① conical surface, $V(1,0,3)$, director curve:

$$\mathcal{C}: \begin{cases} \frac{x^2}{4} + y^2 + z^2 = 1 \\ 2x + 3z = 0 \end{cases}$$

$$d_{\lambda,\mu} \in V \Rightarrow d_{\lambda,\mu}: \frac{x-1}{a} = \frac{y-0}{b} = \frac{z-3}{c}$$

$$\Leftrightarrow \begin{cases} \frac{x-1}{a} = \frac{y}{b} \\ \frac{x-1}{a} = \frac{z-3}{c} \end{cases} \Leftrightarrow \begin{cases} b(x-1) = a \cdot y \\ c(x-1) = a(z-3) \end{cases}$$

$$\Leftrightarrow \begin{cases} x-1 = \overset{\lambda}{\frac{a}{b}} \cdot y \\ x-1 = \underset{\mu}{\frac{a}{c}}(z-3) \end{cases} \qquad \left(\Rightarrow \begin{cases} \lambda = \frac{x-1}{y} \\ \mu = \frac{x-1}{z-3} \end{cases} \right)$$

$$\Rightarrow d_{\lambda,\mu}: \begin{cases} x-1 = \lambda \cdot y \\ x-1 = \mu \cdot (z-3) \end{cases}$$

$$\begin{cases} 2x + 3z = 0 \\ x - 1 = \lambda \cdot y \\ x - 1 = \mu(z-3) \\ \frac{x^2}{4} + y^2 + z^2 = 1 \end{cases}$$

$$\begin{cases} x = \lambda y + 1 \\ x = \mu z - 3\mu + 1 \Rightarrow \mu z = x + 3\mu - 1 \Rightarrow z = \frac{1}{\mu}x + 3 - \frac{1}{\mu} \\ 2x + 3z = 0 \\ \frac{x^2}{4} + y^2 + z^2 = 1 \end{cases}$$

$$2x + 3z = 0$$

$$2(\lambda y + 1) + 3\left(\frac{1}{\mu}x + 3 - \frac{1}{\mu}\right) = 0$$

$$2\lambda y + 2 + 3\left(\frac{1}{\mu}(\lambda y + 1) + 3 - \frac{1}{\mu}\right) = 0$$

$$2\lambda y + 2 + \frac{3\lambda}{\mu}y + \cancel{\frac{3}{\mu}} + 9 - \cancel{\frac{3}{\mu}} = 0$$

$$\overset{\mu}{\overline{2\lambda y}} + \frac{3\lambda y}{\mu} + 11 = 0$$

$$\frac{2\lambda\mu y + 3\lambda y}{\mu} = -11$$

$$y(2\lambda\mu + 3\lambda) = -11\mu$$

$$y = -\frac{11\mu}{2\lambda\mu + 3\lambda} \Rightarrow \boxed{y = -\frac{11\mu}{\lambda(2\mu + 3)}}$$

$$\Rightarrow x = \lambda \cdot \left(-\frac{11\mu}{\lambda(2\mu+3)}\right) + 1$$

$$\boxed{x = 1 - \frac{11\mu}{2\mu+3}} = \frac{-9\mu+3}{2\mu+3}$$

$$z = -\frac{2x}{3} = -\frac{2}{3} \cdot \left(1 - \frac{11\mu}{2\mu+3}\right)$$

$$\boxed{z = \frac{22\mu}{6\mu+9} - \frac{2}{3}} = \frac{18\mu-6}{6\mu+9} = \frac{6\mu-2}{2\mu+3}$$

$\Rightarrow$ Compatibility condition:

$$\left(\frac{\overset{3-9\mu}{\cancel{11\mu}}}{2\mu+3}\right)^2 \cdot \frac{1}{4} + \left(\frac{11\mu}{\lambda(2\mu+3)}\right)^2 + \left(\frac{\overset{18\mu-6}{\cancel{22\mu}}}{6\mu+9}\right)^2 = 1$$

