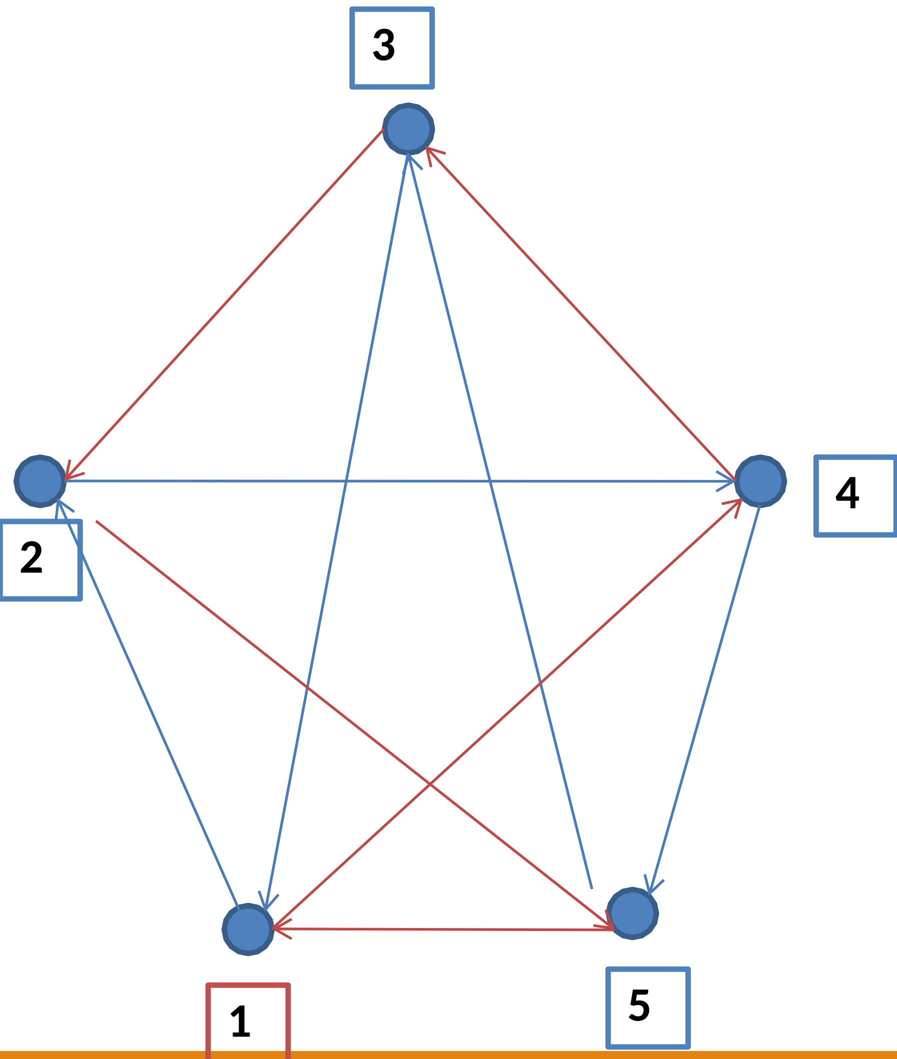
EXPERIMENT 14

AIM:

Implement Travelling salesman program using branch and bound.

Travelling salesman Problem-Definition



- Let us look at a situation that there are 5 cities, Which are represented as NODES
- There is a Person at NODE-1
- This **PERSON HAS TO REACH EACH NODES ONE AND ONLY ONCE AND COME BACK TO ORIGINAL (STARTING)POSITION.**
- This **process has to occur with minimum cost or minimum distance travelled.**
- Note that starting point can start with any Node. For Example:

1-5-2-3-4-1

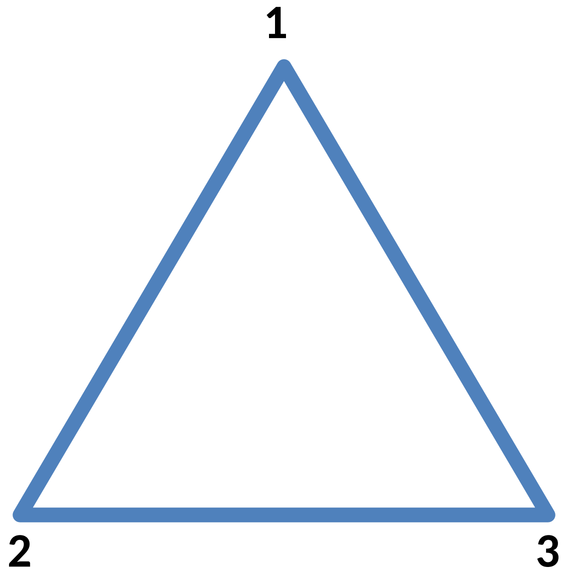
2-3-4-1-5-2

Travelling salesman Problem-Definition

- If there are ‘n’ nodes there are (n-1)! Feasible solutions
- From these (n-1)! Feasible solutions we have to find OPTIMAL SOLUTION.
- This can be related to GRAPH THEORY.
- Graph is a collection of Nodes and Arcs(Edges).

