Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Retake Exam on Dynamical Systems

1. (1.25p) Find a LHDE with constant real coefficients that has as solution the function
$$e^{-3t} + \sin(3t).$$

2. (1p) Find the solution of the IVP
$$x_{k+2} - 13x_{k+1} + 30x_k = 0, \quad x_0 = 1, \quad x_1 = 0.$$

3. Let $g : \mathbb{R}^2 \to \mathbb{R}$ be a C^1 function. Consider the planar differential system
$$\dot{x} = -y + xg(x, y), \quad \dot{y} = x + yg(x, y).$$

(a) (0.25p) Check that $(0, 0)$ is an equilibrium point. There are other equilibrium points?

(b) (1.5p) Using the linearization method, study the stability of the equilibrium point $(0, 0)$. Discuss with respect to the values of $g(0, 0)$.

(c) (0.75p) Prove that any orbit (that does not correspond to an equilibrium point) rotates around $(0, 0)$.

(d) (1p) In the case that g takes the value 0 in any point of the unit circle, check that
$$\varphi(t, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = \left(\cos(t + \frac{\pi}{4}), \sin(t + \frac{\pi}{4})\right), \quad \text{for any} \quad t \in \mathbb{R}.$$

4. (1.25p) Let $a, b \in \mathbb{R}$ and $f(x) = ax^2 + bx + 1$ be such that $f(1) = 2$ and $f(2) = 1$. Study whether the discrete scalar dynamical system $x_{k+1} = f(x_k)$, $k \in \mathbb{N}$ has an attracting 2-periodic orbit.

Test 2. (1p) Describe the properties of the solutions of the scalar dynamical system $\dot{x} = -x(x^2 - \pi^2)$.