Replace λ, μ:

Very good!

$$\left(\frac{11(x-1)}{z-3} \cdot \frac{1}{2\left(\frac{x-1}{z-3}\right) - \cancel{3}}\right)^2 \cdot \frac{1}{4} + \left(\frac{11(x-1)}{z-3} \cdot \frac{y}{x-1} \cdot \frac{1}{2\left(\frac{x-1}{z-3}\right)+3}\right)^2$$

$$+ \left(\frac{22(x-1)}{z-3} \cdot \frac{1}{6\left(\frac{x-1}{z-3}\right)+9}\right)^2 = 0$$

$$\left(\frac{11(x-1)}{\cancel{z-3}}\cdot\frac{\cancel{z-3}}{2(x-1)-3(z-3)}\right)^2\cdot\frac{1}{4}+\left(\frac{11\cancel{(x-1)}}{\cancel{z-3}}\cdot\frac{y}{\cancel{x-1}}\cdot\frac{\cancel{z-3}}{2(x-1)-3(z-3)}\right)^2$$

$$+\left(\frac{22(x-1)}{\cancel{z-3}}\cdot\frac{\cancel{z-3}}{6(x-1)+9(z-3)}\right)^2=0$$

$$\left(\frac{11(x-1)}{2(x-1)-3(z-3)}\right)^2\cdot\frac{1}{4}+\left(\frac{11y}{2(x-1)-3(z-3)}\right)^2+\left(\frac{22(x-1)}{6(x-1)+9(z-3)}\right)^2=0$$

Final equation

Close enough. In any case, you took care not to make the typical mistake.

2) cylindrical surface, parallel to d:

$$d: \frac{x-1}{2} = \frac{y}{3} = z \qquad \vec{\ell}(2,3,1)$$

and ~~director~~ curve:

$$\mathcal{C}: \begin{cases} -x^2 - \frac{y^2}{4} + z^2 = 1 \\ 3x + y - z = 0 \end{cases}$$

$$d_{\lambda,\mu}: \frac{x-x_0}{2} = \frac{y-y_0}{3} = \frac{z-z_0}{1}$$

$$d_{\lambda,\mu}: \begin{cases} x - x_0 = 2(z - z_0) \\ 3(x-x_0) = 2(y-y_0) \end{cases}$$

✓

$$d_{\lambda,\mu}: \begin{cases} x - 2z = x_0 - 2z_0 \quad \lambda \\ 3x - 2y = 3x_0 - 2y_0 \quad \mu \end{cases}$$

✓

$$d_{\lambda,\mu}: \begin{cases} x - \cancel{3}^{2} z = \lambda \\ 3x - 2y = \mu \end{cases}$$

Ouch, be careful at the exam!

$$\begin{cases} x - 3z = \lambda \\ 3x - 2y = \mu \\ 3x + y - z = 0 \Rightarrow y = z - 3x \\ -x^2 - \frac{y^2}{4} + z^2 = 1 \end{cases}$$

$$\begin{cases} x - 3z = \lambda \\ 3x - 2(z - 3x) = \mu \end{cases}$$

$$\begin{cases} x - 3z = \lambda & \cdot \mid -2 \\ 9x - 2z = \mu & \cdot \mid 3 \end{cases}$$

$$\begin{cases} -2x + 6z = -2\lambda \\ 27x - 6z = 3\mu \end{cases} \quad \oplus$$

$$25x = 3\mu - 2\lambda$$

$$\boxed{x = \frac{3\mu - 2\lambda}{25}}$$

✓

$$2y = 3x - \mu$$

$$y = \frac{3x - \mu}{2} = \frac{1}{2} \cdot \left(\frac{3(3\mu - 2\lambda)}{25} - \frac{\mu}{\cancel{2}} \right) =$$

$$= \frac{1}{2} \cdot \left(\frac{9\mu - 6\lambda}{25} - \frac{\mu}{2} \right) = \frac{1}{2} \left(\frac{18\mu - 12\lambda - 25\mu}{50} \right)$$

$$= \frac{-12\lambda - 7\mu}{100} \Rightarrow \boxed{y = -\frac{12\lambda + 7\mu}{100}}$$