Travelling salesman Problem-Definition

- Let us say there are Nodes Connected as shown
- We can find a Sub graph as 1-3-2-1.Hence this **GRAPH IS HAMILTONIAN**

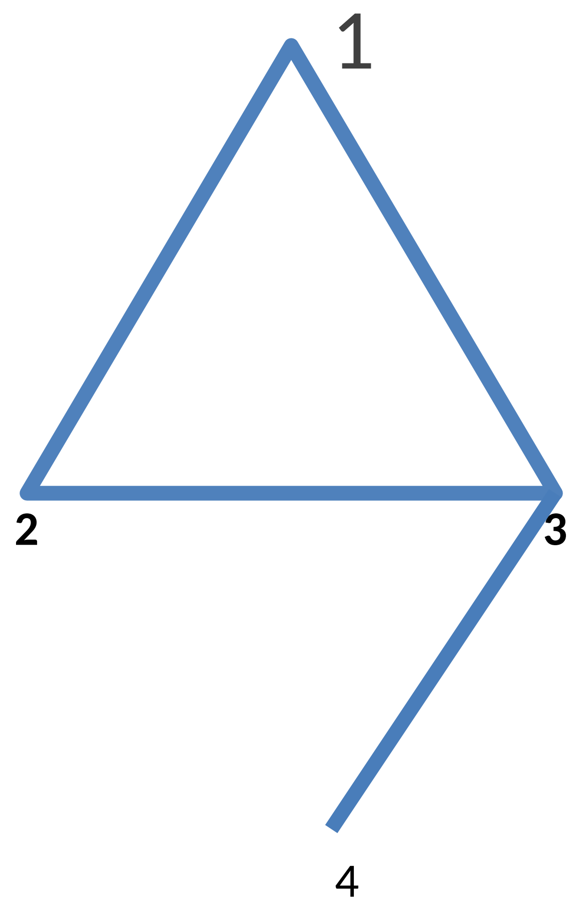


Travelling salesman Problem-Definition

But let us consider this graph

- We can go to

1-3-4-3-2-1



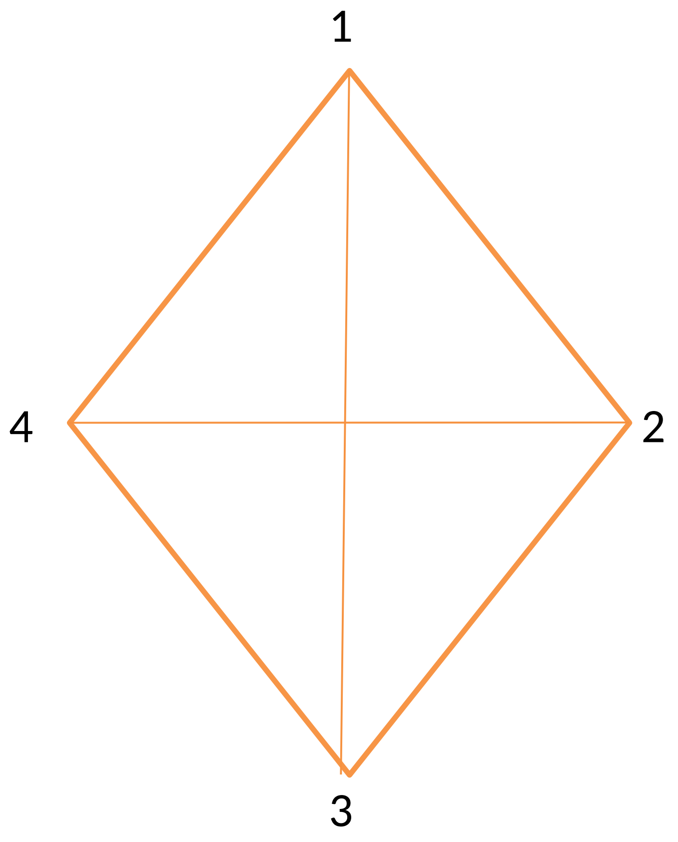
But we are reaching 3 again to make a cycle. **HENCE THIS GRAPH IS NOT HAMILTONIAN**

HAMILTONIAN GRAPHS

- The Given Graph is Hamiltonian
- If a graph is Hamiltonian, it may have more than one Hamiltonian Circuits.
- For eg:

