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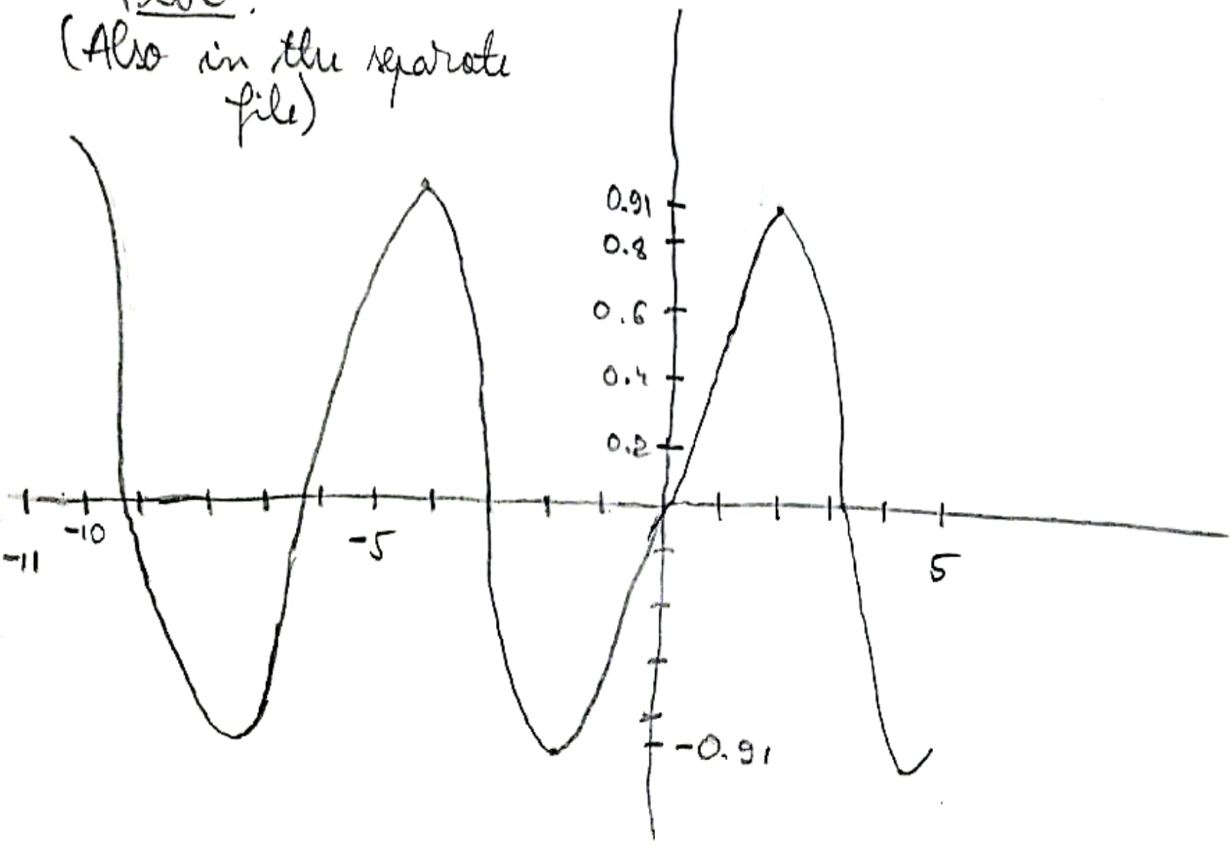
= LAB TEST =

1) $u'' + 5u' - 7u = 5\cos x - 7\sin x$

A periodic solution: $\boxed{\varphi(x) = \frac{81}{89}\sin(x) - \frac{5}{89}\cos(x)}$

Plot:
(Also in the separate file)



$\varphi\left(\frac{\pi}{2}\right) = \frac{81}{89}$ OR using an approximation ("evalf") $\varphi\left(\frac{\pi}{2}\right) = 0.9101123596$

