Vlad Bogdan-Tudor, 914
VlaB

10) $$T(n) = \begin{cases} 1, \text{ if } n \le 1 \\ 1 + 3T(n/3), \text{ otherwise} \end{cases}$$

Assume $n \overset{\text{much bigger}}{\gg} 3$ and that $n = 3^k$, $k \in \mathbb{N}$
$$\Downarrow$$
$$k = \log_3 n$$

$$\begin{cases} T(n) = 1 + 3T\left(\frac{n}{3}\right) \\ T\left(\frac{n}{3}\right) = 1 + 3T\left(\frac{n}{9}\right) \\ T\left(\frac{n}{9}\right) = 1 + 3T\left(\frac{n}{27}\right) \\ \dots \\ T(1) = 1 \end{cases}$$

$$\Rightarrow T(n) = 1 + 3T\left(\frac{n}{3}\right)$$
$$= 1 + 3\left(1 + 3T\left(\frac{n}{9}\right)\right) = 1 + 3 + 9T\left(\frac{n}{9}\right)$$
$$= 1 + 3 + 9\left(1 + 3T\left(\frac{n}{27}\right)\right) = 1 + 3 + 9 + 27T\left(\frac{n}{27}\right)$$

By induction $\Rightarrow T(n) = 1 + 3^1 + 3^2 + \dots + 3^{k-1} + 3^k \underbrace{T\left(\frac{n}{3^k}\right)}_{= 1, \text{ since } n = 3^k}$

$$\Rightarrow T(n) = 1 + 3^1 + 3^2 + \dots + 3^k \Rightarrow \boxed{T(n) \in \Theta(\log_3 n)}$$