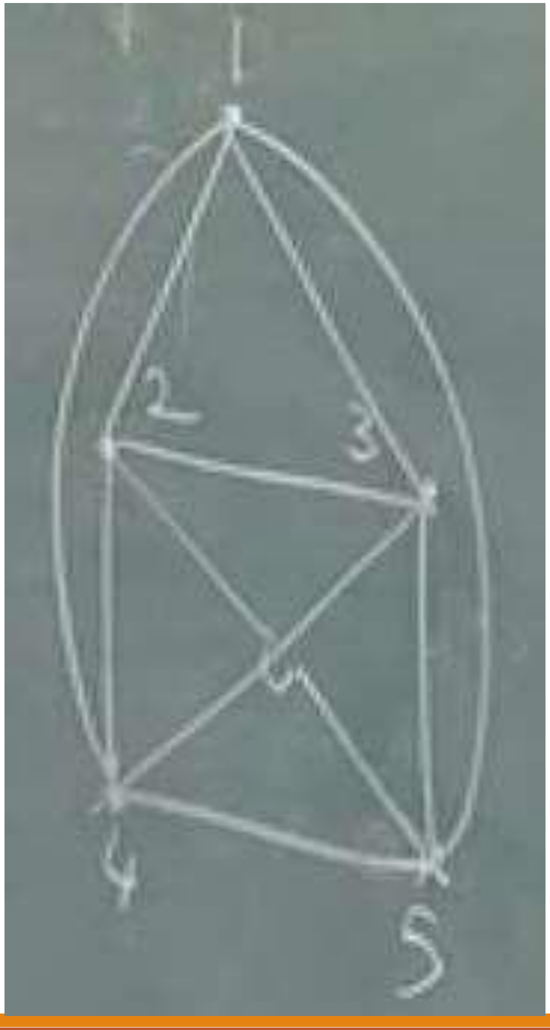
1-4-2-3-1

1-2-3-4-1 etc.,

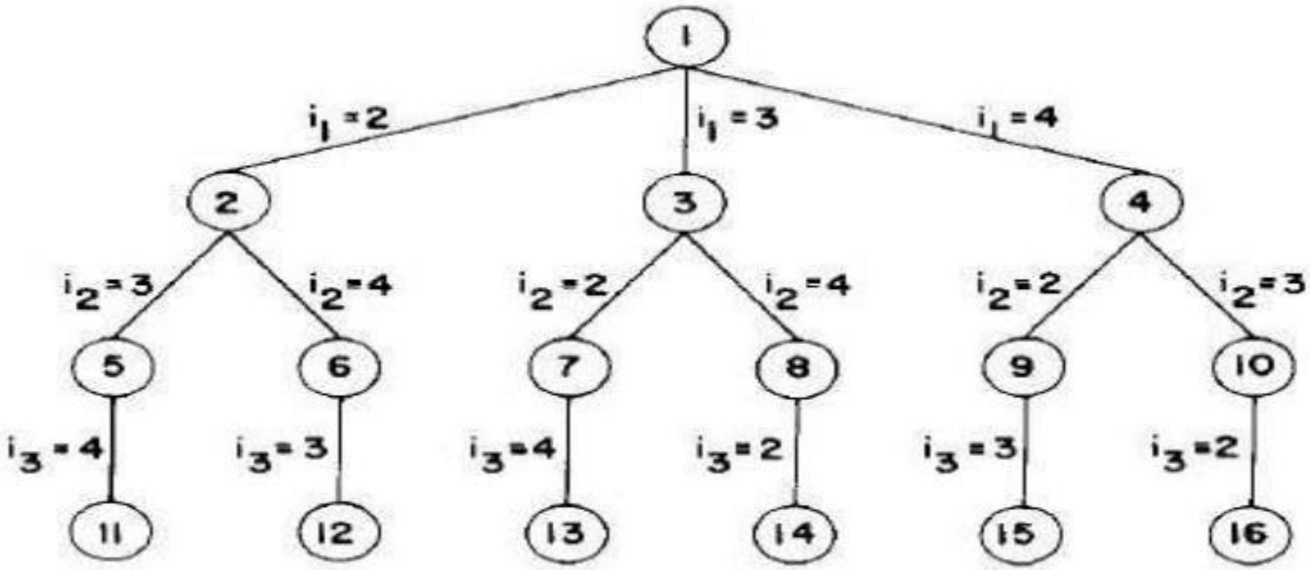


Hamiltonian Graphs And Travelling Salesman Problem

- Graphs Which are Completely Connected i.e., if we have Graphs with every vertex connected to every other vertex, then Clearly That graph is HAMILTONIAN.
- So Travelling Salesman Problem is nothing but finding out LEAST COST HAMILTONIAN CIRCUIT



Here is the state space tree for n=4 (Using Branch And Bound)



State space tree for the traveling salesperson problem with $n = 4$ and $i_0 = i_4 = 1$

Procedure for solving traveling sales person problem

1.RowReduction

Reduce the given cost matrix. A matrix is reduced if every row and column is reduced. A row (column) is said to be reduced if it contain at least one zero and all-remaining entries are non-negative. This can be done as follows:

a) *Row reduction:* Take the minimum element from first row, subtract it from all elements of first row, next take minimum element from the second row and subtract it from second row. Similarly apply the same procedure for all rows.

b) Find the sum of elements, which were subtracted from rows.

2.Column Reduction:

Apply column reductions for the matrix obtained after row reduction.

Column reduction: Take the minimum element from first column, subtract it from all elements of first column, next take minimum element from the second column and subtract it from second column. Similarly apply the same procedure for all columns.

