Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems

Seminar Test

1. We consider the nonlinear planar system

$$\dot{x} = ax - 2y - 2y(x^2 + y^2), \quad \dot{y} = 2x + ay,$$

depending on the parameter $a \in \mathbb{R}$.

(i) (2.5p) Study the stability of the equilibrium point $(0,0)$, discussing with respect to the parameter a.

(ii) (0.5p) How many equilibrium points has the system?

(iii) (2.5p) Take $a = 0$. Check that $H : \mathbb{R}^2 \to \mathbb{R}$, $H(x,y) = (x^2 + y^2)e^{y^2}$ is a first integral of the system. Show that the level curves of H are bounded.

(iv) (2p) Take $a = 0$. Pass to polar coordinates.

(v) (0.5p) What can we deduce from (iii) and (iv) about the shape of the orbits of the system when $a = 0$?

2. (2p) Represent the phase portrait of $\dot{x} = 1 - 2x^2$. Specify the properties of $\varphi(t, \eta)$ for $\eta \in \{-100,\ 0,\ 100\}$.