Vlad Bogdan-Tudor,
917
VBT

- $r \to p \vee q,\ p \to (r \to q),\ q \to p \vdash r \to p \wedge q \wedge r$

We will use resolution.

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Refutation is a refutation proof method, since we are working with the negation of the conclusion (or of a formula). We can check whether or not the conclusion holds according to the following theorem:

T1: Soundness and completness of resolution (propos. logic)
A set S of propositional clauses is inconsistent iff $S \vdash_{Res} \square$.

We also use the following:

T2: If $U_1, U_2, U_3, \dots, U_n, V$ are propositional formulas,

$U_1, U_2, \dots, U_n \vdash V$ iff

$U_1, U_2, \dots, U_n \vDash V$ iff

$CNF(U_1 \wedge U_2 \wedge \dots U_n \wedge \neg V) \vdash_{Res} \square$

We find the propositional clauses, replace the '$\to$' connectives, and find the CNF.

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$H_1 = r \to p \vee q \equiv \neg r \vee p \vee q : C_1$

$H_2 = p \to (r \to q) \equiv p \to (\neg r \vee q) \equiv \neg p \vee \neg r \vee q : C_2$

$H_3 = q \to p \equiv \neg q \vee p : C_3$

$\neg C = \neg (r \to p \wedge q \wedge r) \equiv \neg(\neg r \vee (p \wedge q \wedge r))$

$\equiv r \wedge (\neg p \vee \neg q \vee \neg r) : C_4 \wedge C_5$

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$$\begin{cases} C_1 : \neg r \vee p \vee q \\ C_2 : \neg p \vee \neg r \vee q \\ C_3 : \neg q \vee p \\ C_4 : r \\ C_5 : \neg p \vee \neg q \vee \neg r \end{cases}$$

$S = \{ C_1, C_2, C_3, C_4, C_5 \}$

We now check if the set S is inconsistent or not.

$C_6 = Res_r(C_1, C_4) = p \vee q$

$C_7 = Res_q(C_6; C_3) = p$

$C_8 = Res_p(C_7, C_2) = \neg r \vee q$

$C_9 = Res_r(C_8, C_4) = q$

$C_{10} = Res_q(C_9, C_5) = \neg p \vee \neg r$

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$C_{11} = Res_p(C_7, C_{10}) = \neg r$

$C_{12} = Res_r(C_{11}, C_4) = \square$

$S \vdash_{Res} \square \Rightarrow S$ is inconsistent $\Rightarrow$ the deduction holds.