Find the sum of elements, which were subtracted from columns.

3. Cumulative Sum

Obtain the cumulative sum of row wise reduction and column wise reduction.

Cumulative reduced sum = Row wise reduction sum + column wise reduction sum.

Associate the cumulative reduced sum to the starting state as lower bound and $\propto$ as upper bound.

4. Calculate the reduced cost matrix for every node R. Let A is the reduced cost matrix for node R. Let S be a child of R such that the tree edge (R, S) corresponds to including edge <i, j> in the tour. If S is not a leaf node, then the reduced cost matrix for S may be obtained as follows:

a) Change all entries in row i and column j of A to $\propto$.

b) Set A (j, 1) to $\propto$.

c) Reduce all rows and columns in the resulting matrix except for rows and column containing only $\propto$. Let r is the total amount subtracted to reduce the matrix.

c) Find $\hat{c}(S) = \hat{c}(R) + A(i, j) + r$, where 'r' is the total amount subtracted to reduce the matrix, $\hat{c}(R)$ indicates the lower bound of the i^{th} node in (i, j) path and $\hat{c}(S)$ is called the cost function.

5. Repeat step3 until all nodes are visited

Find the LC branch and bound solution for the traveling sale person problem whose cost matrix is as follows:

Given the following cost matrix:

$$\begin{bmatrix} inf & 20 & 30 & 10 & 11 \\ 15 & inf & 16 & 4 & 2 \\ 3 & 5 & inf & 2 & 4 \\ 19 & 6 & 18 & inf & 3 \\ 16 & 4 & 7 & 16 & inf \end{bmatrix}$$

STEP 1: ROW REDUCTION

$$\begin{bmatrix} inf & 20 & 30 & 10 & 11 \\ 15 & inf & 16 & 4 & 2 \\ 3 & 5 & inf & 2 & 4 \\ 19 & 6 & 18 & inf & 3 \\ 16 & 4 & 7 & 16 & inf \end{bmatrix}$$

Apply row reduction method:
Deduct 10 (which is the minimum) from all values in the 1st row.
Deduct 2 (which is the minimum) from all values in the 2nd row.
Deduct 2 (which is the minimum) from all values in the 3rd row.
Deduct 3 (which is the minimum) from all values in the 4th row.
Deduct 4 (which is the minimum) from all values in the 5th row.

$$\begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 15 & inf & 16 & 4 & 2 \\ 3 & 5 & inf & 2 & 4 \\ 19 & 6 & 18 & inf & 3 \\ 16 & 4 & 7 & 16 & inf \end{bmatrix} \begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 13 & inf & 14 & 2 & 0 \\ 3 & 5 & inf & 2 & 4 \\ 19 & 6 & 18 & inf & 3 \\ 16 & 4 & 7 & 16 & inf \end{bmatrix} \begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 13 & inf & 14 & 2 & 0 \\ 1 & 3 & inf & 0 & 2 \\ 19 & 6 & 18 & inf & 3 \\ 16 & 4 & 7 & 16 & inf \end{bmatrix} \begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 13 & inf & 14 & 2 & 0 \\ 1 & 3 & inf & 0 & 2 \\ 16 & 3 & 15 & inf & 0 \\ 16 & 4 & 7 & 16 & inf \end{bmatrix} \begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 13 & inf & 14 & 2 & 0 \\ 1 & 3 & inf & 0 & 2 \\ 16 & 3 & 15 & inf & 0 \\ 12 & 0 & 3 & 12 & inf \end{bmatrix}$$

ROW1 ROW2 ROW3 ROW4 ROW5

Row wise reduction sum = 10 + 2 + 2 + 3 + 4 = 21

STEP 2: COLUMN REDUCTION

$$\begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 13 & inf & 14 & 2 & 0 \\ 1 & 3 & inf & 0 & 2 \\ 16 & 3 & 15 & inf & 0 \\ 12 & 0 & 3 & 12 & inf \end{bmatrix}$$

Now apply column reduction for the above matrix EXCEPT WHICH CONTAINS 0

Deduct 1 (which is the minimum) from all values in the 1st column.

Deduct 3 (which is the minimum) from all values in the 3rd column.

$$\begin{bmatrix} inf & 10 & 20 & 0 & 1 \\ 12 & inf & 14 & 2 & 0 \\ 0 & 3 & inf & 0 & 2 \\ 15 & 3 & 15 & inf & 0 \\ 11 & 0 & 3 & 12 & inf \end{bmatrix}$$

COLUMN 1

$$\begin{matrix} inf & 10 & 17 & 0 & 1 \\ 12 & inf & 11 & 2 & 0 \\ 0 & 3 & inf & 0 & 2 \\ 15 & 3 & 12 & inf & 0 \\ 11 & 0 & 0 & 12 & inf \end{matrix}$$

