Dynamical Systems 2020/21
Lab Test

1. Find a periodic solution of the differential equation

$$u'' + 5u' - 7u = 5\cos x - 7\sin x,$$

   then plot its graph on the interval $[-11, 5]$, and finally compute, for it and for its first order derivative, approximate values in $\pi/2$. Note that the unknown is the function denoted by $u(x)$.

2. a) Plot the planar curve of parametric equations $x = \cos(2t) + 3\sin(2t)$, $y = \sin(2t)$ for $t \in [0, 4]$.

   b)* Can $\varphi(t) = (\cos(2t) + 3\sin(2t), \sin(2t))$, $t \in \mathbb{R}$, be a solution of a linear planar system $\dot{X} = AX$?

3. Introduce the matrix A corresponding to the linear system $x' = -y$, $y' = 5x$. Compute its determinant and eigenvalues. Compute e^{tA}. Specify the type and stability of the linear system.

4. We consider the nonlinear system $x' = 2x + 3y - 2xy$, $y' = 4x + 6y + xy^2$. Is $(0, 0)$ the unique equilibrium point? Is $(0, 0)$ a hyperbolic equilibrium point?

5. We consider the map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 - 0.5$. Find its fixed points, preferably as decimals (*hint*: all of them are in the interval $(-2, 2)$). Describe your opinion on the behavior of the sequences of iterations starting with 0, 1.2 and, respectively, -1.1.