Reliability design problem

In **reliability design**, the problem is to design a system that is composed of several devices connected in series.

If we imagine that r1 is the reliability of the device.

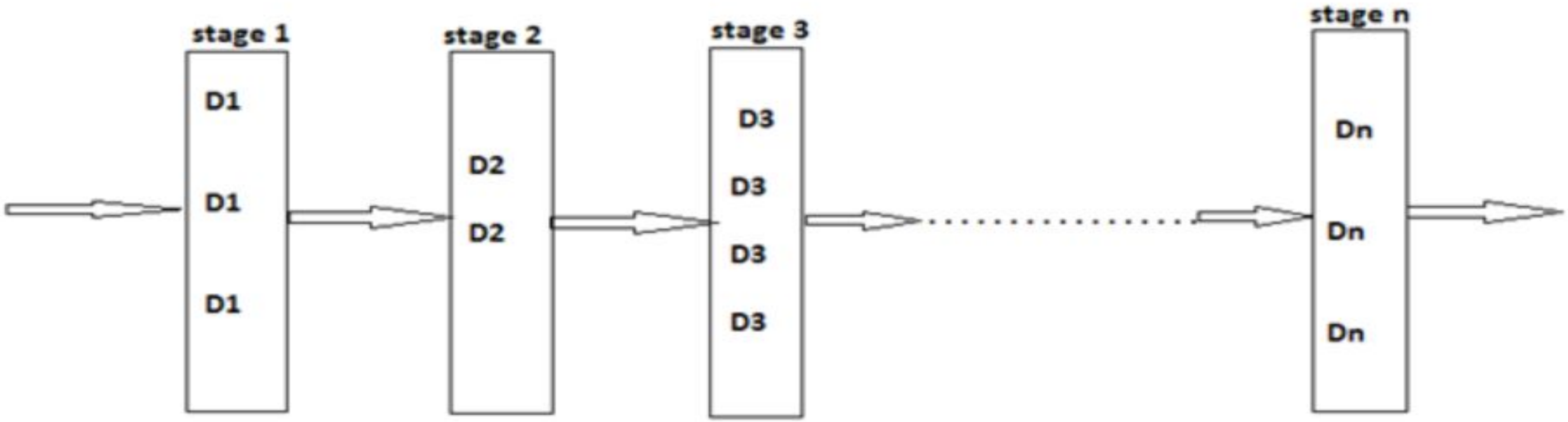
Then the reliability of the function can be given by **πr1**.

If **r1 = 0.99** and **n = 10** that **n** devices are set in a series, **1 <= i <= 10**, then reliability of the whole system **πri** can be given as: **Πri = 0.904**

So, if we duplicate the devices at each stage then the reliability of the system can be increased.

It can be said that multiple copies of the same device type are connected in parallel through the use of switching circuits. Here, switching circuit determines which devices in any given group are functioning properly. Then they make use of such devices at each stage, that result is increase in reliability at each stage. If at each stage, there are **mi** similar types of devices **Di**, then the probability that all **mi** have a malfunction is **(1 - ri)^mi**, which is very less.

And the reliability of the stage **I** becomes **(1 – (1 - ri) ^mi)**. Thus, if **ri = 0.99** and **mi = 2**, then the stage reliability becomes **0.9999** which is almost equal to **1**. Which is much better than that of the previous case or we can say the reliability is little less than **1 - (1 - ri) ^mi** because of less reliability of switching circuits.



Multiple Devices Connected in Parallel in Each Stage

In reliability design, we try to use device duplication to maximize reliability. But this maximization should be considered along with the cost.

Let **c** is the maximum allowable cost and **ci** be the cost of each unit of device **i**. Then the maximization problem can be given as follows:

Maximize π Øi (mi) for 1 <= I <= n

Subject to:

$$\sum_{i=1}^{n} ci\,mi <= \mathrm{c}$$

mi>= 1 and integer 1 <= i <= n

Here, **Øi (mi)** denotes the reliability of the stage **i**.

The reliability of the system can be given as follows:

Π Øi (mi) for 1 <= i <= n

If we increase the number of devices at any stage beyond the certain limit, then also only the cost will increase but the reliability could not increase.

Design a 3-stage system with devices (D1,D2,D3) with cost values (C1,C2,C3)=(30,15,20) and reliablities (r1,r2,r3)=(0.9,0.8,0.5). The cost of the total system shoud not exceed 105.

- First calculate Upper bound which tells us how many maximum devises of same type can be considered

Upper bound of each divice

$$u_i = (c + c_i - \sum c_j)/c_i$$

D_i	c_i	r_i	u_i
D_1	30	0.9	2
D_2	15	0.8	3
D_3	20	0.5	3

Max $(0.648, 100)$
$D_3 = 2$
$D_2 = 2$
$D_1 = 1$

without considering any device: $S^0 = \{(1,0)\}$

consider D_1
$S_1^1 = \{(0.9, 30)\}$
$S_2^1 = \{(0.99, 60)\}$
$\Rightarrow S^1 = \{(0.9, 30), (0.99, 60)\}$

consider D_2
$S_1^2 = \{(0.72, 45), (0.792, 75)\}$
$S_2^2 = \{(0.864, 60), (0.9504, \cancel{90})\}$
(cost is exceeding 105) so reject
$S_3^2 = \{(0.8928, 75)\}$
remaining reject
$S^2 = \{(0.72, 45), (\underline{0.792}, \cancel{75}), (\underline{0.864}, 60), (0.8928, 75)\}$ Dominance rule

if 2 Device 2 copies then reliability is
$1-(1-r_2)^2$
$1-(1-0.8)^2$
$= \underline{0.96}$,
cost $= 15+15 = \underline{30}$

$r = (1-(1-r_2)^3)$
$= (1-(1-0.8)^3)$
$= 0.992$
$c = 45$

consider D_3
$S_1^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$
$S_2^3 = \{(0.54, 85), (\underline{0.648}, 100), \times\}$ $r = 0.75$, $c = 40$
$S_3^3 = \{(0.63, 105), — —\}$ $r = 0.875$, $c = 60$
$S^3 = \{(0.36, 65), (0.432, 80), (0.54, 85), (0.648, 100)\}$