COLUMN 3

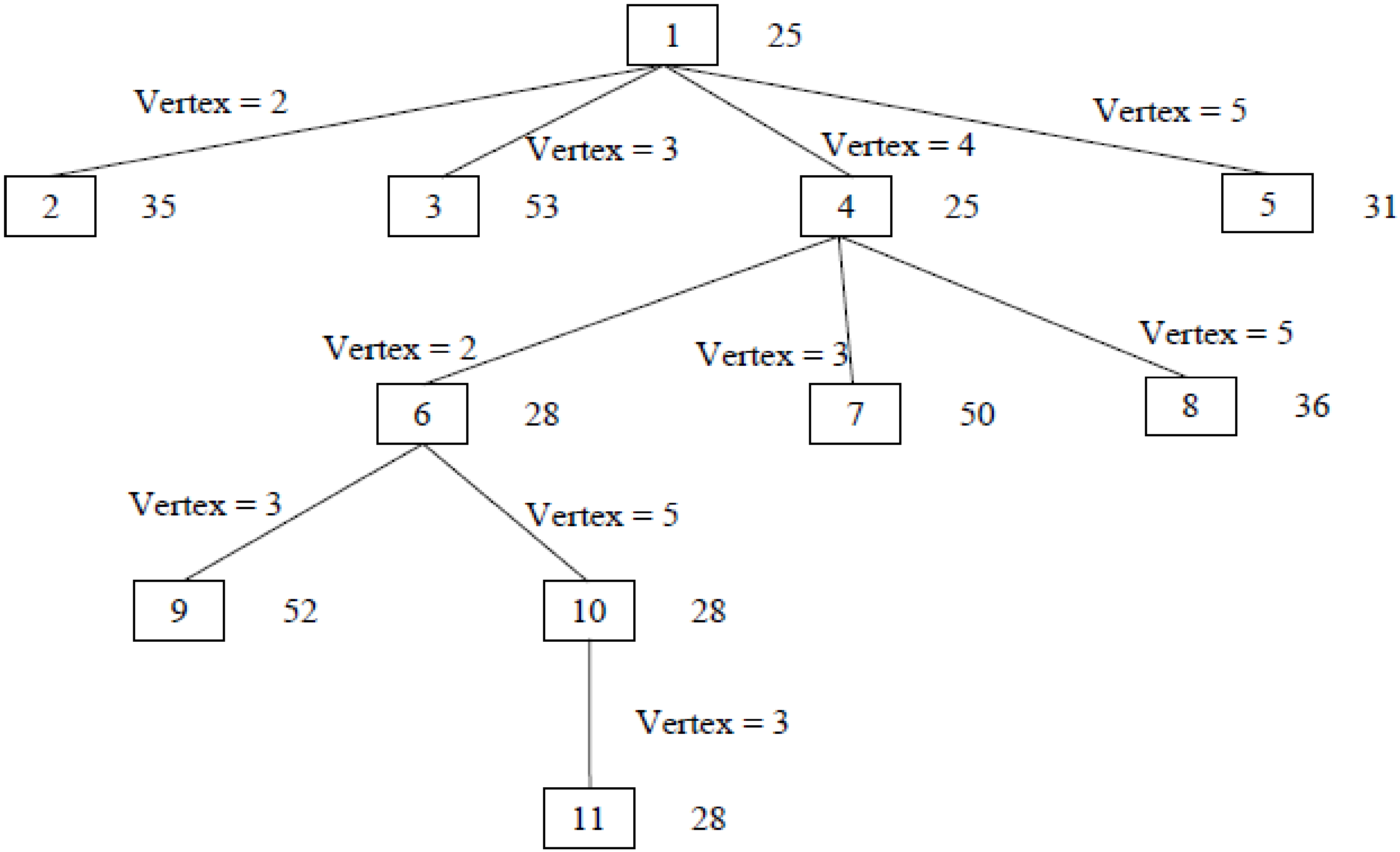
ROW 2 ,ROW 4 AND ROW 5 are already reduced as it contains 0

Column wise reduction sum = 1 + 0 + 3 + 0 + 0 = 4

Cumulative reduced sum = row wise reduction + column wise reduction sum.

= 21 + 4 = 25.

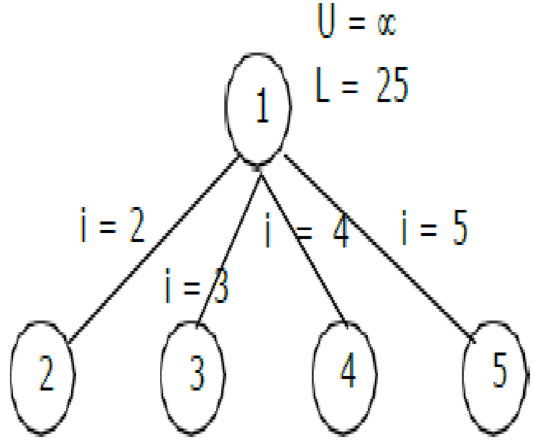
Its time to construct state space tree as follows



Cumulative reduced sum i.e 25 is the cost of a root i.e., node 1, because this is the initially reduced cost matrix

Starting from node 1, we can next visit 2, 3, 4 and 5 vertices. So, consider to explore the paths (1, 2), (1, 3), (1, 4) and (1,5).

