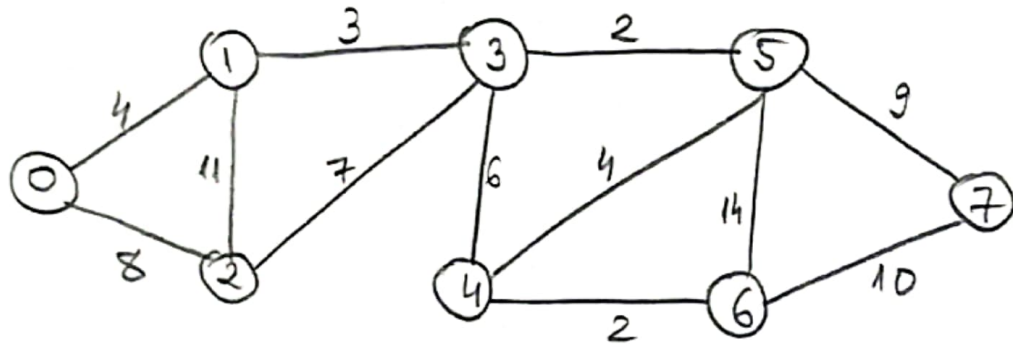


# ① Minimum Spanning Tree (Prim's Algorithm), Manual Execution



INPUT FILE:

8	12		3	5	2
0	1	4	4	5	4
0	2	8	4	6	2
1	2	11	5	6	14
1	3	3	5	7	9
2	3	7	5	7	9
3	4	6	6	7	10

start = 0

	tree-edges	current vertex	neighbour	queue	prev	dist	processed																																																
init	[ ]			←	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td><td>-</td></tr></table>	0	1	2	3	4	5	6	7	-	-	-	-	-	-	-	-	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>0</td><td>∞</td><td>∞</td><td>∞</td><td>∞</td><td>∞</td><td>∞</td><td>∞</td></tr></table>	0	1	2	3	4	5	6	7	0	∞	∞	∞	∞	∞	∞	∞	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td></tr></table>	0	1	2	3	4	5	6	7	T	F	F	F	F	F	F	F
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it2.1	[(0,1)]	1	0				<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td></tr></table>	0	1	2	3	4	5	6	7	T	T	F	F	F	F	F	F																																
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	tree-edges	current vertex	neighbour	queue	prev	dist	processed																													
it3.1	[(0,1), (1,3)]	3	1				<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>T</td><td>T</td><td>F</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td></tr></table>	0	1	2	3	4	5	6	7	T	T	F	T	F	F	F	F													
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it3.4	5	<del>(2,5)</del> (6,4) (7,2) (8,2)	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>-</td><td>0</td><td>3</td><td>1</td><td>3</td><td>3</td><td>-</td><td>-</td></tr></table>	0	1	2	3	4	5	6	7	-	0	3	1	3	3	-	-	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>0</td><td>4</td><td>7</td><td>3</td><td>6</td><td>2</td><td>∞</td><td>∞</td></tr></table>	0	1	2	3	4	5	6	7	0	4	7	3	6	2	∞	∞
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	tree-edges	current vertex	neighbour	queue	prev	dist	processed																																
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it5.1	[(0,1),(1,3), (3,5),(5,4)]	4	3				<table> <tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr><td>T</td><td>T</td><td>F</td><td>T</td><td>T</td><td>T</td><td>F</td><td>F</td></tr> </table>	0	1	2	3	4	5	6	7	T	T	F	T	T	T	F	F																
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	tree-edges	current vertex	neighbour	queue	prev	dist	processed																
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it 8.1	<div> <div>(0,1), (1,3)</div> <div>(3,5), (5,4)</div> <div>(4,6), (3,2)</div> </div>	2	○	<div> <div>← (8,2) (9,7) (14,6) ←</div> </div>			<table> <tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>F</td></tr> </table>	0	1	2	3	4	5	6	7	T	T	T	T	T	T	T	F
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it 10.1	<div> <div>(0,1), (1,3)</div> <div>(3,5), (5,4)</div> <div>(4,6), (3,2)</div> <div>(5,7)</div> </div>	7	5	<div> <div>← (14,6) ←</div> </div>			<table> <tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td></tr> </table>	0	1	2	3	4	5	6	7	T	T	T	T	T	T	T	T
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it 10.2			6																				
it 10.3		6		<div> <div>=====</div> </div>																			
				THE QUEUE IS EMPTY ⇒ FINISH																			

With the edges saved in  $\langle \text{tree-edges} \rangle$  we can reconstruct the Minimum Spanning Tree:

$$\text{tree-edges} = [(0,1), (1,3), (3,5), (5,7), (4,6), (3,2), (5,4)]$$