✓ (ignoring the 2)

$$3z = x - \lambda$$

$$z = \frac{1}{3}(x - \lambda)$$

$$z = \frac{1}{3}\left(\frac{3\mu - 2\lambda}{25} - \frac{25}{\lambda}\right)$$

$$z = \frac{1}{3}\left(\frac{3\mu - 2\lambda - 25\lambda}{25}\right)$$ ✓

$$z = \frac{1}{3}\left(\frac{3\mu - 27\lambda}{25}\right)$$

$$\boxed{z = \frac{\mu - 9\lambda}{25}}$$ ✓

Compatibility condition:

$$-\left(\frac{3\mu - 2\lambda}{25}\right)^2 - \frac{1}{4}\left(\frac{12\lambda + 7\mu}{100}\right)^2 + \left(\frac{\mu - 9\lambda}{25}\right)^2 = 1$$

Replace λ, μ:

$$-\left(\frac{3(3x-2y) - 2(x-3z)}{25}\right)^2 - \frac{1}{4}\left(\frac{12(x-3z) + 7(3x-2y)}{100}\right)^2$$

$$+\left(\frac{3x - 2y - 9(x-3z)}{25}\right)^2 = 1$$ ✓

Okay! But you had 2 mistakes that most likely led you to a wrong answer

③ $\ell: \frac{x+1}{2} = \frac{y-1}{3} = \frac{z}{4}$ $\vec{\ell}(2,3,4)$

conoidal surface, generatrices intersect and are $\perp$ to ℓ, director curve:

$$\mathcal{C}: \begin{cases} x+1=0 \\ z=(y+1)^2+x^2 \end{cases}$$

HINT PLEASE ?

For a conoidal surface you expect to see:

- the line that the generatrices intersect
- the plane that the generatrices are parallel to
- the director curve

In this case what you received is that:

- $d_{\lambda,\mu}$ intersect ℓ
- $d_{\lambda,\mu}$ are perpendicular to ℓ
- the director curve.

Clearly the two orange conditions should correspond. Indeed they do: $\boxed{d_{\lambda,\mu} \perp \ell \Leftrightarrow d_{\lambda,\mu} \parallel \Pi}$, where Π is a plane that is perpendicular to ℓ.

Since $\vec{\ell}(2,3,4)$ it suffices to choose $\Pi: 2x+3y+4z=0$.

From now on, the exercise can be solved like the similar one from the lecture.

④ revolution surface,

$$\mathcal{C}: \begin{cases} y = \frac{1}{x} \\ z = 0 \end{cases}$$

around the x-axis.

$$Ox: \begin{cases} y = 0 \\ z = 0 \end{cases} \qquad \vec{O_x}(1,0,0)$$

✓

Take $P(1,0,0) \in Ox$.

Very good! $O(0,0,0)$ would have also been a good choice

$$\begin{cases} (x-1)^2 + y^2 + z^2 = \lambda \\ x = \mu \end{cases}$$

✓

Now use the curve:

$$\begin{cases} x = \mu \\ z = 0 \\ y = \frac{1}{\mu} \\ (x-1)^2 + y^2 + z^2 = \lambda \end{cases}$$

We get the compatibility condition:

$$(\mu-1)^2 + \frac{1}{\mu^2} = \lambda$$

Now replace λ, μ:

$$\cancel{(x-1)^2} + \frac{1}{x^2} = \cancel{(x-1)^2} + y^2 + z^2$$

$$\frac{1}{x^2} - y^2 - z^2 = 0 \qquad \cdot / x^2$$

$$1 - x^2 y^2 - x^2 z^2 = 0$$ Final equation

Yes! Well done!