The tree organization up to this point is as follows:



Variable 'i' indicates the next node to visit.

REDUCED MATRIX:

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

Step 2:

Consider the path (1, 2):

Change all entries of row 1 and column 2 of A to ∝ and also set A(2, 1) to ∝.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows and columns are not completely ∝.

Then the resultant matrix is
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

Row reduction sum = 0 + 0 + 0 + 0 = 0
Column reduction sum = 0 + 0 + 0 + 0 = 0
Cumulative reduction (r) = 0 + 0 = 0

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(1, 2) + r$

$\hat{c}(S)$ = 25 + 10 + 0 = 35

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

Consider the path (1, 3):

Change all entries of row 1 and column 3 of A to $\propto$ and also set A(3, 1) to $\propto$.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows and columns are not completely $\propto$.

Then the resultant matrix is $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$

Row reduction sum = 0
Column reduction sum = 11
Cumulative reduction (r) = 0 + 11 = 11

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(1, 3) + r$

$\hat{c}(S)$ = 25 + 17 + 11 = 53

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

Consider the path (1, 4):

Change all entries of row 1 and column 4 of A to $\propto$ and also set A(4, 1) to $\propto$.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows and columns are not completely $\propto$.

Then the resultant matrix is $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$

Row reduction sum = 0
Column reduction sum = 0
Cumulative reduction (r) = 0 + 0 = 0

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(1, 4) + r$

$\hat{c}(S)$ = 25 + 0 + 0 = 25

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

nsider the path (1, 5):

Change all entries of row 1 and column 5 of A to $\propto$ and also set A(5, 1)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows columns are not completely $\propto$.

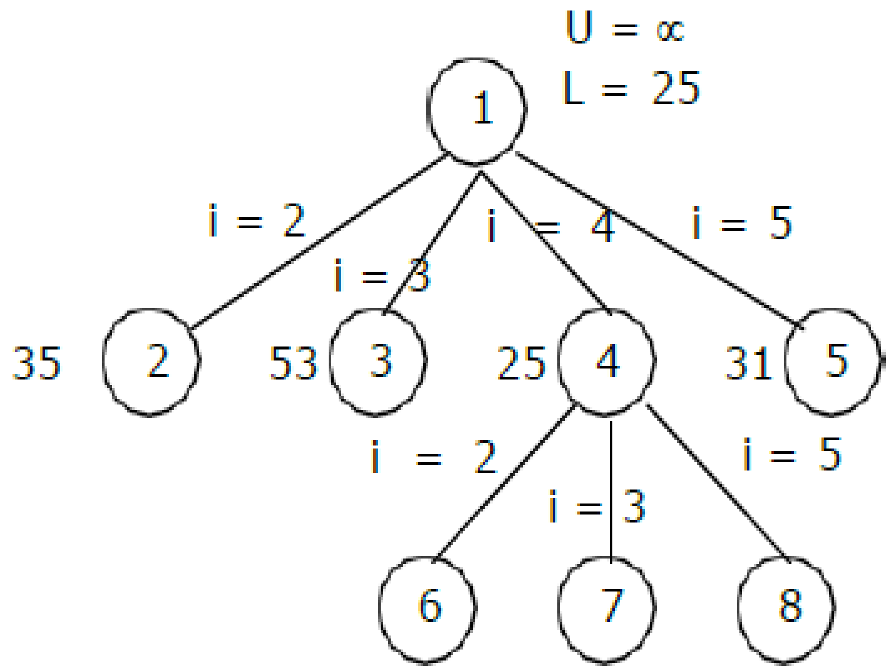
Then the resultant matrix is
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

Row reduction sum = 5
Column reduction sum = 0
Cumulative reduction (r) = 5 + 0 = 0

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(1, 5) + r$

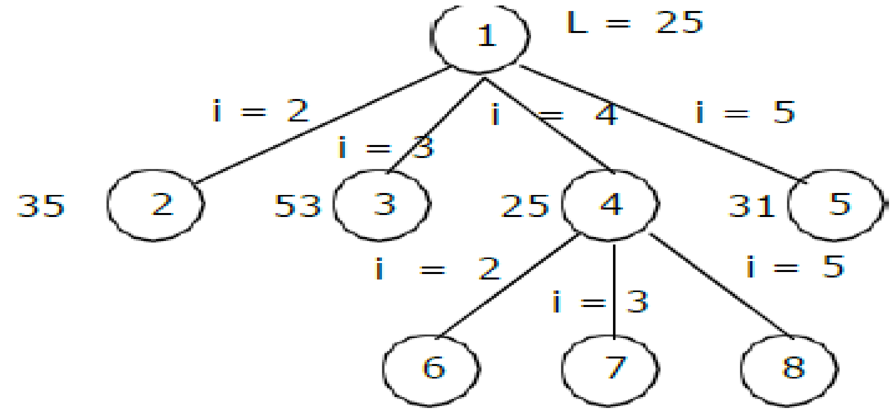
$\hat{c}(S)$ = 25 + 1 + 5 = 31

The tree organization up to this point is as follows:



The cost of the paths between (1, 2) = 35, (1, 3) = 53, (1, 4) = 25 and (1, 5) = 31. The cost of the path between (1, 4) is minimum. Hence the matrix obtained for path (1, 4) is considered as reduced cost matrix.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$



The new possible paths are (4, 2), (4, 3) and

Consider the path (4, 2):

Change all entries of row 4 and column 2 of A to ∝ and also set A(2, 1) to ∝.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows and columns are not completely ∝.