$\varphi'(x) = \frac{5}{89}\sin(x) + \frac{81}{89}\cos(x)$

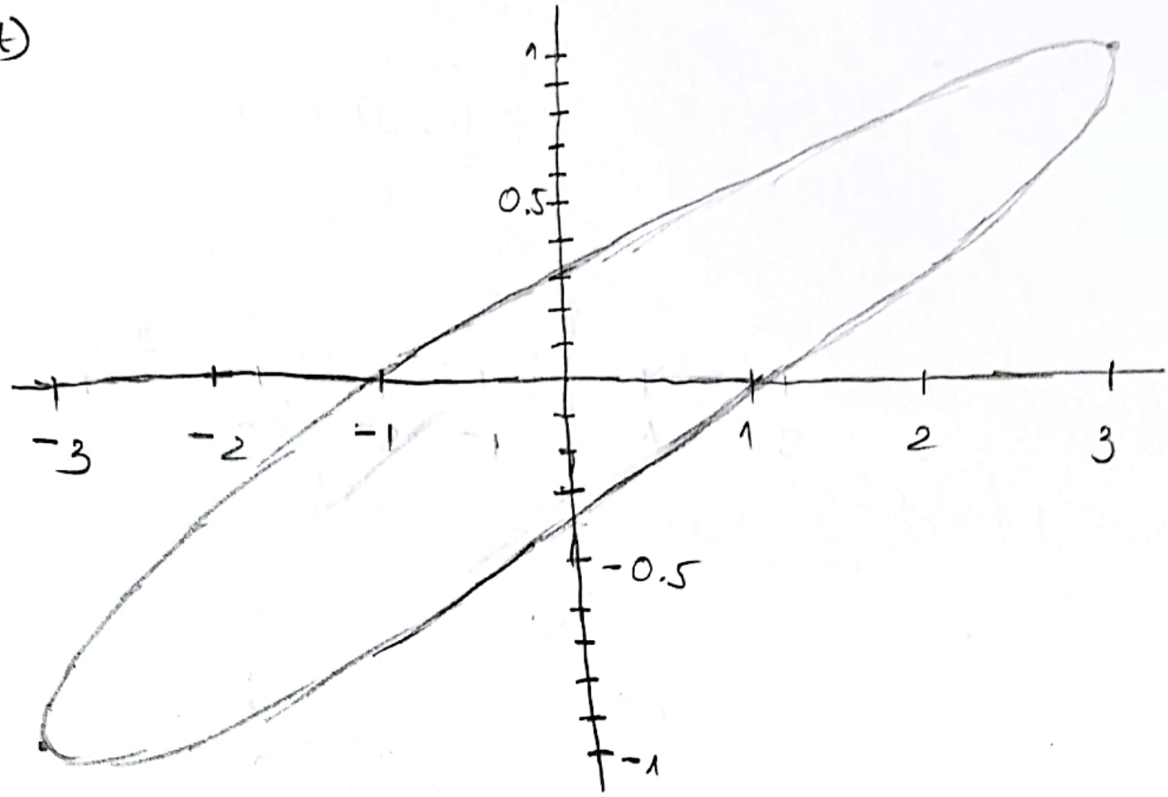
$\varphi'\left(\frac{\pi}{2}\right) = \frac{5}{89}$ OR using an approximation $\varphi'\left(\frac{\pi}{2}\right) = 0.0561797752$

2) $\begin{cases} x = \cos(2t) + 3\sin(2t) \\ y = \sin(2t) \end{cases}$

$t \in [0, 4]$

a) <u>Plot</u>:

(Also in the separate file)



b)* We can see that the solution is a <u>slanted ellipse</u>.

$\Rightarrow$ it <u>cannot</u> be the solution of a linear system $\dot{X} = AX$. In order for it to be one, we would expectect a circle or an ellipse that is not slanted.

3) $\begin{cases} x' = -y \\ y' = 5x \end{cases}$

$$A = \begin{pmatrix} 0 & -1 \\ 5 & 0 \end{pmatrix}$$

$\det(A) = 5$

eigenvalues of A: $\lambda_{1,2} = \pm i\sqrt{5} \Rightarrow$ CENTER, stable

The linear system is a CENTER and is stable.

$$e^{tA} = \begin{pmatrix} \cos(t\sqrt{5}) & -\frac{1}{5}\sin(t\sqrt{5})\sqrt{5} \\ \sin(t\sqrt{5})\sqrt{5} & \cos(t\sqrt{5}) \end{pmatrix}$$

4) $\begin{cases} x' = 2x + 3y - 2xy \\ y' = 4x + 6y + xy^2 \end{cases}$

Eq. points : $\{(0,0), (\frac{6}{5}, -4)\}$

$\Rightarrow$ <u>NO</u>, $(0,0)$ is <u>not</u> the unique equilibrium point

$$Jf(x,y) = \begin{pmatrix} 2-2y & 3-2x \\ 4+y^2 & 6+2xy \end{pmatrix}$$

For $x^* = (0,0)$, $A = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

eigenvalues of A : $\begin{cases} \lambda_1 = 0 \\ \lambda_2 = 8 \end{cases}$

$\lambda_1 = 0 \Rightarrow (0,0)$ is <u>not</u> hyperbolic

5) $f:\mathbb{R}\to\mathbb{R}$ $f(x)=x^2-0.5$

Fixed points: $\begin{cases} x_1 = -0.3660254038 \\ x_2 = 1.366025404 \end{cases}$

We will use 30 iterations (seems to be enough)
The graphs of the iterations are attached as "plot_exercises_" + "first x", "second x", "third x" for the values 0, 1.2, and -1.1 of x, respectively.

For $x_0=0$:

We can see that it converges to a fixed point. By checking with Maple, it looks to be converging towards the fixed point $x_1=-0.3660254038$
Also, the iterations seem to have oscillatory behavior.

For $x_0=1.2$:

The first 2 iterations have high values, far away from the rest of the iterations. But still, the iterations look to be converging towards the same fixed point, $x_1=-0,3660254038$

For $x_0=-1.1$:

Again the first 2 iterations seem to be away from the rest of the iterations but again, the iterations quickly converge towards the fixed point $x_1=-0.3660254038$.
Like for the case of $x_0=0$, the iterations seem to have oscillatory behavior.