Then the resultant matrix is $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$

Row reduction sum = 0
Column reduction sum = 0
Cumulative reduction (r) = 0 + 0 = 0

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(4, 2) + r$

$\hat{c}(S) = 25 + 3 + 0 = 28$

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Consider the path (4, 3):

Change all entries of row 4 and column 3 of A to ∞ and also set A(3, 1) to ∞.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows a
columns are not completely ∞.

Then the resultant matrix is
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

Row reduction sum = 2
Column reduction sum = 11
Cumulative reduction (r) = 2 + 11 = 13

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(4, 3) + r$

$\hat{c}(S) = 25 + 12 + 13 = 50$

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Consider the path (4, 5):

Change all entries of row 4 and column 5 of A to ∝ and also set A(5, 1) to ∝.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows and columns are not completely ∝.

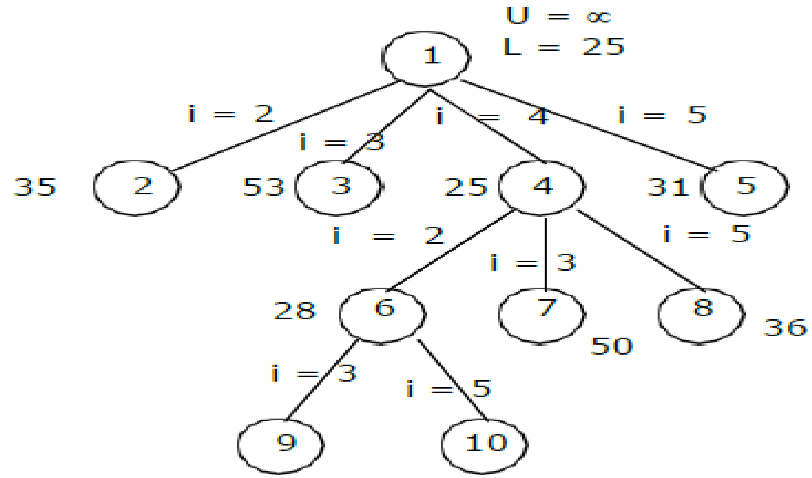
Then the resultant matrix is
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

Row reduction sum = 11
Column reduction sum = 0
Cumulative reduction (r) = 11+0 = 11

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(4, 5) + r$

$\hat{c}(S)$ = 25 + 0 + 11 = 36

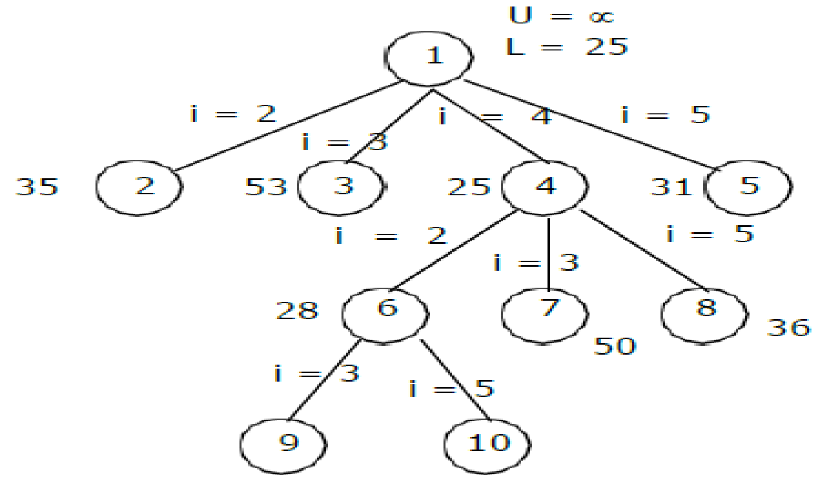
The tree organization up to this point is as follows:



The cost of the paths between (4, 2) = 28, (4, 3) = 50 and (4, 5) = 36. The cost of the path between (4, 2) is minimum. Hence the matrix obtained for path (4, 2) is considered as reduced cost matrix.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

The new possible paths are (2, 3) and (2, 5).



Consider the path (2, 3):

Change all entries of row 2 and column 3 of A to ∝ and also set A(3, 1) to ∝.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix}$$

Apply row and column reduction for the rows and columns whose rows and columns are not completely ∝.

Then the resultant matrix is $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$

Row reduction sum = 2
Column reduction sum = 11
Cumulative reduction (r) = 2 + 11 = 13

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(2, 3) + r$

$\hat{c}(S)$ = 28 + 11 + 13 = 52

Consider the path (2, 5):

Change all entries of row 2 and column 5 of A to ∝ and also set A(5, 1) to ∝.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

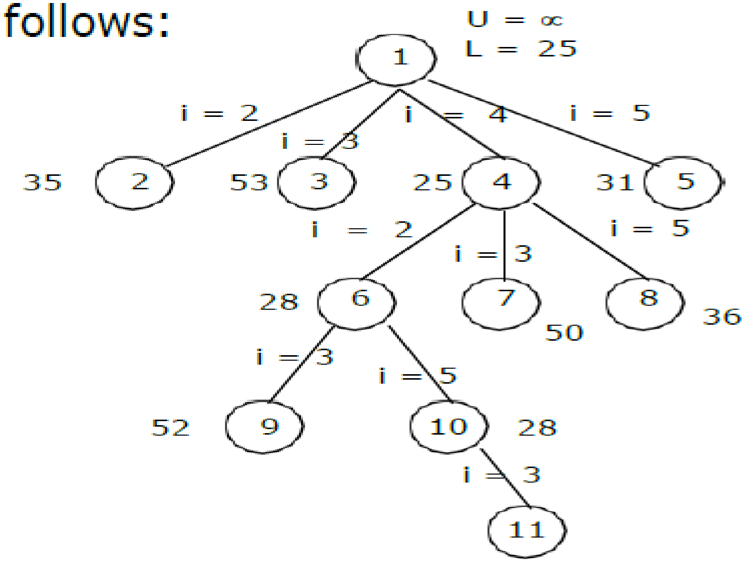
Apply row and column reduction for the rows and columns whose rows and columns are not completely ∝.

Then the resultant matrix is $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$

Row reduction sum = 0
Column reduction sum = 0
Cumulative reduction (r) = 0 + 0 = 0

Therefore, as $\hat{c}(S) = \hat{c}(R) + A\,(2, 5) + r$

The tree organization up to this point is as follows:



The cost of the paths between (2, 3) = 52 and (2, 5) = 28. The cost of the path between (2, 5) is minimum. Hence the matrix obtained for path (2, 5) is considered as reduced cost matrix.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

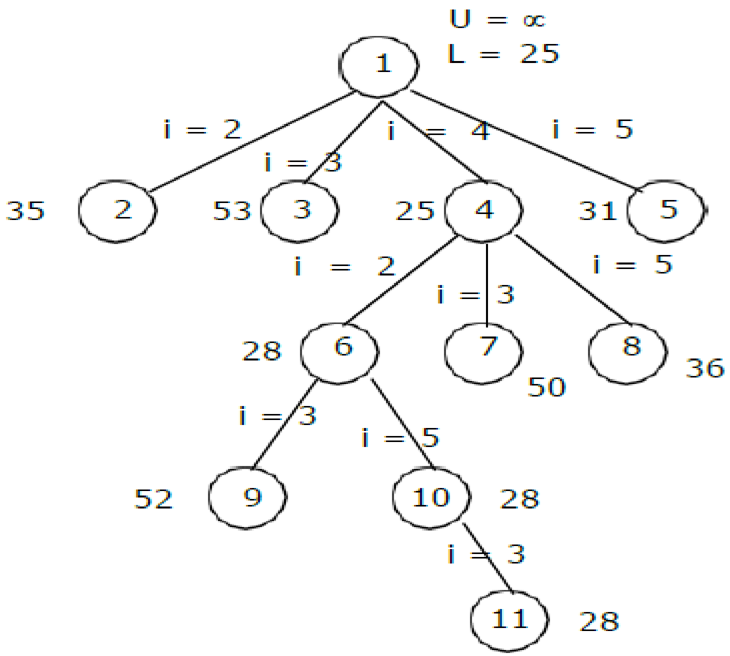
Consider the path (5, 3):

Change all entries of row 5 and column 3 of A to $\propto$ and also set A(3, 1) to $\propto$. Apply row and column reduction for the rows and columns whose rows and columns are not completely $\propto$.

Then the resultant matrix is $\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$

Row reduction sum = 0
Column reduction sum = 0
Cumulative reduction (r) = 0 + 0 = 0

Therefore, as $\hat{c}(S) = \hat{c}(R) + A(5, 3) + r$
$\hat{c}(S)$ = 28 + 0 + 0 = 28



The path of traveling sale person problem is:

1 ⟶ 4 ⟶ 2 ⟶ 5 ⟶ 3 ⟶ 1

The minimum cost of the path is: 10 + 6 +2+ 7 + 3 = 28.

VIVA

1. State Travelling Salesman Problem using branch and bound
2. Define E-node, Live node and Dead node
3. Distinguish LIFO and FIFO branch and bound
4. Which data structure is useful for FIFO branch and bound strategy
5. Mention the time complexity of TSP using branch and bound strategy